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THE EARTH'S LONGITUDE GRAVITY FIELD AS SENSED BY THE DRIFT OF THREE SYNCHRONOUS SATELLITES

by C. A. Wagner

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Greenbelt, Md.

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ABSTRACT

One hundred and fifty-two orbits of three synchronous communications satellites have been analyzed for sensitivity to earth longitude gravity components through third order which are in resonance with them. Eighty-seven orbits are of Syncom 2 with inclination between 32° and 33° and distributed (but not uniformly) in mean geographic longitude between 66° and 305°. Nineteen orbits, distributed with fair uniformity between 173° and 180°, are of the nearly geostationary Syncom 3. Forty-six orbits are of the nearly geostationary Early Bird satellite between 330° and 332° longitude. These orbits were calculated without consideration of resonant gravitational effects.

The orbit data was reduced to give a set of essentially nine well determined long term longitude accelerations for these satellites between 66° and 332°. From this reduced acceleration record, after extensive testing, four earth longitude gravity harmonics of second and third order appear to be well discriminated. These harmonics with their standard errors, for which adjustments for sun and moon effects and the probable influence of neglected higher order earth gravity have been made, are

$J_{22} = -(1.816 \pm 0.020)\times 10^{-6}$ (This corresponds to a difference in major and minor axes of the earth's elliptical equator of 69.4 ± 0.8 meters.)

$\lambda_{22} = -(15.4 \pm 0.3)^\circ$

$J_{33} = -(0.171 \pm 0.017)\times 10^{-6}$

$\lambda_{33} = (24.9 \pm 3.3)^\circ$.

In addition to these harmonics, a third pair (J_{31}, λ_{31}) was poorly discriminated from the limited acceleration record. The data shows tentatively that

$J_{31} = -\left(1.4^{+1.0}_{-0.2}\right)\times 10^{-6}$

$\lambda_{31} = -(168 \pm 26)^\circ$.

Tests of the satellite data were also made to try and reveal the influence of resonant fourth order earth gravity. These tests were inconclusive.

The above results show that an equatorial 24-hour satellite can be in uncontrolled long term east-west equilibrium at only the following four longitude locations:

$\lambda_1 = 76.7 \pm 0.8^\circ$ (dynamically stable east-west equilibrium)

$\lambda_2 = 161.8 \pm 0.7^\circ$ (statically stable east-west equilibrium)

$\lambda_3 = -108.1 \pm 1.0^\circ$ (dynamically stable east-west equilibrium)

$\lambda_4 = -12.2 \pm 0.7^\circ$ (statically stable east-west equilibrium).

According to the analysis of 24-hour satellite drift thus far, the maximum longitude acceleration due to earth gravity which could be experienced by the nearly geostationary satellite is

$$\ddot{\lambda} = -(1.83 \pm 0.05)\times 10^{-3} \text{ degrees/day}^2$$

at about 118° east of Greenwich. To correct continuously for this east-west acceleration would require, conservatively, a velocity increment of $\Delta V = 6.38$ ft/sec/year.

CONTENTS

Page

THE EARTH'S LONGITUDE GRAVITY FIELD AS SENSED BY THE DRIFT OF THREE SYNCHRONOUS SATELLITES

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INTRODUCTION

This report summarizes the results of a two year investigation of the "resonant" earth gravity drift of the world's first three operational synchronous satellites. Previous reports (References 1 through 6) in this series have dealt with the accelerated drift theory for 24-hour satellites and its applications to the reduction of range and range rate orbit data returned from Syncom 2 during its "free" gravity drift over Brazil and the Pacific Ocean.

The specific objective of this summary report is to determine those longitude dependent components of the earth's gravity field which can be fairly said to explain the longitude acceleration record of these satellites (Syncom 2 [1963 31A], Syncom 3 [1964 47A], and Early Bird [1965 28A]) from August 1963 to June 1965. The previous investigations by the author cited above and earlier pioneer theoretical studies such as References 7, 8, 9, and 10 have established beyond reasonable doubt now that the principal long term longitude disturbance on the 24-hour satellite arises from second order earth longitude gravity, associated with the ellipticity of the earth's Equator. The studies in References 1, 4, 5, and 6 indicated that earth longitude gravity effects of higher than second order on the 24-hour satellite are at maximum, about an order of magnitude less than the maximum second order effect. Frick and Garber (Reference 9) and the author in the present report and in the simulation studies of References 3, 5, and 6 have shown the long term sun and moon gravity effects on the near circular orbit synchronous satellite to be about two orders of magnitude less than the maximum second tesseral effect for periods of record in excess of about two months.

Experiment Plan and Error Sources

The basic orbit data of this gravity determining experiment are ascending Equator crossings, orbit vectors and subsatellite points which were reported by the Tracking and Data Systems Directorate of GSFC and the Communications Satellite Corporation without consideration of longitude gravity. Generally, the experiment plan and error control aspects follow that for the early Syncom 2 drift analyses implicit in References 3, 5, and 6. This plan is designed to insure as accurate a result as possible with the limited orbit data available.

The gravity experiment is carried out in two stages. In the first stage the basic orbit data is reduced by an appropriate model, and long term (periods greater than a day) longitude accelerations of the satellites are derived. These are called the "measured" accelerations. In this gravity experiment one hundred and fifty-two independently determined orbits of Syncom 2 and 3 and Early Bird have been reduced to give essentially nine well determined long term longitude accelerations for these satellites. These reduced accelerations are the basic, measured data for the second stage of the experiment. The second stage tests these reduced accelerations for sensitivity to the earth's longitude gravity which is assumed to be responsible for them. In the second stage of the experiment, then, we have a set of "actual" data which consists of measured longitude accelerations (long term) due to the following causes:

1. Physical agencies
   a. True earth longitude gravity
   b. Sun, moon, planetary and earth zonal gravity
   c. Other assumed (or proven) negligibly small physical agencies (nongravitational) such as:
      (1) micrometeorite drag or impact
      (2) random or systematic outgassing from the satellite control jets
      (3) solar wind and radiation pressure
      (4) magnetic field interactions.
2. First stage experiment-induced accelerations (errors). Compared to accelerations from true earth longitude gravity, presumably small errors in "measured" accelerations will be due to errors in
   a. the basic data (orbit determination) from which the accelerations were deduced
   b. the model used to derive the "measured" accelerations from the basic orbit data (first stage model error)

In the second stage of the experiment two error sources are separated. The "basic data error" (in the "measured" accelerations) is considered to be the sum of all the acceleration producing sources in the measured data except true earth longitude gravity. The "model error" in the second stage is considered to be due entirely to the necessarily limited earth model which can be assumed to explain the limited number of measurements in the experiment.

Error Control

First Experiment Stage (Acceleration Analysis)

The aim of this stage of the experiment is to obtain long term satellite accelerations as "basic data" which are as free from nonearth longitude gravity effects as possible. A number of nongravitational disturbances on the satellite have been computed theoretically and shown to be negligibly small in this experiment (Appendix E). The others are assumed to be negligibly small also (Reference 3).

In one or two instances basic orbit data has been used at the beginning or end of a long free gravity drift arc which is known to have been disturbed by commanded control gas jet pulsing. In these instances it is evident by inspection that the use of this data does not significantly disturb the previous or following free drift record. This "overlap" data, when used, reduces significantly the standard experiment error over the limited record of that drift arc.

In order to keep the long term sun and moon (principally moon) disturbances almost negligibly small, it was found that free drift arc lengths of about two months or more were required. It also turned out that this was the length of the typical Syncom drift period between orbit corrections. It was also generally necessary to consider a time period of at least two months to obtain reasonably small standard errors for the "measured" Syncom accelerations. Thus, the length of time and record were the major means of error control in the first stage (acceleration analysis) of the experiment.

The model error in the acceleration analysis stage was also under somewhat independent control. The chief consideration for model error control was the mean drift rate. In those arcs (arcs 1, 2, 6, 7, 8, and 9) where the mean drift rate was ±0.1 degree/day or less, the geographic longitude excursion was limited to less than 10°. It was found that with this "slow drift" regime a simple polynomial of third degree in the time could adequately describe the longitude drift of the ascending Equator crossing (Appendix D and Reference 3). In those arcs (arcs 3, 4, and 5) where the mean drift rate was greater than ±0.1 degree/day ("fast drift" regime) it was found that a simple three parameter function of the longitude of the ascending Equator crossing of the satellite could adequately describe (for a reasonable arc length) the change of the drift rate of these crossings. This was in accord with the use of the energy integral of the gravity drift of the 24-hour satellite in a simple second order field (see Equation 8 and Reference 2). For arc lengths of up to about 50°, the three parameter (second order) model appeared theoretically adequate to reproduce true gravity accelerations within a reasonably small standard experimental error which includes sun and moon "gravity noise". However, even for somewhat longer arc lengths, simulations show that the simple three parameter velocity model still gives sufficiently good results to be utilized without adjustment (Table 11). This fact depends on the evident overbearing strength of the second order gravity field compared to higher orders. Nevertheless "velocity arcs" of as short a length as possible (to give reasonable standard errors) were chosen to provide a longitude-acceleration survey of as great an extent as possible. This was necessitated, finally, by the method of acceleration analysis. In order to keep the model prejudice in the measurement of the acceleration to as low a level as possible and also to utilize only the best determined statistic in each arc, only a single acceleration near the center of each arc was finally chosen to represent the mean acceleration for that arc. This being the case, it was found that only by breaking up a long drift arc such as arc 5 into smaller, approximately 50° subarcs, could a reasonably extensive, precise and unprejudiced longitude survey be made with the data at hand.

Second Experiment Stage (Earth Gravity Synthesis)

Error control in this stage was primarily directed toward the refinement of the earth model which reflects the acceleration data. For additional error control, however, experiments with some adjustment of the acceleration data itself for sun, moon and first stage model errors on the basis of simulated 24-hour trajectories have also been made. In addition, various data weighting schemes and random acceleration choices based on the standard acceleration errors in the drift arcs have been tried as error control methods in arriving at a final reasonable earth gravity synthesis as seen by the data. At each stage of the experiment the experimental error was checked or verified externally by simulations of 24-hour satellite drift in a sun, moon, and/or full earth field, numerically calculated, with trajectory conditions close to those actually experienced in the various arcs.

We now proceed with the first experiment stage and analyze the 24-hour orbit data reported by GSFC and Comsat for what it reveals in terms of long term accelerations according to the "resonant gravity" models for slow and fast drift regimes previously discussed.

1. REDUCTION OF THE BASIC ORBIT DATA IN NINE 24-HOUR SATELLITE FREE DRIFT ARCS FOR LONG TERM LONGITUDE ACCELERATIONS

Arc 1, Syncom 2, 18 August 1963 - 18 November 1963

Syncom 2, the world's first operating synchronous communications satellite, was launched into orbit in late July 1963 and reached station over Brazil in mid-August 1963. The orbit inclination was close to 33° and the ascending Equator crossing was near 55° W moving at less than 0.1 degree/day eastward after the last corrective thrust was applied going into free drift arc 1, on 18 August 1963. From this date to 28 November 1963, the "figure of 8" ground track of Syncom 2 drifted freely without orbit correction from 55° W to 59° W when on-board jet pulsing was applied to virtually stop the westward movement of the track. The details of this accelerated free gravity drift are presented in Table 1 and Figure 1.

The acceleration of the ascending Equator crossing in arc 1 was determined from a fit of the orbit data according to the third order polynomial,

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3 , \tag{1}$$

which is shown to apply for slow drift regimes in Appendix D and in the simulations in Tables 1S, 1S/1, etc. The longitude, L, is the ascending Equator crossing east of an arbitrary base longitude, which, for computational accuracy, is preferably near the center of the drift arc. The time, t, is an arbitrary base time which is also preferably located near the center of the arc for computational accuracy.

It is possible to evaluate semi-empirically the error due to sun and moon gravity and model bias implicit in calculating the satellite accelerations due to the earth's longitude gravity in this slow drift arc according to Equation 1. In the second stage of the experiment (gravity synthesis) we will assume long term 24-hour satellite resonant earth gravity acceleration is derivable from the harmonic expansion through fourth order,

$$\ddot{\lambda} = -12\pi^2 \sum_{n=2}^{4} \sum_{m=1}^{n} F_{nm} \sin m\left(\lambda - \lambda_{nm}\right) F(i)_{nm}, \frac{\text{rad}}{(\text{sid. day})^2} \tag{2}$$

for n-m, even

where

$$F_{22} F(i)_{22} = \frac{6J_{22}}{a_s^2} \left[\left(\cos i_s + 1\right)/2\right]^2 , \tag{3}$$

$$F_{31} F(i)_{31} = \frac{-3J_{31}}{2a_s^3} \left\{\frac{\left(1 + \cos i_s\right)}{2} - \frac{5 \sin^2 i_s \left(1 + 3 \cos i_s\right)}{8}\right\} , \tag{4}$$

$$F_{33} F(i)_{33} = \frac{45 J_{33}}{a_s^3} \left\{\left(\cos i_s + 1\right)/2\right\}^3 , \tag{5}$$

$$F_{42} F(i)_{42} = \frac{-15 J_{42}}{a_s^4} \left\{\frac{\left(1 + \cos i_s\right)^2}{4} - \frac{7 \sin^2 i_s \cdot \cos i_s \left(1 + \cos i_s\right)}{4}\right\} , \tag{6}$$

$$F_{44} F(i)_{44} = \frac{420 J_{44}}{a_s^4} \left\{\left(1 + \cos i_s\right)/2\right\}^4 . \tag{7}$$

See Equation 65 in Reference 2 for the derivation of Equation 2 above.

The symbol λ represents the longitude location of the ascending Equator crossing (or mean daily longitude location) for the 24-hour satellite of reasonably small eccentricity and drift rate (Reference 2). The symbols a_s and i_s are the "synchronous" semimajor axis (in earth radii) and inclination of the satellite's orbit. The significance of the gravity constants J_{nm}, λ_{nm} is contained in Appendix B and explained in further detail in References 2 and 11. In evaluating the first stage model error we calculate numerically, particle trajectories closely parallel to the actual one in free drift, including as many relevant perturbation effects as desired. (In References 3, 5, and 6 these simulated trajectories clearly show the necessity of considering earth longitude gravity in the long term orbit determination for the 24-hour satellite.) Then, a direct comparison of the longitude acceleration measured in the simulated trajectory with the theoretical 24-hour satellite resonant gravity acceleration as given by Equation 2 gives an estimate of the bias error from the effects neglected in the real trajectory analysis. These errors include sun and moon gravity accelerations and model error implicit in the limited accuracy of the longitude Equation 1.

Ideally, this numerical approach to assessing nonresonant gravity effects should involve an attempt to duplicate as closely as possible the real trajectory. This is to avoid the criticism that the nonresonant effects in the simulated trajectory may not be exactly or even nearly the same as in the actual trajectory. (Compare, for example, the two trajectory results in Table 4S.) It can be appreciated that the full process of such duplication, or closest duplication (in a least squares sense, for example), must involve adjustment of at least the six initial trajectory parameters, as well as the earth longitude gravity constants, the earth radius, and the principal gravity constant. Such an analysis is considerably beyond the scope of this one. Still, extensive experience with a more limited numerical approach to this problem appears to show that such bias errors can be fairly accurately determined without precise duplication, at least under a reasonably wide range of longitude gravity constants.

In this approach only the semimajor axis of the actual 24-hour orbit at the beginning of each arc was adjusted so that, together with a fixed nominal second order longitude gravity field, the simulated trajectory gives close longitude-time congruence with the actual. Data and results from such trajectories are found in Tables 1S, 2S, 4S, etc. The second order earth longitude field was specified as $J_{22} = -1.68\times10^{-6}$, $\lambda_{22} = -18.0°$ in conformance with the 24-hour gravity results of Reference 3. At a later stage in the analysis, a more accurate set of longitude gravity harmonics through third order was derived. Simulated trajectories utilizing this set with the same initial elements as before were calculated and analyzed. Data and results from these trajectories are found in Tables 1S/1, 2S/1, 4S/1, etc. Except for arc 4S versus 4S/1, the bias results of these two parallel simulated trajectories are consistent. The principal conclusion from the study of the simulated trajectories (not all of which are reported here) is that when arc lengths in excess of two months are considered, the cumulative model bias acceleration errors are within $\pm0.03\times10^{-5}$ radians/sid. day^2, RMS (see Table 11). In Section 2 the net gravity effect of these model bias errors in each acceleration measurement is shown to be small. In fact, there is evidence that the actual data analyzed without bias adjustment gives longitude gravity with greater precision than with such adjustment. The implication is that over the limited acceleration record of the experiment the bias error acts to cancel more often than not the random observation error attributable to the imprecise orbit determination.

The principal results of the acceleration analysis on the actual data in Table 1 and the simulated data in Tables 1S and 1S/1 are listed in Tables 10 and 11 in the next section.

Arc 2, Syncom 2, 28 November 1963 - 18 March 1964

On 28 November 1963, on-board jets were fired to virtually stop the westward drift of Syncom 2 which had built up over the previous three months due to earth longitude gravity. From 28 November 1963 to 18 March 1964 Syncom 2 was allowed to drift freely about 8° in mean longitude from 59° W to 67° W under the westward accelerating influence of earth longitude gravity. The orbit inclination during this time was about 32.8°. The details of this drift are presented in Table 2 and Figure 2. The length of the drift record in this arc should be long enough to extract two well defined acceleration values by considering a t^4 fit (see Appendix D). Unfortunately the orbit determination errors in this arc appear to be too great to allow this finer discrimination.

Table 1

Syncom 2 Osculating Elements at the First Ascending Equator Crossing Past the Tracking Epoch and Related Data for Free Drift Arc 1*.

Orbit Number 1 –	Tracking Epoch** (yr-mo-day-hr-min, UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from January 0.0, 1963 (days)	Geographic Longitude of First Ascending Equator Crossing After Tracking Epoch, λ (degrees)	① Time from January 276.5057, 1963, t (days)	② Longitude of Ascending Equator Crossing East of -56.517°, L (degrees)
1	63-8-18-1-30.0	6.6105587	33.120	-42.357	.00023	37.901	-37.890	230.1302	-55.004	-46.3755	1.513
2	63-8-22-6-12.14	6.6105498	33.081	-42.444	.00024	53.529	-53.508	235.1159	-54.893	-41.3898	1.624
3	63-8-26-17.0	6.6105779	33.091	-42.553	.00018	58.903	-58.888	239.1047	-54.847	-37.4010	1.670
4	63-8-31.0	6.6107824	33.062	-42.526	.00022	45.989	-45.974	243.0937	-54.803	-33.4120	1.714
5	63-9-3-13-23.0	6.6105663	33.082	-42.635	.00026	42.792	-42.773	247.0824	-54.792	-29.4233	1.725
6	63-9-5.0	6.6110747	33.064	-42.639	.00016	22.930	-22.924	248.0796	-54.753	-28.4261	1.764
7	63-9-9.0	6.6107958	33.048	-42.728	.00015	15.313	-15.308	252.0685	-54.774	-24.4372	1.743
8	63-9-12-2.0	6.6110077	33.078	-42.788	.00021	31.588	-31.577	256.0575	-54.812	-20.4482	1.705
9	63-9-17-2.0	6.6108515	33.040	-42.850	.00020	29.394	-29.384	261.0438	-54.862	-15.4619	1.655
10	63-9-20-2.0	6.6109260	33.009	-42.919	.00020	39.841	-39.830	264.0356	-54.932	-12.4701	1.585
11	63-9-27-2.0	6.6111697	33.031	-43.018	.00020	-24.116	24.109	271.0165	-55.037	- 5.4892	1.480
12	63-10-1-2.0	6.6107666	33.023	-43.112	.00020	8.796	- 8.793	275.0056	-55.142	- 1.5001	1.375
13	63-10-8-2.0	6.6114380	33.013	-43.233	.00025	11.624	-11.619	281.9872	-55.519	5.4815	.998
14	63-10-14-2.0	6.6113174	32.979	-43.201	.00030	23.884	-23.872	287.9715	-55.740	11.4658	.777
15	63-10-22-2.0	6.6116625	32.994	-43.411	.00029	-23.336	23.325	295.9503	-56.171	19.4446	.346
16	63-10-30.0	6.6113312	32.946	-43.449	.00024	- 9.384	9.380	303.9297	-56.658	27.4240	- .141
17	63-11-6.0	6.6119680	32.952	-43.683	.00031	15.139	-15.130	310.9116	-57.253	34.4059	- .736
18	63-11-12-5.0	6.6117819	32.919	-43.703	.00030	25.807	-25.792	316.8964	-57.713	40.3907	-1.196
19	63-11-18-13.0	6.6120265	32.925	-43.877	.00019	15.413	-15.409	322.8811	-58.280	46.3754	-1.763
		Average: 6.611113 = a_s	Average: 33.024 = i_s								

*One earth radius = 6378.388 km; the earth gravity constant used in the trajectory program, $\mu_e = 3.986267 \times 10^5$ km^3/sec^2

The osculating elements and the equator crossing data were derived from the satellite vectors reported by the Tracking and Data Systems Directorate of NASA-GSFC in Table A1. The trajectory generator, called "ITEM" (Interplanetary Trajectory by an Encke Method) at GSFC, for this derivation used the same earth, moon, and sun model as the original orbit (vector) determination program (see Table A1).

**The tracking epoch refers to the epoch of the satellite vectors reported in Table A1.

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

$a_1 = 1.2720 \pm 1.032 \times 10^{-2}$ degrees

$a_2 = -(3.4899 \pm 0.0589) \times 10^{-2}$ degrees/sol. day

$a_3 = -(6.4947 \pm 0.0938) \times 10^{-4}$ degrees/sol. day^2

$a_4 = -(1.765 \pm 3.795) \times 10^{-7}$ degrees/sol. day^3

Standard error of estimate = 0.02825

$\ddot{\lambda}$ (with minimum standard error) = $-(2.253 \pm 0.0325) \times 10^{-5}$ rad/sid. day^2, at t = -0.674 days, L = 1.296°, λ = -55.22°, on January 275.832, 1963. See Figure 1.

Table 1S

Ascending Equator Crossing Orbit Data from a Simulated Syncom 2 Trajectory for Free Drift Arc 1 with Earth Longitude Gravity Through Second Order.*

Orbit Number 1S –	Tracking Epoch (yr-mo-day-hr-min UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1963.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time from January 276.5056, 1963, t (days)	② Longitude of the Ascending Equator Crossing East of -56.537°, L (degrees)
1	63-8-18-1.5	6.6106431	33.120	-42.358	.00023	37.945	-37.930	230.1302	-55.006	-46.3754	1.531
2	63-8-22-6-12.14	6.6105732	33.102	-42.423	.00024	53.526	-53.505	235.1161	-54.888	-41.3895	1.649
3	63-8-26-17.0	6.6105273	33.095	-42.488	.00019	57.150	-57.131	239.1047	-54.814	-37.4009	1.723
4	63-8-31.0	6.6107816	33.091	-42.539	.00017	44.072	-44.063	243.0935	-54.765	-33.4121	1.772
5	63-9-3-13-23.0	6.6108968	33.077	-42.585	.00021	48.646	-48.632	247.0824	-54.736	-29.4232	1.801
6	63-9-5.0	6.6108259	33.072	-42.601	.00020	54.853	-54.836	248.0796	-54.731	-28.4260	1.806
7	63-9-9.0	6.6106796	33.061	-42.673	.00014	61.617	-61.604	252.0685	-54.724	-24.4371	1.813
8	63-9-12-2.0	6.6109677	33.058	-42.729	.00020	38.963	-38.952	256.0575	-54.750	-20.4481	1.787
9	63-9-17-2.0	6.6110100	33.040	-42.793	.00026	52.421	-52.402	261.0438	-54.801	-15.4618	1.736
10	63-9-20-2.0	6.6109082	33.031	-42.844	.00023	59.346	-59.327	264.0356	-54.840	-12.4700	1.697
11	63-9-27-2.0	6.6111994	33.024	-42.949	.00018	46.804	-46.791	271.0166	-54.986	- 5.4890	1.551
12	63-10-1-2.0	6.6112339	33.009	-43.002	.00021	55.489	-55.469	275.0058	-55.098	- 1.4998	1.439
13	63-10-8-2.0	6.6112347	32.996	-43.128	.00016	44.518	-44.505	281.9870	-55.348	5.4814	1.189
14	63-10-14-2.0	6.6113821	32.980	-43.208	.00025	53.893	-53.871	287.9712	-55.616	11.4656	.921
15	63-10-22-2.0	6.6113844	32.970	-43.346	.00017	58.412	-58.401	295.9501	-56.029	19.4445	.508
16	63-10-30.0	6.6114845	32.952	-43.461	.00019	67.886	-67.871	303.9293	-56.530	27.4237	.007
17	63-11-6.0	6.6117444	32.947	-43.581	.00020	42.270	-42.258	310.9113	-57.051	34.4057	- .514
18	63-11-12-5.0	6.6117173	32.932	-43.668	.00023	58.345	-58.325	316.8960	-57.537	40.3904	-1.000
19	63-11-18-13.0	6.6117764	32.930	-43.770	.00017	57.820	-57.803	322.8809	-58.067	46.3753	-1.530
	Average: 6.611104 = a_s	Average: 33.026 = i_s									

*Computed by ITEM with gravity constants the same as in Table A1 with the addition of earth constants, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18^o$.

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

$a_1 = (1.3885 \pm 0.00152)$ degrees

$a_2 = -(3.289 \pm 0.00866) \times 10^{-2}$ degrees/day

$a_3 = -(6.451 \pm 0.0138) \times 10^{-4}$ degrees/day^2

$a_4 = -(7.564 \pm 5.577) \times 10^{-8}$ degrees/day^3

Standard error of estimate = 0.004151 degree

$\ddot{\lambda}$(measured) = $-(2.2389 \pm 0.0048) \times 10^{-5}$ rad/sid. day^2, for t = −0.674 day, L = 1.4104°, λ = -55.127°

$\ddot{\lambda}$(theoretical, from Equation 2) = $-(2.2211) \times 10^{-5}$ rad/sid. day^2, for a_s = 6.611104 earth radii, i_s = 33.026°, λ = -55.127°, $J_{22} = -1.68 \times 10^{-6}$, λ_{22} = .18.0°.

Estimate of measured bias due to sun-moon perturbations and J_{22} model error (exclusive of higher order longitude gravity effects) in $\ddot{\lambda}$ at t = -0.674 day in Syncom 2 arc 1 on January 275.832, 1963: Bias = theoretical - measured

$$= -(2.2211) \times 10^{-5} + 2.2389 \times 10^{-5} = +(0.0178) \times 10^{-5} \text{ rad/sid. day}^2.$$

Table 1S/1

Ascending Equator Crossing Data from a Simulated Syncom 2 Trajectory for Free Drive Arc 1 Computed by ITEM with Earth Longitude Gravity through Third Order.*

Orbit Number 1S/1–	Tracking Epoch (yr-mo-day-hr-min UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Time from 1963.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time from January 276.5055, 1963, t (days)	② Longitude of the Ascending Equator Crossing East of -56.395°, L (degrees)
1	63-8-18-1.5	6.6106432	33.120	230.1302	-55.006	-46.3753	1.389
2	63-8-22-6-12.14	6.6105732	33.102	235.1161	-54.888	-41.3894	1.507
3	63-8-26-17.0	6.6105276	33.095	239.1048	-54.814	-37.4007	1.581
4	63-8-31.0	6.6107816	33.091	243.0936	-54.765	-33.4119	1.630
5	63-9-3-13-23.0	6.6108969	33.077	247.0824	-54.736	-29.4231	1.659
6	63-9-5.0	6.6108260	33.072	248.0796	-54.731	-28.4259	1.664
7	63-9-9.0	6.6106800	33.061	252.0685	-54.725	-24.4370	1.670
8	63-9-12-2.0	6.6109680	33.058	256.0575	-54.751	-20.4480	1.644
9	63-9-17-2.0	6.6110104	33.040	261.0438	-54.802	-15.4617	1.593
10	63-9-20-2.0	6.6109087	33.031	264.0356	-54.842	-12.4699	1.553
11	63-9-27-2.0	6.6112000	33.024	271.0166	-54.988	- 5.4889	1.407
12	63-10-1-2.0	6.6112345	33.009	275.0058	-55.100	- 1.4997	1.295
13	63-10-8-2.0	6.6112351	32.996	281.9871	-55.351	5.4816	1.044
14	63-10-14-2.0	6.6113824	32.980	287.9712	-55.619	11.4657	.776
15	63-10-22-2.0	6.6113836	32.970	295.9501	-56.032	19.4446	.363
16	63-10-30.0	6.6114827	32.951	303.9293	-56.532	27.4238	- .137
17	63-11-6.0	6.6117414	32.947	310.9113	-57.052	34.4058	- .657
18	63-11-12-5.0	6.6117129	32.932	316.8960	-57.536	40.3905	-1.141
19	63-11-18-13.0	6.6117706	32.930	322.8808	-58.064	46.3753	-1.669
		Average: 6.6111 = a_S	Average: 33.026 = i_S	(see Figure 1)			

*Gravity constants of this trajectory are the same as those in Table A1 with the addition of the earth constants:

$J_{22} = -1.8 \times 10^{-6}$ $\lambda_{22} = -15.35^{\circ}$ $J_{33} = -0.16 \times 10^{-6}$ $\lambda_{33} = -24^{\circ}$ $J_{31} = -1.5 \times 10^{-6}$ $\lambda_{31} = 0^{\circ}$,

(see Figure B1 for the significance of the constants).

The initial elements of this trajectory, aside from those listed for orbit 1S/1-1, are the same as those in orbit 1S-1 (Table 1S).

Results of least squares fit of data in ① and ②above according to the theory of Equation 1 (for arc 1S/1):

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = (1.2439 \pm 0.00151)$ degrees

$a_2 = -(3.2953 \pm 0.00864) \times 10^{-2}$ degrees/day

$a_3 = -(6.4319 \pm 0.0138) \times 10^{-4}$ degrees/day^2

$a_4 = -(2.94 \pm 5.57) \times 10^{-8}$ degrees/day^3

Standard error of estimate = 0.00414 degree

$\ddot{\lambda}$(measured, with minimum standard error) = -2.2328×10^{-5} rad/sid. day^2, at $\lambda = -55.13^{\circ}$ on $t' = 275.8318^{\circ}$ January 1963.

$\ddot{\lambda}$(theoretical, from Equation 2) = -2.2185×10^{-5} rad/sid. day^2, for $a_s = 6.611$ earth radii, $i_s = 33.026^{\circ}$, $\lambda = -55.13^{\circ}$, $J_{22} - J_{31}$ as noted.

Estimate of acceleration bias at $\lambda = -55.13^{\circ}$ in arc 1S/1 = $\ddot{\lambda}$(theoretical) - $\ddot{\lambda}$(measured) = + 0.0143 x 10^{-5} rad/sid. day^2.

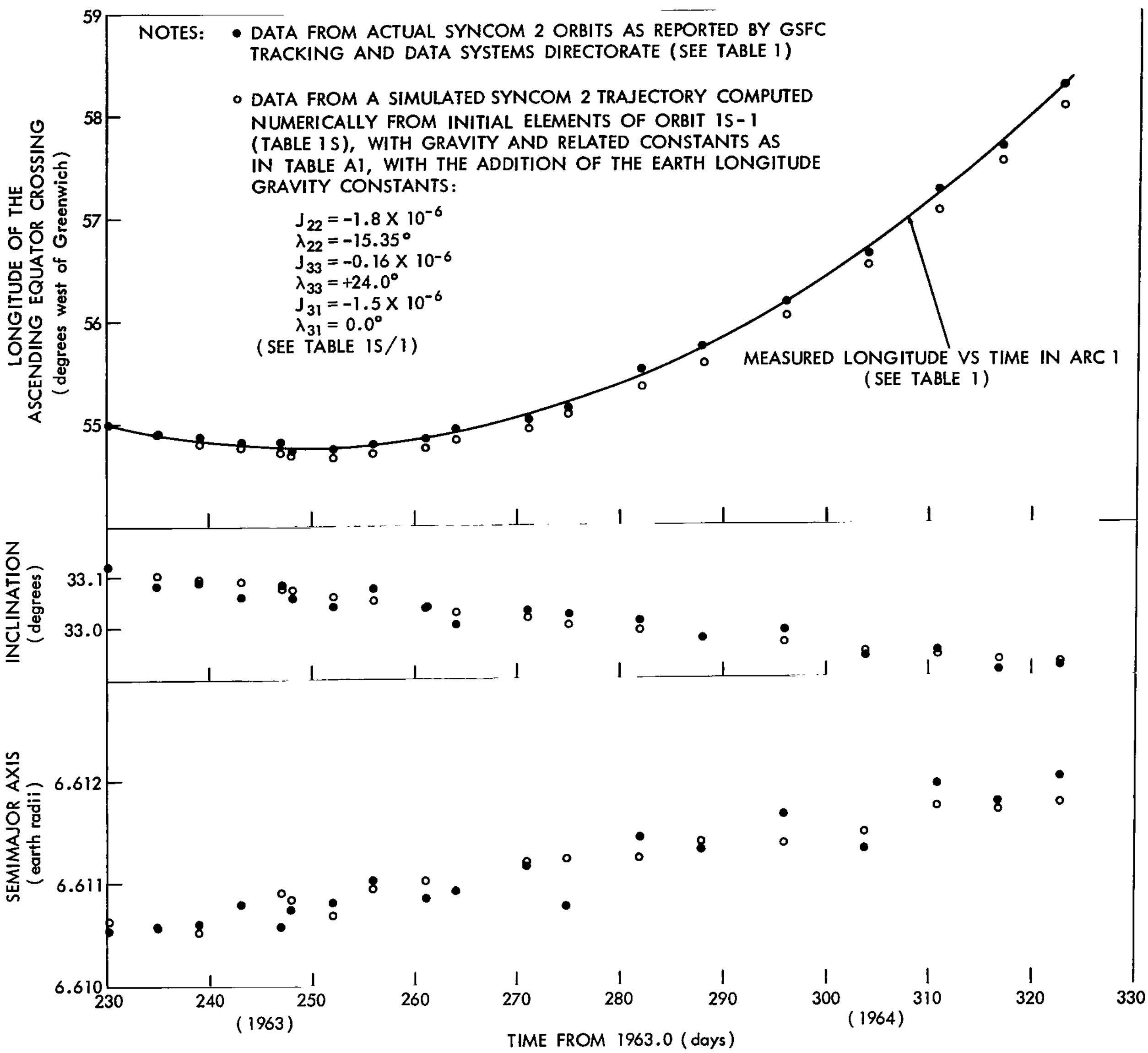

Figure 1—Measured and simulated orbit data at ascending Equator crossings in free drift arc 1 (Syncom 2).

The principal results of the acceleration analysis on the actual data in Table 2 and the simulated data in Tables 2S and 2S/1 are listed in Tables 10 and 11 at the end of the next section.

Arc 3, Syncom 2, 18 March 1964 - 25 April 1964

On 18 March 1964 the westward drift of Syncom 2 was speeded to 1.3 degrees/day by on-board gas jet pulsing. Between 18 March and 25 April, Syncom 2 drifted rapidly (in a "fast drift" regime) from 67° W to 116° W. The orbit inclination during this period was about 32.7°.

Table 2

Syncom 2 Osculating Elements at the First Ascending Equator Crossing Past the Tracking Epoch in Free Drift Arc 2.*

Orbit Number 2–	Tracking Epoch (yr-mo-day-hr-min UT)	Seminajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1963.0 (days)	Geographic Longitude of First Ascending Equator Crossing After Tracking Epoch λ (degrees)	① Time from January 384.2230, 1963, t (days)	② Longitude of the Ascending Equator Crossing East of -62.649°, L (degrees)
1	63-11-28-1.0	6.6103981	32.920	-44.040	.00006	-153.442	153.440	332.8558	-59.161	-51.3672	3.488
2	63-12-4.0	6.6112988	32.892	-44.090	.00019	46.962	- 46.949	338.8395	-59.227	-45.3835	3.422
3	63-12-10.0	6.6108885	32.881	-44.138	.00009	- 2.249	2.249	344.8230	-59.243	-39.4000	3.406
4	63-12-16-17.0	6.6109177	32.873	-44.266	.00010	95.745	- 95.744	350.8067	-59.392	-33.4163	3.257
5	63-12-23-19.0	6.6110073	32.795	-44.242	.00019	81.870	- 81.852	358.7851	-59.465	-25.4379	3.184
6	64-1-6-17.0	6.6111007	32.867	-44.456	.00014	14.514	- 14.511	371.7509	-60.155	-12.4721	2.494
7	64-1-9-6.0	6.6113769	32.857	-44.539	.00013	5.238	- 5.237	374.7431	-60.360	- 9.4799	2.289
8	64-1-15-18.0	6.6117781	32.810	-44.580	.00025	60.341	- 60.322	381.7246	-60.607	- 2.4984	2.042
9	64-1-20-21.0	6.6115475	32.825	-44.713	.00016	68.659	- 68.647	386.7119	-61.112	2.4889	1.537
10	64-1-29-20.0	6.6120395	32.856	-44.797	.00029	40.492	- 40.473	395.6892	-61.863	11.4662	.786
11	64-2-5-16.0	6.6119118	32.800	-44.925	.00026	49.760	- 49.738	401.6738	-62.350	17.4508	.299
12	64-2-10-19.0	6.6123174	32.832	-45.026	.00024	37.741	- 37.727	407.6588	-62.958	23.4358	- .309
13	64-2-17-17.0	6.6120695	32.760	-45.134	.00023	54.393	- 54.376	414.6413	-63.632	30.4183	- .983
14	64-2-25-19.0	6.6124562	32.767	-45.182	.00036	52.806	- 52.775	422.6218	-64.543	38.3988	-1.894
15	64-3-4-23.0	6.6123649	32.723	-45.386	.00017	49.417	- 49.405	430.6019	-65.452	46.3789	-2.803
16	64-3-10-13.0	6.6124118	32.747	-45.391	.00023	25.910	- 25.899	435.5902	-66.136	51.3672	-3.487
		Average: 6.6116178 = a_s	Average: 32.825 = i_s								

*Computed by ITEM with gravity constants the same as in Table A1 with the addition of earth constants, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18°$.

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

$a_1 = 1.7349 \pm 2.273 \times 10^{-2}$ degrees

$a_2 = -(7.118 \pm 0.126) \times 10^{-2}$ degrees/sol. day

$a_3 = -(6.6165 \pm 0.1650) \times 10^{-4}$ degrees/sol. day^2

$a_4 = (1.492 \pm 0.636) \times 10^{-6}$ degrees/sol. day ^{3}

Standard error of estimate = 6.041×10^{-2} degrees

$\ddot{\lambda}$ (with minimum standard error) = $-(2.291 \pm 0.0572) \times 10^{-5}$ rad/sid. day^2, at t = 0.412 day, L = 1.706°, λ = 60.94°. (See Figure 2)

See Table 1 For Additional Notes.

Table 2S

Osculating Elements at the First Ascending Equator Crossing Past the Syncom 2 Tracking Epoch, and Related Data in a Simulated Syncom 2, Arc 2 Trajectory with Earth Longitude Gravity through Second Order.*

Orbit Number 2S–	Tracking Epoch (yr-mo-day-hr-min UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1963.0 (days)	Geographic Longitude of first Ascending Equator Crossing After Tracking Epoch, λ (degrees)	① Time from January 384.2230, 1963, t (days)	② Longitude of the Ascending Equator Crossing East of -62.649°, L (degrees)
1	63-11-28-1.0	6.6105430	32.920	-44.040	.00005	-153.540	153.543	332.8558	-59.160	-51.3672	3.489
2	63-12-4.0	6.6109361	32.920	-44.128	.00008	- 18.896	18.894	338.8393	-59.201	-45.3837	3.448
3	63-12-10.0	6.6108628	32.906	-44.213	.00004	34.301	- 34.307	344.8229	-59.270	-39.4001	3.379
4	63-12-16-17.0	6.6110089	32.907	-44.304	.00004	- 66.460	66.491	350.8066	-59.386	-33.4164	3.263
5	63-12-23-19.0	6.6111138	32.891	-44.404	.00001	- 72.413	72.423	358.7851	-59.615	-25.4379	3.034
6	64-1-6-17.0	6.6113789	32.870	-44.582	.00007	25.588	- 25.594	371.7507	-60.185	-12.4723	2.464
7	64-1-9-6.0	6.6113491	32.867	-44.629	.00004	8.904	- 8.904	374.7428	-60.341	- 9.4802	2.308
8	64-1-15-18.0	6.6117630	32.861	-44.709	.00008	- 20.008	20.007	381.7246	-60.760	- 2.4984	1.889
9	64-1-20-21.0	6.6115831	32.844	-44.778	.00002	- 7.961	7.969	386.7117	-61.093	2.4887	1.556
10	64-1-29-20.0	6.6119796	32.829	-44.893	.00013	14.624	- 14.621	395.6888	-61.801	11.4658	.848
11	64-2-5-16.0	6.6118246	32.811	-44.984	.00006	25.370	- 25.371	401.6735	-62.313	17.4505	.336
12	64-2-10-19.0	6.6121671	32.804	-45.058	.00009	- 13.385	13.385	407.6585	-62.879	23.4355	- .230
13	64-2-17-17.0	6.6120119	32.778	-45.157	.00002	24.230	- 24.248	414.6411	-63.594	30.4181	- .945
14	64-2-25-19.0	6.6124152	32.762	-45.263	.00013	18.256	- 18.252	422.6215	-64.512	38.3985	-1.863
15	64-3-4-23.0	6.6122805	32.739	-45.392	.00006	26.790	- 26.792	430.6020	-65.490	46.3790	-2.841
16	64-3-10-13.0	6.6126075	32.731	-45.455	.00009	- 0.415	0.415	435.5900	-66.150	51.3670	-3.501
		Average: $6.611614 = a_s$	Average: $32.84 = i_s$								

*Computed by ITEM with gravity constants the same as in Table A1 with the addition of the earth constants: $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$.

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = 1.7226 \pm 2.223 \times 10^{-3}$ degrees

$a_2 = -(6.7983 \pm 0.01237) \times 10^{-2}$ degrees/sol. day

$a_3 = -(6.5640 \pm 0.01614) \times 10^{-4}$ degrees/sol. day^2

$a_4 = -(1.493 \pm 62.19) \times 10^{-9}$ degrees/sol. day^3

Standard error of estimate = 5.908×10^{-3} degrees

$\ddot{\lambda}$(measured) = $-(2.2787 \pm 0.00560) \times 10^{-5}$ rad/sid. day^2, at t = 0.412 day and $\lambda = -60.954°$

$\ddot{\lambda}$(theoretical, from Equation 2) = -2.3061×10^{-5} rad/sid. day^2, for $a_s = 6.61161$ earth radii, $i_s = 32.84°$, $\lambda = -60.954°$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$

Estimate of acceleration bias due to sun-moon perturbations and J_{22} model error (exclusive of higher order longitude gravity effects), in $\ddot{\lambda}$ at t = 0.412 day in Syncom arc 2:

Bias = theoretical - measured = $-(2.3061 \times 10^{-5}) + (2.2787 \times 10^{-5}) = -(0.0274) \times 10^{-5}$ rad/sid. day^2

Table 2S/1

Ascending Equator Crossing Data from a Simulated Syncom 2 Trajectory for Free Drift Arc 2, Computed by ITEM with Earth Longitude Gravity through Third Order*.

Orbit Number 2S/1–	Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Time from 1963.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time From January 384.2227, 1963, t (days)	② Longitude of Ascending Equator Crossing East of -62.556°, L (degrees)
1	63-11-28-1.0	6.6105430	32.920	332.8558	-59.160	-51.3669	3.396
2	63-12-4.0	6.6109340	32.920	338.8393	-59.200	-45.3834	3.356
3	63-12-10.0	6.6108585	32.906	344.8229	-59.268	-39.3998	3.288
4	63-12-16-17.0	6.6110021	32.907	350.8066	-59.382	-33.4161	3.174
5	63-12-23-19.0	6.6111039	32.891	358.7851	-59.605	-25.4376	2.951
6	64-1-6-17.0	6.6113633	32.870	371.7506	-60.162	-12.4721	2.394
7	64-1-9-6.0	6.6113321	32.867	374.7427	-60.315	- 9.4800	2.241
8	64-1-15-18.0	6.6117426	32.861	381.7245	-60.723	- 2.4982	1.833
9	64-1-20-21.0	6.6115603	32.844	386.7116	-61.048	2.4889	1.508
10	64-1-29-20.0	6.6119519	32.829	395.6886	-61.737	11.4659	.819
11	64-2-5-16.0	6.6117933	32.811	401.6733	-62.234	17.4506	.322
12	64-2-10-19.0	6.6121317	32.804	407.6583	-62.785	23.4356	- .229
13	64-2-17-17.0	6.6119720	32.778	414.6408	-63.479	30.4181	- .923
14	64-2-25-19.0	6.6123694	32.762	422.6211	-64.368	38.3984	-1.812
15	64-3-4-23.0	6.6122286	32.739	430.6016	-65.315	46.3789	-2.759
16	64-3-10-13.0	6.6125514	32.731	435.5895	-65.952	51.3668	-3.396
		Average: $6.6116 = a_s$	Average: $32.84 = i_s$				

(see Figure 2)

*Gravity constants of this trajectory are the same as that in Table A1, with the addition of the earth constants:

$J_{22} = -1.8 \times 10^{-6}$, $\lambda_{22} = -15.35^o$

$J_{33} = -0.16 \times 10^{-6}$, $\lambda_{33} = 24.0^o$

$J_{31} = -1.5 \times 10^{-6}$, $\lambda_{31} = 0.0^o$

(See Figure B1 for the significance of these constants). The initial elements of this trajectory, aside from those listed for orbit 2 S/1-1, are the same as those in orbit 2S-1 (Table 2S).

Results of least squares fit of data in ① and ② according to the theory of Equation 1: (for arc 2S/1)

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = (1.6705 \pm 0.00214)$ degrees

$a_2 = -(6.6252 \pm 0.0119) \times 10^{-2}$ degrees/sol. day

$a_3 = -(6.3450 \pm 0.0155) \times 10^{-4}$ degrees/sol. day^2

$a_4 = (7.13 \pm 5.98) \times 10^{-8}$ degrees/sol. day^3

Standard error of estimate = 0.00569 degree

$\ddot{\lambda}$ (measured, with minimum standard error) = -2.2024×10^{-5} rad/sid. day^2, at $\lambda = -60.91^o$ on $t' = 384.6346$ January 1963

$\ddot{\lambda}$ (theoretical, from Equation 2) = -2.2330×10^{-5} rad/sid. day^2, for $a_s = 6.6116$ earth radii, $i_s = 32.84^o$, $\lambda = -60.91^o$, $J_{22} - J_{31}$ as noted.

Estimate of acceleration bias in arc S2/1 = $\ddot{\lambda}$ (theoretical) − $\ddot{\lambda}$ (measured) = -0.0306×10^{-5} rad/sid. day^2

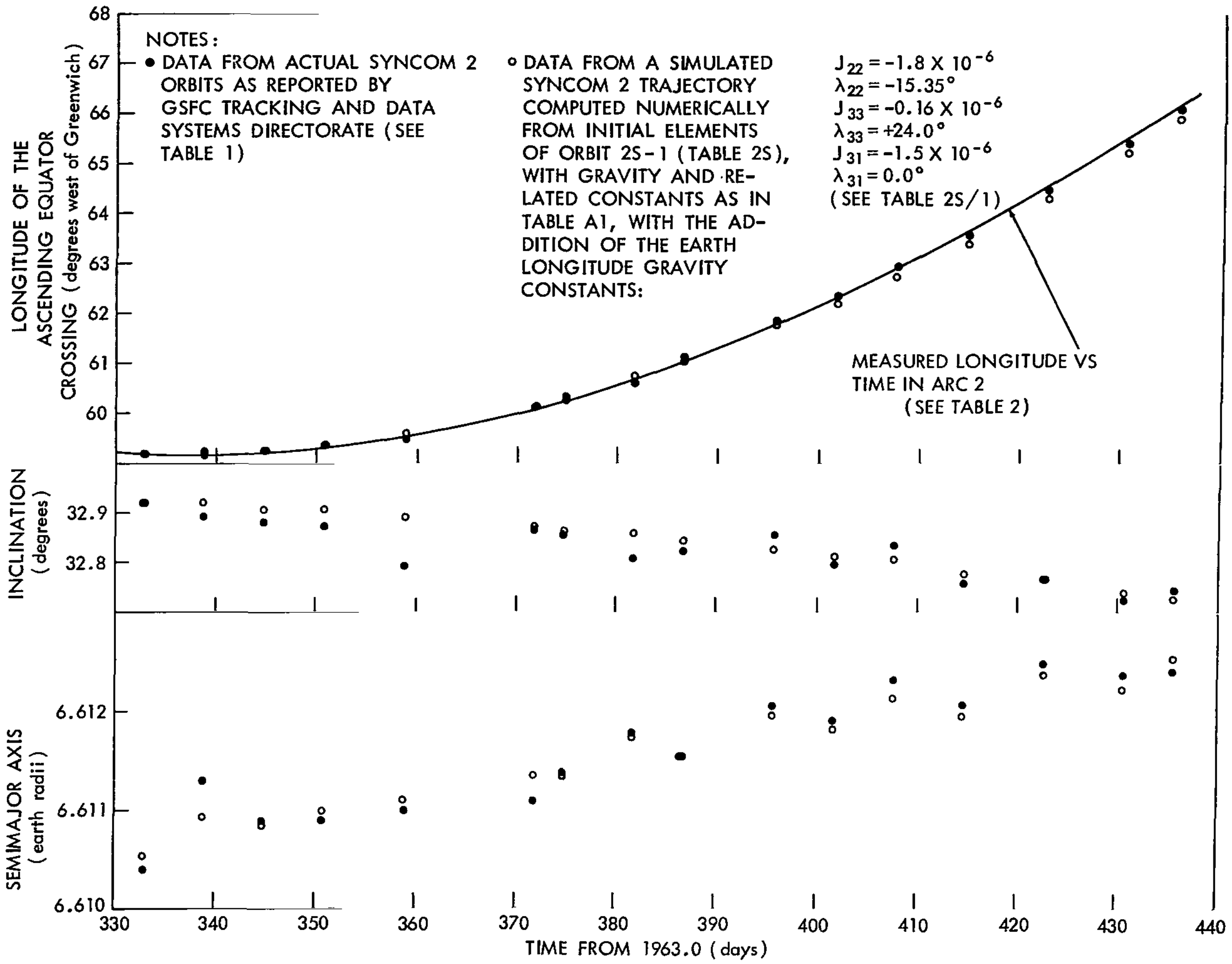

Figure 2—Measured and simulated orbit data at ascending Equator crossings in arc 2 (Syncom 2).

We distinguish two drift regimes for the 24-hour satellite on the basis of mean drift rate as discussed in the introduction. If the mean drift rate in an arc is less than ±0.1 degree/day, the ground track of the synchronous satellite is somewhat arbitrarily said to be in a slow drift regime. In this regime, it has been found in practice (and reconciled theoretically in Appendix D; see also Reference 3) that the mean geographic longitude can be expressed as a low degree polynomial in the time with constant coefficients related to the strength and orientation of the underlying longitude gravity field. If the mean drift rate is greater than ±0.1 degree/day, the satellite is said to be in a "fast drift" regime. In this regime the energy, or first, integral of Equation 2 has been found to give the best representation of the motion as a function of the underlying longitude gravity field. In particular, it has been found that for a realistic earth, if the drift excursion in the arc is limited to about 50°, only the second order H_{22} harmonic terms in Equation 2 need be retained for a sufficiently accurate representation. With this limitation, the energy integral can be shown to be expressable as

$$(\dot{\lambda})^2 = C_0 + C_{22} F(i_s, a_s)_{22} \cos 2\lambda + S_{22} F(i_s, a_s)_{22} \sin 2\lambda \text{ rad/sid. day}^2 \quad (8)$$

where

$$C_{22} = J_{22} \cos 2\lambda_{22}$$

and

$$S_{22} = J_{22} \sin 2\lambda_{22} \quad . \tag{9}$$

Equation 9 gives the dependence of the determinable constants in Equation 8 in terms of the strength (J_{22}) and orientation (λ_{22}) of the (H_{22})earth gravity harmonic. The orbit constant $F(i_s, a_s)_{22}$ is given as

$$F(i_s, a_s)_{22} = 18\left[\pi(\cos i_s + 1)/a_s\right]^2 \quad , \tag{10}$$

where a_s is in units of earth radii (see Equations 76 and 77 in Reference 2).

In References 5 and 6 it was shown that in the fast drift regime, for excursions of about 10 days or 10° or less, sufficient accuracy is maintained if the drift rate is calculated simply from the difference of successive longitudes and assigned to the midlongitude. Unfortunately, in arc 3 the orbit determination appears to be so poor, or the record so brief, that utilization of only a single Equator crossing in each determined orbit has not proved adequate to give even one well determined acceleration for this arc. Some improvement in acceleration discrimination has resulted from the utilization of successive crossings in each orbit to provide additional independent drift velocity data. (See the discussion in Reference 6.) But even with this extra data and also with the use of an estimated crossing just prior to the jet pulsing on 25 April initiating arc 4, the best standard error in the acceleration for this arc is close to 100% of the measured value. The details of this drift are presented in Table 3. The principal results of the acceleration analysis on the actual data in Table 3 are listed in Table 10 in the next section. The data was so inconclusive that the "best" measured acceleration in this arc was ignored in the final gravity synthesis.

Arc 4, Syncom 2, 25 April 1964 - 4 July 1964

On 25 April 1964, on-board jet pulsing slowed the westward drift of Syncom 2 from -1.3 degree/day to -0.8 degree/day. The ascending Equator crossing at this time was at 116° W. The orbit inclination was about 32.6°. From 25 April to 7 July 1964, the "figure of 8" ground track of Syncom 2 moved from 116° W to 164° W in free gravity drift. The principal effect in this two month arc was a deceleration of the drift rate, due to resonant earth longitude gravity, from -0.81 degree/day to -0.75 degree/day. The drift regime is "fast" and the details of the long term acceleration analysis on the measured and simulated ascending Equator crossings according to Equation 8 are presented in Table 4 and Figure 3 and summarized in Tables 10 and 11.

A number of gravity drift simulated trajectories for this arc were calculated with initial semimajor axis, Equator crossing longitude and earth longitude gravity as variables. The results of these (Tables 4S and 4S/1) were only fairly conclusive as to the exact magnitude of long term acceleration bias due to sun and moon gravity and model error in this arc. It would seem that the

Table 3

Syncom 2 Osculating Elements at the First Ascending Equator Crossings Past the Tracking Epoch and Related Data for Free Drift Arc 3.†

Orbit Number 3 –	Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	j	Mean Anomaly (degrees)	Time from 1964.0, T (days)	$\Delta T = T_{j+1} - T_j$ for [bracketed] Data: $T_{j'} - T_j$ (days)	① Longitude of the Ascending Equator Crossing, λ (degrees)	$\Delta\lambda = \lambda_{j+1} - \lambda_j$ for [bracketed] Data: $\lambda_{j'} - \lambda_j$ (degrees)	② $\Delta\lambda/\Delta T$, $\dot{\lambda}$ (degrees/day)
1	64-3-18-2.0	6.6266333	32.682	-45.512	.00205	-173.534	1	173.508	78.5721		- 67.623		
											[- 68.28]*		[-1.3082]
2	64-3-18-2.0	6.6266800	32.681	-45.529	.00203	-173.162	1'	173.134	79.5730	1.0009	- 68.932	- 1.309	
											(- 71.55)**	- 7.848	(-1.3068)
3	64-3-24-13.0	6.6266362	32.724	-45.544	.00190	-169.992	2	169.954	84.5774	6.0053	- 75.471		
											[- 76.11]		[-1.2838]
4	64-3-24-13.0	6.6266143	32.720	-45.558	.00190	-170.431	2'	170.394	85.5782	1.0008	- 76.756	- 1.285	
											(81.31)		(-1.2959)
5	64-4-1-22.0	6.6269968	32.685	-45.699	.00204	-171.037	3	171.000	93.5847	9.0073	- 87.144	-11.673	
											[- 87.81]		[-1.3339]
6	64-4-1-22.0	6.6270420	32.684	-45.714	.00204	-170.705	3'	170.667	94.5856	1.0009	- 88.479	- 1.335	
											(- 91.12)		(-1.3242)
7	64-4-7-15.0	6.6269419	32.701	-45.831	.00197	-172.877	4	172.849	99.5900	6.0053	- 95.096	- 7.952	
											[- 95.75]		[-1.3093]
8	64-4-7-15.0	6.6269010	32.697	-45.846	.00198	-173.240	4'	173.213	100.5909	1.0009	- 96.406	- 1.310	
											(- 99.03)		(-1.3109)
9	64-4-13-19.0	6.6264550	32.599	-45.990	.00213	-172.507	5	172.475	105.5950	6.0050	-102.968	- 7.872	
											[-103.62]		[-1.3001]
10	64-4-13-19.0	6.6264975	32.599	-46.008	.00212	-172.055	5'	172.022	106.5958	1.0008	-104.269	- 1.3011	
											(-109.58)		(-1.3215)
11***	64-4-25-2.0	6.6268369	32.603				6		115.6026	10.0076	-116.193	-13.225	
		Average: $6.62675 = a_s$	Average: $32.67 = i_s$										

$*(\lambda_{j'} - \lambda_j)/2$; for [bracketed] data.

$**(\lambda_{j+1} - \lambda_j)/2$ for other data.

***Data estimated for crossing just prior to Epoch 64-4-25-2.0 hour from arc 4, orbit 4-1, and arc 3, orbit 3-9.

†See notes in Table 1.

Results of least squares fit of (bracketed) data in ① and ② above according to the drift theory of Equation 8:

$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s)\cos 2\lambda + S_{22}F(i_s, a_s)\sin 2\lambda$

$C_{22} = -(1.01 \pm 3.12) \times 10^{-6}$

$S_{22} = (5.85 \pm 6.75) \times 10^{-7}$

Standard error of estimate $= 1.215 \times 10^{-5}$ rad/sid. day^2

$\ddot{\lambda}$ (with minimum standard error) $= -(0.897 \pm 0.888) \times 10^{-5}$ rad/sid. day^2, at $\lambda = -88.0°$, (see Figure 10).

difference between the acceleration biases in trajectories 4S and 4S/1 can be accounted for by the neglect, in the analysis in Table 4S/1, of higher order earth longitude gravity which was responsible for the fine details of the drift. Analysis of long drift arcs (in excess of 30°) in various realistic earth longitude gravity fields has shown that the order of magnitude of this higher order earth gravity bias in an analysis with only second order gravity is about 0.03×10^{-5} rad/sid. day^2.

Evidently, an accurate assessment of the total model bias in a long, fast drift arc can only be made in a numerical analysis which includes at least some third order gravity effects. The most accurate discrimination of the model bias in such a regime would involve a third order reduction of the drift as well as its inclusion in the trajectory generator. Unfortunately, the parallel analysis of the actual orbits with a third order energy integral (similar to Equation 8) appears to lose considerable acceleration discrimination over the second order analysis of the third order trajectories in arcs 4 and 5, due to the large observation errors present in the limited actual data.

After many techniques were tried, the straightforward numerical evaluation of the model bias in the gravity experiment on the data presented here seems to be adequate to the observational precision of that data. Other more analytical and more lengthy iterative techniques have been used by Allan (Reference 12) to evaluate sun, moon, and model bias in Syncom 2 drift data (see Discussion.) The present analysis on the limited nine arc record points to the conclusion that the actual unadjusted accelerations as a whole are, if anything, better measures of earth gravity effects, more precise than bias adjusted measurements on any basis (see Table 12.) Perhaps in the future, when a greater proportion of augmentation (rather than cancellation) of error data is received and processed, it will prove more than academic to examine these bias removal techniques with thoroughness.

Arc 5, Syncom 2, 4 July 1964 - 19 February 1965

On 4 July 1965 the westward drift of Syncom 2, at a mean longitude of 171° W, was slowed from -0.75 degree/day to -0.5 degree/day by ground commanded on-board jet pulsings. For the next 7-1/2 months the satellite, as far as can be determined, drifted freely in the gravity fields of the earth, sun, and moon. The details of this drift as derived from orbits for Syncom 2 determined at GSFC are presented in Table 5 and Figure 4. Tables 5S and 5S/1 give the results of closely paralleling simulated trajectories numerically calculated in the presence of earth longitude gravity through second and third order. The second order trajectory uses gravity constants which were derived from earlier, more limited Syncom 2 data (Reference 3). The constants in the third order trajectory represent a best estimate at an intermediate stage in this analysis. The J_{31}, λ_{31} constants in these "ITEM" computed trajectories are best estimates in a private communication from W. M. Kaula in October 1964.

Greatest precision and fidelity to true earth effects at this stage in the reduction of the actual "noisy" data appears to be preserved when a single acceleration is calculated from a second order longitude gravity model extending over as short an arc as feasible. Model bias effects appear to be most accurately assessed by paralleling these reductions on a third order numerically calculated trajectory which closely follows the actual drift.

Table 4

Syncom 2 Osculating Elements at the First Ascending Equator Crossing Past the Tracking Epoch, and Related Data for Free Drift Arc 4.*

Orbit Number 4 –	Syncom 2 Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccen-tricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1964.0, t (days)	$\Delta t = t_{j+1} - t_j$ (days)	① Longitude of the Ascending Equator Crossing, λ (degrees)	$\Delta\lambda = \lambda_{j+1} - \lambda_j$ (degrees)	② $\frac{\Delta\lambda}{\Delta t}$, $\dot{\lambda}$ (degrees/day)
1	64-4-25-2.0	6.6208880	32.602	-46.131	.00120	-168.118	168.090	116.6039		-117.182		
								(118.6030)		(-118.81)**		(-0.8134)
2	64-4-28-15.0	6.6208306	32.594	-46.142	.00116	-163.317	163.279	120.6020	3.9981	-120.434	-3.252	
								(124.1001)		(-123.26)		(-0.8086)
3	64-5-5-16.0	6.6206780	32.560	-46.274	.00117	-165.263	165.229	127.5982	6.9962	-126.091	-5.657	
								(131.0963)		(-128.91)		(-0.8072)
4	64-5-12-16.0	6.6206500	32.626	-46.424	.00118	-159.744	159.697	134.5943	6.9961	-131.738	-5.647	
								(137.5927)		(-134.11)		(-0.7926)
5	64-5-19-14.0	6.6205248	32.577	-46.452	.00113	-163.604	163.567	140.5910	5.9967	-136.491	-4.753	
								(144.0889)		(-139.25)		(-0.7898)
6	64-5-25-15.0	6.6205311	32.600	-46.614	.00122	-162.158	162.115	147.5867	6.9957	-142.016	-5.525	
								(151.5844)		(-145.13)		(-0.7791)
7	64-6-2-21.0	6.6204625	32.576	-46.678	.00123	-163.458	163.418	155.5820	7.9953	-148.245	-6.229	
								(159.0798)		(-150.94)		(-0.7711)
8	64-6-9-21.0	6.6198435	32.580	-46.763	.00119	-159.072	159.023	162.5776	6.9956	-153.639	-5.394	
								(166.0753)		(-156.30)		(-0.7604)
9	64-6-16-15.0	6.6199213	32.565	-46.874	.00118	-162.909	162.870	169.5729	6.9953	-158.958	-5.319	
								(173.0705)		(-161.59)		(-0.7518)
10	64-6-23-15.0	6.6201029	32.562	-46.991	.00122	-161.998	161.955	176.5680	6.9951	-164.217	-5.259	
		Average: 6.6204433 = a_s	Average: 32.584 = i_s									

Results of least squares fit of (bracketed) data in ① and ② above according to the drift theory of Equation 8:

$$(\dot{\lambda})^2 = C_0 + C_{22} F(i_s, a_s) \cos 2\lambda + S_{22} F(i_s, a_s) \sin 2\lambda$$

$$C_{22} = -(1.4347 \pm 0.0788) \times 10^{-6}$$

$$S_{22} = (0.8114 \pm 0.2823) \times 10^{-6}$$

Standard error of estimate = 1.152×10^{-6} rad/sid. day^2

$\ddot{\lambda}$ (with mimimum standard error) = $(2.138 \pm 0.0842) \times 10^{-5}$ rad/sid. day^2, at $\lambda = -140.00^\circ$ (see Figure 3)

*See notes in Table 1.

**$\Delta\lambda/2$ for (bracketed) longitudes

Table 4S-A

Osculating Elements at the First Equator Crossing Past the Syncom 2 Tracking Epoch, and Related Data in Two Simulated Syncom 2 Arc 4 Trajectories with Earth Longitude Gravity through Second Order.*

Orbit Number 4S-A –	Syncom 2 Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1964.0, t (days)	$\Delta t = t_{j+1} - t_j$ (days)	① Longitude of the Ascending Equator Crossing, λ (degrees)	$\Delta\lambda = \lambda_{j+1} - \lambda_j$ (degrees)	② $\Delta\lambda/\Delta t$, $\dot{\lambda}$ (degrees/day)
1	64-4-25-2.0	6.6206462	32.602	-46.131	.00120	-168.120	168.093	116.6039		-117.182		
								(118.6029)		(-118.81)**		(-.8167)
2	64-4-28-15.0	6.6205913	32.601	-46.201	.00123	-166.322	166.289	120.6019	3.9980	-120.447	-3.265	
								(124.1001)		(-123.29)		(-.8138)
3	64-5-5-16.0	6.6207620	32.592	-46.293	.00120	-165.756	165.722	127.5983	6.9964	-126.141	-5.694	
								(131.0964)		(-128.97)		(-.8087)
4	64-5-12-16.0	6.6204145	32.582	-46.404	.00127	-164.655	164.617	134.5945	6.9962	-131.799	-5.658	
								(137.5929)		(-134.21)		(-.8046)
5	64-5-19-14.0	6.6205767	32.572	-46.495	.00115	-167.061	167.031	140.5913	5.9968	-136.624	-4.825	
								(144.0893)		(-139.40)		(-.7940)
6	64-5-25-15.0	6.6202807	32.566	-46.613	.00122	-165.700	165.665	147.5872	6.9959	-142.179	-5.555	
								(151.5849)		(-145.320)		(-.7856)
7	64-6-2-21.0	6.6203666	32.557	-46.711	.00119	-165.422	165.387	155.5825	7.9953	-148.460	-6.281	
								(159.0803)		(-151.18)		(-.7765)
8	64-6-9-21.0	6.6201344	32.551	-46.829	.00122	-163.006	162.966	162.5781	6.9956	-153.892	-5.432	
								(166.0759)		(-156.58)		(-.7682)
9	64-6-16-16.0	6.6200798	32.535	-46.914	.00115	-166.916	166.887	169.5736	6.9955	-159.266	-5.374	
								(173.0713)		(-161.91)		(-.7571)
10	64-6-23-15.0	6.6199550	32.533	-47.020	.00120	-164.171	164.134	176.5689	6.9953	-164.562	-5.296	
		Average: $6.6203807 = a_s$	Average: $32.57 = i_s$									

*Computed by ITEM with gravity constants the same as in Table A1 with the addition of earth constants: $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$.

**$\Delta\lambda/2$ for (bracketed) longitudes.

Results of least squares fit of (bracketed) data for arc S4-A in ① and ② above according to the drift theory of Equation 8:

$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s) \cos 2\lambda + S_{22}F(i_s, a_s) \sin 2\lambda$

$C_0 = 1.7987 \times 10^{-4}$ rad²/sid. day²

$C_{22} = -(1.3234 \pm 0.0390) \times 10^{-6}$

$S_{22} = (1.0770 \pm 0.1372) \times 10^{-6}$

Standard error of estimate = 5.685×10^{-7} rad²/sid. day²

$\ddot{\lambda}$ (measured, at $\lambda = -140.0°$) = $(2.051 \pm 0.042) \times 10^{-5}$ rad/sid. day²

$\ddot{\lambda}$ (theoretical, from Equation 2) = 2.078×10^{-5} rad/sid. day², where $a_s = 6.6203807$ earth radii, $i_s = 32.57°$, $\lambda = -140.0°$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$.

Bias (theoretical-measured) = $2.078 \times 10^{-5} - 2.051 \times 10^{-5} = +\,0.027 \times 10^{-5}$ rad/sid. day²

Table 4S-B

Osculating Elements at the First Equator Crossing Past the Syncom 2 Tracking Epoch, and Related Data in Two Simulated Syncom 2 Arc 4 Trajectories with Earth Longitude Gravity through Second Order.*

Orbit Number 4S-B –	Syncom 2 Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1964.0, t (days)	$\Delta t = t_{j+1} - t_j$ (days)	① Longitude of the Ascending Equator Crossing, λ (degrees)	$\Delta\lambda = \lambda_{j+1} - \lambda_j$ (degrees)	② $\Delta\lambda/\Delta t$, $\dot{\lambda}$ (degrees/day)
1	64-4-25-2.0	6.6206118	32.602	-46.131	.00120	-163.674	163.635	116.6039		-117.160		
										(-118.79)		(-.8139)
2	64-4-28-15.0	6.6205573	32.601	-46.201	.00124	-161.997	161.953	120.6018	3.9979	-120.414	-3.254	
										(-123.25)		(-.8110)
3	64-5-5-16.0	6.6207281	32.592	-46.292	.00120	-161.312	161.268	127.5981	6.9963	-126.088	-5.674	
										(-128.91)		(-.8060)
4	64-5-12-16.0	6.6204353	32.582	-46.420	.00125	-159.660	159.611	134.5943	6.9962	-131.727	-5.639	
										(-134.13)		(-.8019)
5	64-5-19-14.0	6.6205429	32.572	-46.495	.00115	-162.421	162.381	140.5910	5.9967	-136.536	-4.809	
										(-139.30)		(-.7913)
6	64-5-25-15.0	6.6202474	32.566	-46.613	.00122	-161.330	161.285	147.5869	6.9959	-142.072	-5.536	
										(-145.20)		(-.7827)
7	64-6-2-21.0	6.6203333	32.557	-46.711	.00119	-160.950	160.906	155.5821	7.9952	-148.330	-6.258	
										(-151.04)		(-.7739)
8	64-6-9-21.0	6.6201010	32.551	-46.829	.00122	-158.629	158.578	162.5777	6.9956	-153.744	-5.414	
										(-156.42)		(-.7653)
9	64-6-16-15.0	6.6200466	32.535	-46.914	.00115	-162.291	162.252	169.5732	6.9955	-159.098	-5.354	
										(-161.74)		(-.7544)
10	64-6-23-15.0	6.6199205	32.533	-47.020	.00120	-159.742	159.695	176.5684	6.9952	-164.375	-5.277	
		Average: $6.6203524 = a_s$	Average: $32.57 = i_s$									

*See notes in Table 4S-A.

Results of least squares fit of (bracketed) data for arc 4S-B in ① and ② above according to the drift theory of Equation 8:

$(\dot{\lambda})^2 = C_0 + C_{22} F(i_s, a_s) \cos 2\lambda + S_{22} F(i_s, a_s) \sin 2\lambda$

$C_{22} = -(1.3250 \pm 0.0388) \times 10^{-6}$

$S_{22} = (1.0787 \pm 0.1378) \times 10^{-6}$

Standard error of estimate $= 5.669 \times 10^{-7}$ rad^2/sid. day^2

$\ddot{\lambda}$ (measured, at $\lambda = -140.0°$) $= (2.054) \pm 0.0413) \times 10^{-5}$ rad/sid, day^2

$\ddot{\lambda}$ (theoretical, from Equation 2) $= 2.078 \times 10^{-5}$ rad/sid. day^2, where $a_s = 6.620352$ earth radii, $i_s = 32.57°$, $\lambda = 140.0°$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$.

Bias (theoretical-measured) $= 2.078 \times 10^{-5} - 2.054 \times 10^{-5} = +0.024 \times 10^{-5}$ rad/sid. day^2.

Table 4S/1

Osculating Elements at the First Ascending Equator Crossing Past the Syncom 2 Tracking Epoch, and Related Data in a Simulated Syncom 2 Arc 4 Trajectory with Earth Longitude Gravity through Third Order.*

Orbit Number 4S/1 –	Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Time from 1964.0, t (days)	$\Delta t = t_{j+1} - t_j$ (days)	① Longitude of the Ascending Equator Crossing, λ (degrees)	$\Delta\lambda = \lambda_{j+1} - \lambda_j$ (degrees)	② $\Delta\lambda/\Delta t, \dot{\lambda}$ (degrees/day)
1	64-4-25-2.0	6.6206454	32.602	116.6039		-117.182		
						(-118.81)**		(-.8167)
2	64-4-28-15.0	6.6205886	32.601	120.6019	3.9980	-120.447	-3.265	
						(-123.29)		(-.8134)
3	64-5-5-16.0	6.6207557	32.592	127.5983	6.9964	-126.138	-5.691	
						(-128.96)		(-.8080)
4	64-5-12-16.0	6.6204594	32.583	134.5945	6.9962	-131.791	-5.653	
						(-134.20)		(-.8038)
5	64-5-19-14.0	6.6205639	32.572	140.5912	5.9967	-136.611	-4.820	
						(-139.38)		(-.7929)
6	64-5-25-15.0	6.6202643	32.566	147.5871	6.9959	-142.158	-5.547	
						(-145.29)		(-.7840)
7	64-6-2-21.0	6.6203440	32.557	155.5824	7.9953	-148.426	-6.268	
						(-151.13)		(-.7743)
8	64-6-9-21.0	6.6201058	32.551	162.5780	6.9956	-153.843	-5.417	
						(-156.52)		(-.7655)
9	64-6-16-15.0	6.6200444	32.535	169.5734	6.9954	-159.198	-5.355	
						(-161.84)		(-.7539)
10	64-6-23-15.0	6.6199117	32.533	176.5686	6.9952	-164.472	-5.274	
		Average: $6.6204 = a_s$	Average: $32.57 = i_s$					

*Gravity constants of this trajectory computed by ITEM are the same as those in Table A1, with the addition of these earth constants:

$J_{22} = -1.8 \times 10^{-6}$, $\lambda_{22} = -15.35°$

$J_{33} = -0.16 \times 10^{-6}$, $\lambda_{33} = 24.0°$

$J_{31} = -1.5 \times 10^{-6}$, $\lambda_{31} = 0.0°$

See Figure B1 for the significance of these constants. The initial elements of this trajectory, aside from those listed for orbit 4S/1-1, are the same as those in orbit S4A-1 (Table 4S).

**$(\lambda_{j+1} - \lambda_j/2)$ for (bracketed) longitudes

Results of least squares fit of the (bracketed) data in ① and ② above according to the theory of Equation 8:

$(\dot{\lambda})^2 = C_0 + C_{22}\, F(i_s, a_s) \cos 2\lambda + S_{22}\, F(i_s, a_s) \sin 2\lambda_{22}$

$C_0 = 1.786 \times 10^{-4}$ rad²/sid. day²

$C_{22} = -(1.3905 \pm 0.0386) \times 10^{-6}$

$S_{22} = (1.1362 \pm 0.1362) \times 10^{-6}$

Standard error of estimate = 5.627×10^{-7} rad²/sid. day²

$\ddot{\lambda}$ (with minimum standard error) = 2.156×10^{-5} rad/sid. day², measured, at $\lambda = -140.0°$

$\ddot{\lambda}$ (theoretical, from Equation 2) = 2.163×10^{-5} rad/sid. day², where $a_s = 6.6204$ earth radii, $i_s = 32.57°$, $\lambda = -140.0°$, and $J_{22} - J_{31}$ as noted

Estimate of acceleration bias at $\lambda = -140.0°$ in arc S4/1 = $\ddot{\lambda}$ (theoretical) − $\ddot{\lambda}$ (actual) = $+0.007 \times 10^{-5}$ rad/sid. day².

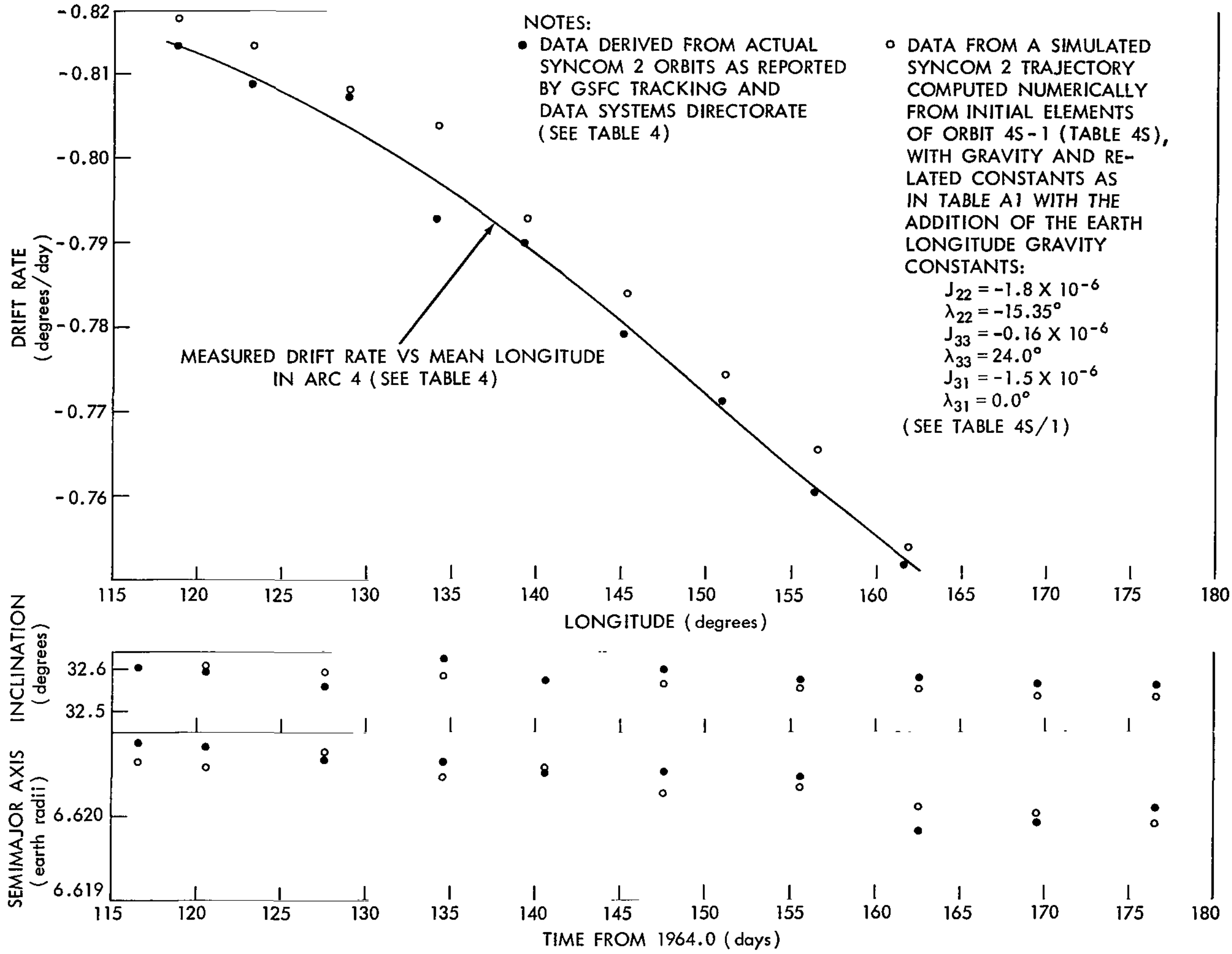

Figure 3—Measured and simulated orbit data at ascending Equator crossings in arc 4 (Syncom 2).

In line with this reasoning and also with the object in mind of obtaining as wide and precise a longitude survey as possible with the single-acceleration reduction technique, arc 5 and one simulated arc 5 trajectory were split up into 18 consecutive sub-arcs. (See Tables 10 and 11 in the next section). The first such sub-arc spans the first 9 (bracketed) longitude-drift rate data points in Tables 5 and 5S/1. Each succeeding overlapped sub-arc drops the leading point of the previous one and adds a new point consecutively until the entire arc is covered. Each of these sub-arcs are analyzed for acceleration in exactly the same way as in arc 4, except as noted below.

Between 17 November 1964 and 10 January 1965 no orbits for Syncom 2 were calculated. The range and range rate transponder on board the satellite was not used then to conserve battery power, as the satellite was spending a considerable time in the earth's shadow in this period. As a result, the first few new orbits in January 1965 were particularly ill determined. In addition extensive data testing has shown that a few orbits in November 1964 also gave poorly defined single Equator crossing information.

On the other hand, it appeared that such suspect single crossing orbits gave reasonably good drift rates from successive crossings. This is equivalent to saying that the satellite position at epoch was poorly determined for these orbits (21, 24, and 27 in Table 5), but the semimajor axis was reasonably well determined. The reverse is the usual situation. In Table 5 it will be seen that the orbits mentioned above have been utilized for drift rate information without association with neighboring orbits. While the mean crossing longitudes for these are probably in error by about 0.2°, it is sufficiently accurate in this reduction where rate information demands the greater precision.

It is regretted that the standard errors in the semimajor axes and longitude locations reported by GSFC for these and other Syncom 2 and Syncom 3 orbits did not always agree with the evident errors revealed by the full arc analysis and simulations in this investigation. An effort was made to get better (smoother) information by using the complete record in these arcs without exception. Various a priori data weighting schemes were tried on the basis of these reported errors without noticeable gain in accuracy. Simple data rejection on the basis of a 3σ criteria, after a trial reduction, was used in this and other arcs to arrive at the final acceptable data record.

In the future, further smoothing of the data in all the Syncom arcs may be possible by a separate analysis of the semimajor axis drift according to the resonant formulations in References 2 and 3. (See Discussion).

Arc 6, Syncom 3, 31 October 1964 - 21 December 1964

Syncom 3, the first geostationary satellite was launched in August 1964 and reached station in the Pacific over the International Date Line in October. From 31 October to 21 December 1964, the satellite was permitted to drift from 180° to 178° at less than 0.1 degree/day westward without correction maneuvers. The details of this free gravity accelerated drift are found in Table 6 and Figure 5 and summarized in Tables 10 and 11 in the next section.

The actual data analysis for acceleration of the geostationary or slow moving Syncom 3 drift in this arc follows the t^3 fit technique used in arcs 1 and 2 for Syncom 2. While the inertial location of Syncom 3's Equator crossing (determined by the crossing time) was poorly defined (the orbit being nearly equatorial), the geographic location had good definition in this arc since the subsatellite point was nearly stationary. This is brought out best by the low value of the standard error of the estimate of the Equator crossing drift under a t^3 formulation, compared to the Syncom 2 arcs 1 and 2.

The poor definition of the inertial location of Syncom 3 in arc 6 has caused some difficulty in finding an acceptably close parallel simulated trajectory. Comparison of the results of the two simulations in Table 6S shows the discrepancy even half a day can make in the sun, moon, and model bias errors. However, it is noted that in arc 6, as in arcs 1 and 2, there is good agreement in the bias results in the two simulations from identical initial elements between second order and

Table 5

Syncom 2 Osculating Elements at the First Ascending Equator Crossings Past the Tracking Epoch and Related Data in Free Drift Arc 5*.

Orbit Number 5 –	Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1964.0, t (days)	$\Delta t = t_{j+1} - t_j$ (days)	① Longitude of the Ascending Equator Crossing, λ (degrees)	$\Delta\lambda = \lambda_{j+1} - \lambda_j$ (degrees)	② $\Delta\lambda/\Delta t$, $\dot{\lambda}$ (degrees/day)
1	64-7-4-2.0	6.6165754	32.541	-47.118	.00086	-154.569	154.527	186.5599		-171.261		
										(-171.97)**		(-.4734)
2	64-7-7-3.0	6.6169477	32.537	-47.161	.00094	-150.702	150.649	189.5555	2.9956	-172.679	- 1.418	
										(-174.36)		(-.4805)
3	64-7-13-17.0	6.6166919	32.500	-47.211	.00079	-153.849	153.810	196.5455	6.9900	-176.038	- 3.359	
										(-177.91)		(-.4692)
4	64-7-21-21.0	6.6164887	32.515	-47.363	.00087	-147.458	147.404	204.5337	7.9882	-179.791	- 3.753	
										(178.81)		(-.4677)
5	64-7-27-16.0	6.6165999	32.500	-47.422	.00084	-150.162	150.114	210.5249	5.9912	177.407	- 2.802	
										(175.79)		(-.4624)
6	64-8-3-17.0	6.6166087	32.476	-47.555	.00081	-145.700	145.648	217.5144	6.9895	174.175	- 3.232	
										(172.59)		(-.4548)
7	64-8-11-1.0	6.6163047	32.445	-47.606	.00077	-149.337	149.292	224.5039	6.9895	170.996	- 3.179	
										(169.43)		(-.4494)
8	64-8-17-19.0	6.6164211	32.408	-47.700	.00089	-145.420	145.362	231.4932	6.9893	167.855	- 3.141	
										(166.27)		(-.4522)
9	64-8-25-10.0	6.6164814	32.443	-47.739	.00089	-138.379	138.311	238.4828	6.9896	164.694	- 3.161	
										(163.13)		(-.4466)
10	64-9-1-10.0	6.6163260	32.397	-47.933	.00080	-144.436	144.382	245.4718	6.9890	161.573	- 3.121	
										(159.54)		(-.4521)
11	64-9-9-14.0	6.6160374	32.372	-48.100	.00090	-145.744	145.686	254.4580	8.9862	157.510	- 4.063	
										(156.17)		(-.4487)
12	64-9-15-12.0	6.6165134	32.360	-48.148	.00083	-140.900	140.840	260.4489	5.9909	154.822	- 2.688	
										(153.46)		(-.4562)
13	64-9-22-10.0	6.6164083	32.302	-48.256	.00087	-141.927	141.866	266.4398	5.9909	152.089	- 2.733	
										(150.48)		(-.4605)
14	64-9-29-6.0	6.6165573	32.327	-48.344	.00083	-140.203	140.143	273.4294	6.9896	148.870	- 3.219	
										(147.29)		(-.4534)
15	64-10-6-5.0	6.6164264	32.321	-48.414	.00081	-135.504	135.440	280.4189	6.9895	145.701	- 3.169	
										(144.05)		(-.4733)
16	64-10-13.0	6.6167308	32.312	-48.565	.00090	-135.824	135.753	287.4085	6.9896	142.393	- 3.308	
										(140.50)		(-.4746)
17	64-10-20-16.0	6.6163209	32.289	-48.662	.00090	-139.171	139.104	295.3969	7.9884	138.602	- 3.791	
										(137.16)		(-.4817)
18	64-10-26-16.0	6.6169645	32.250	-48.815	.00072	-133.261	133.202	301.3881	5.9912	135.716	- 2.886	
										(134.21)		(-.5012)
19	64-11-2-5.0	6.6164857	32.249	-48.864	.00079	-139.157	139.097	307.3799	5.9918	132.713	- 3.003	
		Average: 6.6165206 = a_s	Average: 32.397 = i_s	arc 5A data								
										(130.41)		(-.5125)
20	64-11-11-2.0	6.6171112	32.287	-48.990	.00080	-128.016	127.945	316.3677	8.9878	128.107	- 4.606	
21	64-11-17-6.0	6.6169067	32.230	-49.018	.00081	-137.326	137.264	322.3588		125.370		
										(125.1)		(-.5177)
22	64-11-17-6.0	6.6168678	32.229	-49.038	.00083	-136.946	136.882	323.3575	0.9987	124.853	- 0.517	
										(112.0)†		(-.5570)
23	65-1-10-6.0	6.6173810	32.131	-49.815	.00066	-120.051	119.987	376.2941	59.9264	94.729	-33.378	
24	65-1-13-16.0	6.6180715	32.164	-49.942	.00071	-130.815	130.754	380.2891		92.461		
										(92.16)		(-.6057)
25	65-1-13-16.0	6.6181446	32.163	-49.954	.00070	-128.832	128.771	381.2880	0.9989	91.856	- 0.605	
										(91.45)		(-.5977)
26	65-1-20-12.0	6.6185305	32.117	-50.105	.00060	-319.867	139.823	387.2814	10.9873	88.162	- 6.567	
27	65-1-27-4-5.0	6.6181841	31.956	-49.833	.00066	160.176	-160.151	393.2771		84.089		
										(83.79)		(-.6087)
28	65-1-27-4-5.0	6.6182309	31.955	-49.845	.00065	160.915	-160.891	394.2760	0.9989	83.48	- 0.608	
										(80.21)†		(-.6127)
29	65-2-16-4-5.0	6.6183288	32.111	-50.406	.00061	-127.648	127.592	413.2537	25.9723	72.248	-15.914	
										(71.02)		(-.6169)
30	65-2-19-23.0	Average: 6.617425 = a_s	Average: 32.16° = i_s	arc 5B data (includes data in orbits 5-17, 18, 19)				417.2496	3.9959	69.783††	- 2.465	
		Average: 6.617 = a_s	Average: 32.33° = i_s	full arc 5 data								

See Footnotes Page 25.

*See notes in Table 1.

**$(\lambda_{j+1} - \lambda_j)/2$ for (bracketed) longitudes between [bracketed] or un-[bracketed] data.

†Data adjusted for long arc effect (Reference 5).

††This data is from an orbit not listed in Table A1 for which no elements are available but which, when calculated at GSFC, appeared to show little mean motion effects from commanded but poorly executed jet pulsings just prior to the listed epoch.

Results of least squares fit of (bracketed) data for the full arc 5 in ① and ② according to the drift theory of Equation 8:

$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s) \cos 2\lambda + S_{22}F(i_s, a_s) \sin 2\lambda$

$C_0 = 8.789 \times 10^{-5}$ rad/sid. day^2

$C_{22} = -(1.6261 \pm 0.0288) \times 10^{-6}$

$S_{22} = (1.0500 \pm 0.0448) \times 10^{-6}$

Standard error of estimate = 1.415×10^{-6} rad/sid. day^2

$\ddot{\lambda}$ (with minimum standard error) = $-(2.295 \pm 0.0397) \times 10^{-5}$ rad/sid. day^2, at $\lambda = 134.0°$, (see Figure 4)

Results of least squares fit of bracketed data for arc 5A in ① and ② above according to the drift theory of Equation 8:

$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s) \cos 2\lambda + S_{22}F(i_s, a_s) \sin 2\lambda$

$C_0 = 8.7719 \times 10^{-5}$ rad/sid. day^2

$C_{22} = -(1.5996 \pm 0.1465) \times 10^{-6}$

$S_{22} = (1.0667 \pm 0.1161) \times 10^{-6}$

Standard error of estimate = 1.465×10^{-6} rad/sid. day^2

$\ddot{\lambda}$ (with minimum standard error) = $-(0.1991 \pm 0.0661) \times 10^{-5}$ rad/sid. day^2, at $\lambda = 161.0°$

Results of least squares fit of bracketed data for arc 5B in ① and ② according to the drift theory of Equation 8:

$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s) \cos 2\lambda + S_{22}F(i_s, a_s) \sin 2\lambda$

$C_{22} = -(1.5746 \pm 0.1653) \times 10^{-6}$

$S_{22} = (1.0523 \pm 0.1126) \times 10^{-6}$

Standard error of estimate = 1.603×10^{-6} rad/sid. day^2

$\ddot{\lambda}$ (with minimum standard error) = $-(2.389 \pm 0.0724) \times 10^{-5}$ rad/sid. day^2, at $\lambda = 106.0°$

Table 5S

Osculating Elements at Ascending Equator Crossings Past the Syncom 2 Tracking Epochs and Related Data in a Simulated Syncom 2 Arc 5 Trajectory with Earth Longitude Gravity through Second Order*.

Arc	Orbit Number 5S –	Syncom 2 Tracking Epoch (yr-mo-day-hr UT)	Semimajor axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1964.0, t (days)	$\Delta t = t_{j+1} - t_j$ (days)	① Longitude of the Ascending Equator Crossing, λ (degrees)	$\Delta\lambda = \lambda_{j+1} - \lambda_j$ (degrees)	② $\Delta\lambda/\Delta t$, $\dot{\lambda}$ (degrees/day)
	1	64-7-4-2.0	6.6165691	32.541	-47.118	.00085	-154.597	154.556	186.5599		-171.261		
											(-171.99)**		(-.4847)
	2	64-7-7-3.0	6.6166139	32.541	-47.165	.00084	-150.913	150.867	189.5566	2.9957	-172.713	- 1.452	
											(-174.39)		(-.4800)
	3	64-7-13-7.0	6.6166423	32.522	-47.244	.00074	-155.410	155.375	196.5455	6.9899	-176.068	- 3.355	
											(-177.95)		(-.4708)
	4	64-7-21-21.0	6.6165608	32.514	-47.359	.00080	-151.734	151.691	204.5338	7.9883	-179.829	- 3.761	
											(178.78)		(-.4649)
	5	64-7-27-16.0	6.6165572	32.497	-47.424	.00078	-153.685	153.646	210.5249	5.9911	177.386	- 2.785	
											(175.77)		(-.4614)
	6	64-8-3-17.0	6.6164269	32.485	-47.534	.00080	-150.289	150.243	217.5145	6.9896	174.161	- 3.225	
											(172.56)		(-.4583)
	7	64-8-11-1.0	6.6163983	32.460	-47.618	.00073	-155.785	155.751	224.5040	6.9895	170.958	- 3.203	
											(169.37)		(-.4537)
	8	64-8-17-19.0	6.6163760	32.450	-47.723	.00079	-152.098	152.056	231.4934	6.9894	167.787	- 3.171	
											(166.21)		(-.4515)
	9	64-8-25-10.0	6.6163634	32.426	-47.808	.00077	-154.648	154.610	238.4828	6.9894	164.631	- 3.156	
											(163.05)		(-.4521)
Arc 5S-A	10	64-9-1-10.0	6.6163724	32.414	-47.922	.00077	-150.313	150.270	245.4721	6.9893	161.471	- 3.160	
											(159.44)		(-.4518)
	11	64-9-9-14.0	6.6162137	32.382	-48.055	.00076	-155.592	155.556	254.4584	8.9863	157.411	- 4.060	
											(156.05)		(-.4530)
	12	64-9-15-12.0	6.6164386	32.375	-48.140	.00077	-151.417	151.375	260.4493	5.9909	154.697	- 2.714	
											(153.34)		(-.4540)
	13	64-9-22-10.0	6.6163175	32.352	-48.226	.00079	-155.322	155.285	266.4402	5.9909	151.977	- 2.720	
											(150.37)		(-.4594)
	14	64-9-29-6.0	6.6164698	32.343	-48.345	.00074	-150.534	150.432	273.4297	6.9895	148.766	- 3.211	
											(147.15)		(-.4623)
	15	64-10-6-5.0	6.6163509	32.318	-48.453	.00075	-155.976	155.942	280.4192	6.9895	145.535	- 3.231	
											(143.90)		(-.4674)
	16	64-10-13.0	6.6165564	32.312	-48.566	.00078	-151.320	151.278	287.4088	6.9896	142.268	- 3.267	
											(140.38)		(-.4731)
	17	64-10-20-16.0	6.6164131	32.290	-48.694	.00082	-154.741	154.701	295.3971	7.9883	138.489	- 3.779	
											(137.04)		(-.4834)
	18	64-10-26-16.0	6.6167452	32.286	-48.789	.00073	-150.410	150.369	301.3885	5.9914	135.593	- 2.896	
											(134.14)		(-.4867)
	19	64-11-2-5.0	6.6166390	32.268	-48.884	.00074	-155.158	155.122	307.3799	5.9914	132.677	- 2.916	
			Average: 6.616475 = a_s	Average: 32.409 = i_s	Arc 5SA data								
											(130.45)		(-.4960)
	20	64-11-11-2.0	6.6169669	32.264	-49.021	.00076	-150.154	150.110	316.3673	8.9874	128.219	- 4.458	
	21	64-11-17-6.0	6.6168007	32.249	-49.117	.00081	-154.039	153.998	322.3590		125.198		
											(124.94)		(-.5097)
Arc 5S-B	22	64-11-17-6.0	6.6167780	32.248	-49.137	.00083	-153.308	153.265	323.3576	.9986	124.689	- .509	
											(112.6)***		(-.5404)
	23	65-1-10-6.0	6.6177912	32.170	-49.867	.00075	-152.191	152.151	376.2908	59.9235	95.834	-32.385	
	24	65-1-13-16.0	6.6178060	32.166	-49.932	.00077	-148.777	148.732	380.2861		93.522		
											(93.23)		(-.5836)
	25	65-1-13-16.0	6.6178901	32.166	-49.944	.00075	-147.449	147.403	381.2850	.9989	92.939	- .583	
											(92.63)		(-.5828
	26	65-1-20-12.0	6.6180467	32.145	-50.009	.00065	-152.333	152.298	387.2782	10.9874	89.430	- 6.404	
	27	65-1-27-4-5.0	6.6179582	32.134	-50.104	.00072	-150.493	150.452	393.2713		85.910		
											(85.62)		(-.5907)
	28	65-1-27-4-5.0	6.6180122	32.133	-50.116	.00073	-149.632	149.590	394.2701	.9988	85.320	- .590	
											(81.73)***		(-.5929)
	29	65-2-16-4-5.0	6.6182753	32.080	-50.358	.00062	-150.957	150.922	413.2489	25.9707	74.032	-15.398	
											(72.84)		(-.5957)
			Average: 6.617394 = a_s	Average: 32.20 = i_s	arc 5SB data				417.2443	3.9954	71.652	- 2.380	
			Average: 6.6169 = a_s	Average: 32.33 = i_s	full arc 5S data								

*See Footnotes Page 27.

*Computed by ITEM with gravity constants the same as in Table A1, with the addition of the earth constants:

$J_{22} = -1.68 \times 10^{-6}$

$\lambda_{22} = -18°$

**$(\lambda_{j+1} - \lambda_j)/2$ for (bracketed) longitudes between [bracketed] or un-[bracketed] data.

***Data adjusted for long arc effects (Reference 5) (see Table 11 for summary of sub-arc analysis).

Results of least squares fit of bracketed data for arc 5S-A in ① and ② above according to the drift theory of Equation 8:

$$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s) \cos 2\lambda + S_{22}F(i_s, a_s) \sin 2\lambda.$$

$$C_0 = 8.5845 \times 10^{-5} \text{ rad/sid. day}^2$$

$$C_{22} = -(1.4109 \pm 0.0293) \times 10^{-6}$$

$$S_{22} = (1.0378 \pm 0.0233) \times 10^{-6}$$

Standard error of estimate = 2.939×10^{-7} rad/sid. day^2

$\ddot{\lambda}$ (measured at $\lambda = 161.0°$) = $-(0.0702 \pm 0.0132) \times 10^{-5}$ rad/sid. day^2

$\ddot{\lambda}$ (theoretical, according to Equation 2), = $-(0.0809) \times 10^{-5}$ rad/sid. day^2,

for $a_s = 6.616475$ earth radii, $i_s = 32.409°$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$, $\lambda = 161.0°$

Estimate of measured bias in $\ddot{\lambda}$ at 161.0° in Syncom 2 arc 5A:

Bias = theoretical-measured

$= -(0.0809) \times 10^{-5} + (0.0702) \times 10^{-5}$

$= -(0.0107) \times 10^{-5}$ rad/sid. day^2

Results of least squares fit of bracketed data for arc 5S-B in ① and ② above according to the drift theory of Equation 8:

$$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s) \cos 2\lambda + S_{22}F(i_s, a_s) \sin 2\lambda$$

$$C_{22} = -(1.4219 \pm 0.0550) \times 10^{-6}$$

$$S_{22} = (0.9296 \pm 0.0392) \times 10^{-6}$$

Standard error of estimate = 5.152×10^{-7} rad/sid. day^2

$\ddot{\lambda}$ (measured at $\lambda = 106.0°$) = $-(2.132 \pm 0.0240) \times 10^{-5}$ rad/sid. day^2

$\ddot{\lambda}$ (theoretical, according to Equation 2), = $-(2.154) \times 10^{-5}$ rad/sid. day^2,

for $a_s = 6.61739$ earth radii, $i_s = 32.20°$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$, $\lambda = 106.0°$

Estimate of measured bias in $\ddot{\lambda}$ at $\lambda = 106.0°$ in Syncom 2 arc 5B, due to sun-moon perturbations and J_{22} model error exclusive of higher order longitude gravity:

Bias = theoretical-measured

$= -(2.154) \times 10^{-5} + (2.132) \times 10^{-5}$

$= -(0.022) \times 10^{-5}$ rad/sid. day^2

Results of least squares fit of bracketed data for the full arc 5S in ① and ② above according to the theory of Equation 8:

$$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s) \cos 2\lambda + S_{22}F(i_s, a_s) \sin 2\lambda$$

$$C_{22} = -(1.3664 \pm 0.00725) \times 10^{-6}, \; S_{22} = (1.0008 \pm 0.0115) \times 10^{-6}$$

Standard error of estimate = 3.566×10^{-7} rad/sid. day^2

$\ddot{\lambda}$ (measured at $\lambda = 134.0°$) = $-(19344 \pm 0.0100) \times 10^{-5}$ rad/sid. day^2

$\ddot{\lambda}$ (theoretical, $i_s = 32.33°$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$, $\lambda = 134.0°$)= $-(1{,}3237) \times 10^{-5}$ rad/sid. day^2, for $a_s = 6.6169$ earth radii.

Estimate of measured bias in $\ddot{\lambda}$ at $\lambda = 134.0°$ in Syncom 2, arc 5, due to sun-moon perturbations and J_{22} model error exclusive of higher order longitude gravity:

Bias = Theoretical-measured

$= -(1.9237) \times 10^{-5} + 1.9344 \times 10^{-5} = +0.0107 \times 10^{-5}$ rad/sid. day^2

Table 5S/1

Osculating Elements at Ascending Equator Crossings Past the Syncom 2 Tracking Epochs and Related Data in a Simulated Syncom 2 Arc 5 Trajectory with Earth Longitude Gravity through Third Order*.

Orbit Number 5S/1 –	Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Time from 1964.0, t (days)	$\Delta t = t_{j+1} - t_j$ (days)	① Longitude of the Ascending Equator Crossing, λ (degrees)	$\Delta\lambda = \lambda_{j+1} - \lambda_j$ (degrees)	② $\Delta\lambda/\Delta t$, $\dot{\lambda}$ (degrees/day)
1	64-7-4-2.0	6.6165686	32.541	186.5599		188.739		
						(188.01)**		(-.4844)
2	64-7-7-3.0	6.6166101	32.541	189.5556	2.9957	187.288	- 1.451	
						(185.61)		(-.4794)
3	64-7-13-7.0	6.6166312	32.522	196.5455	6.9899	183.937	- 3.351	
						(182.06)		(-.4694)
4	64-7-21-21.0	6.6165417	32.514	204.5338	7.9883	180.187	- 3.750	
						(178.80)		(-.4632)
5	64-7-27-16.0	6.6165335	32.497	210.5249	5.9911	177.412	- 2.775	
						(175.81)		(-.4593)
6	64-8-3-17.0	6.6163985	32.485	217.5144	6.9895	174.202	- 3.210	
						(172.61)		(-.4557)
7	64-8-11-1.0	6.6163664	32.460	224.5038	6.9894	171.017	- 3.185	
						(169.44)		(-.4510)
8	64-8-17-19.0	6.6163418	32.450	231.4932	6.9894	167.865	- 3.152	
						(166.30)		(-.4487)
9	64-8-25-10.0	6.6163290	32.427	238.4825	6.9893	164.729	- 3.136	
						(163.16)		(-.4493)
10	64-9-1-10.0	6.6163389	32.414	245.4718	6.9893	161.589	- 3.140	
						(159.57)		(-.4491)
11	64-9-9-14.0	6.6161849	32.382	254.4580	8.9862	157.553	- 4.036	
						(156.20)		(-.4509)
12	64-9-15-12.0	6.6164149	32.375	260.4489	5.9909	154.852	- 2.701	
						(153.50)		(-.4525)
13	64-9-23-10.0	6.6163004	32.352	266.4398	5.9909	152.141	- 2.711	
						(150.54)		(-.4583)
14	64-9-29-6.0	6.6164625	32.343	273.4292	6.9894	148.938	- 3.203	
						(147.32)		(-.4621)
15	64-10-6-5.0	6.6163551	32.318	280.4187	6.9895	145.708	- 3.230	
						(144.07)		(-.4683)
16	64-10-13.0	6.6165741	32.312	287.4084	6.9897	142.435	- 3.273	
						(140.54)		(-.4752)
17	64-10-20-16.0	6.6164494	32.290	295.3967	7.9883	138.639	- 3.796	
						(137.18)		(-.4869)
18	64-10-26-16.0	6.6167966	32.286	301.3881	5.9914	135.722	- 2.917	
						(134.25)		(-.4917)
19	64-11-2-5.0	6.6167065	32.268	307.3796	5.9915	132.776	- 2.946	
						(130.52)		(-.5026)
20	64-11-11-2.0	6.6170613	32.264	316.3672	8.9876	128.259	- 4.517	
21	64-11-17-6.0	6.6169132	32.249	322.3590		125.187		
						(124.93)		(-.5187)
22	64-11-17-6.0	6.6168937	32.248	323.3577	[.9987]	124.669	[- .518]	
						(112.2)***		(-.5548)
23	65-1-10-6.0	6.6180217	32.170	376.2943	59.9271	95.013	-33.246	
24	65-1-13-16.0	6.6180377	32.166	380.2886		92.625		
						(92.32)		(-.6037)
25	65-1-13-16.0	6.6181219	32.165	381.2875	[.9989]	92.022	[- .603]	
						(91.71)		(-.6018)
26	65-1-20-12.0	6.6182756	32.144	387.2810	10.9867	88.401	- 6.612	
27	65-1-27-4-5.0	6.6181815	32.133	393.2744		84.769		
						(84.47)		(-.6077)
28	65-1-27-4-5.0	6.6182343	32.133	394.2733	[.9989]	84.162	[- .607]	
						(80.48)***		(-.6100)
29	65-2-16-4-5.0	6.6184524	32.079	413.2529	25.9719	72.557	-15.844	
						(71.34)		(-.6094)
30	65-2-19-23.0			417.2486	3.9957	70.122	- 2.435	
		Average: $6.61647 = a_s$	Average: $32.41 = i_s$	Arc 5S/1-A data				
		Average: $6.61755 = a_s$	Average: $32.20 = i_s$	Arc 5S/1-B data				
		Average: $6.61693 = a_s$	Average: $32.33 = i_s$	full Arc 5S/1 data				

*Gravity constants of this trajectory (computed by ITEM) are the same as those in Table A1, with the addition of the earth constants: $J_{22} = -1.8 \times 10^{-6}$, $\lambda_{22} = -15.35°$, $J_{33} = -0.16 \times 10^{-6}$, $\lambda_{33} = 24°$, $J_{31} = -1.5 \times 10^{-6}$, $\lambda_{31} = 0.0°$. The initial elements of this trajectory, aside from those listed for orbit 5S/1-1, are the same as those in orbit 5S-1 (Table 5S).

**$(\lambda_{j+1} - \lambda_j)/2$ for (bracketed) longitudes between [bracketed] or un-[bracketed] data.

***Data adjusted for long arc effects (Reference 5)

Results of least squares fit of (bracketed) data for the full arc 5S/1, in ① and ② above, according to Equation 8:

$(\dot{\lambda})^2 = C_0 + C_{22}F(i_s, a_s)_{22} \cos 2\lambda + S_{22}F(i_s, a_s)_{22} \sin 2\lambda$

$C_0 = 8.797 \times 10^{-5}$ rad/sid. day^2, $C_{22} = -(1.6095 \pm 0.0108) \times 10^{-6}$, $S_{22} = (1.0981 \pm 0.0169) \times 10^{-6}$

Standard error of estimate = 5.318×10^{-7} rad/sid. day^2, (see Table 11 for complete 18 sub arcs analysis)

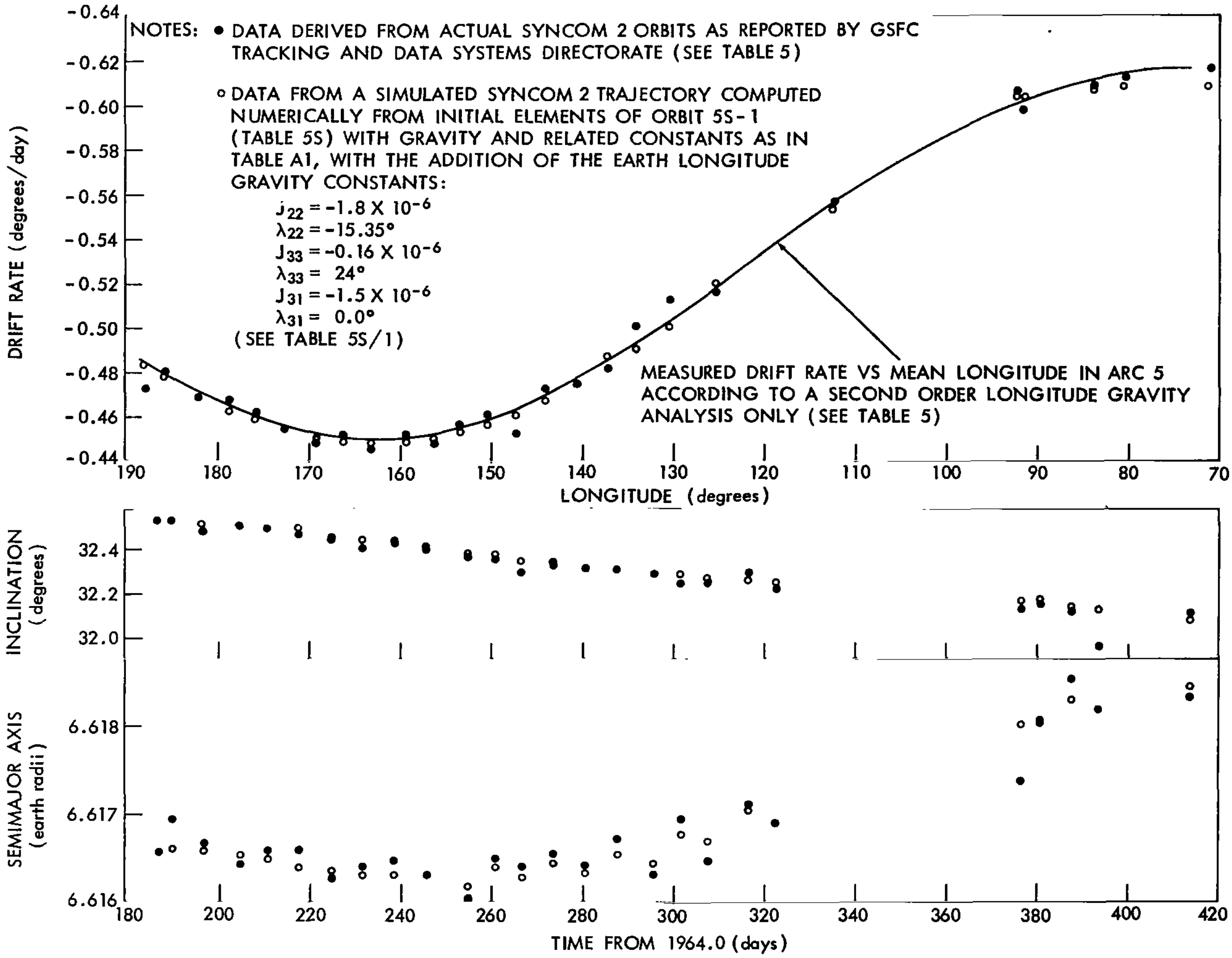

Figure 4—Measured and simulated orbit data at ascending Equator crossings in arc 5 (Syncom 2).

full third order trajectories. In this case the initial elements of orbit 6S-B were chosen to give the best representation of the time characteristics (and thus the more faithful bias estimate) over the whole arc. Orbit 6S-A (with GSFC reported "mean" elements for epoch 6S-A-1) appears to give a better overall representation of the inclination and eccentricity in arc 6.

Arc 7, Syncom 3, 14 January 1965 - 16 March 1965

Between 20 December 1964 and 14 January 1965 a number of orbit change maneuvers were performed on Syncom 3, repositioning the mean longitude of the satellite back near the International Date Line and increasing the orbit inclination to about 1°. Between 14 January and 30 January 1965 a number of attitude and inclination change maneuvers were performed which apparently had little effect on the mean motion of the satellite. Between 30 January and 16 March 1965 the control jets

Table 6

Syncom 3 Osculating Elements at the First Ascending Equator Crossing Past the Tracking Epoch and Related Data for Free Drift Arc 6*.

Orbit Number 6 –	Tracking Epoch (hr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1964.0, t (days)	Geographic Longitude of the First Ascending Equator Crossing After the Tracking Epoch λ (degrees)	① Time from January 331.2059, 1964, t (days)	② Longitude of the Ascending Equator Crossing East of 179.017°, L (degrees)
1	64-10-31-2.0	6.6119343	.097	-149.455	.00026	- 45.737	45.719	305.9723	180.219	-25.2336	1.202
2	64-11-3-13-18.0	6.6116163	.066	135.469	.00005	- 13.946	13.948	308.7567	180.028	-22.4492	1.011
3	64-11-7-8.0	6.6116896	.057	165.876	.00008	- 23.052	23.052	312.8308	179.737	-18.3751	.720
4	64-11-16-3.0	6.6115890	.038	106.668	.00012	151.303	-151.297	321.6437	179.191	- 9.5622	.174
5	64-11-24-3.0	6.6115203	.080	112.569	.00008	43.507	- 43.514	329.6393	178.798	- 1.5666	- .219
6	64-11-30-10-35.0	6.6114447	.090	68.895	.00001	140.529	-140.585	335.5028	178.509	4.2969	- .508
7	64-12-8-12-45.0	6.6111913	.261	54.788	.00017	-137.715	137.706	344.4400	178.171	13.2341	- .846
8	64-12-15-12.0	6.6113693	.127	74.467	.00003	-157.902	157.913	351.4760	177.968	20.2701	-1.049
9	64-12-21-9.0	6.6110148	.204	66.016	.00009	- 8.119	8.118	356.4394	177.814	25.2335	-1.203
		Average: $6.611474 = a_s$	Average: $0.113 = i_s$								

*See notes in Table 1.

Results of least squares fit of data in ① and ② according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = -(3.080 \pm 0.067) \times 10^{-1}$ degrees

$a_2 = -(4.629 \pm 0.073) \times 10^{-2}$ degrees/day

$a_3 = (4.915 \pm 0.170) \times 10^{-4}$ degrees/day^2

$a_4 = -(2.12 \pm 1.40) \times 10^{-6}$ degrees/day^3

$\ddot{\lambda}$ (with minimum standard error) = $+(1.707 \pm 0.0591) \times 10^{-5}$ rad/sid. day^2, at $t = -0.0332$ days, $t' = 331.1727$ January 1964, $L = -0.310°$, $\lambda = 178.707°$.
(see Figure 5)

Table 6S-A

Osculating Elements at the First Ascending Equator Crossing Past the Syncom 3 Tracking Epoch and Related Data, in Two Simulated Syncom 3 Arc 6 Trajectories with Earth Longitude Gravity through Second Order.*

Orbit Number 6S-A–	Syncom 3 Tracking Epoch (yr-mo-day-hr- UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1964.0, t (days)	Geographic Longitude of the First Ascending Equator Crossing After the Tracking Epoch, λ (degrees)	① Time From 330.9708 January 1964, t (days)	② Longitude of the Ascending Equator Crossing East of 179.003°, L (degrees)
1	64-10-31-2.0	6.6118485	.066	37.760	.00011	-130.250	130.248	305.4935	180.275	-25.4773	1.272
2	64-11-3-13-18.0	6.6119239	.069	37.793	.00004	-133.732	133.752	308.4860	180.053	-22.4848	1.050
3	64-11-7-8.0	6.6117741	.081	39.858	.00006	153.835	-153.831	312.4816	179.779	-18.4892	0.776
4	64-11-16-3.0	6.6116883	.095	51.145	.00010	-162.428	162.426	321.4898	179.226	- 9.4810	0.223
5	64-11-24-3.0	6.6114084	.123	56.365	.00014	-179.030	179.030	329.4837	178.790	- 1.4871	-0.213
6	64-11-30-10-35.0	6.6115566	.132	60.210	.00005	-165.418	165.417	335.4787	178.495	4.5079	-0.508
7	64-12-8-12-45.0	6.6112925	.161	64.207	.00011	144.315	-144.310	344.4663	178.125	13.4955	-0.878
8	64-12-15-12.0	6.6114751	.174	66.575	.00007	-178.506	178.505	351.4544	177.880	20.4836	-1.123
9	64-12-21-9.0	6.6111823	.198	69.060	.00014	158.894	-158.891	356.4480	177.731	25.4772	-1.272
		Average: 6.611572 = a_s	Average: 0.122 = i_s								

Results of least squares fit of data in ① and ② according to the theory of Equation 1, for arc 6S-A

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = -(0.29025 \pm 0.00217)$ degrees

$a_2 = -(4.947 \pm 0.0234) \times 10^{-2}$ degrees/day

$a_3 = (4.438 \pm 0.0546) \times 10^{-4}$ degrees/day^2

$a_4 = -(6.082 \pm 4.435) \times 10^{-7}$ degrees/day^3

Standard error of estimate = 3.938×10^{-3} degrees

$\ddot{\lambda}$ (measured) = $(1.5394 \pm 0.0190) \times 10^{-5}$ rad/sid. day^2, at $\lambda = 178.703°$ and $t' = 331.1727$ January 1964

$\ddot{\lambda}$ (theoretical) = $+(1.5036) \times 10^{-5}$ rad/sid. day^2 for $a_s = 6.611572$ earth radii, $i_s = 0.122°$, $\lambda = 178.703°$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$

Bias = theoretical–measured = $(1.5036 - 1.5394) \times 10^{-5} = -0.0358 \times 10^{-5}$ rad/sid. day^2

*Computed by ITEM with gravity constants the same as in Table A1, with the addition of the earth constants: $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18°$.

Table 6S-B

Osculating Elements at the First Ascending Equator Crossing Past the Syncom 3 Tracking Epoch and Related Data, in Two Simulated Syncom 3 Arc 6 Trajectories with Earth Longitude Gravity through Second Order.*

Orbit Number 6S-B–	Syncom 3 Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1964.0, t (days)	Geographic Longitude of the First Ascending Equator Crossing After the Tracking Epoch, λ (degrees)	① Time From 330.9708 January 1964, t (days)	② Longitude of the Ascending Equator Crossing East of 179.003°, L (degrees)
1	64-10-31-2.0	6.6119240	.097	-149.455	.00026	-45.719	45.698	305.9723	180.219	-24.9985	1.216
2	64-11-3-13-18.0	6.6119574	.093	-149.660	.00026	-41.421	41.403	308.9642	180.003	-22.0066	1.000
3	64-11-7-8.0	6.6116738	.082	-153.621	.00022	-45.310	45.294	312.9430	179.735	-18.0278	0.732
4	64-11-16-3.0	6.6117460	.077	-168.885	.00028	-18.764	18.754	321.8777	179.173	- 9.0931	0.170
5	64-11-24-3.0	6.6115233	.070	167.455	.00022	- 8.705	8.701	329.7916	178.729	- 1.1792	-0.274
6	64-11-30-10-35.0	6.6113283	.075	159.890	.00020	1.339	- 1.338	335.7550	178.435	4.7842	-0.568
7	64-12-8-12-45.0	6.6114224	.088	138.974	.00028	9.174	- 9.169	344.6736	178.064	13.7028	-0.939
8	64-12-15-12.0	6.6110825	.099	132.866	.00017	31.826	-31.819	351.6382	177.805	20.6930	-1.198
9	64-12-21-9.0	6.6114383	.118	124.209	.00025	27.235	-27.225	356.6010	177.654	25.6302	-1.349
		Average: $6.611566 = a_s$	Average: $0.089 = i_s$								

Results of least squares fit of data in ① and ② according to the theory of Equation 1, for arc 6S-B

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = -(3.310 \pm 0.0237) \times 10^{-1}$ degrees

$a_2 = -(5.082 \pm 0.0257) \times 10^{-4}$ degrees /day

$a_3 = (4.388 \pm 0.0604) \times 10^{-6}$ degrees/day^2

$a_4 = -(1.930 \pm 4.926) \times 10^{-7}$ degrees/day^3

Standard error of estimate = 4.290×10^{-3}

$\ddot{\lambda}$(measured) = $(1.5231 \pm 0.0209) \times 10^{-5}$ rad/sid. day^2 at $\lambda = 178.662^\circ$ and $t' = 331.1727$ January 1964

$\ddot{\lambda}$(theoretical) = 1.5003×10^{-5} rad/sid. day^2 for $a_s = 6.611566$ earth radii, $i_s = 0.089^\circ$, $\lambda = 178.662^\circ$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0^\circ$

Bias = theoretical–measured = $1.5003 \times 10^{-5} - 1.5231 \times 10^{-5} = -(0.0228) \times 10^{-5}$ rad/sid. day^2

*Computed by ITEM with gravity constants the same as in Table A1, with the addition of the earth constants: $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18^\circ$.

Table 6S/1

Ascending Equator Crossing Data From a Simulated Syncom 3 Trajectory for Free Drift Arc 6, Computed by ITEM With Earth Longitude Gravity through Third Order.*

Orbit Number 6S/1	Tracking Epoch (yr-mo-day-hr UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Time from 1964.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time from January 331.2865, 1964 t (days)	② Longitude of the Ascending Equator Crossing East of 179.002°, L (degrees)
1	64-10-31-2.0	6.6119228	.097	305.9723	180.219	-25.3142	1.217
2	64-11-3-13-18.0	6.6119523	.093	308.9642	180.004	-22.3223	1.002
3	64-11-7-8.0	6.6116643	.082	312.9430	179.738	-18.3435	.736
4	64-11-16-3.0	6.6117257	.078	321.8777	179.188	- 9.4088	.186
5	64-11-24-3.0	6.6114943	.070	329.7915	178.760	- 1.4950	- .242
6	65-11-30-10-35.0	6.6112925	.075	335.7549	178.482	4.4684	- .520
7	64-12-8-12-45.0	6.6113766	.088	344.6733	178.141	13.3868	- .861
8	64-12-15-12.0	6.6110288	.099	351.6379	177.911	20.3514	-1.091
9	64-12-21-9.0	6.6113790	.118	356.6007	177.784	25.3142	-1.218
		Average: 6.611537 = a_s	Average: 0.089 = i_s				

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = -(3.1148 \pm 0.0233) \times 10^{-1}$ degrees

$a_2 = -(4.8012 \pm 0.0253) \times 10^{-2}$ degrees/day

$a_3 = (4.8565 \pm 0.0594) \times 10^{-4}$ degrees/day^2

$a_4 = -(1.362 \pm 4.856) \times 10^{-7}$ degrees/day^3

Standard error of estimate = 0.00423 degrees

$\ddot{\lambda}$ (with minimum standard error) = 1.6860×10^{-5} rad/sid. day^2, at $t = -0.0091$ day, $t' = 331.2774$ Jan. 1964, $\lambda = 178.69^\circ$, measured

$\ddot{\lambda}$ (theoretical from Equation 2, $i_s = 0.089^\circ$, $\lambda = 178.69^\circ$, $J_{22} - J_{31}$ as noted) = 1.6615×10^{-5} rad/sid. day^2, for $a_s = 6.61154$ earth radii

Estimate of acceleration bias at $\lambda = 178.69^\circ$ in arc 6S/1 = $\ddot{\lambda}$ (theoretical) − $\ddot{\lambda}$ (measured) = -0.0245×10^{-5} rad/sid. day^2

*Gravity constants of this trajectory are the same as that in Table A1, with the addition of the earth constants:

$J_{22} = -1.8 \times 10^{-6}$, $\lambda_{22} = -15.35^\circ$; $J_{33} = -0.16 \times 10^{-6}$, $\lambda_{33} = 24^\circ$

$J_{31} = -1.5 \times 10^{-6}$, $\lambda_{31} = 0^\circ$.

The initial elements of this trajectory, aside from those listed for orbit 6S/1-1, are the same as those in orbit 6S-B-1 (Table 6S).

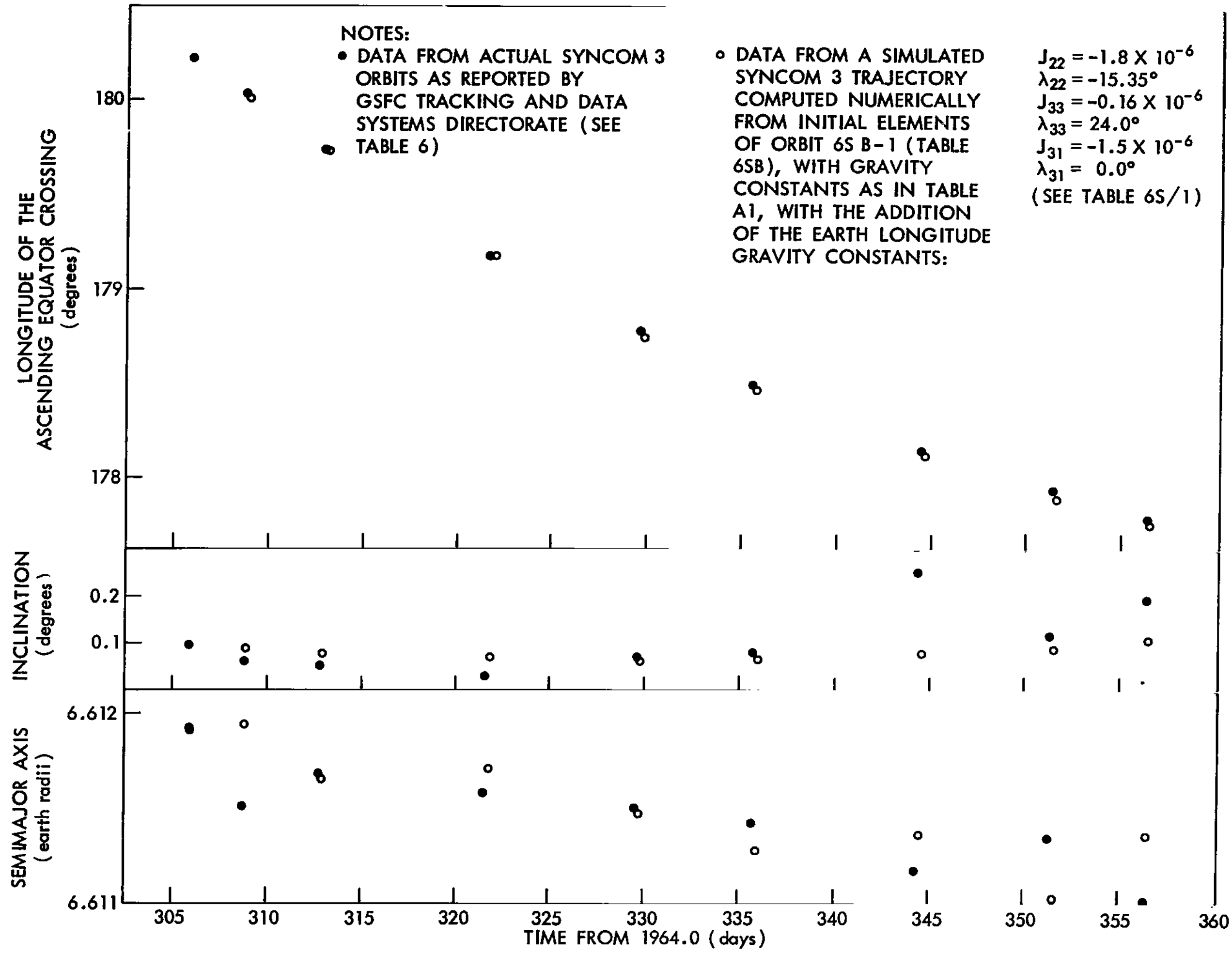

Figure 5—Measured and simulated orbit data at ascending Equator crossings in free drift arc 6 (Syncom 3).

on Syncom 3 were inactive. During this two month period the mean longitude of the satellite moved westward from about 181° to 173.5° with a mean drift rate of about -0.1 degree/day. The details of this gravity accelerated drift are found in Table 7 and Figure 6 and summarized in Tables 10 and 11 in the next section.

The acceleration analysis during this relatively slow westward drift utilized the same t^3 fit method as in previous slow drift 24-hour satellite arcs 1, 2, and 6. The efficacy of this method is once again attested to by the small bias results evident in the close arc 7 simulated trajectories in Tables 7S and 7S/1. In these simulations, no attempt was made to break the arc at the end of January 1965 to account for the evident inclination change maneuver. Near-equatorial 24-hour orbits, theoretically, suffer resonant gravity effects with small sensitivity to inclination changes (see Equations 3 through 7).

It is noted that the standard error of estimate for the t^3 longitude fit in this arc is over four times that in the arc 6 experiment for the same amount of data. The results of attempts to reduce this error by both a priori and a posteriori weighting have been inconclusive, in large part because of the scarcity of the data in this arc. In arc 2, which apparently suffers from similarly ill determined orbits, the standard error in the best measured acceleration is relatively small because more data is available in that arc than in arc 7. Nevertheless, the best measured acceleration in arc 7 seems to be much less in error from true earth resonant acceleration than its standard first stage experiment error (in Table 7) would indicate, (compare the results of the gravity synthesis in the next section). In the future, a more detailed analysis of the day-by-day drift of Syncom 3 in this arc, after the method used for Early Bird (arc 9), promises a far better discrimination of the acceleration, and with greater longitude separation from arc 6 than presently attained (see Discussion).

Arc 8, Syncom 2, 25 February 1965 - 10 May 1965

Between 19 and 24 February 1965, gas jets were pulsed on board Syncom 2 to reorient the satellite and adjust its mean motion to as close to synchronous as possible. The last of this final series of Syncom 2 maneuvers took place on 24 February 1965 and left the satellites ground track with a westward drift rate of 0.05 degree/day (reduced from 0.6 degree/day westward on 19 February) at a mean longitude of 67.7°. Under the influence of earth longitude gravity, the ground track drift rate westward was further reduced till a momentarily stationary condition was reached in late May 1965 near 65.2°. The details of this gravity decelerated drift are found in Table 8 and Figure 7 and summarized in Tables 10 and 11 in the next section.

Once again, the slow drift "t^3" theory was utilized for the acceleration analysis in this arc, both for the actual data and the closely parallel simulated data (see Tables 8S and 8S/1). The orbit determination over this slow drift arc appears to be about as good as in arc 1 (Syncom 2), but not as precise as in arc 6 (Syncom 3). The analysis of the two simulated trajectories with different earth models shows reasonably consistent model bias acceleration results for this arc considering the different lengths of these trajectories. The results of the parallel acceleration analysis on the simulated trajectories are summarized in Tables 10 and 11 in the next section.

According to the 24-hour satellite earth gravity drift theory summarized in this report (see Conclusions), a point of stable drift equilibrium exists for the Equatorial Geostationary Satellite, at about 77° (and at about the same longitude for a 32° inclined orbit satellite). Thus Syncom 2 is now probably forever in a long period oscillatory drift regime between mean longitudes of about 65° and 89°. The exact description of this oscillation (with a period of about 2-1/2 years initially) depends on the long term change in the inclination of Syncom 2's orbit due to the gravitational attractions of the sun and moon. At present (summer 1965) the orbit inclination is about 31.7° and is being reduced at about 0.8 degree/year (see Table 8S/1).

Table 7

Syncom 3 Osculating Elements at the First Ascending Equator Crossing Past the Tracking Epoch, and Related Data for Drift Arc 7.*

Orbit Number 7–	Syncom 3 Tracking Epoch (yr-mo-day-hr-min UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1965.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time from January 45.4138, 1965, t (days)	② Longitude of the Ascending Equator Crossing East of 177.164° E, L (degrees)
1**	65-1-14-23-30.0	6.6121095	1.137	- 34.097	.00181	-97.425	97.220	15.0855	180.803	-30.3283	3.639
2	65-1-30-13-10.0	6.6124038	.173	- 61.562	.00029	-64.872	64.844	30.9720	178.543	-14.4418	1.379
3	65-2-2-6.0	6.6129132	.101	- 35.767	.00013	-64.858	64.847	34.0364	178.124	-11.3774	.960
4	65-2-9-11.0	6.6124858	.166	- 81.989	.00025	-76.958	76.934	40.8915	177.300	- 4.5223	.136
5	65-2-16-12.0	6.6118440	.060	-112.788	.00001	-30.762	30.809	47.7895	176.446	2.3757	- .718
6	65-2-23.0	6.6123345	.138	-100.918	.00019	-82.895	82.879	54.8054	175.677	9.3916	-1.487
7	65-3-2.0	6.6119364	.205	-118.912	.00034	-87.998	87.963	61.7382	175.021	16.3244	-2.143
8	65-3-9.0	6.6123015	.074	- 71.138	.00020	-76.296	76.279	68.8539	174.130	23.4401	-3.034
9	65-3-16.0	6.6120906	.355	-105.212	.00070	-96.150	96.072	75.7421	173.525	30.3283	-3.639
		Average: 6.612269 = a_S	Average: 0.268 = i_S								

*See notes in Table 1.

**A number of attitude and orbit inclination change maneuvers were performed on Syncom 2 between 14 January and 30 January 1965. These did not appear to affect the mean motion of the satellite significantly.

Results of least squares of data in ① and ② above according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = -(0.4168 \pm 0.0244)$ degrees

$a_2 = -(0.1166 \pm 0.0023)$ degree/day

$a_3 = (4.409 \pm 0.505) \times 10^{-4}$ degrees/day^2

$a_4 = -(4.131 \pm 3.042) \times 10^{-6}$ degrees/day^3

Standard error of estimate = 0.0515°

$\ddot{\lambda}$(with minimum standard error) = $(1.550 \pm 0.175) \times 10^{-5}$ rad/sid. day^2, at t = -0.4507 day, t′ = 44.9631 January 1965, L = -0.3634°, λ = 176.801° (see Figure 6).

Table 7S

Ascending Equator Crossing Data from a Simulated Syncom 3 Trajectory for Free Drift Arc 7, Computed by ITEM in the Presence of Earth Longitude Gravity*.

Orbit Number 7S	Syncom 3 Tracking Epoch (yr-mo-day-hr-min UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1965.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time From January 45.4138, 1965, t (days)	② Longitude of the Ascending Equator Crossing East of 177.164° E, L (degrees)
1	65-1-14-23-30.0	6.6127641	.200	-64.894	.00030	- 98.417	98.416	15.0024	180.836	-30.4094	3.672
2	65-1-30-13-10.0	6.6126603	.148	-63.708	.00025	-111.274	111.249	30.9658	178.628	-14.4480	1.464
3	65-2-2-6.0	6.6125307	.140	-65.126	.00024	-117.018	116.994	33.9548	178.244	-11.4590	1.080
4	65-2-9-11.0	6.6123492	.126	-66.027	.00030	-105.735	105.705	40.9356	177.363	- 4.4782	.199
5	65-2-16-12.0	6.6122952	.100	-67.046	.00021	-111.429	111.408	47.9160	176.519	2.5022	- .645
6	65-2-23.0	6.6122513	.091	-66.187	.00028	-116.269	116.241	54.9015	175.710	9.4877	-1.454
7	65-3-2.0	6.6121399	.071	-68.756	.00025	-119.829	119.804	61.8774	174.951	16.4636	-2.213
8	65-3-9.0	6.6120920	.065	-67.449	.00029	-104.308	104.279	68.8639	174.210	23.4501	-2.954
9	65-3-16.0	6.6119327	.044	-70.595	.00023	-115.776	115.755	75.8381	173.502	30.4243	-3.662
		Average: 6.612335 = a_s	Average: 0.109 = i_s								

*$J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18^{\circ}$, only earth longitude gravity used in this simulation. All other gravity constants are the same as those used in Table A1.

Results of least squares fit of data in ① and ② above (Table 7S) according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$ $\quad a_1 = -(3.4626 \pm 0.0182) \times 10^{-1}$ degrees $\quad a_2 = -(1.1971 \pm 0.0017) \times 10^{-1}$ degrees/day

$a_3 = (3.8156 \pm 0.0374) \times 10^{-4}$ degrees/day^2 $\quad a_4 = -(8.980 \pm 2.241) \times 10^{-7}$ degrees/day^3

Standard error of estimate = 0.00383 degree

$\ddot{\lambda}$ (measured, at 176.871°) = $(1.3289 \pm 0.0129) \times 10^{-5}$ rad/sid. day^2, at $t = -0.4507$ day, $t^1 = 44.9631$ January 1965.

$\ddot{\lambda}$(theoretical) = 1.3546×10^{-5} rad/sid. day^2, for $a_s = 6.612335$ earth radii, $i_s = 0.109^{\circ}$, $\lambda = 176.871^{\circ}$, $J_{22} = -1.68 \times 10^{-6}$, λ_{22} -18.0°.

Bias = Theoretical − Measured = $1.3546 \times 10^{-5} - 1.3289 \times 10^{-5} = +0.0257 \times 10^{-5}$ rad/sid. day^2

Table 7S/1

Ascending Equator Crossing Data from a Simulated Syncom 3 Trajectory for Free Drift Arc 7, Computed by ITEM in the Presence of Earth Longitude Gravity*.

Orbit Number 7S/1 –	Syncom 3 Tracking Epoch (yr-mo-day-hr-min UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Time from 1965.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time From January 45.4138, 1965, t (days)	② Longitude of the Ascending Equator Crossing East of 177.164° E, L (degrees)
1	65-1-14-23-30.0	6.6127641	.200	15.0002	180.836	-30.4136	3.672
2	65-1-30-13-10.0	6.6126416	.148	30.9658	178.641	-14.4480	1.477
3	65-2-2-6.0	6.6125089	.137	33.9547	178.262	-11.4591	1.098
4	65-2-9-11.0	6.6123196	.126	40.9355	177.396	- 4.4783	.232
5	65-2-16-12.0	6.6122585	.100	47.9158	176.571	2.5020	- .593
6	65-2-23.0	6.6122077	.091	54.9013	175.786	9.4875	-1.378
7	65-3-2.0	6.6120897	.071	61.8771	175.054	16.4633	-2.110
8	65-3-9.0	6.6120352	.065	68.8635	174.344	23.4497	-2.820
9	65-3-16.0	6.6118698	.044	75.8376	173.671	30.4238	-3.493
		Average: 6.6123 = a_s	Average: 0.109 = i_s				

*$J_{22} = -1.8 \times 10^{-6}$, $\lambda_{22} = -15.35°$, $J_{33} = -.16 \times 10^{-6}$, $\lambda_{33} = +24°$, $J_{31} = -1.5 \times 10^{-6}$, $\lambda_{31} = 0°$, only earth longitude gravity used in this simulation. Initial elements as in orbit 7S-1 above. All other gravity constants as in Table A1.

Results of least squares fit of data in ① and ② above (Table 7S/1) according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$ $\quad a_1 = -(3.0135 \pm .0178) \times 10^{-1}$ degrees $\quad a_2 = -(1.1687 \pm 0.00166) \times 10^{-1}$ degrees/day

$a_3 = (4.2397 \pm 0.0366) \times 10^{-4}$ degrees/day^2 $\quad a_4 = (9.599 \pm 2.192) \times 10^{-7}$ degrees/day^3

Standard error of estimate = 0.00375 degree

$\ddot{\lambda}$(measured, with minimum standard error) = 1.4726×10^{-5} rad/sid. day^2, at t = -0.4433 days, λ = 176.915°.

$\ddot{\lambda}$(theoretical, from Equation 2, for a_s = 6.6123 E.R., i_s = 0.109°, λ = 176.915°) = 1.501×10^{-5} rad/sid. day^2

Estimated bias in arc S7/1 at λ = 176.915° = theoretical – measured = $1.501 \times 10^{-5} - 1.473 \times 10^{-5}$ =

0.028×10^{-5} rad/sid. day^2

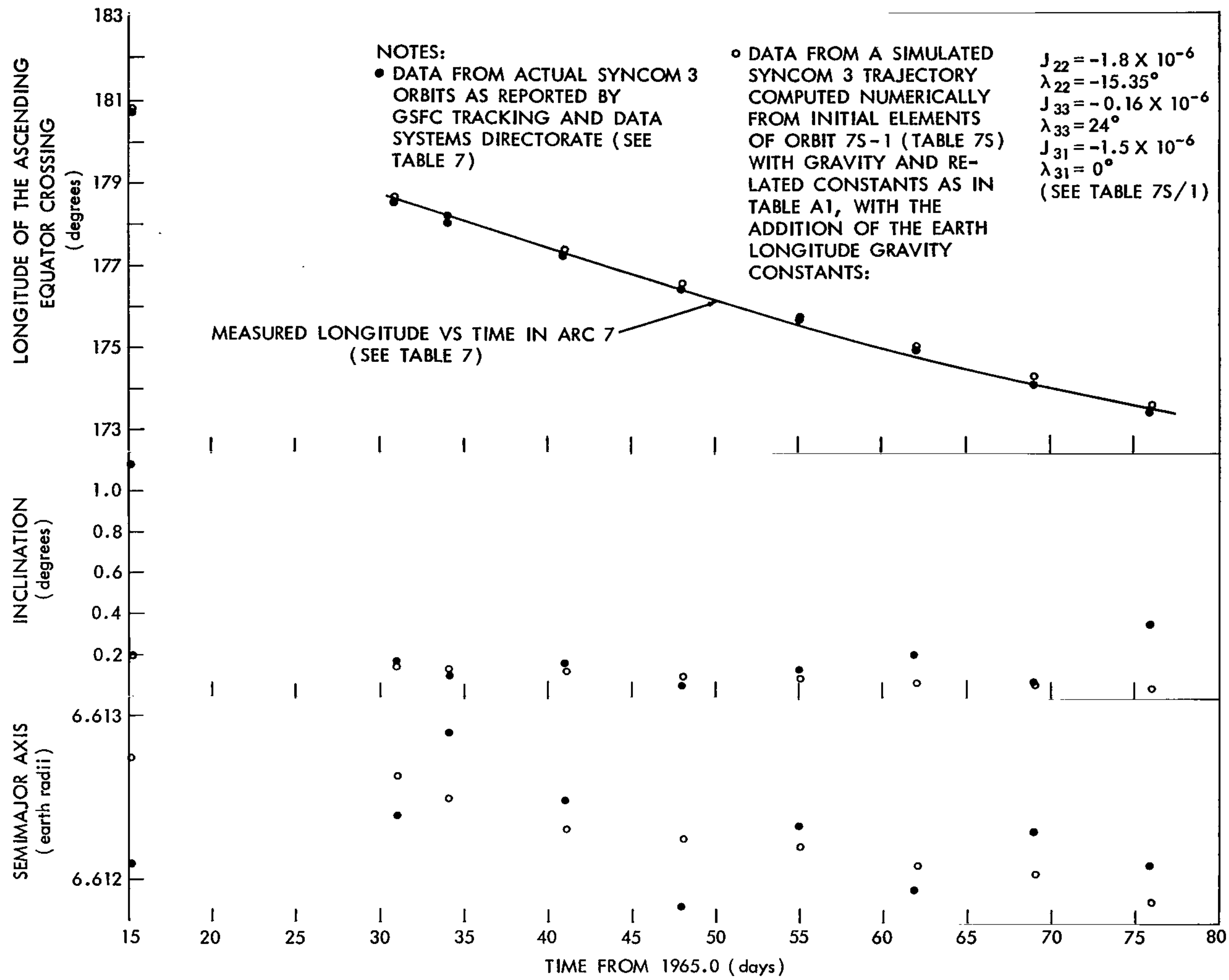

Figure 6—Measured and simulated orbit data at ascending Equator crossings in drift arc 7 (Syncom 3).

Arc 9, Early Bird, 23 April 1965 - 21 June 1965

The Communications Satellite Corporation's first 24-hour satellite, Early Bird, was launched in March 1965 and was brought to station at 30° W in late April 1965. Free gravity drift of this nearly geostationary satellite commenced on 23 April with the mean longitude at 30° W and the drift rate about +0.06 degree/day. Tracking has been maintained on nearly an around-the-clock basis since this time from the A.T. and T. facility at Andover, Maine. Figure 8 shows Early Bird reached a momentarily stationary configuration at 28° W in late June 1965.

Extremely fine precision in defining the long term drift of this satellite has been achieved by averaging the results of a large number of daily subsatellite position determinations converted directly from simultaneous range, azimuth and elevation fixes on Early Bird from Andover. The technique of utilizing this large amount of directly observed data almost every day in arc 9 can be followed in Table 9.

Table 8

Syncom 2 Mean Elements at the Tracking Epoch, and Related Data for Free Drift Arc 8*.

Orbit Number 8–	Tracking Epoch (yr-mo-day-hr-min, UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from January 0.0,1965.0 (days)	Geographic Longitude of the Ascending Equator Crossing After Tracking Epoch, λ (degrees)	① Time from January 86.1612, 1965, t (days)	② Longitude of Ascending Equator Crossing East of 66.634° L (degrees)
**1	65-2-25.0	6.6115151	31.956	-50.539	.00075	-28.510	28.469	56.2412	67.733	-29.9200	1.099
**2	65-3-3.0	6.6112551	31.930	-50.539	.00079	-25.098	25.060	62.2257	67.423	-23.9355	.789
3	65-3-6.0	6.6112849	31.939	-50.630	.00076	-26.952	308.359	65.2177	67.270	-20.9435	.636
4	65-3-13.0	6.6113429	31.912	-50.905	.00069	-38.553	326.845	72.1986	67.007	-13.9626	.373
**5	65-3-29.0	6.6114858	31.955	-50.921	.00077	-18.507	18.479	88.1567	66.314	1.9955	- .320
6	65-4-5.0	6.6111445	31.830	-51.104	.00069	-19.366	329.612	95.1378	66.071	8.9766	- .563
7	65-4-12.0	6.6111334	31.848	-51.053	.00073	-11.053	328.269	102.1195	65.822	15.9583	- .812
8	65-4-19.0	6.6110585	31.867	-51.414	.00065	-29.162	353.146	109.0997	65.712	22.9385	- .922
9	65-4-26.0	6.6110267	31.787	-51.420	.00063	-22.995	353.717	116.0811	65.534	29.9199	-1.100
		Average: 6.611250 = a_s	Average: 31.892 = i_s	← Data through orbit 8-9.							
10	65-5-3.0	6.6110035	31.807	-51.583	.00061	-22.243	359.897	123.0619	65.404	36.9007	-1.230
11	65-5-10.0	6.6109407	0.731	-51.665	.00059	-15.485	0.012	130.0428	65.288	43.8816	-1.346
		Average: 6.611199 = a_s	Average: 31.869 = i_s	← Data through orbit 8-11.							

*Mean (Brouwer) elements and longitude data as reported by GSFC Tracking and Data Systems Directorate (except as noted). Longitude data also reported by GSFC is at the first ascending Equator crossing past the Syncom 2 tracking epoch.

**Osculating elements and longitude data at the first ascending Equator crossing past the tracking epoch (see notes in Table 1).

Results of least squares fit of data in ① and ② above acording to the theory of Equation 1: (through orbit 8-9).

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$ $\quad a_1 = -(2.430 \pm 0.168) \times 10^{-1}$ degrees $\quad a_2 = -(3.857 \pm 0.131) \times 10^{-2}$ degrees/day

$a_3 = (2.699 \pm 0.319) \times 10^{4}$ degrees/day^2 $\quad a_4 = (2.445 \pm 1.873) \times 10^{-6}$ degrees/day^3

Standard error of estimate = 0.0291 degree

$\ddot{\lambda}$(with minimum standard error) = $(0.9566 \pm 0.1097) \times 10^{-5}$ rad./sid. day^2, at t = 0.7759 days, t = 86.9371 January 1965, L = -0.273°, λ = 66.361°.

Revised data: (through orbit 8-11)

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$ $\quad a_1 = -(2.403 \pm .159) \times 10^{-1}$ degrees $\quad a_2 = -(3.741 \pm 0.0864) \times 10^{-2}$ degrees/day

$a_3 = (2.6082 \pm 0.2845) \times 10^{-4}$ degrees/day^2 $\quad a_4 = (5.471 \pm 9.487) \times 10^{-7}$ degrees/day^3

Standard error of estimate = 0.0280 degree

$\ddot{\lambda}$(with minimum standard error) = $(0.9500 \pm .0616) \times 10^{-5}$ rad./sid. day^2, at t = 7.8135 days, t^{1} = 93.9747 January 1965, λ = 66.115°.

(see figure 7)

Table 8S

Ascending Equator Crossing Data From a Simulated Syncom 2 Trajectory for Free Drift Arc 8, Computed by ITEM in the Presence of Earth Longitude Gravity*.

Orbit Number 8S –	Tracking Epoch (yr-mo-day UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Right Ascension of the Ascending Node (degrees)	Eccentricity	Argument of Perigee (degrees)	Mean Anomaly (degrees)	Time from 1965.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time from January 86.1612, 1965, t (days)	② Longitude of the Ascending Equator Crossing East of 66.634°, L (degrees)
1	65-2-25.0	6.6114062	31.956	-50.538	.00075	-28.528	28.488	56.2412	67.735	-29.9200	1.101
2	65-3-3.0	6.6114759	31.936	-50.611	.00075	-25.625	25.588	62.2254	67.456	-23.9358	.822
3	65-3-6.0	6.6112778	31.925	-50.663	.00070	-27.225	27.188	65.2175	67.321	-20.9437	.687
4	65-3-13.0	6.6114573	31.913	-50.770	.00079	-24.836	24.798	72.1990	66.993	-13.9622	.359
5	65-3-29.0	6.6113880	31.871	-51.015	.00075	-25.549	25.512	88.1565	66.318	1.9953	- .316
6	65-4-5.0	6.6110762	31.853	-51.142	.00070	-29.586	29.547	95.1377	66.048	8.9765	- .586
7	65-4-12.0	6.6112985	31.839	-51.239	.00076	-20.941	20.910	102.1191	65.770	15.9579	- .864
8	65-4-19.0	6.6110384	31.826	-51.369	.00071	-26.757	26.721	109.1003	65.525	22.9391	-1.109
9	65-4-26.0	6.6112287	31.816	-51.462	.00074	-25.117	25.081	116.0816	65.291	29.9204	-1.343
		Average: 6.611289 = a_s	Average: 31.882 = i_s								

*J_{22} = -1.68 x 10^{-6}, λ_{22} = -18° only earth longitude gravity used in this simulation. All other gravity constants as in Table A1.

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

$a_1 = -(2.337 \pm 0.0336) \times 10^{-1}$ degree

$a_2 = -(4.126 \pm 0.0262) \times 10^{-2}$ degrees/day

$a_3 = (1.264 \pm 0.0638) \times 10^{-4}$ degrees/day^2

$a_4 = (4.151 \pm 3.743) \times 10^{-7}$ degrees/day^3

Standard error of estimate = 0.00581 degree

$\ddot{\lambda}$ (measured) = $(0.4421 \pm 0.0219) \times 10^{-5}$ rad/sid. day^2, for t = 0.7759 day, t' = 86.9371 Jan. 1965, L = -0.266°, λ = 66.368°

$\ddot{\lambda}$ (theoretical, from Equation 2, i_s = 31.882°, λ = 66.368°, $J_{22} = -1.68 \times 10^{-6}$, λ_{22} = -18.0°)= $(0.4560) \times 10^{-5}$ rad/sid. day^2, for a_s = 6.611289 earth radii.

Estimate of measured bias due to sun-moon perturbations and J_{22} model error (exclusive of higher order longitude gravity effects) in $\ddot{\lambda}$ at t = 0.7759 days in Syncom 2 arc 8, on January 86.9371, 1965: Bias = theoretical-measured

$= 0.4560 \times 10^{-5} - 0.4421 \times 10^{-5}$

$= +0.0139 \times 10^{-5}$ rad/sid day^2

Table 8S/1

Ascending Equator Crossing Data From a Simulated Syncom 2 Trajectory for Free Drift Arc 8, Computed by ITEM in the Presence of Earth Longitude Gravity*.

Orbit Number 8S/1 –	Tracking Epoch (yr-mo-day UT)	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Time from 1965.0 (days)	Geographic Longitude of the Ascending Equator Crossing, λ (degrees)	① Time from January 86.1612, 1965, t (days)	② Longitude of the Ascending Equator Crossing East of 66.634°, L (degrees)
1	65-2-25.0	6.6114053	.956	56.2412	67.735	-29.9200	1.101
2	65-3-3.0	6.6114581	.936	62.2254	67.460	-23.9358	.826
3	65-3-6.0	6.6112516	.925	65.2174	67.331	-20.9438	.697
4	65-3-13.0	6.6114112	.913	72.1989	67.023	-13.9623	.389
5	65-3-29.0	6.6112959	.871	88.1561	66.438	1.9949	- .196
6	65-4-5.0	6.6109642	.853	95.1372	66.226	8.9760	- .408
7	65-4-12.0	6.6111669	.839	102.1184	66.018	15.9572	- .616
8	65-4-19.0	6.6108871	.827	109.0994	65.853	22.9382	- .781
9	65-4-26.0	6.6110582	.816	116.0804	65.711	29.9192	- .923
10	65-5-3.0	6.6107742	.805	123.0613	65.597	36.9001	-1.037
11	65-5-10.0	6.6109303	.792	130.0422	65.492	43.8810	-1.142
		Average: $6.611146 = a_s$	Average: $31.867 = i_s$				

*$J_{22} = -1.8 \times 10^{-6}$ $J_{33} = -0.16 \times 10^{-6}$ $J_{31} = -1.5 \times 10^{-6}$

$\lambda_{22} = -15.35°$ $\lambda_{33} = 24°$ $\lambda_{31} = 0°$

Only earth longitude gravity used in this simulation. Initial elements as in orbit 8S-1 above. All other gravity constants as in Table A1.

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

$a_1 = -(1.2826 \pm 0.0336) \times 10^{-1}$ degrees

$a_2 = -(3.4062 \pm 0.0183) \times 10^{-2}$ degrees/day

$a_3 = (2.4390 \pm 0.0601) \times 10^{-4}$ degrees/day^2

$a_4 = (1.826 \pm 2.005) \times 10^{-7}$ degrees/day^3

Standard error of estimate = 0.00591 degree

$\ddot{\lambda}$ *(with minimum standard error) measured* = 0.8616 x 10^{-5} rad/sid. day^2, at 66.255°, on 93.9747 January 1965.

$\ddot{\lambda}$ (theoretical, from Equation 2, $i_s = 31.87°$, $\lambda = 66.255°$)= 0.884 x 10^{-5} rad/sid. day^2, for $a_s = 6.61115$ earth radii.

Estimated bias in arc 8S/1 at $\lambda = 66.255°$

= theoretical-measured

= $0.884 \times 10^{-5} - 0.862 \times 10^{-5} = +0.022 \times 10^{-5}$ rad/sid. day^2

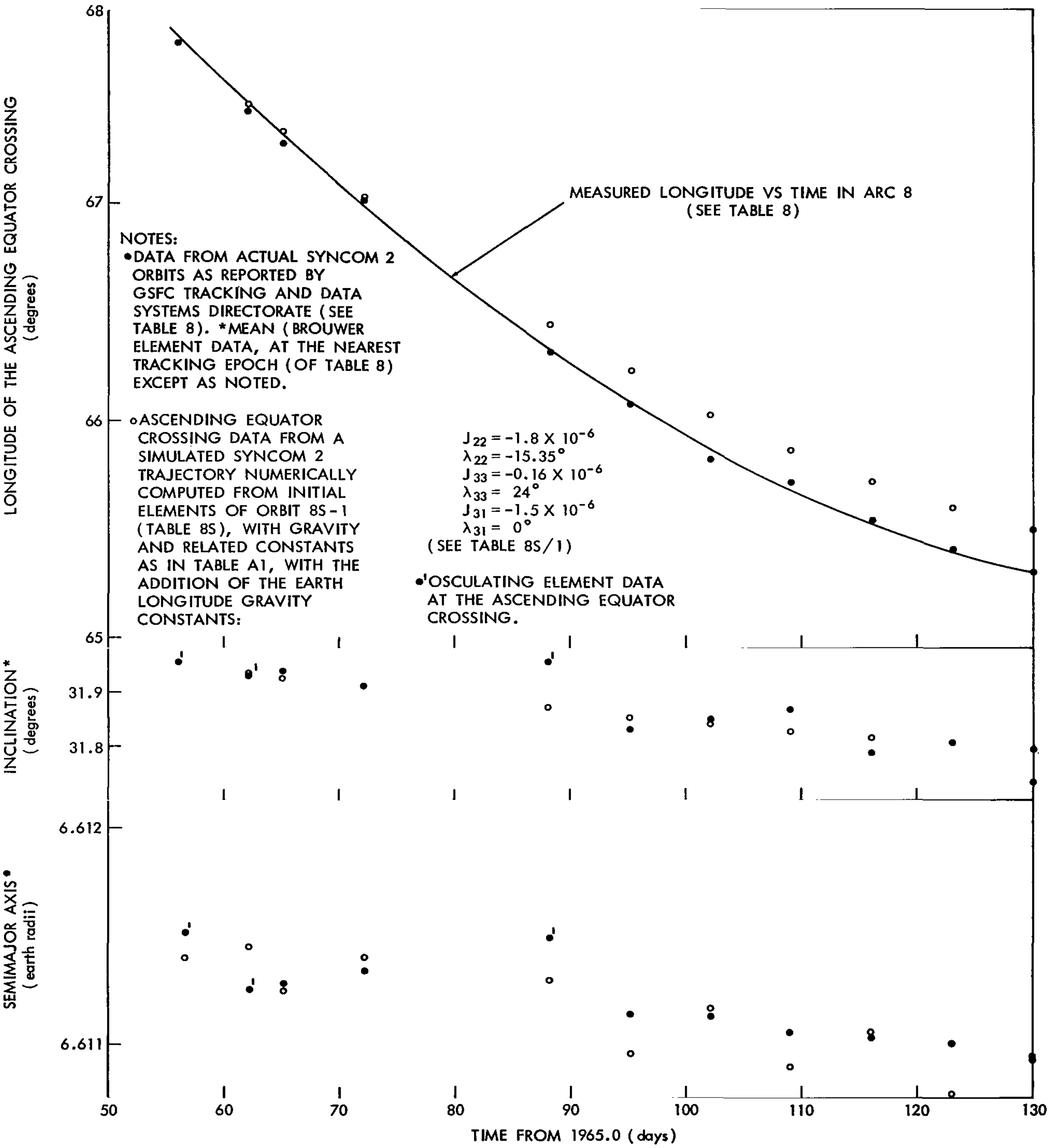

Figure 7—Measured and simulated orbit data in free drift arc 8 (Syncom 2).

In the first three columns of that table, the densely observed reference orbit ground track of Early Bird for 23 April is listed. The theory of 24-hour satellite drift used in this report references such drift specifically to the ascending Equator crossing of the satellite (see References 2 and 3, for example). However, the results apply equally well to orbit averaged drift from any nodal argument. If the satellite suffered no orbit perturbations, an orbit period would be specified precisely by the time between two similarly directed latitude passes. Thus, we can measure the 24-hour orbit longitude drift by comparing a given measured longitude-latitude point in an actual orbit with its longitude in the reference orbit by way of the indicated latitude. Unfortunately, perturbations caused the inclination of Early Bird to grow by about 100% in the two months of arc 9. For a large number of measured ground points in this history of Early Bird, there was no clear reference latitude match to indicate orbit longitude drift. This problem was especially aggravated by the preponderance of Early Bird observations at the maximum north and south points. Fortunately, the observation time alone provides a second and always clear reference orbit match criteria, even in the presence of perturbations, providing the orbit period (nodal) is reasonably well known. In the case of Early Bird in the spring of 1965, the nodal period is obviously close to synchronous, or one revolution in about 4 minutes short of 24 hours. Thus, the reference orbit longitude which corresponds to N number of orbit periods from the observed longitude occurs at a reference orbit time approximately 4N minutes later in the day than the observed longitude for a nearly synchronous satellite. Many of the comparative longitude matchings in Table 9 were made solely on this daily orbit time basis, especially where the observations were in the maximum north and south regions. Near equatorial observations were generally matched by both methods and an average reference longitude is listed in Table 9 for these cases. It is noted that the observed reference orbit ground track is "biased" north by about 0.06°. Most of this bias is probably due to error in the Andover antenna altitude calibration. A similar bias in the azimuth calibration would shift the mean longitude of the satellite by a small constant amount, insignificant in this acceleration analysis.

The average of the days estimated drift from the reference orbit is then applied to the ascending Equator crossing longitude in the reference orbit, to arrive at an estimated ascending Equator crossing longitude for that day (see Table 9). The subsequent acceleration analysis of this derived crossing data follows the same technique as used in the other slow drift arcs (1, 2, 6, 7, and 8). The average semimajor axis for Early Bird during arc 9 was not observed but inferred from the closely fitting simulated trajectory of orbit 9S/1 (Table 9S/1). The north-south excursion in the observed reference orbit (Table 9) gave a nominal value of 0.14° for the initial orbit inclination of Early Bird. The average orbit inclination in this arc was also inferred from the close simulated trajectory of orbit 9S/1.

The results of the acceleration analysis on the actual data and on the simulated data are found in Tables 9, 9S and 9S/1, and also summarized in Tables 10 and 11 in the next section.

Table 9

Ground Track and Related Ascending Equator Crossing Data for Free Drift Arc 9 (Early Bird)*

Ground Track Reference Orbit Data								**		
Time (April 1965) Day 23 Hr	Latitude (degrees)	Geographic Longitude (degrees west)	Time Day Hr	Latitude (degrees)	Longitude (degrees west)	Comparative Longitude in Reference Orbit (degrees west)	Drift from Reference Orbit (degrees east)	Estimated Ascending Equator Crossing Longitude (degrees west)	Time of Ascending Equator Crossing During Day: Estimate (hours)	Time (sid. day Integers From 23 April 1965)
2	.15	30.04	(April 1965)							
3	.14	30.045	23 -	-.00	30.005	30.005	0.00	30.005	15.8	0
3	.13	30.05	24 2	.15	29.98	30.04	0.06			
3	.12	30.05	3	.13	29.99	30.05	0.06			
4	.11	30.05	6	.035	30.015	30.075	0.06			
4	.10	30.055	11	-.07	30.00	30.06	0.06			
4	.09	30.06	15	-.02	29.95	30.01	0.06			
5	.08	30.065	19	+.115	29.90	29.96	0.06			
5	.07	30.065				avg:	0.06	29.945	16	1
5	.06	30.07	26 6	.05	29.90	30.075	.175			
6	.05	30.075	10	-.07	29.895	30.065	.170			
6	.04	30.075	14	-.04	29.85	30.025	.175			
6	.03	30.075	18	.07	29.81	29.975	.165			
6	.02	30.08	22	.18	29.79	29.96	.170			
7	.01	30.08				avg:	.170	29.835	16	3
7	.00	30.08	27 6	.03	29.845	30.075	.230			
8	-.01	30.075	13	-.07	29.83	30.045	.215			
8	-.02	30.075	18	.08	29.75	29.975	.225			
8	-.03	30.075	24/0	.19	29.74	29.97	.230			
9	-.04	30.075				avg:	.225	29.780	16	4
9	-.05	30.075	28 6	.04	29.785	30.075	.290			
10	-.06	30.07	12	-.07	29.77	30.05	.280			
11	-.07	30.06	18	.08	29.70	29.975	.275			
11	-.075	30.06	24/0	.19	29.685	29.97	.285			
12	-.07	30.05				avg:	.285	29.720	16	5
12	-.06	30.04	29 6	.03	29.74	30.075	.335			
13	-.05	30.035	12	-.07	29.72	30.05	.330			
14	-.04	30.025	17	.03	29.65	29.99	.340			
14	-.03	30.015				avg:	.335	29.670	16	6
15	-.02	30.01	30 2	.17	29.66	30.04	.380			
15	-.01	30.01	6	.04	29.685	30.075	.390			
15	-.00	30.005	13	-.07	29.67	30.045	.375			
16	.01	29.995	19	.02	29.61	29.98	.370			7
16	.02	29.995				avg:	.380	29.625	16	
17	.03	29.99	(May 1965)							
17	.04	29.985	1 3	.14	29.615	30.045	.430			
17	.05	29.98	6	.02	29.64	30.08	.440			
18	.06	29.98				avg:	.435	29.570	15	8
18	.07	29.975	2 2	.15	29.57	30.04	.470			
18	.08	29.975	9	-.07	29.59	30.407	.480			
18	.09	29.97				avg:	.475	29.530	15	9

Results of least squares fit of data in ① and ② (pages 47-48) according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = (3.6180 \pm 0.01023) \times 10^{-1}$ degrees

$a_2 = (3.202 \pm 0.0107) \times 10^{-2}$ degrees/sid. day

$a_3 = -(4.127 \pm 0.0279) \times 10^{-4}$ degrees/sid. day^2

$a_4 = (1.039 \pm 1.799) \times 10^{-7}$ degrees/sid. day^3

Standard error of estimate = 4.95×10^{-3} degrees

$\ddot{\lambda}$(with minimum standard error) = $-(1.4408 \pm 0.0097) \times 10^{-5}$ rad/sid day^2, at t = -0.3003 sid. day, t′ = 22 May 1965, $\lambda = -28.703°$.

*Ground track (subsatellite point) information on Early Bird, from Andover Maine Tracking, supplied through the Offices of Robert Greene, Comsat Corp., Washington, D.C.

**Calculated as 30.005° minus average drift from the reference orbit.

Ground Track and Related Ascending Equator Crossing Data for Free Drift Arc 9 (Early Bird)*

Ground Track Reference Orbit Data: Time (April 1965) Day Hr (Day 23)	Ground Track Reference Orbit Data: Latitude (degrees)	Ground Track Reference Orbit Data: Geographic Longitude (degrees west)	(May 1965) Time Day Hr	Latitude (degrees)	Longitude (degrees west)	Comparative Longitude in Reference Orbit (degrees west)	Drift from Reference Orbit (degrees east)	** Estimated Ascending Equator Crossing Longitude (degrees west)	Time of Ascending Equator Crossing During Day: Estimate (hours)	Time (sid. day integers from 23 April 1965)
19	.10	29.97	4 6	.01	29.495	30.08	.585			
19	.11	29.965	12	-.08	29.465	30.055	.590			
19	.12	29.96	21	.185	29.385	29.96	.575			
20	.13	29.955	24/0	.20	29.40	29.97	.570			
20	.14	29.955				avg:	.580	29.425	15	11
20	.15	29.95	5 6	.00	29.445	30.08	.635			
21	.16	29.955	12	-.09	29.42	30.05	.630			
21	.17	29.955	24/6	.19	29.35	29.97	.620			
22	.18	29.96				avg:	.630	29.375	15	12
23	.19	29.965	6 6	.005	29.40	30.08	.680			
23/24 24/0	.18	29.97				avg:	.680	29.325	15	13
24 1	.17	29.975	7 9	-.085	29.35	30.07	.720			
2	.16	29.98				avg:	.270	29.285	15	14
2	.15	29.98	8 22	.22	29.205	29.965	.760			
						avg:	.760	29.245	15	15
			10 8	-.075	29.23	30.07	.840			
			24/0	.21	29.13	29.97	.840			
						avg:	.830	29.165	15	17
			11 7	-.035	29.18	30.075	.895			
			24/0	.21	29.085	29.97	.885			
						avg:	.890	29.115	15	18
			12 8	-.065	29.14	30.07	.930			
			24/0	.215	29.045	29.97	.925			
						avg:	.930	29.075	15	19
			13 8	-.075	29.105	30.065	.960			
			24/0	.205	29.01	29.965	.955			
						avg:	.960	29.045	15	20
			14 8	-.08	29.065	30.075	1.010			
			24/0	.21	28.97	29.97	1.00			
						avg:	1.005	29.000	15	21
			15 8	-.09	29.02	30.075	1.055			
			24/0	.21	28.93	29.965	1.035			
						avg:	1.045	28.960	15	22
			16 8	-.10	28.985	30.065	1.080			
			24/0	.21	28.895	29.97	1.075			
						avg:	1.080	28.925	15	23
			18 8	-.11	28.91	30.065	1.155			
			23	.225	28.81	29.97	1.160			
						avg:	1.155	28.850	14	25
			19 8	-.12	28.87	30.065	1.195			
			24/0	.21	28.74	29.97	1.230			
			25/1	.18	28.79	29.975	1.185			
						avg:	1.190	28.815	14	26

Results of least squares fit of data in ① and ② (pages 47-48) according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = (3.6180 \pm 0.01023) \times 10^{-1}$ degrees

$a_3 = -(4.127 \pm 0.0279) \times 10^{-4}$ degrees/sid. day^2

$a_2 = (3.202 \pm 0.0107) \times 10^{-2}$ degrees/sid. day

$a_4 = (1.039 \pm 1.799) \times 10^{-7}$ degrees/sid. day^3

Standard error of estimate = 4.95×10^{-3} degrees

$\ddot{\lambda}$(with minimum standard error) = $-(1.4408 \pm 0.0097) \times 10^{-5}$ rad/sid day^2, at $t = -0.3003$ sid. day, $t' = 22$ May 1965, $\lambda = -28.703^\circ$.

*Ground track (subsatellite point) information on Early Bird, from Andover Maine Tracking, supplied through the Offices of Robert Greene, Comsat Corp., Washington, D.C.

**Calculated as 30.005° minus average drift from the reference orbit.

Table 9 (Cont.)

Ground Track and Related Ascending Equator Crossing Data for Free Drift Arc 9 (Early Bird)*

(May 1965) Time Day	Hr	Latitude (degrees)	Longitude (degrees west)	Computed Longitude in Reference Orbit (degrees west)	Drift from Reference Orbit (degrees east)	** Estimated Ascending Equator Crossing Longitude (degrees west)	Time of Ascending Equator Crossing During Day: Estimate (hours)	Time (sid. day integers from 23 April 1965)	① Time from 23.6583 April 1965 + 29.5 Sid. Days, t (sid. days)	② Ascending Equator Crossing Longitude East of -29.055°, L (degrees)
20	8	-.12	28.835	30.065	1.230				-29.5	-.950
	24/0	.205	28.75	29.97	1.220				-28.5	-.890
				avg:	1.225	28.780	14	27	-26.5	-.780
21	8	-.115	28.80	30.065	1.265				-25.5	-.725
				avg:	1.265	28.740	14	28	-24.5	-.665
22	2	.145	28.73	30.045	1.315				-23.5	-.615
	8	-.13	28.765	30.065	1.300				-22.5	-.570
	24/0	.205	28.685	29.975	1.290				-21.5	-.515
				avg:	1.300	28.705	14	29	-20.5	-.475
23	8	-.135	28.73	30.065	1.335				-18.5	-.370
				avg:	1.335	28.670	14	30	-17.5	-.320
24	8	-.14	28.70	30.065	1.365				-16.5	-.270
				avg:	1.365	28.640	14	31	-15.5	-.230
25	2	.12	28.64	30.05	1.410				-14.5	-.190
	8	-.14	28.67	30.065	1.395				-12.5	-.110
	23	.23	28.58	29.97	1.390				-11.5	-.060
				avg:	1.400	28.605	14	32	-10.5	-.020
26	8	-.14	28.635	30.065	1.430				- 9.5	+.010
	23	.225	28.555	29.97	1.415				- 8.5	+.055
				avg:	1.425	28.580	14	33	- 7.5	.095
27	8	-.14	28.605	30.065	1.460				- 6.5	.130
	25/1	.165	28.545	29.98	1.435				- 4.5	.205
				avg:	1.450	28.555	14	34	- 3.5	.240
28	8	-.145	28.585	30.065	1.480				- 2.5	.275
	24/0	-.205	28.51	29.975	1.465				- 1.5	.315
				avg:	1.475	28.530	14	35	- 0.5	.350
									+ 0.5	.385
29	8	-.15	28.55	30.065	1.515				+ 1.5	.415
	23	.225	28.485	29.97	1.485				2.5	.450
				avg:	1.500	28.505	14	36	3.5	.475
30	8	-.15	28.53	30.065	1.535				4.5	.500
	24/0	.19	28.465	29.975	1.510				5.5	.525
				avg:	1.525	28.480	14	37	6.5	.550
31	8	-.15	28.50	30.065	1.565				7.5	.575
	25/1	.17	28.445	29.98	1.535				8.5	.600
				avg:	1.550	28.455	14	38	9.5	.625
(June 1965)									10.5	.650

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = (3.6180 \pm 0.01023) \times 10^{-1}$ degrees $\qquad a_3 = -(4.127 \pm 0.0279) \times 10^{-4}$ degrees/sid. day^2

$a_2 = (3.202 \pm 0.0107) \times 10^{-2}$ degrees/sid. day $\qquad a_4 = (1.039 \pm 1.799) \times 10^{-7}$ degrees/sid. day^3

Standard error of estimate = 4.95×10^{-3} degrees

$\ddot{\lambda}$ (with minimum standard error) = $-(1.4408 \pm 0.0097) \times 10^{-5}$ rad/sid day^2, at t = -0.3003 sid. day, t' = 22 May 1965, λ = -28.703°.

*Ground track (subsatellite point) information on Early Bird, from Andover Maine Tracking, supplied through the Offices of Robert Greene, Comsat Corp., Washington, D.C.

**Calculated as 30.005° minus average drift from the reference orbit.

Table 9 (Cont.)

Ground Track and Related Ascending Equator Crossing Data for Free Drift Arc 9 (Early Bird)*

(June 1965) Time Day	Hr	Latitude (degrees)	Longitude (degrees west)	Computed Longitude in Reference Orbit (degrees west)	Drift from Reference Orbit (degrees east)	** Estimated Ascending Equator Crossing Longitude (degrees east)	Time of Ascending Equator Crossing During Day: Estimate (hours)	Time (sid. day integers from 23 April 1965)	① Time from 23.6583 April 1965 + 29.5 Sid. Days, t (sid. days)	② Ascending Equator Crossing Longitude East of -29.055° L (degrees)
1	8	-.16	28.475	30.07	1.595				11.5	.670
	24/0	.185	28.42	29.975	1.555				14.5	.745
				avg:	1.575	28.430	14	39	15.5	.760
2	8	-.17	28.45	30.065	1.615				16.5	.780
	23	.24	28.385	29.97	1.585				17.5	.800
				avg:	1.600	28.405	14	40	22.5	.870
3	9	-.175	28.425	30.06	1.635				25.5	.916
	25/1	.135	28.38	29.985	1.605				27.5	.930
				avg:	1.620	28.385	14	41	29.5	.950
6	9	-.18	28.35	30.06	1.710					
				avg (est.)	1.695	28.310	14	44		
7	9	-.18	28.335	30.06	1.725					
	24/0	.205	28.285	29.98	1.695					
				avg:	1.710	28.295	14	45		
8	8	-.18	28.325	30.06	1.735					
	23	.255	28.255	29.975	1.720					
				avg:	1.730	28.275	13	46		
9	9	-.18	28.305	30.06	1.755					
	23	.255	28.240	29.98	1.740					
				avg:	1.750	28.255	13	47		
10	10	-.18	28.29	30.025	1.735					
	24/0	.215	28.225	29.99	1.765					
				avg:	1.750	28.255***	13	48		
14	8	-.20	28.23	30.05	1.82					
				avg:	1.82	28.185	13	52		
15	10	-.15	28.225	30.025	1.80					
	24/0	.18	28.23	29.995	1.765					
				avg:	1.785	28.22***	13	53		
16	24/0	.27	28.115	29.985	1.870					
				avg:	1.870	28.135***	13	54		
17	9	-.21	28.21	30.04	1.830					
	23	.32	28.085	29.98	1.895					
				avg:	1.875	28.140	13	55		
18	9	-.25	28.215	30.045	1.830					
	21	.32	28.09	29.97	1.880					
				avg:	1.855	28.150***	13	56		
19	9	-.19	28.155	30.035	1.880					
	23	.25	28.095	29.98	1.885					
				avg:	1.880	28.125	12	57		
21	9	-.205	28.135	30.035	1.900					
				avg:	1.900	28.105	12	59		

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$

$a_1 = (3.6180 \pm 0.01023) \times 10^{-1}$ degrees

$a_3 = -(4.127 \pm 0.0279) \times 10^{-4}$ degrees/sid. day^2

$a_2 = (3.202 \pm 0.0107) \times 10^{-2}$ degrees/sid. day

$a_4 = (1.039 \pm 1.799) \times 10^{-7}$ degrees/sid. day^3

Standard error of estimate = 4.95×10^{-3} degrees

$\ddot{\lambda}$(with minimum standard error) = $-(1.4408 \pm 0.0097) \times 10^{-5}$ rad/sid day^2, at $t = -0.3003$ sid. days, $t' =$ 22 May 1965, $\lambda = -28.703°$.

*Ground track (subsatellite point) information on Early Bird, from Andover Maine Tracking, supplied through the Offices of Robert Greene, Comsat Corp., Washington, D.C.

**Calculated as 30.005° minus average drift from the reference orbit.

***Data not used in acceleration analysis as inclusion gives unacceptably large residuals for this arc.

Table 9S

Ascending Equator Crossing Orbit Data from a Simulated Early Bird Trajectory for Free Drift Arc 9, with Earth Longitude Gravity through Second Order*.

Time, (April 1965) Day Hour	Semimajor Axis, a (earth radii)	Inclination, i (degrees)	Eccentricity	Longitude of the Ascending Equator Crossing (degrees)	Time (sid. day integers from 23 April 1965)	① Time from 23.6583 April 1965 +29.5 sid. days, t (sid. days)	② Longitude of the Ascending Equator Crossing East of -29.055°, L (degrees)
23-15.8	6.6101480	0.140	.00035	-30.000	0	-29.5	-.945
24-15.8	6.6101187	0.141	.00034	-29.941	1	-28.5	-.886
26-15.7	6.6101023	0.143	.00032	-29.828	3	-26.5	-.773
27-15.7	6.6101270	0.144	.00032	-29.773	4	-25.5	-.718
28-15.6	6.6101804	0.144	.00032	-29.719	5	-24.5	-.664
29-15.5	6.6102617	0.145	.00034	-29.667	6	-23.5	-.612
30-15.5	6.6103572	0.146	.00035	-29.616	7	-22.5	-.561
(May 1965)							
1-15.4	6.6104416	0.149	.00037	-29.565	8	-21.5	-.510
2-15.3	6.6104874	0.152	.00038	-29.515	9	-20.5	-.460
4-15.2	6.6104227	0.161	.00036	-29.413	11	-18.5	-.358
5-15.1	6.6103416	0.165	.00033	-29.362	12	-17.5	-.307
6-15.1	6.6102623	0.169	.00031	-29.311	13	-16.5	-.256
7-15.1	6.6102046	0.171	.00030	-29.261	14	-15.5	-.206
8-15.1	6.6101804	0.173	.00030	-29.212	15	-14.5	-.157
10-15.0	6.6102312	0.176	.00033	-29.118	17	-12.5	-.063
11-15.0	6.6102955	0,176	.00035	-29.073	18	-11.5	-.018
12-14.9	6.6103682	0.177	.00038	-29.030	19	-10.5	+.025
13-14.8	6.6104367	0.179	.00040	-28.988	20	- 9.5	+.067
14-14.7	6.6104899	0.182	.00041	-28.946	21	- 8.5	.109
15-14.6	6.6105205	0.185	.00041	-28.904	22	- 7.5	.151
16-14.6	6.6105278	0.189	.00041	-28.861	23	- 6.5	.194
18-14.5	6.6104863	0.196	.00040	-28.774	25	- 4.5	.281
19-14.4	6.6104493	0.200	.00038	-28.730	26	- 3.5	.325
20-14.4	6.6104093	0.203	.00037	-28.687	27	- 2.5	.368
21-14.4	6.6103715	0.206	.00036	-28.644	28	- 1.5	.411
22-14.3	6.6103412	0.208	.00034	-28.602	29	- 0.5	.453
23-14.3	6.6103236	0.209	.00033	-28.562	30	+ 0.5	.493
24-14.3	6.6103243	0.211	.00033	-28.522	31	+ 1.5	.533
25-14.2	6.6103491	0.211	.00032	-28.484	32	2.5	.571
26-14.1	6.6104025	0.213	.00033	-28.447	33	3.5	.608
27-14.1	6.6104817	0.214	.00034	-28.410	34	4.5	.645
28-14.0	6.6105750	0.217	.00036	-28.374	35	5.5	.681
29-13.9	6.6106568	0.220	.00038	-28.339	36	6.5	.716
30-13.8	6.6106987	0.224	.00038	-28.304	37	7.5	.751
31-13.7	6.6106832	0.229	.00037	-28.269	38	8.5	.786
(June 1965)							
1-13.7	6.6106173	0.234	.00035	-28.234	39	9.5	.821
2-13.7	6.6105271	0.238	.00033	-28.199	40	10.5	.856
3-13.7	6.6104437	0.242	.00031	-28.163	41	11.5	.892
6-13.6	6.6103943	0.249	.00032	-28.064	44	14.5	.991
7-13.5	6.6104430	0.250	.00034	-28.034	45	15.5	1.021
8-13.5	6.6105081	0.251	.00037	-28.005	46	16.5	1.050
9-13.4	6.6105763	0.253	.00039	-27.977	47	17.5	1.078
14-13.0	6.6106906	0.270	.00040	-27.837	52	22.5	1.218
17-12.9	6.6105811	0.281	.00036	-27.751	55	25.5	1.304
19-12.9	6.6105121	0.286	.00034	-27.697	57	27.5	1.358
21-12.8	6.6104972	0.289	.00032	-27.648	59	29.5	1.407
	Average: 6.610410 = a_s	Average: 0.200 = i_s					

*Computed by ITEM with gravity constants the same as in Table A1, with the addition of the earth constants: $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18°$

The other initial elements of this orbit besides those listed in the top line are: argument of perigee =0°, mean anomaly =0°, longitude of launch =30° West.

Results of least squares fit of data in ① and ② above according to the theory of Equation 1:

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

$a_1 = (4.7042 \pm 0.00667) \times 10^{-1}$ degrees, $a_2 = (3.9577 \pm 0.00695) \times 10^{-2}$ degrees/sid. day

$a_3 = -(2.7366 \pm 0.01821) \times 10^{-4}$ degrees/sid. day^2, $a_4 = (1.039 \pm 1.799) \times 10^{-7}$ degrees/sid. day^3. Standard error of estimate = 3.23×10^{-3} degrees.

$\ddot{\lambda}$ (measured) = $-(0.9559 \pm 0.0064) \times 10^{-5}$ rad/sid. day^2, at $\lambda = -28.597°$ and $t = -0.3003$ days

$\ddot{\lambda}$ (theoretical, from Equation 2) = -0.9877×10^{-5} rad/sid. day^2, for $a_s = 6.61041$ earth radii, $i_s = 0.2°$, $\lambda = -28.597°$, $J_{22} = -1.68 \times 10^{-6}$, $\lambda_{22} = -18.0°$.

Bias = theoretical-measured
$= (-0.9877) \times 10^{-5} - (-0.9559) \times 10^{-5}$
$= -(0.0318) \times 10^{-5}$ rad/sid. day^2

Table 9S/1

Ascending Equator Crossing Orbit Data from a Simulated Early Bird Trajectory for Free Drift Arc 9 Computed by "ITEM" with Earth Longitude Gravity Through Third Order.*

Semimajor Axis, a (earth radii)	Inclination i (degrees)	Longitude of the Ascending Equator Crossing (degrees)	Time (sidereal day integers from 23 April 1965)	① Time from 23.6583 April 1965 + 29.5 Sid. Days, t (sid. days)	② Longitude of the Ascending Equator Crossing East of −29.017° L (degrees)
6.6101480	.140	-30.000	0	-29.5	-.983
6.6101214	.141	-29.941	1	-28.5	-.924
6.6101100	.143	-29.829	3	-26.5	-.812
6.6101374	.144	-29.775	4	-25.5	-.758
6.6101937	.144	-29.722	5	-24.5	-.705
6.6102773	.145	-29.671	6	-23.5	-.654
6.6103750	.147	-29.621	7	-22.5	-.604
6.6104624	.149	-29.572	8	-21.5	-.555
6.6105106	.152	-29.524	9	-20.5	-.507
6.6104513	.161	-29.427	11	-18.5	-.410
6.6103728	.165	-29.378	12	-17.5	-.361
6.6102961	.169	-29.330	13	-16.5	-.313
6.6102414	.171	-29.282	14	-15.5	-.265
6.6102194	.173	-29.236	15	-14.5	-.219
6.6102767	.175	-29.149	17	-12.5	-.132
6.6103427	.176	-29.108	18	-11.5	-.091
6.6104180	.177	-29.069	19	-10.5	-.052
6.6104895	.179	-29.031	20	- 9.5	-.014
6.6105452	.182	-28.994	21	- 8.5	+.023
6.6105787	.185	-28.956	22	- 7.5	+.061
6.6105884	.188	-28.918	23	- 6.5	+.099
6.6105524	.196	-28.842	25	- 4.5	+.175
6.6105180	.200	-28.803	26	- 3.5	+.214
6.6104808	.203	-28.766	27	- 2.5	+.251
6.6104460	.206	-28.729	28	- 1.5	+.288
6.6104184	.208	-28.694	29	- 0.5	+.323
6.6104035	.209	-28.660	30	+ 0.5	+.357
6.6104067	.211	-28.627	31	+ 1.5	+.390
6.6104345	.211	-28.595	32	+ 2.5	+.422
6.6104905	.213	-28.565	33	+ 3.5	+.452
6.6105730	.214	-28.536	34	+ 4.5	+.481
6.6106686	.217	-28.508	35	+ 5.5	+.509
6.6107537	.220	-28.480	36	+ 6.5	+.537
6.6107980	.224	-28.453	37	+ 7.5	+.564
6.6107854	.229	-28.427	38	+ 8.5	+.590
6.6107224	.234	-28.400	39	+ 9.5	+.617
6.6106350	.238	-28.374	40	+10.5	+.643
6.6105545	.242	-28.347	41	+11.5	+.670
6.6105136	.249	-28.276	44	+14.5	+.741
6.6105652	.250	-28.256	45	+15.5	+.761
6.6106331	.251	-28.238	46	+16.5	+.779
6.6107041	.253	-28.220	47	+17.5	+.797
6.6108330	.270	-28.135	52	+22.5	+.882
6.6107324	.281	-28.085	55	+25.5	+.932
6.6106693	.286	-28.057	57	+27.5	+.960
6.6106604	.289	-28.033	59	+29.5	+.984
Average: $6.6105 = a_s$	Average: $0.200 = i_s$				

*Gravity constants of this trajectory, the same as in Table A1, with the addition of the earth constants:

$J_{22} = -1.8 \times 10^{-6}$ $\qquad J_{33} = -0.16 \times 10^{-6}$ $\qquad J_{31} = -1.5 \times 10^{-6}$

$\lambda_{22} = -15.35°$ $\qquad \lambda_{33} = 24°$ $\qquad \lambda_{31} = 0°$

The initial elements of this trajectory, aside from those listed in the top line are the same as those in the top line of Table 9S.
Results of least squares fit of the data in ① and ② according to the theory of Equation 1:

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

$a_1 = (3.3794 \pm 0.00661) \times 10^{-1}$ degrees

$a_2 = (3.3134 \pm 0.00688) \times 10^{-2}$ degrees/sid. day

$a_3 = -(3.8650 \pm 0.0180) \times 10^{-4}$ degrees/sid. day^2

$a_4 = (1.101 \pm 1.162) \times 10^{-7}$ degrees/sid. day^3 Standard error of estimate $= 3.198 \times 10^{-3}$ degrees

$\ddot{\lambda}$ (measured, with minimum standard error) $= -1.3488 \times 10^{-5}$ rad/sid. day^2, at $t = -0.3003$ day, $t' =$ 22 May 1965, $\lambda = -28.69°$.

$\ddot{\lambda}$ (theoretical from Equation 2), $i_s = 0.2°$, $\lambda = -28.69°$, $J_{22} - J_{31}$ as noted $= -1.380 \times 10^{-5}$ rad/sid. day^2, for $a_s = 6.6105$ earth radii.

Estimate of acceleration bias $= \ddot{\lambda}$ (theoretical) $- \ddot{\lambda}$ (measured) at $\lambda = -28.69°$ in arc S 9/1

$= -1.380 \times 10^{-5} + 1.349 \times 10^{-5}$

$\approx -0.031 \times 10^{-5}$ rad/sid. day^2.

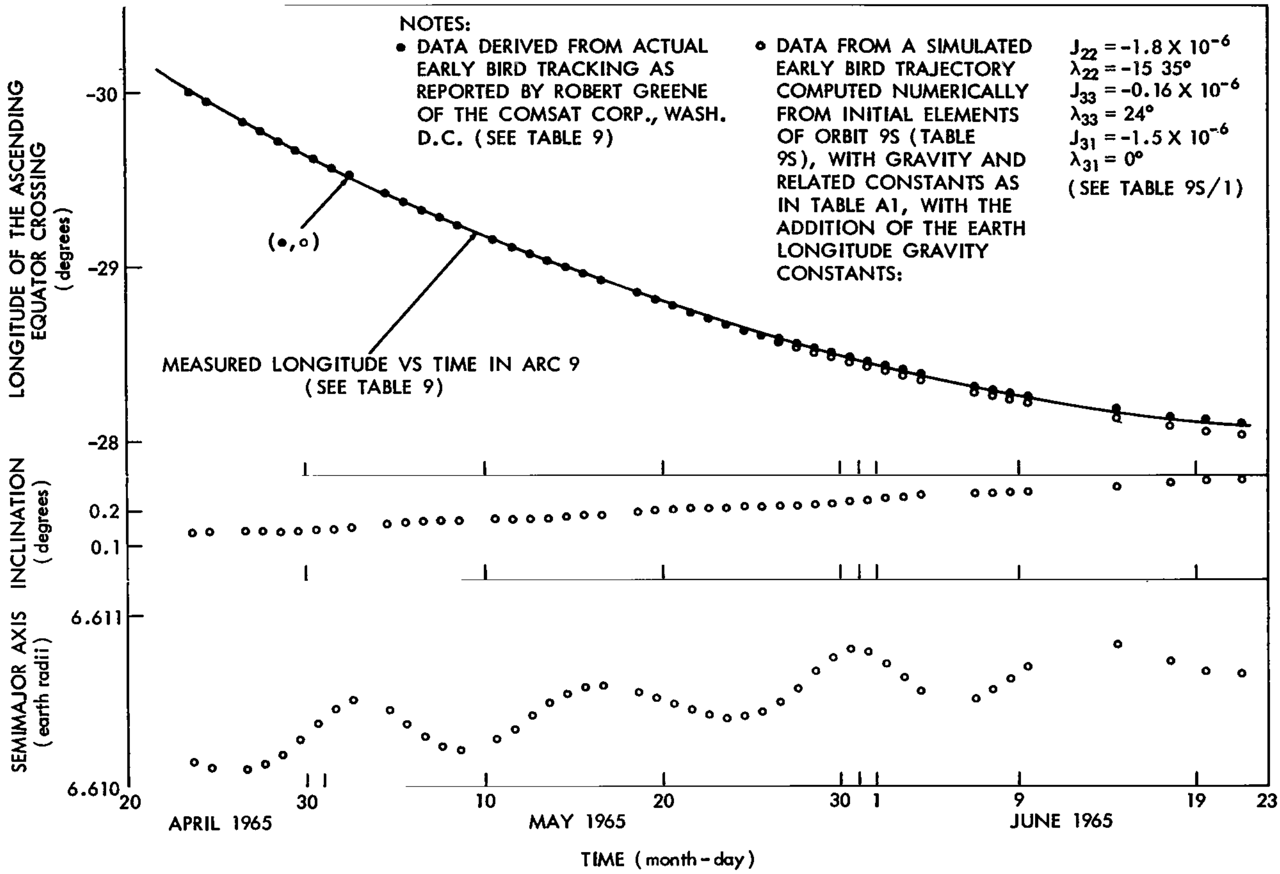

Figure 8—Measured and simulated orbit data at ascending Equator crossings in free drift arc 9 (Early Bird).

2. SYNTHESIS OF THE LONGITUDE ACCELERATION RECORD TO REVEAL COMPONENTS IN THE EARTH'S LONGITUDE GRAVITY FIELD

According to Equation 2, the long term resonant earth gravity accelerated longitude drift of the 24-hour satellite is given very closely through fourth order, by

$$\begin{aligned}\ddot{\lambda} = -12\pi^2 \Bigg[& \frac{6}{a_s^2} F(i)_{22}\left\{C_{22} \sin 2\lambda - S_{22} \cos 2\lambda\right\} \\ & + \frac{45}{a_s^3} F(i)_{33}\left\{C_{33} \sin 3\lambda - S_{33} \cos 3\lambda\right\} - \frac{3F(i)_{31}}{2\, a_s^3}\left\{C_{31} \sin \lambda - S_{31} \cos \lambda\right\} \\ & + \frac{420\, F(i)_{44}}{a_s^4}\left\{C_{44} \sin 4\lambda - S_{44} \cos 4\lambda\right\} - \frac{15\, F(i)_{42}}{a_s^4}\left\{C_{42} \sin 2\lambda - S_{42} \cos 2\lambda\right\}\Bigg] \end{aligned} \tag{11}$$

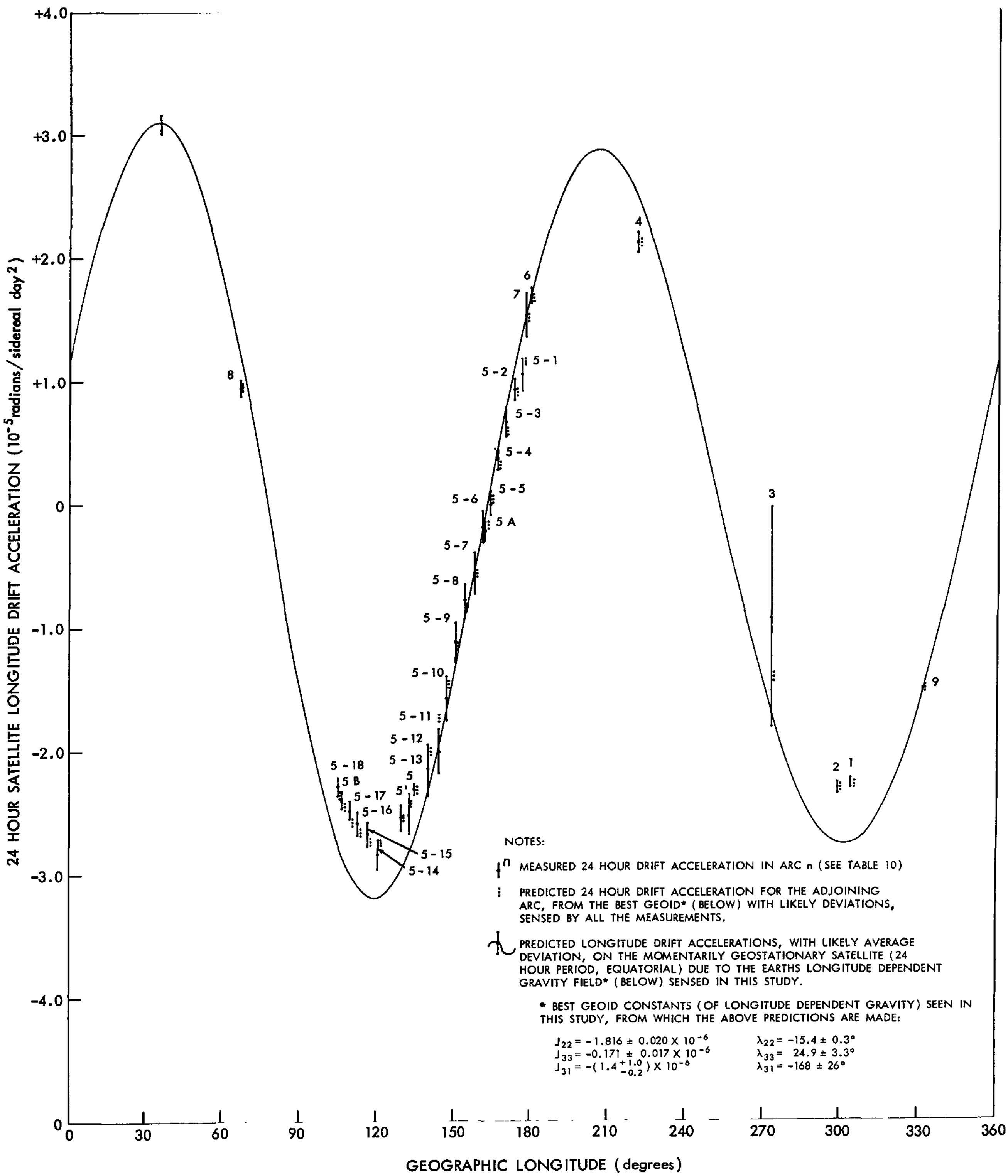

Figure 9—Measured and geoid-predicted 24-hour satellite longitude drift accelerations.

where $\ddot{\lambda}$ is the longitude acceleration in units of radians/sid. day^2 and a_s is the "synchronous" semimajor axis of the satellite in earth radii. The $F(i)_{nm}$ functions of the satellite inclination i's are

$$\left.\begin{aligned}
F(i)_{22} &= \left[\frac{1}{2}\left(\cos i_s + 1\right)\right]^2 \\
F(i)_{33} &= \left[\frac{1}{2}\left(\cos i_s + 1\right)\right]^3 \\
F(i)_{31} &= \left[\frac{1}{2}\left(\cos i_s + 1\right) - \frac{5}{8}\sin^2 i_s\left(1 + 3\cos i_s\right)\right] \\
F(i)_{44} &= \left[\frac{1}{2}\left(\cos i_s + 1\right)\right]^4 \\
F(i)_{42} &= \left[\frac{1}{4}\left(\cos i_s + 1\right)^2 - \frac{7}{4}\sin^2 i_s\left(\cos i_s + 1\right)\right] .
\end{aligned}\right\} \tag{12}$$

The C_{nm}, S_{nm} gravity coefficients are given in terms of the J_{nm}, λ_{nm} coefficients by

$$\left.\begin{aligned}
C_{nm} &= J_{nm}\cos m\lambda_{nm} \\
S_{nm} &= J_{nm}\sin m\lambda_{nm} .
\end{aligned}\right\} \tag{13}$$

In Equations 13, the J_{nm} are all negative so that the λ_{nm} gravity harmonic phase angles with respect to Greenwich are interpreted physically as in Appendix B. Thus the proper quadrant for λ_{nm} from the C_{nm} and S_{nm} is determined by

$$\lambda_{nm} = \frac{1}{m}\tan^{-1}\left[\frac{-S_{nm}}{-C_{nm}}\right] . \tag{14}$$

The J_{nm} are given from the C_{nm} and S_{nm} by

$$J_{nm} = -\left[C_{nm}^2 + S_{nm}^2\right]^{1/2} . \tag{15}$$

From a set of measured long term 24-hour satellite accelerations $\ddot{\lambda}$ at longitudes λ, with inclinations i_s and semimajor axes a_s, it is possible to determine the C_{nm}, S_{nm} in Equation 11 which best satisfy this set of accelerations. If the set of measurements numbers the same as the C_{nm}, S_{nm}

coefficients, then clearly the C_{nm}, S_{nm} can be determined from them by a simple simultaneous solution of the set of Equations 11 with the specified measurements. However, such a straightforward solution for the underlying gravity field assumes that Equation 11 is an exact equation, if, for example, ten measurements are available. Actually, as discussed in the introduction, the measurements of $\ddot{\lambda}$ will be in error from true resonant gravity acceleration due to a number of sources.

A more realistic model from which to determine the dominating gravity effects from the measured accelerations is the general linear least squares model. In this model, we allow each of the measurements j to be conditioned by Equation 11 (with specified C_{nm}'s and S_{nm}'s as unknowns) with the addition of a small unknown error ϵ_j. We require more measurements than unknowns in order to allow for these additional unknown ϵ_j's. In the least squares solution, the otherwise overdetermined C_{nm} and S_{nm} are adjusted so that the sums of the squares of the residuals of $\ddot{\lambda}$ (measured, or the left hand side of Equation 11) and $\ddot{\lambda}$ (theoretical, or the right hand side of Equation 11) are a minimum. These residuals are an estimate of the unknown ϵ_j. Furthermore, if we assume that the ϵ_j are random normally distributed with mean of zero and constant variance σ, we can estimate σ and make statistically significant statements about the likely variation of the C_{nm} and S_{nm} coefficients in any given test of the measured data according to Equation 11. (For general treatments of the least squares model, see References 13 and 14.) In Tables 10 and 11 we list the results of the acceleration analysis in the actual and simulated 24-hour satellite gravity drift arcs. Since we have available less than ten independent acceleration measurements, we cannot hope to determine all ten resonant gravity harmonics from a solution of Equation 11. In addition, of course, we tacitly ignore in this analysis the effects on the data of the infinite set of earth resonant gravity harmonics of order higher than fourth. It seems evident, even before looking at the data, that from four to six harmonics is all that might reasonably be extracted from a least squares solution of Equation 1 (ignoring selected coefficients).

In Appendix C we have calculated maximum resonant gravity effects from a recent (1965) geoid due to W. H. Guier (Reference 15). From this calculation, it is evident that C_{22}, S_{22}, C_{33} and S_{33} should be the dominating harmonics on the three 24-hour satellites in this study. The next most influential set of harmonics on all the satellites appears to be C_{31} and S_{31}, although on Syncom 2, C_{44} and S_{44} might rival it in influence according to other recent geoids listed in Table B1.

It is interesting to compare the standard acceleration error in the actual experiment with these maximum theoretical effects. Except for arc 3 (not used), arc 7 and about half of the 18 sub-arcs in arc 5, the standard acceleration error measures between 0.03×10^{-5} and 0.10×10^{-5} rad/sid. day^2. These levels are over an order of magnitude below the maximum theoretical J_{22} caused accelerations, and significantly below the theoretical maximum J_{33} effects on all the satellites. We are consequently encouraged to believe that the four harmonics C_{22}, S_{22}, C_{33} and S_{33} should be the minimum yield from the synthesis (or explanation) of the measured accelerations in Table 10 according to the theory of Equation 11. The theoretical maximum J_{31} effects appear to be at or just above the average noise level of this experiment, while the J_{42} and J_{44} effects appear to be somewhat below that level. While it would seem that not enough data is yet available to distinguish these harmonics with good precision, we are encouraged to hope that the apparently low errors in the widely separated equatorial arcs 6 and 9 will allow at least a tentative discrimination of C_{31} and S_{31} (see Discussion).

We now proceed to test the actual data in Table 10 for sensitivity to C_{22}, S_{22} alone and then together with C_{33}, S_{33}, and finally in combination with C_{31}, S_{31} as well, according to a least squares model based on Equation 11 (see Table 12). The blanks in Table 12 indicate that particular harmonic was not considered in the test.

The first three tests in Table 12, with the unadjusted data weighted according to the measured standard errors in Table 10, shows the general trend of the results. The weighting scheme chosen for these tests assumes that samples with standard errors less than 0.05×10^{-5} rad/sid. day^2 contain predominantly model bias errors and (in this unadjusted test) carry equal, unit weight. There is some justification for this assumption based on the bias results in Table 11. In this test, arc 5 was assigned a weight of 1.0 on the basis of the sum of the independent arcs 5A and 5B weights (column 3 of Table 10). This unit weight was divided among the 18 sub-arcs according to an independent arc 5 weighting scheme which gave unit weight to the best determined sample, that of sub-arc 5-18.

There are two strong conclusions which can be drawn at once from the first three tests in Table 12. The first is that the 24-hour longitude coverage around the equator is now so complete that we can almost define the dominant C_{22}, S_{22} harmonics without regard for the presence of higher order effects. (See also tests 28-30.) Theoretically, since the potential is an infinite series of orthogonal Legendre functions, a least squares fit of the actual potential through all space with respect to any combination of potential harmonics with determinable coefficients will yield the true Legendre coefficients of the potential. It appears a reasonable conjecture that, because of the natural suppression of higher order effects, near convergence to the true low order gravity potential coefficients should be possible from a complete longitude survey at 24-hour altitudes.

The second conclusion from tests 1-3 is that 24-hour satellite drift to date is now strongly sensitive to C_{33} and S_{33}, or at least third order earth longitude gravity. The sensitivity to higher order gravity is shown by the over fivefold reduction in the standard error of these tests when higher order effects are allowed. In fact, the higher order tests bring the standard acceleration error of the test to the level of the individual acceleration standard errors.

In addition to this dramatic reduction of the residuals upon allowance for third order effects, we note a similarly large reduction in the standard errors of the H_{22} coefficients. If we can attribute the residuals of the H_{22}, H_{33} test 2 purely to "observation noise," then we would expect the inclusion of the H_{31} harmonic (test 3) to make negligible change in the previously determined coefficients. We could also expect no clear H_{31} result, as well as an increase in the test- H_{22} and H_{33} standard errors, because the degrees of freedom of this limited sample would have been reduced at no comparable improvement of the fit.

Table 10

Longitude Accelerations in 24-Hour Satellite Arcs 1 to 9.

(Satellite)-Arc	① Longitude Acceleration Measured, $\ddot{\lambda}$ (10^{-5} rad/sid.day^2)	Longitude, λ (degrees)	Semimajor Axis, a_s (earth radii)	Orbit Inclination, i_s (degrees)	Standard Error of Longitude Acceleration, σ (10^{-5} rad/sid. day^2)	② Estimated Acceleration Model Bias* (10^{-5} rad/sid. day^2)	①+② Bias Adjusted Acceleration, $\ddot{\lambda}$ (adj.) (10^{-5} rad/sid. day^2)	③ Weight of Sample (If $\sigma \leq 0.05 \times 10^{-5}$ rad/sid. day^2, wt. = 1.0)	④ Weight of Sample in Arc 5 (If $\sigma \leq 0.616 \times 10^{-5}$ rad/sid. day^2, wt. = 1.0)	Number of Orbits Considered in Acceleration Determination	Arc Time (span)	Arc Time (months)
(SYNCOM 2) 1	-2.253	- 55.22	6.611113	33.024	0.0325	+ .015	-2.238	1.000		19	AUG-DEC 1963	3
(SYNCOM 2) 2	-2.291	- 60.94	6.611618	32.825	0.0572	- .029	-2.320	.765		16	DEC-MAR 1963/64	3 1/2
(SYNCOM 2) ***3	-0.897	- 88.00	6.62675	32.67	0.888			.000		6	MAR-APR 1964	1
(SYNCOM 2) 4	2.138	-140.00	6.620443	32.584	0.0842	+ .015	2.153	.353		10	APR-JULY 1964	2
(SYNCOM 2) 5A	-0.199	161.00	6.616521	0.397	0.0661	- .011	- .210	.570		17	JULY-NOV 1964	4 1/2
(SYNCOM 2) 5	-2.295	134.00	6.617	0.33	0.0397	+ .011	-2.284	1.000		26	JULY-FEB 1964/65	7 1/2
(SYNCOM 2) 5B	-2.389	106.00	6.617425	0.16	0.0724	- .022	-2.411	.479		10	NOV-FEB 1964/65	3 1/2
(SYNCOM 2) 5-1	1.066	175.5	6.6165	0.47	0.131	- .045	1.021	****	0.252	10	JULY-SEPT 1964	2
(SYNCOM 2) 5-2	0.954	172.5	6.6165	0.45	0.0887	- .029	0.925		0.550	10	JULY-SEPT 1964	2
(SYNCOM 2) 5-3	0.672	169.0	6.6164	0.43	0.0926	+ .006	0.678		0.501	10	JULY-SEPT 1964	2
(SYNCOM 2) 5-4	0.394	166.0	6.6164	0.40	0.0819	- .021	0.373		0.645	10	JULY-SEPT 1964	2
(SYNCOM 2) 5-5	0.016	163.0	6.6164	0.39	0.0822	- .006	0.010		0.640	10	AUG-OCT 1964	2
(SYNCOM 2) 5-6	-0.157	160.0	6.6164	0.38	0.129	- .002	-0.159		0.260	10	AUG-OCT 1964	2
(SYNCOM 2) 5-7	-0.541	157.0	6.6164	0.37	0.167	+ .019	-0.522		0.155	10	AUG-OCT 1964	2
(SYNCOM 2) 5-8	-0.788	153.5	6.6164	0.35	0.163	- .003	-0.791		0.162	10	AUG-OCT 1964	2
(SYNCOM 2) 5-9	-1.103	150.0	6.6165	0.34	0.160	+ .036	-1.067		0.169	10	SEPT-NOV 1964	2
(SYNCOM 2) 5-10	-1.560	146.5	6.6166	0.32	0.185	+ .034	-1.526		0.126	10	SEPT-NOV 1964	2
(SYNCOM 2) 5-11	-1.990	143.5	6.6167	0.31	0.181	+ .030	-1.960		0.132	10	SEPT-NOV 1964	2
(SYNCOM 2) 5-12	-2.138	139.5	6.6167	0.30	0.213	+ .018	-2.120		0.095	10	SEPT-NOV 1964	2
(SYNCOM 2) 5-13	-2.501	132.0	6.6169	0.22	0.167	- .033	-2.534		0.154	10	OCT-JAN 1964/65	3 1/2
(SYNCOM 2) 5-14	-2.828	120.0	6.6171	0.22	0.122	+ .037	-2.791		0.290	10	OCT-JAN 1964/65	3 1/2
(SYNCOM 2) 5-15	-2.663	116.0	6.6172	0.20	0.102	+ .025	-2.638		0.415	10	OCT-JAN 1964/65	3 1/2
(SYNCOM 2) 5-16	-2.584	112.0	6.6173	0.18	0.0907	- .005	-2.589		0.525	10	OCT-JAN 1964/65	3 1/2
(SYNCOM 2) 5-17	-2.474	109.0	6.6174	0.16	0.0847	- .033	-2.507		0.600	10	NOV-FEB 1964/65	3 1/2
(SYNCOM 2) 5-18	-2.278	104.5	6.6176	0.15	0.0656	- .041	-2.319		1.000	10	NOV-FEB 1964/65	3 1/2
(SYNCOM 3) 6	1.707	178.707	6.611474	0.113	0.0591	- .024	1.683	0.715		10	NOV-DEC 1964	2
(SYNCOM 3) 7	1.550	176.801	6.612269	0.268	0.175	+ .027	1.577	0.082		9	JAN-MAR 1965	2
(SYNCOM 2) 8	0.950	66.115	6.611199	31.869	0.0616	+ .018	0.968	0.660		11	FEB-MAY 1965	2 1/2
(EARLY BIRD) 9	-1.441	- 28.703	6.6105	0.200	0.010	- .031	-1.472	1.000		46	APR-JUNE 1965	2
**5'	-2.528	129.00	6.617	32.33	0.11					26	JULY-FEB 1964/65	7 1/2

*From a compromise of the biases reported in Table 11.

**Independent full arc 5 acceleration from drift rate data reduced by the J_{22} model, Equation 8, determined from successive equator crossings in each orbit only (velocity data).

***Not used in the gravity synthesis.

****In weighted analyses interdependent arcs 5-1→5-18 were given total wt. = 1.0 (replacing arcs 5A & 5B), distributed according to column 4 weights. In "unweighted" analyses, arcs 5-1→5-18 were given total weight = 2.0 without predjudice, all other independent arcs used being given wt. = 1.0.

Table 11

Longitude Accelerations in Simulated 24-Hour Satellite Arcs 1S/1 to 9S/1*.

Arc	① Longitude Acceleration, $\ddot{\lambda}$ 10^{-5} rad/sid. day^2)	Longitude, λ (degrees)	Semimajor Axis, a_s (earth radii)	Orbit Inclination, i_s (degrees)	Standard Error of Longitude Acceleration, σt (10^{-5} rad/sid. day^2)	② Theoretical $\ddot{\lambda}$ for Given a_s, i_s, and λ** (10^{-5} rad/sid. day^2)	Column ① minus Column ②, Longitude Acceleration Model Bias (10^{-5} rad/sid. day^2)	Bias from a Second Order Earth Longitude Gravity Trajectory†† (10^{-5} rad/sid. day^2)
1S/1	-2.2328	- 55.13	6.6111	33.03	.0048	-2.2185	+.0143	+.018
2S/1	-2.2024	- 60.91	6.6116	32.84	.0056	-2.2330	-.0306	-.027
4S/1	2.156	-140.00	6.6204	32.57	.0413	2.163	+.007	+.025
6S/1	1.686	178.69	6.6115	0.09	.0209	1.6615	-.0245	-.023
7S/1	1.473	176.915	6.6123	0.11	.0129	1.501	+.028	+.026
8S/1	0.862	66.255	6.61115	31.87	.0219	.884	+.022	+.014
9S/1	-1.349	-28.69	6.6105	0.20	.0064	-1.380	-.031	-.032
5S/1-1	1.1915	175.5	6.6164	32.47	.0305	1.146	-.0455	
5S/1-2	.927	172.5	6.6164	32.46	.0304	.898	-.029	
5S/1-3	.5885	169.0	6.6164	32.44	.0230	.594	+.0055	
5S/1-4	.341	166.0	6.6164	32.42	.0232	.321	-.021	
5S/1-5	.049	163.0	6.6164	32.40	.0266	.043	-.006	
5S/1-6	- .237	160.0	6.6164	32.39	.0274	- .239	-.002	
5S/1-7	- .540	157.0	6.6164	32.38	.0243	- .521	+.019	
5S/1-8	- .842	153.5	6.6164	32.36	.0248	- .845	-.003	
5S/1-9	-1.194	150.0	6.6165	32.35	.0361	-1.1585	+.0355	
5S/1-10	-1.449	147.0	6.6165	32.34	.0427	-1.415	+.034	
5S/1-11	-1.726	143.5	6.6166	32.32	.0420	-1.6955	+.0305	
5S/1-12	-2.003	139.5	6.6167	32.31	.0392	-1.985	+.018	
5S/1-13	-2.3835	132.0	6.6171	32.26	.0310	-2.416	-.0325	
5S/1-14	-2.768	120.0	6.6172	32.25	.0266	-2.731	+.037	
5S/1-15	-2.752	116.5	6.6173	32.23	.0272	-2.727	+.025	
5S/1-16	-2.662	112.5	6.6174	32.22	.0380	-2.667	-.005	
5S/1-17	-2.535	109.0	6.6175	32.19	.0404	-2.568	-.033	
5S/1-18	-2.363	105.0	6.6176	32.16	.0350	-2.404	-.041	
***5S-A					.0132			-.0107
***5S					.0100			+.0107
***5S-B					.0240			-.022

*Trajectories computed numerically by ITEM in the presence of sun and moon gravity through third order earth longitude gravity field given by:

$J_{22} = -1.8 \times 10^{-6}$, $\lambda_{22} = -15.35^\circ$

$J_{33} = -0.16 \times 10^{-6}$, $\lambda_{33} = 24^\circ$

$J_{31} = -1.5 \times 10^{-6}$, $\lambda_{31} = 0^\circ$.

Accelerations derived by a second order gravity drift model (see Tables 1S/1 through 9S/1)

**Computed from Equation 2 with the gravity and orbit constants above.

***Data from Table 5S; simulated trajectory with sun and moon gravity and second order earth longitude gravity

††See Tables 1S–9S.

†Data from arcs 1S–9S except for 5S/1-1 through 5S/1-18

Table 12

Tests of 24-Hour Satellite Accelerations for Sensitivity to Resonant Earth Gravity Harmonics.*

(All values in units of 10^{-6} except as noted)

Arcs	Test No.	C_{22}	$S(C_{22})$	S_{22}	$S(S_{22})$	C_{33}	$S(C_{33})$	S_{33}	$S(S_{33})$	C_{31}	$S(C_{31})$	S_{31}	$S(S_{31})$	C_{44}	$S(C_{44})$	S_{44}	$S(S_{44})$	Standard Error of Test (10^{-7} rad/sid. day^2)
1,2,4,5-1–5-18, 6,7,8,9	1	-1.526	.019	.951	.022													20.96
	2	-1.564	.004	.927	.004	-.045	.004	-.160	.003									3.96
	3	-1.545	.004	.932	.004	-.024	.005	-.159	.003	2.08	.38	.21	.18					3.28
	4	-1.536	.019	.943	.022													21.18
	5	-1.573	.0042	.919	.0043	-.050	.0046	-.160	.0035									4.09
	6	-1.556	.0046	.924	.0048	-.031	.0054	-.159	.0034	2.02	.41	.14	.20					3.52
1,2,4,5A, 5B,6,7, 8,9	7	-1.52/-1.50/-1.52	.07/.08/.08	.99/.96/1.02	.07/.07/.08													21.6/23.2/24.2
	8	-1.57/-1.54/-1.57	.02/.03/.04	.92/.88/.94	.02/.03/.04	-.02/-.06/-.02	.02/.03/.04	-.16/-.15/-.17	.01/.03/.03									4.8/9.1/11.4
	9	-1.56/-1.54/-1.54	.01/.04/.04	.90/.92/.90	.02/.05/.05	-.01/-.04/-.00	.01/.04/.05	-.15/-.16/-.14	.01/.03/.04	-.23/4.1/-.55	1.2/3.5/4.0	1.4/-1.7/2.6	.6/1.7/2.0					3.3/9.5/11.0
1,2,4,5A, 5',5B,6, 7,8,9	10	-1.537	.067	1.004	.069													22.2
	11	-1.557	.012	.920	.014	-.039	.014	-.161	.011									4.04
	12	-1.549	.011	.917	.016	-.021	.015	-.159	.010	1.08	1.19	.55	.58					3.37
1,2,4,5-1–5-18, 6,7,8,9	13	-1.525	.021	1.005	.021													21.17
	14	-1.565	.0041	.930	.0043	-.047	.0046	-.164	.0035									4.11
	15	-1.554	.0047	.935	.0055	-.029	.0061	-.164	.0036	1.72	.44	.02	.21					3.79
1S/1, 2S/1, 4S/1, 5S/1-1–5S/1-18,6S/1, 7S/1, 8S/1, 9S/1	16	-1.494	.018	.993	.018													18.7
	17	-1.529	.0029	.926	.0030	-.026	.0032	-.150	.0025									2.90
	18	-1.542	.0027	.922	.0032	-.048	.0035	-.151	.0021	-1.95	.26	-.06	.12					2.19
		(-1.548)	(J_{22} = -1.8 x 10^{-6})	(.919)	(λ_{22} = -15.35°)	(-.0494)	(J_{33} = -.16 x 10^{-6})	-.152	(λ_{33} = 24°)	-1.500	(J_{31} = -1.5 x 10^{-6})	0.	(λ_{31} = 0°)					
	19	-1.386	.012	1.096	.012													12.23
	20	-1.409	.0026	1.053	.0028	-.0131	.0030	-.0975	.0023									2.68
	21	-1.421	.0007	1.036	.0008	-.0355	.0009	-.0936	.0005	-2.991	.062	.545	.029					.53
		(-1.425)	(J_{22} = -1.77 x 10^{-6})	1.050	(λ_{22} = -18.2°)	-.0371	(J_{33} = -.105 x 10^{-6})	-.0982	(λ_{33} = 23.1°)	-2.111	(J_{31} = 2.12 x 10^{-6})	.200	(λ_{31} = -5.4°)					
	22	-1.483	.017	.846	.017													17.89
	23	-1.517	.0023	.783	.0024	-.0601	.0026	-.135	.0019									2.30
	24	-1.533	.0007	.787	.0008	-.0849	.0009	-.140	.0005	-1.63	.06	-.52	.03					.55
		(-1.535)	(J_{22} = -1.72 x 10^{-6})	.776	(λ_{22} = -13.4°)	-.0920	(J_{33} = -.165 x 10^{-6})	-.137	(λ_{33} = 18.7°)	-1.996	(J_{31} = -2.01 x 10^{-6})	-.235	(λ_{31} = 6.7°)					
	25	-1.516	.019	.991	.019													19.8
	26	-1.554	.0017	.921	.0018	-.057	.0019	-.153	.0015									1.74
	27	-1.545	.00068	.930	.00080	-.042	.00088	-.155	.00053	1.88	.06	-.28	.03					.55
		(-1.548)	(J_{22} = -1.8 x 10^{-6})	(.919)	(λ_{22} = -15.35°)	(-.049)	(J_{33} = -.16 x 10^{-6})	(-.152)	(λ_{33} = 24°)	(1.50)	(J_{31} = -1.5 x 10^{-6})	(0.)	(λ_{31} = 180°)					
1,2,4,5', 5-1–5-18,6,7, 8,9	28	-1.549	.0032	.916	.0045	-.044	.0038	-.153	.0028					.0035	.0025	.018	.003	3.14
	29	-1.552	.0046	.964	.023	-.025	.0052	-.169	.0075	3.37	1.19	-1.03	.65	-.016	.010	.0058	.0046	2.83
1,2,4,5-1–5-18, 6,7,8,9	30	-1.555	.006	.954	.023	-.020	.006	-.168	.007	2.94	1.20	-1.14	.67	-.02	.01	.015	.005	2.98

Test No.	Comments
1	Reduced from unadjusted data: weighted according to σ of measured accelerations: relative arc 5 wt. = 1.0: (see Table 10 and text): total samples = 100
2	Reduced from unadjusted data: weighted according to σ of measured accelerations: relative arc 5 wt. = 1.0: (see Table 10 and text): total samples = 100
3	Reduced from unadjusted data: weighted according to σ of measured accelerations: relative arc 5 wt. = 1.0: (see Table 10 and text): total samples = 100
4	Reduced from bias adjusted data: weighted according to σ of measured accelerations: relative arc 5 wt. = 1.0: (see Table 10 and text): total samples = 100
5	Reduced from bias adjusted data: weighted according to σ of measured accelerations: relative arc 5 wt. = 1.0: (see Table 10 and text): total samples = 100
6	Reduced from bias adjusted data: weighted according to σ of measured accelerations: relative arc 5 wt. = 1.0: (see Table 10 and text): total samples = 100
7	(A/B/C) reduced from unadjusted data: 3 random choices (A/B/C) from normal distributions specified by $\ddot{\lambda}$ and σ in Table 10 (9 data for each test): total samples = 9
8	(A/B/C) reduced from unadjusted data: 3 random choices (A/B/C) from normal distributions specified by $\ddot{\lambda}$ and σ in Table 10 (same data as test 7): total samples = 9
9	(A/B/C) reduced from unadjusted data: 3 random choices (A/B/C) from normal distributions specified by $\ddot{\lambda}$ and σ in Table 10 (same data as test 7): total samples = 9
10	Reduced from unadjusted data: uses Table 10 values (unweighted), includes independent arc 5' measurement: total samples = 10
11	Reduced from unadjusted data: uses Table 10 values (unweighted), includes independent arc 5' measurement: total samples = 10
12	Reduced from unadjusted data: uses Table 10 values (unweighted), includes independent arc 5' measurement: total samples = 10
13	Reduced from unadjusted data: uses Table 10 values, unweighted, without arc 5-11: total samples = 100
14	Reduced from unadjusted data: uses Table 10 values, unweighted, without arc 5-11: total samples = 100
15	Reduced from unadjusted data: uses Table 10 values, unweighted, without arc 5-11: total samples = 100
16	Reduced from simulated data in Table 11 (geoid: Wagner-Kaula combined 1964/65 through 3rd order), includes sun and moon effects; unweighted, without arc S5/1-11: total samples = 100
17	Reduced from simulated data in Table 11 (geoid: Wagner-Kaula combined 1964/65 through 3rd order), includes sun and moon effects; unweighted, without arc S5/1-11: total samples = 100
18	Reduced from simulated data in Table 11 (geoid: Wagner-Kaula combined 1964/65 through 3rd order), includes sun and moon effects; unweighted, without arc S5/1-11: total samples = 100
	(Actual geoid harmonics: Wagner-Kaula combined (1964/65))
19	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Kaula (1964); unweighted, without arc S5/1-11: total samples = 100
20	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Kaula (1964); unweighted, without arc S5/1-11: total samples = 100
21	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Kaula (1964); unweighted, without arc S5/1-11: total samples = 100
	(Actual geoid harmonics: Kaula (1964), including: $J_{42} = -0.117 \times 10^{-6}$, $\lambda_{42} = 42.3°$, $J_{44} = -0.0104 \times 10^{-6}$, $\lambda_{44} = 14.5°$)
22	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Guier (1965); unweighted, without arc S5/1-11: total samples = 100
23	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Guier (1965); unweighted, without arc S5/1-11: total samples = 100
24	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Guier (1965); unweighted, without arc S5/1-11: total samples = 100
	(Actual geoid harmonics: Guier (1965), including: $J_{42} = -0.193 \times 10^{-6}$, $\lambda_{42} = 23.4°$, $J_{44} = -0.006 \times 10^{-6}$, $\lambda_{44} = 34.5°$)
25	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Wagner-Guier (1965); unweighted, without arc S5/1-11: total samples = 100
26	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Wagner-Guier (1965); unweighted, without arc S5/1-11: total samples = 100
27	Reduced from simulated data calculated according to Equation 2 from harmonic coefficients of geoid of Wagner-Guier (1965); unweighted, without arc S5/1-11: total samples = 100
	(Actual geoid harmonics: Wagner-Guier (1965), including: $J_{42} = -0.19 \times 10^{-6}$, $\lambda_{42} = 42.3°$, $J_{44} = -0.006 \times 10^{-6}$, $\lambda_{44} = 34.5°$)
28	Reduced from unadjusted data in Table 10, unweighted, includes independent arc 5' measurement; without arcs 5-1, 5-10, 5-11: total samples = 111
29	Reduced from unadjusted data in Table 10, unweighted, includes independent arc 5' measurement; without arcs 5-1, 5-10, 5-11: total samples = 111
30	Reduced from unadjusted data in Table 10, weighted according to σ of measured accelerations: relative arc 5 wt. = 1.0 (see Table 10): total samples = 100

*All data reductions by "least squares" fit from condition equations calculated from Equation 2 according to λ, a_s, i_s in the given satellite arcs of Tables 10 or 11.

The results of test 3 (see also tests 28-30) are strongly suggestive but not conclusive as to the sensitivity of 24-hour satellite drift thus far to other than second and third order sectorial ($n = m$) earth gravity. Test 3 (and almost all the other H_{22}, H_{33}, and H_{31} tests in Table 12) does produce small changes in the H_{22} and H_{33} coefficients and a slight reduction in the test standard error, as well as a fairly strong C_{31} coefficient with a standard error of about 20%. These signs point to some significance in the H_{31} results of test 3. On the other hand, since the longitude and sample survey is limited in these tests, the small changes in the H_{22} and H_{33} coefficients might be the result of random "observation" or model bias error in the data. The slight decrease in the test standard error may be equally fortuitous. The increase in the standard errors of the H_{22} and H_{33} coefficients is the strongest indication that the H_{31} harmonic is not yet clearly sensed by the 24-hour data so far. In the future, with more data of similar quality, we would expect the standard test error to remain in the vicinity of $(3\text{-}5)\times10^{-7}$ rad/sid. day^2 upon the inclusion of third and fourth order terms. At the same time, we would expect the standard errors of the lower order coefficients to decline consistent with coefficient convergence as the dominating higher order terms are brought into the synthesis one by one or in combinations.

We now want to determine as closely as possible the effects on these acceleration data reductions which can be attributed to sun and moon gravity as well as other model bias errors inherent in the method of acceleration analysis. First, we rerun tests 1-3, adjusting the acceleration samples by the probable model biases determined from the simulated trajectories in the previous section (see Table 10). The results of these data adjusted tests (Table 12, 4-6) show minor and insignificant changes from the unadjusted results. In fact, the unadjusted accelerations appear to be even closer, on the average, to true resonant gravity accelerations, judging by the smaller standard errors throughout the unadjusted data tests. Evidently, the model errors in the actual data have acted to cancel the "observation" errors (in the orbit determinations) more often than not. As a result, the conclusions of this study have been drawn primarily from the unadjusted data.

Before proceeding with independent tests of the simulated data, we would like to make a few more tests of the actual data to better judge the true latitude in the harmonics which is allowed by the measurements. In these tests (7-12 in Table 12), we use the unweighted, unadjusted accelerations in Table 10 and replace the interdependent 18 sub-arcs of arc 5 by the independent measurements for arcs 5A and 5B. In tests 10-12 the independently determined arc 5′ measurement is also included. Tests 7-9 each used three random samplings (A/B/C) of a normal distribution given by the mean $\ddot{\lambda}$ and σ values for these accelerations in Table 10. On the assumption that there is no bias in the accelerations and all the measured σ's arise from random observation error, tests 7-9 give an example of the widest latitude permitted by the measurements. From the results of tests 4-6, there is no reason to expect randomly chosen bias adjusted data to yield significantly more divergent results. Indeed, since the sun, moon, and insufficient model introduce pseudo-random "noise" (of up to $\sigma/2$) into the accelerations as well as biases (see Table 11), the "noise" level attributable to the observations alone should be somewhat less than the σ's reported in Table 10. Thus, random tests 7-9 should be reasonably conservative as to the divergent results permitted by the data. In view of the apparent oddness of λ_{31} that is implicit in the "mean" accelerations of Table 10 (see Conclusions), it is interesting and perhaps significant that two of three random samplings in these tests (A and C) yield best λ_{31}'s in the neighborhood of -80°. This is about 90° from the best λ_{31} determined from the "mean" accelerations. Test 9A, in fact, shows a standard test error as low as similar tests with the "mean" accelerations. The relatively great range of H_{31} harmonics revealed in test 9 further emphasizes the tentative nature of the best reported values for these quantities. The random tests resulted in a far smaller divergence of H_{22} and H_{33} harmonics which increase our confidence in the best reported values for them (see Conclusions). Tests 10-12 with unweighted data are consistent with the previous actual weighted data tests which used 18 sub-arcs of arc 5.

Finally, a series of unweighted data tests was made using the dense longitude coverage of arc 5 provided by sub-arcs 5-1 to 5-18. In these (tests 13-15), arc 5 was counted with relative weight = 2.0 since the acceleration σ's of independent arcs 5A and 5B were each near the average σ for the independent arcs in Table 10. Tests 13-15 ignore the influence of arc 5-11 because of its excessive residual (4σ) when it is included in the tests for the harmonics to and through third order. This series of tests, relatively unprejudiced by arbitrary weighting and giving complete coverage to arc 5, was chosen as the basis for drawing final conclusions in this gravity experiment.

We have already computed the effects on the gravity synthesis of previously determined acceleration model biases in the various arcs (compare tests 1-3 with tests 4-6). There is another way to present this result which reveals the likely model errors in both stages of this experiment directly in terms of the harmonic coefficients and the test standard errors. We merely repeat precisely the same gravity synthesis on the simulated parallel acceleration data in Table 11 as we did in the preceding tests on the actual data in Table 10. However, the simulated 24-hour trajectories summarized in Table 11 contained no effects from resonant earth gravity of higher than third order. To gage the likely effects of higher order gravity on the second and third order gravity syntheses, or rather to obtain a wider range of likely second stage model error in the actual data, we have also simulated satellite drift over four geoids taken from recent studies. In these simulations, no full gravity trajectories were calculated. Instead, simple drift accelerations from Equation 2 were computed for the satellite arcs in Table 11 according to the gravity constants of the four geoids. These accelerations were then combined in the same way as the actual data (reversing the solution of Equation 11 with selected coefficients ignored) to yield gravity fields whose bias is readily apparent.

The results of these parallel gravity syntheses on the simulated data are found in Table 12, tests 16-27. Two of the geoids chosen for these simulations come from single comprehensive studies of satellite perturbations. The geoid of Kaula (1964) (see Table B1) is derived from camera observations of five to ten medium altitude, medium and high inclination satellites. The geoid of Guier (1965) was determined from comprehensive, worldwide Doppler radar tracking of five medium altitude satellites (see Table B1). These two geoids are considered representative of the best available geodetic results from independent satellite tracking to date. The geoid labeled Wagner-Kaula combined 1964/65 (used in the ITEM trajectory computations) used H_{22} and H_{33} harmonic coefficients derived at an earlier stage of the present study. The H_{31} coefficients are from an "average" geoid due to W. M. Kaula (private communication) recommended as a "first guess" in geodetic studies when there is no better evidence of different values. The geoid labeled Wagner-Guier (1965) uses the H_{22}, H_{33} values derived earlier in this study, H_{31} values near those finally chosen as best representing 24-hour satellite drift to date, and H_{42}, H_{44} values from the geoid of Guier (1965). This latter geoid was thought to be the most accurate known at the time this study was initiated (spring, 1965). These tests illustrate the kind of convergence to true harmonic values, which should be evident and apparently is, in the actual data reductions for C_{22} and S_{22}, when higher order gravity is introduced without constraint into the tests. Test 18 shows clearly that sun, moon, and first stage model bias has almost negligibly small effect on these harmonic reductions. This is also evident from the low values of the acceleration biases in Table 10.

The harmonic biases (theoretical-measured values) shown by these simulated data tests are calculated in Table 13. Except for the effects on H_{31}, they are all reasonably consistent. This illustrates, more than anything else, the agreement in the geoids themselves. As more harmonics are permitted, the sharp reduction in the H_{22} and H_{33} biases shows that, observation errors excluded, these harmonics should be essentially determined from the arcs in the data test through the third order. In Table 14 the model biases in the harmonics determined from the simulated arcs in Table 13 are added to the harmonics derived from the actual data (Table 12, tests 13-15) to arrive at a reasonable range of bias free harmonics at all stages in the reductions.

It appears noteworthy that all H_{22} and H_{33} harmonics except C_{33} are relatively unchanged through the reductions when they are corrected at each stage through third order by the average biases. This result, which could not be anticipated in advance (for two of the four geoid simulations), gives added assurance in both the overall quality of the basic data and the H_{22}, H_{33} gravity field implicit in that drift data after only a third order reduction. As a further test of the conjecture that H_{22} and H_{33} are essentially determined by only a third order reduction of this wide coverage 24-hour data, we allow a fourth order harmonic into the reductions (tests 28 and 29, Table 12). Appendix C makes it fairly clear that H_{44} is the next strongest harmonic after H_{31} in its influence on the 24-hour satellites in this study.

It is encouraging to find that these tests show essentially the same H_{22} and H_{33} results as previously in the all third order reductions (i.e., tests 13-15). It is interesting that $J_{44} \simeq -0.02\times10^{-6}$ in these tests since this checks reasonably well with recent results for this harmonic determined from lower altitude data (see Table B1). The fact that both J_{44} and λ_{44} determined from tests 28 and 29 are almost identical with and without the inclusion of H_{31} seems to be coincidental. The C_{31} from test 29 seems unrealistically high from the results of this and other recent geodetic investigations from satellite motions (see Table B1). As a further confirmation of this, in test 30 we repeat the weighted data test 3 through third order with the inclusion of H_{44}. While all the other harmonics are virtually unchanged from test 29, the S_{44} harmonic has increased significantly. With only 9 or 10 well determined 24-hour satellite accelerations at this point in the analysis, it should not be surprising that only four resonant gravity coefficients appear well determined.

The best estimate of these coefficients is presented at the bottom of Table 14, including a very preliminary estimate of H_{31}, as far as can be judged from the many gravity tests in this section. In general, mean values and standard errors were chosen together to encompass as widely as possible the results of all the actual data tests in Table 12 and bias adjusted reductions in Table 14. The single reduction most heavily relied upon in this judgement was the average bias-added gravity synthesis (in Table 14) from unweighted basic acceleration data with dense arc 5 coverage. It appears that a more balanced estimate of S_{33} should be $S_{33} = -(0.16 \pm 0.01)\times 10^{-6}$, since not one test showed S_{33} to be greater than -0.17×10^{-6}. In this one instance, balance over all the tests was ignored in favor of the bias-adjusted results on the unweighted dense arc 5 coverage data reductions. The reason for this exception is to be found in the remarkably consistent bias-adjusted S_{33}'s in Table 14. This seems to arise in large part from the consistency in S_{33} between H_{22}, H_{33}, and H_{22}, H_{33}, H_{31} reductions when arc 5 is densely covered (compare tests 14 and 15, dense coverage tests with tests 11 and 12 in Table 12). To guard against error in this somewhat special judgement of the test results, we have allowed for a 0.015×10^{-6} standard error in S_{33} which is probably higher than is strictly seen in this experiment.

The judgement of the H_{31} harmonics in Table 14 also calls for some comment. Again, a balanced view over all the tests was the criterion of choice. The results of the random sampling tests 7A, 7B, 7C (in Table 12) were strongly relied upon in the H_{31} estimates. This was in spite of the fact that arc 5 was sparsely covered in these tests and there were only nine samples to test for six coefficients (independent arc 5′ not being used, for example). In particular, test 7A shows that viewed probablistically, the basic data allows low overall acceleration residuals and H_{22}, H_{33} values reasonably consistent with other "best values" tests and, in addition, H_{31} values considerably divergent from the "best value" results. The standard errors of the H_{31} harmonics were, in fact, taken from test 7A. The best estimates of H_{31} were considered to be the last two bias adjusted values in Table 14. Together with the estimated standard errors, a fair measure of the range of H_{31} seen in these tests is covered. The standard error in J_{31} was not estimated from $S(C_{31})$ and $S(S_{31})$ through Equation 15, assuming uncorrelated coefficients (Reference 5, Appendix E), since C_{31} and S_{31} appear to be significantly correlated in these tests. Clearly J_{31}, seen in the experiment, is bounded between about -10^{-6} and -3×10^{-6}. The estimated J_{31} deviation in Table 14 reflects this result.

Finally, in Figure 9, we display the measured (unadjusted) 24-hour satellite accelerations (in solid) in this study (from table 10) and match them against the accelerations (in dots) from the geoid (at the bottom of Table 14) which the measured accelerations have determined in large part. The standard errors in the matching geoid accelerations (about 0.03×10^{-5} rad/sid. day^2 for Syncom 2 arcs) were calculated from somewhat smaller H_{nm} deviations than those finally chosen in Table 14. Actual geoid accelerations may differ from the best values calculated through Equation 2 with the best estimated coefficients in Table 14 by about 0.08×10^{-5} rad/sid. day^2 for equatorial 24-hour satellites. This number reflects only the long term acceleration uncertainty remaining in our knowledge of the effect of the earth's field on the distant synchronous satellite. But we know also from this study that, beyond two months of drift, long term sun and moon gravity effects may continue to be responsible for as much as $\pm 0.03\times 10^{-5}$ rad/sid. day^2 in the drift acceleration of the 24-hour satellite.

The solid curve in Figure 9 represents the best estimated geoid accelerations on the geostationary satellite around the equator, as determined from this study.

Table 13

Model Biases in Gravity Harmonics Reduced from Simulated 24-Hour Satellite Drifts with Four Recent Geoids*.
(all bias values in units of 10^{-6})

Reductions for Harmonics H_{nm} (C_{nm}, S_{nm})		Bias in Reduced C_{22}	RMS Bias Dev. from Average Bias $\pm S(C_{22})$	Bias S_{22}	RMS Bias Dev. $\pm S(S_{22})$	Bias C_{33}	RMS Bias Dev. $\pm S(C_{33})$	Bias S_{33}	RMS Bias Dev. $\pm S(S_{33})$	Bias C_{31}	RMS Bias Dev. $\pm S(C_{31})$	Bias S_{31}	RMS Bias Dev. $\pm S(S_{31})$
H_{22} only	(1)	-.054		-.074									
	(2)	-.039		-.046									
	(3)	-.052		-.070									
	(4)	-.032		-.072									
(Average)		(-.044)	(.011)	(-.065)	(.013)								
H_{22} and H_{33}	(5)	-.019		-.007		-.023		-.002					
	(6)	-.016		-.003		-.024		-.000					
	(7)	-.018		-.007		-.032		-.002					
	(8)	+.006		-.002		+.008		+.001					
(Average)		(-.012)	(.012)	(-.005)	(.003)	(-.018)	(.018)	(-.001)	(.002)				
H_{22}, H_{33} and H_{31}	(9)	-.006		-.003		-.001		-.001		+.45		+.06	
	(10)	-.004		+.014		-.001		-.004		+.88		-.35	
	(11)	-.002		-.011		-.007		+.003		-.37		+.28	
	(12)	-.003		-.012		-.007		+.003		-.33		+.28	
(Average)		(-.004)	(.002)	(-.003)	(.009)	(-.004)	(.003)	(.000)	(.003)	(+.16)	(.61)	(+.07)	(.30)

*Data from least squares reductions of simulations over arcs 1, 2, 4, 5-1 to 5-18, 6, 7, 8 and 9: see Table 12 and text.

GEOIDS USED

(1) Wagner-Kaula combined 1964/65 (3^{rd} order) plus sun, moon, and 1^{st} stage experiment bias effects: see test 16, Table 12
(2) Kaula 1964: see test 13, Table 12.
(3) Guier 1965: see test 22, Table 12
(4) Wagner-Guier 1965: see test 25, Table 12
(5) Wagner-Kaula comb. (1964/65) plus sun and moon
(6) Kaula 1964
(7) Guier 1965
(8) Wagner-Guier 1965
(9) Wagner-Kaula comb. (1964/65) plus sun and moon
(10) Kaula 1964
(11) Guier 1965
(12) Wagner-Guier 1965

Table 14

Gravity Synthesis, from Actual 24-Hour Satellite Data and Likely Model Bias Errors.
(all harmonic values in units of 10^{-6} except as noted)

Reductions for Harmonics H_{nm} (C_{nm}, S_{nm})		C_{22}	$\pm S(C_{22})$	S_{22}	$\pm S(S_{22})$	C_{33}	$\pm S(C_{33})$	S_{33}	$\pm S(S_{33})$	C_{31}	$\pm S(C_{31})$	S_{31}	$\pm S(S_{31})$
H_{22} only	(1)	-1.525		1.005									
H_{22} and H_{33}	(2)	-1.565		.930		-.047		-.164					
H_{22}, H_{33} and H_{31}	(3)	-1.554	.005	.935	.006	-.029	.006	-.164	.004	1.72	.4	.02	.2
H_{22} only	(4)	-1.569		.940									
H_{22} and H_{33}	(4)	-1.577		.925		-.065		-.165					
H_{22}, H_{33} and H_{31}	(4)	-1.558		.932		-.033		-.164		1.88		.09	
(Average)	(4)	(-1.568)	(.012)*	(.932)	(.013)*	(-.049)	(.018)*	(-.165)	(.003)*	(1.88)	(.6)*	(.09)	(.3)*
H_{22} and H_{33}	(5)	-1.584		.932		-.070		-.166					
H_{22}, H_{33} and H_{31}	(6)	-1.560		.932		-.030		-.165		2.17		.08	
H_{22} and H_{33}	(7)	-1.581		.927		-.071		-.164					
H_{22}, H_{33} and H_{31}	(8)	-1.558		.949		-.030		-.168		2.60		-.33	
H_{22} and H_{33}	(9)	-1.583		.923		-.079		-.166					
H_{22}, H_{33} and H_{31}	(10)	-1.556		.924		-.036		-.161		1.35		.30	
H_{22} and H_{33}	(11)	-1.559		.928		-.039		-.163					
H_{22}, H_{33} and H_{31}	(12)	-1.557		.923		-.036		-.161		1.39		.30	
	(13)	-1.56	.02	.93	.02	-.045	.030	-.165	.015	1.4	1.2	.3	.6
		J_{22}	$S(J_{22})$	λ_{22}	$S(\lambda_{22})$	J_{33}	$S(J_{33})$	λ_{33}	$S(\lambda_{33})$	J_{31}	$S(J_{31})$**	λ_{31}	$S(\lambda_{31})$
	(14)	-1.816	.02	-15.40	.32°	-.171	.017	24.92°	3.3°	-1.4	+0.2 -1.0	-167.9°	25.8°

*Maximum RMS bias dev. from average biases in Table 13.

**Estimated from a range of $(C_{31}^2 + S_{31}^2)^{1/2}$ values in Tables 12, 13 and 14.

$\overline{C_{22}}$ (best) = 2.42 ± .03 × 10^{-6}

$\overline{S_{22}}$ (best) = -1.44 ±.03 × 10^{-6}

$\overline{C_{33}}$ (best) = 0.322 ± .215 × 10^{-6}

$\overline{S_{33}}$ (best) = 1.183 ± .108 × 10^{-6}

GEOIDS USED

(1) From actual data (unweighted): see Table 12, test 13
(2) From actual data (unweighted): see Table 12, test 14
(3) From actual data (unweighted): see Table 12, test 15
(4) From actual data reduction above and average biases (see Table 13)
(5) From actual data and Wagner-Kaula comb. (1964/65) biases (see Table 13)
(6) From actual data and Wagner-Kaula comb. (1964/65) biases (see Table 13)
(7) From actual data reduction above and Kaula (1964) biases (see Table 13)
(8) From actual data reduction above and Kaula (1964) biases (see Table 13)
(9) From actual data reduction above and Guier (1965) biases (see Table 13)
(10) From actual data reduction above and Guier (1965) biases (see Table 13)
(11) From actual data reduction above and Wagner-Guier (1965) biases (see Table 13)
(12) From actual data reduction above and Wagner-Guier (1965) biases (see Table 13)
(13) Estimated best values: C_{nm}, S_{nm} (see text)
(14) Estimated best values: J_{nm}, λ_{nm}: from C_{nm}, S_{nm} values above through Equations 13 and 14

3. EAST-WEST EQUILIBRIUM LONGITUDES AND MAXIMUM EAST-WEST STATION KEEPING REQUIREMENTS FOR THE GEOSTATIONARY SATELLITE

This study confirms the results of many recent satellite-geoid reductions (see Table B1 and Reference 1) that the dominance of the J_{22} harmonic in the earth's field establishes only four narrow longitude zones at which a geostationary satellite may be placed and kept for long periods of time without the necessity of east-west station keeping. As a study of Figure 9 shows, initially geostationary satellites placed near geoid acceleration zeros over the Indian Ocean and the eastern Pacific will be forever trapped in the field under the influence only of the small longitude dependent components of the earth's gravity potential. Similar satellites placed near the east-west acceleration zeros over the Atlantic and western Pacific will tend to drift away from their initial positions, but initially only very slowly, at least due to perturbations of the earth's field (see also Reference 1 and 8). The present study of the drift of three 24-hour satellites over a period of two years indicates these east-west equilibrium points in the earth's field at synchronous altitudes are at:

$$\begin{aligned}
\lambda_1 &= 76.7 \pm 0.8^\circ \text{ (dynamically stable equilibrium longitude)} \\
\lambda_2 &= 161.8 \pm 0.7^\circ \text{ (statically stable equilibrium longitude)} \\
\lambda_3 &= -108.1 \pm 1.0^\circ \text{ (dynamically stable equilibrium longitude)} \\
\lambda_4 &= -12.2 \pm 0.7^\circ \text{ (statically stable equilibrium longitude).}
\end{aligned} \tag{16}$$

These longitudes and their standard errors have been calculated through Equation 2 from the geoid harmonics and their likely deviations at the bottom of Table 14. The derived geoid in Table 14, reflecting the drift record of the 24-hour satellites, is as free as we can make it from model bias. This includes higher order earth effects, as well as sun and moon gravity effects, and the error sources in the data reduction methods. Therefore these equilibrium longitude zones are believed to be absolute measures. Barring a much stronger higher order earth longitude dependent field than appears likely now, the true equilibrium zones should fall within the limits above.

To calculate the maximum east-west station keeping requirements for geostationary satellites, we compare Equation B4 with Equation C1. The longitude perturbing force per unit mass on the circular orbit equatorial 24-hour satellite given in terms of the drift acceleration (in radians/sid. day^2) it produces, is

$$F_\lambda = \frac{\left(-\ddot{\lambda}\ \mu_e/a_s^2\right)}{12\pi^2} = -0.00621\ \ddot{\lambda}\ \text{ft/sec}^2 \ , \tag{17}$$

for $a_s = 6.611$ earth radii, with $\ddot{\lambda}$ in units of rad /sid. day^2. The maximum longitude acceleration which might be experienced by such a satellite is (from Figure 9)

$$\begin{aligned}
\ddot{\lambda}_{max} &= -(3.18 \pm 0.08)\times 10^{-5} \text{ rad/sid. day}^2 \\
&= -(1.83 \pm 0.05)\times 10^{-3} \text{ degrees/day}^2 \ ,
\end{aligned} \tag{18}$$

at $\lambda = 118°$ E (over Indonesia). Using the conservative upper bound of Equation 18 in Equation 17, we calculate the maximum east-west station keeping requirements for the geostationary as (following Reference 3, p. 31):

$$\Delta V_{T \cdot max} = F_{\lambda \cdot max} \times \Delta T\,(1\,yr)$$

$$= 0.00621 \times 3.26 \times 86{,}400\ (sec/day) \times 365\ (days/yr) \times 10^{-5}$$

$$= 6.38\ ft/sec\text{-}yr) \quad . \tag{19}$$

Other longitudes where near maximum east-west station keeping requirements on geostationary satellites would exist, occur roughly halfway between λ_2 and λ_3, λ_3 and λ_4, and λ_4 and λ_1 in Equations 16.

DISCUSSION

Perhaps the best check on the validity of the geodetic results of this study (in Figure 9 and Table 14) is a recent private communication from R. R. Allan to the author (June 1965). Mr. Allan of the Royal Aircraft Establishment in England reports the following gravity harmonics as seen by Syncom 2 drift in arcs 1-5:

$$J_{22} = -1.80 \times 10^{-6}$$

$$\lambda_{22} = -15.0°$$

$$J_{33} = 0.178 \times 10^{-6}$$

$$\lambda_{33} = 24.7°$$

$$J_{44} = -0.017 \times 10^{-6}$$

$$\lambda_{44} = 37.9° \quad .$$

Mr. Allan performed his independent reductions on essentially the same orbit data in arcs 1-5 as found in this report (in Appendix A). Allan apparently used a somewhat different analysis than here. He removed sun and moon perturbations by a semi-analytic technique and solved for all the harmonics directly from the drift in the arcs by an iterative method. His best results for H_{22} and H_{33} are well within the standard deviations reported for the final values in Table 14 except for λ_{22} which does not differ significantly.

The present study could come to no firm conclusion on H_{44} except to estimate broadly that $0.01 < |J_{44} \times 10^6| < 0.03$ (see Conclusions). The best external check on the results, (at least for H_{22} and H_{33}) is found in the recent geodetic reductions in Table B1. For the two dominant sectorials through third order, the results of this study agree most closely individually and as a set with the Doppler - satellite geoids of Guier (1965) and Anderle (1965), and the camera - satellite geoid of Kaula (1964). The only surprising result of this study is the phase angle λ_{31} of the lowest order "mixed" or tesseral harmonic, which was sensed to be almost 180° from the consensus of recent observations (see Table B1). However, as pointed out in Section 2 of this report, the 24-hour data thus far appears to allow for a considerably greater range of λ_{31}'s than settled on in the final geoid values of Table 14. The random sampling tests on the 24-hour data (Table 12) showed that λ_{31} could be less than 90° (west) without seriously affecting the H_{22} and H_{33} harmonics determined from the estimated "best" data. Since the inclusion of H_{44} into the gravity synthesis of that data did not seem to improve the situation with regard to H_{31}, we must wait for more acceleration data, particularly from the equatorial 24-hour satellites, to clear up the mystery of the apparent discrepancy in this harmonic. It should be emphasized again, however, that the reported values of H_{22} and H_{33} as a set, even without the inclusion of H_{31}, give nearly as good a reproduction of the "best measured" accelerations as the complete third order geoid reported. In fact, it is believed that the reported H_{22} and H_{33} values are *individually* absolute within their stated deviations, and are not expected to vary significantly from these ranges when the 24-hour data is complete enough to reveal fourth order gravity as well.

The greater part of the effort in this study has been to obtain long term accelerations with as small likely deviations as possible. As Appendix C shows, not a great deal could be expected of obtaining meaningful results on third or fourth order harmonics from the limited record, unless accuracies of the order of 0.1×10^{-5} rad/sid.day^2 in the longitude drift accelerations were obtained. For the most part this goal was met and exceeded. But for much of the Syncom 2 and 3 record it was not an easy task because the orbit determinations were often of much reduced quality as can be judged from a comparison of the standard test errors among arcs 1, 2, and 8. In the main, use of a single well determined longitude location for each reported orbit gave sufficiently precise accelerations for the Syncom arcs, provided the drift exceeded about two months and more than eight individual orbits were available for the arc. But in many Syncom free drift arcs the longitude data alone was not good enough to obtain sufficiently precise accelerations in spite of the arc length. This was especially true for arcs 3, 5B and 7 (see Table 7). For arc 3, the length was short of two months, but even with the use of two longitudes in each GSFC reported orbit to help determine the drift velocity changes, unacceptable precision of about 1×10^{-5} rad/sid. day^2 was all that could be attained. In arc 5B (and the latter sub-arcs of arc 5) some of the single longitude estimations in November 1964 and January 1965 proved so poor they were discarded altogether as drift velocity indicators and replaced for this purpose by successive longitude estimations for those orbits.

In Reference 6 the use of such drift rate data from a single orbit was discussed. In theory it should provide a semi-independent determination of the accelerations in the Syncom arcs since an orbit is specified by both independent position and velocity information. However, except for isolated regions in the 24-hour record, the velocity (or semimajor axis) measurements were not as "smooth" for the purposes of the analysis as the position (longitude) measurements. In theory this is understandable since the differences of two errored equator positions and crossing times over a small time span gives the velocity from one independent orbit determination. The satellite velocity determined from two independent "best" position measurements over a longer base time (between orbits) should have superior precision. For example, in arc 5′, we could only measure the best acceleration from such single orbit velocity data with a precision of 0.11×10^{-5} rad/sid. day^2 compared to 0.4×10^{-5} rad/sid. day^2 in arc 5 covering essentially the same Syncom 2 orbits (see also the results of the semimajor axis analysis in Reference 3). It appears on inspection that the mean semimajor axes for these orbits, reported by GSFC but not presented here, change with sufficient smoothness over arc 5 to permit independent acceleration determinations to be made from them within 0.1×10^{-5} rad/sid. day^2 accuracy. Alternately, one could also use for this purpose the average drift rate from three or four successive Equator crossings generated from a single set of vector elements in the presence of sun and moon and zonal gravity perturbations (such as in Appendix A). The "mean" semimajor axis implies such velocity smoothing over the perturbations for a number of days past the orbit epoch.

Both of these velocity-from-single-orbit approaches will be tried in the future on a more thoroughgoing basis than here to augment and refine the rather inadequate 24-hour acceleration record presently available for the Syncom satellites. It is hoped that such an augmentation of the past record, together with new equatorial data from Syncom 3, Early Bird, and near future 24-hour satellites will produce a clear picture of H_{31} as well as show conclusive evidence of fourth order resonant earth gravity. The earth harmonics of higher than fourth order appear to be beyond reasonable discrimination from 24-hour altitudes until far more data is available than is foreseen for the next few years.

With a few exceptions, noted in the tables of Section 1, the orbit data used in the arc analyses are believed to be substantially free from all but gravity perturbations (see also the Discussion in Reference 3). For example, in Appendix E, are calculated likely magnitudes for residual atmospheric drag and solar radiation pressure accelerations on the 24-hour Syncom satellites. These are found to be entirely negligible compared to resonant earth gravity accelerations.

CONCLUSIONS

From this comprehensive investigation of the long term gravity drift of three 24-hour "synchronous" satellites over a period of two years (1963-1965), the following conclusions are drawn:

1. Virtually all of the east-west geographic acceleration of these satellites can be accounted for by the second and third order sectorial harmonics of the earth's gravitational field which resonate with them.

2. With due adjustment for small effects of sun and moon gravity and the neglect of likely higher order resonant earth gravity, these dominant sectorial harmonics are estimated to be

$$J_{22} = -(1.816 \pm 0.020)\times10^{-6} \quad ,$$

which corresponds to a difference in major and minor axes of 69.4 ± 0.9 meters in the earth's elliptical Equator, and

$$\lambda_{22} = -(15.4 \pm 0.3)^\circ \quad ,$$

which is the longitude location of the major axis of the elliptical Equator, and

$$J_{33} = -(0.171 \pm 0.017) \times 10^{-6}$$

$$\lambda_{33} = 24.9 \pm 3.3^\circ \quad .$$

3. The sectorial harmonics above, within their ranges, are believed to be absolute or true measures of those individual components of the earth's field.

4. A third pair of third order resonant earth harmonics was just evident but poorly discriminated from the limited acceleration record. The data shows tentatively

$$J_{31} = -\left(1.4 \begin{smallmatrix} +1.0 \\ -0.2 \end{smallmatrix}\right) \times 10^{-6} \quad ,$$

$$\lambda_{31} = -(168 \pm 26)^\circ \quad .$$

5. Tests of the complete 24-hour satellite record for the effects of individual fourth order earth resonant gravity harmonics were inconclusive but gave some evidence that

$$0.01 < |J_{44} \times 10^6| < 0.03 \quad .$$

6. The geoid resulting from this close study of operating 24-hour satellites implies that an equatorial synchronous satellite can be in uncontrolled long term east-west equilibrium at only the following four longitude locations:

   $\lambda_1 = 76.7 \pm 0.8^\circ$ (dynamically stable east-west equilibrium)

   $\lambda_2 = 161.8 \pm 0.7^\circ$ (statically stable east-west equilibrium)

   $\lambda_3 = -108.1 \pm 1.0^\circ$ (dynamically stable east-west equilibrium)

   $\lambda_4 = -12.2 \pm 0.7^\circ$ (statically stable east-west equilibrium).

7. The maximum long term longitude acceleration due to earth gravity which can be experienced by the nearly geostationary satellite, according to the 24-hour acceleration record thus far, is conservatively $\ddot{\lambda} = -1.88 \times 10^{-3}$ degrees/day^2, at about 118° east of Greenwich.

8. To correct continuously for this east-west acceleration would require a velocity increment of

$$\Delta V_{max} = 6.38 \text{ ft/(sec-yr)} \quad .$$

9. Further study of the drift of both present and near future 24-hour satellites should be rewarded in a few years by the first unambiguous picture of the earth's resonant longitude gravity field through fourth order.

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Appendix A

Basic Orbit Data Used in This Report

Vector and mean elements for Syncom 2 (arcs 1, 2, 3, 4, 5, and 8) and Syncom 3 (arcs 6 and 7) were used as basic data in this report and are reported in Tables A1 and A2 below. These elements were calculated by the GSFC Tracking and Data Systems Directorate from radar and minitrack observations on the satellite made over a period of about three days per orbit following the listed epochs. The orbit determination program used for the calculation of these elements employed a gravity-earth model with the following constants (see Appendix B):

$$\mu_{earth} = 3.98627\times10^{5}\,km^{3}/sec^{2}$$

$$R_0 = 6378.388 \text{ km/mean equatorial earth radius}$$

$$J_{20} = 1082.21\times10^{-6}$$

$$J_{30} = -2.29\times10^{-6}$$

$$J_{40} = -2.10\times10^{-6}$$

$$\mu_{sun} = 332.490\ \mu_{earth}$$

$$\mu_{moon} = 0.01229491\ \mu_{earth}$$

The Equator crossing data in Section 1 of this report was derived (mainly) by generating numerically a short trajectory from the vector elements of Tables A1 and A2 utilizing the gravity-earth constants above. The absence of longitude earth gravity in the orbit determinations for the Syncom satellites generally limited the time for which data could be applied to each orbit to less than a week.

The numerical trajectory generator employed in deriving the Equator crossing data and in running the long simulated longitude gravity trajectories in Section 1, is called "ITEM" at Goddard Space Flight Center. Details of this generator can be found in GSFC Document X-640-63-71, "Interplanetary Trajectory Encke Method (ITEM) Program Manual", May 1963.

The basic subsatellite position data for arc 9 (Early Bird) was supplied by Robert H. Green of the Comsat Corporation (see Table 9). It was determined by simultaneous range-azimuth-elevation observations on Early Bird from the A. T. and T. tracking facility at Andover, Maine.

Table A1

Inertial Position and Velocity Coordinates for Syncom 2 and Syncom 3 as Reported by GSFC.*

Arc	Tracking Epoch (Yr-mo-day-hr-min UT)	X (10^4 km)	Y (10^4 km)	Z (10^4 km)	$\dot{X}$ (km/sec)	$\dot{Y}$ (km/sec)	$\dot{Z}$ (km/sec)
	63-8-18-1-30.0	1.8517253	-3.6656408	-0.95346425	2.5197630	0.87570544	1.5296730
	63-8-22-6-12.14	3.8192813	0.19653916	1.7732339	-0.64139671	2.8111679	1.0698498
	63-8-26-17.0	-3.9190365	0.71434463	-1.3838910	-0.030659063	-2.7659652	-1.3414586
	63-8-31.0	1.2517473	-3.8173186	-1.2805028	2.7082374	0.42130417	1.3937010
	63-9-3-13-23.0	-2.6683093	3.2430659	0.37724660	-2.0920997	-1.5282086	-1.6556391
	63-9-5.0	1.5690433	-3.7540418	-1.1058861	2.6179496	0.66119258	1.4710835
	63-9-9.0	1.8101775	-3.6842867	-0.96169979	2.5333080	0.84735057	1.5232783
	63-9-12-2.0	3.4100141	-2.4566117	0.33394973	1.4036497	2.1745471	1.6606540
	63-9-17-2.0	3.5605032	-2.1949356	0.52762629	1.1881299	2.3199229	1.6320863
Arc 1	63-9-20-2.0	3.6381029	-2.0310580	0.64240062	1.0551433	2.3993342	1.6084890
	63-9-27-2.0	3.7828113	-1.6282253	0.90343513	0.73324914	2.5577582	1.5413518
	63-10-1-2.0	3.8398851	-1.3915057	1.0449696	0.54792106	2.6322531	1.4928190
	63-10-8-2.0	3.8997221	-0.96844341	1.2765410	0.22281006	2.7325580	1.3929604
	63-10-14-2.0	3.9114166	-0.59213158	1.4569605	-0.059266662	2.7895298	1.2936179
	63-10-22-2.0	3.8655564	-0.089237507	1.6821569	-0.43011588	2.8231687	1.1399466
	63-10-30.0	3.7935711	-1.5763511	0.94831895	0.69205550	2.5804974	1.5229227
	63-11-6.0	3.8724273	-1.1762815	1.1818209	0.38097748	2.6938419	1.4335987
	63-11-12-5.0	1.1446010	3.4554430	2.1294725	-2.7244144	1.2840822	-0.61733366
	63-11-18-13.0	-3.7038702	-0.58064695	-1.9332132	0.90759960	-2.7961257	-0.89794623
	63-11-28-1.0	3.4781437	1.1376687	2.0944723	-1.2933144	2.7059520	0.67780023
	63-12-4.0	3.7151749	0.53514140	1.9201218	-0.87415171	2.8043813	0.90960269
	63-12-10.0	3.5731662	0.92615699	2.0379487	-1.1461780	2.7500585	0.76033287
	63-12-16-17.0	1.0227024	-3.8691081	-1.3292554	2.7458357	0.25931246	1.3585847
	63-12-23-19.0	3.0804533	-2.8787714	0.055186822	1.7485346	1.9038213	1.6650503
	64-1-6-17.0	2.2436953	-3.5177574	-0.60704656	2.3262615	1.2062956	1.6089899
	64-1-9-6.0	-3.0926247	2.8665793	-0.080953346	-1.7324029	-1.9159081	-1.6667974
Arc 2	64-1-15-18.0	3.3299438	-2.5658898	0.32807382	1.4797710	2.1321920	1.6489369
	64-1-20-21.0	3.7030072	0.56000451	1.9369923	-0.88759725	2.8080493	0.88485319
	64-1-29-20.0	3.8204264	0.093177361	1.7811029	-0.55721471	2.8372181	1.0470801
	64-2-5-16.0	2.8698222	-3.0876991	-0.10282631	1.9209748	1.7308532	1.6640514
	64-2-10-19.0	3.8650196	-0.19769561	1.6738813	-0.34675337	2.8365031	1.1356011
	64-2-17-17.0	3.7268026	-1.7536007	0.90292256	0.82362190	2.5381818	1.5281483
	64-2-25-19.0	3.6533259	0.69908427	1.9851452	-0.98120516	2.7965023	0.82109998
	64-3-4-23.0	0.18113658	3.8087583	1.8019013	-2.8347353	0.61701878	-1.0178939
	64-3-10-13.0	2.0644779	-3.6124046	-0.68632246	2.4052556	1.0737921	1.5862758
	64-3-18-3.0	-3.6249117	2.0233288	-0.74902788	-1.0365741	-2.4353607	-1.5693664
	64-3-24-13.0	2.3283566	-3.4999056	-0.50715805	2.2721038	1.2746548	1.6158241
Arc 3	64-4-1-22.0	0.76296064	3.6490419	1.9857054	-2.7806607	1.0205339	-0.81923611
	64-4-7-15.0	3.4461234	-2.4088100	0.50885287	1.3277141	2.2415368	1.6144312
	64-4-13-19.0	3.3872421	1.3385265	2.1522760	-1.4069923	2.6725021	0.54080057
	64-4-25-2.0	-2.5603443	3.3390780	0.29994066	-2.1318214	-1.4887792	-1.6428786
	64-4-28-15.0	3.2628949	-2.6693162	0.32101150	1.5457882	2.0856009	1.6366277
	64-5-5-16.0	3.7257132	-1.7632128	0.94048572	0.82412264	2.5449717	1.5038900
	64-5-12-16.0	3.7475309	-1.6867283	0.99296812	0.76273664	2.5725322	1.4891387
Arc 4	64-5-19-14.0	2.7849331	-3.1784434	-0.10945226	1.9747957	1.6719452	1.6505955
	64-5-25-15.0	3.4307022	-2.4126224	0.53392194	1.3319460	2.2493385	1.6072617
	64-6-2-21.0	1.7406646	3.1597779	2.1946670	-2.5288299	1.6899106	-0.43439828
	64-6-9-21.0	1.6500071	3.2194181	2.1774956	-2.5601432	1.6304633	-0.47782909
	64-6-16-15.0	3.5655116	-2.1536100	0.72108034	1.1248629	2.3871212	1.5668312
	64-6-23-15.0	3.6075927	-2.0587512	0.78733380	1.0494437	2.4320607	1.5495642
	64-7-4-2.0	-3.2093967	2.7165420	-0.32063943	-1.5841089	-2.0660850	-1.6378223
	64-7-7-3.0	-3.6954496	1.7950587	-0.94983861	-0.85046154	-2.5458831	-1.5023067
	64-7-13-17.0	3.7354217	0.41539202	1.9259618	-0.76727529	2.8431082	0.87209834
	64-7-21-21.0	0.68011374	3.6978808	1.9157389	-2.7798310	-0.96706741	-0.88586596
	64-7-27-16.0	3.8417868	-0.11824881	1.7510381	-0.39126192	2.8593634	1.0493355
	64-8-3-17.0	3.4664817	1.1480683	2.1210692	-1.2649191	2.7375684	0.58214636
	64-8-11-1.0	-3.3701691	2.4805650	-0.51870386	-1.3894809	-2.2250586	-1.6062097
	64-8-17-19.0	1.6959231	3.2024028	2.1645995	-2.5377008	1.6647695	-0.48003497
	64-8-25-10.0	1.5330007	-3.8258893	-0.91436182	2.5905352	0.67495556	1.5073075
	64-9-1-10.0	1.7622665	-3.7590544	-0.76800569	2.5108491	0.85857627	1.5478150
	64-9-9-14.0	3.8729406	-0.56806750	1.5864037	-0.064200095	2.8378419	1.1715632
	64-9-15-12.0	3.4973065	-2.2642849	0.69291203	1.2114165	2.3483095	1.5648130
	64-9-22-10.0	2.3957332	-3.4619283	-0.32713684	2.2142000	1.3764322	1.6238887
Arc 5	64-9-29-6.0	-1.2057564	-3.4918173	-2.0387967	2.6768512	-1.3377786	0.70292893
	64-10-6-5.0	-1.9027758	-3.0642587	-2.1871975	2.4522966	-1.8112966	0.39976832
	64-10-13.0	-3.7909750	1.3890374	-1.2159474	-0.54220619	-2.6929405	-1.3843106
	64-10-20-16.0	2.3418688	2.7061755	2.2406431	-2.2408225	2.0926322	-0.18928830
	64-10-26-16.0	2.1737218	2.8588045	2.2200917	-2.3278640	1.9849102	-0.28044993
	64-11-2-5.0	-1.0575392	-3.5703165	-1.9843669	2.7067256	-1.2347065	0.77366371
	64-11-11-2.0	-3.2269498	-1.5931919	-2.1991397	1.5433675	-2.6360297	-0.35716605
	64-11-17-6.0	0.44169064	-3.9493268	-1.4227193	2.7947443	-0.14738721	1.2691720
	65-1-10-6.0	1.8733482	-3.7329199	-0.61394837	2.4551255	0.97133874	1.5716495
	65-1-13-16.0	-0.024904471	3.9169200	1.5734233	-2.8064159	0.44943751	-1.1686893
	65-1-20-12.0	3.2361444	1.5906403	2.1989944	-1.5260383	2.6448765	0.33028481
	65-1-27-4-5.0	0.42866892	-3.9652039	-1.3910958	2.7914381	-0.14522612	1.2721610
	65-2-2-13-30.0**	1.8326846	3.1377111	2.1684747	-2.4504221	1.7730384	-0.49949115
	65-2-16-4-5.0	0.94696655	-3.9573969	-1.1249388	2.7174166	0.24662704	1.4128023

*From the Data Systems and Tracking Directorate, computed from range and range rate and Minitrack Data by Robert Chaplick, Gerald Repass, and Carleton Carver, from an orbit determination program due to Dr. Joseph Siry (see text in Appendix A for earth-gravity constants used in this program.) The inertial orthogonal system x, y, z has the x axis pointing towards the vernal equinox of epoch and the z axis pointing towards the North Celestial Pole.

**This orbit gave unacceptably large residuals in the acceleration analysis in Section 1 and was consequently ignored there in the final reduction of arc 5 data.

Table A2

Inertial Position and Velocity Coordinates or Mean Elements for Syncom 2 and Syncom 3 as Reported by GSFC*.

	Tracking Epoch (yr-mo-day-hr-min UT)	x (10^4 km)	y (10^4 km)	z (10^4 km)	$\dot{x}$ (km/sec)	$\dot{y}$ (km/sec)	$\dot{z}$ (km/sec)	Semimajor Axis (earth radii)	Eccentricity	Inclination (degrees)	Mean Anomaly (degrees)	Argument of Perigee (degrees)	Right Ascension of the Ascending Node (degrees)
Arc 7	65-1-14-23.5	1.2675646	-4.0179928	-0.051959215	2.9359926	0.92060244	-0.047842619						
	65-1-30-13-10.0	-3.4792181	2.3828935	-0.0058904292	-1.7368621	-2.5371972	-0.0083528082						
	65-2-2-6.0	3.2099039	2.7368950	0.0073851190	-1.9943285	2.3390669	0.0012901180						
	65-2-9-11.0	-2.2119906	3.5906580	-0.0049486959	-2.6173528	-1.6131485	-0.0081798989						
	65-2-16-12.0	-3.3564297	2.5533931	-0.0042520595	-1.8613403	-2.4471176	-0.0086970709						
	65-2-23.0	3.5938828	-2.2084394	0.0096558595	1.6094139	2.6188512	0.0026358856						
	65-3-2.0	3.8120235	-1.8069874	0.015075846	1.3165671	2.7773230	-0.00057799012						
	65-3-9.0	3.9810232	-1.3940918	0.0045517934	1.0162757	2.9011202	0.0024674641						
	65-3-16.0	4.1104734	-0.95605039	0.026135089	0.69589465	2.9925915	-0.00060597968						
Arc 8	65-2-25.0	-2.6260533	-2.4306852	-2.2283266	2.0528730	-2.2888731	0.081154582						
	65-3-3.0	-2.3397165	-2.7254249	-2.2051595	2.2300224	-2.1051269	0.23910311						
	65-3-6.0							6.6112847	.00076	31.939	308.359	333.048	309.370
	65-3-13.0							6.6113429	.00069	31.912	326.845	321.447	309.095
	65-3-29.0	-0.85799562	-3.6815475	-1.8631946	2.7341821	-1.0890101	0.89574242						
	65-4-5.0							6.6111445	.00069	31.830	329.612	340.634	308.896
	65-4-12.0							6.6111335	.00073	31.848	328.269	348.585	308.947
	65-4-19.0							6.6110585	.00065	31.867	353.146	330.838	308.586
	65-4-26.0							6.6110267	.00063	31.787	353.717	337.005	308.580
	65-5-3.0							6.6110035	.00061	31.807	359.897	337.757	308.417
	65-5-10.0							6.6109407	.00059	31.731	0.012	344.515	308.335

*From the Data Systems and Tracking Directorate (see note in Table A1). The mean elements, good for more than one orbit (smoothing out the periodic sun, moon, and zonal gravity effects) are calculated from the vector elements of position and velocity according to a theory due to D. Brouwer.

Appendix B

Earth Gravity Potential and Force Field Used in This Report: Comparison with Recent Investigations

The gravity potential used as the basis for the data reduction in this study is the exterior potential of the earth derived (in Reference B1 at the end of this appendix) for geocentric spherical coordinates referenced to the earth's spin axis and its center of mass (Equation B1). The infinite series of spherical harmonics is truncated after J_{44} because the great height of the synchronous satellite makes it very insensitive to higher orders of earth gravity. The zonal gravity and other earth constants used in this study (and illustrated in Equation B1) are from Reference B2 and represent a somewhat outdated set used by GSFC in the orbit determination program for the Syncom satellites (see Appendix A). They are (with the corresponding mean equatorial radius):

$$R_0 = 6378.388 \text{ km/mean equator earth radius (Reference B3)},$$

$$\mu_{earth} = 3.98627\times10^5 \text{ km}^3/\text{sec}^2 \text{ (Reference B3)},$$

$$J_{20} = 1082.21\times10^{-6},$$

$$J_{30} = 2.29\times10^{-6},$$

$$J_{40} = -2.10\times10^{-6}.$$

Though these values individually are not the most accurate known to date (1965), they were chosen for the trajectory generations in this study to insure consistency with the published orbits. The longitude gravity reductions themselves are not significantly affected by the probable errors in these zonal gravity and other principal earth constants. The most accurate "zonal geoid" is still probably that of Kozai (1962) in Reference B4, with the following constants:

$$R_0 = 6378.2 \text{ km},$$

$$\mu_{earth} = 3.98603\times10^5 \text{ km}^3/\text{sec}^2,$$

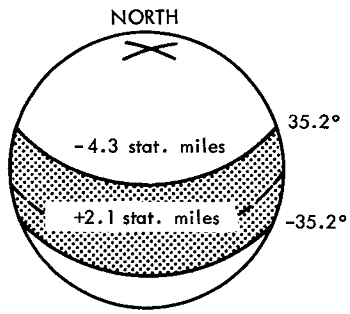
$$J_{20} = 1082.48\times10^{-6},$$

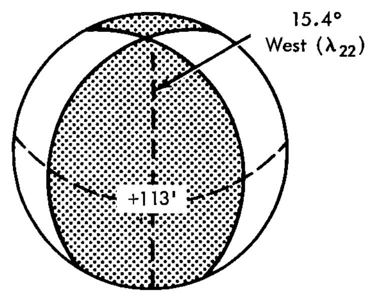
$$J_{30} = -2.56\times10^{-6},$$

$$J_{40} = -1.84\times10^{-6},$$

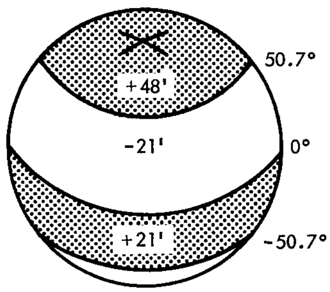
plus higher order terms.

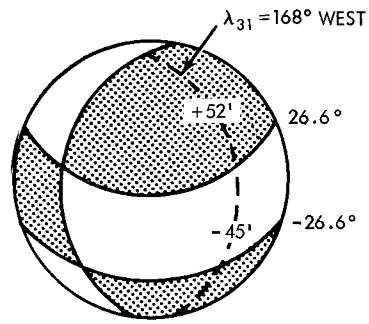
A good review of zonal gravity investigations from satellite observations through 1965 is to be found in Reference B5.



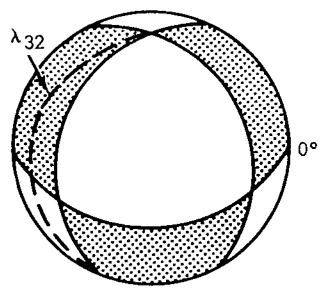


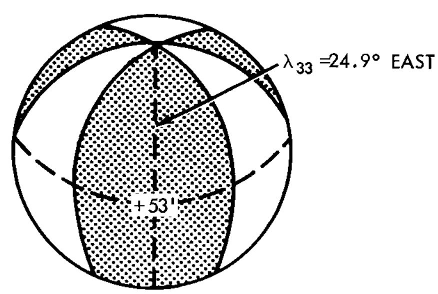
$$V_E = \frac{\mu_E}{r}\left[1 - \frac{J_{20}\,R_0^2}{2r^2}\left(3\sin^2\phi - 1\right) - 3J_{22}\,\frac{R_0^2}{r^2}\cos^2\phi\cos 2\left(\lambda - \lambda_{22}\right)\right.$$



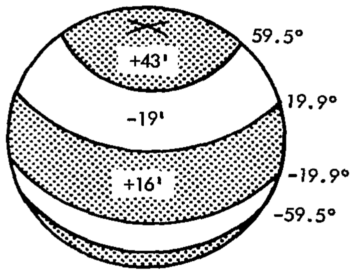


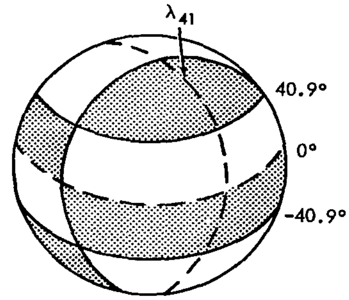
$$- \frac{J_{30}\,R_0^3}{2r^3}\left(5\sin^3\phi - 3\sin\phi\right) - \frac{J_{31}\,R_0^3}{2r^3}\cos\phi\left(15\sin^2\phi - 3\right)\cos\left(\lambda - \lambda_{31}\right)$$



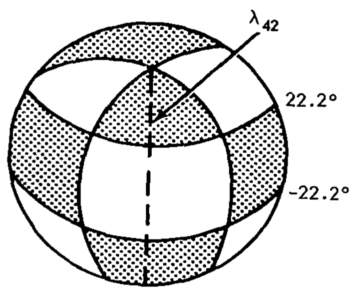


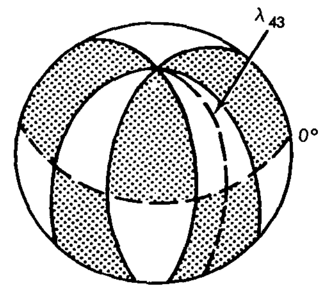
$$- 15J_{32}\,\frac{R_0^3}{r^3}\cos^2\phi\sin\phi\cos 2\left(\lambda - \lambda_{32}\right) - 15J_{33}\,\frac{R_0^3}{r^3}\cos^3\phi\cos 3\left(\lambda - \lambda_{33}\right)$$



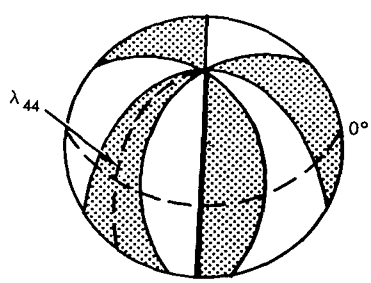


$$- \frac{J_{40} R_0^4}{8r^4} \left(35 \sin^4 \phi - 30 \sin^2 \phi + 3\right) - \frac{J_{41} R_0^4}{8r^4} \left(140 \sin^3 \phi - 60 \sin \phi\right) \cos \phi \cos \left(\lambda - \lambda_{41}\right)$$





$$- \frac{J_{42} R_0^4}{8r^4} \left(420 \sin^2 \phi - 60\right) \cos^2 \phi \cos 2\left(\lambda - \lambda_{42}\right) - \frac{J_{43} R_0^4}{8r^4} 840 \sin \phi \cos^3 \phi \cos 3\left(\lambda - \lambda_{43}\right)$$



$$- \frac{J_{44} R_0^4}{8r^4} 840 \cos^4 \phi \cos 4\left(\lambda - \lambda_{44}\right)\Bigg] \ . \qquad \text{(B1)}$$

The earth-gravity field (per unit test mass), whose potential is Equation B1, is given as the gradient of B1, or

$$\bar{F} = \hat{r}F_r + \hat{\lambda}F_\lambda + \hat{\phi}F_\phi = \nabla V_E = \hat{r}\frac{\partial V_E}{\partial r} + \frac{\hat{\lambda}}{r\cos\phi}\frac{\partial V_E}{\partial \lambda} + \frac{\hat{\phi}}{r}\frac{\partial V_E}{\partial \phi} \; ; \tag{B2}$$

or

$$\begin{aligned} F_r = \frac{\mu_E}{r^2}\Big\{ -1 + (R_0/r)^2 \Big[& 3/2 J_{20} \left(3\sin^2\phi - 1\right) + 9J_{22}\cos^2\phi\cos 2(\lambda - \lambda_{22}) \\ & + 2(R_0/r)\, J_{30}\left(5\sin^2\phi - 3\right)(\sin\phi) + 6(R_0/r)\, J_{31}\left(5\sin^2\phi - 1\right)\cos\phi\cos(\lambda - \lambda_{31}) \\ & + 60(R_0/r)\, J_{32}\cos^2\phi\sin\phi\cos 2(\lambda - \lambda_{32}) + 60(R_0/r)\, J_{33}\cos^3\phi\cos 3(\lambda - \lambda_{33}) \\ & + 5/8(R_0/r)^2 J_{40}\left(35\sin^4\phi - 30\sin^2\phi + 3\right) \\ & + 25/2(R_0/r)^2 J_{41}\left(7\sin^2\phi - 3\right)\cos\phi\sin\phi\cos(\lambda - \lambda_{41}) \\ & + 75/2(R_0/r)^2 J_{42}\left(7\sin^2\phi - 1\right)\cos^2\phi\cos 2(\lambda - \lambda_{42}) \\ & + 525(R_0/r)^2 J_{43}\cos^3\phi\sin\phi\cos 3(\lambda - \lambda_{43}) + 525(R_0/r)^2 J_{44}\cos^4\phi\cos 4(\lambda - \lambda_{44}) \Big] \Big\} , \end{aligned} \tag{B3}$$

$$\begin{aligned} F_\lambda = \frac{\mu_E}{r^2}(R_0/r)^2 \Big\{ & 6J_{22}\cos\phi\sin 2(\lambda - \lambda_{22}) + 3/2(R_0/r)\, J_{31}\left(5\sin^2\phi - 1\right)\sin(\lambda - \lambda_{31}) \\ & + 30(R_0/r)\, J_{32}\cos\phi\sin\phi\sin 2(\lambda - \lambda_{32}) + 45(R_0/r)\, J_{33}\cos^2\phi\sin 3(\lambda - \lambda_{33}) \\ & + 5/2(R_0/r)^2 J_{41}\left(7\sin^2\phi - 3\right)\sin\phi\sin(\lambda - \lambda_{41}) + 15(R_0/r)^2 J_{42}\left(7\sin^2\phi - 1\right)\cos\phi\sin 2(\lambda - \lambda_{42}) \\ & + 315(R_0/r)^2 J_{43}\cos^2\phi\sin\phi\sin 3(\lambda - \lambda_{43}) \\ & + 420(R_0/r)^2 J_{44}\cos^3\phi\sin 4(\lambda - \lambda_{44}) \Big\} , \end{aligned} \tag{B4}$$

$$
\begin{aligned}
F_\phi = \frac{\mu_E}{r^2}(R_0/r)^2 \Big\{ & -3J_{20}\sin\phi\cos\phi + 6J_{22}\cos\phi\sin\phi\cos 2(\lambda-\lambda_{22}) \\
& -3/2(R_0/r)J_{30}(5\sin^2\phi-1)\cos\phi + 3/2(R_0/r)J_{31}(15\sin^2\phi-11)\sin\phi\cos(\lambda-\lambda_{31}) \\
& +15(R_0/r)J_{32}(3\sin^2\phi-1)\cos\phi\cos 2(\lambda-\lambda_{32}) \\
& +45(R_0/r)J_{33}\cos^2\phi\sin\phi\cos 3(\lambda-\lambda_{33}) - 5/2(R_0/r)^2J_{40}(7\sin^2\phi-3)\sin\phi\cos\phi \\
& +5/2(R_0/r)^2J_{41}(28\sin^4\phi-27\sin^2\phi+3)\cos(\lambda-\lambda_{41}) \\
& +30(R_0/r)^2J_{42}(7\sin^2\phi-4)\cos\phi\sin\phi\cos 2(\lambda-\lambda_{42}) \\
& +105(R_0/r)^2J_{43}(4\sin^2\phi-1)\cos^2\phi\cos 3(\lambda-\lambda_{43}) \\
& +420(R_0/r)^2J_{44}\cos^3\phi\sin\phi\cos 4(\lambda-\lambda_{44})\Big\} \quad . \qquad \text{(B5)}
\end{aligned}
$$

The actual sea-level surface of the earth is to be conceptualized through Equation B1 as a sphere of radius 6378 km, around which are superimposed the sum of the separate spherical harmonic deviations illustrated. To these static gravity deviations, of course, must be added a centrifugal earth-rotation potential at the earth's surface, to get the true sea-level surface.

Table B1 gives longitude coefficients for this earth-gravity field form as reported by geodesists from 1942 to 1965.

The longitude gravity represented in Equation B1 represents only that gravity determined in the final longitude geoid of this report (see Table 14). (The longitude gravity drift simulations used slightly different values than these as reported in Sections 1 and 2.) The coefficients of this longitude geoid are

$$
\begin{aligned}
J_{22} &= -1.816\times10^{-6} \ , \\
\lambda_{22} &= -15.4^\circ \ , \\
J_{31} &= -1.4\times10^{-6} \ , \\
\lambda_{31} &= -168^\circ \ , \\
J_{33} &= -0.171\times10^{-6} \ , \\
\lambda_{33} &= +24.9^\circ \ .
\end{aligned}
$$

Other longitude coefficients from recent studies are reported in Table B1 for purposes of comparison. The maximum geoid heights and depressions illustrated in Equation B1 are proportional to the J_{nm} amplitudes of the corresponding gravity harmonics.

Table B-1

Longitude Coefficients in the Earth's Gravity Potential $\left\{V_E = \frac{\mu}{r} \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left[1 - (R_0/r)^n P_n^m(\sin\phi) J_{nm} \cos m(\lambda - \lambda_{nm})\right]\right\}^1$, as Reported 1942-1965[2].

Longitude Geoid Reference	J_{22}	λ_{22}	J_{31}	λ_{31}	J_{32}	λ_{32}	J_{33}	λ_{33}	J_{41}	λ_{41}	J_{42}	λ_{42}	J_{43}	λ_{43}	J_{44}	λ_{44}
(1) Wagner (1965a)[3]	-1.81×10^{-6}	-15.4°	-1.4×10^{-6}	-168.0°			$-.171 \times 10^{-6}$	24.9°								
(2) Izsak (1965)[4]	-1.57	-15.2°	-1.72	-1.0°	$-.300 \times 10^{-6}$	-32.5°	-.199	32.4°	$-.519 \times 10^{-6}$	-134.0°	$-.138 \times 10^{-6}$	35.5°	$-.0413 \times 10^{-6}$	-2.7°	$-.0076 \times 10^{-6}$	27.2°
(3) Guier (1965)[3]	-1.72	-13.4°	-2.01	6.7°	-.477	-14.6°	-.165	18.7°	-.679	-142.0°	-.193	23.4°	-.0506	0.2°	-.0060	34.5°
(4) Anderle (1965)[3]	-1.86	-16.0°	-2.32	7.0°	-.455	-21.3°	-.242	23.5°	-.723	-130.0°	-.158	33.7°	-.0626	-4.3°	-.0116	34.6°
(5) Kaula (1964)[4]	-1.77	-18.2°	-2.12	-5.4°	-.379	10.5°	-.105	23.1°	-.263	-239.0°	-.117	42.3°	-.0473	15.0°	-.0104	14.5°
(1966)	-1.82	-14.9	-2.27	5.5	-.36	-21.9	-.194	22.6			-.184	31.2			-.0081	33.8
(6) Uotila (1964)[5]	-1.52	-36.5°	-0.685	-81.0°	-.409	-5.2°	-.398	19.5°	-.238	-127.0°	-.211	14.6°	-.082	-9.3°	-.0142	-2.6°
(7) Kozai (1962)[4]	-1.2	-26.4°	-1.9	4.6°	-.14	-16.8°	-.10	42.6°	-.52	-122.5	-.062	65.2°	-.035	0.5°	-.031	14.9°
(8) Zhongolovitch (1961)[5]	-5.95	-7.7°	-2.21	-25.7°	-.628	-26.4°	-.54	13.0°	-.78	-149.1	-.080	45.0°	-.051	-3.8°	-.0224	15.9°
(9) Jeffreys (1942)[5]	-4.1	0.0	-2.1	0.0	-.66	0.0	-.24	33.3°								

[1] r is the radial distance of the field point to the center of mass of the earth, μ the earth's Gaussian gravity constant $\doteq 3.9860 \times 10^{20}$ cm^3/sec^2, R_0 the mean equatorial radius of the earth $\doteq$ 6378.2 km. ϕ is the geocentric latitude of the field point. λ is the geographic longitude of the field point. $J_{21} \doteq 0$, since the polar axis is very nearly a principal axis of inertia for the earth. $P_n^m(\sin\phi) = \cos^m\phi \sum_{t=0}^{K} T_{nmt} \sin^{n-m-2t}\phi$, where K is the integer part of (n - m)/2 and $T_{nmt} = \frac{(-1)^t (2n-2t)!}{2^n t!(n-t)!(n-m-2t)!}$ (See Kaula, 1965 [Reference 16]). The longitude coefficients are those for which $m \neq 0$.

[2] The J_{nm}'s and λ_{nm}'s in this table, except in one or two instances, have been converted from the original author's set of gravity coefficients. The blanks indicate the author did not consider that particular harmonic in fitting an earth potential to the observed data.

[3] Satellite - radar geoid.

[4] Satellite-camera geoid.

[5] Surface-gravimetric geoid.

Table B2

References and Notes to Longitude Geoid Data of Table B1

Longitude Geoid	Reference	Notes
(1)	This report	Considers the drift of three 24-hour satellites, two geo-stationary and one of medium inclination, with fair global longitude coverage.
(2)	Data due to I. Izsak quoted in: "Y. Kozai," *Summary of* Numerical Results Derived from Satellite Observations," paper presented at the 2nd International Symposium on the Use of Artificial Satellites for Geodesy, Athens, Greece, April, 1965.	Uses 10-15 medium altitude, medium and high inclination satellites; data reduced from Baker-Nunn camera observations.
(3)	In: "Recent Progress in Satellite Geodesy," Johns Hopkins University, APL Report TG-659, Feb. 1965	Uses data from about 5 "Transit" medium altitude, medium and high inclination satellites, reduced from Doppler observations.
(4)	Data due to Anderle quoted in the Y. Kozai paper above [for longitude geoid (2)]	Uses data from both "Transit" and "Anna" satellites; reduction uses cross track perturbations as well as along track.
(5)	Private communication to the author (July 1964) from W. M. Kaula	Uses about 10 medium altitude, medium and high inclination satellites.
(6)	Data due to Uotila quoted in a private communication to the author (July 1964) from W. M. Kaula	Believed by Kaula to be the most comprehensive-coverage gravimetric geoid to date (1964).
(7)	Private communication to the author (Oct. 1962) from Y. Kozai.	Uses about 5 medium altitude, medium inclination satellites.
(8)	Data due to Zhongolovitch quoted in: Y. Kozai, "Tesseral Harmonics of the Gravitational Potential of the Earth," Astronom. J. 66(7): Sept. 1961.	Recent Russian gravimetric geoid for comparison purposes.
(9)	Data due to Jeffreys quoted in the paper above [for longitude geoid (8)].	Older gravimetric geoid for comparison purposes.

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B4. Kozai, Y., "Numerical Results from Orbits," Smithsonian Inst. Astrophys. Obs. Spec. Rept. 101, July 31, 1962, pp. 1-21.

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Appendix C

Preliminary Maximum Longitude Accelerations on 24-Hour Satellites Due to the Resonant Gravity Harmonics of the Earth through Fourth Order

The source for this appendix is Reference 15. The studies in Reference 1 and especially References 3 through 6 have already established the dominance of second order earth gravity (represented by the elliptical Equator) on the perturbed drift of the 24-hour satellite. In order to establish a reasonable basis for higher order gravity tests, we calculate here maximum 24-hour satellite accelerations due to resonant gravity harmonics through fourth order as well. The source for the higher order harmonics is Guier (1965), Reference 15 (see also Table B1), considered to be representative of the best of recent high order satellite-geoid determinations.

To cover the three satellites in this study, we calculate maximum drift accelerations for equatorial $(i_s = 0$, Syncom 3 and Early Bird$)$ as well as moderately inclined orbit satellites $(i_s = 32.5°$, Syncom 2$)$.

Equatorial Satellites (Syncom 3 and Early Bird)

From Equation 57B in Reference 1 (or Equation 66 in Reference 2) the long term longitude drift acceleration of the 24-hour equatorial satellite is given through fourth order earth gravity as

$$\ddot{\lambda} = -12\pi^2 \left(R_0/a_s\right)^2 \Big\{ 6 J_{22} \sin 2\left(\lambda - \lambda_{22}\right) - \frac{3}{2}\left(R_0/a_s\right) J_{31} \sin\left(\lambda - \lambda_{31}\right) + 45\left(R_0/a_s\right) J_{33} \sin 3\left(\lambda - \lambda_{33}\right)$$

$$- 15\left(R_0/a_s\right)^2 J_{42} \sin 2\left(\lambda - \lambda_{42}\right) + 420\left(R_0/a_s\right)^2 J_{44} \sin 4\left(\lambda - \lambda_{44}\right)\Big\} \ \text{rad/sid. day}^2 , \quad \text{(C1)}$$

where the following set of harmonic constants are used:

$J_{22} = -1.8\times10^{-6}$ (Wagner, preliminary estimate from this study),
$J_{31} = -2.0\times10^{-6}$ (Guier (1965); Reference 15),
$J_{33} = -0.17\times10^{-6}$ (Guier (1965)),
$J_{42} = -0.19\times10^{-6}$ (Guier (1965)),
$J_{44} = -0.006\times10^{-6}$ (Guier (1965)).

Calculating only the maximum values of the terms in Equation C1 for $(R_0/a_s)^2 = 0.0229$ with $a_s = 6.61$ earth radii gives

$$|\ddot{\lambda}_{22}(\max)|_{\text{equator}} = 2.92\times10^{-5} \text{ rad/sid. day}^2, \tag{C2}$$

$$|\ddot{\lambda}_{31}(\max)|_{\text{equator}} = 0.12\times10^{-5} \text{ rad/sid. day}^2, \tag{C3}$$

$$|\ddot{\lambda}_{33}(\max)|_{\text{equator}} = 0.31\times10^{-5} \text{ rad/sid. day}^2, \tag{C4}$$

$$|\ddot{\lambda}_{42}(\max)|_{\text{equator}} = 0.018\times10^{-5} \text{ rad/sid. day}^2, \tag{C5}$$

$$|\ddot{\lambda}_{44}(\max)|_{\text{equator}} = 0.016\times10^{-5} \text{ rad/sid. day}^2 \quad . \tag{C6}$$

32.5° Inclined Satellite (Syncom 2)

For this satellite the maximum values in Equations C2 and C6 should be multiplied by the inclination factors $F_{nm}(i)$ (Reference 2, Equation 67):

n, m	$F_{nm}(i)$,	
2, 2	$\frac{1}{4}(\cos i + 1)^2$,	(C7)
3, 1	$\frac{1}{2}(\cos i + 1) - \frac{5}{8}\sin^2 i\ (1 + 3\cos i)$,	(C8)
3, 3	$\frac{1}{8}(\cos i + 1)^3$,	(C9)
4, 2	$\frac{1}{4}(\cos i + 1)^2 - \frac{7}{4}\sin^2 i \cos i\ (\cos i + 1)$,	(C10)
4, 4	$\frac{1}{16}(\cos i + 1)^4 \quad .$	(C11)

Evaluating Equations C7-C11 at $i = 32.5°$ gives

$$F_{22}(i) = 0.850, \tag{C12}$$

$$F_{31}(i) = 0.284, \tag{C13}$$

$$F_{33}(i) = 0.785, \tag{C14}$$

$$F_{42}(i) = 0.063, \tag{C15}$$

$$F_{44}(i) = 0.723 \quad . \tag{C16}$$

Equations C12-C16 as multiplying factors of Equations C2-C6 give the maximum accelerations on the Syncom 2 satellite due to the gravity harmonics through fourth order as

$$|\ddot{\lambda}_{22}(\max)|_{\text{Syncom 2}} = 2.48\times10^{-5} \text{ rad/sid. day}^2, \tag{C17}$$

$$|\ddot{\lambda}_{31}(\max)|_{\text{Syncom 2}} = 0.03\times10^{-5} \text{ rad/sid. day}^2, \tag{C18}$$

$$|\ddot{\lambda}_{33}(\max)|_{\text{Syncom 2}} = 0.24\times10^{-5} \text{ rad/sid. day}^2, \tag{C19}$$

$$|\ddot{\lambda}_{42}(\max)|_{\text{Syncom 2}} = 0.001\times10^{-5} \text{ rad/sid. day}^2, \tag{C20}$$

$$|\ddot{\lambda}_{44}(\max)|_{\text{Syncom 2}} = 0.012\times10^{-5} \text{ rad/sid. day}^2 \quad . \tag{C21}$$

Conclusions

Seven arcs of longitude-acceleration data (arcs 1, 2, 4, 5A, 5′, 5B and 8 in Table 10) are available from Syncom 2 drift with standard errors from $(0.03 - 0.11)\times10^{-5}$ rad/sid. day^2. With this data alone, only both harmonic coefficients of H_{22} and H_{33} should be well discriminated. Additionally, one arc of longitude-acceleration data (arc 9) from Early Bird is available with a standard error of 0.01×10^{-5} rad/sid. day^2. Sun, moon, and model bias accelerations average about 0.02×10^{-5} rad/sid. day^2 for all of the above arcs.

It is anticipated then, that if all the data without model bias adjustment are used, H_{22} and H_{33} should be well discriminated, and the effects of H_{31} should be marginally apparent. The separate effects of fourth order resonant earth gravity are probably at or below the average noise level of this experiment. Only an indication of the probable order of magnitude of fourth order effects should be realizable from the available limited acceleration record.

Appendix D

The Approximate Longitude Excursion of a Slowly Drifting 24-Hour Satellite

Introduction

A very close approximation to the geographic drift excursion in a resonant gravity field of a 24-hour satellite follows the differential equation of motion, Equation 2, (see also Reference 2) and is, evidently, given by an elliptical integral such as that developed in Appendix E of Reference 3. If the excursion itself is limited in extent, a simple closed form of this solution in terms of harmonic functions becomes applicable (References 2 and 9). Unfortunately this solution is essentially non-linear in the unknown gravity constants. This makes the extraction of these constants from time-longitude data somewhat cumbersome statistically. If the time in the excursion is limited, a simple closed form of the elliptic integral solution is applicable in terms of a power series in the time (Reference 3). This solution is essentially linear in the gravity constants and will be developed here, by a Taylor series, to cover the excursion $\Delta\lambda$ from a longitude λ_0 where the drift rate $\dot{\lambda}_0$ is low but not zero (as in Reference 3). This solution (within appropriate limits on the excursion time Δt) should then be suited to describe the slow drift of the 24-hour satellites in arcs 1, 2, 6, 7, 8 and 9 of Section 1.

Development

Expanding the drift of the 24-hour satellite from λ_0 (i.e., the longitude of the ascending Equator crossing) in a Taylor series in the excursion time Δt gives

$$\Delta\lambda = \lambda - \lambda_0 = \dot{\lambda}_0 \Delta t + \frac{\ddot{\lambda}_0 \Delta t^2}{2} + \frac{\lambda_0^{(3)} \Delta t^3}{6} + \frac{\lambda_0^{(4)} \Delta t^4}{24} + \cdots \quad \text{(D1)}$$

When resonant earth terms of higher than second order are ignored, Equation 2 gives

$$\ddot{\lambda} = -A_{22} \sin 2\gamma \;, \quad \text{(D2)}$$

where γ is the longitude of the 24-hour satellite east of the minor axis of the elliptical Equator, and

$$A_{22} = -72\pi^2 J_{22} \left(R_0/a_s\right)^2 \left[\frac{1}{4}\left(\cos i_s + 1\right)^2\right] \text{ rad/sid. day}^2 \quad . \tag{D3}$$

Differentiating Equation D2 with respect to time gives

$$\lambda^{(3)} = -2A_{22}\,\dot{\gamma}\cos 2\gamma = -2A_{22}\,\dot{\lambda}\cos 2\gamma \quad . \tag{D4}$$

Similarly, differentiation of Equation D4 shows that

$$\begin{aligned} \lambda^{(4)} &= -2A_{22}\,\ddot{\lambda}\cos 2\gamma + 4A_{22}\,(\dot{\lambda})^2 \sin 2\gamma \\ &= A_{22}\sin 4\gamma + 4A_{22}\,(\dot{\lambda})^2 \sin 2\gamma \quad . \end{aligned} \tag{D5}$$

Substituting Equations D5, D4 and D2 at $\lambda = \lambda_0$ $\left(\text{or } \gamma = \gamma_0\right)$ into Equation D1 gives the excursion from λ_0 to fourth order in Δt as

$$\begin{aligned} \Delta\lambda &= \dot{\lambda}_0\,\Delta t - \frac{A_{22}}{2}\left(\sin 2\gamma_0\right)\Delta t^2 - \frac{A_{22}}{3}\left(\cos 2\gamma_0\right)\dot{\lambda}_0\,\Delta t^3 \\ &+ \frac{1}{24}\left[A_{22}^2 \sin 4\gamma_0 + 4A_{22}\left(\dot{\lambda}_0\right)^2 \sin 2\gamma_0\right]\Delta t^4 + \cdots \quad . \end{aligned} \tag{D6}$$

The adequacy of a series such as Equation D6 to approximate the actual drift motion stemming from the basic differential equation is discussed at greater length in Reference 3.

A typical value for the "longitude noise" level (due to sun and moon effects and observational error) in the actual gravity experiment is 0.025° (see Section 1). We wish to apply a polynomial fit to the actual data to the third degree in Δt, by way of Equation D6. Such a fit allows the acceleration in any slow drift arc to vary with time as the satellite samples different longitudes in the gravity field. By introducing an additional degree of freedom it also permits the precision of the acceleration determination to vary. Under normal circumstances the "best acceleration" measurement will occur with this fit near the "centroid" of the longitude-time data. This statistical result coincides with our intuition of where the best measured parameters of the drift should occur.

Let us assume that at $\Delta t = 0$, $\lambda = \lambda_0$, $\gamma = \gamma_0$ the drift rate is $\dot{\lambda}_0 = 0.1$ degree/day. Then Equation D6, truncated at the Δt^3 term, will hold to an error of the order of 0.025° at the end of an arc time length Δt if

$$\frac{1}{24}\left(A_{22}^2 \sin 4\gamma_0 + 4A_{22}\left(\frac{0.1}{57.3}\right)^2 \sin 2\gamma_0\right)\Delta t^4 \leq 0.025° \quad , \tag{D7}$$

with A_{22} in units of rad/sid. day^2. In Reference 3, A_{22} for measured Syncom 2 drift over Brazil was

$$A_{22} = 23.2\times10^{-6} \text{ rad/sid. day}^2 \quad ,$$

corresponding to

$$i_s = 33° \quad \text{and} \quad J_{22}\,(\text{measured}) = -1.7\times10^{-6} \quad .$$

We wish to evaluate Equation D7 in the most conservative drift condition (giving the minimum Δt for the inequality to apply). In Appendix F (from Figure 9) it is estimated that for drift of an equatorial satellite over the Indian Ocean, J_{22} (measured) $\simeq -1.93\times10^{-6}$, giving the strongest longitude acceleration on the 24-hour satellite. From Equation D3, the A_{22} measured for this satellite location would be

$$A_{22}(\max) = 23.2\times10^{-6}\times\left(\frac{1.94}{1.7}\right)\times\left(\frac{2}{\cos 33.0° + 1}\right)^2 = 31.3\times10^{-6} \text{ rad/sid. day}^2 \quad . \tag{D8}$$

Equation D8 in Equation D7 gives the critical inequality as

$$\left(9.797\times10^{-10} \sin 4\gamma_0 + 3.814\times10^{-10} \sin 2\gamma_0\right)\Delta t^4 \leq 0.01047 \tag{D9}$$

with Δt in units of days. The factor of Δt^4 in Equation D9 is maximum when $2\gamma_0 \doteq 48.7°$, or the satellite is about 24.4° east of one of the two minor equatorial axis longitudes. At this longitude the Δt^4 factor is 12.58×10^{-10}. From Equation D9, the minimum time for inclusion of the Δt^4 term in Equation D6 according to the above criteria is

$$\Delta t_{min}\,(\text{for fourth order term inclusion in slow 24-hour satellite drift}) = (0.0832\times10^8)^{1/4}$$

$$= \pm 53.7 \text{ days.} \tag{D10}$$

The conservative time criteria in Equation D10 shows that Equation D6 should give an adequate description of the slow drift regime in all the 24-hour satellite arcs of Section 1. The excellent agreement of theoretical accelerations with measured results from Δt^3 analyses of the simulated data in these arcs confirms the adequacy of this description. (See the bias results in Table 11.)

Let the drift time from an arbitrary base time $\underline{t}$ (i.e. the beginning of the year or the middle of the arc for statistical convenience) be given by t. Let $t = 0$ be the time (with respect to the base time) when the satellite is at $\lambda = \lambda_0$ moving at drift rate $\dot{\lambda}_0$. Let the drift be given from an arbitrary base longitude by L. Let the longitude λ_0 with respect to the base longitude $\underline{L}$ be L_0. Then

$$\left.\begin{aligned} \Delta\lambda &= \lambda - \lambda_0 = L - L_0 , \\ \Delta t &= t . \end{aligned}\right\} \tag{D11}$$

Equations D11 into Equation D6 truncated at the Δt^3 term gives the longitude excursion L as

$$L = L_0 + \left(\dot{\lambda}_0\right) t - \left(\frac{A_{22}}{2} \sin 2\gamma_0\right) t^2 - \left(\frac{A_{22}}{3} \dot{\lambda}_0 \cos 2\gamma_0\right) t^3 . \tag{D12}$$

Let

$$\left.\begin{aligned} a_1 &= L_0 , \\ a_2 &= \dot{\lambda}_0 , \\ a_3 &= -\frac{A_{22}}{2} \sin 2\gamma_0 , \\ a_4 &= -\frac{A_{22}\dot{\lambda}_0}{3} \cos 2\gamma_0 . \end{aligned}\right\} \tag{D13}$$

Equations D13 in Equation D12 gives the drift from the arbitrary longitude $\underline{L}$ in the terms of the time from an arbitrary base time $\underline{t}$ as

$$L = a_1 + a_2 t + a_3 t^2 + a_4 t^3 , \tag{D14}$$

where the four arbitrary constants of Equation D14 serve to define the elements L_0, $\dot{\lambda}_0$, A_{22} and γ_0 of the arc dynamics in terms of a purely second order resonant gravity drift, presumably dominant at 24-hour altitudes.

It is easily seen from Equation 2 that the effect of higher order resonant gravity on the slow drift expansion, Equation D6, is to merely alter the coefficients of the Δt^2 and higher order terms. The fundamental polynomial representation does not change.

Appendix E

The Secular Accelerations on Syncom 24-Hour Satellites Due to Particle Atmospheric Drag and Solar Radiation Pressure

Introduction

We have seen previously (Section 1) that the long term tracking of the three 24-hour satellites (modeled after Syncom) has provided a discrimination of drift acceleration of the order of 0.1×10^{-5} rad/sid. day^2. It will be instructive to compare this figure with the likely long term accelerations on these satellites due to particle-atmospheric drag and solar radiation pressure. In performing this calculation, we will follow the work of D. D. Williams in Reference 7. We shall assume that long term secular accelerations in drift from these sources will be negligible on the gravity analysis here if these accelerations are of the order of magnitude of 0.01×10^{-5} rad/sid. day^2 or less.

Atmospheric-Particle Drag

With respect to the drag caused by a continuum of nonmeteoritic particles in the upper atmosphere, Williams distinguishes two cases. The first is drag caused by particles of residual earth atmosphere, at rest but not rotating with respect to the earth. The second is drag caused by the interplanetary particles, at rest with respect to the sun in the earth's orbital path.

Residual Atmospheric Drag

For a conservative Syncom spacecraft mass of only 0.75 slug and fully elastic collisions, Williams calculates the density of residual atmosphere at 24-hour altitudes necessary to produce one degree of longitude drift in a year to be

$$\rho = 0.1826\times10^{-17} \text{ gm/cc (with drift proportional to density).}$$

An estimate of the order of magnitude of the residual atmosphere at 24-hour altitudes is (Reference E1 at the end of this appendix, pp. 2-8)

$$\rho \doteq 10^{-21} \text{ gm/cc .}$$

The acceleration, in radians per sidereal day^2, necessary to give a drift of one degree in a year is (from $\Delta\lambda = 1/2\,\ddot{\lambda}\,\Delta t^2$):

$$\ddot{\lambda} = \frac{2\Delta\lambda}{\Delta t^2} = \frac{2\times 1^\circ}{57.3^\circ/\text{rad}\times(366 \text{ sid. day})^2} = 0.261\times 10^{-6} \text{ rad/day}^2 \; .$$

Thus, for a Syncom spacecraft (including empty apogee motor) of 2.34 slugs, the drift acceleration due to residual atmosphere drag should be of the order of

$$\ddot{\lambda} \text{ (residual atmosphere)} \simeq 0.261\times 10^{-6}\times \left(10^{-21}/0.1826\times 10^{-17}\right)\times(0.75/2.34) \text{ rad/sid. day}^2$$

$$= 0.46\times 10^{-10} = 0.0000046\times 10^{-5} \text{ rad/sid. day}^2 \; .$$

Clearly, the long term acceleration effects of residual earth atmosphere on the 24-hour satellite gravity experiment are negligible.

Interplanetary Particles (Non Meteoritic)

Again, for a Syncom spacecraft mass of 0.75 slug and fully elastic collisions, Williams calculates the density of (solar stationary) interplanetary particles necessary to produce one degree of longitude drift in a year to be

$$\rho = 0.2432\times 10^{-18} \text{ slug/ft}^3 = 0.1257\times 10^{-18} \text{ gm/cc} \; ,$$

with the drift proportional to the density. From Reference E1, pp. 2-8, the interplanetary density is of the order of 10^{-22} gm/cc. Thus, for a Syncom spacecraft of 2.34 slugs, the drift acceleration due to solar stationary interplanetary particles should be of the order of

$$\ddot{\lambda} \text{ (interplanetary particles, non meteoritic)} \simeq 0.261\times 10^{-6}\times \left(10^{-22}/0.1257\times 10^{-18}\right)\times(0.75/2.34)$$

$$= 0.0000067\times 10^{-5} \text{ rad/sid. day}^2 \; .$$

In summary, the effects of residual atmospheric and solar-stationary interplanetary particle drag on the 24-hour satellite gravity experiment appear to be insignificant.

Solar Radiation Pressure

In Reference E2, Appendix F, it was shown that the solar radiation force on Syncom was about five orders of magnitude less than the solar gravity force. But the conclusion does not follow that, because of this, the solar radiation pressure *perturbations* are insignificant compared to solar gravity *perturbations*. The reason is that solar gravity perturbations of an earth satellite arise from the slightly different solar gravity accelerations experienced by the earth and its satellite. Solar radiation pressure caused accelerations of the earth, on the other hand, are extremely small because the earth presents to the sun a very small projected area/mass ratio. Thus between the two effects, we ought to compare the differences (between earth and satellite) of large accelerations due to solar gravity, with the small acceleration on the satellite alone due to the solar radiation pressure. It is by no means certain which effect will predominate for any given satellite. Experience with the Echo satellites has shown that for satellites with sufficiently large area/mass ratios, radiation pressure effects cannot be ignored.

Instead of making a direct comparison of solar gravity and radiation effects, since they do not act analogously (as was tacitly assumed in Reference E2), we shall merely calculate here the secular effect of radiation pressure pertinent to the 24-hour satellite gravity experiment.

In Reference E2, Appendix F, it is calculated that in August 1963 the radiation force on Syncom 2 (for full radiant energy absorption) was 4.13×10^{-7} pounds. With a spacecraft mass of 2.34 slugs, this gives an acceleration caused by radiation pressure of $4.13\times10^{-7}/2.34 = 1.77\times10^{-7}$ ft/sec^2. In Reference 7 on p. 13, for a spacecraft mass of 0.75 slug, the radiation acceleration is calculated as 4.30×10^{-7} ft/sec^2 for a fully reflecting equatorial spacecraft at the equinoxes. For a 2.34 slug equatorial Syncom, fully reflecting (with spin axis pointing north) at the equinoxes, the radiation acceleration would be $4.30\times10^{-7}\times(0.75/2.34) = 1.38\times10^{-7}$ ft/sec^2, since the radiation pressure acceleration is inversely proportional to the mass (at constant projected area). This latter acceleration actually represents the most conservative value for Syncom for our purposes since in this configuration the sun is in the orbit plane and no part of the solar pressure is wasted on "plane change effects".

Consider in Figure E1 the Syncom-earth-sun orbital geometry in this latter configuration (from Figures 2-4, Reference 7), where a is the earth rate about the sun, and θ_s is the orbital argument of Syncom with respect to the moving earth-sun line. At $t = 0$, $\theta_s = \theta_0$ which is the initial sun-satellite argument, for example, at the first ascending Equator crossing in an arc. Clearly, when θ_s is positive, solar pressure acts to retard the motion of the satellite. When θ_s is negative, the pressure adds energy to the satellite. In fact, from Figure E1 the energy adding (or tangential) force F (per unit mass)

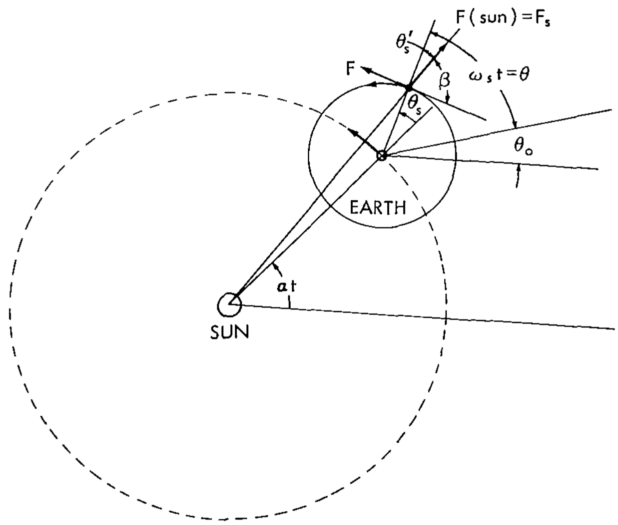

Figure E1—Equatorial Syncom-earth-sun orbital configuration at the equinoxes, looking south (after Williams).

on Syncom due to solar pressure is

$$F = -F_s \cos\beta = -F_s \sin\theta_s' \doteq -F_s \sin\theta_s$$

since $\theta_s' \doteq \theta_s$ due to the small "parallax" of the sun from the 24-hour orbit (~ 1 min of arc). From Figure E1 also

$$\theta_s = \omega_s t - \alpha t + \theta_0 \quad .$$

Thus

$$F \doteq -F_s \sin\left[(\omega_s - \alpha)\ t + \theta_0\right] = -F_s \left[\cos\theta_0 \sin(\omega_s - \alpha)\ t + \sin\theta_0 \cos(\omega_s - \alpha)\ t\right] \ .$$

The orbit averaged $(\omega_s\ t = 2\pi)$ solar pressure perturbing force (per unit mass) is then

$$\overline{F} = -\frac{F_s}{1 \text{ sid. day}} \int_0^{2\pi/\omega_s} \left[\cos\theta_0 \sin(\omega_s - \alpha)\ t + \sin\theta_0 \cos(\omega_s - \alpha)\ t\right] dt$$

$$= \frac{+F_s}{\omega_s - \alpha} \left|\cos\theta_0 \cos(\omega_s - \alpha)\ t - \sin\theta_0 \sin(\omega_s - \alpha)\ t\right|_0^{2\pi/\omega_s}$$

$$= \frac{F_s\left[\cos\left(\frac{2\pi\alpha}{\omega_s}\right) - 1\right]\cos\theta_0 + F_s\left[\sin\left(\frac{2\pi\alpha}{\omega_s}\right)\right]\sin\theta_0}{(\omega_s - \alpha)}$$

$$\doteq \frac{F_s}{\omega_s - \alpha}\left[\sin\left(\frac{2\pi\alpha}{\omega_s}\right)\right]\sin\theta, \quad \text{for } \frac{2\pi\alpha}{\omega_s} \text{ small} \ ,$$

with ω_s and α in units of rad/sid. day. But $\omega_s = 2\pi$ and $\alpha \doteq 2\pi/366 = 0.01715$. Therefore $2\pi\alpha/\omega_s \doteq 0.01715$ radians and $\omega_s - \alpha \doteq 6.26$ rad/sid. day. Thus

$$\overline{F} \text{ (solar pressure)} \doteq \frac{F_s \times 0.01715 \sin\theta_0}{6.26} = 2.74\times10^{-3}\ F_s \sin\theta \ ,$$

which is in units of acceleration (F_s). It is assumed in these calculations that F_s is a constant over the 24-hour orbit. For $F_s = 1.38\times10^{-7}$ ft/sec^2.

$\bar{F}_{max}$ (solar pressure on Syncom) $\doteq 3.78\times10^{-10}$ ft/sec^2, when $\theta_0 = 90°$. From Equation 10 in Reference E2, the long term longitude acceleration on the 24-hour satellite is given from $\bar{F}$ by

$$\ddot{\lambda} = \frac{-12\pi^2 \bar{F}}{\mu_e / a_s^2} = -\frac{12\pi^2 \bar{F}}{g_s} \left(\text{rad/sid. day}^2\right) \quad ,$$

where g_s is the radial gravity acceleration ($\doteq 0.7355$ ft/sec^2) on Syncom. Thus

$$\ddot{\lambda}_{max} (\text{due to solar pressure on Syncom satellites}) \doteq \frac{+12\pi^2}{0.7355} \times 3.26\times10^{-10} = 0.00609\times10^{-5} \text{ rad/sid. day}^2$$

It is evident that solar radiation pressure has negligible effect on the 24-hour satellite gravity experiment. The maximum total longitude excursion due to solar pressure in this conservative case accumulates over half a year and is approximately

$$\Delta\lambda_{max} \doteq \frac{1}{2}\left(0.637\times6.09\times10^{-8}\right) (183)^2 \times 57.3 \doteq 0.0372° \quad .$$

The factor 0.637 is the average of sin θ_0 for $0 < \theta_0 < \pi$, which is the range of θ_0 over half a year.

Summary

The effects of high altitude atmospheric particle drag and solar radiation pressure on the 24-hour satellite experiment should be entirely negligible.

REFERENCES

E1. Koelle, H. H., "Handbook of Astronautical Engineering," New York: McGraw-Hill, 1961.

E2. Wagner, C. A., "Determination of the Triaxiality of the Earth from Observations on the Drift of the Syncom 2 Satellite," GSFC Document X-621-64-90, April 1964.

Appendix F

Average Second Order Resonant Gravity Fields on the Geostationary Satellite

In the near future, many 24-hour equatorial satellites (mainly for communication purposes) will be placed and maintained in orbit at selected longitudes around the Equator. It will usually be necessary to keep these satellites within a longitude band about 20° wide or less depending on the location of the ground stations and the sophistication of the transmission and tracking equipment. Since the J_{22} gravity field is dominant at 24-hour altitudes, it may be convenient, in predicting the trajectory of these nearly "fixed" satellites between orbit corrections, to consider the average J_{22} field on the geostationary satellite over wide longitude arcs.

Average J_{22} gravity fields on the geostationary satellite are most naturally grouped into four longitude zones (of about 90° each) surrounding the four equilibrium longitudes (see Figure 9 and Section 3). To determine these average fields we first find the average of the peak accelerations in Figure 9 (without sign) on either side of these equilibrium longitudes. The average J_{22} in the region between these relative maxima is then determined by solving Equation 2 for J_{22} with all other gravity coefficients zero, $i_s = 0°$, $a_s = 6.611$ earth radii and $2(\lambda - \lambda_{22}) = 90°$. The effective λ_{22} in this region is given by the corresponding equilibrium longitude λ_e (see Section 3). The results of this calculation follow in Table F1 below.

Table F1

Average Second Order Resonant Gravity Fields on the Geostationary Satellite*.

Longitude Region	$\bar{J}_{22}$	$\bar{\lambda}_{22}$ (degrees)	Equilibrium Longitude (degrees)
$34° < \lambda < 118°$	-1.93×10^{-6}	-13.3	76.7
$118° < \lambda < 204°$	-1.87×10^{-6}	-18.2	161.8
$204° < \lambda < 300° (-60°)$	-1.73×10^{-6}	-18.1	251.9 (-108.1)
$-60° < \lambda < 34°$	-1.79×10^{-6}	-12.2	-12.2

*Representing the average effects of a geoid without higher order longitude gravity on a 24-hour equatorial satellite ($a_s = 6.611$ earth radii) in the given longitude region.

Appendix G

List of Symbols

a, a_s	Semimajor axis and synchronous semimajor axis of the orbit of a 24-hour satellite. (More specifically, a_s is defined in the text and tables of Section 1 as the average semimajor axis of the satellite during a drift arc.)
C_{nm}, S_{nm}	The cosine and sine parameters of the n, m gravity harmonic.
F	A gravity force per unit mass acting on a 24-hour satellite.
$F_{nm}(i)$	Inclination factor corresponding to the 24-hour drift caused by the n, m resonant gravity harmonic.
$F_{22}\left(i_s, a_s\right)$	A "constant" of the 24-hour drift motion caused by the H_{22} resonant gravity harmonic, a function of the average inclination and semimajor axis in the drift arc.
H_{nm}	Specifying the n, m earth gravity harmonic.
i, i_s	Orbit inclination and synchronous orbit inclination during a 24-hour drift arc.
J_{nm}, λ_{nm}	Amplitude and geographic phase of the n, m earth gravity harmonic.
R_0	The mean equatorial radius of the earth ($\doteq$ 6378.2 km).
s( )	Standard error of the bracketed quantity ( ).
λ, r, ϕ	Geographic longitude, geocentric radius, and geocentric latitude of the 24-hour satellite position.
μ_e	Earth's Gaussian gravity constant $\left(\doteq 3.986\times10^5 \frac{km^3}{sec^2}\right)$.

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