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PURDUE UNIVERSITY
SCHOOL OF ELECTRICAL ENGINEERING
ELECTRONIC SYSTEMS RESEARCH LABORATORY

SEMI-ANNUAL REPORT OF RESEARCH
PERFORMED UNDER GRANT N5G-553

January 1, through July 1, 1966

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TABLE OF CONTENTS


| | Page |
|--|------|
| FOREWORD | i |
| SECTION I - Technical Papers and Internal Memoranda. | 1 |
| SECTION II - Abstracts of Interim Reports. | 2 |
| SECTION III - Research Summaries | 4 |
| A. Experimental Results on Communication Systems Subject to Intersymbol Interference | 4 |
| B. Phase Lock Loop Studies | 10 |
| C. Purdue Channel Simulator. | 17 |
| D. 1. On Optimum Binary Signal-Sets for Certain Scatter-Multi- path Channel Models. | 19 |
| 2. Families of Equal-Length Shift Register Sequences Obtainable From A New Class of Cyclic Codes. | 21 |

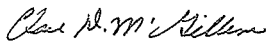
Foreword

This report summarizes work carried out at the Electronic Systems Research Laboratory of Purdue University under NASA Grant NsG-553 during the period January 1, 1966 through June 30, 1966.

In keeping with NASA's policy for administration of research grants the report has been kept as concise as possible, and when appropriate reference has been made to interim reports, internal memoranda, and technical papers resulting from research carried out under this grant.

The format of the report consists of a listing of technical papers and internal memoranda which have been submitted to NASA during the period covered by the report, abstracts of interim reports submitted during the reporting period, and appropriate extracts from the current Purdue University, School of Electrical Engineering, Semi-Annual Research Summary.


John C. Lindenlaub,
Principal Investigator


Clare D. McGillem, Director
Electronic Systems Research Laboratory

I. Technical Papers and Internal Memoranda

Three technical papers were presented at the International Symposium on Information Theory at U.C.L.A., Los Angeles, California, January 31 - February 2, 1966. The titles of these papers are:

"On Optimum Signal-Sets for Certain Scatter-Multipath Channel Models," by Douglas R. Anderson.

"The Effect of Phase Detector Characteristics on Phase Lock Loop Design Parameters", by John J. Uhran and John C. Lindenlaub.

"Adaptive Detection of Non-Synchronous Signals", by Thomas L. Stewart and John C. Hancock.

A fourth paper

"A Property of the L_2 -Norm of a Convolution", by Douglas R. Anderson was published in the May 1966 issue of the Bulletin of the American Mathematical Society.

Copies of these papers were submitted to NASA Headquarters on January 24, 1966.

II. Abstracts of Interim Reports

An interim technical report entitled "Jointly Optimum Waveforms and Receivers for Channels with Memory", by J. C. Hancock and E. A. Quincy was completed during the period covered by this report. An abstract of this report is given below:

This research considers the problem of finding the jointly optimum set of transmitted waveforms and receiver structure which minimize average probability of error, where errors are due to additive noise and inter-symbol interference. The channel characteristics are assumed to be known and time-invariant.

The approach used here differs from other investigations of the joint problem in that: (1) the receiver is not restricted to the linear class and (2) the performance criterion is minimum average probability of error. The memoryless, non-linear bayes receiver structure for M bauds of pulse overlap is developed. The average probability of error is also formulated. Then the channel memory is restricted to adjacent-baud overlap ($M = 1$) in order to evaluate the probability of error. An equivalent criterion to minimum average probability of error is derived for signal design from the error curves. This criterion is: (1) maximize energy transferred through the channel while (2) constraining the cross-correlation energy between the head and tail of the channel out-put signal. The optimum signal for an arbitrary channel is given as the eigenfunction corresponding to the maximum eigenvalue of a symmetric integral operator.

A numerical algorithm is given which was used to solve the integral equation for the optimum signal when supplied sampled values of an

experimental channel impulse response. This procedure was most effectively demonstrated with experimental telephone channel data.

Experimental, optimum input-output waveforms are shown for an experimentally simulated second-order channel. Computed, optimum input-output waveforms are shown for experimental telephone channel data.

The jointly optimum transmitter and receiver performance is given for: (1) an analytic first-order channel, (2) an experimentally simulated second-order channel and (3) data representing an experimental telephone channel with quadratic delay. The performance of the jointly optimum system for practical channels, such as the telephone channel, is shown to achieve ultimate performance. That is, maximum energy is transferred by the optimum signal while the receiver eliminates the effect of intersymbol interference.

III. Research Summaries

A. Experimental Results on Communication Systems Subject to Intersymbol Interference

J. C. Lindenlaub

C. C. Bailey

A digital computer simulation program has been used to investigate the effects of channel-induced phase distortion on digital communication systems. A method of reducing these effects was also simulated and evaluated. The investigation was confined to consideration of one type of channel phase distortion-linear delay distortion. Cascades of all-pass networks were used as phase correction devices to reshape the received signals before detection. Results were obtained indicating the degradation in system performance (measured in terms of probability of error) as a function of the severity of the linear delay distortion introduced by the channel. Also, the improvement in system performance resulting from the use of the all-pass phase correction networks was determined.

The communication system which was investigated is shown in the following block diagram

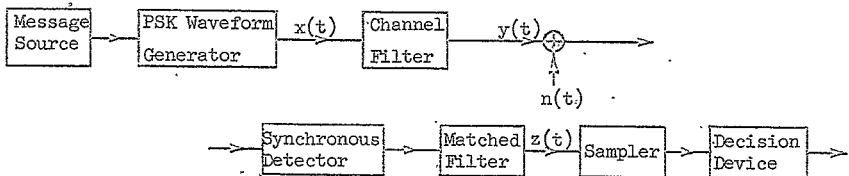


Figure 1

The following assumptions were made concerning this system:

- 1) Phase-shift keying (PSK) modulation is employed at the transmitter.
- 2) The transmitted signal is a pulse of energy E whose spectrum is of the raised cosine form, i.e., the Fourier transform of a transmitted pulse, $X(f)$, is given by

$$X(f) = \begin{cases} \frac{E}{f_b} \cos\left(\frac{\pi(f-f_o)}{2f_b}\right) & f_o - f_b < f < f_o + f_b \\ 0 & \text{elsewhere} \end{cases}$$

- 3) The channel filter is a time-invariant linear filter whose transfer function (in the pass band of interest) has magnitude unity and phase function of the form

$$\beta(\omega) = K(\omega - \omega_o)^2$$

- 4) The additive noise $n(t)$ is zero-mean, white gaussian noise of spectral density N_o .
- 5) The receiver employs a phase-coherent synchronous demodulator.
- 6) The receiver employs an ideal matched filter detector which is perfectly synchronized to the waveform generator.
- 7) The signal to noise ratio (E/N_o) for the system remains constant.

The simulation in this study was made for an equivalent low-pass model of the above system. A description of the digital computer simulation program which was used is given in (1).

The severity of the linear delay distortion introduced by the channel was measured by the ratio D/T , where

D = difference in envelope delays at the edge of the communication pass band and at the center of the band. That is, if envelope delay, $T_d(\omega)$, is defined as $T_d(\omega) = \frac{d}{d\omega} \beta(\omega)$, D is given by $T_d(2\pi f_b) - T_d(0)$ for the equivalent low-pass system

T = baud length associated with the transmitted pulse sequence

$$(T = 1/f_b)$$

Note that since the signal-to-noise ratio was held constant, an increase in D/T in this experiment can be interpreted as an increase in channel distortion severity for a constant transmitted data rate, or an increase in the transmitted data rate (with transmitted energy per bit being held constant) and a fixed channel filter.

The phase correction filters, when used, were inserted in front of the synchronous detector. They consisted of identical one-pole all-pass networks arranged in cascade. Two parameters are required to completely specify a one-pole all-pass network, and similarly, for the phase correction filters used here, two conditions on the system will specify each of the required all-pass networks. The two conditions placed on the filter were:

- 1) The second derivative of the phase function of the correction system at the center frequency of the pass band must equal the negative of the second derivative of the phase function of the channel filter.
- 2) The third derivative of the phase function of the correction filter at the center of the pass band must be zero.

These conditions insured that, for frequencies near the pass band center frequency, the phase function of the correction filter will appear to be a quadratic function whose curvature will exactly cancel the curvature of the quadratic phase function of the channel filter.

Some results of this investigation are shown in Figures 2 through 4. Figure 2 shows the pulse transmission characteristic (response at the output of the matched filter resulting from one transmitted pulse) for various degrees of distortion severity in the channel. This figure points out

the fact that the phase distortion degrades the performance of the communication system in two ways. First, the peak of the pulse transmission characteristic at the sampling instant is reduced. Second, the values of the pulse transmission characteristic at the adjacent sampling instants ($t = \pm T, \pm 2T, \text{etc.}$) become nonzero, introducing intersymbol effects.

The effects of the linear delay distortion on system probability of error are shown in Figure 3. For this experiment, the signal-to-noise ratio was set so that the probability of error with no phase distortion was $1/10$. The degradation in probability of error is shown as a function of the degree of severity of the linear delay distortion (D/T) for three cases. The first case is for no phase correction filter present at the receiver. The other two cases are for correction filters consisting of two and five all-pass networks being present at the receiver. This figure shows that all phase distortion effects are reduced up to $D/T = 1$ for the two all-pass network correction filter and up to $D/T = 1.5$ for the five all-pass network filter.

Figure 4 is an example of the "sensitivity to mismatch" exhibited by the phase-corrected receiver system. For this experiment, the channel filter distortion was fixed at $D/T = 2$, and the correction filter was set for optimum correction of distortion factors from $D/T = 1$ to 4. Five all-pass networks were used in the phase correction network. It should be noted that the uncorrected system's probability of error for $D/T = 2$ is .195.

Reference:

1. Bailey, C. C. and Lindenlaub, J. C., "Experimental Research on Communication Systems Subject to Intersymbol Interference", Semi-Annual Report of Research Performed Under NASA Grant NsG-553, July 1, 1965 thru December 31, 1965.

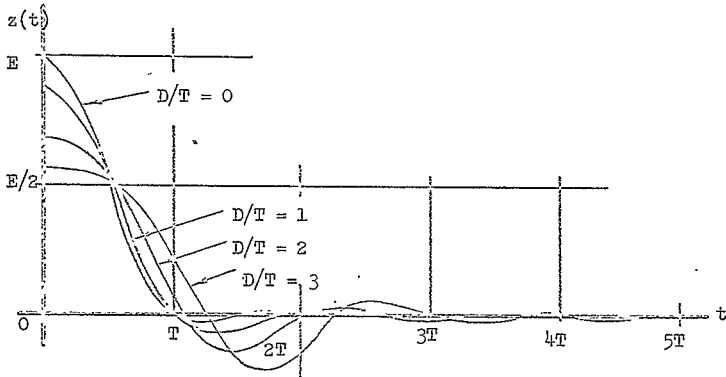


Figure 2 - Pulse Transmission Characteristics for Various Phase Distortions

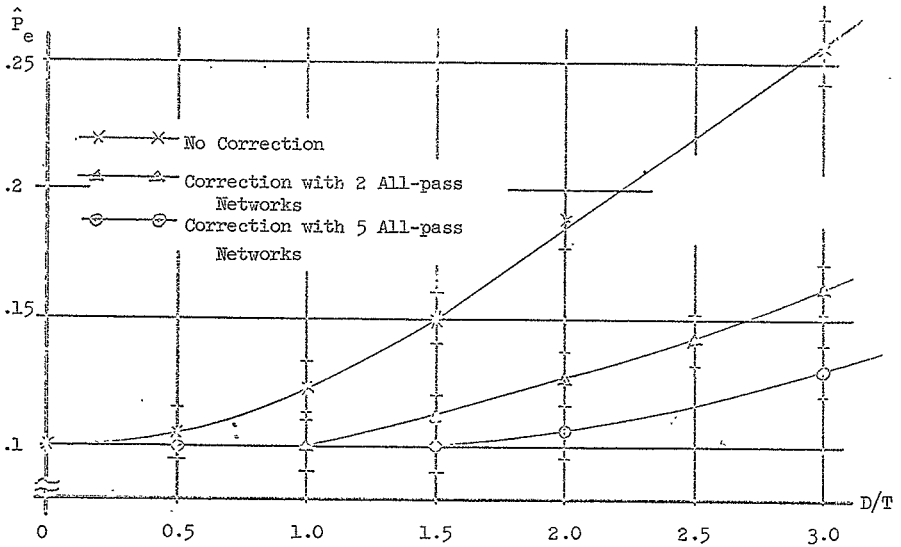


Figure 3 - Probability of Error Degradation versus Linear Delay Distortion

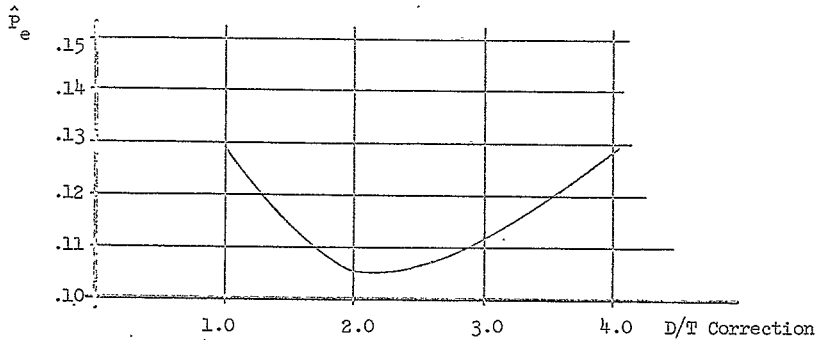


Figure 4 - Sensitivity of All-Pass Correction Filter to "Mismatch".

B. Phase Lock Loop Studies

J. C. Lindenlaub

J. J. Uhran

A theoretical study of the threshold properties of phase lock loops using various phase detectors has proven impossible in all but special cases. However, previous work indicated that other phase detectors chosen ad hoc might give improved performance. To check this conclusion extensive laboratory tests on the experimental tanlock model reported in the previous semi-annual report have been conducted.

The tanlock system provides an interesting device for study because of the wide range of non-linearity that can be introduced in the system with a change in the parameter k . The physical system can not be analyzed analytically because of the divider which also severely limits the noise performance causing a high threshold. A modified tanlock system has been built to overcome this problem. No interference of the signal characteristics occurs, but noise performance is improved.

Most of the measurements have been made on the modified system but performance will be shown with and without the modification for comparison. Four values of the parameter k were used; $k = 0, .5, .75, .9$. The measured lock range was within 2% of the theoretical values in each case. The equivalent noise bandwidth was kept constant for all measurements at a value of 345 cps and the open loop gain, G , was 690 cps.

Figure 1 is a plot of the total output noise power in a 600 cps band. Each system has the same output power for high S/N , but as k approaches one, the noise rises more sharply at a higher S/N . This break from the linear result is an indicator of the threshold and the onset of cycle

slipping pulses. It should be noted that the threshold occurs at a much higher S/N for the unmodified system. Thus the modified tanlock system was used in all the remaining work.

Figure 2 is a plot of the rate of cycle slipping vs. the input S/N ratio. In the range of measurement, the curves are uniformly spaced and thus give an excellent measure of the difference in threshold for any of these systems. The loss in the threshold amounts to 4.5 db for the case of $k = .9$, which is bought at the price of increases lock range shown in Figure 3.

The tanlock system was previously shown to have improved lock range and synchronization time in the noiseless case. In any system of this type, noise will tend to reduce the usable stability range. This is seen by realizing that the closer operation is the extremes of the stability range, the more the system is susceptible to noise in terms of slipping a cycle. Noise peaks need be much smaller to exceed the maximum stable voltage.

Figure 3 is a plot of the maximum usable lock range for each tanlock system. Each curve is based on a criterion of 10 slips/sec. If a higher slip rate is permitted the curves will be slightly higher. The improved range for larger k is maintained until the threshold occurs, but the falloff is faster. It is also evident that the higher values of k have a sharper threshold.

Figure 4 demonstrates the linearity of the various systems to a signal fm modulated by a 100 cps sine wave. Linearity exists until break-up occurs and the maximum increases as k , due to the increased range.

Figure 5 is a histogram of the times between cycle slipping. A definite dead time exists; then a sharp rise occurs followed by an exponential

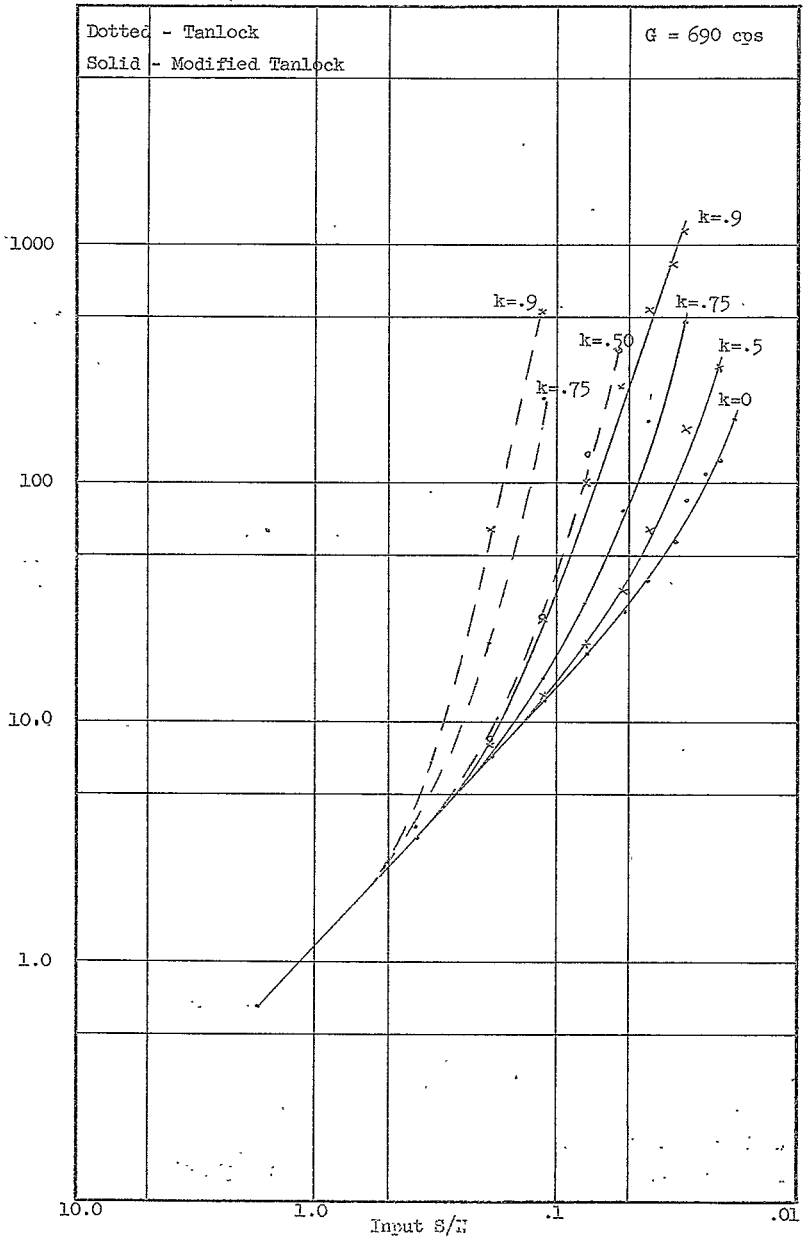


Figure 1

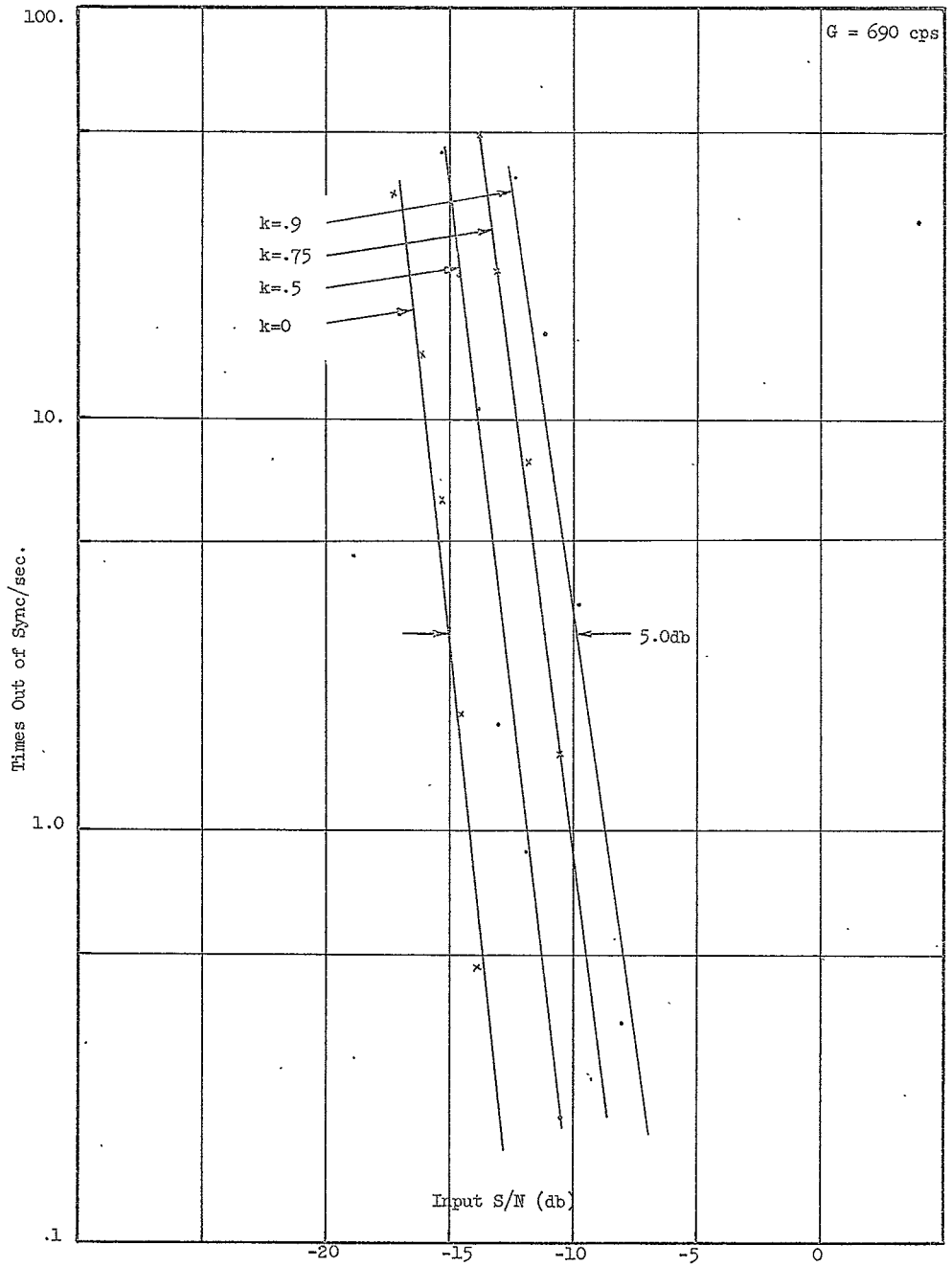


Figure 2

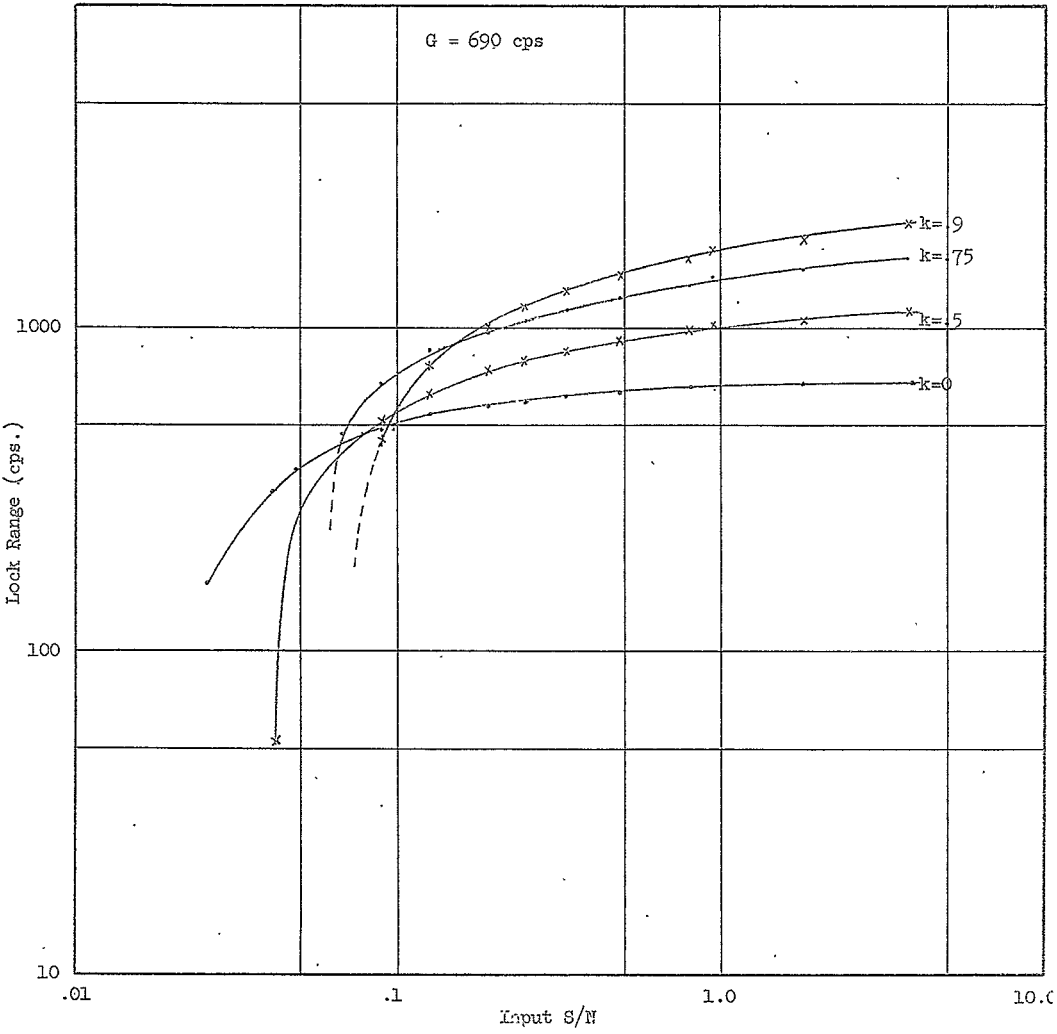


Figure 5

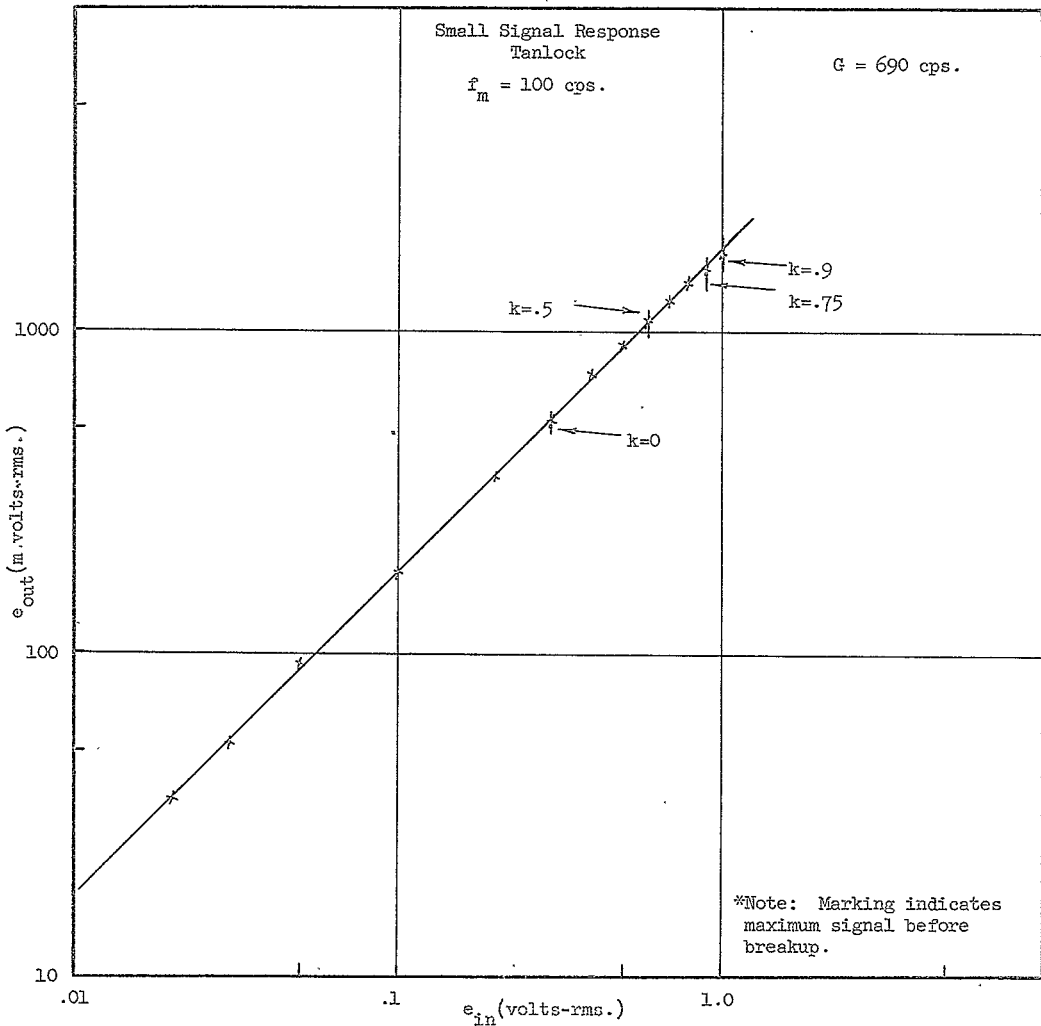


Figure 4

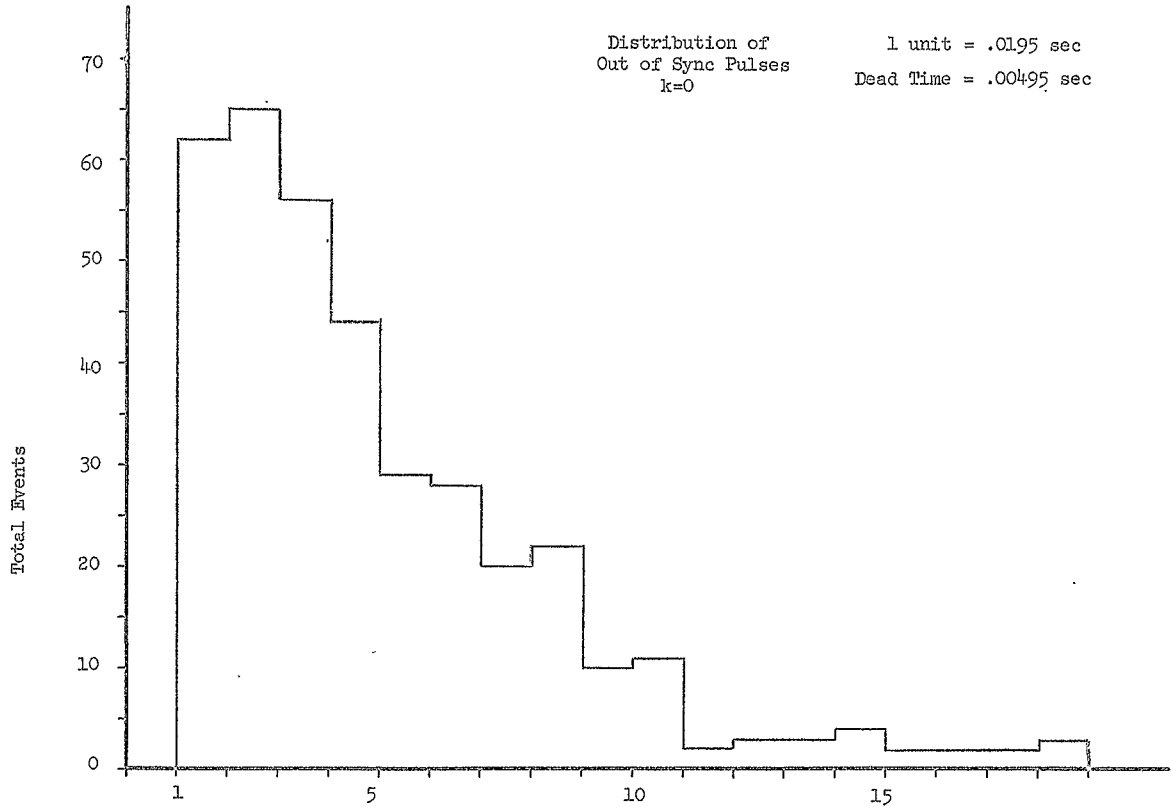


Figure 5

decay. The shape is similar to that reported by Lindsey. Although Figure 5 was done for $k = 0$, similar plots have been obtained for the other cases considered. A plot of the second order statistics indicates that the events are independent. The assumption of others that the events are Poisson seems to be borne out, though only a necessary condition has been shown.

A second order filter of the simple RC type has also been tried. Results are the same as the first order independent of the damping factor for the same equivalent noise bandwidth. A slight improvement in the lock range and the threshold for a given S/N is noticed, however.

It should be noted that the results will differ depending on the equivalent noise bandwidth. If the input S/N is normalized to the equivalent noise bandwidth, a universal curve is obtained, thus making this parameter very critical.

The effects introduced by a second order imperfect filter and a limiter at the input to the system remain to be studied on this investigation.

C. Purdue Channel Simulator

P. A. Wintz

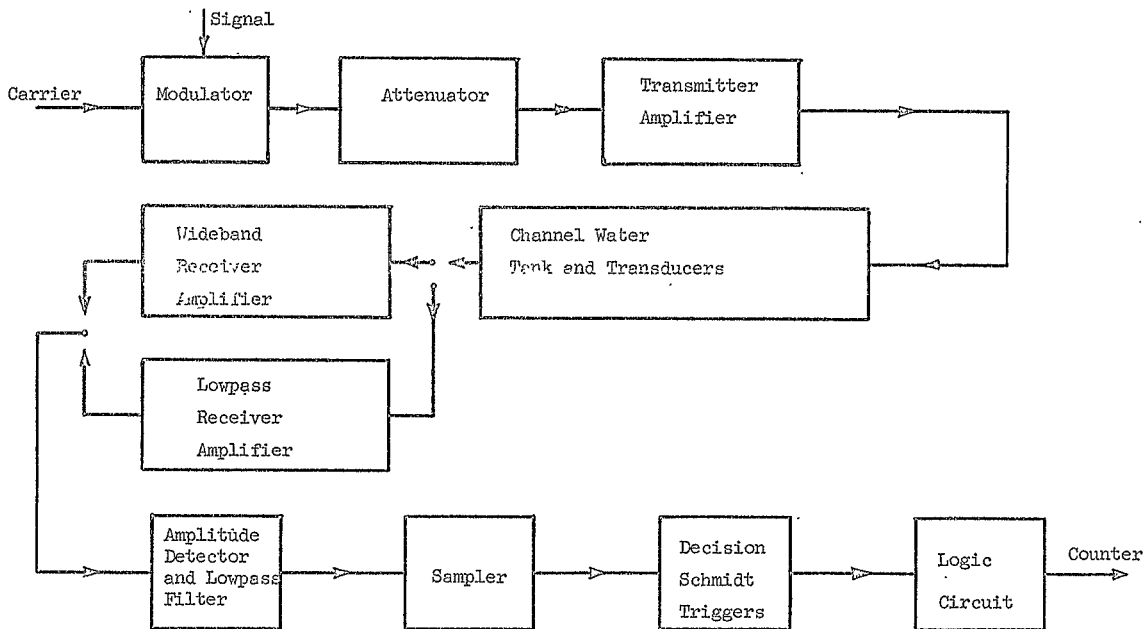
R. A. Markley

A description of the channel simulator and some of its applications has been previously reported in the third semi-annual research summary.

Briefly, the system presently consists of a water tank "channel", hydrophone "antennas", and some of the electronic equipment necessary for signal generation and processing. The advantage of using the water tank comes from the scaling of path length, antenna beamwidths, etc. by a

Purdue Channel Simulator

System Block Diagram



factor that allows the entire channel and test equipment to be contained in the laboratory.

Scatter components can be introduced by controlled release of air bubbles from a system of nozzles near the bottom of the tank. Doppler shifts can also be introduced by moving the water relative to the hydrophones. A device for measuring the statistics of the channel output is presently in the final stages of design.

A system block diagram is included for the purposes of illustrating one possible experimental setup and to list the available electronic components designed for use with the channel simulator. Two different types of hydrophones have been mounted and tested in the tank and are available for use at 3.5 mc, 1.0 mc, 500 kc, and 50 kc.

The Purdue Channel Simulator is available for use by any interested person. It is expected that some users will attempt to simulate real-life channels. Other users will use the simulator as a convenient source of perturbed signals to test adaptive receivers designed to operate efficiently for a variety of channel environments.

D.1 On Optimum Binary Signal-Sets for Certain Scatter-Multipath Channel Models

D. R. Anderson

It has been conjectured by Turin⁽¹⁾ that an optimum binary signal-set for a scatter-multipath channel with random path delays is characterized by the following property: the envelope of the cross-correlation function has the smallest possible peak value. This conjecture can now be shown to be true for his channel model whenever specular received components

are entirely lacking and the path delays are independent. What is more, for the case of equal-energy signals occupying the same frequency-interval, a pair of chirp-like signals has been discovered to have approximately the required minimal correlation-envelope peak.

Suppose one considers Turin's model of a scatter-multipath channel in the case of no received specular components and independent path delays. Then, using methods of error-probability calculation developed by Price in (2), it follows that a binary signal-set whose cross-correlation envelope has a minimal peak will minimize the maximum probability of error over all choices of joint distribution functions for the path delays. This min-max approach is valuable even when one considers the path delays known, since it yields a signal-design criterion independent of quantitative channel properties. A similar assertion is true for the analogous case of Kailath's time-varying filter channel model. (See Chapter 7 of (3)).

Moreover, if equal energy and joint occupancy of the same frequency-interval are required, the peak of the envelope of the normalized cross-correlation must be at least $(2TB)^{-\frac{1}{2}}$, T and B being the common time width and common bandwidth, respectively. (See (4)). On the other hand, for any choice of T and B, a pair of chirp-like signals can always be constructed such that the cross-correlation envelope has a peak of no more than $2/(2TB)^{-\frac{1}{2}}$, so that the signal-set is essentially optimal for time-bandwidth products above 10^4 . The generic pair, $x_1(t)$ and $x_2(t)$ has the following form ω_0 being chosen much larger than KT .

$$x_k(t) = \begin{cases} \cos \left[\omega_0 t + (-1)^k K(t-T/2)^2 \right], & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

References:

1. Turin, G. L., "A Review of Statistical Multipath Communication Theory," Hughes Aircraft Company, Research Report 104, April, 1959.
2. Price, R., "Error Probability for Adaptive Multichannel Reception of Binary Signals," Lincoln Laboratory Technical Report No. 258, July, 23, 1952.
3. Kailath, T., "Communication via Randomly Varying Channels," Sc.D. Thesis Department of Electrical Engineering, M.I.T., Cambridge, Massachusetts, June, 1961.
4. Anderson, D. R., "A Property of the L_2 -Norm of a Convolution," May, 1966, Bulletin of the American Mathematical Society.

D.2 Families of Equal-Length Shift Register Sequences Obtainable From A New Class of Cyclic Codes

D. R. Anderson

As pointed out by Levitt in (1), each cyclic code, particularly a Bose-Chaudhuri-Hocquenghem code, naturally generates a family of equal length shift register sequences with relatively good correlation properties. In fact, all cross-correlation functions of pairs of such sequences and all out-of-phase autocorrelations are at most $1 - (2d/n)$, d being the minimum distance and n the code length.

A new class of cyclic codes is considered here. The generic p -ary code of this new class (p any prime) has as generating polynomial $(X^{(p^m-1)}-1)/p_1(X)\dots p_r(X)$ where $p_1(X), \dots, p_r(X)$ is any collection of distinct primitive polynomials of degree m over $GR(p)$ (unless p^m-1 is a prime in which case $p_1(X), \dots, p_r(X)$ must exclude at least one such primitive polynomial). The code so defined turns out to have length p^m-1 , p^m-1-mr checkbits, and minimum distance at least $\left\lfloor \frac{p^m-1}{mr} \right\rfloor + 1$. It is of some interest that this code is the intersection of at most mr distinct Bose-Chaudhuri-Hocquenghem codes all of length p^m-1 .

While all such codes yield a corresponding family of p-ary shift register sequences, there is for each p and each m a set of these codes with particularly good properties. These are the codes defined in the case that $p_1(X), \dots, p_r(X)$ are the primitive polynomials satisfied by, respectively, $\beta^{m_1}, \dots, \beta^{m_r}$, where β is any primitive element, $m < m_1 < \dots < m_r < 1 + p^{-m/2}(p^m - 1)$, and $(m_i, p) = (m_i, p^m - 1) = 1$ for all i. (It can be proved that $p_1(X), \dots, p_r(X)$ are distinct, but the proof is not elementary.) It can be shown that such a cyclic code generates a family of $(p^{mr} - 1)/(p^m - 1)$ p-ary shift register sequences $\{s_k\}$ of length p-ary shift register sequences whose recursion relationships are determined by $p_1(X), \dots, p_r(X)$. Further, for any $\{s_k\}$ in the above family one has the following auto-correlation formula.

$$1) \quad \left| \sum_{k=1}^{p^m - 1} \exp \left[\frac{2\pi i}{p} (s_k - s_{k+l}) \right] \right| \leq 1 + (m_r - 1) p^{m/2}, \quad l \neq 0$$

In addition, for any two distinct such sequences, $\{s_k\}$ and $\{s'_k\}$, one gets the following cross-correlation formula.

$$2) \quad \left| \sum_{k=1}^{p^m - 1} \exp \left[\frac{2\pi i}{p} (s_k - s'_{k+l}) \right] \right| \leq 1 + (m_r - 1) p^{m/2}, \quad \text{all } l$$

Formulas 1) and 2) permit one to say that for p^m large enough one can find an arbitrarily large family of p-ary shift register sequences with all cross-correlations and all out-of-phase auto-correlations in that family at most $p^{(m/2)(1+\delta)}$ for any fixed positive δ . This is true since for any fixed positive δ and p^m large enough one can choose the exponents m_1, \dots, m_r so as to meet the additional requirements that a) $m_r \leq (p^{m/2})^\delta$ and b) $r \geq \frac{1}{2}(p^{m/2})^\delta$. Thus, one can find arbitrarily large families of sequences with what might be called approximate pseudo-noise properties relative to themselves and each other. Of course, as a direct consequence of 1) and 2), there are many moderate-sized families of sequences which in

a usual sense have pseudo-noise properties relative to themselves and each other. Finally, the fact that each of the above families contains r maximal length p -ary shift register sequences of the same length yields, in the light of 2), previously unknown cross-correlation properties of such sequences.

Reference:

1. Levitt, Karl N., "Correlation Properties of Multi-level Cyclic Sequences," Technical Report 400-125, Laboratory for Electrosience Research, Department of Electrical Engineering, New York University.