RADIATION AND CURRENT PROPERTIES OF THE RESISTIVELY LOADED TRAVELING WAVE V-ANTENNA

by

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SECTION I

INTRODUCTION

This report is the first of two presentations which summarize the research on the traveling wave V-antenna over the period from July 1964 to the present.

The traveling wave V-antenna consists of two resistively loaded sections placed in a V configuration and fed at the apex. If the value of load resistance is properly chosen, only the outward going traveling wave is present on the section between the driving point and the termination. The antenna operates best when the driving frequency is such that the resistance is a quarter wavelength from the end of the antenna. Two separate cases may be considered depending on whether or not a resonant or non-resonant section is between the driving point and the resistive termination. The value of the optimum load resistance was based on the analysis of a single leg of the antenna. The analysis is valid so long as the current distribution is not affected by the proximity of the other leg. Experiments have, thus, substantiated the validity of this assumption. These radiation properties are used in calculating the effective area of the antenna.

In order to calculate the equivalent temperature of the antenna the driving point impedance is required. This impedance is determined in Section III based on the physically reasonable model of an infinite traveling wave V-antenna. Numerical results are presented for a wide variety of antenna diameters and apex angles. Detailed calculations are also given for the RAE antenna of this assumption for normally used apex angle (i.e. greater than 20^{o}). The results for optimum load resistances are presented here for the case when the traveling wave section is anti-resonant.

A general discussion of the relevant antenna parameters needed to describe the traveling wave V-antenna is given in Section II. A summary of the radiation properties is also given in this section based on the formulation presented in Scientific Report No. 2 of this grang in Section IV, the Fortram programs for the theoretical results are listed.

SECTION II

The Radioastronomical Measurement Properties of the Satellite Borne V-antenna

Some care must be employed in interpreting measurements made on space probes (1). Here it is important to relate the properties of the radio-astronomical source to physically observable quantities on the satellite. General formulas given in the literature are specialized to a particular antenna. For example, the valid formulas given by Bracewell (2) are not concerned with the driving point impedance of an antenna or the difference between this impedance and the radiation impedance.

The following two formulas given by Sandler (1) will be applied to the satellite borne V-antenna.

EFFECTIVE AREA $A(m^2)$

$$A = R\,e \left\{ 8 \left| \frac{\cos\psi \, h_e(\theta)}{Z_L + Z_o} \right|^2 \xi_o Z_L \right. \qquad (1)$$

where R e is the Real part

$\cos\psi$ = tilt angle between incident electric field and antenna

$h_e(\theta)$ = complex effective length of the receiving antenna

Z_L = load impedance = $R_L + j X_L$

$\xi_o \doteq 120\pi$ ohms

(1) S. S. Sandler, "Effective Area of Satellite-Borne Antennas for Radio Astronomy", Planetary and Space Science, 1963, Vol. 11, pp. 817 - 822.

(2) R. N. Bracewell, "Radio Astronomy Techniques", in Encyclopedia of Physics, S. Fluegge, ed., Spunger Verlag, Berlin, Vol. 54, pp. 42 - 129, 1962.

Z_o = driving point impedance = $R_o + j\, X_o$

ζ_o = characteristic impedance of free space

RADIO SOURCE TEMPERATURE T (^oK) for a Rayleigh-Jeans Source

$$T = \frac{2}{32\pi k \zeta_o} \quad Re\left\{\frac{(Z_L + Z_o)^2}{Z_L\, h_e^2(\theta)}\right\} P^r_{RL} \tag{2}$$

where

λ = wavelength in meters

k = Boltzmann's constant

k = 1.3803×10^{-23} joules / degree

P^r_{RL} = real power measured at load impedance of antenna with a randomly polarized field.

The above formulas were derived for a single linear antenna in free space. For a single linear antenna the radiation field is independant of the angle ϕ in spherical coordinates if the antenna is along the z-axis.

The complex effective length of this antenna is also proportional to the complex field function of the antenna when used for transmission. Thus

$$h_e(\theta) = \frac{1}{\beta_o h} F_o\ (\theta, \beta_o h) \tag{3}$$

where

$$F_o(\theta, h) = \frac{\beta_o}{2I_o} \quad I(z') \int_{-h}^{h} e^{j\beta_o z' \cos\theta} \sin\theta\, dz' \tag{4}$$

I(z) = antenna current

h = half length of antenna

$\beta_o = 2\pi / \lambda_o$

In the general case when the dipole is arbitrarily oriented the complex, the complex field function may be replaced by a space radiation function $K(\theta,\phi)$. Since K is proportional to the vector potential A and h is proportional to the electric field in spherical coordinates, a factor $\sin^2\theta$ will appear in the relation between h^2 and K^2. This relation is given by $h^2(\theta) \Longleftrightarrow \frac{\sin^2\theta}{4\beta_o^2} K^2(\theta,\phi)$ (5)

The space radiation factor for the V-antenna $K^2(\theta,\phi)$ is given by Sandler (3) and has been machine computed for a wide variety of traveling wave V-antennas. The spatial temperature distribution for a Rayleigh-Jeans source is then T(θ,) where

$$T(\theta,\phi) = \frac{\lambda^2}{32\pi k} \frac{4\beta_o^2}{\sin^2\theta} \mathrm{Re}\left\{\frac{(Z_L + Z_o)^2}{Z_L K^2 (\theta,\phi)}\right\} P^r_{RL} \tag{6}$$

$$T(\theta,\phi) = \frac{\pi}{2k \sin^2\theta} \mathrm{Re}\left\{\frac{(Z_L + Z_o)^2}{Z_L K^2(\theta,\phi)}\right\} P^r_{RL} \tag{7}$$

The source temperature as measured by the antenna must also be corrected to include only that temperature which is enclosed by the main beam. A correction factor B is defined by the following relation (4)

$$B = \frac{W_{beam}}{W_{in}} \tag{8}$$

where W_{beam} = "power" in the main beam.

(3) S. S Sandler, "Theory of the Resistively Loaded Traveling Wave V-antenna", Science R·port, No. 2, Prepared for the National Aeronautics and Space Administration Grant Ns G-355, Northeastern University, Boston, Mass.

(4) See Reference 2, Section 38.

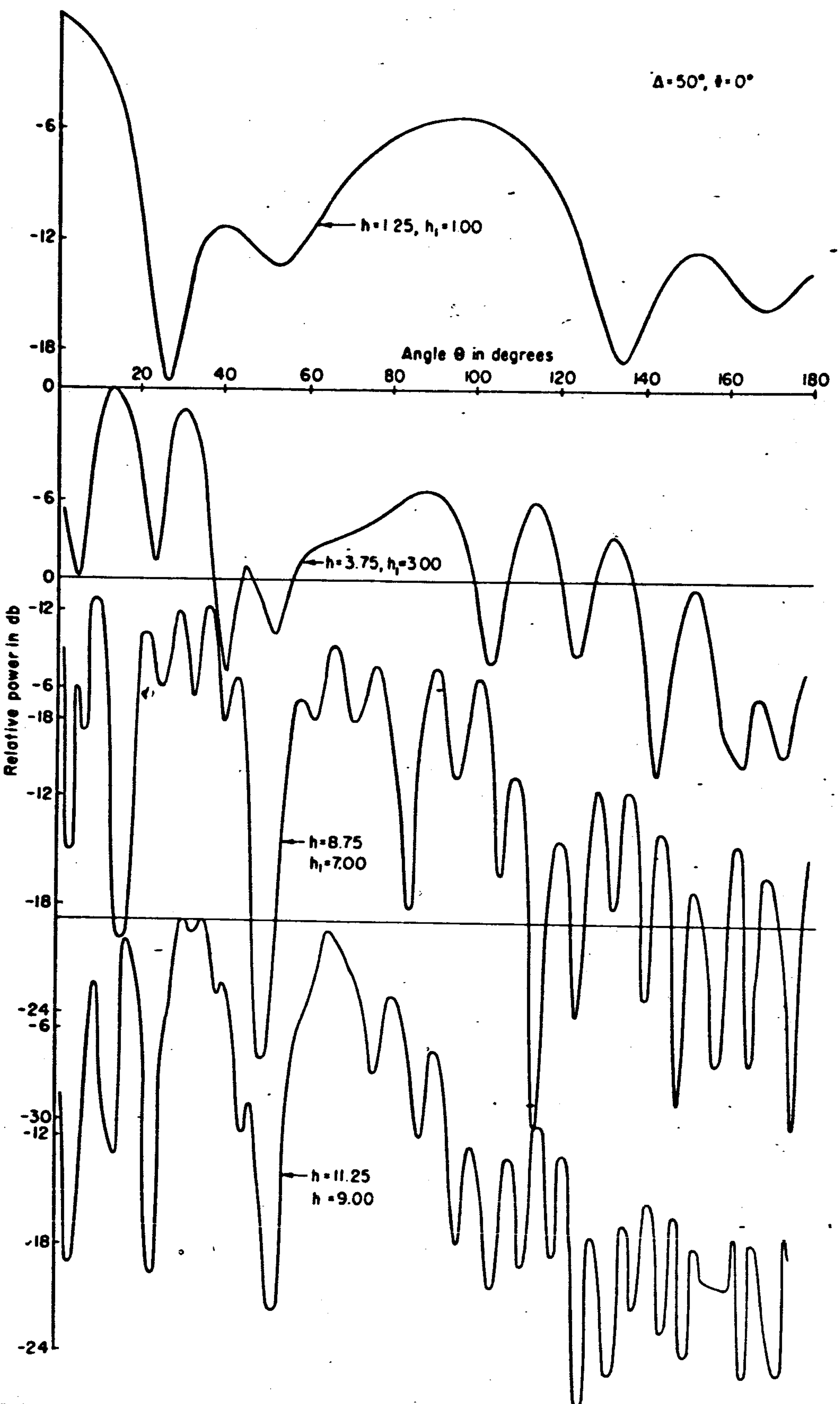


Figure 1. Radiation patterns for a traveling wave V-antenna in the plane perpendicular to the antenna (5/4 series)

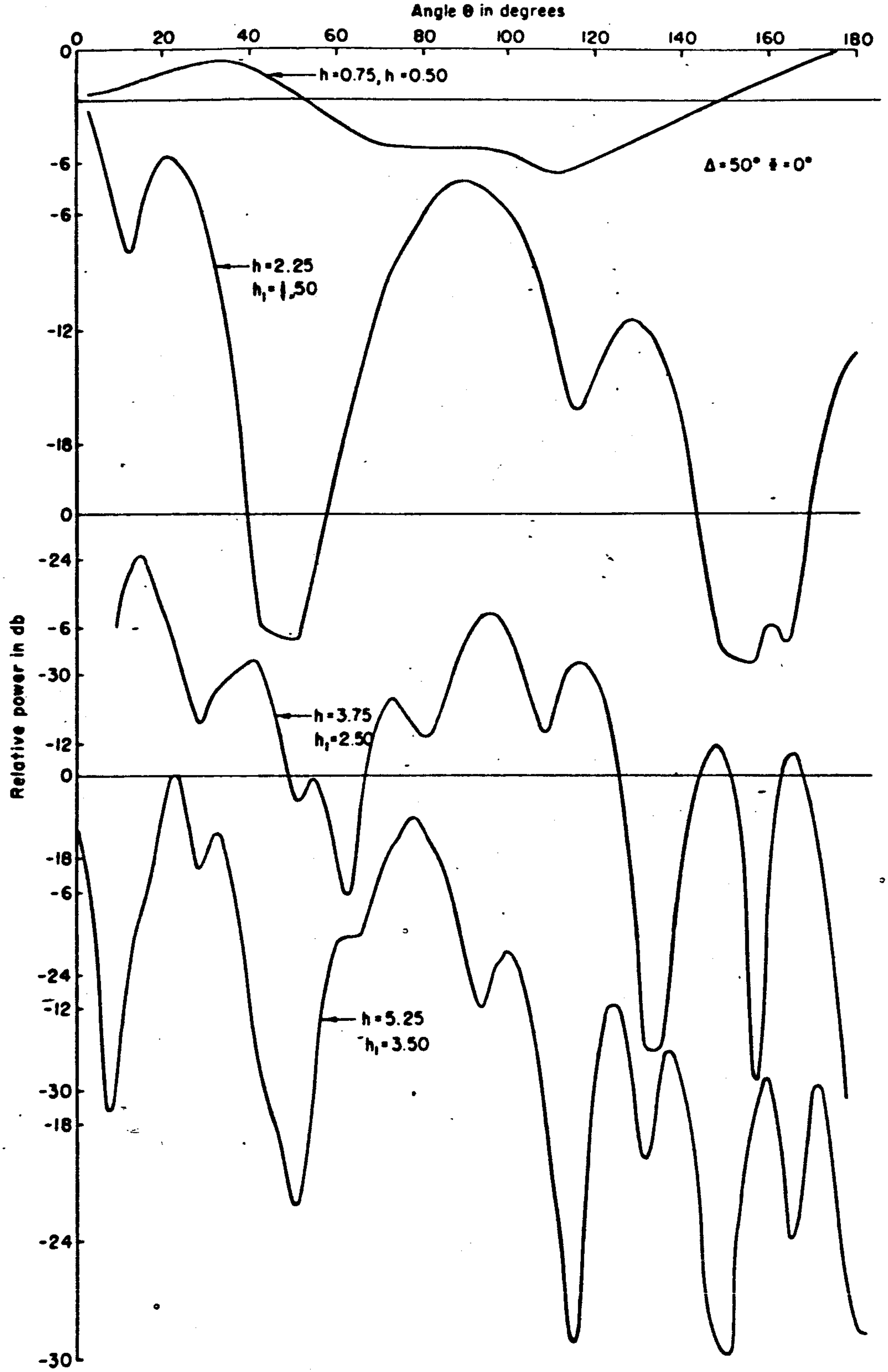
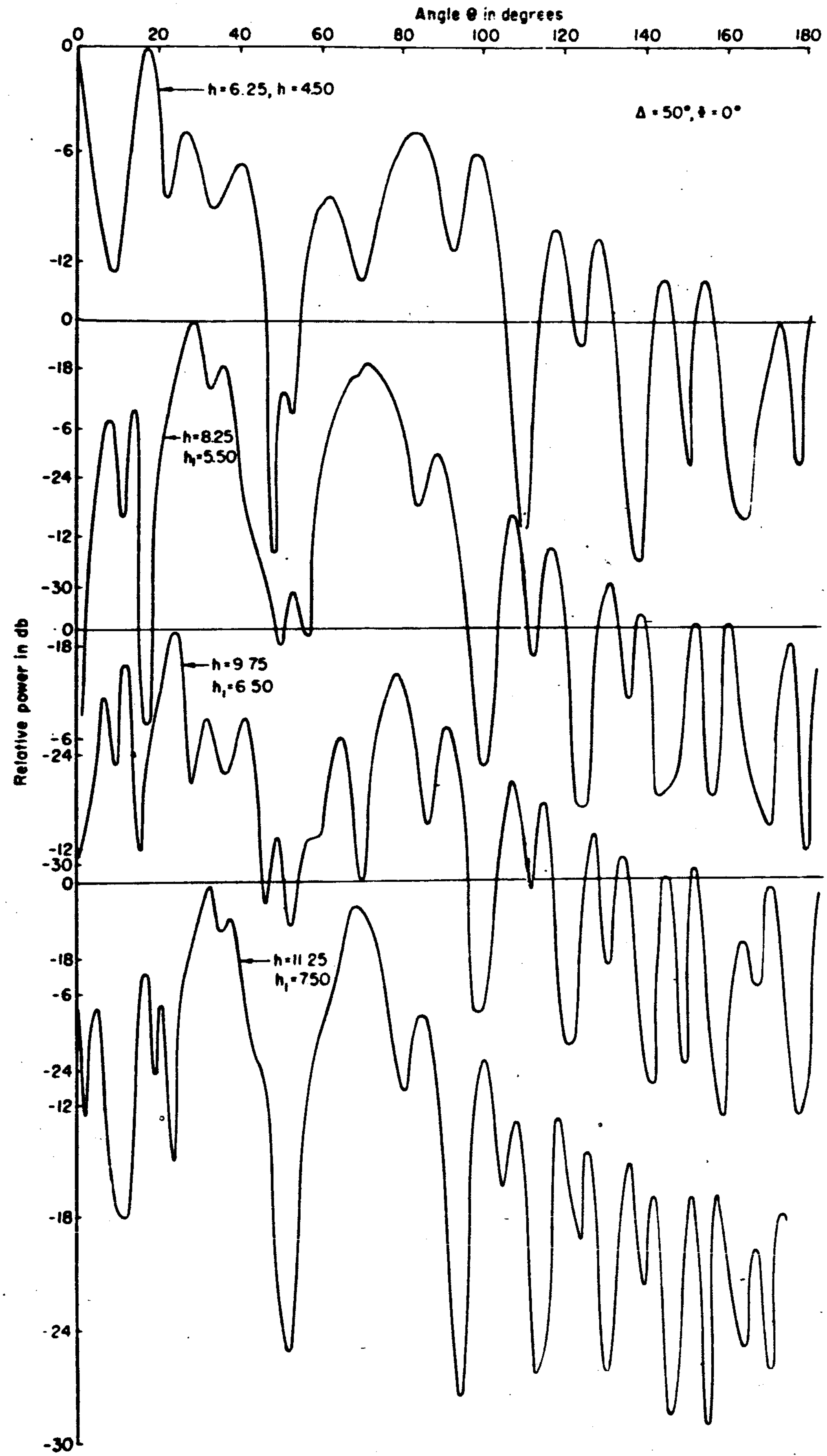


Figure 2. Radiation patterns for the traveling wave V-antenna in the plane perpendicular to the antenna (3/2 series)



Angle θ in degrees
0 20 40 60 80 100 120 140 160 180
h=6.25, h=4.50
Δ=50°, Φ=0°
h=8.25
h1=5.50
h=9.75
h1=6.50
h=11.25
h1=7.50
Relative power in db

SERIES I

DELTA= 20.00

H	H1	K(0,0)**2
1.00	0.75	0.3701E 01
3.00	2.25	0.3289E 02
5.00	3.75	0.9037E 02
7.00	5.25	0.1743E 03
9.00	6.75	0.2822E 03
11.00	8.25	0.4105E 03

DELTA= 40.00

H	H1	K(0,0)**2
1.00	0.75	0.1455E 02
3.00	2.25	0.1196E 03
5.00	3.75	0.2803E 03
7.00	5.25	0.4220E 03
9.00	6.75	0.4806E 03
11.00	8.25	0.4329E 03

DELTA= 45.00

H	H1	K(0,0)**2
1.00	0.75	0.1825E 02
3.00	2.25	0.1427E 03
5.00	3.75	0.3010E 03
7.00	5.25	0.3801E 03
9.00	6.75	0.3280E 03
11.00	8.25	0.1876E 03

DELTA= 60.00

H	H1	K(0,0)**2
1.00	0.75	0.3103E 02
3.00	2.25	0.1812E 03
5.00	3.75	0.1921E 03
7.00	5.25	0.5287E 02
9.00	6.75	0.1192E 02
11.00	8.25	0.7782E 02

DELTA= 80.00

H	H1	K(0,0)**2
1.00	0.75	0.4875E 02
3.00	2.25	0.1022E 03
5.00	3.75	0.3408E 01
7.00	5.25	0.5077E 02
9.00	6.75	0.3357E 02
11.00	8.25	0.3719E 02

DELTA= 120.00

H	H1	K(0,0)**2
1.00	0.75	0.5752E 02
3.00	2.25	0.1981E 02
5.00	3.75	0.1981E 02
7.00	5.25	0.5752E 02
9.00	6.75	0.5752E 02
11.00	8.25	0.1981E 02

DELTA= 160.00

H	H1	K(0,0)**2
1.00	0.75	0.1875E 02
3.00	2.25	0.1137E 02
5.00	3.75	0.7580E 01
7.00	5.25	0.9172E 01
9.00	6.75	0.3664E 02
11.00	8.25	0.6125E 00

SERIES II

DELTA= 20.00

H	H1	K(0,0)**2
1.25	1.00	0.6096E 01
3.75	3.00	0.5417E 02
6.25	5.00	0.1480E 03
8.75	7.00	0.2828E 03
11.25	9.00	0.4521E 03

DELTA= 40.00

H	H1	K(0,0)**2
1.25	1.00	0.2383E 02
3.75	3.00	0.1871E 03
6.25	5.00	0.3950E 03
8.75	7.00	0.4987E 03
11.25	9.00	0.4276E 03

DELTA= 45.00

H	H1	K(0,0)**2
1.25	1.00	0.2979E 02
3.75	3.00	0.2158E 03
6.25	5.00	0.3816E 03
8.75	7.00	0.3481E 03
11.25	9.00	0.1582E 03

DELTA= 60.00

H	H1	K(0,0)**2
1.25	1.00	0.4963E 02
3.75	3.00	0.2220E 03
6.25	5.00	0.1012E 03
8.75	7.00	0.7605E 01
11.25	9.00	0.1208E 03

DELTA= 80.00

H	H1	K(0,0)**2
1.25	1.00	0.7320E 02
3.75	3.00	0.4395E 02
6.25	5.00	0.6117E 02
8.75	7.00	0.2272E 02
11.25	9.00	0.2661E 02

DELTA= 120.00

H	H1	K(0,0)**2
1.25	1.00	0.5752E 02
3.75	3.00	0.1981E 02
6.25	5.00	0.1981E 02
8.75	7.00	0.5752E 02
11.25	9.00	0.5752E 02

DELTA= 160.00

H	H1	K(0,0)**2
1.25	1.00	0.2209E 01
3.75	3.00	0.9632E 01
6.25	5.00	0.1051E 02
8.75	7.00	0.2891E 01
11.25	9.00	0.3088E 02

(Note: B in (8) above is the quantity G as computed in reference 3 for a particular value of θ_1).

A quantity G has been computed for the V-antenna which measures the ratio of the electromagnetic energy transfer into space of a cone of half angle θ_1 to the total energy transfer.

$$G = \frac{\int_0^{2\pi}\int_0^{\theta_1} K^2(\theta,\phi)\sin\theta \; d\theta \; d\phi}{\int_0^{2\pi}\int_0^{\pi} K^2(\theta,\phi)\sin\theta \; d\theta \; d\phi} \tag{9}$$

Plots of $K^2(\theta, O)$ for a representative V-antenna are shown in Figs. 1 and 2. The plot of K^2 represents the power pattern of the antenna. Note also that the power pattern contains no sharp maxima. As the length of the V-antenna increases, the plot of K^2 will become increasingly complicated and it will become difficult to determine the extent of the main beam. If the actual source temperature is T_a then T_a is given by a convolution of the corrected terminal temperature and the source distribution (5).

If the source is uniform over the main beam and the background temperature is negligible then

$$T_a = T/\beta$$

and the Flux density S is given by

$$S = \frac{2k}{\beta\lambda^2} \iint_{beam} T_a \; dx \; dy$$

(5) See Reference 2, Section 50.

SECTION III

The Driving-Point Impedance of an Infinite Length V-antenna

Introduction

The V-antenna is constructed by bending the two halves of a dipole about the driving point. This configuration is shown in Fig. 1. The theoretical analysis of this structure is complicated by the two-dimensional coupling of the fields and by peculiar singularities which appear near the driving point. An example of the peculiar behavior near the driving point is the consideration of the impedance of the structure with a delta function generator (i.e. $\delta = 0$). This condition is not physically meaningful since the conductors are in actual contact. Also if the structure is treated entirely as an antenna, potential contributions near the driving point are inordinately large. These contributions are mainly due to the large proximity effect which is not accounted for in the quasi-one-dimensional form for the potential integrals.

A physically more meaningful approach to the impedance determination is to consider the driving terminals as having a finite, but small, separation. The separation distance is the width of a two wire line which drives the structure. Since the directions of the currents on the two halves of the antenna are opposite, the antenna behaves like a non-homogeneous transmission line near the driving points. When the electrical spacing between

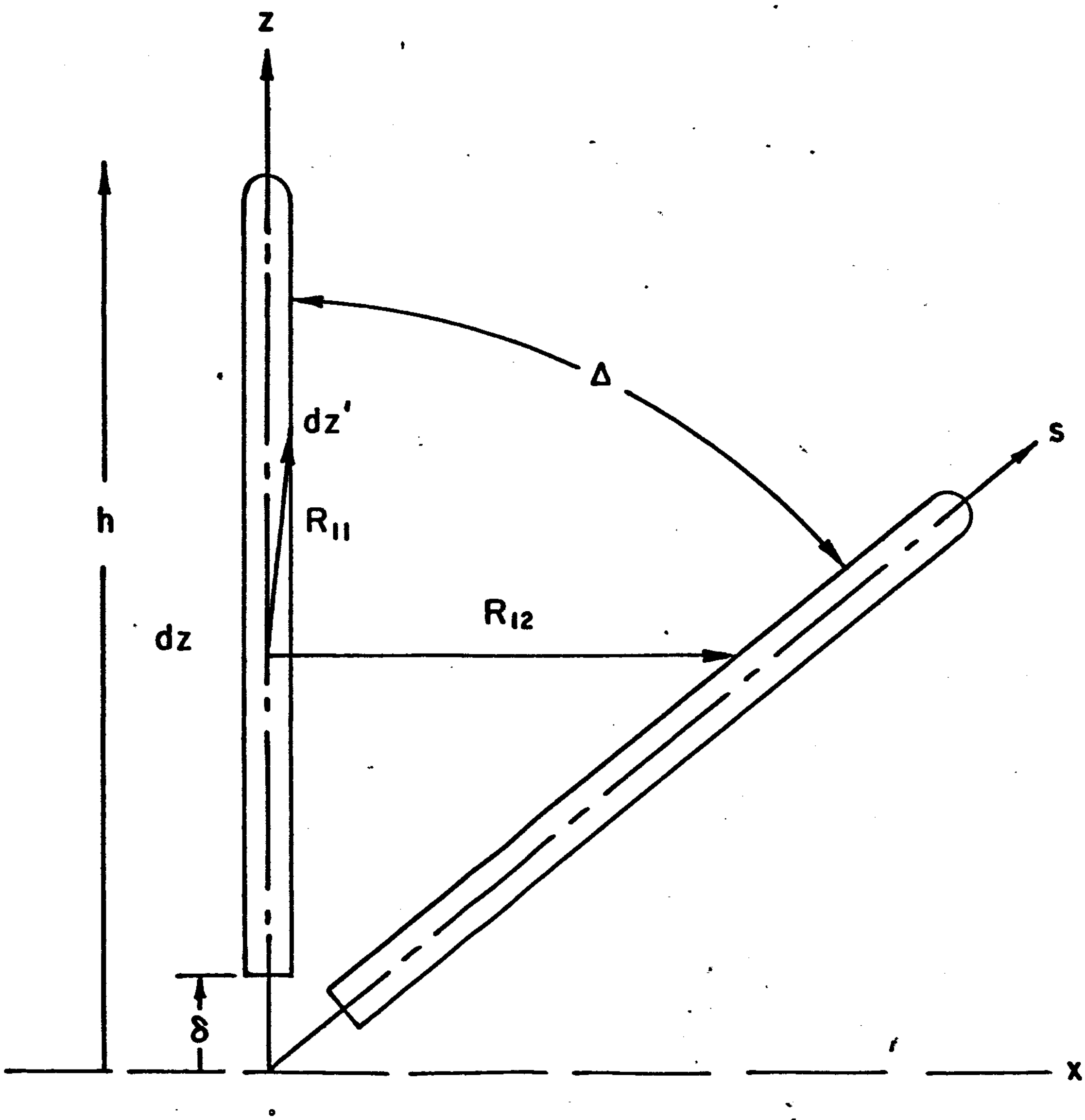


Figure 1. V-antenna geometry

opposite parts of antenna is not small compared to a wavelength the radiation properties become important. Thus the section near the driving point can be analysed in terms of transmission line theory, and the remaining section in terms of antenna theory.

The V-antenna of infinite length is chosen as an example since only one outward travelling wave of current is involved. This type of antenna is approximated by a finite length V-antenna which is properly resistively loaded near the end. It is possible for this case to find an approximate analytical solution which illustrates the physical principles. The approximate solution is first derived for the limiting case of an apex angle of 180^{o} which corresponds to the infinite length dipole. The presen approximate theory compares favorably with the well kn own results for the driving-point impedance of the infinite length dipole. The method is then extended to the V-antenna with an arbitrary apex angle.

<u>The driving point impedance of a Thin Infinite Cylindrical Dipole Antenna</u>

The Hallen integral equation for an infinite antenna is given by

$$\int_{-\infty}^{\infty} I_z(z') \frac{e^{-j\beta_o R_1}}{R_1} \, dz' = \frac{2\pi}{\zeta_o} V_o \, e^{-j\beta_o |z|} \qquad (3\text{-}1)$$

where $I_z(z)$ is the current along the antenna, $\zeta_o = 120\pi$ ohms,

$\beta_o = 2\pi/\lambda_o$, λ_o is the free space wavelength,

$$R_1 = \sqrt{(z - z')^2 + a^2} \; .$$

The inward traveling wave has been deleted on the right hand side of (1) since the antenna is infinite and no reflections can occur. Details for the derivation of (1) are given in King.

The vector potential on the left hand side of (1) behaves like the current,

$$4\pi\nu_0 \, A_z = \int_{-\infty}^{\infty} I_z(z) \frac{e^{-j\beta_o R_1}}{R_1} \, dz' \sim I_z(z) \qquad (3\text{-}2)$$

or

$$4\pi\nu_0 \, A_z = \Psi(z) \, I_z(z) \qquad (3\text{-}3)$$

In (3-3) the function $\Psi(z)$ is slowly varying and will be approximated at a particular value of z, thus

$$\Psi(z) \cong \Psi(z_o) \qquad (3\text{-}4)$$

The value of the constant $\Psi(z_o)$ depends on the particular choice for the zeroth-order current and also on the value of z_o. Since interest here is in the driving point impedance, the Ψ function in (3-3) will be specified at the base (i.e. z = o). The simplest zero-order assumption for the current will also be chosen. It is the outward traveling wave

$$I_z(z) = I_o\, e^{-j\beta_0|z|} \qquad (3\text{-}5)$$

Far away from the source (3-5) is incorrect since it is well known that the current for large z has an amplitude dependence proportional to $(\ln \beta_o z)^{-1}$. However, here the form of the current near the base is of prime importance. The value of $\Psi(z_o) = \Psi(o)$ is given by (3-3) with (3-5), thus

$$\Psi(o) = \int_{-\infty}^{\infty} I_z(z')\, \frac{e^{-j\,\beta_o\, R_{10}}}{R_{10}} \cdot dz' \qquad (3\text{-}6)$$

$$\Psi(0) = 2[-\, Ci(\beta_o a) - j\,(\frac{\pi}{2} - Si\,(\beta_o a)] \qquad (3\text{-}7a)$$

$$\Psi(o) = 2[\ln \frac{1}{\gamma\, \beta_o a} - j\,\frac{\pi}{2}\,],\ (\beta_o a) << 1 \qquad (3\text{-}7b)$$

where

$$\gamma = 1.781\ , \quad \ln\gamma = 0.5772.$$

The integral equation (3-1) is reduced to the following algebraic equation with (3-3), thus

$$e^{-j\beta_o|z|} \quad I_o\, \Psi(o) = \frac{2\pi}{\zeta_o}\ Vo\ e^{-j\beta_0|z|}$$

or

$$Z_o = \frac{\xi_o}{2\pi}\ \Psi_0 = \frac{\zeta_0}{\pi}\,[\ln \frac{1}{\gamma\, \beta_0 a}\ -j\,\frac{\pi}{2}\,],\ (\beta_0 a) << 1 \qquad (3\text{-}8)$$

The value in (3-8) agrees with the rigorous impedance formula given by Wu for a thin infinite antenna.

The Infinite Length V-antenna

The integral equation for the V-antenna has been formulated by King. After some minor manipulations the integral equation for the finite V-antenna can be placed in the following form:

$$\int_{\delta}^{h} I_{1z}(z')\, K_v(z,z')dz' = \frac{4\pi}{\delta_o} [(C_2\, e^{-j\beta_o\delta} + \frac{1}{2}\bar{V}_\delta)e^{j\beta_o\delta}\, e^{-j\beta_o z} + C_2\, e^{j\beta_o z} - \nu(z)] \tag{3-9}$$

where

$$K_v(z,z') = \frac{e^{-j\beta_o R_1}}{R_1} - \cos\Delta\, \frac{e^{-j\beta_o R_{12}}}{R_{12}} \tag{3-10}$$

$$R_1 = \sqrt{(z_v - z')^2 + a_v^2} \tag{3-11}$$

$$R_{12} = \sqrt{(z_v - z')^2 + a_v^2}$$

$$z_v = z\cos\Delta \tag{3-12}$$

$$a_v^2 = z^2 \sin^2\Delta + a^2, \quad a = \text{antenna radius}$$

$$\zeta_o = 120\pi \text{ ohms} \tag{3-13}$$

$$\zeta = \text{gap distance at apex of V-antenna}$$

$$1/2\ \bar{V}_\delta = 1/2\ V_\delta - \phi_{1x}(\delta) \qquad (3\text{-}14)$$

$$\phi_{1x}(z) = \frac{j\zeta_o}{4\pi}\, p(z)\, \sin^2 \Delta \qquad (3\text{-}15)$$

$$p(z) = \beta^2 \int_\delta^h I_{1z}(z')\, K_{12}(z,z')\, z'\, dz' \qquad (3\text{-}16)$$

$$K_{12}(z,z') = \frac{e^{-j\beta R_{12}}}{\beta^3\, R^3_{12}}\, (1 + j\beta\, R_{12}) \qquad (3\text{-}17)$$

The term $\nu(z)$ on the R.H.S. of (3-13) vanishes at the driving point $z = \delta$. Away from the driving point it will influence the form of the current. From experimental measurements the form of the current remains sensibly constant as a function of the apex angle Δ. Since the interest is in the driving point impedance the term $\nu(z)$ will be neglected. If desired the square bracketed term can be included at the risk of increasing the complexity of the formulation.

The constant C_2 vanishes for an infinite length antenna since there is only an outward going traveling wave. The simplified form of (3-9) for an infinite length V-antenna is

$$\int_\delta^\infty I_{1z}(z')\, K_v(z,z')dz' \simeq \frac{4\pi}{\zeta_o}\, [(\frac{1}{2}\, V_o - \phi_{1x}(\delta))\, e^{-j\beta_o \delta}\, e^{-j\beta_o z}] \qquad (3\text{-}18)$$

The vector potential on the left hand side behaves like the current providing the legs of the V-antenna are not too close. Experimental results also indicate that the form of the current on a traveling wave V-antenna does not differ appreciably from the current distribution on a traveling wave dipole. It is therefore reasonable to assume that an outward traveling wave of current exists on the antenna as a zeroth order approximation. As with the infinite length dipole the Ψ function is approximated by the value at the driving point, thus

$$\int_{\delta}^{\infty} I_{1z}(z') \; K_v(z,z') = \Psi_v(o) \; I_{1z}(z') \tag{3-19}$$

where

$$\Psi_v(\delta) = \int_{\delta}^{\infty} I_{1z}(z') \; K_v(\delta,z')dz' \tag{3-20}$$

After integration (3-20) may be expressed in terms of integral sine and cosine functions

$$\Psi_v(\delta) = [- Ci \; (\beta_o a) - j(\tfrac{\pi}{2} - Si \; (\beta_o a)]$$

$$- \cos \Delta[- Ci \; (\beta_o \delta_v) - j(\tfrac{\pi}{2} - Si(\beta_o \delta_v)] \tag{3-21}$$

where

$$\delta_v = \bar{\delta} + \sqrt{\bar{\delta}^2 + a_v^2} \quad . \tag{3-22}$$

$$\bar{\delta} = \delta(1 - \cos \Delta)$$

For $\delta^2 >> a^2$ a simplified form for δ_v is possible.

$$\delta_v = 2\delta(\sin^2 \frac{\Delta}{2} + \sin \frac{\Delta}{2}), \quad \delta^2 >> a^2 \tag{3-23}$$

The scalar potential contribution $\phi_{1x}(\delta)$ is more difficult to evaluate. It has the following approximate value:

$$\phi_{1x}(\delta) = \frac{j\zeta_o}{4} p(\delta) \sin^2 \Delta \tag{3-24}$$

where for

$$I_{1z}(z') = I_o e^{-j\beta_o z}$$

$$\frac{p(\delta)}{I_o} = e^{-j\beta_o\delta \cos \Delta} \quad I_o \frac{e^{-j2\beta_o \bar{a}_v}}{\beta_o \bar{a}_v} - j [- Ci (2\beta_o \bar{a}_v)$$

$$- j (\frac{\pi}{2} - Si(2\beta_o \bar{a}_v))]$$

$$+ \delta \cos [e^{-j\beta_o \bar{a}_v} \quad \frac{a_v^2}{8\beta_o \bar{a}_v} + \frac{\beta_o a_v^2}{12 \bar{a}_v^2} + j \frac{a_v^2}{12 \bar{a}_v^3} - j \frac{\beta_o^2 a_v^2}{6 \bar{a}_v}$$

$$- \frac{\beta_o^3 a_v^2}{3} (- Ci(2\beta_o \bar{a}_v) - j \frac{\pi}{2} + j Si(2\beta_o \bar{a}_v)] \tag{3-34}$$

$$\bar{a}_v = \bar{a}_v^2 (z = \delta) = \delta^2 \sin^2 \Delta + a^2 + \bar{\delta}^2 \tag{3-25}$$

When (3-24), (3-21) and (3-19) are substituted in the integral equation (3-18), then an algebraic equation for the current or voltage results.

$$I_o \Psi_v(\delta)\, e^{-j\beta_o z} = \frac{4\pi}{\zeta_o} \left[\frac{1}{2} V_o - \frac{j\beta_o}{4} p(\delta) \sin^2 \Delta \right] e^{j\beta_o} \quad e^{-j\beta_o z} \tag{3-26}$$

The driving point impedance of the V-antenna is

$$Z_o = \frac{V_o}{I_o} = \frac{\zeta_o}{2\pi} \left[\Psi_v(\delta)\, e^{-j\beta_o \delta} + j \frac{p(\delta)}{I_o} \sin^2 \Delta \right] \tag{3-27}$$

The Input Impedance of a short

V - Transmission Line

Under the conditions that the line spacing is large compared to the wire radius and the line length is large compared with the spacing, simplified expressions are available for the line parameters.

The capacitance per unit length is given by c (z) where

$$c\ (z) = 2\pi\epsilon_0\ /\ k_o\ (z) \tag{3-28}$$

The function k_o (z) is the scalar potential difference between the two conductors and to a high order of approximation may be written as

$$k_o(z) = \int_{\delta}^{\ell} \left(\frac{1}{R_1} - \frac{1}{R_{12}}\right) dz' \tag{3-29}$$

where

$$R_1 = \sqrt{(z - z')^2 + a^2} \tag{3-30}$$

$$R_{12} = \sqrt{(z' - z_v)^2 + a_v^2} \tag{3-31}$$

$$z_v = z \cos \Delta$$

$$a_v^2 = a^2 + w^2 \sin^2 \tag{3-32}$$

By elementary methods the integrals may be evaluated to give the following result

$$\int_{\delta}^{\ell} \frac{1}{R_1}\, dz' = \ln \quad (z - \delta) + \sqrt{(z - \delta)^2 + a^2}$$

$$+ \ln \quad (\ell - z) + \sqrt{(\ell - z)^2 + a^2} \quad - 2 \ln a \tag{3-33}$$

The preceeding integral can be simplified by noting its value at the extremals of z(i.e. $z = \delta$ and $z = \ell$), thus 3-34

$$\text{at } z = \delta, \quad \int_{\delta}^{\ell} \frac{1}{R_1}\, dz' = \ln 2\ell/a, \qquad \ell \gg \delta \gg a$$

$$\text{at } z = \ell \quad \int_{\delta}^{\ell} \frac{1}{R_1}\, dz' = \ln 2\ell/a, \qquad \ell \gg \delta \gg a$$

Within the region of the approximations a simple linear representation for 3-33 is

$$\int_{\delta}^{\ell} \frac{1}{R_1}\, dz' \cong 2\ell/a, \quad \delta \leq z \leq 1 \qquad \text{3-34}$$

Similary the integral for the other conductor is

$$\int_{\delta}^{\ell} \frac{1}{R_{12}}\, dz' = \ln\left[(z_v - \delta) + \sqrt{(z_v - \delta)^2 + a_v^{\,2}}\right] + \ln\left[(\ell - z_v) + \sqrt{(\ell - z_v)^2 + a_v^{\,2}}\right] - 2 \ln a_v \qquad \text{3-35}$$

The integral (3-35) is evaluated at the extremal values in order to find a simple linear approximation.

$$\int_{\delta}^{\ell} \frac{1}{R_{12}}\, dz' \Big|_{z=\delta} = \ln\left[\ell(\cos\Delta + 1)\right] + \ln\left[2\ell\left(\sin^2\frac{\Delta}{2} + \sin\frac{\Delta}{2}\right)\right] - 2 \ln(\ell \sin\Delta) \qquad \text{3-36}$$

$$\int_{\delta}^{\ell} \frac{1}{R_{12}}\, dz' \Big|_{z=\ell} = \ln 2\ell + \ln\left[2\delta\left(\sin^2\frac{\Delta}{2} + \sin\frac{\Delta}{2}\right)\right] - 2 \ln(\delta \sin\Delta) \qquad \text{3-37}$$

A linear function which includes the end points in (3-36), (3-37) and (3-33) is

$$k_o(z) \cong \ln \frac{(2z \sin\frac{\Delta}{2})}{a} - \frac{(\ell - z)}{(\ell - \delta)} \ln\left(\cos^2 \frac{\Delta}{2}\right) + \ln\left(1 + \sin\frac{\Delta}{2}\right) \qquad \text{3-38}$$

A numerical calculation for $z \geq 10\, a$ shows that only the first term in (3-38) is important, thus

$$c(z) \cong 2\pi\mathcal{E}_o \,/\, \ln(\frac{2z \sin\frac{\Delta}{2}}{a}) \qquad 3\text{-}39$$

$$\mathcal{E}_o = 8.85 \times 10^{-12}\ \mathcal{F}/m$$

If the length of the line is small compared with a wavelength, the line can be represented as a lumped element. The total capacitance C is given by

$$C = \int_{\delta}^{\ell} c\,(z)\,dz\,, \qquad 3\text{-}40$$

$$C = 2\pi\mathcal{E}_o \int_{\delta}^{\ell} \frac{1}{\ln\frac{(2z \sin\Delta/2)}{a}} \cdot dz \qquad 3\text{-}41$$

The integral in (3-41) is related to the exponential integral given in Jahnke-Emde, thus

$$C = \frac{2\pi\mathcal{E}_o a}{2\sin(\Delta/2)} \quad \text{li}\ \frac{2\ell}{a}\sin(\Delta/2) - \text{li}\ \frac{2\delta}{a}\sin(\Delta/2) \qquad 3\text{-}42a$$

where

$$\text{li}\,(u) = \text{Ei}\,(\ln x) \qquad 3\text{-}42b$$

The total lumped inductance for a short section of V-transmission line is given by

$$L = \int_{\delta}^{\ell} \ell\,(z)\,dz \qquad 3\text{-}43a$$

where

$\ell(z)$ = inductance per unit length

$$\ell\,(z) = \frac{1}{2\pi\nu_o} \int_{\delta}^{\ell} \frac{1}{R_1} - \cos\Delta \cdot \frac{1}{R_{12}}\ dz' \qquad 3\text{-}44a$$

$$= \frac{1}{2\pi\nu_o}\left[\cos\Delta\ \ln\,(2z \sin\frac{\Delta}{2}) - \ln a\right] \qquad 3\text{-}44b$$

With (3-44b) and (3-43a) the lumped inductance L is readily evaluated.

$$2\pi\nu_o L = \ell \ln\,(2\ell \sin\frac{}{2}) - \ \ln\,(2\ \ \sin\frac{}{2}) - (\ell - \delta) \qquad 3\text{-}45$$

The Input Impedance of an

Arbitrary Length V-Transmission Line

The computation of the input impedance of an arbitrary length V-transmission line is based here on the conventional transmission line equations given by

$$\frac{d^2V}{dz^2} - \left(\frac{d}{dz} \ln \gamma Z_c\right) \frac{dV}{dz} - \gamma^2 V = 0 \qquad 3\text{-}46$$

and

$$\frac{d^2i}{dz^2} - \left(\frac{d}{dz} \ln \frac{Z_c}{\gamma}\right) \frac{di}{dz} - \gamma^2 i = 0 \qquad 3\text{-}47$$

where

V = potential difference

i = current

Z_c = charactenstic impedance

γ = propagation constant

The charactenstic impedance for the V-transmission line is given approximately by (3-39) and (3-45).

$$Z_c = \sqrt{\frac{\ell}{c}} = 120 \ln \frac{2 z \sin \frac{\Delta}{2}}{a} \qquad 3\text{-}48$$

An approximate solution of the transmission line equations is found by linearizing 3-48, thus

$$Z_c = cz + d, \quad z_o \leq z \leq z_1 \qquad 3\text{-}49$$

where $c = \dfrac{120 \ln (z_1/z_o)}{(z_1 - z_o)}$

and $d = \dfrac{120}{(z_1 - z_o)} \quad z_1 \ln\left(\dfrac{2 z_o \sin\frac{\Delta}{2}}{a}\right) - z_o \ln\left(\dfrac{2z \sin\frac{\Delta}{2}}{a}\right.$

With (3-49) in (3-46) and (3-47) the transmission line equations are reduced to

$$\frac{d^2V}{dz^2} - \frac{1}{(z+\bar{z})}\frac{dV}{dz} - \gamma^2 V = 0 \qquad 3\text{-}50$$

$$\frac{d^2i}{dz^2} - \frac{1}{(z+\bar{z})}\frac{di}{dz} - \gamma^2 i = 0 \qquad 3\text{-}51$$

where

$$\bar{z} = \frac{\ln(z_1/z_o)}{z_1 \ln\left[\frac{2z_o\sin(A/2)}{a}\right] - z_o \ln\left[\frac{2z_1 \sin(A/2)}{a}\right]}$$

Assuming an $e^{j\omega t}$ time dependance the solution of (3-50) is

$$V = e^{j\omega t} J_o\left[\gamma(z+\bar{z})\right]$$

$$V(z,t) = \frac{e^{j\omega t}}{2}\left\{H_o^{(1)}\left[\gamma(z+\bar{z})\right] + H_o^{(2)}\left[\gamma(z+\bar{z})\right]\right\}$$

where J_o = Bessel function of the first kind and zeroth order.
$H_o^{(1)}$ = Hankel function of the first kind and zeroth order
$H_o^{(2)}$ = Hankel function of the second kind and zeroth order.

The outward traveling wave solution is concerned only with $H_o^{(1)}$.

Numerical Results

The computation of the driving point impedance of an inifinite length V-antenna consists of two parts. The first is the input impedance of a truncated section and the second part is the contribution of the remaining transmission line section. When the transmission line section is short compared to a wavelength, it can be replaced by a lumped reactance.

The truncated section was chosen to begin at a distance of one-tenth of wavelength from the apex. Although this distance is arbitrary, a tenth wavelength long transmission is easily replaced by a lumped capacitance. The driving point impedance of the truncated antenna section is proportional to ψ_v and the scalar potential contribution which is proportional to $p(\varsigma)$.

The value of ψ_v has been given in terms of related sine and cosine integral functions. An approximate expression has been given for $p(\varsigma)$ and will be compared to numerically integrated values.

Computation of the Input Impedance

The relevant formula is (3-27)

$$Z_o = \frac{\zeta_o}{2\pi}\left[\psi_v(\varsigma)\ e^{-j\beta_o\varsigma} + j\ \frac{p(\varsigma)}{I_o}\ \sin^2\Delta\right]$$

where $\psi_v = \psi_{v1} - \cos\Delta\ \psi_{v2}$

Table I ψ_{v1} and ψ_{v2} for $(\delta/\lambda) = 0.1$

(a/λ)	ψ_{v1}
10^{-2}	2.1910 - j 1.5071
10^{-3}	2.5975 - j 1.5637
10^{-4}	6.8124 - j 1.5693
10^{-5}	9.1150 - j 1.5699
10^{-6}	11.4176 - j 1.5699

apex angle Δ	$\cos\Delta$ ψ_{v2}
20°	0.7570 -j 1.2364
45°	-0.0421 -j 0.6520
60°	-0.1518 -j 0.3366
90°	-0.4673 -j 0.2406
135°	+0.2591 +j 0.1129

The scalar potential term $p(\delta)$ was numerically computed and compared to the approximate value in (3-34). For two sample values the error is about 20 per cent.

Table II, Values of $p(\delta)$ for $(\delta/\lambda) = 0.1$

apex angle Δ	$p(\delta)$ (I.B.M.7094)	$p(\delta)$ from (3-34)
20°	8.166 -j 10.52	5.209 -j 10.42
45°	0.72 -j 2.63	
60°	0.10 -j 1.507	
90°	-0.178 -j 0.639	-0.325 -j 0.544
135°	-0.018 -j 0.276	
180°	-0.178 -j 0.205	

The values given in Tables I and II were used to compute the input resistance and impedance of the truncated V-antenna section. Representative values as sumarized below

Table III, Z_{in} and Y_{in} for $(\delta/\lambda) = 0.1$

a/λ	Δ	Z_{in} (ohms)	Y_{in} (10^{-3} mho)
10^{-2}	20°	133.80 -j 7.8180	7.45 + j 0.435
	45°	154.94 -j 101.71	4.51 +j 2.96
	60°	138.05 -j 137.99	5.09 +j 4.91
	90°	120.52 -j 196.6	2.26 +j 3.70
	135°	43.03 -j 150.37	0.18 +j 0.61
10^{-3}	20°	244.40 -j 90.85	3.594 +j 1.336
	45°	267.77 -j 183.12	2.545 +j 1.740
	60°	250.89 -j 219.39	2.258 +j 1.974
	90°	233.35 -j 261.04	1.903 +j 2.129
	135°	155.56 -j 231.78	1.993 +j 2.975
10^{-4}	20°	307.50 -j 137.08	2.714 +j 1.209
	45°	330.81 -j 229.30	2.042 +j 1.415
	60°	319.72 -j 265.58	1.852 +j 1.536
	90°	296.39 -j 292.18	1.711 +j 1.686
	135°	218.61 -j 277.96	1.747 +j 2.221
10^{-5}	20°	467.84 -j 253.50	1.652 +j 1.248
	45°	491.16 -j 345.71	1.361 +j 0.958
	60°	480.07 -j 382.90	1.275 +j 1.014
	90°	456.73 -j 412.95	1.204 +j 1.089
	135°	378.91 -j 394.38	1.266 +j 1.318

a/λ	Δ	Z_{in} (ohms)	Y_{in}(10^{-3}mho)
10^{-6}	20^{o}	597.96 -j 334.67	1.273 +j 0.712
	45^{o}	602.95 -j 426.89	1.104 +j 0.782
	60^{o}	591.86 -j 463.17	1.047 +j 0.820
	90^{o}	568.53 -j 494.12	0.988 +j 0.870
	135^{o}	490.75 -j 475.54	1.050 +j 1.018

Representative values of the cappacitive correction C_T were computed numerically from (3-28) and (3-29) and are compared for one case to the approximate analytical value of (3-42).

Table IV

Lumped Capacitive correction $C_T = \int_{\delta}^{\ell} c(w)\, dw$

$(\ell/\lambda) = 0.1, \quad \varepsilon_o = 8.857 \times 10^{-12}$ S/m

(δ/λ)	(a/λ)	Δ	$(C_T/2\pi\varepsilon_o)$
10^{-4}	10^{-4}	20^{o}	7.325×10^{-3}
		45^{o}	6.665×10^{-3}
		60^{o}	6.493×10^{-3}
		90^{o}	6.301×10^{-3}
		135^{o}	6.177×10^{-3}
		150^{o}	6.157×10^{-3}
10^{-2}	10^{-3}	20^{o}	1.816×10^{-2}
		45^{o}	1.446×10^{-2}
		60^{o}	1.367×10^{-2}
		90^{o}	1.288×10^{-2}
		135^{o}	1.241×10^{-2}
		150^{o}	1.234×10^{-2}

(δ/λ)	(a/λ)	Δ	$(C_T/2\pi\epsilon_o)$
10^{-3}	10^{-4}	20^o	1.095×10^{-2}
		45^o	0.949×10^{-2}
		60^o	0.914×10^{-2}
		90^o	0.875×10^{-2}
		135^o	0.848×10^{-2}

Comparison of Numerical and Analytical values for $(\delta/\lambda) = 6.7 \times 10^{-2}$, $(a/\lambda) = 6.7 \times 10^{-3}$

Δ	(Numer.) $(C_T/2\pi\epsilon_o)$	(Anal.)
20^o	1.747×10^{-2}	2.23×10^{-2}
45^o	1.365	1.44
60^o	1.303	1.31
90^o	1.246	1.30
135^o	1.216	1.04

A specific example of the theory is concerned with a satelite borne V-antenna (see N.A.S.A. drawing titled "RAE LIBRATION DAMPER ARRANGEMENT). The relevant parameters are

Feed point distance $\delta = 0.65406$ meters

Radius of antenna $a = 6.33 \times 10^{-3}$ meters

apex angle $\Delta = 60^o$

Table V - Summary of Important Parameters for RAE-V-antenna

Fre-(mc/s)	λ(m)	(a/λ)	(δ/ℓ)
1.31	229.00	2.7729×10^{-5}	2.8561×10^{-3}
2.20	136.36	4.6567×10^{-5}	4.7965×10^{-3}
3.43	87.46	7.2604×10^{-5}	7.4784×10^{-3}
4.70	63.83	9.9483×10^{-5}	1.0247×10^{-2}
6.55	45.80	1.3864×10^{-4}	1.4281×10^{-2}
9.18	32.68	1.9430×10^{-4}	2.0014×10^{-2}

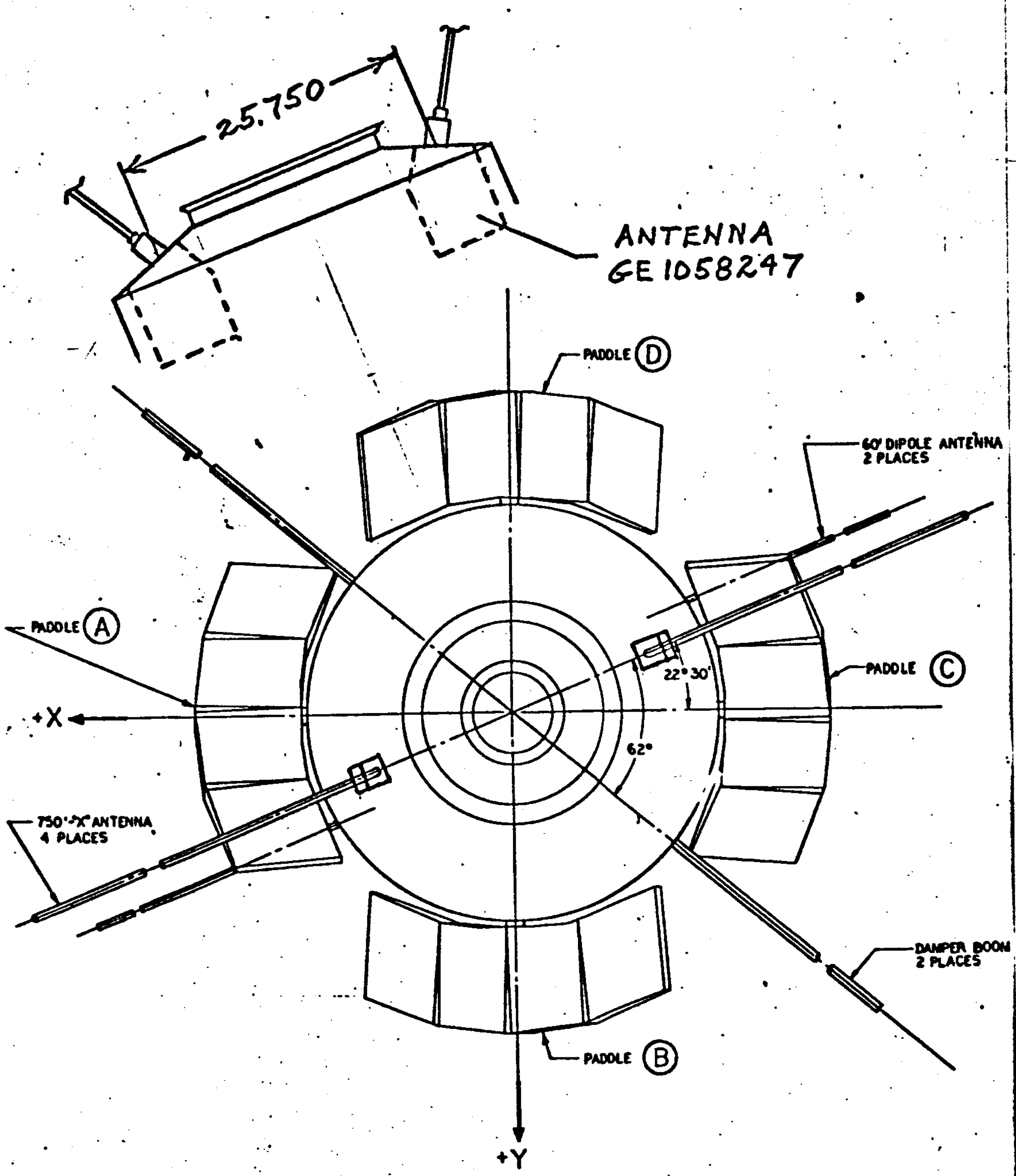


Figure 2. RAE libration damper arrangement.

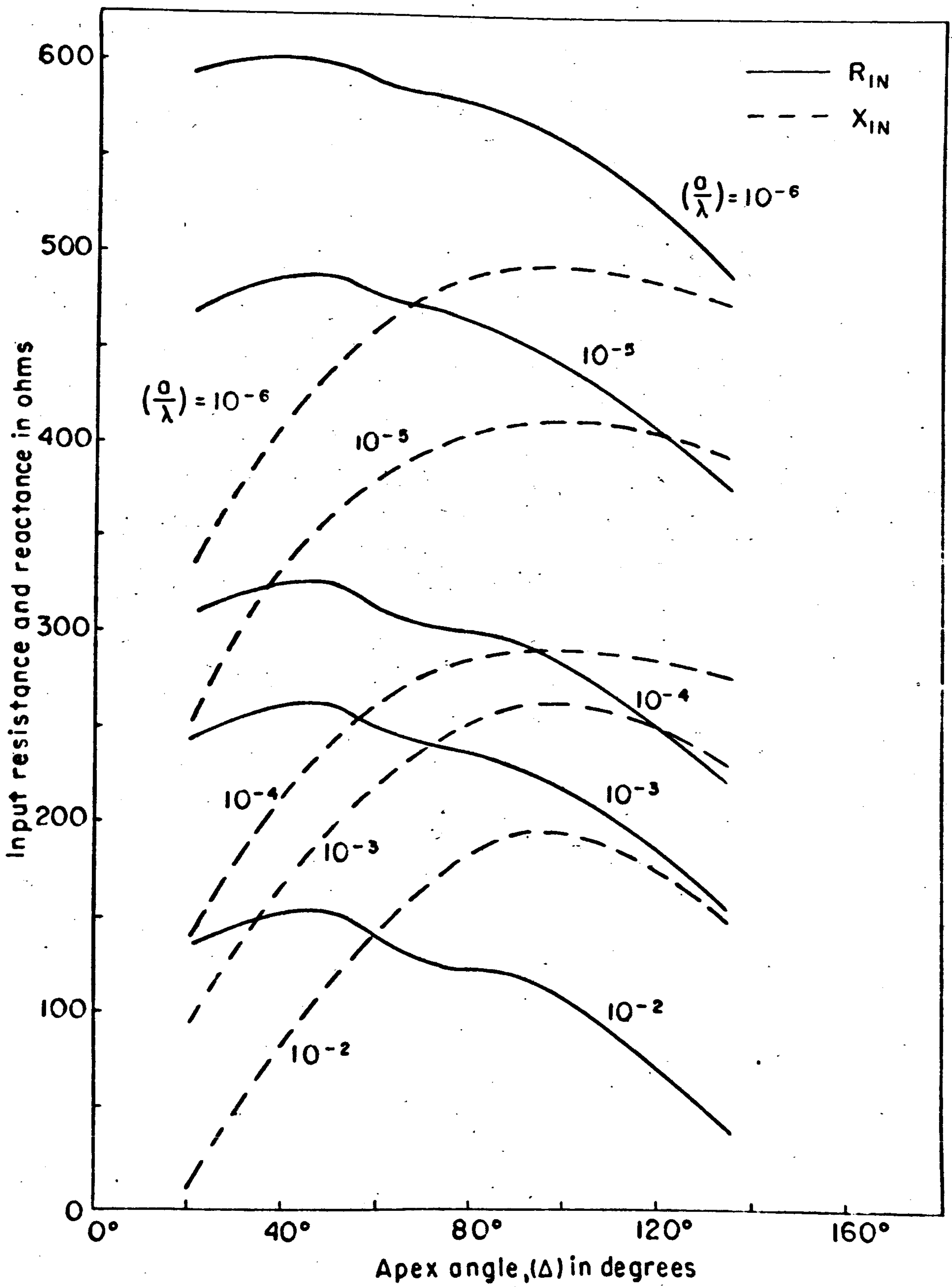


Figure 3. Input impedances corresponding to Table III.

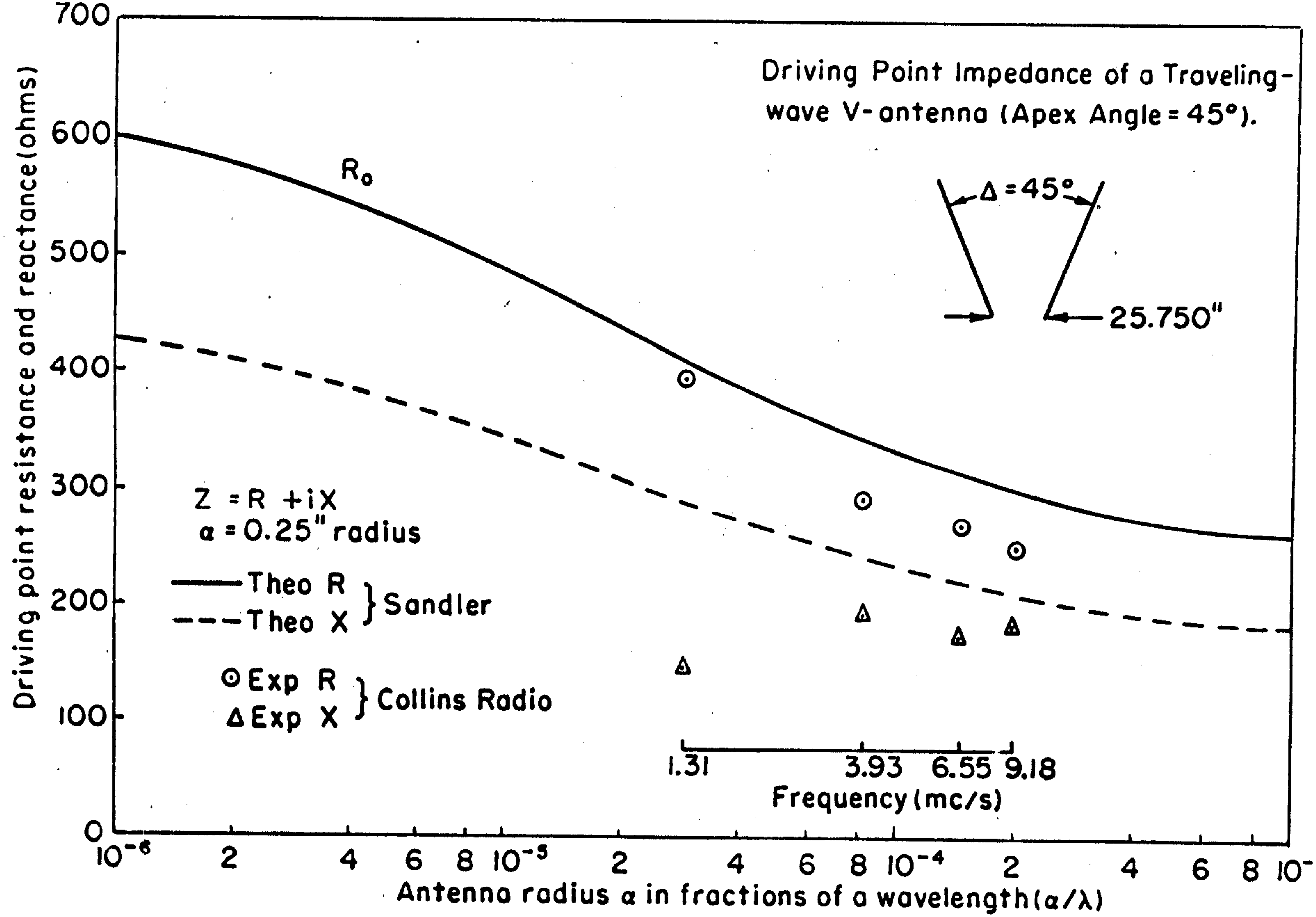


Fig. 4. Comparison of Theory and Experiment for Traveling Wave V-antenna.

Impedances and Admittances

(mc/s)	Z_{in} (ohms)	Y_{in}(x 10^{-3})
1.31	424.5 -j 355.2	1.385 +j 1.159
2.20	398.5 -j 336.3	1.466 +j 1.237
3.43	377.0 -j 316.1	1.558 +j 1.306
4.20	361.4 -j 309.4	1.597 +j 1.367
6.55	345.7 -j 297.9	1.660 +j 1.430
9.18	329.2 -j 285.3	1.735 +j 1.503

Transmission line correction and final Driving point impedance and admittance

mc/s	C_T(x 10^{-3}mho)	Y_o(x 10^{-3}mho)	Z_o(ohms)
1.31	.007	1.385 +j 1.166	422.5 -j 355.7
2.20	.012	1.466 +j 1.249	395.2 -j 336.7
3.43	.018	1.558 +j 1.324	372.6 -j 316.7
4.70	.023	1.597 +j 1.390	356.2 -j 310.1
6.55	.036	1.660 +j 1.466	338.4 -j 298.9
9.18	.049	1.735 +j 1.552	-320.1 -j 286.3

The computational procedure is straight forward. The input admittance of the truncated section is first computed. The lumped transmission line admittance is then added to give the actual driving point admittance.

Appendix

Evaluation of integrals occuring in the Hallen integral equation for the Traveling-Wave antenna.

Consider just the following integral

$$\int_b^c \exp \frac{p(\pm j\,{}_0 u - j\sqrt{u^2 + a^2})}{(u^2 + a^2)^{1/2}}\, du = J_1 \qquad \text{(A-1)}$$

In J_1 let

$$\eta = \pm u - \sqrt{u^2 + a^2} \qquad \text{(A-2)}$$

then $$d\eta = \pm 1 - \frac{u}{(u^2 + a^2)^{1/2}} du = \frac{\pm(u^2 + a^2)^{1/2} - u}{(u^2 + a^2)^{1/2}} du$$

$$d\eta = \mp \eta\, du / \sqrt{(u^2 + a^2)}^{1/4} \qquad \text{(A-3)}$$

Since $(du/\sqrt{u^2 + a^2})$ appears in (A-1)

$$\mp \frac{d\eta}{\eta} = \frac{du}{(u^2 + a^2)^{1/2}} \qquad \text{(A-4)}$$

With (A-4) in (A-1) with (A-2)

$$= \mp \int_{\pm b - \sqrt{b^2+a^2}}^{\pm c - \sqrt{c^2+a^2}} \frac{e^{j\beta_o \eta}}{\eta}\, d\eta \qquad \text{(A-5)}$$

With the minus sign in (A-1) let $J_1(-) = \psi_o$

$$\psi_o = \int_{-a}^{-\infty} \frac{e^{j\beta_o \eta}}{\eta} = \int \frac{e^{-j\beta_o \eta}}{\eta}\, d\eta \qquad \text{(A-6)}$$

$$\psi_o = \text{Ci}\,(\beta_o \eta) - j\,\text{Si}\,(\beta_o \eta) \qquad \text{(A-7)}$$

where

$$Ci(x) = -\int_x^{\infty} \frac{\cos t}{t}\, dt \tag{A-8}$$

and

$$Si(x) = \int_0^{x} \frac{\sin t}{t}\, dt \tag{A-9}$$

Since Ci $(\infty) = 0$ and Si $(\infty) = \pi/2$, then for ψ_o the result is

$$\psi_0 = - Ci\ (\beta_o a) - j\ (\pi/2 - Si\ (\beta_o a)\ ) \tag{A-10}$$

with $(\beta_o a) \ll 1$

$$\psi_0 \simeq -\ln \frac{1}{\gamma \beta_o a} - j\ (\pi/2) \tag{A-11}$$

where $\gamma = 1.781$

$\ln \gamma = 0.5772$

Consider now the evaluation of $\psi_v(\delta)$ which appears in (21)

$$\psi_v(\delta) = \int_{\delta}^{\infty} I_{1z}(z')\ K_v\ (\delta, z')\ dz' \tag{A-12}$$

where

$$K_v(z_1 z') = \frac{e^{-j\beta_o R1}}{R_1} - \cos\Delta \frac{e^{-j\beta_o R12}}{R_{12}} \tag{A-13}$$

Let $\psi_v = \psi_{v1} - \cos\Delta\ \psi_{v2}$ where ψ_{v1} is the part of ψ_v which contains R_1 and ψ_{v2} is the part which contains R_2. In ψ_v the substitution $u = z-z'$ is made in $R_1 = \sqrt{(z-z')^2 + a^2}$ when $z > z'$ and the substitution $u = z'-z$ when $z' < z$. Thus for ψ_{v1}

$$\psi_{v1}(z) = \int_{\delta}^{\infty} I(z') \frac{e^{-j\beta_o R_1}}{R_1} dz' \qquad \text{(A-14)}$$

$$_{v1}(z) = \left[\int_{0}^{(z-\delta)} I(z-u) \frac{e^{-j\beta_o (u^2 + a^2)^{1/2}}}{(u^2 + a^2)^{1/2}} du\right.$$

$$+ \int_{0}^{\infty} I(z+u) \frac{e^{-j\beta_o (u^2 + a^2)^{1/2}}}{(u^2 + a^2)^{1/2}} du \qquad \text{(A-15)}$$

The function $\psi_v(z)$ is approximated by its value at $z = \delta$. With the assumption $I_z(z) = I_o e^{-j\beta_o z}$ then (A-15) is given by (A-10).

$$\psi_{v1}(\delta) = -Ci({}_o a) - j(\pi/2 - Si({}_o a)) \qquad \text{(A-16)}$$

The evaluation of ψ_{v2} follows from the same substitution used for ψ_{v1}, thus

$$\psi_{v2}(z) = \int_{\delta}^{\infty} I_{1z}(z') \frac{e^{-j\beta_o R12}}{R12} dz'$$

$$= \int_{\bar{\delta}}^{\infty} I(u + z_v) \frac{e^{-j\beta_o \sqrt{u^2 + a^2}}}{(u^2 + a^2)^{1/2}} du \qquad \text{(A-17)}$$

where $R_{12} = \sqrt{(z - z_v)^2 + a_v^2}$, $z_v = z \cos\Delta$ (A-18)

$$a_v^2 = a^2 + z^2 \sin^2 \Delta$$

and $\bar{\delta} = \delta(1 - \cos\Delta)$ (A-19)

Note that in (A-17) it has been assumed that $z' > z_v$ with $I(z) = I_o e^{-j\beta_o z}$ then

$$\psi_{v2}(z) = e^{-j\beta_o z_v} \int_{\delta_v}^{\infty} \frac{e^{-j\beta_o \eta}}{\eta} d\eta \qquad \text{(A-20)}$$

$$\Psi_{v2}\,(z = \mathcal{S}) = -\mathrm{Ci}\,(\beta_o \mathcal{S}_v) - j(\pi/2 - \mathrm{Si}\,(\beta_o \mathcal{S}_v)) \qquad \text{(A-21)}$$

where
$$\mathcal{S}_v = \bar{\mathcal{S}} + \sqrt{\bar{\mathcal{S}}^2 + a_v^{\,2}} \qquad \text{(A-22)}$$

for $\mathcal{S}^2 \gg a^2$
$$\mathcal{S}_v \cong 2\mathcal{S}\left[\sin^2(\Delta/2) + \sin(\Delta/2)\right] \qquad \text{(A-23)}$$

The integrals which appear in the potential contribution in the integral equation are more difficult to evaluate. The integral $p(\mathcal{S})$ in (16) is given by $p(z)$ by replacing z with $\mathcal{S}$ and making the substitutions $u = z' - z_v = z' - \mathcal{S}\cos\Delta$, and $I_z(z) = I_o\, e^{-j\beta_o z}$

$$p(\mathcal{S}) = \frac{1}{\beta_o}\int_{\mathcal{S}}^{\infty} e^{-j\beta_o z'}\,\frac{e^{-j\beta_o R_{12}}}{(R_{12\mathcal{S}})^3}\,(1 + j\beta_o R_{12\mathcal{S}})\,z'\,dz' \qquad \text{(A-24)}$$

where

$$R_{12\mathcal{S}} = \sqrt{(z' - \mathcal{S}_v)^2 + a_v^{\,2}} \qquad \text{(A-25)}$$

$$\mathcal{S}_v = \mathcal{S}\cos\Delta$$

$$a_v^{\,2} = a^2 + \mathcal{S}^2\sin^2\Delta \qquad \text{(A-26)}$$

After the substitution $u = z' - \mathcal{S}_v = z' - \mathcal{S}\cos\Delta$

$$p(\mathcal{S}) = e^{-j\beta_o \mathcal{S}\cos\Delta}\left\{\frac{1}{\beta_o}\int_{\bar{\mathcal{S}}}^{\infty} u e^{-j\beta_o u}\,\frac{e^{-j\beta_o (u^2 + a^2)^{1/2}}}{(u^2 + a_v^{\,2})^{3/2}}\,du\right.$$

$$\left. + j\int_{\mathcal{S}}^{\infty} u e^{-j\beta_o u}\,\frac{e^{-j\beta_o (u^2 + a_v^{\,2})^{1/2}}}{(u^2 + a_v^{\,2})}\,du\right\}$$

$$+ \delta \cos\Delta \, e^{-j\beta_o \delta \cos\Delta} \left\{ \int_{\bar{\delta}}^{\infty} \frac{e^{-j(\beta_o u)} e^{-j\,o\sqrt{}}}{(\sqrt{})^3} \, du \right.$$

$$\left. + j \int_{\bar{\delta}}^{\infty} \frac{e^{-j\beta_o u} e^{-j\beta_o \sqrt{}}}{(u^2 + a_v^2)} \, du \right\} \qquad \text{(A-27)}$$

The substitution $v^2 = u^2 + a_v^2$ is made in (A-27), thus $\frac{v\,dv}{v^2 - a_v^2} = du$

$$p(\varsigma) = e^{-j\beta_o \delta \cos\Delta} \left\{ \int_{\sqrt{\bar{\delta}^2 + a_v^2}}^{\infty} e^{-j\beta_o (v^2 - a_v^2)^{1/2}} \frac{e^{-j\beta_o v}}{v^2} \, dv \right.$$

$$+ j \int_{\sqrt{\bar{\delta}^2 + a_v^2}}^{\infty} \frac{e^{-j\beta_o \sqrt{v^2 - a_v^2}}}{v} e^{-j\beta_o v} \, dv$$

$$+ \delta \cos\Delta \, e^{-j\beta_o \delta \cos\Delta} \left\{ \frac{1}{\beta_o} \int_{\sqrt{\bar{\delta}^2 + a_v^2}}^{\infty} e^{-j\beta_o \sqrt{u^2 - a_v^2}} \, v \frac{e^{-j\,o v}}{v^3} \, dv \right.$$

$$\left. + j \int_{\sqrt{\bar{\delta}^2 + a_v^2}}^{\infty} e^{-j\beta_o \sqrt{u^2 - a_v^2}} \, v \frac{e^{-j\beta_o v}}{v^2 (v^2 - a_v^2)^{1/2}} \, dv \right\}. \qquad \text{(A-28)}$$

where $\varsigma = \delta(1 - \cos\Delta)$ (A-29)

Since over the range of interest $v^2 \gg a_v^2 = a^2 + \delta^2 \sin^2\Delta$ except near the lower limit (where the contribution is small) the following approximation is made

$$e^{-j\beta_o (v^2 - a_v^2)^{1/2}} = e^{-j\beta_o v}$$

and $$\frac{v}{(v^2 - a_v^2)^{1/2}} = 1 + \frac{1}{2}\frac{a_v^2}{v^2} + \dots \tag{A-30}$$

With (A-30) the approximate value for (A-28) is

$$p(\delta) = e^{-j(\beta_o \delta \cos\Delta)} \left\{ \frac{1}{\beta_o} \int_{\bar{a}_v}^{\infty} \frac{e^{-j2\beta_o v}}{v^2}\, dv + j\int_{\bar{a}_v}^{\infty} \frac{e^{-j2\beta_o v}}{v}\, dv \right.$$

$$+ \delta\cos\Delta\, e^{-j\beta_o \delta\cos\Delta} \left\{ \frac{1}{\beta_o} \int_{\bar{a}_v}^{\infty} \frac{e^{-j2\ o v}}{v^3}\, dv \right.$$

$$+ j\int_{\bar{a}_v}^{\infty} \frac{e^{-j2\beta_o v}}{v^2}\, dv + \frac{1}{2}\frac{a_v^2}{\beta_o} \int_{\bar{a}_v}^{\infty} \frac{e^{-j2\beta_o v}}{v^5}$$

$$\left. + \int_{\bar{a}_v}^{\infty} \frac{e^{-j2\beta_o v}}{v^4}\, dv \right\} \tag{A-31}$$

where $\bar{a}_v = \sqrt{\bar{\delta}^2 + a_v^2}$

After integration by parts

$$p(\delta) = e^{-j(\beta_o \delta \cos\Delta)} \left\{ \frac{e^{-j2\beta_o \bar{a}_v}}{\beta_o \bar{a}_v} -j \left[-Ci\,(2\beta_o \bar{a}_v) \right. \right.$$

$$\left. -j\,(\pi/2 - Si(2\beta_o \bar{a}_v) \right] + \frac{\delta \cos}{2\beta_o \bar{a}_v{}^2} e^{-j2\beta_o \bar{a}_v}$$

$$+ \frac{\delta\, a_v{}^2 \cos\Delta}{2} \left\{ e^{-j2\beta_o \bar{a}_v} \quad \frac{1}{4\beta_o \bar{a}_v{}^4} + \frac{j}{6\bar{a}_v{}^3} + \frac{\beta_o}{6\bar{a}_v{}^2} - \frac{j\beta_o{}^2}{3\,\bar{a}_v} \right.$$

$$- \frac{2\beta_o{}^3}{3} \left[-Ci(2\beta_o \bar{a}_v) -j\,(\pi/2 - Si\;\; 2\beta_o \bar{a}_v) \right] \qquad \text{(A-32)}$$

Where $\bar{a}_v{}^2 = \bar{a}_v{}^2\,(z = \delta) = \delta^2 \sin^2\Delta + a^2 + \delta^2$

Driving-Point Impedance and Current for Long Resonant Antennas

Most analyses of the distribution of current and impedances of cylindrical antennas yield accurate results only for antennas that are not much longer than about two wavelengths [1]. An important exception is the work of Wu [2] which utilizes the Wiener-Hopf technique to analyze very long antennas. However, no simple formula for the current is given. For some applications, such as problems in superposition, a simple trigonometric representation of the current and the corresponding driving-point impedance of very long antennas is useful. Such a representation has been obtained for resonant antennas.

An examination of measurements made on long antennas [3-6] shows that for resonant antennas [6] the current is approximately trigonometric in form and nearly constant in amplitude. However, no such simple form obtains for the currents along antiresonant antennas [6],

The desired simple formula must be an approximate solution of the following integral equation for the current in a cylindrical antenna, of length 2h and radius a, center driven by a delta-function generator V_o:

$$\int_{-h}^{h} I_z(z')K(z,z')dz' = -j\frac{4\pi}{\zeta_o}(C \cos\beta_o z + \frac{1}{2} V_o \sin\beta_o|z|) \qquad (1)$$

where $K(z,z') = R^{-1}\exp(-j\beta_o R)$ with $R = \sqrt{(z-z')^2+a^2}$, ζ_o = 120 ohms, $\beta_o = 2\pi/\lambda$, and a is the radius of the antenna. The experimental results [6] shown in Fig. 1 suggest that the part of the current that gives rise to the conductance is cosinusoidally distributed when $\beta_o h = n\pi/2$, n odd, with a maximum at z = 0. The part of the current that determines the susceptance resembles a cosine with its maximum shifted by an angle $\beta_o s$ at the base. Values of $\beta_o s$ as obtained from Altshuler's curves are $\beta_o s = 0.14\pi$, 0.20π, 0.23π, 0.25π, 0.25π, 0.25π, respectively, for $\beta_o h = 1.5\pi$, 2.5π, 3.5π, 4.5π, and 5.5π. An approximate value, especially for longer antennas, is $\beta_o s = \pi/4$ or 45°. Since a cosine with a phase shift

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Fig. 1

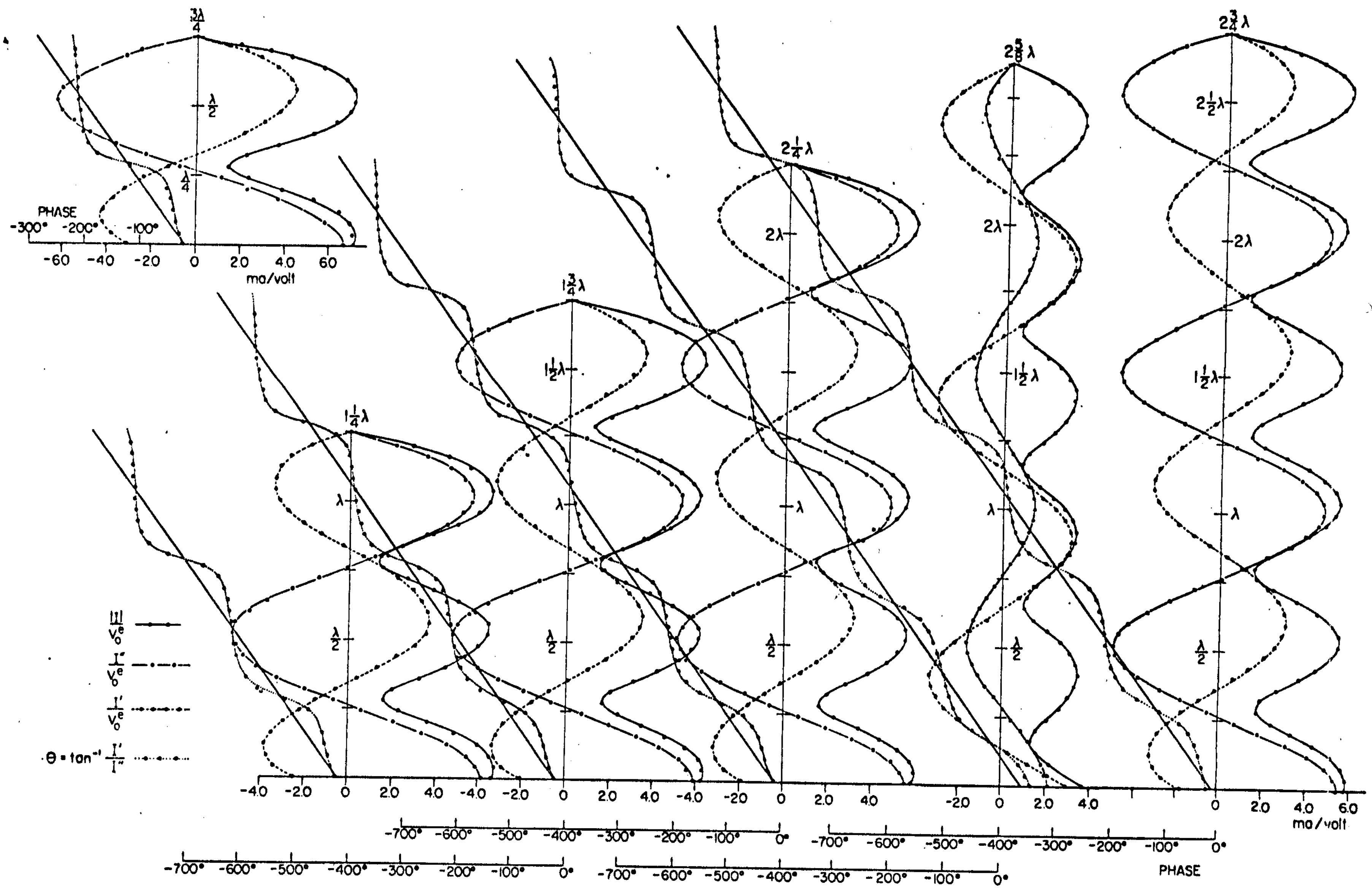

Figure 1. Currents on resonant antennas.

is not zero at $z = \pm h$, this distribution must be modified near the ends so that it vanishes at $z = \pm h$. A suggested approximate form for the current in the range $0 \leq z \leq h$ is

$$I_z(z) = V_o[jB \sec\beta_o s \cos\beta_o(z-s) + G \cos\beta_o z + f(z)] \tag{2}$$

where B is the susceptance and G the conductance of the antenna. Note that symmetry requires that $I(-z) = I(z)$. In (2) $f(z)$ is to be constructed so that $I_z(z)$ vanishes at $z = \pm h$. The distribution $\cos\beta_o(z-s)$ is valid in the range $0 \leq z \leq h$ whereas $f(z)$ must be non-zero only in the range $(h-s) \leq z \leq h$. Since the coefficient of $\cos\beta_o z$ must be complex in the solution of (1) whereas G in (2) is real, a somewhat generalized form of the current is required. It is

$$I_z(z) = jA\{-\cos\beta_o(z-\ell) + H(z-\ell)[\cos\beta_o(z-\ell) - \frac{\cos\beta_o z}{\cos\beta_o \ell}]\} + (D_r + jD_i)\cos\beta_o z \tag{3}$$

This is valid in the range $0 \leq z \leq h$. In (3) $H(t)$ is the Heaviside function defined by $H(t) = 1,\ t \geq 0;\ H(t) = 0,\ t < 0$. In (3) the phase shift $\beta_o \ell$ is related to the observed shift $\beta_o s$ in the quadrature component of the current by the relations, $A \sin\beta_o \ell = -V_o B \sin\beta_o s \sec\beta_o s$ and $A \cos\beta_o \ell = D_i - V_o B$. These lead to

$$\cos \beta_o \ell - \sin \beta_o \ell \cot \beta_o s = D_i/A \tag{4}$$

or, with the approximate value $\beta_o s = \pi/4$, to

$$\cos \beta_o \ell - \sin \beta_o \ell = D_i/A. \tag{5}$$

The first step in the solution of (1) is the rearrangement of the right-hand side into a form which resembles the current in (3). Thus

$$\int_{-h}^{h} I_z(z')K(z,z')dz' = -j\frac{4\pi}{\zeta_o}[C_1 \cos\beta_o z + \frac{V_o}{\sin\beta_o \ell} \cos\beta_o(z-\ell)] \tag{6}$$

The next step is to substitute the current (3) in (6). The only part of the resulting integral which behaves like $\cos\beta_o(z-\ell)$ may be shown to be (a discussion of the integrals involved in (6) is given in Reference 7)

$$\mathrm{Re} \int_{-h}^{h} \cos\beta_o(z'-\ell)K(z,z')dz' \sim \cos \beta_o(z-\ell). \tag{7}$$

If, with (7) the coefficients of $\cos\beta_o(z-\ell)$ in (6) are equated and higher-order terms are neglected, A is given by

$$A = \frac{4\pi}{\zeta_o} \frac{V_o}{\Psi_{CR}(\ell)\ \sin\beta_o\ell} \tag{8}$$

where

$$\Psi_{CR}(\ell) = \mathrm{Re}[\cos\beta_o\ell\ C_a(h,\ell) + \sin\beta_o\ell\ S_a(h,\ell)] \tag{9}$$

and

$$C_a(h,z) = \int_{-h}^{h} \cos\beta_o z' \frac{e^{-j\beta_o R_1}}{R_1} dz' \qquad S_a(h,z) = \int_{-h}^{h} \sin\beta_o|z'|\frac{e^{-j\beta_o R_1}}{R_1} dz' \tag{10}$$

The constant $\Psi_{CR}(\ell)$ is the proportionality factor in (7). Thus, the integral is replaced by a trigonometric function with the amplitude $\Psi_{CR}(\ell)$, which is evaluated at a point of maximum current, $z = \ell$.

The coefficient $D = D_r + jD_i$ in (3) is determined by evaluating the integral equation at $z = h$ with $\beta_o h = \frac{n\pi}{2}$ with n odd. With (10) the result is

$$-jA \int_{-(h-\ell)}^{h-\ell} \cos\beta_o(z'-\ell)K(h,z')dz' + D\int_{-h}^{h} \cos\beta_o z'K(h,z')dz' - j\frac{A}{\cos\beta_o\ell}\left(\int_{-h}^{h} - \int_{-(h-\ell)}^{h-\ell}\right)\cos\beta_o z'K(h,z')dz'$$
$$= -j\frac{4\pi}{\zeta_o}\frac{V_o}{\sin\beta_o\ell}\cos\beta_o(h-\ell) = -j\frac{4\pi}{\zeta_o}V_o\sin\frac{n\pi}{2} \tag{11a}$$

since $\beta_o h = \frac{n\pi}{2}$ with n odd. Or, in a compact form,

$$-jA\Psi_{C1}(h) + D\Psi_{C2}(h) - jA\Psi_C^1 = -jA\Psi_{CR}(\ell)\ \sin\beta_o\ell\ \sin\frac{n\pi}{2} \tag{11b}$$

where, with $h = n\lambda/4$, n odd,

$$\Psi_{C1}(h) = \cos\beta_o\ell\ C_a(h-\ell,h) + \sin\beta_o\ell\ S_a(h-\ell,h); \tag{12}$$

$$\Psi_{C2}(h) = C_a(h,h); \qquad \Psi_C^1 = [C_a(h,h)-C_a(h-\ell,h)]\sec\beta_o\ell \tag{13}$$

The D coefficient is given by (11) with n odd in $\beta_o h = n\pi/2$. Thus

$$D = -jA[\Psi_{C1}(h) + \Psi_C^1 - \Psi_{CR}(\ell)\sin\beta_o\ell\ \sin\frac{n\pi}{2}]/\Psi_{C2}(h) \tag{14}$$

The expression (3) has three unknown parameters, the coefficients A, $D = D_r + jD_i$, and the phase shift $\beta_o\ell$. With the observed phase shift $\beta_o s$ known, the phase shift $\beta_o\ell$ is found by equating (4) to the value of (D_i/A) given by (14). The result is the transcendental equation

$$\frac{D_1}{A} = -\mathrm{Re}\left\{\frac{[\Psi_{C1}(h) + \Psi_C^1 - \Psi_{CR}(\ell)\sin\beta_o\ell\,\sin\frac{n\pi}{2}]}{\Psi_{C2}(h)}\right\}\cos\beta_o\ell - \sin\beta_o\ell\,\cot\beta_o s. \qquad (15)$$

The value of $\beta_o\ell$ given by (15) is then used to compute A in (8) and D in (1). Thus, all parameters can be computed explicitly.

The transcendental equation (15) was programmed on the IBM 7094 computer with $\beta_o s = \frac{\pi}{4}$. The Newton-Raphson method was used with phase angles restricted to the first quadrant. The relevant integrations were performed by a Romberg method in which the maximum error can be limited to a predetermined value. The results are valid to four significant figures. The driving-point impedance and admittance and the current were calculated for $\Omega = 2\ln\frac{2h}{a} = 10$ and 20 in the range $0.25 \leq \frac{h}{\lambda} \leq 10.75$. The values of Z_o, Y_o, A, D and $\frac{\ell}{\lambda}$ are given in the Table. The complete current can be constructed from this table with (3).

The present theory is comparable in accuracy with the second-order King-Middleton theory [1] and the long antenna theory of Wu [2]. For example, the King-Middleton theory for $\Omega = 10$, $\frac{h}{\lambda} = 0.75$ gives $Z_o = 127.6 + j43$; the present theory $Z_o = 118.9 + j39.77$; for $\Omega = 10$, $\frac{h}{\lambda} = 0.25$ the King-Middleton [8] theory gives $Y_o = (9.27 - j4.62)\times10^{-3}$, the present theory $Y_o = (10.34 - j4.79)\times10^{-3}$. For longer antennas the theory of Wu [2] gives $Z_o = 191.35 + j50.48$ for $\Omega = 20$, $\frac{h}{\lambda} = 5.25$, and $Z_o = 218.75 + j41.95$ for $\Omega = 20$, $\frac{h}{\lambda} = 10.75$. The present theory gives, respectively, $Z_o = 185.93 + j49.14$ and $Z_o = 220.86 + j50.82$. Some of the error in the present theory is due to the fact that the phase angle $\beta_o s$ is not exactly equal to $\frac{\pi}{4}$ and is also generally a function of Ω and $\frac{h}{\lambda}$. The variation of this phase angle is shown in Fig. 2 based on the experimental work of Altshuler [9] and the theory of Wu [2].

A sample theoretical current for $\Omega = 10$ and $\frac{h}{\lambda} = 0.75$ is shown in Fig. 3 together with experimental results by Altshuler [6]. The agreement is quite good. It has been verified that comparable results obtain when $\frac{h}{\lambda} = 0.25$. A comparison of the new approximation with the more exact theory of Wu for $\Omega = 20$ and $\frac{h}{\lambda} = 5.25$ is given in Fig. 4. The excellent agreement indicates that a simple approximate representation of the current and impedance of long resonant antennas has been determined.

Mrs. Patricia Comella of NASA Goddard Space Flight Laboratory assisted in programming the new theory. The impedance and current predicted by the theory of Wu was programmed by Dr. C. L. Chen of Harvard University.

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Table of Driving-Point Impedance, Admittance and Current Parameters for Resonant Antennas

A, D and Y_o in millimhos, Z_o in ohms

$\Omega = 10$

$\frac{h}{\lambda}$	$\frac{\ell}{\lambda}$	A	D	Y_o	Z_o
0.25	0.084	3.92	10.34 - j1.41	10.34 - j4.80	79.5 + j36.9
0.75	0.125	3.57	7.56 - j0.00	7.56 - j2.53	119.0 + j39.8
1.25	0.143	3.79	6.44 + j0.62	6.44 - j1.73	144.7 + j38.9
1.75	0.154	4.09	5.72 + j1.06	5.72 - j1.25	166.8 + j36.4
2.25	0.162	4.43	5.17 + j1.43	5.17 - j0.90	187.8 + j32.7
2.75	0.167	4.82	4.69 + j1.78	4.69 - j0.62	209.6 + j27.6
3.25	0.171	5.19	4.29 + j2.09	4.29 - j0.38	231.5 + j20.8
3.75	0.174	5.59	3.91 + j2.39	3.91 - j0.18	255.5 + j12.0
4.25	0.176	6.01	3.55 + j2.69	3.55 - j0.00	282.0 + j 0.3
4.75	0.178	6.45	3.20 + j2.98	3.20 + j0.16	311.9 - j15.5
5.25	0.179	6.92	2.86 + j3.28	2.86 + j0.31	346.1 - j37.3
5.75	0.181	7.41	2.51 + j3.58	2.51 + j0.45	385.7 - j68.5

$\Omega = 20$

$\frac{h}{\lambda}$	$\frac{\ell}{\lambda}$	A	D	Y_o	Z_o
0.25	0.045	3.19	10.08 - j2.16	10.08 - j5.23	78.2 + j40.6
0.75	0.072	2.23	7.70 - j1.03	7.70 - j3.04	112.3 + j44.4
1.25	0.085	2.00	6.86 - j0.70	6.86 - j2.42	129.7 + j45.8
1.75	0.094	1.90	6.37 - j0.52	6.37 - j2.09	141.7 + j46.6
2.25	0.101	1.84	6.03 - j0.40	6.03 - j1.88	151.1 + j47.2
2.75	0.106	1.80	5.78 - j0.31	5.78 - j1.73	158.9 + j47.6
3.25	0.110	1.78	5.57 - j0.24	5.57 - j1.62	165.6 + j48.0
3.75	0.052	3.36	8.68 - j2.11	8.68 - j5.29	84.0 + j51.2
4.25	0.053	3.36	8.62 - j2.09	8.62 - j5.27	84.4 + j51.6
4.75	0.119	1.76	5.14 - j0.10	5.14 - j1.38	181.5 + j48.9
5.25	0.121	1.75	5.03 - j0.06	5.03 - j1.33	185.9 + j49.1
5.75	0.123	1.75	4.93 - j0.03	4.93 - j1.28	190.0 + j49.4
6.25	0.125	1.75	4.84 + j0.00	4.84 - j1.24	193.9 + j49.6
6.75	0.127	1.76	4.76 + j0.03	4.76 - j1.20	197.5 + j49.7
7.25	0.081	2.40	6.90 - j0.92	6.90 - j3.02	121.6 + j53.2
7.75	0.055	3.36	8.36 - j2.02	8.36 - j5.18	86.5 + j53.6
8.25	0.056	3.36	8.33 - j2.01	8.33 - j5.17	86.7 + j53.8
8.75	0.082	2.42	6.83 - j0.91	6.83 - j3.01	122.7 + j54.1
9.25	0.133	1.77	4.45 + j0.13	4.45 - j1.05	213.0 + j50.5
9.75	0.134	1.78	4.39 + j0.15	4.39 - j1.03	215.7 + j50.6
10.25	0.135	1.78	4.35 + j0.17	4.35 - j1.01	218.3 + j50.7
10.75	0.136	1.79	4.30 + j0.18	4.30 - j0.99	220.9 + j50.8

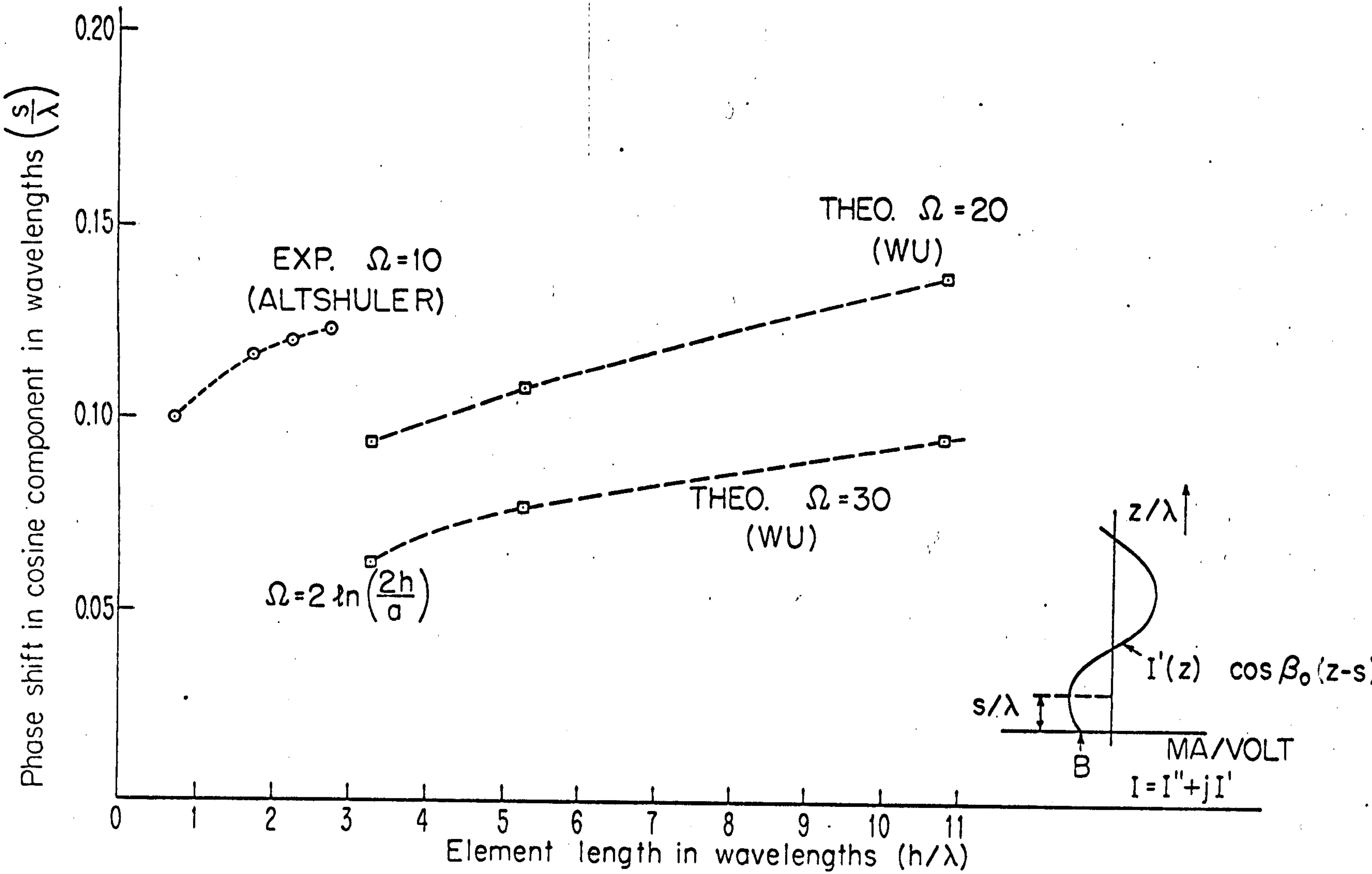

Figure 2. Phase shift of imaginary component of current as a function of element length and radius

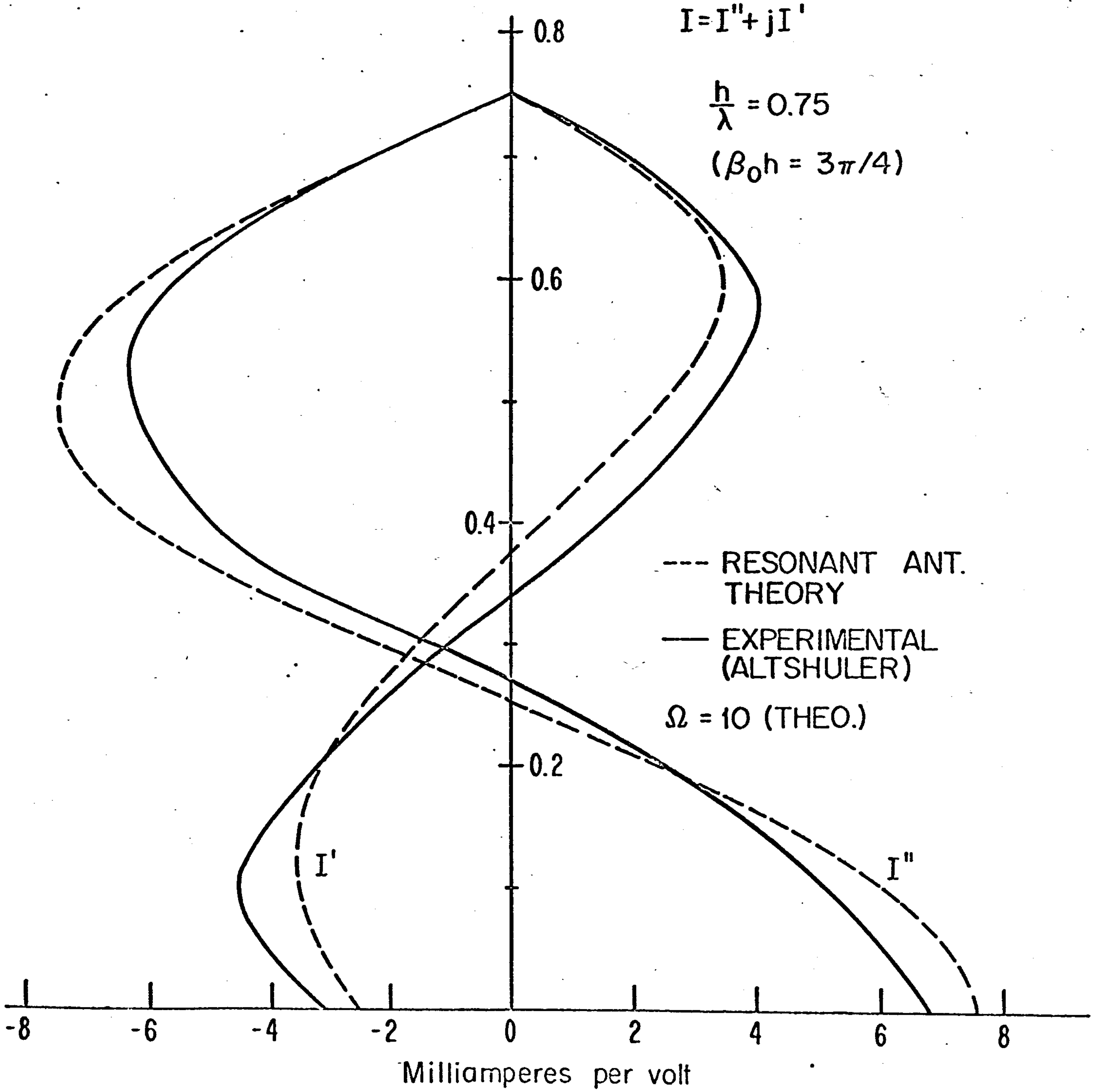

Figure 3. Comparison of resonant antenna theory with experiment for $\beta_o h = 3\pi/4$, $\Omega = 10$.

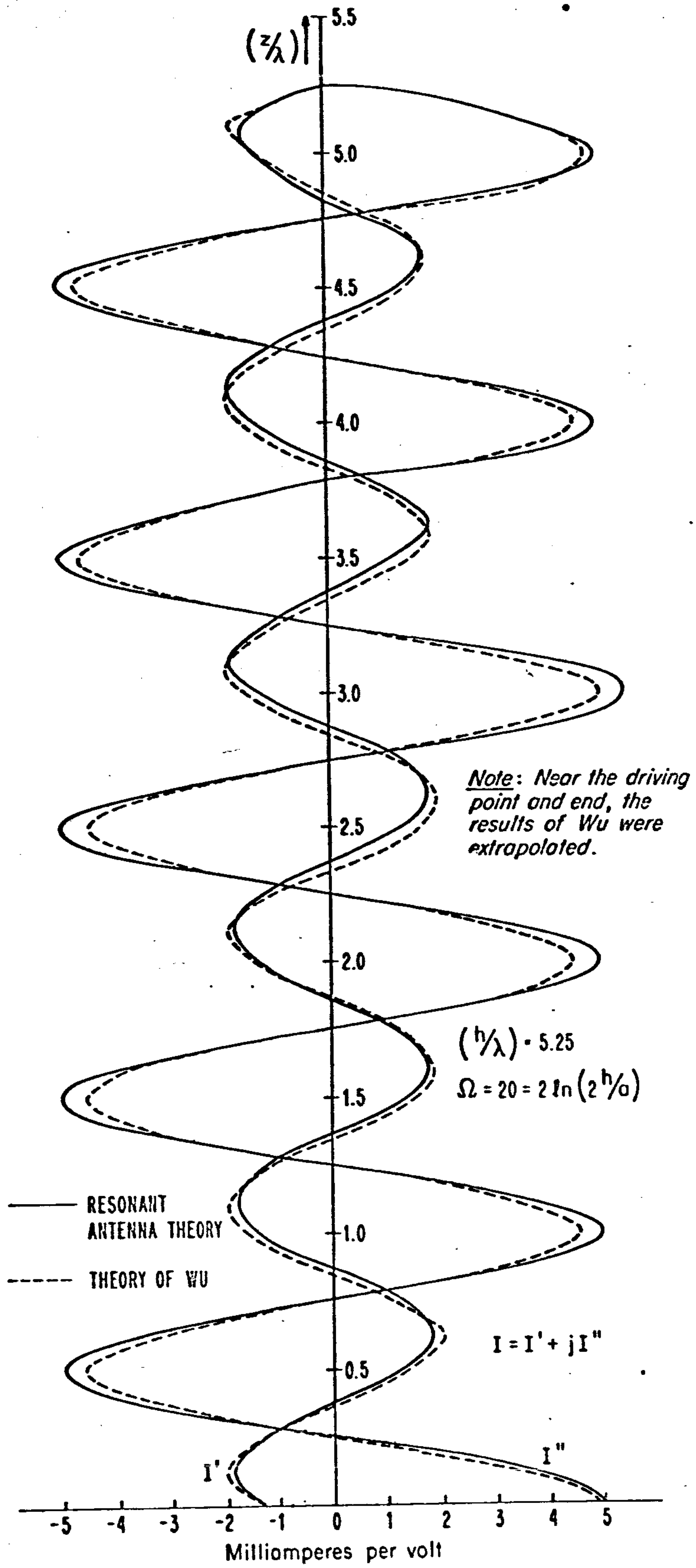

Figure 4. Comparison of resonant antenna theory with the theory of WU.

Optimum Loading Resistance

Values for the V-antenna

The computation of the optimum value of loading resistance for the V-antenna is based on values computed for the single leg. The optimum value computed here is based on the criteria of a maximum outward traveling wave distribution of current on the section between the driving point and the loading resistance. This use of the single leg calculations for the V-antenna is valid when the coupling does not effect the distribution of current. Two basic types of resistively loaded traveling wave antennas may be distinguished. One type has a resonant section before the resistance and the other type an anti-resonant section. The former type may be completely analysed with a superposition of resonant dipoles and the other by antiresonant dipoles. The theory of the arbitrary length resonant antenna is given in this report. Based on this analysis and the superposition outlined by Altshuler (E.E Altshuler, "The Travelling-Wave Linear Antenna", Cruft Laboratory Sci. Rpt. No.7 (Series 2), Harvard University, May 5, 1960) the current and driving point impedance were computed for different values of antenna length and diameter. An example of the results for the single leg when the total length $h = 0.75$ and the resistance is located at $h_1 = 0.50$ is shown in Figure 1. The magnitude of the total antenna current is plotted for different values of the load resistor R.

Note that a value of R = 350 ohms produces the best traveling wave on the section from the driving point to the resistive termination. This antenna is moderately thin ($\Omega = 10 = 2 \ln(2h/a)$) and the value of R-opt. will be seen to be not only dependant on the value of but also on the electrical length. The value of R-opt = 350 ohms compares favorably with the experimentally determined value of R opt = 320 ohms found by Altshuler. A more detailed plot for a longer antenna is shown in Figure 2. Note that the standing wave is minimised at the value of R = Ropt. The accompaning shows the values of input impedance and Ropt for a single antenna for various values of h, h_1 and Ω .

Series I, Omega = 20

h	h_1	Optimum R	Z (T)	
0.75	0.50	750.00	935.349	23.482
2.25	1.50	550.00	751.098	36.669
3.75	2.50	550.00	786.944	33.224
5.25	3.50	550.00	770.823	43.100
6.75	4.50	400.00	661.349	36.722
8.25	5.50	650.00	895.843	40.018
10.75	6.50	350.00	619.948	41.442

Series II, Omega = 20

1.25	1.00	900.00	1119.487	28.117
3.75	3.00	550.00	767.075	139.602
6.25	5.00	700.00	970.081	30.727
8.75	7.00	350.00	630.937	36.041
11.25	9.00	750.00	977.872	47.945

Series I, Omega = 30

0.75	0.50	700.00	922.419	-0.142
2.25	1.50	550.00	751.497	36.910
3.75	2.50	500.00	792.156	16.558
5.25	3.50	450.00	763.805	-52.370
6.75	4.50	400.00	666.946	36.355
8.25	5.50	750.00	1002.231	35.617
10.75	6.50	350.00	604.272	36.426

Series II, Omega = 30

1.25	1.00	950.00	1106.166	43.404
3.75	3.00	550.00	782.123	36.712
6.25	5.00	950.00	1128.387	41.120
8.75	7.00	300.00	657.889	-38.995
11.25	9.00	950.00	1159.387	51.487

Series I, Omega = 40

0.75	0.50	700.00	923.007	-0.552
2.25	1.50	550.00	751.401	36.764
3.75	2.50	500.00	793.175	16.325
5.25	3.50	400.00	723.553	346.741
6.75	4.50	400.00	667.020	36.379
8.25	5.50	750.00	1003.696	38.973
10.75	6.50	350.00	621.361	42.933

Series II, Omega = 40

1.25	1.00	950.00	1105.632	44.400
3.75	3.00	550.00	782.209	36.963
6.25	5.00	950.00	1126.735	41.922
8.75	7.00	350.00	643.257	34.009
11.25	9.00	950.00	1159.084	51.342

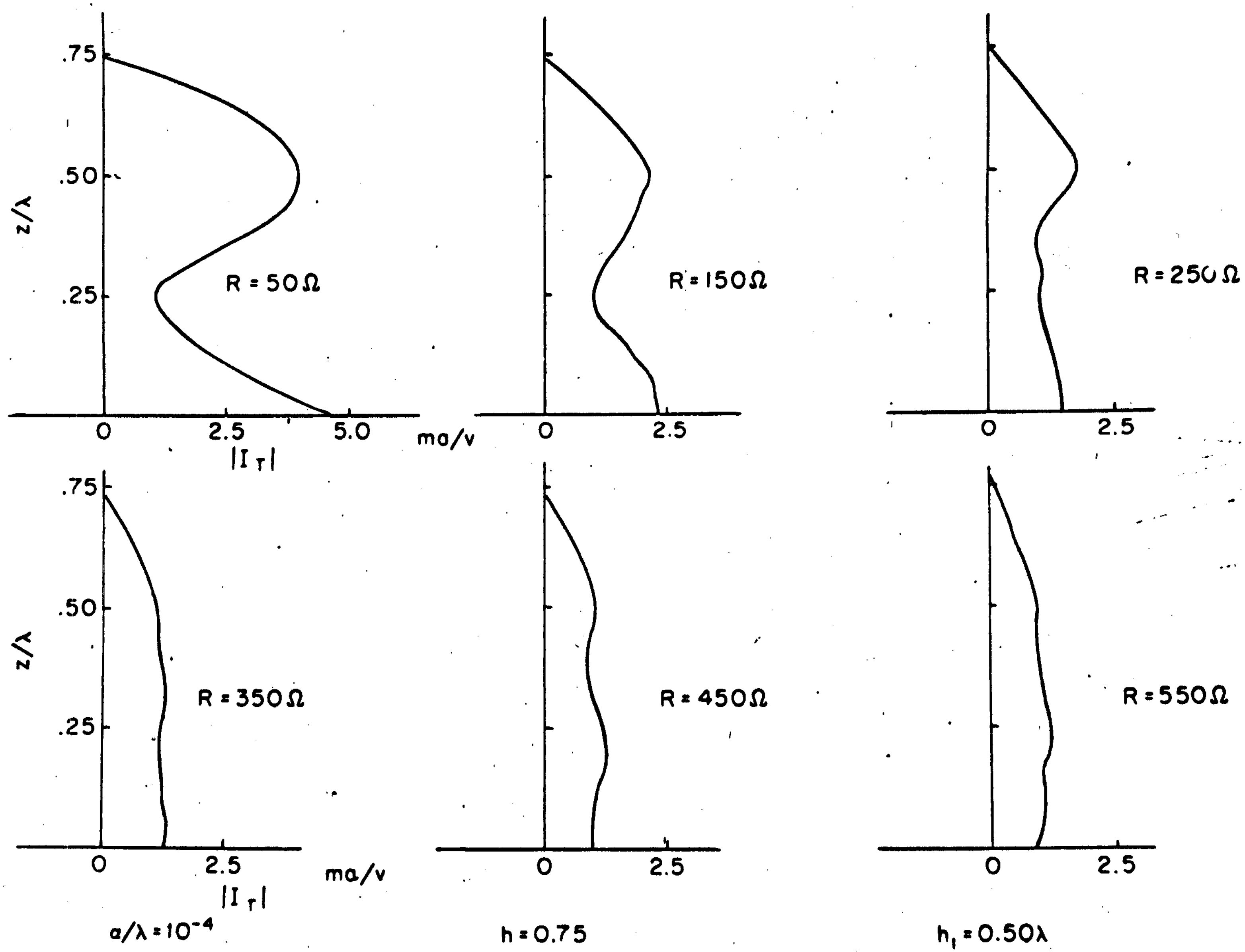

Figure 1. Variation of antenna current with load resistance ($h/\lambda = 0.75$).

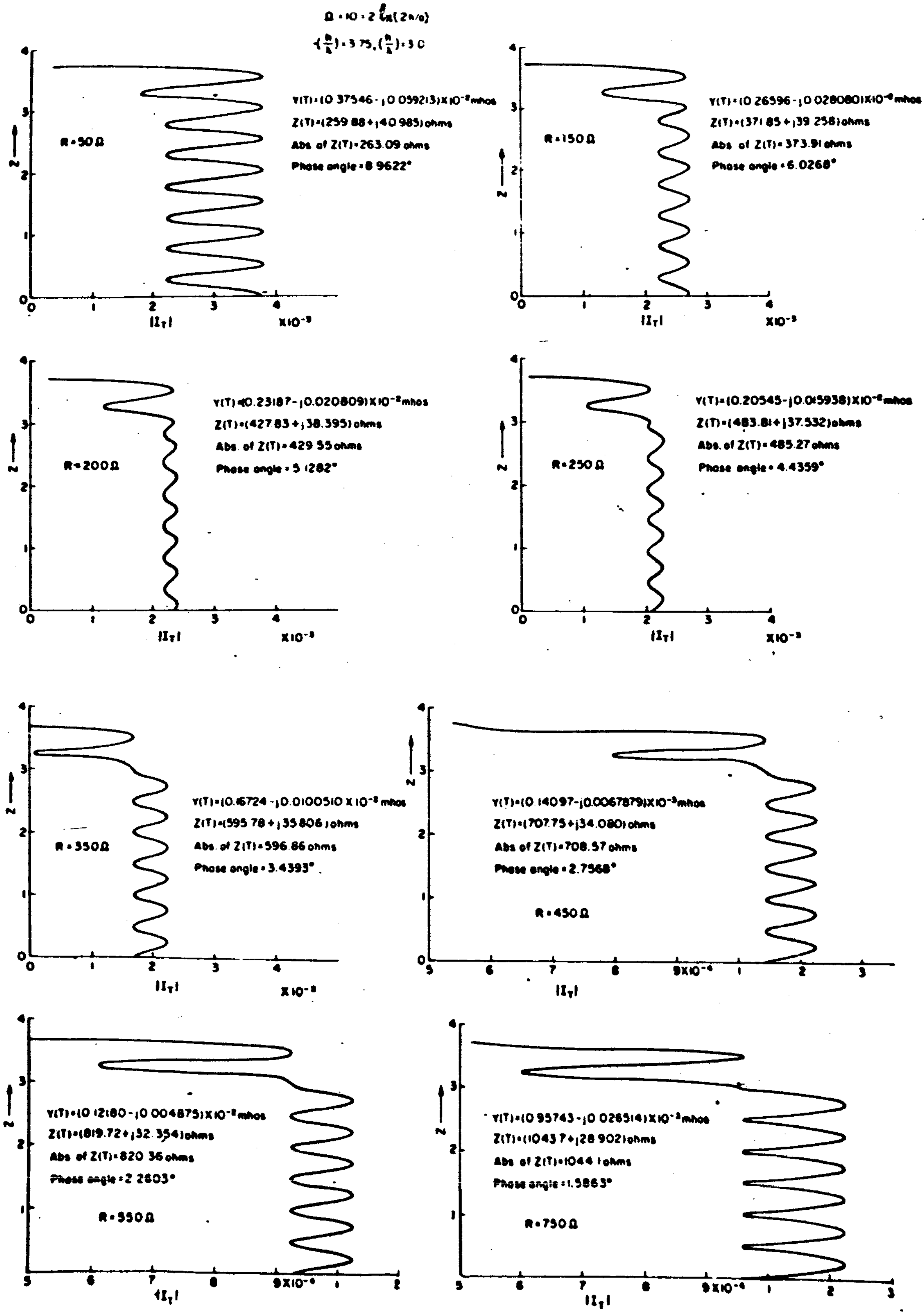

Figure 2. Variation of total antenna current with load resistance (h/λ = 3.75).

SECTION IV

LINEAR TRAVELING WAVE ANTENNA

<u>CALCULATE</u> $Y_T = I_T(o)$, $Z_T = 1/Y_T$

and $I_T(z)$, in steps of $z/ = 0.25$ to $z/\lambda = h/\lambda$

<u>STEP 1</u>

$I_s(z)$ is calculated for a given h and a from the general current and impedance calculation

<u>NEED</u> $Y_s = I_s(o)$ and $I_s(z)$, for $(z) = h$

<u>STEP 2</u>

$I_{ul}(z)$ is calculated for a given $(h - h_1)$ = value of height from the general formulas with z replaced by $z - h_1$

<u>NEED</u> = 2 table value $Y_{ul} = 2\ I_{ul}\ (h_1)\ z = h_1$ admittance at

and $I_{ul}(z)$ for $h_1 \leq z \leq h$

<u>STEP 3</u>

$I_{L1}(z)$ is calculated for a height $= h + h_1$ from the general formulas replace z by $h_1 - z$,

<u>STEP 4</u>

Calculate $Y_{A1} = \dfrac{Y_{u1}\ Y_{L1}}{Y_{u1} + Y_{L1}}$

<u>STEP 5</u>

Calculate $\dfrac{I_{AU1}}{V_D} = \dfrac{Y_{A1}}{Y_{u1}}\ I_{u1}(z)$

<u>STEP 6</u>

Calculate $\dfrac{I_{AL1}}{V_D} = \dfrac{Y_{A1}}{Y_{L1}}\ I_{L1}(h)$

STEP 7

Note $$\frac{I_{AL2}}{V_D} = \frac{I_{Au1}(-z)}{V_D}$$

and $$\frac{I_{Au2}(h)}{V_D} = \frac{I_{AL1}(-h)}{V_D}$$

STEP 8

$$\frac{I_D(h)}{V_D} = \frac{I_{AL1}(h)}{V_D} + \frac{I_{Au1}(h)}{V_D}$$

STEP 9

$$\frac{I_D(z)}{V_S} = \frac{-Y_s \dfrac{I_s(h_1)}{I_s(o)}}{Y_D + \dfrac{1}{R}} \quad \frac{I_D(z)}{V_D}$$

where R is given in ohms

$$Y_D \doteq \frac{I_D(h)}{V_D}$$

STEP 10

Finally $I_T(z) = I_s(z) + \dfrac{I_D(z)}{V_s}$

compute $Y_T = I_T(o)$ and $Z_T = \dfrac{1}{I_T(o)}$

and plot $I_T(z)$ for $0 \leq z \leq h$

R, h, h_1 given (a/λ) radius/ wavelength

Note Y_{u1} = twice the admittance of the corresponding depole

Y_{L1} = depole

small z = distance along antenna

large Z = impedance

a-Al = radius of antenna cross section

Current and Impedance for Resonant Antennas

Input:

Coefficients A and B as computed in "Grunwald-Weber Program"

Resistance R

Computations involve use of the following formula to compute the three component currents:

$$I(z) = -jA \cos \beta (z - .125) + B \cos \beta z + f(z),$$

where f(z) is a correction factor effective only near the end of the antenna.

Succeeding formulas for the computation of the final current can be traced through the program using the attached sheets.

Output consists of

1) the current for each component antenna after the coordinate transformation;

2) intermediate values at $Z = h_1$;

3) final results - Z, I, (z), and $\frac{I_D(z)}{V_S}$;

4) plot of Z vs. I_T (z)

The program may be run for as many values of R and as many values of h as desired.

```
      MAIN
      EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA      07/09/66

C
C
C      SANDLER - CURRENT AND IMPEDANCE CALCULATIONS - ORIGINAL DECK
C      A/LAMBDA = 2H/3 X .1E-03
C
       DIMENSION IV(500),IS(500),IL(500),ILM(500),ITZ(500),ID(500),
      XIDS(500),ABV(500),ZR(500),GRAPH(2000),NSCALE(5),AB(500),ZRR(500)
       DIMENSION ZDR(500),AD(500)
       DIMENSION RZ(500),ABB(500)
       COMPLEX H,H1,A,B,YO,ZO,TWOPI,R,ARG3,ARG4,Z,ARG1,ARG2,FZ,IVH1,YV,
      XYS,ILH1,YL,YA,ISH1,F,IZ,IV,IL,IS,IAV,IAL,ID,IDS,ITZ,YTZ,ZTZ,YD,
      XILM,IAV2,ILH,ILMH1,IALH1,IAV2H1,Z1,Z2,Z3,MZ
       REAL MAX,MIN
       DATA RAD,TWOPI/57.29578,(6.2831853,0.)/
       ARG1(X)=TWOPI*(X-.125)
       ARG2(X)=TWOPI*X
       FZ(X)=-(0.,1.)*A*(((CCOS(ARG3)/CCOS(ARG4))-CCOS(TWOPI/8.))*CCOS
      X(ARG2(X))-CSIN(TWOPI/8.)*CSIN(ARG2(X)))
    11 READ (5,101) HH,HH1,FRR
       WRITE (6,102) HH,HH1
       READ (5,100) A1,BRP1,BIP1,YOR1,YOI1,ZOR1,ZOI1,A2,BRP2,BIP2,YOR2,
      XYOI2,ZOR2,ZOI2,A3,BRP3,BIP3,YOR3,YOI3,ZOR3,ZOI3
       K=1
    10 READ (5,103) RR
       R=CMPLX(RR,0.)
       IF (K.NE.1) GO TO 4
       H=CMPLX(HH,0.)
       H1=CMPLX(HH1,0.)
       FI=REAL(20.*H+1.)
       N=FI*2.-1.
       NN=N/2+1
       NH=NN/2
       ARG3=TWOPI*(H-.25)
       ARG4=TWOPI*(H-.125)
C      INDEX (M) DETERMINES COEFFICIENTS A AND B OUTSIDE Z-LOOP
C      INDEX (N) DETERMINES RANGE OF Z-LOOP
     1 M=1
       WRITE (6,107)
       A=CMPLX(A1,0.)
       B=CMPLX(BRP1,BIP1)
       Z1=(0.,0.)
       IVH1=-(0.,1.)*A*CCOS(ARG1(Z1))+B*CCOS(ARG2(Z1))
       YV=2.*IVH1
       Z=-(H+H1)
       GO TO 4
     2 M=2
       WRITE (6,108)
       A=CMPLX(A2,0.)
       B=CMPLX(BRP2,BIP2)
       ISH1=-(0.,1.)*A*CCOS(ARG1(H1))+B*CCOS(ARG2(H1))
       Z=(0.,0.)
       YS=-(0.,1.)*A*CCOS(ARG1(Z))+B*CCOS(ARG2(Z))
       Z=-H
       GO TO 4
     3 M=3
```

```
      WRITE (6,109)
      A=CMPLX(A3,0.)
      B=CMPLX(BRP3,BIP3)
      Z2=(2.,0.)*H1
      ILMH1=-(0.,1.)*A*CCOS(ARG1(Z2 ))+B*CCOS(ARG2(Z2 ))
      Z3=(0.,0.)
      ILH1=-(0.,1.)*A*CCOS(ARG1(Z3))+B*CCOS(ARG2(Z3))
      YL =2.*ILH1
      YA=(YV*YL)/(YV+YL)
      IALH1=(YA/YL)*ILH1
      IAV2H1=(YA/YL)*ILMH1
      YD=IALH1+IAV2H1
      Z=H1+H
    4 DO 5 I=1,N
      IF (K.NE.1) GO TO 20
      F=(0.,0.)
      IF (ABS(REAL(Z)).GT.(REAL(H)-.125)) F=FZ(Z)
      IZ=-(0.,1.)*A*CCOS(ARG1(Z))+B*CCOS(ARG2(Z))+F
      IF (M-2) 6,7,8
    6 IV(I)=IZ
      IF (REAL(Z).LT.0.) IV(I)=(0.,0.)
      IF (REAL(IV(I)).NE.0.) WRITE (6,105) Z,IV(I)
      GO TO 9
    7 IS(I)=IZ
      ZR(I)=REAL(Z)
      WRITE (6,105) Z,IS(I)
      GO TO 9
    8 IL(I)=IZ
      TEST=-0.001
      IF (REAL(Z).GT..5.OR.REAL(Z).LT.TEST) IL(I)=(0.,0.)
      IF (REAL(IL(I)).NE.0.) WRITE (6,105) Z,IL(I)
      MZ=-Z
      ILM(I)=-(0.,1.)*A*CCOS(ARG1(MZ))+B*CCOS(ARG2(MZ))+F
      IAV=(YA/YV)*IV(I)
      IAL=(YA/YL)*IL(I)
      IAV2=(YA/YL)*ILM(I)
      ID(I)=(2.*YA/YL)*CCOS(TWOPI*Z)*(-(0.,1.)*A3*CCOS(TWOPI*(H1-.125))
     X+B3*CCOS(TWOPI*H1))
      IF (REAL(Z).GE.REAL(H1)) ID(I)=IAV+IAV2
   20 IDS(I)=(-ISH1/(    YD+1./R))*ID(I)
      ITZ(I)=IS(I)+    IDS(I)
      ABV(I)=REAL(CABS(ITZ(I)))
      IF (I-NN) 23,22,21
   21 MAX=AMAX1(MAX,ABV(I))
      MIN=AMIN1(MIN,ABV(I))
      GO TO 23
   22 MAX=ABV(I)
      MIN=ABV(I)
   23 Z=Z-.1
      IF (FLOAT(I).NE.FI) GO TO 9
      YTZ=ITZ(I)
      ZTZ=1./YTZ
      ABSVAL=REAL(CABS(ZTZ))
      TANPHI=AIMAG(ZTZ)/REAL(ZTZ)
      PHI=ATAN(TANPHI)*RAD
    9 IF (I.NE.N) GO TO 5
```

```
      IF (M-2) 2,3,5
    5 Z=Z+.05
      IF (K.EQ.1) WRITE (6,111) IVH1,ILH1,ISH1,YV,YS,YL,ILMH1,YA,YD
      WRITE (6,104) RR
      WRITE (6,110) YTZ,ZTZ,ABSVAL,PHI
      WRITE (6,106) (ZR(I),ITZ(I),IDS(I),ABV(I),I=NN,N)
      DO 12 I=1,NN
      J=(I-1)+NN
      ZDR(I)=ZR(J)
   12 AD(I)=ABV(J)
      DO 13 K=1,NH
      ZRR(K)=ZDR(2*K-1)
   13 AB(K)=AD(2*K-1)
      NSCALE(1)=1
      NSCALE(2)=0
      NSCALE(3)=3
      NSCALE(4)=0
      NSCALE(5)=5
      CALL PLOT1 (NSCALE,10,10,10,10)
      CALL PLOT2 (GRAPH,MAX,MIN,H,0.)
      IF (HH-3.75) 14,15,15
   14 CALL PLOT3 (1H*,AD,ZDR,NN)
      GO TO 16
   15 CALL PLOT3 (1H*,AB,ZRR,NH)
   16 CALL FPLOT4 (33,33H GRAPH OF ABS VAL OF CURRENT VS Z)
  100 FORMAT (6E12.5)
  101 FORMAT (3F12.3)
  102 FORMAT (1H1,20H COMPUTATIONS FOR H=F8.5,8H AND H1=F8.5,19H BEGIN O
     XN THIS PAGE)
  103 FORMAT (F8.2)
  104 FORMAT (1H1,4H R =F8.2)
  105 FORMAT (4E20.5)
  106 FORMAT (1H0,4H   Z,29X,5H I(T),39X,8H I(D)/VS,24X,12H ABS OF I(T)/
     X(F7.2,5X,2E20.5,5X,2E20.5,5X,E20.5))
  107 FORMAT (1H0,19X,7H Z=Z-H1,36X,6H I(U1))
  108 FORMAT (22X,2H Z,38X,5H I(S))
  109 FORMAT (19X,7H Z=H1-Z,36X,6H I(L1))
  110 FORMAT(1H0,7H Y(T) =2E18.5/7H Z(T) =2E18.5/15H CABS OF Z(T) = E
     X18.5/14H PHASE ANGLE = E18.5)
  111 FORMAT (1H0/9H IU1(H1)=2E18.5/9H IL1(H1)=2E18.5/8H IS(H1)=2E18.5/
     X7H Y(U1)=2E18.5/6H Y(S)=2E18.5/7H Y(L1)=2E18.5/10H IL1(-H1)=2E18.5
     X/7H Y(A1)=2E18.5/6H Y(D)=2E18.5)
      K=K+1
      IF (RR .EQ. FRR) GO TO 11
      GO TO 10
      END
```

V-ANTENNA RADIATION PATTERNS

IDEAL CURRENT, EXACT SOLUTION

INPUT* : VARIABLE	FORMAT
CARD 1 : BETA	E16.8
CARD 2 :H, DH, HNO	2F7.2,I5
CARD 3 : H1, DH1	2F7.2
CARD 4 : HALF, DD2, DNO	2F7.2,I5
CARD 5 : THET(1), DTHET, THENO	2F7.2,I5
CARD 6 : PHID(1), DPHID, PHINO	2F7.2,I5
CARD 7 : TORP	I5

* INPUT EXPLAINED AT BEGINNING OF PROGRAM

TO PLOT PATTERNS: ADD THE FOLLOWING CARDS

```
        DIMENSION GRAPH (2000)
AFTER 1175 and WRITE (6,5).....
        WRITE (6,12)
        CALL PLOT 2 (GRAPH, 180.,0.,0.,-30.)*
```

* LOGK2 assumed $\geq$ - 30.

```
        CALL PLOT 3 ([illegible] PHID, LOGK2, PHINO)
        CALL FPLOT4 (29, 29H...DECIBELS...)
        WRITE(6,8)
      8 FORMAT (30X, 3HPHI)
AFTER 1275 and WRITE (6,6)...
        WRITE (6,12)
        CALL PLOT 2 (GRAPH, 180, 0., 0., -30.)
```

```
        CALL PLOT 3 ( H*, THET, LOGK2, THENO)

        CALL FPLOT4 (29, 29H...DECIBELS...)

THIS IS THE UMPLOT PLOTTING ROUTINE
```

```
C     V-ANTENNA IN XZ-PLANE
C     PHI-MEASURED IN XY-PLANE FROM X-AXIS
C     THETA- ANGLE BETWEEN Z-AXIS AND ANY LINE THROUGH ORIGIN
C     H-ANTENNA LENGTH/LAMBDA
C     H1-ANTENNA LENGTH UP TO RESISTANCE /LAMBDA
C     DEL2-HALF ANGLE
C     AS1=INT(0.,BH1) OF (EXP(J*Y*(-1+STH*CPH*SD2+CTH*CD2))DY)
C     +(EXP(-J*BH1)/SBHH1)*INT(BH1,BH) OF (SIN(BH-Y)*EXP(J*Y*( STH*CPH*SD2
C                        +CTH*CD2))DY)
C     AS2=-INT(0.,BH1) OF (EXP(J*Y*(-1-STH*CPH*SD2+CTH*CD2))DY)
C     -(EXP(-J*BH1)/SBHH1)*INT(BH1,BH) OF (SIN(BH-Y)*EXP(J*Y*(-STH*CPH*SD2
C                        +CTH*CD2))DY)
C     (K(THETA,PHI))**2=ATH*CONJ(ATH)+APH*CONJ(APH)
C     ATH=THS1*AS1+THS2*AS2
C     APH=PHS1*AS1+PHS2*AS2
C     THS1=SD2*CTH*CPH-CD2*STH
C     THS2=-SD2*CTH*CPH-CD2*STH
C     PHS1=-SPH*SD2
C     PHS2=SPH*SD2
      DIMENSION PHID(200),KSQ(200),THET(200),LOGK2(200)
      REAL KSQ,LOGK2,MAXK2
      INTEGER HNO,H1NO,DNO,THENO,PHINO ,TORP
      COMPLEX AS1,AS2,ATH,APH,JATH,JAPH,KQ,FACTOR,REAL1,COMP1,REAL2,
     1 COMP2
C     TORP=1 THETA CONSTANT. PHI VARIES
C     TORP=2 PHI CONSTANT. THETA VARIES
C     HNO - NUMBER OF H(N) S.
C     DNO - NUMBER OF DELTA/2
C     THENO - NUMBER OF THETA S
C     PHINO - NUMBER OF PHI S
      DATA  RAD/1.74532925E-02/
      WRITE(6,12)
   50 READ(5,1) BETA
   60 READ(5,2) H,DH,HNO,H1,DH1
   70 READ(5,2) HALF,DD2,DNO,THET(1),DTHET,THENO,PHID(1),DPHID,PHINO
      READ(5,3) TORP
      WHOLE=2.*HALF
      DO  1500 I= 1,HNO
      BH=BETA*H
      CBH=COS(BH)
      SBH=SIN(BH)
      BH1 = BETA*H1
      CBH1= COS(BH1)
      SBH1=SIN(BH1)
      BHH1= BH-BH1
      BHH15=.5*BHH1
      BHH12= 2.*BH1-BH
      SBHH1=SIN(BHH1)
      CBHH1= COS(BHH1)
      FACTOR=CMPLX(CBH1,(-SBH1))/SBHH1
      SBHH12=SIN(BHH12)
      CBHH12=COS(BHH12)
      BH12=2.*BH1
      SBH12=SIN(BH12)
      CBH12=COS(BH12)
```

```
DEL2=HALF
DO  1300 K=1,DNO
D2 = DEL2*RAD
CD2 = COS(D2)
SD2 = SIN(D2)
SEC=1./CD2
BSEC=BETA*SEC
BSEC4= .25/BSEC
CD2H1= H1*CD2
DO  1200 L=1,THENO
THETA=THET(L)*RAD
STH=SIN(THETA)
CTH = COS(THETA)
TERM2= CTH*CD2
TERM4=STH*CD2
DO 1100 M=1,PHINO
PHI=PHID(M)*RAD
SPH = SIN(PHI)
CPH=COS(PHI)
CPSD= CPH*SD2
TERM1=STH*CPSD
TERM3=CTH*CPSD
TERM5=SPH*SD2
C11=TERM1+TERM2
C21=C11-1.
C31=C11+1.
C12=-TERM1+TERM2
C22= C12-1.
THS1=TERM3-TERM4
THS2=-TERM3-TERM4
PHS1=-TERM5
PHS2=TERM5
BOT11=(BSEC*C21*C31)**(-1)
BOT21= (BSEC*C22)**(-1)
BOT22=(BSEC*(1.-C12)*(1.+C12))**(-1)
BH1C21=BH1*C21
SH1C21=SIN(BH1C21)
CH1C21=COS(BH1C21)
BH1C11=BH1*C11
SH1C11=SIN(BH1C11)
CH1C11=COS(BH1C11)
BH1C22=BH1*C22
SH1C22=SIN(BH1C22)
CH1C22=COS(BH1C22)
BH1C12=BH1*C12
SH1C12=SIN(BH1C12)
CH1C12=COS(BH1C12)
BHC12=BH*C12
SHC12=SIN(BHC12)
CHC12=COS(BHC12)
BHC11=BH*C11
CHC11=COS(BHC11)
SHC11=SIN(BHC11)
CPROD1=CBHH1*CH1C11
SPROD1=SBHH1*SH1C11
CSPRO1=CBHH1*SH1C11
```

```
      SCPRO1=SBHH1*CH1C11
      IF(C11.EQ. 0.) GO TO 660
      IF(C31.EQ.0.) GO TO 670
      IF (C21 .EQ .0.) GO TO 600
      REAL1=(C31*SH1C21+FACTOR*(-CHC11+CPROD1-C11*SPROD1))* BOT11
      COMP1=(C31*(1.-CH1C21)+FACTOR*(-SHC11+CSPRO1+C11*SCPRO1)) *BOT11
      GO TO 650
  600 REAL1=CD2H1+FACTOR*(BHH15*SBH/BSEC+BSEC4*(CBH-CBHH12))
      COMP1= -BHH15*CBH/BSEC+BSEC4*(SBH-SBHH12)
      COMP1=COMP1*FACTOR
  650 AS1=BSEC*(REAL1+(0.,1.)*COMP1)
      GO TO 690
  660 REAL1= SBH1+FACTOR *(-CBHH1+1.)
      COMP1=CBH1-1.
      AS1=REAL1+(0.,1.)*COMP1
      GO TO 690
  670 REAL1=.5*SBH12+FACTOR*(.25*(CBH-CBHH12)+SBH*BHH15)
      COMP1=.5*(CBH12-1.)+FACTOR*(.25*(SBHH12-SBH)+CBH*BHH15)
      AS1=REAL1+(0.,1.)*COMP1
  690 IF(C12.EQ.0.) GO TO 760
      IF((C12+1.).EQ.0.) GO TO 765
      IF(C22.EQ.0.) GO TO 700
      REAL2= -SH1C22*BOT21+FACTOR*(CBHH1*CH1C12-C12*SBHH1*SH1C12-
     1  CHC12)*BOT22
      COMP2= (CH1C22-1.)*BOT21+FACTOR*(CBHH1*SH1C12+C12*SBHH1*CH1C12
     1      -SHC12)* BOT22
      GO TO 750
  700 IF(C21.EQ.0.) GO TO 725
      REAL2=-CD2H1+FACTOR*(-BHH15*SBH/BSEC+BSEC4*(CBHH12-CBH))
      COMP2= FACTOR * ( BHH15*CBH/BSEC+BSEC4*(SBHH12-SBH)        )
      GO TO 750
  725 REAL2=-REAL1
      COMP2=-COMP1
  750 AS2=BSEC*(REAL2+(0.,1.)*COMP2)
      GO TO 770
  760 REAL2 = -SBH1 +FACTOR*( CBHH1-1.)
      COMP2= FACTOR *(-CBH1+1.)
      AS2=REAL2+(0.,1.)*COMP2
      GO TO 770
  765 REAL2=-.5*SBH12+FACTOR*(.25*(CBHH12-CBH)-SBH* BHH15)
      COMP2=-.5*(CBH12-1.)+FACTOR*(-.25*(SBHH12-SBH)-CBH*BHH15)
      AS2=REAL2+(0.,1.)*COMP2
  770 ATH=THS1*AS1+THS2*AS2
      APH=PHS1*AS1+PHS2*AS2
      JATH=CONJG(ATH)
      JAPH=CONJG(APH)
      KQ = ATH*JATH+APH*JAPH
      GO TO (775,800),TORP
  775 KSQ(M)=REAL(KQ)
      GO TO 1100
  800 KSQ(L)=REAL(KQ)
      GO TO 1200
 1100 PHID(M+1)=PHID(M)+DPHID
      MAXK2=KSQ(1)
      DO 1150 M=1,PHINO
      TEMP=KSQ(M)
```

```
1150 MAXK2=AMAX1(MAXK2,TEMP)
     DO 1175M=1,PHINO
1175 LOGK2(M)=10.*ALOG10(KSQ(M)/MAXK2)
     WRITE(6,5) DEL2,THET(1),H,H1,BETA,MAXK2,(PHID(M),KSQ(M),LOGK2(M),
    1 M=1,PHINO)

1200 THET(L+1)=THET(L)+DTHET
     GO TO (1300,1225),TORP
1225 MAXK2=KSQ(1)
     DO 1250 M=1,THENO
     TEMP=KSQ(M)
1250 MAXK2=AMAX1(MAXK2,TEMP)
     DO 1275 M=1,THENO
1275 LOGK2(M)=10.*ALOG10(KSQ(M)/MAXK2)
     WRITE(6,6) DEL2,PHID(1),H,H1,BETA,MAXK2,(THET(M),KSQ(M),LOGK2(M),
    1 M=1,THENO)

1300 DEL2=DEL2+DD2
      WRITE(6,10) WHOLE
     WRITE(6,11) H,H1,KSQ(1)
1400 H1=H1+DH1
1500 H=H+DH
     GO TO 50
   1 FORMAT(5E16.8)
   2 FORMAT(2F7.2,I5)
   3 FORMAT(14I5)
   5 FORMAT(6H1DEL2=F6.2,10X,7H THETA=F6.2,10X,3H H=F6.2,10X,4H H1=F6.2,
    1,10X,6H BETA=F9.5,7X,7H MAXK2=E16.8/
    2                              3X4H PHI5X5H K**28X12H LOG10(K**2)5X4H PHI5X5H K
    25H K**28X12H LOG10(K**2)5X4H PHI5X5H K**28X12H LOG10(K**2)/
    53(F7.2,2E16.4))
   6 FORMAT(6H1DEL2=F6.2,10X,5H PHI=F6.2,10X,3H H=F6.2,10X,4H H1=F6.2,
    110X,6H BETA=F9.2,7X,7H MAXK2=E16.8/
    A                              2X5HTHETA5X5H K**28X12H LOG10(K**2)4X5HTHET
    2A5X5H K**28X12H LOG10(K**2)4X5HTHETA5X5H K**28X12H LOG10(K**2)    /
    53(F7.2,2E16.4))
   7 FORMAT(6H1DEL2=F6.2,10X,7H THETA=F6.2,10X,3H H=F6.2,10X,4H H1=F6.2
    1,10X,6H BETA=F9.5,7X,7H MAXK2=E16.8)
   9 FORMAT(30X,6H THETA)
  10 FORMAT(7H0DELTA=F7.2/6X1HH5X2HH117X9HK(0,0)**2)
  11 FORMAT(2F7.2,E26.4)
  12 FORMAT(1H1)
     END
```

PROGRAM TO COMPUTE P(z)

$$P(z) = -\beta^2 \int_{\delta}^{h} I(z')\, K_{12}\, (z,z')z'\, dz'$$

$$I(z') = \exp\,(-j\beta z')$$

$$K_{12}(z,z') = -\exp\,(-j\,\beta R_{12}) \left[1 + j\beta R_{12}\right] \quad (\beta R_{12})^3$$

$$R_{12} = \sqrt{(z' - z\cos\Delta)^2 + a^2 + z^2\sin^2\Delta}$$

INPUT : VARIABLE	FØRMAT
CARD 1 : E1, E2, E3, E4 (tolerances for Simpson sums)	4E 16.8
CARD 2: DH, LOWER , DELTA, Z	4E 16.8

DH $\simeq H - \delta$
LOWER $\simeq \delta$
DELTA $\simeq \Delta$

```
C     P(Z)=-BETA**2*INT(DEL,H) OF I(ZP)*K12(Z,ZP)*ZP*DZP
C     I(ZP)=EXP(-J*BETA*ZP)
C     K12(Z,ZP)=-EXP(-J*BETA*R12)*(1+J*BETA*R12)/(BETA*R12)**3
C     R12=SQRT((ZP-Z*COS(DELTA))**2+A**2+Z**2*SIN(DELTA)**2)
      COMMON H,DELTA,CDEL,SDELQ,AQ      ,BETA,LOWER,DH,DH2,DH3,PHONY,
     1    E1,E2,S,E3,E4,A    ,DEL,I1,I2
      COMPLEX PHONY
      DIMENSION S(4)
      REAL LOWER
      DATA PI,RAD      /3.14159265,1.74532925E-2/
      BETA=6.28318530
      BETAQ=BETA*BETA
      READ(5,1) E1,E2,E3,E4
      WRITE(6,2) E1,E2,E3,E4
      READ(5,1) A
      AQ=A*A
   50 READ(5,1) DH,LOWER,DELTA
      H=LOWER+DH
      DH2=.5*H
      DH3=.333333333*DH
      DRAD=RAD*DELTA
      CDEL=COS(DRAD)
      SDEL=SIN(DRAD)
      SDELQ=SDEL*SDEL
      I1=1
      I2=1
      CALL PINT(LOWER)
      PHONY=CMPLX(S(2),S(4))*BETA*BETA
      WRITE(6,3) DELTA,H,A,LOWER,PHONY
      GO TO 50
    1 FORMAT(4E16.8)
    2 FORMAT(28H1TOLERANCES FOR SIMPSON SUMS/4E13.3)
    3 FORMAT(7H0DELTA=F7.2,5X,2HH=F7.2,5X,2HA=E10.3,5X,4HDEL=F7.2,
     15X,7HP(DEL)=2E13.5)
      END
```

```
      SUBROUTINE PINT(Z)
      COMMON H,DELTA,CDEL,SDELQ,AQ      ,BETA,LOWER,DH,DH2,DH3,PHONY,
     1    E1,E2,S,E3,E4,A    ,DEL,I1,I2
      COMPLEX PHONY
      REAL LOWER
      INTEGER UPPER,WHERE
      DIMENSION  T(4),S(4),TEMP(2)
   40 ZCDEL=Z*CDEL
      SUM = AQ+Z*Z*SDELQ
      ZP=LOWER
      ZZP = ZP-ZCDEL
      R12 = SQRT(ZZP*ZZP+SUM)
      BR = BETA*R12
C     I(ZP)=EXP(-J*BETA*ZP)
      CBR = COS(BR)
      SBR= SIN(BR)
      ZPBR3=ZP/(BR*BR*BR)
      BL=BETA*LOWER
      SBHZ=SIN(BL)*ZPBR3
      CBHZ=COS(BL)*ZPBR3
      T(1)=       SBHZ*(CBR+BR*SBR)
      T(3)=       SBHZ*(CBR*BR-SBR)
      T1=CBHZ*(CBR+BR*SBR)
      T3=CBHZ*(CBR*BR-SBR)
      ZP=H
      ZZP=ZP-ZCDEL
      R12=SQRT(ZZP*ZZP+SUM)
      BR=BETA*R12
      CBR=COS(BR)
      SBR=SIN(BR)
      BZP=BETA*ZP
      ZPBR3=ZP/(BR*BR*BR)
      SBHZ=SIN(BZP)*ZPBR3
      CBHZ=COS(BZP)*ZPBR3
      T(1)=.5*(T(1)+SBHZ*(CBR+BR*SBR)   )
      T(3)=.5*(T(3)+SBHZ*(CBR*BR-SBR))
      T1=.5*(T1+CBHZ*(CBR+BR*SBR))
      T3=.5*(T3+CBHZ*(CBR*BR-SBR))
C      END OF CHANGES
      ZP=LOWER+DH2
      ZZP=ZP-ZCDEL
      R12 = SQRT(ZZP*ZZP+SUM)
      BR=BETA*R12
      CBR = COS(BR)
      SBR = SIN(BR)
      BZP=BETA*ZP
      ZPBR3=ZP/(BR*BR*BR)
      SBHZ=SIN(BZP)*ZPBR3
      CBHZ=COS(BZP)*ZPBR3
      T(2)= .5*( T(1)+ SBHZ * (CBR+BR*SBR))
      T(4)= .5*(T(3)+SBHZ*(CBR*BR-SBR))
      T2=.5*(T1+CBHZ*(CBR+BR*SBR))
      T4=.5*(T3+CBHZ*(CBR*BR-SBR))
      S(1)=  4.*T(2)-T(1)
      S(3)= 4.*T(4)-T(3)
```

```
      S1=4.*T2-T1
      S3=4.*T4-T3
      T(1)= T(2)
      T(3)= T(4)
      T1=T2
      T3=T4
      I3=1
      I4=1
      UPPER=16
      WHERE=1
      FLAG=1.
      N1=2
      N2=1
   50 DO 1000 N=3,UPPER
      K = 2**N2
      XK=FLOAT(K)*FLAG
      XK1 = (2.** N1)*FLAG
      DELXK1=DH/XK1
      TEMP(1)= 0.
      TEMP(2)= 0.
      TEMP1=0.
      TEMP2=0.
      XJ=1.
  110 ZP=LOWER+XJ*DELXK1
      ZZP=ZP-ZCDEL
      R12=SQRT(ZZP*ZZP+SUM)
      BR=BETA*R12
C      12/28/65 VERSION DIRECTLY BELOW
C     SBHZ=SIN(BETA*(H-ZP))*ZP/(BR*BR*BR)
C       3/24/66 VERSION HERE
C     SBHZ=SIN(BETA*ZP)*ZP/(BR*BR*BR)
C     4/20/66
C     I(ZP)=EXP(-J*BZP)
C
      CBR=COS(BR)
      SBR=SIN(BR)
      BZP=BETA*ZP
      ZPBR3=ZP/(BR*BR*BR)
      SBHZ=SIN(BZP)*ZPBR3
      CBHZ=COS(BZP)*ZPBR3
      GO TO ( 120,130),I1
  120 TEMP(1)=TEMP(1)+SBHZ*(CBR+BR*SBR)
  130 GO TO (140,150),I2
  140 TEMP(2)= TEMP(2)+ SBHZ*(CBR*BR-SBR)
  150 GO TO (151,152),I3
  151 TEMP1=TEMP1+CBHZ*(CBR+BR*SBR)
  152 GO TO (153,155),I4
  153 TEMP2=TEMP2+CBHZ*(CBR*BR-SBR)
  155 XJ=XJ+2.
      IF (XJ.LT. XK1) GO TO 110
      GO TO (160,165),I1
  160 TEMP(1)=TEMP(1)/XK
      T(2)=.5* (T(1)+TEMP(1))
      S(2)= 4.*T(2)-T(1)
      T(1)=T(2)
      IF(ABS((S(2)-S(1))/S(1)) .LT.E1) I1=2
```

```
      S(1)=S(2)
 165  GO TO (170,175),I2
 170  TEMP(2)=TEMP(2)/XK
      T(4)=.5*(T(3)+TEMP(2))
      S(4)= 4.*T(4)-T(3)
      T(3)=T(4)
      IF(ABS((S(4)-S(3))/S(3)) .LT. E2) I2=2
      S(3)= S(4)
 175  GO TO (180,185),I3
 180  TEMP1=TEMP1/XK
      T2=.5*(T1+TEMP1)
      S2=4.*T2-T1
      T1=T2
      IF(ABS((S2-S1)/S1).LT.E1) I3=2
      S1=S2
 185  GO TO (190,195),I4
 190  TEMP2=TEMP2/XK
      T4=.5*(T3+TEMP2)
      S4=4.*T4-T3
      T3=T4
      IF(ABS((S4-S3)/S3).LT.E2)I4=2
      S3=S4
 195  IF(I1.EQ.1.OR.I2.EQ.1.OR.I3.EQ.1.OR.I4.EQ.1)      GO TO 240
      GO TO 1300
 240  N2=N1
      N1=N
1000  CONTINUE
      GO TO (1100,1200,1300),WHERE
1100  WHERE = 2
      FLAG=K
      N1=2
      N2=1
      GO TO 50
1200  WHERE=3
      FLAG=FLAG*FLAG
      UPPER=8
      N2=1
      N1=2
      GO TO 50
1300  S(2)=S(2)*DH3
      S(4)=S(4)*DH3
      S2=S2*DH3
      S4=S4*DH3
      ANSR=S2+S(4)
      ANSI=S4-S(2)
      S(2)=ANSR
      S(4)=ANSI
      RETURN
      END
```

TOLERANCES FOR SIMPSON SUMS

1.000E-02 1.000E-02 1.000E-02 1.000E-02

DELTA= 20.00	H= 10.10	A= 0.670E-02	DEL= 0.10	P(DEL)= 0.81686E
DELTA= 45.00	H= 10.10	A= 0.670E-02	DEL= 0.10	P(DEL)= 0.72297E
DELTA= 60.00	H= 10.10	A= 0.670E-02	DEL= 0.10	P(DEL)= 0.10173E-
DELTA= 90.00	H= 10.10	A= 0.670E-02	DEL= 0.10	P(DEL)= -0.17776E-
DELTA= 135.00	H= 10.10	A= 0.670E-02	DEL= 0.10	P(DEL)= -0.21921E-
DELTA= 180.00	H= 10.10	A= 0.670E-02	DEL= 0.10	P(DEL)= -0.18190E-

ANTENNA 4/3 SERIES

RATIO= 0.750 LENGTH= 228.60

DEL2= 22.50 H= 1.00 H1= 0.75

RADIUS= 0.18684E 01 GAIN= 0.11168E 02 AREA= 0.46441E 05

TOTAL VOLUME= 0.27321E 02 MAX(K**2)= 0.20866E 02

TOTAL INTEGRAL= 0.27321E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.39748E-02	0.39748E-02	0.10860E-00
5.00	10.00	0.15664E-01	0.11689E-01	0.42796E-00
10.00	15.00	0.34439E-01	0.18775E-01	0.94092E 00
15.00	20.00	0.59488E-01	0.25049E-01	0.16253E 01
20.00	25.00	0.90092E-01	0.30604E-01	0.24614E 01
25.00	30.00	0.12587E-00	0.35777E-01	0.34389E 01
30.00	35.00	0.16690E-00	0.41034E-01	0.45600E 01
35.00	40.00	0.21368E-00	0.46778E-01	0.58381E 01
40.00	45.00	0.26684E-00	0.53158E-01	0.72904E 01
45.00	50.00	0.32677E-00	0.59928E-01	0.89278E 01
50.00	55.00	0.39320E-00	0.66431E-01	0.10743E 02
55.00	60.00	0.46491E-00	0.71713E-01	0.12702E 02
60.00	65.00	0.53967E 00	0.74760E-01	0.14745E 02
65.00	70.00	0.61444E 00	0.74770E-01	0.16787E 02
70.00	75.00	0.68583E 00	0.71389E-01	0.18738E 02
75.00	80.00	0.75066E 00	0.64829E-01	0.20509E 02
80.00	85.00	0.80649E 00	0.55836E-01	0.22035E 02
85.00	90.00	0.85202E 00	0.45524E-01	0.23278E 02
90.00	95.00	0.88715E 00	0.35131E-01	0.24238E 02
95.00	100.00	0.91291E 00	0.25764E-01	0.24942E 02
100.00	105.00	0.93110E 00	0.18190E-01	0.25439E 02
105.00	110.00	0.94384E 00	0.12733E-01	0.25787E 02
110.00	115.00	0.95312E 00	0.92862E-02	0.26041E 02
115.00	120.00	0.96054E 00	0.74194E-02	0.26243E 02
120.00	125.00	0.96709E 00	0.65529E-02	0.26422E 02
125.00	130.00	0.97322E 00	0.61307E-02	0.26590E 02
130.00	135.00	0.97897E 00	0.57473E-02	0.26747E 02
135.00	140.00	0.98417E 00	0.51980E-02	0.26889E 02
140.00	145.00	0.98863E 00	0.44564E-02	0.27011E 02
145.00	150.00	0.99223E 00	0.36063E-02	0.27109E 02
150.00	155.00	0.99500E 00	0.27628E-02	0.27185E 02
155.00	160.00	0.99701E 00	0.20149E-02	0.27240E 02
160.00	165.00	0.99841E 00	0.14005E-02	0.27278E 02
165.00	170.00	0.99932E 00	0.91110E-03	0.27303E 02
170.00	175.00	0.99983E 00	0.51267E-03	0.27317E 02
175.00	180.00	0.10000E 01	0.16554E-03	0.27321E 02

$$\text{Integral} = T = \int_0^{\pi} \int_0^{\theta} K^2(\theta, \phi)\, d\phi$$

ANTENNA 4/3 SERIES

RATIO= 0.750   LENGTH= 228.60

DEL2= 60.00   H= 1.00   H1= 0.75

RADIUS= 0.20389E 01   GAIN= 0.28213E 02   AREA= 0.11732E 06

TOTAL VOLUME= 0.35504E 02   MAX(K**2)= 0.57523E 02

TOTAL INTEGRAL= 0.35504E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.94661E-02	0.94661E-02	0.33608E-00
5.00	10.00	0.35376E-01	0.25910E-01	0.12560E 01
10.00	15.00	0.71623E-01	0.36246E-01	0.25429E 01
15.00	20.00	0.11157E-00	0.39946E-01	0.39612E 01
20.00	25.00	0.15080E-00	0.39228E-01	0.53539E 01
25.00	30.00	0.18799E-00	0.37190E-01	0.66743E 01
30.00	35.00	0.22397E-00	0.35981E-01	0.79518E 01
35.00	40.00	0.26009E-00	0.36127E-01	0.92344E 01
40.00	45.00	0.29712E-00	0.37028E-01	0.10549E 02
45.00	50.00	0.33500E-00	0.37876E-01	0.11894E 02
50.00	55.00	0.37327E-00	0.38273E-01	0.13253E 02
55.00	60.00	0.41159E-00	0.38324E-01	0.14613E 02
60.00	65.00	0.44999E-00	0.38396E-01	0.15977E 02
65.00	70.00	0.48881E-00	0.38822E-01	0.17355E 02
70.00	75.00	0.52855E 00	0.39737E-01	0.18766E 02
75.00	80.00	0.56961E 00	0.41060E-01	0.20224E 02
80.00	85.00	0.61216E 00	0.42547E-01	0.21734E 02
85.00	90.00	0.65602E 00	0.43866E-01	0.23292E 02
90.00	95.00	0.70068E 00	0.44659E-01	0.24877E 02
95.00	100.00	0.74527E 00	0.44585E-01	0.26460E 02
100.00	105.00	0.78864E 00	0.43367E-01	0.28000E 02
105.00	110.00	0.82948E 00	0.40846E-01	0.29450E 02
110.00	115.00	0.86651E 00	0.37033E-01	0.30765E 02
115.00	120.00	0.89866E 00	0.32143E-01	0.31906E 02
120.00	125.00	0.92524E 00	0.26578E-01	0.32850E 02
125.00	130.00	0.94608E 00	0.20846E-01	0.33590E 02
130.00	135.00	0.96153E 00	0.15445E-01	0.34138E 02
135.00	140.00	0.97230E 00	0.10776E-01	0.34521E 02
140.00	145.00	0.97940E 00	0.71017E-02	0.34773E 02
145.00	150.00	0.98398E 00	0.45748E-02	0.34935E 02
150.00	155.00	0.98721E 00	0.32315E-02	0.35050E 02
155.00	160.00	0.99012E 00	0.29076E-02	0.35153E 02
160.00	165.00	0.99326E 00	0.31384E-02	0.35265E 02
165.00	170.00	0.99648E 00	0.32225E-02	0.35379E 02
170.00	175.00	0.99902E 00	0.25436E-02	0.35469E 02
75.00	180.00	0.10000E 01	0.97793E-03	0.35504E 02

ANTENNA 4/3 SERIES

RATIO= 0.750 LENGTH= 228.60

DEL2= 22.50 H= 3.00 H1= 2.25

RADIUS= 0.21066E 01 GAIN= 0.67756E 02 AREA= 0.31307E 05

TOTAL VOLUME= 0.391615 02 MAX(K**2)= 0.14274E 03

TOTAL INTEGRAL= 0.39161E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.20930E-01	0.20930E-01	0.81964E 00
5.00	10.00	0.74663E-01	0.53732E-01	0.29238E 01
10.00	15.00	0.14231E-00	0.67650E-01	0.55731E 01
15.00	20.00	0.21005E-00	0.67737E-01	0.82257E 01
20.00	25.00	0.27414E-00	0.64093E-01	0.10736E 02
25.00	30.00	0.33496E-00	0.60814E-01	0.13117E 02
30.00	35.00	0.39271E-00	0.57751E-01	0.15379E 02
35.00	40.00	0.44971E-00	0.56999E-01	0.17611E 02
40.00	45.00	0.50908E 00	0.59375E-01	0.19936E 02
45.00	50.00	0.56732E 00	0.58241E-01	0.22217E 02
50.00	55.00	0.61478E 00	0.47457E-01	0.24075E 02
55.00	60.00	0.64713E 00	0.32348E-01	0.25342E 02
60.00	65.00	0.66892E 00	0.21795E-01	0.26196E 02
65.00	70.00	0.68458E 00	0.15660E-01	0.26809E 02
70.00	75.00	0.69579E 00	0.11207E-01	0.27248E 02
75.00	80.00	0.70734E 00	0.11549E-01	0.27700E 02
80.00	85.00	0.72467E 00	0.17333E-01	0.28379E 02
85.00	90.00	0.74657E 00	0.21901E-01	0.29236E 02
90.00	95.00	0.76784E 00	0.21270E-01	0.30069E 02
95.00	100.00	0.78645E 00	0.18610E-01	0.30798E 02
100.00	105.00	0.80440E 00	0.17945E-01	0.31501E 02
105.00	110.00	0.82529E 00	0.20888E-01	0.32319E 02
110.00	115.00	0.85158E 00	0.26297E-01	0.33349E 02
115.00	120.00	0.88112E 00	0.29534E-01	0.34505E 02
120.00	125.00	0.90825E 00	0.27128E-01	0.35568E 02
125.00	130.00	0.93001E 00	0.21762E-01	0.36420E 02
130.00	135.00	0.94778E 00	0.17770E-01	0.37116E 02
135.00	140.00	0.96289E 00	0.15114E-01	0.37708E 02
140.00	145.00	0.97470E 00	0.11810E-01	0.38170E 02
145.00	150.00	0.98284E 00	0.81367E-02	0.38489E 02
150.00	155.00	0.98830E 00	0.54571E-02	0.38702E 02
155.00	160.00	0.99219E 00	0.38920E-02	0.38855E 02
160.00	165.00	0.99511E 00	0.29230E-02	0.38969E 02
165.00	170.00	0.99746E 00	0.23512E-02	0.39061E 02
170.00	175.00	0.99927E 00	0.18087E-02	0.39132E 02
175.00	180.00	0.10000E 01	0.72930E-03	0.39161E 02

V ANTENNA 4/3 SERIES
RATIO= 0.750 LENGTH= 228.60
DEL2= 60.00 H= 7.00 H1= 5.25
RADIUS= 0.24189E 01 GAIN= 0.67419E 02 AREA= 0.57218E 04
TOTAL VOLUME= 0.59284E 02 MAX(K**2)= 0.16308E 03
TOTAL INTEGRAL= 0.59284E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.26869E-02	0.26869E-02	0.15929E-00
5.00	10.00	0.72795E-02	0.45926E-02	0.43156E-00
10.00	15.00	0.17265E-01	0.99859E-02	0.10236E 01
15.00	20.00	0.28816E-01	0.11550E-01	0.17083E 01
20.00	25.00	0.44956E-01	0.16140E-01	0.26652E 01
25.00	30.00	0.60269E-01	0.15313E-01	0.35730E 01
30.00	35.00	0.86054E-01	0.25786E-01	0.51017E 01
35.00	40.00	0.13294E-00	0.46882E-01	0.78810E 01
40.00	45.00	0.20486E-00	0.71927E-01	0.12145E 02
45.00	50.00	0.27267E-00	0.67806E-01	0.16165E 02
50.00	55.00	0.33806E-00	0.65388E-01	0.20041E 02
55.00	60.00	0.39734E-00	0.59284E-01	0.23556E 02
60.00	65.00	0.45712E-00	0.59776E-01	0.27100E 02
65.00	70.00	0.51893E 00	0.61816E-01	0.30765E 02
70.00	75.00	0.59839E 00	0.79452E-01	0.35475E 02
75.00	80.00	0.67813E 00	0.79743E-01	0.40202E 02
80.00	85.00	0.72366E 00	0.45534E-01	0.42902E 02
85.00	90.00	0.75113E 00	0.27467E-01	0.44530E 02
90.00	95.00	0.77808E 00	0.26947E-01	0.46128E 02
95.00	100.00	0.80457E 00	0.26495E-01	0.47698E 02
100.00	105.00	0.82572E 00	0.21152E-01	0.48952E 02
105.00	110.00	0.84495E 00	0.19223E-01	0.50092E 02
110.00	115.00	0.86335E 00	0.18403E-01	0.51183E 02
115.00	120.00	0.88402E 00	0.20670E-01	0.52408E 02
120.00	125.00	0.90038E 00	0.16360E-01	0.53378E 02
125.00	130.00	0.91745E 00	0.17073E-01	0.54390E 02
130.00	135.00	0.93149E 00	0.14039E-01	0.55223E 02
135.00	140.00	0.94519E 00	0.13694E-01	0.56035E 02
140.00	145.00	0.95675E 00	0.11564E-01	0.56720E 02
145.00	150.00	0.96757E 00	0.10815E-01	0.57361E 02
150.00	155.00	0.98005E 00	0.12486E-01	0.58101E 02
155.00	160.00	0.98905E 00	0.90017E-02	0.58635E 02
160.00	165.00	0.99458E 00	0.55317E-02	0.58963E 02
165.00	170.00	0.99814E 00	0.35586E-02	0.59174E 02
170.00	175.00	0.99976E 00	0.16182E-02	0.59270E 02
175.00	180.00	0.10000E 01	0.23833E-03	0.59284E 02

V ANTENNA 4/3 SERIES

RATIO= 0.750 LENGTH= 228.60

DELZ= 22.50 H= 9.00 H1= 6.75

RADIUS= 0.23260E 01 GAIN= 0.14104E 03 AREA= 0.72408E 04

TOTAL VOLUME= 0.52712E 02 MAX(K**2)= 0.32805E 03

TOTAL INTEGRAL= 0.52712E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.29124E-01	0.29124E-01	0.15352E 01
5.00	10.00	0.89417E-01	0.60293E-01	0.47133E 01
10.00	15.00	0.17075E-00	0.81331E-01	0.90005E 01
15.00	20.00	0.24564E-00	0.74894E-01	0.12948E 02
20.00	25.00	0.31579E-00	0.70145E-01	0.16646E 02
25.00	30.00	0.38597E-00	0.70185E-01	0.20345E 02
30.00	35.00	0.47666E-00	0.90688E-01	0.25126E 02
35.00	40.00	0.55005E 00	0.73392E-01	0.28994E 02
40.00	45.00	0.60986E 00	0.59807E-01	0.32147E 02
45.00	50.00	0.63257E 00	0.22712E-01	0.33344E 02
50.00	55.00	0.64902E 00	0.16451E-01	0.34211E 02
55.00	60.00	0.66457E 00	0.15554E-01	0.35031E 02
60.00	65.00	0.68345E 00	0.18877E-01	0.36026E 02
65.00	70.00	0.69918E 00	0.15726E-01	0.36855E 02
70.00	75.00	0.71716E 00	0.17982E-01	0.37803E 02
75.00	80.00	0.73022E 00	0.13060E-01	0.38492E 02
80.00	85.00	0.74253E 00	0.12305E-01	0.39140E 02
85.00	90.00	0.75410E 00	0.11573E-01	0.39750E 02
90.00	95.00	0.76654E 00	0.12441E-01	0.40406E 02
95.00	100.00	0.77983E 00	0.13296E-01	0.41107E 02
100.00	105.00	0.79207E 00	0.12236E-01	0.41752E 02
105.00	110.00	0.80505E 00	0.12982E-01	0.42436E 02
110.00	115.00	0.81799E 00	0.12933E-01	0.43118E 02
115.00	120.00	0.83391E 00	0.15928E-01	0.43957E 02
120.00	125.00	0.85518E 00	0.21270E-01	0.45079E 02
125.00	130.00	0.87531E 00	0.20124E-01	0.46139E 02
130.00	135.00	0.89630E 00	0.20992E-01	0.47246E 02
135.00	140.00	0.91686E 00	0.20562E-01	0.48330E 02
140.00	145.00	0.93144E 00	0.14576E-01	0.49098E 02
145.00	150.00	0.94516E 00	0.13722E-01	0.49822E 02
150.00	155.00	0.95913E 00	0.13973E-01	0.50558E 02
155.00	160.00	0.97319E 00	0.14052E-01	0.51299E 02
160.00	165.00	0.98414E 00	0.10959E-01	0.51876E 02
165.00	170.00	0.99377E 00	0.96299E-02	0.52384E 02
170.00	175.00	0.99835E 00	0.45777E-02	0.52625E 02
175.00	180.00	0.10000E 01	0.16475E-02	0.52712E 02

V ANTENNA 4/3 SERIES

RATIO= 0.750 LENGTH= 228.60

DEL2= 60.00 H= 9.00 H1= 6.75

RADIUS= 0.24746E 01 GAIN= 0.77459E 02 AREA= 0.39767E 04

TOTAL VOLUME= 0.63478E 02 MAX(K**2)= 0.19168E 03

TOTAL INTEGRAL= 0.63478E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.20118E-02	0.20118E-02	0.12771E-00
5.00	10.00	0.75503E-02	0.55385E-02	0.47928E-00
10.00	15.00	0.15929E-01	0.83783E-02	0.10111E 01
15.00	20.00	0.26558E-01	0.10629E-01	0.16858E 01
20.00	25.00	0.41892E-01	0.15335E-01	0.26592E 01
25.00	30.00	0.62467E-01	0.20574E-01	0.39652E 01
30.00	35.00	0.81207E-01	0.18741E-01	0.51548E 01
35.00	40.00	0.11832E-00	0.37108E-01	0.75104E 01
40.00	45.00	0.18017E-00	0.61853E-01	0.11437E 02
45.00	50.00	0.25573E-00	0.75562E-01	0.16233E 02
50.00	55.00	0.32833E-00	0.72601E-01	0.20842E 02
55.00	60.00	0.39125E-00	0.62923E-01	0.24836E 02
60.00	65.00	0.45430E-00	0.63050E-01	0.28838E 02
65.00	70.00	0.52549E 00	0.71186E-01	0.33357E 02
70.00	75.00	0.59966E 00	0.74170E-01	0.38065E 02
75.00	80.00	0.66128E 00	0.61618E-01	0.41976E 02
80.00	85.00	0.71393E 00	0.52650E-01	0.45318E 02
85.00	90.00	0.74076E 00	0.26829E-01	0.47021E 02
90.00	95.00	0.77195E 00	0.31188E-01	0.49001E 02
95.00	100.00	0.80245E 00	0.30506E-01	0.50938E 02
100.00	105.00	0.82692E 00	0.24471E-01	0.52491E 02
105.00	110.00	0.84860E 00	0.21681E-01	0.53867E 02
110.00	115.00	0.86890E 00	0.20299E-01	0.55156E 02
115.00	120.00	0.88723E 00	0.18327E-01	0.56319E 02
120.00	125.00	0.90318E 00	0.15951E-01	0.57332E 02
125.00	130.00	0.91847E 00	0.15291E-01	0.58302E 02
130.00	135.00	0.93334E 00	0.14863E-01	0.59246E 02
135.00	140.00	0.94819E 00	0.14858E-01	0.60189E 02
140.00	145.00	0.96035E 00	0.12160E-01	0.60961E 02
145.00	150.00	0.97267E 00	0.12315E-01	0.61743E 02
150.00	155.00	0.98296E 00	0.10290E-01	0.62396E 02
155.00	160.00	0.98935E 00	0.63900E-02	0.62801E 02
160.00	165.00	0.99540E 00	0.60512E-02	0.63185E 02
165.00	170.00	0.99816E 00	0.27565E-02	0.63360E 02
170.00	175.00	0.99978E 00	0.16235E-02	0.63464E 02
175.00	180.00	0.10000E 01	0.22095E-03	0.63478E 02

V ANTENNA 4/3 SERIES
RATIO= 0.750 LENGTH= 228.60
DEL2= 22.50 H= 11.00 H1= 8.25
RADIUS= 0.23603E 01 GAIN= 0.12559E 03 AREA= 0.43163E 04
TOTAL VOLUME= 0.55078E 02 MAX(K**2)= 0.29643E 03
TOTAL INTEGRAL= 0.55078E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.17889E-01	0.17889E-01	0.98530E 00
5.00	10.00	0.82026E-01	0.64136E-01	0.45178E 01
10.00	15.00	0.16843E-00	0.86407E-01	0.92769E 01
15.00	20.00	0.24704E-00	0.78612E-01	0.13607E 02
20.00	25.00	0.32145E-00	0.74407E-01	0.17705E 02
25.00	30.00	0.40514E-00	0.83693E-01	0.22315E 02
30.00	35.00	0.49313E-00	0.87982E-01	0.27160E 02
35.00	40.00	0.57310E 00	0.79975E-01	0.31565E 02
40.00	45.00	0.61276E 00	0.39662E-01	0.33750E 02
45.00	50.00	0.63187E 00	0.19103E-01	0.34802E 02
50.00	55.00	0.65412E 00	0.22249E-01	0.36027E 02
55.00	60.00	0.67211E 00	0.17993E-01	0.37018E 02
60.00	65.00	0.68769E 00	0.15583E-01	0.37877E 02
65.00	70.00	0.70717E 00	0.19482E-01	0.38950E 02
70.00	75.00	0.72150E 00	0.14325E-01	0.39739E 02
75.00	80.00	0.73411E 00	0.12607E-01	0.40433E 02
80.00	85.00	0.74432E 00	0.10211E-01	0.40995E 02
85.00	90.00	0.75500E 00	0.10685E-01	0.41584E 02
90.00	95.00	0.76885E 00	0.13852E-01	0.42347E 02
95.00	100.00	0.78246E 00	0.13609E-01	0.43096E 02
100.00	105.00	0.79329E 00	0.10828E-01	0.43693E 02
105.00	110.00	0.80267E 00	0.93837E-02	0.44210E 02
110.00	115.00	0.81509E 00	0.12417E-01	0.44894E 02
115.00	120.00	0.83077E 00	0.15682E-01	0.45757E 02
120.00	125.00	0.84635E 00	0.15576E-01	0.46615E 02
125.00	130.00	0.86491E 00	0.18558E-01	0.47637E 02
130.00	135.00	0.88670E 00	0.21797E-01	0.48838E 02
135.00	140.00	0.91035E 00	0.23645E-01	0.50140E 02
140.00	145.00	0.92756E 00	0.17208E-01	0.51088E 02
145.00	150.00	0.94208E 00	0.14524E-01	0.51888E 02
150.00	155.00	0.95491E 00	0.12830E-01	0.52595E 02
155.00	160.00	0.96783E 00	0.12920E-01	0.53306E 02
160.00	165.00	0.98117E 00	0.13336E-01	0.54041E 02
165.00	170.00	0.99084E 00	0.96767E-02	0.54574E 02
170.00	175.00	0.99814E 00	0.72954E-02	0.54975E 02
175.00	180.00	0.10000E 01	0.18605E-02	0.55078E 02

V ANTENNA 4/3 SERIES

RATIC= 0.750 LENGTH= 228.60

DEL2= 60.00 H= 11.00 H1= 8.25

RADIUS= 0.24983E 01 GAIN= 0.80439E 02 AREA= 0.27645E 04

TOTAL VOLUME= 0.65315E 02 MAX(K**2)= 0.20096E 03

TOTAL INTEGRAL= 0.65315E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.11105E-02	0.11105E-02	0.72531E-01
5.00	10.00	0.59595E-02	0.48490E-02	0.38925E-00
10.00	15.00	0.14951E-01	0.89916E-02	0.97653E 00
15.00	20.00	0.27543E-01	0.12592E-01	0.17990E 01
20.00	25.00	0.42131E-01	0.14588E-01	0.27518E 01
25.00	30.00	0.60572E-01	0.18441E-01	0.39563E 01
30.00	35.00	0.82479E-01	0.21907E-01	0.53872E 01
35.00	40.00	0.11188E-00	0.29397E-01	0.73072E 01
40.00	45.00	0.17012E-00	0.58246E-01	0.11112E 02
45.00	50.00	0.24885E-00	0.78730E-01	0.16254E 02
50.00	55.00	0.32578E-00	0.76931E-01	0.21279E 02
55.00	60.00	0.39369E-00	0.67904E-01	0.25714E 02
60.00	65.00	0.46451E-00	0.70817E-01	0.30339E 02
65.00	70.00	0.54003E 00	0.75527E-01	0.35272E 02
70.00	75.00	0.61944E 00	0.79404E-01	0.40459E 02
75.00	80.00	0.68404E 00	0.64602E-01	0.44678E 02
80.00	85.00	0.71123E 00	0.27195E-01	0.46454E 02
85.00	90.00	0.74073E 00	0.29494E-01	0.48381E 02
90.00	95.00	0.76619E 00	0.25458E-01	0.50044E 02
95.00	100.00	0.79751E 00	0.31320E-01	0.52089E 02
100.00	105.00	0.82411E 00	0.26607E-01	0.53827E 02
105.00	110.00	0.84656E 00	0.22443E-01	0.55293E 02
110.00	115.00	0.86745E 00	0.20891E-01	0.56658E 02
115.00	120.00	0.88448E 00	0.17037E-01	0.57770E 02
120.00	125.00	0.90101E 00	0.16532E-01	0.58850E 02
125.00	130.00	0.91806E 00	0.17044E-01	0.59963E 02
130.00	135.00	0.93472E 00	0.16664E-01	0.61052E 02
135.00	140.00	0.94911E 00	0.14384E-01	0.61991E 02
140.00	145.00	0.96143E 00	0.12323E-01	0.62796E 02
145.00	150.00	0.97438E 00	0.12952E-01	0.63642E 02
150.00	155.00	0.98420E 00	0.98181E-02	0.64283E 02
155.00	160.00	0.99143E 00	0.72271E-02	0.64755E 02
160.00	165.00	0.99467E 00	0.32387E-02	0.64967E 02
165.00	170.00	0.99708E 00	0.24107E-02	0.65124E 02
170.00	175.00	0.99912E 00	0.20472E-02	0.65258E 02
175.00	180.00	0.10000E 01	0.87558E-03	0.65315E 02

ANTENNA 4/3 SERIES

RATIO= 0.750 LENGTH= 228.60

IEL2= 60.00 H= 3.00 H1= 2.25

RADIUS= 0.22698E 01 GAIN= 0.35077E 02 AREA= 0.16208E 05

TOTAL VOLUME= 0.48981E 02 MAX(K**2)= 0.79617E 02

TOTAL INTEGRAL= 0.48981E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.20482E-02	0.20482E-02	0.10032E-00
5.00	10.00	0.59443E-02	0.38961E-02	0.29116E-00
10.00	15.00	0.11538E-01	0.55941E-02	0.56516E 00
15.00	20.00	0.25159E-01	0.13621E-01	0.12323E 01
20.00	25.00	0.51066E-01	0.25907E-01	0.25013E 01
25.00	30.00	0.83767E-01	0.32701E-01	0.41030E 01
30.00	35.00	0.13084E-00	0.47072E-01	0.64087E 01
35.00	40.00	0.19299E-00	0.62150E-01	0.94529E 01
40.00	45.00	0.24929E-00	0.56298E-01	0.12210E 02
45.00	50.00	0.29671E-00	0.47421E-01	0.14533E 02
50.00	55.00	0.34583E-00	0.49120E-01	0.16939E 02
55.00	60.00	0.39740E-00	0.51570E-01	0.19465E 02
60.00	65.00	0.45060E-00	0.53203E-01	0.22071E 02
65.00	70.00	0.50875E 00	0.58150E-01	0.24919E 02
70.00	75.00	0.57194E 00	0.63187E-01	0.28014E 02
75.00	80.00	0.63554E 00	0.63604E-01	0.31130E 02
80.00	85.00	0.69478E 00	0.59241E-01	0.34031E 02
85.00	90.00	0.74649E 00	0.51703E-01	0.36564E 02
90.00	95.00	0.78806E 00	0.41578E-01	0.38600E 02
95.00	100.00	0.81796E 00	0.29896E-01	0.40065E 02
100.00	105.00	0.83783E 00	0.19870E-01	0.41038E 02
105.00	110.00	0.85281E 00	0.14982E-01	0.41772E 02
110.00	115.00	0.86787E 00	0.15062E-01	0.42509E 02
115.00	120.00	0.88400E 00	0.16123E-01	0.43299E 02
120.00	125.00	0.89929E 00	0.15296E-01	0.44048E 02
125.00	130.00	0.91304E 00	0.13747E-01	0.44722E 02
130.00	135.00	0.92644E 00	0.13399E-01	0.45378E 02
135.00	140.00	0.93964E 00	0.13200E-01	0.46025E 02
140.00	145.00	0.95087E 00	0.11229E-01	0.46575E 02
145.00	150.00	0.96161E 00	0.10744E-01	0.47101E 02
150.00	155.00	0.97365E 00	0.12038E-01	0.47691E 02
155.00	160.00	0.98213E 00	0.84802E-02	0.48106E 02
160.00	165.00	0.98839E 00	0.62575E-02	0.48412E 02
165.00	170.00	0.99379E 00	0.54052E-02	0.48677E 02
170.00	175.00	0.99739E 00	0.35924E-02	0.48853E 02
175.00	180.00	0.10000E 01	0.26141E-02	0.48981E 02

ANTENNA 4/3 SERIES

RATIO= 0.750 LENGTH= 226.60

DEL2= 22.50 H= 5.00 H1= 3.75

RADIUS= 0.22226E 01 GAIN= 0.13542E 03 AREA= 0.22526E 05

TOTAL VOLUME= 0.45990E 02 MAX(K**2)= 0.30098E 03

TOTAL INTEGRAL= 0.45990E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.35158E-01	0.35158E-01	0.16169E 01
5.00	10.00	0.10650E-00	0.71340E-01	0.48979E 01
10.00	15.00	0.17224E-00	0.65744E-01	0.79214E 01
15.00	20.00	0.23161E-00	0.59372E-01	0.10652E 02
20.00	25.00	0.29111E-00	0.59491E-01	0.13388E 02
25.00	30.00	0.35270E-00	0.61593E-01	0.16221E 02
30.00	35.00	0.41671E-00	0.64013E-01	0.19165E 02
35.00	40.00	0.48686E-00	0.70150E-01	0.22391E 02
40.00	45.00	0.55995E 00	0.73084E-01	0.25752E 02
45.00	50.00	0.60652E 00	0.46572E-01	0.27894E 02
50.00	55.00	0.63044E 00	0.23924E-01	0.28994E 02
55.00	60.00	0.65142E 00	0.20982E-01	0.29959E 02
60.00	65.00	0.66706E 00	0.15638E-01	0.30678E 02
65.00	70.00	0.68590E 00	0.18834E-01	0.31544E 02
70.00	75.00	0.70178E 00	0.15880E-01	0.32275E 02
75.00	80.00	0.71440E 00	0.12626E-01	0.32855E 02
80.00	85.00	0.72603E 00	0.11629E-01	0.33390E 02
85.00	90.00	0.73983E 00	0.13795E-01	0.34025E 02
90.00	95.00	0.75884E 00	0.19011E-01	0.34899E 02
95.00	100.00	0.77465E 00	0.15818E-01	0.35627E 02
100.00	105.00	0.79614E 00	0.21481E-01	0.36614E 02
105.00	110.00	0.82160E 00	0.25462E-01	0.37785E 02
110.00	115.00	0.84135E 00	0.19752E-01	0.38694E 02
115.00	120.00	0.85982E 00	0.18465E-01	0.39543E 02
120.00	125.00	0.87916E 00	0.19341E-01	0.40433E 02
125.00	130.00	0.89975E 00	0.20594E-01	0.41380E 02
130.00	135.00	0.91873E 00	0.18976E-01	0.42252E 02
135.00	140.00	0.93566E 00	0.16938E-01	0.43031E 02
140.00	145.00	0.95094E 00	0.15280E-01	0.43734E 02
145.00	150.00	0.96488E 00	0.13938E-01	0.44375E 02
150.00	155.00	0.97762E 00	0.12738E-01	0.44961E 02
155.00	160.00	0.98744E 00	0.98150E-02	0.45412E 02
160.00	165.00	0.99360E 00	0.61680E-02	0.45696E 02
165.00	170.00	0.99662E 00	0.30141E-02	0.45835E 02
170.00	175.00	0.99874E 00	0.21180E-02	0.45932E 02
175.00	180.00	0.10000E 01	0.12649E-02	0.45990E 02

ANTENNA 4/3 SERIES

ATIO= 0.750 LENGTH= 228.60

EL2= 60.00 H= 5.00 H1= 3.75

ADIUS= 0.23751E 01 GAIN= 0.47991E 02 AREA= 0.79829E 04

TOTAL VOLUME= 0.56119E 02 MAX(K**2)= 0.11398E 03

TOTAL INTEGRAL= 0.56119E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.14423E-02	0.14423E-02	0.80939E-01
5.00	10.00	0.50297E-02	0.35874E-02	0.28226E-00
10.00	15.00	0.16368E-01	0.11339E-01	0.91857E 00
15.00	20.00	0.27502E-01	0.11134E-01	0.15434E 01
20.00	25.00	0.39299E-01	0.11797E-01	0.22054E 01
25.00	30.00	0.64932E-01	0.25632E-01	0.36439E 01
30.00	35.00	0.10120E-00	0.36270E-01	0.56793E 01
35.00	40.00	0.15753E-00	0.56333E-01	0.88407E 01
40.00	45.00	0.22017E-00	0.62632E-01	0.12356E 02
45.00	50.00	0.27981E-00	0.59644E-01	0.15703E 02
50.00	55.00	0.33357E-00	0.53759E-01	0.18720E 02
55.00	60.00	0.38873E-00	0.55162E-01	0.21815E 02
60.00	65.00	0.44643E-00	0.57701E-01	0.25053E 02
65.00	70.00	0.50739E 00	0.60960E-01	0.28474E 02
70.00	75.00	0.56488E 00	0.57490E-01	0.31701E 02
75.00	80.00	0.62742E 00	0.62543E-01	0.35211E 02
80.00	85.00	0.69668E 00	0.69257E-01	0.39097E 02
85.00	90.00	0.75425E 00	0.57570E-01	0.42328E 02
90.00	95.00	0.78908E 00	0.34824E-01	0.44282E 02
95.00	100.00	0.81034E 00	0.21268E-01	0.45476E 02
100.00	105.00	0.82998E 00	0.19633E-01	0.46578E 02
105.00	110.00	0.84877E 00	0.18796E-01	0.47632E 02
110.00	115.00	0.86608E 00	0.17313E-01	0.48604E 02
115.00	120.00	0.88300E 00	0.16918E-01	0.49553E 02
120.00	125.00	0.89838E 00	0.15372E-01	0.50416E 02
125.00	130.00	0.91544E 00	0.17063E-01	0.51374E 02
130.00	135.00	0.93247E 00	0.17032E-01	0.52329E 02
135.00	140.00	0.94499E 00	0.12520E-01	0.53032E 02
140.00	145.00	0.95933E 00	0.14336E-01	0.53837E 02
145.00	150.00	0.96796E 00	0.86287E-02	0.54321E 02
150.00	155.00	0.97579E 00	0.78367E-02	0.54761E 02
155.00	160.00	0.98384E 00	0.80439E-02	0.55212E 02
160.00	165.00	0.99160E 00	0.77645E-02	0.55648E 02
165.00	170.00	0.99582E 00	0.42164E-02	0.55884E 02
170.00	175.00	0.99843E 00	0.26088E-02	0.56031E 02
175.00	180.00	0.10000E 01	0.15742E-02	0.56119E 02

ANTENNA 4/3 SERIES
RATIO= 0.750 LENGTH= 228.60
DEL2= 22.50 H= 7.00 H1= 5.25
RADIUS= 0.22768E 01 GAIN= 0.16693E 03 AREA= 0.14167E 05
TOTAL VOLUME= 0.49438E 02 MAX(K**2)= 0.38006E 03
TOTAL INTEGRAL= 0.49438E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.38006E-01	0.38006E-01	0.18789E 01
5.00	10.00	0.10272E-00	0.64719E-01	0.50785E 01
10.00	15.00	0.17145E-00	0.68729E-01	0.84763E 01
15.00	20.00	0.24224E-00	0.70790E-01	0.11976E 02
20.00	25.00	0.30819E-00	0.65946E-01	0.15236E 02
25.00	30.00	0.37821E-00	0.70016E-01	0.18698E 02
30.00	35.00	0.45094E-00	0.72730E-01	0.22293E 02
35.00	40.00	0.52169E 00	0.70751E-01	0.25791E 02
40.00	45.00	0.59933E 00	0.77638E-01	0.29629E 02
45.00	50.00	0.63351E 00	0.34189E-01	0.31320E 02
50.00	55.00	0.65134E 00	0.17829E-01	0.32201E 02
55.00	60.00	0.66656E 00	0.15219E-01	0.32954E 02
60.00	65.00	0.68018E 00	0.13620E-01	0.33627E 02
65.00	70.00	0.69538E 00	0.15193E-01	0.34378E 02
70.00	75.00	0.70868E 00	0.13308E-01	0.35036E 02
75.00	80.00	0.72666E 00	0.17976E-01	0.35925E 02
80.00	85.00	0.74253E 00	0.15867E-01	0.36709E 02
85.00	90.00	0.75668E 00	0.14152E-01	0.37409E 02
90.00	95.00	0.76865E 00	0.11967E-01	0.38000E 02
95.00	100.00	0.77740E 00	0.87583E-02	0.38433E 02
100.00	105.00	0.78871E 00	0.11309E-01	0.38992E 02
105.00	110.00	0.80060E 00	0.11889E-01	0.39580E 02
110.00	115.00	0.82103E 00	0.20432E-01	0.40590E 02
115.00	120.00	0.84674E 00	0.25701E-01	0.41861E 02
120.00	125.00	0.86913E 00	0.22398E-01	0.42968E 02
125.00	130.00	0.88683E 00	0.17701E-01	0.43843E 02
130.00	135.00	0.90448E 00	0.17645E-01	0.44716E 02
135.00	140.00	0.92094E 00	0.16462E-01	0.45530E 02
140.00	145.00	0.93657E 00	0.15624E-01	0.46302E 02
145.00	150.00	0.95275E 00	0.16183E-01	0.47102E 02
150.00	155.00	0.96699E 00	0.14238E-01	0.47806E 02
155.00	160.00	0.97872E 00	0.11735E-01	0.48386E 02
160.00	165.00	0.98888E 00	0.10157E-01	0.48888E 02
165.00	170.00	0.99595E 00	0.70694E-02	0.49238E 02
170.00	175.00	0.99844E 00	0.24950E-02	0.49361E 02
175.00	180.00	0.10000E 01	0.15565E-02	0.49436E 02

V ANTENNA 4/3 SERIES

RATIO= 0.750 LENGTH= 228.60

DEL2= 22.50 H= 7.00 H1= 5.25

RADIUS= 0.22768E 01 GAIN= 0.16693E 03 AREA= 0.14167E 05

TOTAL VOLUME= 0.49438E 02 MAX(K**2)= 0.38006E 03

TOTAL INTEGRAL= 0.49438E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.38006E-01	0.38006E-01	0.18789E 01
5.00	10.00	0.10272E-00	0.64719E-01	0.50785E 01
10.00	15.00	0.17145E-00	0.68729E-01	0.84763E 01
15.00	20.00	0.24224E-00	0.70790E-01	0.11976E 02
20.00	25.00	0.30819E-00	0.65946E-01	0.15236E 02
25.00	30.00	0.37821E-00	0.70016E-01	0.18698E 02
30.00	35.00	0.45094E-00	0.72730E-01	0.22293E 02
35.00	40.00	0.52169E 00	0.70751E-01	0.25791E 02
40.00	45.00	0.59933E 00	0.77638E-01	0.29629E 02
45.00	50.00	0.63351E 00	0.34189E-01	0.31320E 02
50.00	55.00	0.65134E 00	0.17829E-01	0.32201E 02
55.00	60.00	0.66656E 00	0.15219E-01	0.32954E 02
60.00	65.00	0.68018E 00	0.13620E-01	0.33627E 02
65.00	70.00	0.69538E 00	0.15193E-01	0.34378E 02
70.00	75.00	0.70868E 00	0.13308E-01	0.35036E 02
75.00	80.00	0.72666E 00	0.17976E-01	0.35925E 02
80.00	85.00	0.74253E 00	0.15867E-01	0.36709E 02
85.00	90.00	0.75668E 00	0.14152E-01	0.37409E 02
90.00	95.00	0.76865E 00	0.11967E-01	0.38000E 02
95.00	100.00	0.77740E 00	0.87583E-02	0.38433E 02
100.00	105.00	0.78871E 00	0.11309E-01	0.38992E 02
105.00	110.00	0.80060E 00	0.11889E-01	0.39580E 02
110.00	115.00	0.82103E 00	0.20432E-01	0.40590E 02
115.00	120.00	0.84674E 00	0.25701E-01	0.41861E 02
120.00	125.00	0.86913E 00	0.22398E-01	0.42968E 02
125.00	130.00	0.88683E 00	0.17701E-01	0.43843E 02
130.00	135.00	0.90448E 00	0.17645E-01	0.44716E 02
135.00	140.00	0.92094E 00	0.16462E-01	0.45530E 02
140.00	145.00	0.93657E 00	0.15624E-01	0.46302E 02
145.00	150.00	0.95275E 00	0.16183E-01	0.47102E 02
150.00	155.00	0.96699E 00	0.14238E-01	0.47806E 02
155.00	160.00	0.97872E 00	0.11735E-01	0.48386E 02
160.00	165.00	0.98888E 00	0.10157E-01	0.48888E 02
165.00	170.00	0.99595E 00	0.70694E-02	0.49238E 02
170.00	175.00	0.99844E 00	0.24950E-02	0.49361E 02
175.00	180.00	0.10000E 01	0.15565E-02	0.49438E 02

V ANTENNA 5/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 22.50 H= 1.25 H1= 1.00

RADIUS= 0.19346E 01 GAIN= 0.15399E 02 AREA= 0.40984E 05

TOTAL VOLUME= 0.30328E 02 MAX(K**2)= 0.29790E 02

TOTAL INTEGRAL= 0.30328E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.58290E-02	0.58290E-02	0.17678E-00
5.00	10.00	0.22793E-01	0.16964E-01	0.69125E 00
10.00	15.00	0.49528E-01	0.26735E-01	0.15021E 01
15.00	20.00	0.84370E-01	0.34843E-01	0.25588E 01
20.00	25.00	0.12600E-00	0.41625E-01	0.38212E 01
25.00	30.00	0.17387E-00	0.47878E-01	0.52732E 01
30.00	35.00	0.22831E-00	0.54437E-01	0.69242E 01
35.00	40.00	0.29000E-00	0.61692E-01	0.87952E 01
40.00	45.00	0.35925E-00	0.69244E-01	0.10895E 02
45.00	50.00	0.43515E-00	0.75899E-01	0.13197E 02
50.00	55.00	0.51517E 00	0.80028E-01	0.15624E 02
55.00	60.00	0.59534E 00	0.80171E-01	0.18056E 02
60.00	65.00	0.67097E 00	0.75630E-01	0.20349E 02
65.00	70.00	0.73775E 00	0.66774E-01	0.22374E 02
70.00	75.00	0.79270E 00	0.54955E-01	0.24041E 02
75.00	80.00	0.83480E 00	0.42097E-01	0.25318E 02
80.00	85.00	0.86496E 00	0.30155E-01	0.26232E 02
85.00	90.00	0.88560E 00	0.20649E-01	0.26859E 02
90.00	95.00	0.89998E 00	0.14374E-01	0.27295E 02
95.00	100.00	0.91130E 00	0.11325E-01	0.27638E 02
100.00	105.00	0.92210E 00	0.10801E-01	0.27966E 02
105.00	110.00	0.93377E 00	0.11667E-01	0.28319E 02
110.00	115.00	0.94648E 00	0.12714E-01	0.28705E 02
115.00	120.00	0.95949E 00	0.13009E-01	0.29099E 02
120.00	125.00	0.97162E 00	0.12129E-01	0.29467E 02
125.00	130.00	0.98180E 00	0.10177E-01	0.29776E 02
130.00	135.00	0.98942E 00	0.76228E-02	0.30007E 02
135.00	140.00	0.99446E 00	0.50415E-02	0.30160E 02
140.00	145.00	0.99735E 00	0.28915E-02	0.30248E 02
145.00	150.00	0.99875E 00	0.13983E-02	0.30290E 02
150.00	155.00	0.99931E 00	0.55719E-03	0.30307E 02
155.00	160.00	0.99952E 00	0.20890E-03	0.30313E 02
160.00	165.00	0.99966E 00	0.13772E-03	0.30318E 02
165.00	170.00	0.99981E 00	0.15266E-03	0.30322E 02
170.00	175.00	0.99994E 00	0.13586E-03	0.30326E 02
175.00	180.00	0.10000E 01	0.55154E-04	0.30328E 02

V ANTENNA 5/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 60.00 H= 1.25 H1= 1.00

RADIUS= 0.21051E 01 GAIN= 0.27326E 02 AREA= 0.72728E 05

TOTAL VOLUME= 0.39073E 02 MAX(K**2)= 0.57523E 02

TOTAL INTEGRAL= 0.39073E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.84814E-02	0.84814E-02	0.33140E-00
5.00	10.00	0.30539E-01	0.22057E-01	0.11933E 01
10.00	15.00	0.59082E-01	0.28544E-01	0.23086E 01
15.00	20.00	0.89202E-01	0.30120E-01	0.34854E 01
20.00	25.00	0.12090E-00	0.31702E-01	0.47241E 01
25.00	30.00	0.15685E-00	0.35947E-01	0.61287E 01
30.00	35.00	0.19847E-00	0.41618E-01	0.77548E 01
35.00	40.00	0.24435E-00	0.45884E-01	0.95477E 01
40.00	45.00	0.29157E-00	0.47221E-01	0.11393E 02
45.00	50.00	0.33780E-00	0.46227E-01	0.13199E 02
50.00	55.00	0.38232E-00	0.44516E-01	0.14938E 02
55.00	60.00	0.42569E-00	0.43369E-01	0.16633E 02
60.00	65.00	0.46888E-00	0.43197E-01	0.18321E 02
65.00	70.00	0.51267E 00	0.43786E-01	0.20032E 02
70.00	75.00	0.55742E 00	0.44746E-01	0.21780E 02
75.00	80.00	0.60318E 00	0.45765E-01	0.23568E 02
80.00	85.00	0.64981E 00	0.46626E-01	0.25390E 02
85.00	90.00	0.69691E 00	0.47099E-01	0.27230E 02
90.00	95.00	0.74377E 00	0.46860E-01	0.29061E 02
95.00	100.00	0.78927E 00	0.45502E-01	0.30839E 02
100.00	105.00	0.83190E 00	0.42636E-01	0.32505E 02
105.00	110.00	0.86995E 00	0.38051E-01	0.33992E 02
110.00	115.00	0.90184E 00	0.31881E-01	0.35238E 02
115.00	120.00	0.92654E 00	0.24702E-01	0.36203E 02
120.00	125.00	0.94402E 00	0.17484E-01	0.36886E 02
125.00	130.00	0.95538E 00	0.11364E-01	0.37330E 02
130.00	135.00	0.96263E 00	0.72473E-02	0.37613E 02
135.00	140.00	0.96801E 00	0.53830E-02	0.37824E 02
140.00	145.00	0.97320E 00	0.51873E-02	0.38026E 02
145.00	150.00	0.97875E 00	0.55502E-02	0.38243E 02
150.00	155.00	0.98429E 00	0.55363E-02	0.38460E 02
155.00	160.00	0.98923E 00	0.49393E-02	0.38653E 02
160.00	165.00	0.99336E 00	0.41313E-02	0.38814E 02
165.00	170.00	0.99672E 00	0.33630E-02	0.38945E 02
170.00	175.00	0.99911E 00	0.23856E-02	0.39039E 02
175.00	180.00	0.10000E 01	0.89200E-03	0.39073E 02

V ANTENNA 5/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 22.50 H= 3.75 H1= 3.00

RADIUS= 0.21567E 01 GAIN= 0.10008E 03 AREA= 0.29597E 05

TOTAL VOLUME= 0.42018E 02 MAX(K**2)= 0.21585E 03

TOTAL INTEGRAL= 0.42018E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.28803E-01	0.28803E-01	0.12102E 01
5.00	10.00	0.96460E-01	0.67657E-01	0.40530E 01
10.00	15.00	0.17009E-00	0.73630E-01	0.71468E 01
15.00	20.00	0.23552E-00	0.65427E-01	0.98959E 01
20.00	25.00	0.29540E-00	0.59879E-01	0.12412E 02
25.00	30.00	0.35369E-00	0.58292E-01	0.14861E 02
30.00	35.00	0.41590E-00	0.62210E-01	0.17475E 02
35.00	40.00	0.48652E-00	0.70620E-01	0.20443E 02
40.00	45.00	0.55827E 00	0.71749E-01	0.23457E 02
45.00	50.00	0.62027E 00	0.62004E-01	0.26063E 02
50.00	55.00	0.66846E 00	0.48191E-01	0.28087E 02
55.00	60.00	0.69771E 00	0.29252E-01	0.29317E 02
60.00	65.00	0.70974E 00	0.12030E-01	0.29822E 02
65.00	70.00	0.72083E 00	0.11089E-01	0.30288E 02
70.00	75.00	0.73610E 00	0.15267E-01	0.30929E 02
75.00	80.00	0.74678E 00	0.10680E-01	0.31378E 02
80.00	85.00	0.75633E 00	0.95551E-02	0.31780E 02
85.00	90.00	0.77231E 00	0.15978E-01	0.32451E 02
90.00	95.00	0.79009E 00	0.17781E-01	0.33198E 02
95.00	100.00	0.80746E 00	0.17364E-01	0.33928E 02
100.00	105.00	0.82979E 00	0.22331E-01	0.34866E 02
105.00	110.00	0.85506E 00	0.25270E-01	0.35928E 02
110.00	115.00	0.87731E 00	0.22248E-01	0.36863E 02
115.00	120.00	0.89719E 00	0.19889E-01	0.37698E 02
120.00	125.00	0.91514E 00	0.17941E-01	0.38452E 02
125.00	130.00	0.93030E 00	0.15163E-01	0.39089E 02
130.00	135.00	0.94607E 00	0.15770E-01	0.39752E 02
135.00	140.00	0.96307E 00	0.17002E-01	0.40466E 02
140.00	145.00	0.97641E 00	0.13342E-01	0.41027E 02
145.00	150.00	0.98434E 00	0.79315E-02	0.41360E 02
150.00	155.00	0.98990E 00	0.55527E-02	0.41594E 02
155.00	160.00	0.99474E 00	0.48444E-02	0.41797E 02
160.00	165.00	0.99789E 00	0.31506E-02	0.41929E 02
165.00	170.00	0.99917E 00	0.12732E-02	0.41983E 02
170.00	175.00	0.99973E 00	0.56950E-03	0.42007E 02
175.00	180.00	0.10000E 01	0.26535E-03	0.42018E 02

V ANTENNA 5/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 60.00 H= 3.75 H1= 3.00

RADIUS= 0.23124E 01 GAIN= 0.42703E 02 AREA= 0.12628E 05

TOTAL VOLUME= 0.51791E 02 MAX(K**2)= 0.98744E 02

TOTAL INTEGRAL= 0.51791E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.21826E-02	0.21826E-02	0.11304E-00
5.00	10.00	0.83434E-02	0.61608E-02	0.43211E-00
10.00	15.00	0.15782E-01	0.74382E-02	0.81734E 00
15.00	20.00	0.23277E-01	0.74953E-02	0.12055E 01
20.00	25.00	0.40110E-01	0.16833E-01	0.20773E 01
25.00	30.00	0.75161E-01	0.35052E-01	0.38927E 01
30.00	35.00	0.11661E-00	0.41448E-01	0.60393E 01
35.00	40.00	0.17714E-00	0.60528E-01	0.91741E 01
40.00	45.00	0.24332E-00	0.66184E-01	0.12602E 02
45.00	50.00	0.29700E-00	0.53678E-01	0.15382E 02
50.00	55.00	0.35215E-00	0.55148E-01	0.18238E 02
55.00	60.00	0.40919E-00	0.57042E-01	0.21192E 02
60.00	65.00	0.46750E-00	0.58307E-01	0.24212E 02
65.00	70.00	0.52936E 00	0.61861E-01	0.27416E 02
70.00	75.00	0.59027E 00	0.60913E-01	0.30571E 02
75.00	80.00	0.64827E 00	0.58005E-01	0.33575E 02
80.00	85.00	0.70512E 00	0.56843E-01	0.36519E 02
85.00	90.00	0.75776E 00	0.52641E-01	0.39245E 02
90.00	95.00	0.79880E 00	0.41043E-01	0.41371E 02
95.00	100.00	0.82541E 00	0.26603E-01	0.42749E 02
100.00	105.00	0.84371E 00	0.18300E-01	0.43696E 02
105.00	110.00	0.86189E 00	0.18184E-01	0.44638E 02
110.00	115.00	0.88072E 00	0.18832E-01	0.45613E 02
115.00	120.00	0.89604E 00	0.15317E-01	0.46407E 02
120.00	125.00	0.90805E 00	0.12011E-01	0.47029E 02
125.00	130.00	0.91946E 00	0.11412E-01	0.47620E 02
130.00	135.00	0.93026E 00	0.10794E-01	0.48179E 02
135.00	140.00	0.94101E 00	0.10757E-01	0.48736E 02
140.00	145.00	0.95190E 00	0.10890E-01	0.49300E 02
145.00	150.00	0.96249E 00	0.10583E-01	0.49848E 02
150.00	155.00	0.97208E 00	0.95935E-02	0.50345E 02
155.00	160.00	0.98366E 00	0.11578E-01	0.50944E 02
160.00	165.00	0.99050E 00	0.68464E-02	0.51299E 02
165.00	170.00	0.99479E 00	0.42828E-02	0.51521E 02
170.00	175.00	0.99787E 00	0.30836E-02	0.51681E 02
175.00	180.00	0.10000E 01	0.21302E-02	0.51791E 02

V ANTENNA 5/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 22.50 H= 6.25 H1= 5.00

RADIUS= 0.22721E 01 GAIN= 0.16797E 03 AREA= 0.17882E 05

TOTAL VOLUME= 0.49133E 02 MAX(K**2)= 0.38164E 03

TOTAL INTEGRAL= 0.49133E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.39310E-01	0.39310E-01	0.19314E 01
5.00	10.00	0.10726E-00	0.67952E-01	0.52701E 01
10.00	15.00	0.17217E-00	0.64912E-01	0.84594E 01
15.00	20.00	0.24258E-00	0.70408E-01	0.11919E 02
20.00	25.00	0.31151E-00	0.68923E-01	0.15305E 02
25.00	30.00	0.37987E-00	0.68360E-01	0.18664E 02
30.00	35.00	0.45285E-00	0.72985E-01	0.22250E 02
35.00	40.00	0.52356E 00	0.70706E-01	0.25724E 02
40.00	45.00	0.58970E 00	0.66140E-01	0.28973E 02
45.00	50.00	0.64090E 00	0.51203E-01	0.31489E 02
50.00	55.00	0.65808E 00	0.17185E-01	0.32333E 02
55.00	60.00	0.67474E 00	0.16651E-01	0.33152E 02
60.00	65.00	0.69193E 00	0.17191E-01	0.33996E 02
65.00	70.00	0.70950E 00	0.17572E-01	0.34859E 02
70.00	75.00	0.72130E 00	0.11806E-01	0.35440E 02
75.00	80.00	0.73307E 00	0.11770E-01	0.36018E 02
80.00	85.00	0.74638E 00	0.13303E-01	0.36671E 02
85.00	90.00	0.75839E 00	0.12008E-01	0.37261E 02
90.00	95.00	0.77579E 00	0.17404E-01	0.38117E 02
95.00	100.00	0.79326E 00	0.17473E-01	0.38975E 02
100.00	105.00	0.81399E 00	0.20729E-01	0.39994E 02
105.00	110.00	0.83621E 00	0.22218E-01	0.41085E 02
110.00	115.00	0.85369E 00	0.17477E-01	0.41944E 02
115.00	120.00	0.87099E 00	0.17298E-01	0.42794E 02
120.00	125.00	0.88841E 00	0.17428E-01	0.43650E 02
125.00	130.00	0.90507E 00	0.16653E-01	0.44468E 02
130.00	135.00	0.92079E 00	0.15725E-01	0.45241E 02
135.00	140.00	0.93743E 00	0.16637E-01	0.46058E 02
140.00	145.00	0.95551E 00	0.18078E-01	0.46947E 02
145.00	150.00	0.96854E 00	0.13032E-01	0.47587E 02
150.00	155.00	0.97988E 00	0.11340E-01	0.48144E 02
155.00	160.00	0.98834E 00	0.84645E-02	0.48560E 02
160.00	165.00	0.99373E 00	0.53851E-02	0.48824E 02
165.00	170.00	0.99762E 00	0.38862E-02	0.49015E 02
170.00	175.00	0.99922E 00	0.16021E-02	0.49094E 02
175.00	180.00	0.10000E 01	0.78281E-03	0.49133E 02

V ANTENNA 3/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 60.00 H= 6.25 H1= 5.00

RADIUS= 0.24216E 01 GAIN= 0.60283E 02 AREA= 0.64177E 04

TOTAL VOLUME= 0.59484E 02 MAX(K**2)= 0.14598E 03

TOTAL INTEGRAL= 0.59484E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.17949E-02	0.17949E-02	0.10677E-00
5.00	10.00	0.53090E-02	0.35141E-02	0.31580E-00
10.00	15.00	0.13064E-01	0.77550E-02	0.77710E 00
15.00	20.00	0.25789E-01	0.12725E-01	0.15341E 01
20.00	25.00	0.42935E-01	0.17145E-01	0.25539E 01
25.00	30.00	0.59952E-01	0.17018E-01	0.35662E 01
30.00	35.00	0.89046E-01	0.29093E-01	0.52968E 01
35.00	40.00	0.14154E-00	0.52494E-01	0.84194E 01
40.00	45.00	0.20635E-00	0.64810E-01	0.12275E 02
45.00	50.00	0.27459E-00	0.68237E-01	0.16334E 02
50.00	55.00	0.33512E-00	0.60535E-01	0.19935E 02
55.00	60.00	0.39292E-00	0.57795E-01	0.23373E 02
60.00	65.00	0.45447E-00	0.61552E-01	0.27034E 02
65.00	70.00	0.52073E 00	0.66259E-01	0.30975E 02
70.00	75.00	0.58409E 00	0.63359E-01	0.34744E 02
75.00	80.00	0.65957E 00	0.75485E-01	0.39234E 02
80.00	85.00	0.72963E 00	0.70058E-01	0.43402E 02
85.00	90.00	0.76864E 00	0.39009E-01	0.45722E 02
90.00	95.00	0.79066E 00	0.22014E-01	0.47032E 02
95.00	100.00	0.81616E 00	0.25500E-01	0.48548E 02
100.00	105.00	0.84073E 00	0.24572E-01	0.50010E 02
105.00	110.00	0.85914E 00	0.18415E-01	0.51105E 02
110.00	115.00	0.87578E 00	0.16641E-01	0.52095E 02
115.00	120.00	0.89161E 00	0.15823E-01	0.53037E 02
120.00	125.00	0.90703E 00	0.15421E-01	0.53954E 02
125.00	130.00	0.92091E 00	0.13884E-01	0.54780E 02
130.00	135.00	0.93506E 00	0.14145E-01	0.55621E 02
135.00	140.00	0.94693E 00	0.11875E-01	0.56328E 02
140.00	145.00	0.95960E 00	0.12671E-01	0.57081E 02
145.00	150.00	0.96967E 00	0.10066E-01	0.57680E 02
150.00	155.00	0.97872E 00	0.90541E-02	0.58219E 02
155.00	160.00	0.98479E 00	0.60704E-02	0.58580E 02
160.00	165.00	0.99121E 00	0.64179E-02	0.58961E 02
165.00	170.00	0.99640E 00	0.51932E-02	0.59270E 02
170.00	175.00	0.99874E 00	0.23319E-02	0.59409E 02
175.00	180.00	0.10000E 01	0.12642E-02	0.59484E 02

V ANTENNA 5/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 22.50 H= 8.75 H1= 7.00

RADIUS= 0.23278E 01 GAIN= 0.14953E 03 AREA= 0.81217E 04

TOTAL VOLUME= 0.52837E 02 MAX(K**2)= 0.34307E 03

TOTAL INTEGRAL= 0.52837E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.30233E-01	0.30233E-01	0.15974E 01
5.00	10.00	0.90476E-01	0.60242E-01	0.47804E 01
10.00	15.00	0.17662E-00	0.86148E-01	0.93322E 01
15.00	20.00	0.25312E-00	0.76492E-01	0.13374E 02
20.00	25.00	0.32436E-00	0.71242E-01	0.17138E 02
25.00	30.00	0.39708E-00	0.72724E-01	0.20981E 02
30.00	35.00	0.48503E-00	0.87945E-01	0.25627E 02
35.00	40.00	0.55860E 00	0.73568E-01	0.29514E 02
40.00	45.00	0.62525E 00	0.66651E-01	0.33036E 02
45.00	50.00	0.64914E 00	0.23896E-01	0.34299E 02
50.00	55.00	0.66790E 00	0.18758E-01	0.35290E 02
55.00	60.00	0.68644E 00	0.18540E-01	0.36269E 02
60.00	65.00	0.69817E 00	0.11731E-01	0.36889E 02
65.00	70.00	0.71062E 00	0.12444E-01	0.37547E 02
70.00	75.00	0.72498E 00	0.14366E-01	0.38306E 02
75.00	80.00	0.74259E 00	0.17607E-01	0.39236E 02
80.00	85.00	0.76277E 00	0.20178E-01	0.40302E 02
85.00	90.00	0.77806E 00	0.15289E-01	0.41110E 02
90.00	95.00	0.78898E 00	0.10927E-01	0.41687E 02
95.00	100.00	0.79705E 00	0.80678E-02	0.42114E 02
100.00	105.00	0.80601E 00	0.89635E-02	0.42587E 02
105.00	110.00	0.81778E 00	0.11770E-01	0.43209E 02
110.00	115.00	0.83308E 00	0.15297E-01	0.44017E 02
115.00	120.00	0.85367E 00	0.20588E-01	0.45105E 02
120.00	125.00	0.87683E 00	0.23159E-01	0.46329E 02
125.00	130.00	0.89472E 00	0.17891E-01	0.47274E 02
130.00	135.00	0.91003E 00	0.15307E-01	0.48083E 02
135.00	140.00	0.92510E 00	0.15078E-01	0.48880E 02
140.00	145.00	0.94114E 00	0.16038E-01	0.49727E 02
145.00	150.00	0.95546E 00	0.14318E-01	0.50483E 02
150.00	155.00	0.96971E 00	0.14251E-01	0.51236E 02
155.00	160.00	0.98097E 00	0.11254E-01	0.51831E 02
160.00	165.00	0.98990E 00	0.89347E-02	0.52303E 02
165.00	170.00	0.99558E 00	0.56832E-02	0.52603E 02
170.00	175.00	0.99862E 00	0.30322E-02	0.52764E 02
175.00	180.00	0.10000E 01	0.13846E-02	0.52837E 02

V ANTENNA 5/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 60.00 H= 8.75 H1= 7.00

RADIUS= 0.24621E 01 GAIN= 0.82881E 02 AREA= 0.45018E 04

TOTAL VOLUME= 0.62521E 02 MAX(K**2)= 0.20407E 03

TOTAL INTEGRAL= 0.62521E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.21260E-02	0.21260E-02	0.13292E-00
5.00	10.00	0.78409E-02	0.57149E-02	0.49022E-00
10.00	15.00	0.16026E-01	0.81850E-02	0.10020E 01
15.00	20.00	0.27059E-01	0.11034E-01	0.16918E 01
20.00	25.00	0.41125E-01	0.14065E-01	0.25712E 01
25.00	30.00	0.61575E-01	0.20450E-01	0.38497E 01
30.00	35.00	0.80188E-01	0.18613E-01	0.50134E 01
35.00	40.00	0.11833E-00	0.38138E-01	0.73979E 01
40.00	45.00	0.18840E-00	0.70072E-01	0.11779E 02
45.00	50.00	0.26533E-00	0.76932E-01	0.16589E 02
50.00	55.00	0.33665E-00	0.71316E-01	0.21048E 02
55.00	60.00	0.40105E-00	0.64408E-01	0.25074E 02
60.00	65.00	0.46720E-00	0.66151E-01	0.29210E 02
65.00	70.00	0.53791E 00	0.70702E-01	0.33631E 02
70.00	75.00	0.61902E 00	0.81109E-01	0.38702E 02
75.00	80.00	0.69756E 00	0.78546E-01	0.43612E 02
80.00	85.00	0.73356E 00	0.36003E-01	0.45863E 02
85.00	90.00	0.76140E 00	0.27833E-01	0.47603E 02
90.00	95.00	0.78851E 00	0.27116E-01	0.49299E 02
95.00	100.00	0.80927E 00	0.20756E-01	0.50596E 02
100.00	105.00	0.83048E 00	0.21205E-01	0.51922E 02
105.00	110.00	0.85232E 00	0.21849E-01	0.53288E 02
110.00	115.00	0.87290E 00	0.20573E-01	0.54575E 02
115.00	120.00	0.89107E 00	0.18172E-01	0.55711E 02
120.00	125.00	0.90697E 00	0.15903E-01	0.56705E 02
125.00	130.00	0.92160E 00	0.14631E-01	0.57620E 02
130.00	135.00	0.93419E 00	0.12589E-01	0.58407E 02
135.00	140.00	0.94634E 00	0.12145E-01	0.59166E 02
140.00	145.00	0.95888E 00	0.12547E-01	0.59951E 02
145.00	150.00	0.97020E 00	0.11317E-01	0.60658E 02
150.00	155.00	0.98023E 00	0.10026E-01	0.61285E 02
155.00	160.00	0.98950E 00	0.92777E-02	0.61865E 02
160.00	165.00	0.99497E 00	0.54637E-02	0.62207E 02
165.00	170.00	0.99864E 00	0.36684E-02	0.62436E 02
170.00	175.00	0.99960E 00	0.95493E-03	0.62496E 02
175.00	180.00	0.10000E 01	0.40387E-03	0.62521E 02

V ANTENNA 3/4 SERIES

RATIO= 0.800 LENGTH= 228.60

DEL2= 22.50 H= 11.25 H1= 9.00

RADIUS= 0.23763E 01 GAIN= 0.14875E 03 AREA= 0.48875E 04

TOTAL VOLUME= 0.56208E 02 MAX(K**2)= 0.35347E 03

TOTAL INTEGRAL= 0.56208E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.15211E-01	0.15211E-01	0.85495E 00
5.00	10.00	0.83016E-01	0.67806E-01	0.46661E 01
10.00	15.00	0.17143E-00	0.88415E-01	0.96357E 01
15.00	20.00	0.25249E-00	0.81062E-01	0.14192E 02
20.00	25.00	0.32970E-00	0.77208E-01	0.18532E 02
25.00	30.00	0.40960E-00	0.79898E-01	0.23023E 02
30.00	35.00	0.50660E 00	0.97002E-01	0.28475E 02
35.00	40.00	0.58263E 00	0.76026E-01	0.32748E 02
40.00	45.00	0.62800E 00	0.45369E-01	0.35298E 02
45.00	50.00	0.64681E 00	0.18815E-01	0.36356E 02
50.00	55.00	0.66541E 00	0.18595E-01	0.37401E 02
55.00	60.00	0.68036E 00	0.14953E-01	0.38241E 02
60.00	65.00	0.69684E 00	0.16477E-01	0.39168E 02
65.00	70.00	0.71547E 00	0.18635E-01	0.40215E 02
70.00	75.00	0.73153E 00	0.16056E-01	0.41117E 02
75.00	80.00	0.74564E 00	0.14110E-01	0.41910E 02
80.00	85.00	0.76030E 00	0.14662E-01	0.42735E 02
85.00	90.00	0.77389E 00	0.13590E-01	0.43498E 02
90.00	95.00	0.78605E 00	0.12158E-01	0.44182E 02
95.00	100.00	0.79770E 00	0.11652E-01	0.44837E 02
100.00	105.00	0.80979E 00	0.12090E-01	0.45516E 02
105.00	110.00	0.82016E 00	0.10374E-01	0.46099E 02
110.00	115.00	0.82973E 00	0.95666E-02	0.46637E 02
115.00	120.00	0.84356E 00	0.13833E-01	0.47415E 02
120.00	125.00	0.86115E 00	0.17588E-01	0.48403E 02
125.00	130.00	0.88467E 00	0.23521E-01	0.49725E 02
130.00	135.00	0.90359E 00	0.18921E-01	0.50789E 02
135.00	140.00	0.92086E 00	0.17262E-01	0.51759E 02
140.00	145.00	0.93523E 00	0.14376E-01	0.52567E 02
145.00	150.00	0.94878E 00	0.13548E-01	0.53329E 02
150.00	155.00	0.96227E 00	0.13488E-01	0.54087E 02
155.00	160.00	0.97508E 00	0.12809E-01	0.54807E 02
160.00	165.00	0.98576E 00	0.10681E-01	0.55407E 02
165.00	170.00	0.99345E 00	0.76954E-02	0.55840E 02
170.00	175.00	0.99831E 00	0.48563E-02	0.56113E 02
175.00	180.00	0.10000E 01	0.16899E-02	0.56208E 02

V ANTENNA 5/4 SERIES
RATIO= 0.800 LENGTH= 228.60
DEL2= 60.00 H= 11.25 H1= 9.00
RADIUS= 0.25140E 01 GAIN= 0.98905E 02 AREA= 0.32498E 04
TOTAL VOLUME= 0.66556E 02 MAX(K**2)= 0.24865E 03
TOTAL INTEGRAL= 0.66556E 02

THETA1	THETA2	INTEGRAL FROM 0 TO THETA2	NORMALIZED INTEGRAL FROM THETA1 TO THETA2	UN-NORMALIZED INTEGRAL FROM 0 TO THETA2
0.	5.00	0.16807E-02	0.16807E-02	0.11186E-00
5.00	10.00	0.74778E-02	0.57972E-02	0.49769E-00
10.00	15.00	0.14767E-01	0.72889E-02	0.98281E 00
15.00	20.00	0.25627E-01	0.10860E-01	0.17056E 01
20.00	25.00	0.39390E-01	0.13763E-01	0.26216E 01
25.00	30.00	0.58705E-01	0.19315E-01	0.39072E 01
30.00	35.00	0.81690E-01	0.22984E-01	0.54369E 01
35.00	40.00	0.10718E-00	0.25494E-01	0.71336E 01
40.00	45.00	0.16435E-00	0.57167E-01	0.10938E 02
45.00	50.00	0.24613E-00	0.81777E-01	0.16381E 02
50.00	55.00	0.32675E-00	0.80625E-01	0.21747E 02
55.00	60.00	0.39737E-00	0.70612E-01	0.26447E 02
60.00	65.00	0.46740E-00	0.70030E-01	0.31108E 02
65.00	70.00	0.54389E 00	0.76497E-01	0.36199E 02
70.00	75.00	0.63052E 00	0.86630E-01	0.41965E 02
75.00	80.00	0.68496E 00	0.54439E-01	0.45588E 02
80.00	85.00	0.72136E 00	0.36403E-01	0.48011E 02
85.00	90.00	0.75232E 00	0.30955E-01	0.50071E 02
90.00	95.00	0.78485E 00	0.32531E-01	0.52236E 02
95.00	100.00	0.80870E 00	0.23845E-01	0.53823E 02
100.00	105.00	0.83161E 00	0.22915E-01	0.55348E 02
105.00	110.00	0.85387E 00	0.22257E-01	0.56830E 02
110.00	115.00	0.87587E 00	0.22007E-01	0.58294E 02
115.00	120.00	0.89364E 00	0.17766E-01	0.59477E 02
120.00	125.00	0.90835E 00	0.14715E-01	0.60456E 02
125.00	130.00	0.92157E 00	0.13216E-01	0.61336E 02
130.00	135.00	0.93521E 00	0.13639E-01	0.62243E 02
135.00	140.00	0.94956E 00	0.14350E-01	0.63198E 02
140.00	145.00	0.96337E 00	0.13812E-01	0.64118E 02
145.00	150.00	0.97367E 00	0.10301E-01	0.64803E 02
150.00	155.00	0.98346E 00	0.97861E-02	0.65455E 02
155.00	160.00	0.98988E 00	0.64237E-02	0.65882E 02
160.00	165.00	0.99545E 00	0.55674E-02	0.66253E 02
165.00	170.00	0.99821E 00	0.27657E-02	0.66437E 02
170.00	175.00	0.99969E 00	0.14786E-02	0.66535E 02
175.00	180.00	0.10000E 01	0.30693E-03	0.66556E 02