N66 38724
20 pages
code 1
TMX-57914
CAT 08
HC $1.00
MF .50

# COMPUTER DERIVATIONS OF EQUATIONS OF MOTION IN A GENERAL ORTHOGONAL CURVILINEAR COORDINATE SYSTEM USING THE FORMAC LANGUAGE

by

James C. Howard
Ames Research Center, NASA
Moffett Field, Calif.

## SUMMARY

The use of digital computers in performing non-numeric operations is rendered possible by the evolution of new computer languages such as Formac. In this paper the problem of using a digital computer to derive the equations of motion of a particle in a general orthogonal curvilinear coordinate system is considered. Since this operation involves a formulation in terms of first and second order differential coefficients, it provides a good demonstration of a computer's capability to do non-numeric work, and to assist in the formulation process which normally precedes the numerical data processing stage. Moreover, this particular problem serves to illustrate the advantages of the mathematical techniques employed. Because of the invariant nature of the formulation with respect to coordinate transformations, these techniques can be used to reduce complicated formulation problems to routine computer operations. In applying this procedure, the user need only know the coordinate transformation equations relating the curvilinear coordinates to an orthogonal Cartesian set. A computer program has been written to implement these ideas. When this program is used and the coordinate transformation equations are supplied as input, the computer will output the equations of motion. The equations of motion obtained will be relative to the curvilinear coordinate system specified by the coordinate transformation equations used

as input. Results are presented for the following curvilinear coordinate systems: spherical polar, cylindrical polar, oblate spheroidal and prolate spheroidal.

## INTRODUCTION

Research undertaken with the object of promoting man-computer interaction has directed attention to the use of computers for non-numeric operations. In particular, the possibility of using digital computers to derive the equations of motion and of mathematical physics in a general curvilinear coordinate system has been explored. Traditionally, these functions were considered to be the exclusive preserve of the scientist. Nevertheless, as is shown in this paper, digital computers can participate in the performance of such tasks. However, if the extensive logic and storage capabilities of these computers are to be used to full advantage, a departure from conventional techniques of formulation may be necessary. The extent to which conventional methods should be modified to enable digital computers to participate effectively in non-numeric operations has been examined. For example, when conventional methods are used, the form which the equations of motion and of mathematical physics assumes depends on the coordinate system used to describe the problem. This dependence, which is due to the practice of expressing vectors in terms of their physical components, can be removed by the simple expedient of expressing all vectors in terms of their tensor components. As a consequence of the geometrical simplification inherent in the tensor method, the operations involved in formulating problems in unfamiliar curvilinear coordinate systems can be reduced to routine computer operations. It is this aspect of the tensor method which makes it so attractive for the types of computer applications contemplated in this paper.

## NOMENCLATURE

| | |
|---|---|
| M | mass of particle |
| P(i) | $\frac{d^2x^i}{dt^2}$ |
| R(i) | $\frac{dx^i}{dt}$ |
| t | time |
| $x^i$ | system coordinates |
| $y^i$ | system coordinates |
| $\tau^i$ | physical component of force |
| $\varphi$ | potential function |
| $\nabla\varphi$ | gradient of potential function |

Supercripts

| | |
|---|---|
| $\alpha,i,j,k$ | indices of contravariance |

## ANALYSIS

A formulation of the equations of motion of a point mass, which is valid in all orthogonal curvilinear coordinate systems, may be obtained by expressing all relevant vectors in terms of their tensor components, rather than in terms of their physical components. This formulation gives rise to the tensor equations of motion. In order to indicate the method of dealing with mixed quantities, part of the force system is assumed to be given in the form of the gradient of a potential function. Such a force assumes the covariant form (ref. 1). The existence of forces which are known only in terms of their physical components is also assumed. Moreover, since acceleration and velocity are contravariant vectors, the equations of motion have to be formulated from a system of covariant, contravariant and physical quantities. Since compatibility requires that the two sides of every equation must balance with respect to their covariant or contravariant properties, it is necessary to convert all the force terms to the contravariant form. When appropriate conversions are made, the resulting equations are in a form well suited to non-numeric computer operations.

Omitting the details of the derivation, the procedure may be described as follows: let the coordinate transformation equations relating a curvilinear coordinate system $x$, to an orthogonal Cartesian system $y$, be given by

$$y^i = y^i(x^1x^2x^3) \qquad i = 1,2,3 \tag{1}$$

In terms of the first and second order partial differential coefficients of $y$ with respect to $x$, the equations of motion of a particle of mass $M$ relative to the curvilinear coordinate system $x$ assume the following form:

$$M\left[\left(\frac{\partial y^\alpha}{\partial x^{(i)}}\frac{\partial y^\alpha}{\partial x^{(i)}}\right)\frac{d^2x^i}{dt^2} + \left(\frac{\partial^2 y^\alpha}{\partial x^j \partial x^k}\frac{\partial y^\alpha}{\partial x^i}\right)\frac{dx^j}{dt}\frac{dx^k}{dt}\right] = \left(\frac{\partial\varphi}{\partial x^i} + \sqrt{\frac{\partial y^\alpha}{\partial x^{(i)}}\frac{\partial y^\alpha}{\partial x^{(i)}}}\,\tau^i\right) \tag{2}$$

where $i,j,k,\alpha = 1,2,3$ and the summation convention for indices is assumed to be operative. That is, if a given index occurs twice in any expression, the expression must be summed with respect to that index. An exception to this rule occurs when repeated indices are enclosed in parentheses. Parentheses around an index imply that the summation convention is to be suspended for that index. This means that for each value of "i," equation (2) must be summed on $\alpha, j$, and $k$. For example, when equation (2) is summed on $\alpha$, it appears as follows:

$$M\left[\left(\frac{\partial y^1}{\partial x^{(i)}}\frac{\partial y^1}{\partial x^{(i)}}+\frac{\partial y^2}{\partial x^{(i)}}\frac{\partial y^2}{\partial x^{(i)}}+\frac{\partial y^3}{\partial x^{(i)}}\frac{\partial y^3}{\partial x^{(i)}}\right)\frac{d^2x^i}{dt^2}\right.$$

$$\left.+\left(\frac{\partial^2 y^1}{\partial x^j\partial x^k}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^j\partial x^k}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^j\partial x^k}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^j}{dt}\frac{dx^k}{dt}\right]$$

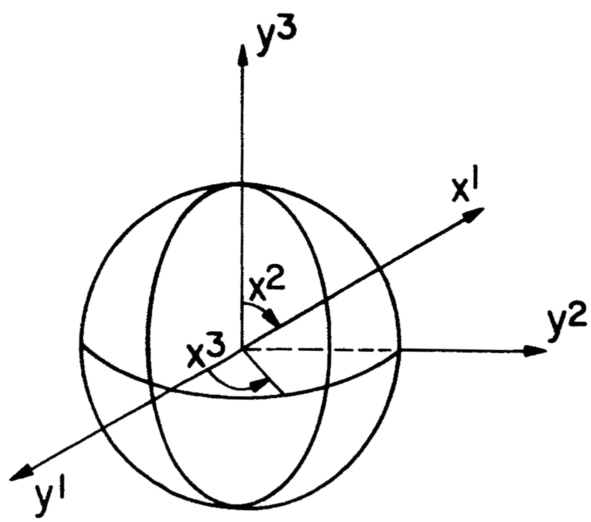
$$=\left(\frac{\partial\varphi}{\partial x^i}+\sqrt{\frac{\partial y^1}{\partial x^{(i)}}\frac{\partial y^1}{\partial x^{(i)}}+\frac{\partial y^2}{\partial x^{(i)}}\frac{\partial y^2}{\partial x^{(i)}}+\frac{\partial y^3}{\partial x^{(i)}}\frac{\partial y^3}{\partial x^{(i)}}}\,\tau^i\right) \tag{3}$$

The left side of this equation must also be summed on $j$ and $k$. When each of these indices is permitted to take the values 1,2,3, in turn, equation (3) assumes the following form:

$$
\begin{aligned}
& M\Bigg[\left(\frac{\partial y^1}{\partial x^{(i)}}\frac{\partial y^1}{\partial x^{(i)}}+\frac{\partial y^2}{\partial x^{(i)}}\frac{\partial y^2}{\partial x^{(i)}}+\frac{\partial y^3}{\partial x^{(i)}}\frac{\partial y^3}{\partial x^{(i)}}\right)\frac{d^2x^i}{dt^2} \\
& +\left(\frac{\partial^2 y^1}{\partial x^1\partial x^1}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^1\partial x^1}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^1\partial x^1}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^1}{dt}\frac{dx^1}{dt} \\
& +\left(\frac{\partial^2 y^1}{\partial x^1\partial x^2}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^1\partial x^2}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^1\partial x^2}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^1}{dt}\frac{dx^2}{dt} \\
& +\left(\frac{\partial^2 y^1}{\partial x^1\partial x^3}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^1\partial x^3}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^1\partial x^3}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^1}{dt}\frac{dx^3}{dt} \\
& +\left(\frac{\partial^2 y^1}{\partial x^2\partial x^1}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^2\partial x^1}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^2\partial x^1}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^2}{dt}\frac{dx^1}{dt} \\
& +\left(\frac{\partial^2 y^1}{\partial x^2\partial x^2}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^2\partial x^2}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^2\partial x^2}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^2}{dt}\frac{dx^2}{dt} \\
& +\left(\frac{\partial^2 y^1}{\partial x^2\partial x^3}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^2\partial x^3}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^2\partial x^3}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^2}{dt}\frac{dx^3}{dt} \\
& +\left(\frac{\partial^2 y^1}{\partial x^3\partial x^1}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^3\partial x^1}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^3\partial x^1}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^3}{dt}\frac{dx^1}{dt} \\
& +\left(\frac{\partial^2 y^1}{\partial x^3\partial x^2}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^3\partial x^2}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^3\partial x^2}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^3}{dt}\frac{dx^2}{dt} \\
& +\left(\frac{\partial^2 y^1}{\partial x^3\partial x^3}\frac{\partial y^1}{\partial x^i}+\frac{\partial^2 y^2}{\partial x^3\partial x^3}\frac{\partial y^2}{\partial x^i}+\frac{\partial^2 y^3}{\partial x^3\partial x^3}\frac{\partial y^3}{\partial x^i}\right)\frac{dx^3}{dt}\frac{dx^3}{dt}\Bigg] \\
& =\left(\frac{\partial \phi}{\partial x^{(i)}}+\sqrt{\frac{\partial y^1}{\partial x^{(i)}}\frac{\partial y^1}{\partial x^{(i)}}+\frac{\partial y^2}{\partial x^{(i)}}\frac{\partial y^2}{\partial x^{(i)}}+\frac{\partial y^3}{\partial x^{(i)}}\frac{\partial y^3}{\partial x^{(i)}}}\;\tau^i\right) \qquad (4)
\end{aligned}
$$

This equation is in a form well suited to routine non-numeric computer operations. The large number of terms appearing in equation (4) is due to the generality of this equation, which is applicable to any space of three dimensions. Moreover, since this equation is applicable to any space of three dimensions, it may be permanently stored in the computer. Hence, in order to obtain the equations of motion in any system of coordinates, the only information required is the special form of equation (1) relating that system of coordinates to the orthogonal Cartesian coordinates $y^i$. For example, consider a transformation of coordinates specifying the relation between the spherical polar coordinates $x^i$ and the orthogonal Cartesian coordinates $y^i$. In this case, equation (1) becomes: See sketch (a).

$$y^1 = x^1 \sin x^2 \cos x^3$$
$$y^2 = x^1 \sin x^2 \sin x^3$$
$$y^3 = x^1 \cos x^2$$




sketch (a)

These coordinate transformation equations were supplied as input to an IBM 7094 computer, which was programmed for non-numeric operations. When the computer was instructed to perform the operations involved in equation (4), the following output was obtained in Fortran language.

COMPUTER OUTPUT

```
FOR I = 1

The expression input for  Y(I) is given below.

X(1)*FMCSIN(X(2))*FMCCOS(X(3)$

FOR I = 2

The expression input for  Y(I) is given below.

X(1)*FMCSIN(X(2))*FMCSIN(X(3))$

FOR I = 3

The expression input for  Y(I) is given below.

X(1)*FMCCOS(X(2))$
```

Equations of Motion

```
FOR I = 1

The equation for  I = 1 is given below.

M*(P(1)-R(2)**2.0*X(1)-R(3)**2.0*X(1)*FMCSIN(X(2))**2.0)$

=DPHI(1)+TAU(1)$

FOR I = 2

The equation for  I = 2 is given below.

M*(P(2)*X(1)**2.0+R(1)*R(2)*X(1)*2.0-R(3)**2.0*X(1)**2.0*FMCSIN(X(2))*FMCCOS(X(2)))$

=DPHI(2)+TAU(2)*X(1)$

FOR I = 3

The equation for  I = 3 is given below.

M*(P(3)*X(1)**2.0*FMCSIN(X(2))**2.0+R(1)*R(3)*X(1)*FMCSIN(X(2))**2.0*2.0+R(2)*R(3)*
X(1)**2.0*FMCSIN(X(2))*FMCCOS(X(2))*2.0)$

=DPHI(3)+TAU(3)*X(1)*FMCSIN(X(2))$
```

JOB ACCOUNTING

| TIME ON | COMP/LOAD TIME MIN. | EXECUTIVE TIME MIN. |
|---|---|---|
| 950.09 | .78 | .41 |

In interpreting these Fortran statements, it must be remembered that:

$$R(i) = \frac{dx^i}{dt}$$

$$P(i) = \frac{d^2x^i}{dt^2}$$

In terms of conventional mathematical symbolism, these equations assume the following form:

$$M \left[ \frac{d^2x^1}{dt^2} - x^1 \left( \frac{dx^2}{dt} \right)^2 - x^1 \left( \sin x^2 \frac{dx^3}{dt} \right)^2 \right] = \frac{\partial \varphi}{\partial x^1} + \tau^1$$

$$M \left[ (x^1)^2 \frac{d^2x^2}{dt^2} + 2x^1 \frac{dx^1}{dt} \frac{dx^2}{dt} - (x^1)^2 \sin x^2 \cos x^2 \left( \frac{dx^3}{dt} \right)^2 \right] = \left( \frac{\partial \varphi}{\partial x^2} + x^1 \tau^2 \right)$$

$$M \left[ \left( x^1 \sin x^2 \right)^2 \frac{d^2x^3}{dt^2} + 2x^1 \sin^2 x^2 \frac{dx^1}{dt} \frac{dx^3}{dt} + 2(x^1)^2 \sin x^2 \cos x^2 \frac{dx^2}{dt} \frac{dx^3}{dt} \right]$$

$$= \left( \frac{\partial \varphi}{\partial x^3} + x^1 \sin x^2 \tau^3 \right)$$

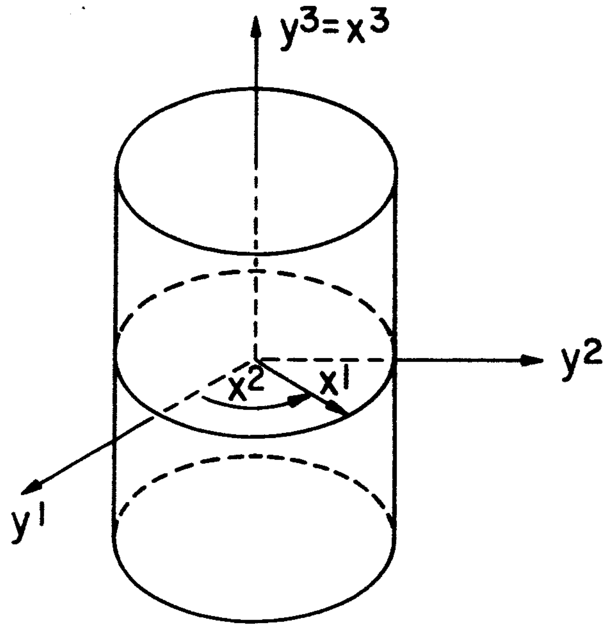
Because of its generality, equation (4) is applicable in all coordinate systems. Therefore, in order to obtain the equations of motion in any other coordinate system, all that is required is to supply the computer with the appropriate coordinate transformation equations. As a further illustration of the procedure involved, consider the equations of motion in a cylindrical polar system of coordinates. In this case, the coordinate transformation equations are: see sketch (b).

$$y^1 = x^1 \cos x^2$$

$$y^2 = x^1 \sin x^2$$

$$y^3 = x^3$$

When these coordinate transformation equations were used to evaluate the terms of equation (4), the following computer output was obtained.




sketch (b)

COMPUTER OUTPUT

FOR I = 1

The expression input for Y(I) is given below.

X(1)*FMCCOS(X(2)) $

FOR I = 2

The expression input for Y(I) is given below.

```
X(1)*FMCSIN(X(2)) $
```

FOR I =3

The expression input for Y(I) is given below.

```
X(3) $
```

EQUATIONS OF MOTION

The equation for I = 1 is given below.

```
M*(P(1)-R(2)**2.0*X(1))$
=DPHI(1)+TAU(1)$
```

The equation for I = 2 is given below.

```
M*(P(2)*X(1)**2.0+R(1)*R(2)*X(1)*2.0)$
=DPHI(2)+TAU(2)*X(1)$
```

The equation for I = 3 is given below.

```
M*P(3)$
=DPHI(3)+TAU(3)$
```

JOB ACCOUNTING

| TIME ON | COMP/LOAD TIME MIN. | EXECUTIVE TIME MIN. |
|---|---|---|
| 037.78 | 1.15 | .14 |

Translating these equations from Fortran language to conventional mathematical symbolism yields the following:

$$\left.\begin{aligned} M\left[\frac{d^2x^1}{dt^2} - x^1\frac{dx^2}{dt}\frac{dx^2}{dt}\right] &= \frac{\partial\varphi}{\partial x^1} + \tau^2 \\ M\left[\left(x^1\right)^2\frac{d^2x^2}{dt^2} + 2x^1\frac{dx^1}{dt}\frac{dx^2}{dt}\right] &= \frac{\partial\varphi}{\partial x^2} + x^1\tau^2 \\ M\left[\frac{d^2x^3}{dt^2}\right] &= \frac{\partial\varphi}{\partial x^3} + \tau^3 \end{aligned}\right\} \qquad (5)$$

Prolate Spheroidal Coordinates

Another interesting system of orthogonal curvilinear coordinates are the prolate spheroidal coordinates. Coordinate surfaces are obtained by rotating a family of confocal ellipses and hyperbolae about their major axes. Rotation of these conic sections gives rise to a system of prolate spheroids and hyperboloids of two sheets. A family of planes through the axis of rotation completes the system of orthogonal surfaces. The curvilinear coordinate systems generated by the curves of intersection of these surfaces are useful in certain quantum mechanical problems. (Ref. 2). The transformation equations relating this system of coordinates to the orthogonal Cartesian system are as follows:

$$y^1 = a \sinh x^1 \sin x^2 \cos x^3$$
$$y^2 = a \sinh x^1 \sin x^2 \sin x^3$$
$$y^3 = a \cosh x^1 \cos x^2$$

In order to obtain the equations of motion relative to a prolate spheroidal system of coordinates, these transformation equations were substituted for equation (1) in the computer program. Execute time was 1.63 minutes. Omitting the print-out in Fortran language, the equations of motion were obtained as follows:

$$M\left[a^2(\sin^2 x^2 + \sinh^2 x^1)\frac{d^2x^1}{dt^2} + 2a^2 \sin x^2 \cos x^2 \frac{dx^1}{dt}\frac{dx^2}{dt}\right.$$

$$+ a^2 \sinh x^1 \cosh x^1 \frac{dx^1}{dt}\frac{dx^1}{dt} - a^2 \sinh x^1 \cosh x^1 \frac{dx^2}{dt}\frac{dx^2}{dt}$$

$$\left. - a^2 \sin^2 x^2 \sinh x^1 \cosh x^1 \frac{dx^3}{dt}\frac{dx^3}{dt}\right]$$

$$= a\sqrt{(\sin^2 x^2 + \sinh^2 x^1)}\,\tau^1 + \frac{\partial\varphi}{\partial x^1}$$

$$M\left[a^2(\sin^2 x^2 + \sinh^2 x^1)\frac{d^2x^2}{dt^2} - a^2 \sin x^2 \cos x^2 \frac{dx^1}{dt}\frac{dx^1}{dt}\right.$$

$$+ 2a^2 \sinh x^1 \cosh x^1 \frac{dx^1}{dt}\frac{dx^2}{dt} + a^2 \sin x^2 \cos x^2 \frac{dx^2}{dt}\frac{dx^2}{dt}$$

$$\left. - a^2 \sin x^2 \cos x^2 \sinh^2 x^1 \frac{dx^3}{dt}\frac{dx^3}{dt}\right]$$

$$= a\left(\sqrt{\sin^2 x^2 + \sinh^2 x^1}\right)\tau^2 + \frac{\partial\varphi}{\partial x^2}$$

$$M\left[a^2 \sin^2 x^2 \sinh^2 x^1 \frac{d^2x^3}{dt^2} + 2a^2 \sin^2 x^2 \sinh x^1 \cosh x^1 \frac{dx^1}{dt}\frac{dx^3}{dt}\right.$$

$$\left. + 2a^2 \sin x^2 \cos x^2 \sinh^2 x^1 \frac{dx^2}{dt}\frac{dx^3}{dt}\right]$$

$$= a \sin x^2 \sinh x^1\, \tau^3 + \frac{\partial\varphi}{\partial x^3}$$

## Oblate Spheroidal Coordinates

If a family of confocal ellipses and hyperbolae are rotated about their minor axes, a system of surfaces is generated. These surfaces are the oblate spheroids and hyperboloids of one sheet (ref. 3). These surfaces, together with a family of planes through the axis of rotation, constitute a family of orthogonal surfaces. The curvilinear coordinate systems generated by the curves of intersection of these surfaces are called oblate spheroidal coordinates. Oblate spheroids are sometimes referred to as planetary ellipsoids, because the Earth and the planet Jupiter are approximately of this form. The transformation equations relating this system of coordinates to the orthogonal Cartesian system are as follows:

$$y^1 = a \cosh x^1 \sin x^2 \cos x^3$$

$$y^2 = a \cosh x^1 \sin x^2 \sin x^3$$

$$y^3 = a \sinh x^1 \cos x^2$$

These transformation equations take the place of equation (1) in the computer. In this case, the time required to execute the program was 1.63 minutes. Omitting the print-out in Fortan language, the equations of motion relative to a system of oblate spheroidal coordinates were obtained in the following form:

$$M\left[a^2(\sinh^2 x^1 + \cos^2 x^2)\frac{d^2x^1}{dt^2} + a^2(\sinh x^1 \cosh x^1)\frac{dx^1}{dt}\frac{dx^1}{dt}\right.$$

$$- 2a^2 \cos x^2 \sin x^2 \frac{dx^1}{dt}\frac{dx^2}{dt} - a^2 \sinh x^1 \cosh x^1 \frac{dx^2}{dt}\frac{dx^2}{dt}$$

$$\left.- a^2 \cosh x^1 \sinh x^1 \sin^2 x^2 \frac{dx^3}{dt}\frac{dx^3}{dt}\right]$$

$$= a\left(\sqrt{\sinh^2 x^1 + \cos^2 x^2}\right)\tau^1 + \frac{\partial\varphi}{\partial x^1}$$

$$M\left[a^2(\sinh^2 x^1 + \cos^2 x^2)\frac{d^2x^2}{dt^2} + a^2 \sin x^2 \cos x^2 \frac{dx^1}{dt}\frac{dx^1}{dt}\right.$$

$$+ 2a^2 \sinh x^1 \cosh x^1 \frac{dx^1}{dt}\frac{dx^2}{dt} - a^2 \sin x^2 \cos x^2 \frac{dx^2}{dt}\frac{dx^2}{dt}$$

$$\left.- a^2 \cosh^2 x^1 \sin x^2 \cos x^2 \frac{dx^3}{dt}\frac{dx^3}{dt}\right]$$

$$= a\sqrt{(\sinh^2 x^1 + \cos^2 x^2)}\,\tau^2 + \frac{\partial\varphi}{\partial x^2}$$

$$M\left[a^2 \cosh^2 x^1 \sin^2 x^2 \frac{d^2x^3}{dt^2} + 2a^2 \sinh x^1 \cosh x^1 \sin^2 x^2 \frac{dx^1}{dt}\frac{dx^3}{dt}\right.$$
$$\left. + 2a^2 \cosh^2 x^1 \sin x^2 \cos x^2 \frac{dx^2}{dt}\frac{dx^3}{dt}\right]$$
$$= a \cosh x^1 \sin x^2 \tau^3 + \frac{\partial\varphi}{\partial x^3}$$

## CONCLUSIONS

Digital computers can be used to perform a wide range of non-numeric operations if they are properly programmed. Research indicates that these computers can be used more effectively for this purpose, if all vector quantities are expressed in terms of their tensor components rather than in terms of their physical components. It is a consequence of the geometrical simplification inherent in the tensor method, that the operations involved in formulating problems in unfamiliar curvilinear coordinate systems can be reduced to routine computer operations. To implement the proposed method, a digital computer program has been written to perform a variety of non-numeric operations. In order to illustrate the ideas embodied in this report, the program has been used to derive the equations of motion of a point mass in any coordinate system requested by the user. The results are presented in Fortran language. However, for the convenience of readers, the Fortran statements are translated to conventional mathematical symbolism. The exploitation and extension of these techniques should lead to a substantial reduction in the man hours required to formulate and process engineering and scientific problems.

## REFERENCES

1. Howard, James C.: Application of Intrinsic Differentiation to Orbital Problems Involving Curvilinear Coordinate Systems. NASA TN D-2861, p. 4.

2. Margenaw, H., and Murphy, G. M.: The Mathematics of Physics and Chemistry. D. Van Nostrand Co., Inc., pp. 5-7.

3. Korn, G. A. and Korn, T. M.: Mathematical Handbook for Scientists and Engineers. McGraw-Hill Book Co., Inc., 1961, p. 177.