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NONSTEADY LIQUID AND GAS FLOW WITH HEAT ADDITION AND SHOCK PERTURBATIONS

by Fred S. Sidransky and Margaret Marie Smith

Lewis Research Center
Cleveland, Ohio

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SUMMARY

The theory of one-dimensional nonhomentropic nonsteady fluid flow is developed independently of a specific form of the equation of state. The method of characteristics is used to develop general compatibility relations which are applicable to either liquids or gases. By assuming specific equations of state, the classical water-hammer equations of Joukowski and Allievi and the nonsteady gas flow relationships of Riemann are deduced from the general compatibility relations.

General numerical methods, based on the method of characteristics, are discussed for the two characteristic network for nonsteady liquid flow and the three characteristic network for nonsteady gas flow. Computer programs, utilizing these numerical procedures, are also given.

To illustrate the versatility and to corroborate in part these methods of solving nonsteady flow problems, three examples were selected: (1) nonsteady liquid flow, (2) nonsteady gas flow with heat addition (or removal), (3) shock perturbations in a supersonic diffuser. The first example is verified by an alternate technique. The second example is verified in part by the Rayleigh analysis for steady flow in constant area ducts with heating or cooling.

INTRODUCTION

The analysis and study of nonsteady liquid flow and nonsteady gas flow have tended to diverge into two branches of fluid mechanics (refs. 1 and 2). This division has arisen quite naturally because of engineering requirements solely in hydraulics such as in the analysis of water-hammer in penstocks, and solely in gas dynamics such as in the analysis of shock tubes and pulse jets. In the analysis of advanced systems, for example, rocket engines and Rankine cycle space power systems, the dynamics of the system is governed in large measure by the interdependent nonsteady flow characteristics of fluids in different phases. A representation of transients in such systems is therefore dependent on an understanding of the dynamics of fluids in both the liquid and gas phases.

In this report, it will be shown, using the method of characteristics, that the classical water-hammer equations of Joukowski and Allievi and the nonsteady gas flow relationships of Riemann can be deduced from a general theory for one-dimensional nonhomentropic nonsteady fluid flow. (In homentropic flow, the entropy of each fluid particle is equal to the entropy of any other particle in the flow region; whereas in isentropic flow, the entropy of each particle is constant but may be different from any other particle. Hence a flow may be isentropic but nonhomentropic since different particles may have different entropies.)

To facilitate the use of the theory for the analysis of nonsteady nonhomentropic fluid flow, numerical procedures, derived from the general theory and adaptable to high-speed computers, are discussed. More particular attention is given to nonhomentropic nonsteady gas flow than to liquid flow, owing to its greater complexity. (A parallel to this characteristic problem may be found in supersonic rotational flow.)

To illustrate and corroborate these numerical methods, computer programs, which are given in appendixes C and D, were developed at the Lewis Research Center, and the three examples selected are as follows: (1) nonsteady liquid flow (or water-hammer), (2) nonsteady gas flow with heat addition (or removal), (3) shock perturbations in a supersonic diffuser. As a check on the accuracy of the numerical methods, the first example is verified by an alternate technique (ref. 3), and the second is checked in part by the Rayleigh analysis for steady flow in constant area ducts with heating and cooling.

THEORY

General Compatibility Relations

The fundamental relation for continuity is given by

$$v \frac{\partial \rho}{\partial y} + \rho \frac{\partial v}{\partial y} + \frac{\partial \rho}{\partial t} = 0 \tag{1}$$

(Symbols are defined in appendix A.) Conservation of momentum (omitting body and dissipative forces) yields

$$v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial y} \tag{2}$$

If reversible heat addition along the path of a fluid particle of fixed identity is assumed, the following is obtained:

$$v \frac{\partial s}{\partial y} + \frac{\partial s}{\partial t} = \frac{1}{T} \frac{DQ}{Dt} = \frac{Ds}{Dt} = \psi \tag{3}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial y}$$

(For a detailed discussion of these basic equations, see refs. 4, 5, and 6.) The density ρ, which is some unspecified function of the entropy s and the pressure P, is expressed as

$$\rho = \rho(s, P) \tag{4}$$

By equation (4), it follows that

$$\frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial P} \frac{\partial P}{\partial y} + \frac{\partial \rho}{\partial s} \frac{\partial s}{\partial y} \tag{5}$$

and

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial P} \frac{\partial P}{\partial t} + \frac{\partial \rho}{\partial s} \frac{\partial s}{\partial t} \tag{6}$$

and inasmuch as

$$\left(\frac{\partial \rho}{\partial P}\right)_s = \frac{1}{a^2} \tag{7}$$

equation (1) becomes

$$\frac{v}{a^2} \frac{\partial P}{\partial y} + v \frac{\partial \rho}{\partial s} \frac{\partial s}{\partial y} + \rho \frac{\partial v}{\partial y} + \frac{1}{a^2} \frac{\partial P}{\partial t} + \frac{\partial \rho}{\partial s} \frac{\partial s}{\partial t} = 0 \tag{8}$$

Following reference 4, let

$$\xi = \xi(y, t) \tag{9}$$

$$\eta = \eta(y, t) \tag{10}$$

Equations (9) and (10) are used to transform equations (2), (8), and (3) to the following:

$$\frac{\partial P}{\partial \xi} \frac{1}{\rho} \frac{\partial \xi}{\partial y} + \frac{\partial v}{\partial \xi}\left(v \frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right) = -\frac{\partial P}{\partial \eta} \frac{1}{\rho} \frac{\partial \eta}{\partial y} - \frac{\partial v}{\partial \eta}\left(v \frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) \tag{11}$$

$$\frac{\partial P}{\partial \xi} \frac{1}{a^2}\left(v \frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right) + \frac{\partial v}{\partial \xi} \rho \frac{\partial \xi}{\partial y} + \frac{\partial s}{\partial \xi} \frac{\partial \rho}{\partial s}\left(v \frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right) = -\frac{\partial P}{\partial \eta} \frac{1}{a^2}\left(v \frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) - \frac{\partial v}{\partial \eta} \rho \frac{\partial \eta}{\partial y} - \frac{\partial s}{\partial \eta}\left(v \frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) \frac{\partial \rho}{\partial s} \tag{12}$$

$$\frac{\partial s}{\partial \xi}\left(v \frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right) = -\frac{\partial s}{\partial \eta}\left(v \frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) + \psi \tag{13}$$

From these equations, the compatibility relations and their corresponding characteristic directions are determined using the theory of characteristics as discussed in reference 4 and in appendix A of reference 6 (vol. I). The compatibility relations will yield the properties of the flow on a characteristic. The intersection of the characteristics indicates the position of a net point on the time-distance plane. The general compatibility relations, as derived in appendix B of this report, are given by

$$\frac{1}{\rho a} \frac{\delta^+ P}{\delta t} + \frac{\delta^+ v}{\delta t} = -a\psi \frac{\partial(\ln \rho)}{\partial s} \tag{14}$$

for the characteristic having a positive slope,

$$\frac{1}{\rho a} \frac{\delta^- P}{\delta t} - \frac{\delta^- v}{\delta t} = -a\psi \frac{\partial(\ln \rho)}{\partial s} \tag{15}$$

for the characteristic having a negative slope in subsonic flow, and

$$\frac{Ds}{Dt} = \psi \tag{16}$$

for the particle path. The directional derivatives $\frac{\delta^+}{\delta t}$ and $\frac{\delta^-}{\delta t}$ appearing in the general compatibility equations (eqs. (14) and (15)) are defined by

$$\frac{\delta^+}{\delta t} = \frac{\partial}{\partial t} + (v + a) \frac{\partial}{\partial y} \tag{17}$$

$$\frac{\delta^-}{\delta t} = \frac{\partial}{\partial t} + (v - a) \frac{\partial}{\partial y} \tag{18}$$

The characteristic slopes corresponding to the general compatibility equations (eqs. (14), (15), and (16)) are defined, respectively, by

$$\frac{dy}{dt} = v + a \tag{19}$$

$$\frac{dy}{dt} = v - a \tag{20}$$

$$\frac{dy}{dt} = v \tag{21}$$

Because the compatibility equations as expressed in equations (14), (15), and (16) are not functions of any particular equation of state, they may be used for the one-dimensional nonsteady nonhomentropic flow analysis of either a liquid or a gas. Indeed, it can be shown that both the classical water-hammer equations of Joukowski and Allievi (ref. 2) and the nonsteady gas flow relations of Riemann (ref. 5) can be deduced from these compatibility relations.

Liquid Dynamics

If it is assumed that in a liquid the percent change of density with entropy change is negligibly small, then it may be assumed that

$$\frac{\partial(\ln \rho)}{\partial s} = 0 \tag{22}$$

Thus the compatibility equations become

$$\frac{1}{\rho a}\frac{\delta^+ P}{\delta t} + \frac{\delta^+ v}{\delta t} = 0 \tag{23}$$

and

$$\frac{1}{\rho a}\frac{\delta^- P}{\delta t} - \frac{\delta^- v}{\delta t} = 0 \tag{24}$$

Since there are only two equations with two unknowns (viz., the pressure P and the velocity v), the third compatibility relation (eq. (16)) may be omitted.

The compatibility relations (eqs. (23) and (24)) may be presented in a more familiar form by defining the head H as the pressure divided by the specific weight of the fluid on the Earth's surface (here, the assumption is that the datum or reference pressure is zero); then

$$\frac{P}{\rho} = g_c H \tag{25}$$

where g_c = 32.2 feet per second squared. If the volume flow is obtained from

$$q = Fv \tag{26}$$

then the compatibility relations become

$$\frac{\delta^+ H}{\delta t} = -\frac{a}{g_c F}\frac{\delta^+ q}{\delta t} \tag{27}$$

$$\frac{\delta^- H}{\delta t} = \frac{a}{g_c F}\frac{\delta^- q}{\delta t} \tag{28}$$

which are the Joukowski water-hammer relations in terms of head and volume flow.

The fundamental or canonical water-hammer relations may be derived from equations (27) and (28). If the velocity v is assumed to be negligible as compared to the acoustic velocity, equations (27) and (28) become, respectively,

$$g_c \frac{\partial H}{\partial t} + \frac{a^2}{F}\frac{\partial q}{\partial y} + ag_c \frac{\partial H}{\partial y} + \frac{a}{F}\frac{\partial q}{\partial t} = 0 \tag{29}$$

$$g_c \frac{\partial H}{\partial t} + \frac{a^2}{F}\frac{\partial q}{\partial y} - ag_c \frac{\partial H}{\partial y} - \frac{a}{F}\frac{\partial q}{\partial t} = 0 \tag{30}$$

after expanding according to the definitions of the directional derivatives. Equations (29) and (30) are obviously true if

$$g_c \frac{\partial H}{\partial t} = -\frac{a^2}{F}\frac{\partial q}{\partial y} \tag{31}$$

$$g_c \frac{\partial H}{\partial y} = -\frac{1}{F}\frac{\partial q}{\partial t} \tag{32}$$

which are known as the canonical water-hammer equations (ref. 2). If the acoustic veloctiy is a constant, these canonical equations may be shown to be but another form of the classical wave equation

$$\frac{\partial^2 H}{\partial y^2} = \frac{1}{a^2}\frac{\partial^2 H}{\partial t^2} \tag{33}$$

Thus for hydraulic systems in which the flow velocity is small as compared to the acoustic velocity and in which the acoustic velocity does not vary along a characteristic, the foregoing system of equations is applicable. This, it should be noted, does not exclude the possibility that different characteristics in the flow region may have different acoustic velocities as may exist in a duct with discrete cross-sectional area discontinuities. In a hydraulic system with a gradually varying area conduit and in which body and dissipative forces are significant, the compatibility relations may be rederived from the fundamental equations (viz., eqs. (1), (2), and (3)) and put in a more general form. The canonical equations (eqs. (31) and (32)) are not applicable in this case

Gas Dynamics

The assumption of equation (22) is, of course, untenable in a perfect gas. To evaluate $\frac{\partial(\ln \rho)}{\partial s}$ for a gas, first consider that

$$ds = C_p\, d(\ln T) - \frac{R}{g_c J}\, d(\ln P) \tag{34}$$

from thermodynamics and the perfect gas law. Inasmuch as

$$C_p = \left(\frac{\gamma}{\gamma - 1}\right) \frac{R}{g_c J} \tag{35}$$

and if γ is assumed constant, equation (34) may be transformed to

$$\frac{g_c J}{R}\, ds = \frac{2\gamma}{\gamma - 1}\, d(\ln a) - d(\ln P) \tag{36}$$

since

$$a = \sqrt{\gamma R T} \tag{37}$$

Now from equation (37) and the equation of state

$$P = \frac{\rho}{\gamma}\, a^2 \tag{38}$$

Hence, instead of equation (36)

$$\frac{g_c J}{R}\, ds = \frac{2\gamma}{\gamma - 1}\, d(\ln a) - 2\, d(\ln a) - d(\ln \rho) \tag{39}$$

is obtained. Equation (39) may be simplified to

$$d(\ln \rho) = \left(\frac{1}{\gamma - 1}\right) \frac{da^2}{a^2} - \frac{g_c J}{R}\, ds \tag{40}$$

By equations (35) and (37), and since for a perfect gas

$$h = C_p T \tag{41}$$

equation (40) may be rewritten as

$$d(\ln \rho) = \frac{g_c J}{\gamma R} \frac{1}{T} dh - \frac{g_c J}{R} ds \tag{42}$$

or

$$\left[\frac{\partial(\ln \rho)}{\partial s}\right]_p = \frac{g_c J}{\gamma R} \frac{1}{T} \left(\frac{\partial h}{\partial s}\right)_p - \frac{g_c J}{R} \tag{43}$$

By Maxwell's thermodynamic relationship

$$\left(\frac{\partial h}{\partial s}\right)_p = T \tag{44}$$

it can be seen that

$$\frac{\partial(\ln \rho)}{\partial s} = \frac{g_c J}{R} \left(\frac{1 - \gamma}{\gamma}\right) = -\frac{1}{C_p} \tag{45}$$

Thus for a perfect gas, the compatibility relations (eqs. (14) and (15)) become

$$\frac{1}{\rho a} \frac{\delta^+ P}{\delta t} + \frac{\delta^+ v}{\delta t} = \frac{a g_c J}{R} \left(\frac{\gamma - 1}{\gamma}\right) \psi = \frac{a}{C_p} \psi \tag{46}$$

for a characteristic with a positive slope (cf. eq. (19)) and

$$\frac{1}{\rho a} \frac{\delta^- P}{\delta t} - \frac{\delta^- v}{\delta t} = \frac{a g_c J}{R} \left(\frac{\gamma - 1}{\gamma}\right) \psi = \frac{a}{C_p} \psi \tag{47}$$

for a characteristic with a negative slope in subsonic flow (cf. eq. (20)) and

$$\frac{Ds}{Dt} = \psi \tag{16}$$

for the particle path.

In the special case of homentropic flow where the entropy is not a function of the conduit location y or the time t, the Riemann variables $\frac{2}{\gamma - 1} a + v$ and $\frac{2}{\gamma - 1} a - v$ are constants; this may be shown from the aforementioned compatibility relations by considering the following form of equation (46):

$$\frac{1}{\rho a} \frac{P}{P} \frac{\delta^{+}P}{\delta t} + \frac{\delta^{+}v}{\delta t} = \frac{a g_c J}{R} \left(\frac{\gamma - 1}{\gamma}\right) \psi \tag{48}$$

With the help of equations (36) and (38), equation (48) becomes

$$\frac{a}{\gamma} \left[\left(\frac{2\gamma}{\gamma - 1}\right) \frac{\delta^{+}(\ln a)}{\delta t} - \frac{g_c J}{R} \frac{\delta^{+}s}{\delta t}\right] + \frac{\delta^{+}v}{\delta t} = \frac{a g_c J}{R} \left(\frac{\gamma - 1}{\gamma}\right) \psi \tag{49}$$

Further manipulation and remembering that $\psi = \frac{Ds}{Dt}$ yields

$$\frac{\delta^{+}}{\delta t}\left(\frac{2}{\gamma - 1} a + v\right) = \frac{a g_c J}{R} \left(\frac{\gamma - 1}{\gamma}\right) \frac{Ds}{Dt} + \frac{a}{\gamma} \frac{g_c J}{R} \frac{\delta^{+}s}{\delta t} \tag{50}$$

For homentropic flow, the right side of equation (50) is zero. Hence,

$$\frac{\delta^{+}}{\delta t}\left(\frac{2}{\gamma - 1} a + v\right) = 0$$

and

$$\frac{2}{\gamma - 1} a + v = \text{const} \tag{51}$$

along the characteristic direction given by equation (19). A corresponding derivation may be made from equation (47) yielding

$$\frac{2}{\gamma - 1} a - v = \text{const} \tag{52}$$

along the characteristic direction given by equation (20). Thus, for homentropic flow, the compatibility relations as presented in equations (46) and (47) give the classical Riemann variables.

As in reference 5, the compatibility relations for nonhomentropic gas flow may be made nondimensional by specifying a reference acoustic velocity a_o, which may be selected from steady-state conditions and a specified reference length y_o. From these, a reference time t_o may be deduced from

$$t_o = \frac{y_o}{a_o} \tag{53}$$

Furthermore, two nondimensional parameters may be defined as

$$\zeta = \frac{y}{y_o} \tag{54a}$$

$$\tau = \frac{t}{t_o} \tag{54b}$$

Multiplying the compatibility relation (eq. (50)) by y_o/a_o^2 results in

$$\frac{\delta^+}{\delta\tau}\left(\frac{2}{\gamma - 1}\mathscr{A} + \mathscr{U}\right) = (\gamma - 1)\mathscr{A}\frac{DS}{D\tau} + \mathscr{A}\frac{\delta^+ S}{\delta\tau} \tag{55}$$

where the nondimensional entropy S is

$$S = \frac{g_c Js}{\gamma R} \tag{56}$$

the nondimensional acoustic velocity is

$$\mathscr{A} = \frac{a}{a_o} \tag{57}$$

and the nondimensional velocity is

$$\mathscr{U} = \frac{v}{a_o} \tag{58}$$

It may also be shown that the other compatibility relations in nondimensional form become

$$\frac{\delta^-}{\delta\tau}\left(\frac{2}{\gamma - 1}\mathscr{A} - \mathscr{U}\right) = (\gamma - 1)\mathscr{A}\frac{DS}{D\tau} + \mathscr{A}\frac{\delta^- S}{\delta\tau} \tag{59}$$

and for the particle path

$$\frac{DS}{D\tau} = \frac{g_c J t_o}{\gamma R}\psi \tag{60}$$

NUMERICAL PROCEDURES

Each net point on the distance-time plane is determined in general by the compatibility relations and their corresponding directions. The properties of the flow such as pressure and entropy can be found from the compatibility relations. The characteristic directions serve to locate the position of the net point on the distance-time plane.

Two options are before the computer in the construction of a characteristic network - either a free or a fixed characteristic network may be developed. In a free characteristic network, the location of a new or unknown characteristic net point, as net point 3' in figure 1, is determined from known end points A and C on the respective left running and right running characteristics. Because it is less laborious, the free characteristic network has been commonly utilized in hand computation (ref. 5, pp. 43 to 45). In a fixed characteristic network, the position of the net point on the distance-time plane is prescribed as net point 3 in figure 1. Consequently, the characteristics passing through this point must be determined, which means that the end points of the characteristics such as 1 and 2 in figure 1 must be found by interpolation. Furthermore, in a fixed network procedure either the time or distance coordinates of the end points must be given, the other coordinate being determined analytically.

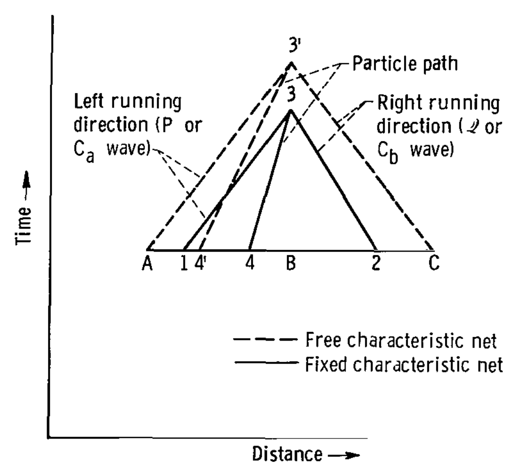


Figure 1. - Basic net point.

In this report a fixed network was utilized because it satisfied typical engineering requirements for information as to dynamic flow conditions at prescribed locations and uniform time increments. The interpolation that is necessary for finding end points 1 and 2 as well as 4 presents no special difficulty with the availability of a high-speed computer. In the examples of this report, the time coordinate of the end points was preselected for nonsteady gas flow while, for the sake of convenience, the end points were prescribed at fixed distance coordinates for nonsteady liquid flow.

Numerical Methods for Liquids

Basic net point. - Integrating equation (27) along the left running characteristic results in

$$H + \frac{a}{g_c F} q = C_a \tag{61}$$

where C_a is the constant of integration or water-hammer variable for the left running wave, which is analogous to the nonsteady gas flow equation (eq. (51)). By equation (61) the following may be written along the left running characteristic:

$$H_1 + \frac{a_1}{g_c F_1} q_1 = H_3 + \frac{a_1}{g_c F_1} q_3 = C_a \tag{62}$$

Integrating equation (28) along the right running direction gives

$$H - \frac{a}{g_c F} q = C_b \tag{63}$$

where C_b is a constant of integration or water-hammer variable for the right running wave, which is similar to equation (52). Furthermore, equation (63) yields

$$H_2 - \frac{a_2}{g_c F_2} q_2 = H_3 - \frac{a_2}{g_c F_2} q_3 = C_b \tag{64}$$

The corresponding characteristic direction for the left running wave is $\frac{dy}{dt} = a$, and for the right running wave, $\frac{dy}{dt} = -a$, since the velocity v is assumed to be negligibly small compared to the acoustic velocity a. Obviously the volume flow q_3 at net point 3 is determined by the simultaneous solution of equations (62) and (64), which is

$$q_3 = \frac{C_a - C_b}{\dfrac{a_1}{g_c F_1} + \dfrac{a_2}{g_c F_2}} \tag{65}$$

The volume flow at net point 3 can then be substituted into either equation (62) or (64) to compute the head at that net point.

Inasmuch as in liquid dynamics the location of end points 1 and 2 is preselected (as mentioned previously), the corresponding time coordinates of the net points must be

$$t_1 = t_3 - \frac{y_3 - y_1}{a_1} \tag{66}$$

$$t_2 = t_3 - \frac{y_2 - y_3}{a_2} \tag{67}$$

the distance coordinates y_2 and y_1 being fixed. Since the acoustic velocities a_1 and a_2 are constants, the times t_1 and t_2 need be computed but once.

Summing up, the computation of H_3 and q_3 may proceed as follows. From known values of t_1 and t_2 and the values of head and volume flow at those times, the water-hammer variables C_a and C_b may be computed as in equations (62) and (64). The volume flow is then defined from equation (65), and the head H_3 may be computed from equation (62) or (64). This process is continued proceeding from one net point to another, net point 3 being always the net point whose position is known on the distance-time plane but whose head and flow are unknown.

Since the characteristic curves may be regarded as patching curves (cf. ref. 6, p. 601), the patching of flow regions which are analytically different is permitted. This allows for the introduction of a valve, for example, at any given distance coordinate, which may be characterized by

$$q_3 = K_\nu \frac{(H_{3\ell} - H_{3r})}{|H_{3\ell} - H_{3r}|^{1/2}} \tag{68}$$

where K_ν is the valve orifice coefficient and $H_{3\ell}$ and H_{3r} are the heads immediately ahead and behind the valve. In this instance, equations (62) and (64) become

$$H_1 + \frac{a_1}{g_c F_1} q_1 = H_{3\ell} + \frac{a_1}{g_c F_1} q_3 = C_a \tag{69}$$

$$H_2 - \frac{a_2}{g_c F_2} q_2 = H_{3r} - \frac{a_2}{g_c F_2} q_3 = C_b \tag{70}$$

Equations (68), (69), and (70) may be solved simultaneously, and using the Quadratic Formula (ref. 7) results in

$$q_3 = K_\nu^2 \frac{(\mathscr{S}_a - \mathscr{S}_b)}{2} \frac{(C_a - C_b)}{|C_a - C_b|} \left[-1 + \sqrt{1 + \frac{4|C_a - C_b|}{K_\nu^2(\mathscr{S}_a - \mathscr{S}_b)^2}}\right] \tag{71}$$

where

$$\mathscr{S}_a = \frac{a_1}{g_c F_1} \tag{72}$$

and

$$\mathscr{S}_b = -\frac{a_2}{g_c F_2} \tag{73}$$

This should be compared to reference 3, page 10 in which the field method is used instead of the lattice point method of this report. These two methods of solving the partial differential equations of nonsteady (and supersonic) flow are discussed in reference 6, pages 491 and 492.

Left and right boundaries. - For the left boundary, only the right running compatibility relation (eq. (64)) need be used. If the end point of the right running wave is represented by net point 2, appropriate values of head and flow, H_2 and q_2 at time t_2, define the water-hammer variable C_b. If net point 3 is chosen as the left boundary point, there are two unknowns (viz., the head and flow at the boundary H_3 and q_3). Hence, either the head or the flow at the boundary must be given; for example, the head at the outlet of a reservoir is typically assumed to be constant. Of course, the boundary may be a valve in which case the functions describing valve performance together with the compatibility relation can be solved either analytically, if possible, or numerically.

For the right boundary, only compatibility relation (eq. (62)) is required, and the past reference time is clearly t_1. For some assumed boundary condition, the procedures are quite similar to the left boundary discussed previously.

Numerical Methods for Gases

Basic net point. - In nonsteady nonhomentropic flow, the procedure for the basic net point (cf. fig. 1, p. 12) is considerably more difficult because of essentially two complications: (1) the particle path or third characteristic cannot be neglected and (2) the characteristic slopes are not constants. In the first instance, the particle path cannot be neglected since the compatibility equations for a perfect gas (viz., eqs. (55) and (59)) depend on the value of the co-moving or substantial derivative $\frac{DS}{D\tau}$, a fundamental parameter for the third characteristic. Thus to find the flow conditions at net point 3 in figure 1, three compatibility equations instead of two must be solved. (Net point 3 always identifies the "later" net point whose position is known on the distance-time plane but whose flow properties are unknown.) Secondly, owing to the compressibility of a perfect gas, the acoustic velocity cannot be assumed to be a constant. In proceeding from net point 1 to net point 3, for example, the acoustic as well as the flow velocities at each net point may be different, requiring an approximation of the characteristic slope (cf. eq. (19)). Although the position of net point 3 on the distance-time plane is preselected, it does not follow that the location of net point 1 on the base line is immediately evident as is the case in liquid dynamics. Rather, due to the variability of the acoustic and flow velocities, the location of net point 1 becomes part of the problem.

The difficulties encountered and the solution to the basic net point procedure in a fixed network may be more readily understood by considering the following example. In figure 1, at points A, B, and C on the initial line all parameters are assumed to be known. Assume that the end points 1 and 4 are between A and B, and that end point 2 is between B and C. Also assume that reasonable guesses for the nondimensional acoustic velocity at net point 3 $\mathscr{A}_3$ and the nondimensional velocity $\mathscr{U}_3$ are $\mathscr{A}_B$ and $\mathscr{U}_B$, respectively, which are the flow velocity and the acoustic velocity at B. By linearly interpolating for $\mathscr{A}_1$ and $\mathscr{U}_1$ between A and B, the correct location of net point 1 on the base line ABC will be that point whose characteristic slope will pass through the preselected net point 3 and net point 1. The characteristic slope in this instance is given by

$$\frac{\tau_3 - \tau_1}{\zeta_3 - \zeta_1} = \frac{2}{(\mathscr{U}_3 + \mathscr{U}_1) + (\mathscr{A}_1 + \mathscr{A}_3)} \qquad (74)$$

which is an inverted finite difference form of equation (19). It may be shown by the application of the elementary principles of analytical geometry that the location of net point 1 ζ_1 may be expressed in general by the quadratic

$$\alpha_\ell \zeta_1^2 + \beta_\ell \zeta_1 + \varphi_\ell = 0 \tag{75}$$

where

$$\alpha_\ell = C_{a1\ell} C_{u1\ell} \tag{76a}$$

$$\beta_\ell = C_{a1\ell} C_{n2\ell} - C_{n1\ell} C_{u1\ell} - 2 \tag{76b}$$

$$\varphi_\ell = 2\zeta_3 - C_{n1\ell} C_{n2\ell} \tag{76c}$$

and

$$C_{a1\ell} = \frac{\tau_A - \tau_B}{\zeta_A - \zeta_B} \tag{77a}$$

$$C_{u1\ell} = \left[\frac{1 + C_{a1\ell}^2}{(\zeta_A - \zeta_B)^2 + (\tau_A - \tau_B)^2}\right]^{1/2} \left[(\mathscr{U}_B - \mathscr{U}_A) + (\mathscr{A}_B - \mathscr{A}_A)\right] \tag{77b}$$

$$C_{n1\ell} = \tau_3 - \tau_A + C_{a1\ell} \zeta_A \tag{77c}$$

$$C_{n2\ell} = \mathscr{U}_3 + \mathscr{A}_3 + \mathscr{U}_A + \mathscr{A}_A - C_{a1\ell} \zeta_A \tag{77d}$$

Likewise, for the location of netpoint 2 ζ_2

$$\alpha_r \zeta_2^2 + \beta_r \zeta_2 + \varphi_r = 0 \tag{78a}$$

and for the slope (cf. eq. (20))

$$\frac{\tau_3 - \tau_2}{\zeta_3 - \zeta_2} = \frac{2}{\mathscr{U}_2 + \mathscr{U}_3 - \mathscr{A}_2 - \mathscr{A}_3} \tag{78b}$$

where

$$\alpha_r = C_{a1r} C_{u1r} \tag{79a}$$

$$\beta_r = C_{a1r} C_{n2r} - C_{n1r} C_{u1r} - 2 \tag{79b}$$

$$\varphi_r = 2\zeta_3 - C_{n1r} C_{n2r} \tag{79c}$$

and

$$C_{a1r} = \frac{\tau_B - \tau_C}{\zeta_B - \zeta_C} \tag{80a}$$

$$C_{u1r} = \left[\frac{1 + C_{a1r}^2}{(\zeta_B - \zeta_C)^2 + (\tau_B - \tau_C)^2}\right]^{1/2} \left[(\mathscr{U}_C - \mathscr{U}_B) - (\mathscr{A}_C - \mathscr{A}_B)\right] \tag{80b}$$

$$C_{n1r} = \tau_3 - \tau_C + C_{a1r}\zeta_B \tag{80c}$$

$$C_{n2r} = \mathscr{U}_3 - \mathscr{A}_3 + \mathscr{U}_B - \mathscr{A}_B - C_{u1r}\zeta_B \tag{80d}$$

For the location of ζ_4 the end point of the particle path on the initial line, a similar quadratic is the result

$$\alpha_m \zeta_4^2 + \beta_m \zeta_4 + \varphi_m = 0 \tag{81a}$$

and for the slope (cf. eq. (21))

$$\frac{\tau_3 - \tau_4}{\zeta_3 - \zeta_4} = \frac{2}{\mathscr{U}_3 + \mathscr{U}_4} \tag{81b}$$

where

$$\alpha_m = C_{a1m} C_{u1m} \tag{81c}$$

$$\beta_m = C_{n2m} C_{a1m} - C_{u1m} C_{n1m} - 2 \tag{81d}$$

$$\varphi_m = 2\zeta_3 - C_{n1m} C_{n2m} \tag{81e}$$

and

$$C_{a1m} = \frac{\tau_A - \tau_B}{\zeta_A - \zeta_B} \tag{82a}$$

$$C_{u1m} = \left[\frac{1 + C_{a1m}^2}{(\zeta_B - \zeta_A)^2 + (\tau_B - \tau_A)^2} \right]^{1/2} (\mathcal{U}_B - \mathcal{U}_A) \tag{82b}$$

$$C_{n1m} = \tau_3 - \tau_A + C_{a1m} \zeta_A \tag{82c}$$

$$C_{n2m} = \mathcal{U}_3 + \mathcal{U}_A - C_{u1m} \zeta_A \tag{82d}$$

These quadratics (viz., eqs. (75), (78a), and (81a)) of course, may be solved for ζ_1, ζ_2, and ζ_4, respectively, by the well-known Quadratic Formula where the correct sign is given by the solution for ζ_1 which is to the left of and nearest to ζ_3, and for ζ_2 which is to the right of and nearest to ζ_3, and for ζ_4 which is to the left of and nearest to ζ_3 if $(\mathcal{U}_3 + \mathcal{U}_4)/2 > 0$. Further it will be seen that, if the slope of the base line is zero, then

$$\alpha_\ell = \alpha_r = \alpha_m = 0$$

and the solutions to equations (75), (78a), and (81a) may be represented by

$$\zeta_1 = -\frac{\varphi_\ell}{\beta_\ell} \tag{83a}$$

$$\zeta_2 = -\frac{\varphi_r}{\beta_r} \tag{83b}$$

$$\zeta_4 = -\frac{\varphi_m}{\beta_m} \tag{83c}$$

respectively.

If the estimated location and associated flow parameters of the end points of the characteristics are determined from the assumed values of $\mathscr{A}_3$ and $\mathscr{U}_3$, a better approximation of the values of $\mathscr{A}_3$ and $\mathscr{U}_3$ can be made by the Method of Iteration for Simultaneous Equations (ref. 8). A new value of $\mathscr{U}_3$ may be determined from the assumed values of $\mathscr{A}_3$ and $\mathscr{U}_3$. If the compatibility relation (eq. (60)) is expressed in finite difference form, the result is as in reference 5

$$S_3 = S_4 + \frac{DS}{D\tau}(\tau_3 - \tau_4) \tag{84}$$

The time increment $\tau_3 - \tau_4$ is obviously known since the basic network is for fixed time and distance coordinates. The nondimensional entropy S_4 is found by linearly interpolating along the base line $\overline{ABC}$ at location ζ_4. If the rate of entropy increase $\frac{DS}{D\tau}$ is assumed to be known, then S_3 is defined. With the interpolated values of the flow parameters at locations ζ_1 and ζ_2, the Riemann variables at location ζ_3 may be found by the finite-difference form of equations (55) and (59), namely,

$$\frac{2}{\gamma - 1}\mathscr{A}_3 + \mathscr{U}_3 = \frac{2}{\gamma - 1}\mathscr{A}_1 + \mathscr{U}_1 + (\gamma - 1)\mathscr{A}_{13}\frac{DS}{D\tau}(\tau_3 - \tau_1) + \mathscr{A}_{13}(S_3 - S_1) \tag{85a}$$

and

$$\frac{2}{\gamma - 1}\mathscr{A}_3 - \mathscr{U}_3 = \frac{2}{\gamma - 1}\mathscr{A}_2 - \mathscr{U}_2 + (\gamma - 1)\mathscr{A}_{23}\frac{DS}{D\tau}(\tau_3 - \tau_2) + \mathscr{A}_{23}(S_3 - S_2) \tag{85b}$$

where the double subscript represents an average value (e.g., $\mathscr{A}_{13} = (\mathscr{A}_1 + \mathscr{A}_3)/2$). If

$$\mathscr{P}_3 = \frac{2}{\gamma - 1}\mathscr{A}_3 + \mathscr{U}_3 \tag{86a}$$

and

$$\mathscr{Q}_3 = \frac{2}{\gamma - 1} \mathscr{A}_3 - \mathscr{U}_3 \tag{86b}$$

Then $\mathscr{U}_3$ is determined by

$$\mathscr{U}_3 = \frac{1}{2} (\mathscr{P}_3 - \mathscr{Q}_3) \tag{87}$$

By following the Method of Iteration for Simultaneous Equations, the entire system is recalculated with the new value of $\mathscr{U}_3$ and the previous value of $\mathscr{A}_3$; this includes finding new locations for ζ_1, ζ_2, and ζ_4. With the new interpolated values of the flow parameters at these locations, equations (84), (85a), and (85b) are recalculated; $\mathscr{A}_3$ is then computed with the new value of $\mathscr{U}_3$ by

$$\mathscr{A}_3 = \frac{\gamma - 1}{2} (\mathscr{Q}_3 + \mathscr{U}_3) \tag{88}$$

This completes a single iteration. The process is repeated until the values of $\mathscr{A}_3$ and $\mathscr{U}_3$ converge within some desired tolerance. With a reasonable tolerance for engineering calculations, this method has rarely required more than two iterations for the computation of any basic net point in the examples of this report.

If, in a problem, net points 1 and 2 have different gas properties, the basic net point procedure is not significantly altered except that in equations (85a) and (85b) the ratio of specific heats γ at corresponding net points must be altered to correspond with the properties of the fluid.

Left boundary. - If a constant pressure P_3 and a constant entropy S_3 at the left boundary are assumed, the nondimensional acoustic velocity $\mathscr{A}_3$ at the boundary may be derived from equation (36) to yield

$$\mathscr{A}_3 = \left(\frac{P_3}{P_o}\right)^{(\gamma-1)/2\gamma} e^{[S_3(\gamma-1)/2]} \tag{89}$$

where the nondimensional entropy at the boundary S_3 is determined from equation (56). The reference pressure P_o must be consistent with the reference acoustic velocity (cf. eq. (54)) and some assumed reference density ρ_o; thus

$$P_o = \frac{a_o^2 \rho_o}{\gamma}$$

as in equation (38). A solution for the nondimensional velocity at the boundary $\mathscr{U}_3$ may now be found by applying the Method of Iteration (cf. ref. 8). Since the location ζ_3 and the time τ_3 at the left boundary are known, the location of ζ_2 on the base line may be found by equation (83b) if $\mathscr{U}_B$ is assumed to be a good initial guess for $\mathscr{U}_3$, the nondimensional velocity at the boundary. If an initial location for ζ_2 is chosen, a linear interpolation along the base line may be made for $\mathscr{U}_2, \mathscr{A}_2$, and S_2. Only a single compatibility relation (i.e., eq. (85b)) is necessary to define the Riemann variable $\mathscr{Q}_3$ inasmuch as only a positive direction of flow is assumed. Then $\mathscr{U}_3$ is given by

$$\mathscr{U}_3 = \frac{2}{\gamma - 1} \mathscr{A}_3 - \mathscr{Q}_3 \tag{90}$$

remembering that $\mathscr{A}_3$ is a constant as in equation (89). If the previous value of $\mathscr{U}_3$ is different from the initial guess, a few iterations are ususally sufficient for convergence to some desired tolerance level.

Later, in the discussion of the supersonic diffuser example, special procedures are covered if the left boundary consists of a supersonic diffuser with a normal shock downstream of the inlet.

Right boundary. - If the right boundary is choked, the ratio of the duct area F to the throat area F^* is given by (ref. 6, pp. 85 to 86)

$$\frac{F}{F^*} = \frac{1}{M_3} \left[\left(\frac{2}{\gamma + 1}\right) \left(1 + \frac{\gamma - 1}{2} M_3^2\right)\right]^{(\gamma+1)/2(\gamma-1)} \tag{91}$$

where M_3 is the subsonic Mach number just upstream of the choking orifice at the right boundary. Hence, for a specified F/F^*, M_3 is constant. For positive flow, two compatibility relations (eqs. (84) and (85a)) and their corresponding end point ζ_1 and ζ_4 together with the boundary condition are necessary to define flow conditions at the boundary. If $\mathscr{A}_B$ is used as an initial guess for $\mathscr{A}_3$, $\mathscr{U}_3$ is computed by

$$\mathscr{U}_3 = \mathscr{A}_3 M_3 \tag{92}$$

The locations ζ_1 and ζ_4 are defined by equations (83a) and (83c). Interpolating along the base line for $\mathscr{U}_1, \mathscr{A}_1$, and S_1 at ζ_1 and for S_4 at ζ_4, the Riemann variable $\mathscr{P}_3$

is known by first determining S_3 from equation (84) and substituting this value together with the aforementioned parameters in equation (85a). The acoustic velocity $\mathscr{A}_3$ is then given by

$$\mathscr{A}_3 = \frac{\mathscr{P}_3}{\dfrac{2}{\gamma - 1} + M_3} \tag{93}$$

This value is then compared to the initial guess for $\mathscr{A}_3$ to assess whether additional iterations are necessary.

Obviously the right boundary condition is not limited to a choking orifice nor is the left boundary limited to a constant pressure. Had the boundary conditions been reversed or other boundary conditions prevailed, similar methods as described previously would apply.

Special net point cases. - Clearly, as in figure 2, there will be instances in the computation of the basic net point 3 when the end point of the $\mathscr{P}$ characteristic, net point 1, or the end point of the $\mathscr{Q}$ characteristic, net point 2, exceed their respective boundaries. In those instances the boundary values must be used in place of the fictitious net points 1 and 2. Consider the case of a net point 3 where ζ_1, the end point of the $\mathscr{P}$ characteristic, falls beyond the left boundary. To find the intersection of the $\mathscr{P}$ characteristic with the boundary, the location of the fictitious net point 1 is first computed by equation (83a). The intersection of the characteristic passing through net points 1 and 3 with

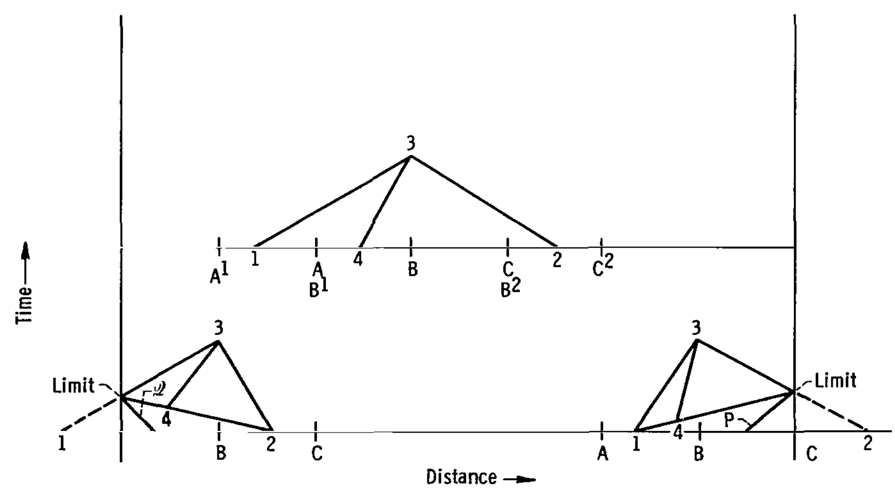


Figure 2. - Special net point cases.

the boundary is then found by

$$\tau_{lim} = \tau_3 - (\zeta_3 - \zeta_{lim}) \frac{\tau_3 - \tau_1}{\zeta_3 - \zeta_1} \tag{94a}$$

where τ_{lim} is the nondimensional time at the boundary and ζ_{lim} is the location of the boundary itself. The $\mathscr{Q}$ characteristic which intersects the limit point on the left boundary as in figure 2 is determined by the methods discussed in the section on left boundary procedures. The values $\mathscr{A}_{lim}$, $\mathscr{U}_{lim}$, and S_{lim} become in effect the end point flow parameters of the left running wave; the line connecting the limit point with net point 2 is used as the base line on which the end point of the particle path, net point 4, is located. Points A and B are the limit point and net point 2, respectively (see fig. 2); quadratic equation (eq. (81a)) must be solved since the slope of the base line is no longer zero.

Similar methods apply to the right boundary where, instead of equation (94a), there is obtained

$$\tau_{lim} = \tau_3 - (\zeta_3 - \zeta_{lim}) \frac{\tau_3 - \tau_2}{\zeta_3 - \zeta_2} \tag{94b}$$

Another difficulty arises when base point 1 or 2 is within the boundaries but falls to the left of point A or to the right of point C, respectively. In this case if net point 1 falls beyond A, as in figure 2, then interpolations may be performed between A^1 and B^1. Similarly, if net point 2 is beyond C, then interpolation between B^2 and C^2 may be necessary.

Orangization of Network Calculations

There are a number of ways in which a characteristic network can be organized. Basically each net point must be identified by some enumerative order; at each identified net point, a scheme must be devised for selecting the correct end points of the characteristics for that net point; and lastly, there must be a method of proceeding from one net point to another. The enumerative order selected in the computer program for nonsteady nonhomentropic gas flow is shown in figure 3. The known base points of net point 5, 2 for ex-

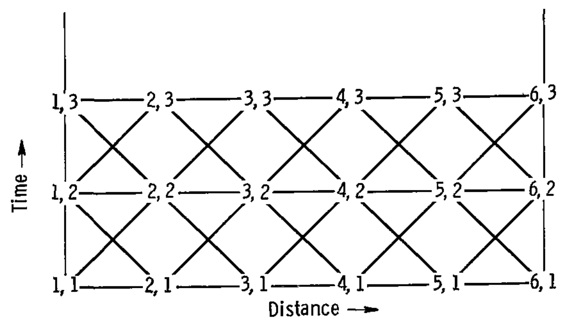


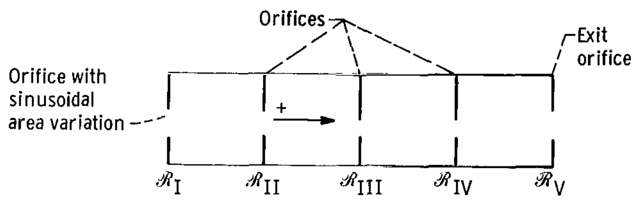
Figure 3. - Organization of network.

ample, are net points 4, 1 and 5, 1 and 6, 1 (between which interpolations will probably be necessary) which correspond to the base points A, B, and C of figure 1 (p. 12). The first subscript, it will be seen, indicates the distance coordinate and the second, the time coordinate. The procedure of going from one net point to another is nothing more than proceeding according to some numerical order along a specific time coordinate (fig. 3).

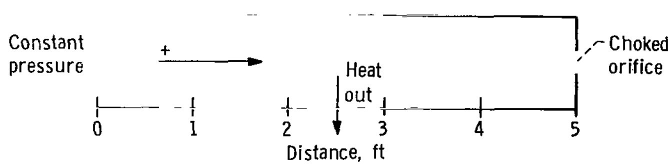
Evidently both problems in water-hammer or gas dynamics may be solved on the same network using the same procedures and in fact using the same computer program if the general compatibility relations (eqs. (14), (15), and (16)) are employed in a region free of compression shocks. But inasmuch as the slopes of the characteristics are constants in liquid dynamics, a simpler water-hammer program may be devised (cf. appendix D or ref. 3).

EXAMPLES

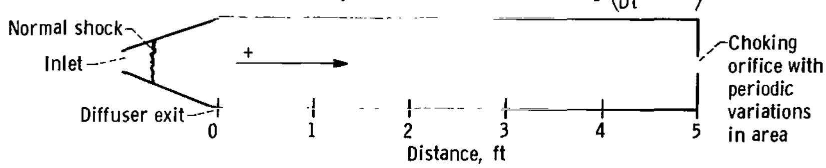
Three examples have been chosen (corresponding to the three parts of fig. 4). The first two have been selected to corroborate the numerical methods of this report (viz., an example in water-hammer or liquid dynamics which can readily be checked against the



(a) Hydraulic duct perturbed by periodic variation of inlet orifice.



(b) Constant area duct subjected to defined rate of cooling $\left(\frac{DS}{D\tau} = -0.5\right)$.



(c) Supersonic diffuser.

Figure 4. - Configurations for examples.

methods of reference 3 and secondly, an example in gas dynamics, the cooling of a perfect gas flowing in a duct of constant cross-sectional area). A partial verification of the latter example is possible inasmuch as at steady-state conditions the results of the nonsteady flow analysis should be consistent with the Rayleigh analysis for cooling or heating of a constant area duct (ref. 6, ch. 7). The third example, shock perturbations in a supersonic diffuser, has been selected to show how the analytical methods of this report may be applied in an approximate manner to the dynamics of a supersonic inlet, the design of which is of current interest.

Example 1: Hydraulic Duct Perturbed by a Periodic Variation of Inlet Orifice

In the configuration shown in figure 4(a), three orifices are inserted at equal intervals of 17 feet in a constant cross-sectional area duct with an inside diameter of 7/8 inch. The values of the orifice coefficients are given by

$$\mathscr{R}_{II} = \mathscr{R}_{III} = \mathscr{R}_{IV} = 2.456 \text{ ft}^{1/2}/\text{sec} \tag{95}$$

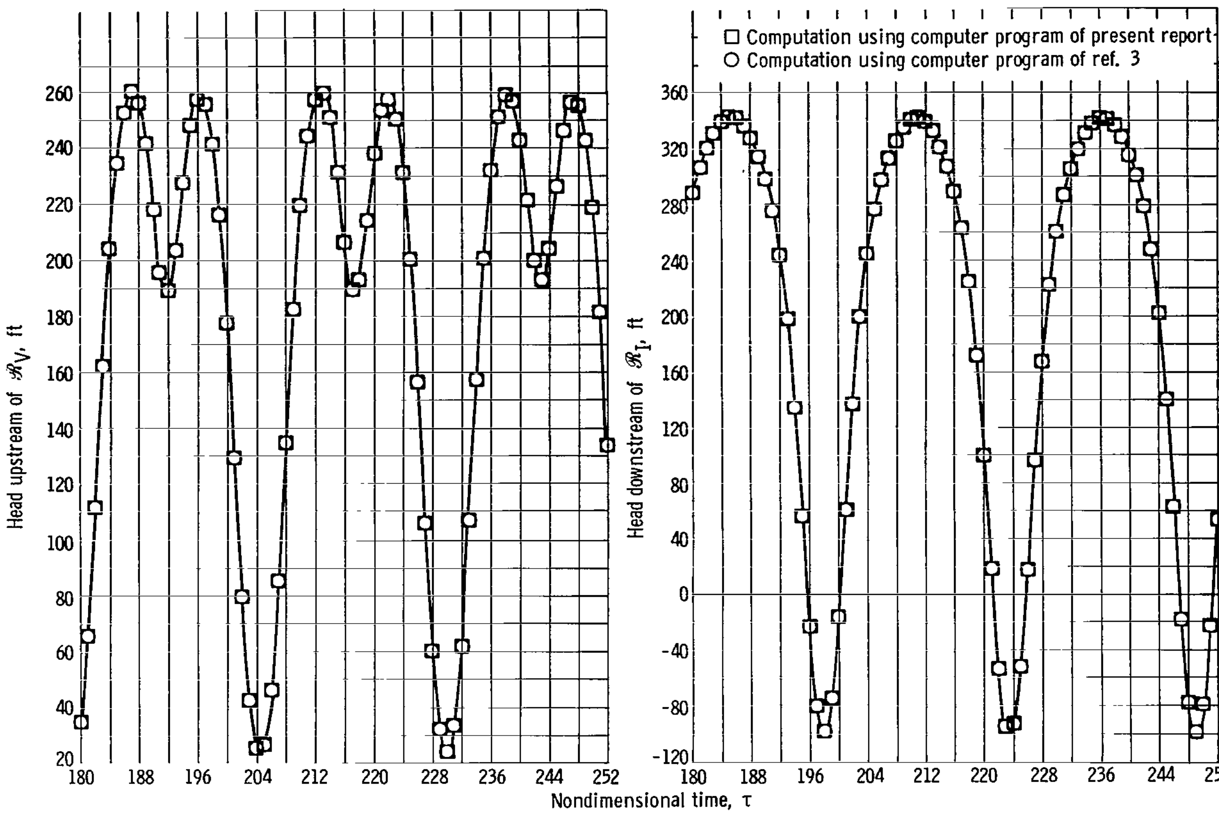
The subscripts II, III, and IV denote the location of these orifices in the duct (cf. fig. 4(a)). The orifice coefficient $\mathscr{R}$ is equal to the orifice coefficient K_ν divided by the duct area (cf. eq. (68)); hence, the velocity v at each orifice can be presented by

$$v = \mathscr{R} \frac{(H_\ell - H_r)}{|H_\ell - H_r|^{1/2}} \tag{96}$$

where H_ℓ and H_r are the heads immediately ahead or upstream and behind or downstream of the orifice. At the left and right boundaries are additional orifices represented by $\mathscr{R}_I = 0.65$ and $\mathscr{R}_V = 0.414$ (foot)$^{1/2}$per second, respectively. Initially, the head just downstream of the first orifice is 260 feet, the head just upstream of the right boundary is 238.6 feet, and the velocity through the duct is 6.5 feet per second. The acoustic velocity is set at 3800 feet per second throughout the duct. Perturbations are introduced into this simple system by varying the orifice coefficient $\mathscr{R}_I$ at the left boundary according to the function

$$\mathscr{R}_I = \mathscr{R}_0 + \mathscr{R}_{amp} \sin \omega t \tag{97}$$

where $\mathscr{R}_0 = 0.65$, $\mathscr{R}_{amp} = 0.5$, and $\omega = 2\pi f$ where $f = 70$ cps. The time t is real



(a) Head at hydraulic line discharge plotted against nondimensional time.

(b) Head immediately downstream of hydraulic inlet plotted against nondimensional time.

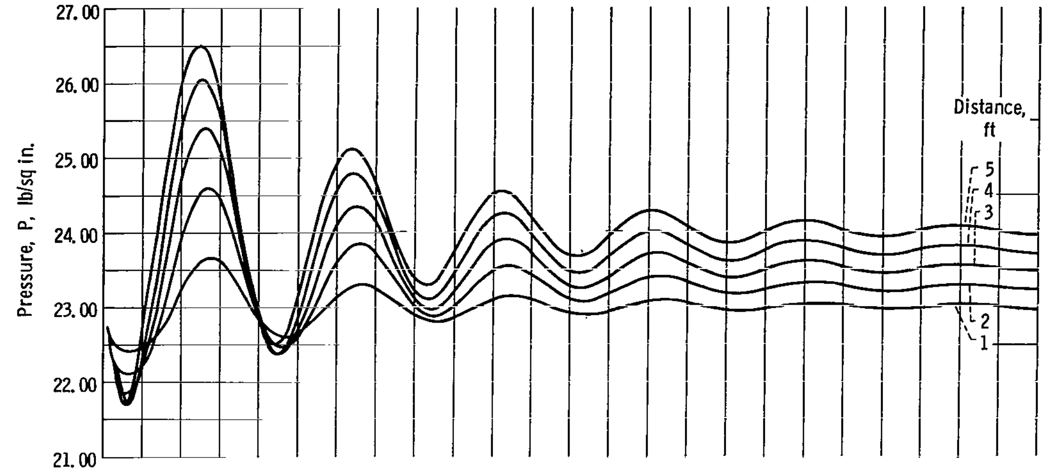
Figure 5. - Nonsteady liquid flow example.

time in seconds.

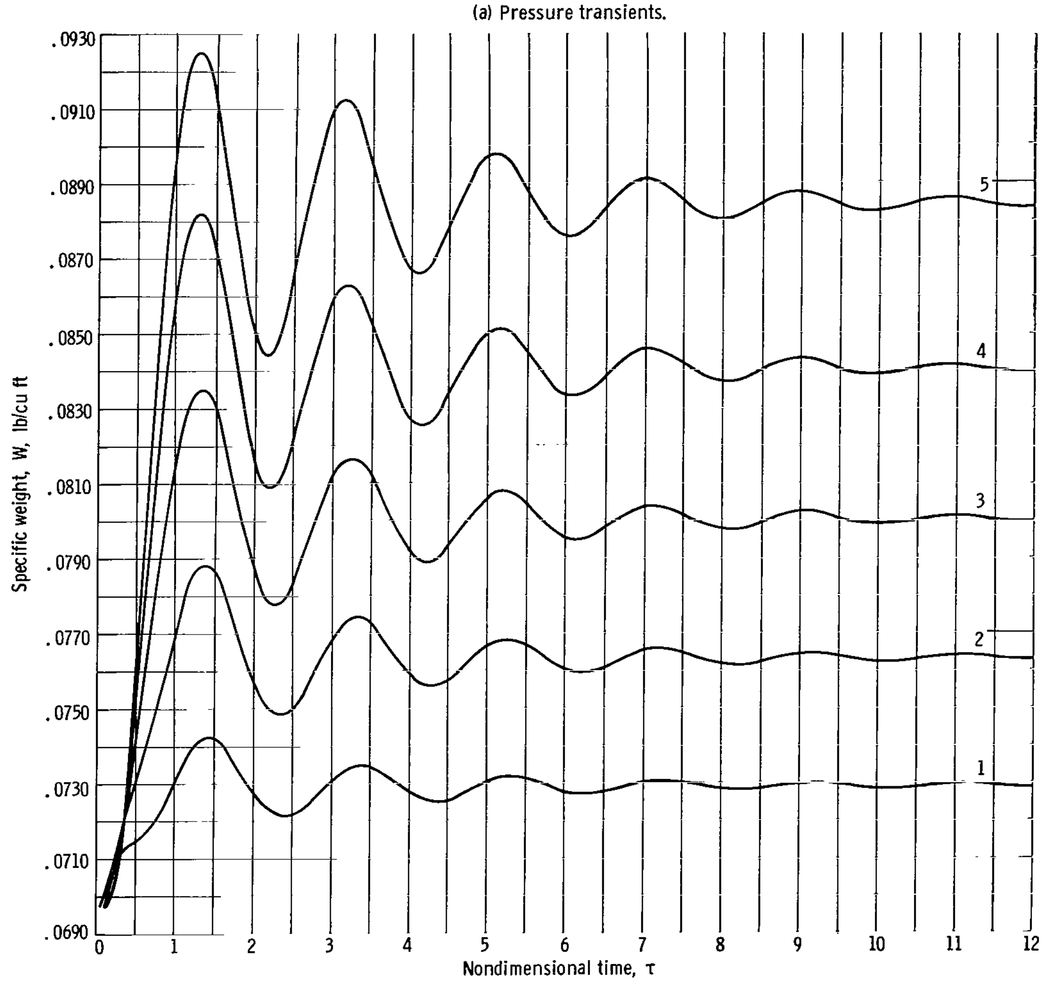
Under these conditions the water-hammer programs of this report and of reference 3 were used to compute the dynamics of this line. The results were practically identical, which may be verified by inspection of figure 5, which represents a short period or time slice of the entire transient.

Example 2: Cooling Along a Constant Area Duct

As seen in figure 4(b), a duct of constant cross-sectional area, 5 feet in length, choked at the right boundary, and having a constant pressure left boundary, contains, prior to any transients, a perfect gas flowing at the same Mach numer of 0.384 at each location upstream of the choking orifice. Initial values of pressure, specific weight, and temperature (assumed to be constant at every upstream location) are $P = 22.74$ pounds per square inch, $W = 0.06977$ pound per cubic foot, and $T = 880.57^{\circ}$ R. A reference

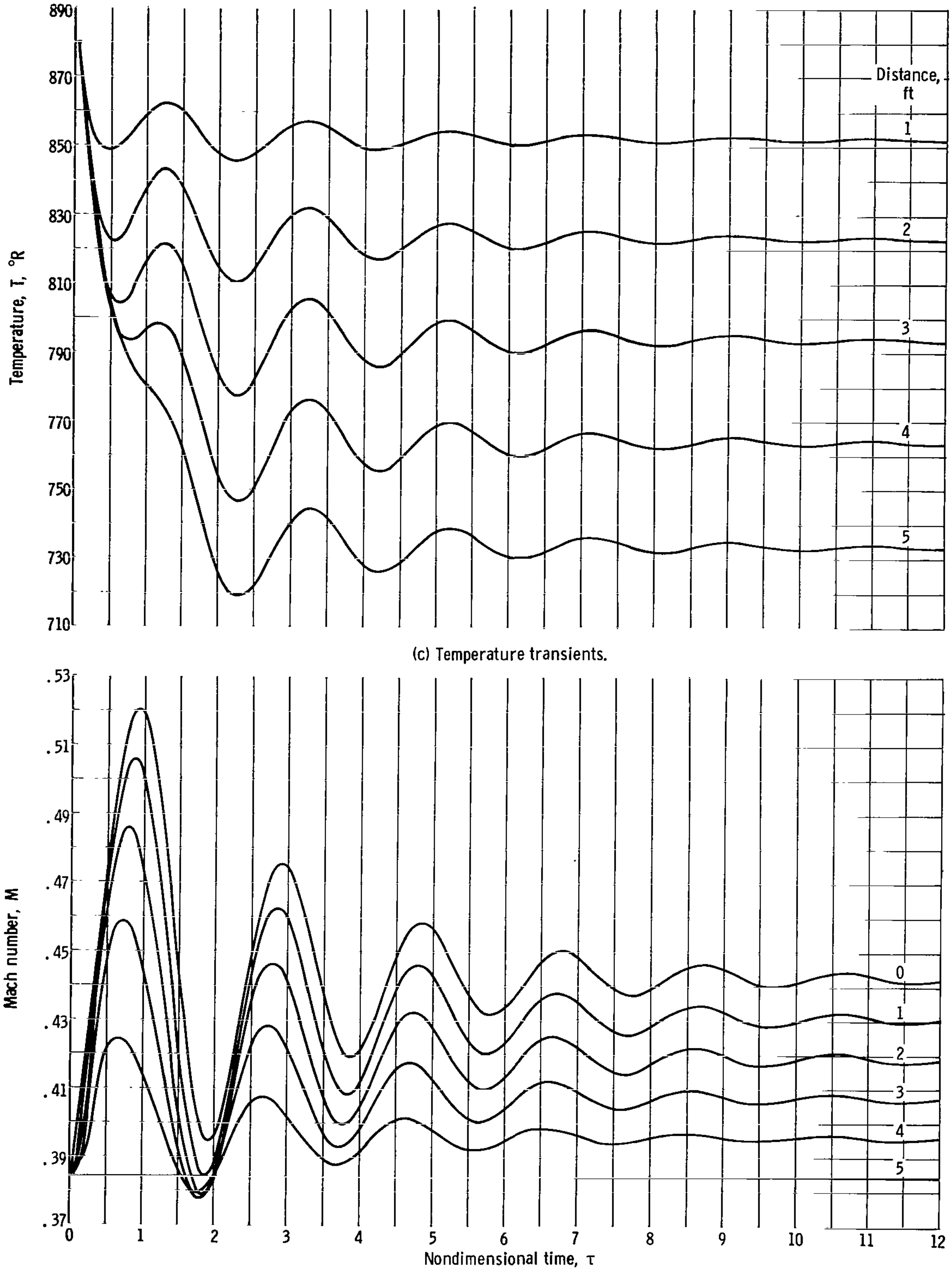


(a) Pressure transients.



(b) Specific weight transients.

Figure 6. - Transients due to cooling of constant area duct. Real time, $\tau(0.8937\times10^{-2})$ second; ratio of specific heats, 1.4; gas constant, 53.3 feet per °R; $\frac{DS}{D\tau} = -0.5$.



(c) Temperature transients.

(d) Mach number transients.

Figure 6. - Concluded.

length y_0 of 10 feet and a reference acoustic velocity a_0 of 1119 feet per second were selected. By equation (54) the reference time t_0 becomes 0.00894 second, and real time in seconds is the nondimensional time multiplied by this factor.

A transient is initiated in this system by instantaneously imposing and sustaining a defined rate of cooling, namely, a $\frac{DS}{D\tau} = -0.5$ (cf. eq. (60)) along the duct. Periodic oscillations follow the transient as can be seen in figure 6; after these oscillations have been spent, the flow settles about altered steady-state conditions at each location, quite different from the initial conditions prevailing before the transient.

It is interesting to compare, as a check, the final steady-state conditions with the Rayleigh steady-state analysis (cf. ref. 6, ch. 7). After the transient has been completed, the temperature at the left boundary is 880.57° R and the Mach number is 0.443. At the right boundary or discharge of the duct, the Mach number is 0.384, with a corresponding temperature of 733.79° R. With the help of table B.5 in reference 6, pages 628 and 629, the critical temperature ratios are (adopting the nomenclature of ref. 6)

$$\left(\frac{T}{T^*}\right)_{M=0.443} = 0.69505 \tag{98a}$$

and

$$\left(\frac{T}{T^*}\right)_{M=0.384} = 0.58282 \tag{98b}$$

by interpolation. Thus the temperature ratio is

$$\frac{\left(\frac{T}{T^*}\right)_{M=0.443}}{\left(\frac{T}{T^*}\right)_{M=0.384}} = \frac{0.69505}{0.58282} = 1.193 \tag{98c}$$

which compares within 1 percent of the temperature ratio of the values computed from the nonsteady flow analysis. A similar check may be made for pressure ratio, and it may be shown that excellent agreement is obtained. In addition, it may be seen that the frequency of the oscillations, about 59 cps, approximates the quarter wave resonant frequency for the duct.

Example 3: Perturbation of a Normal Shock in a Supersonic Diffuser

In figure 4(c) (p. 25), a supersonic diffuser followed by a constant area duct may be seen. In this example the same dimensions and initial conditions prevail in the duct as in the previous cooling example. At the right boundary the system is perturbed by varying the choking throat area with time (fig. 7(a)). In place of the constant pressure boundary, there is a supersonic diffuser with a diffuser exit to inlet area ratio F_{ex}/F_{in} of 1.6. The Mach number at station zero (the inlet of the duct or diffuser exit) M_{ex} is 0.384, and the Mach number at the diffuser inlet M_{in} is 1.5. In this example, the quasi-steady methods of reference 5, pages 99 to 103 are adopted. Hence the weight flow by continuity and the stagnation temperature by conservation of energy are assumed to be invariant from the diffuser inlet to the diffuser exit. By these assumptions if

$$D_{ex} = \frac{M_{ex}}{\left(1 + \frac{\gamma - 1}{2} M_{ex}^2\right)^{(\gamma+1)/2(\gamma-1)}} \tag{99a}$$

$$D_{in} = \frac{M_{in}}{\left(1 + \frac{\gamma - 1}{2} M_{in}^2\right)^{(\gamma+1)/2(\gamma-1)}} \tag{99b}$$

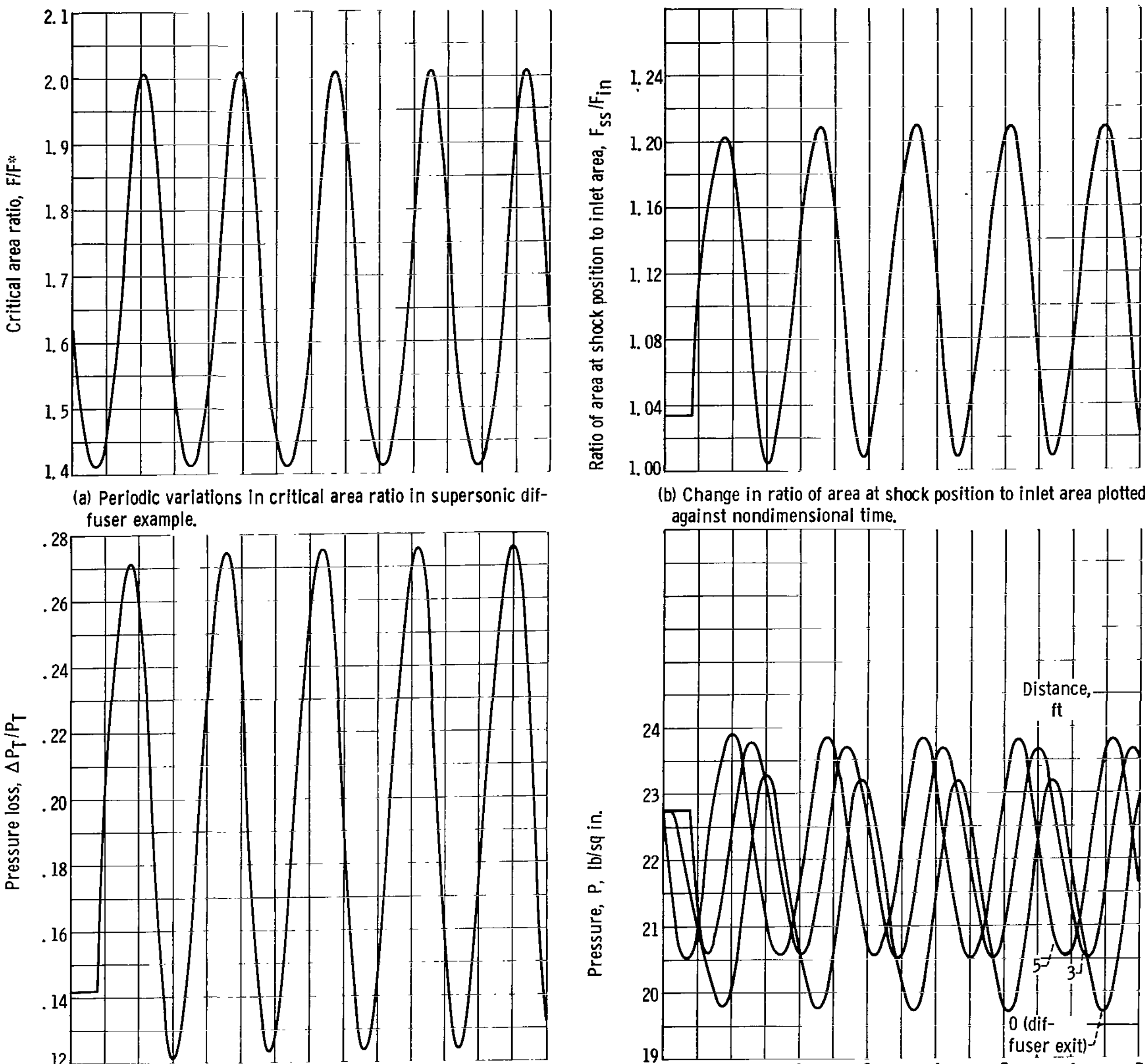
the nondimensional entropy increase across the shock $S_{ex} - S_{in}$ is given by

$$S_{ex} - S_{in} = \frac{g_c J(S_{ex} - S_{in})}{\gamma R} = \frac{1}{\gamma} \ln\left(\frac{F_{ex}}{F_{in}} \frac{D_{ex}}{D_{in}}\right) = \frac{1}{\gamma} \ln\left(\frac{P_{T,in}}{P_{T,ex}}\right) \tag{100}$$

as in reference 5 and reference 6, pages 118 to 121. For the specified M_{ex}, M_{in} and F_{ex}/F_{in}, $S_{ex} - S_{in} = 0.0948$. The supersonic Mach number just upstream of the shock M_{ss} may be found by solving the following equation for M_{ss}:

$$S_{ex} - S_{in} = \frac{1}{\gamma - 1} \ln\left[\frac{2}{(\gamma + 1)M_{ss}^2} + \frac{\gamma - 1}{\gamma + 1}\right] + \frac{1}{\gamma(\gamma - 1)} \ln\left(\frac{2\gamma}{\gamma + 1} M_{ss}^2 - \frac{\gamma - 1}{\gamma + 1}\right) \tag{101}$$

The ratio of shock location area to inlet area F_{ss}/F_{in} is then known by



(a) Periodic variations in critical area ratio in supersonic diffuser example.

(b) Change in ratio of area at shock position to inlet area plotted against nondimensional time.

(c) Pressure loss plotted against nondimensional time.

(d) Pressure perturbations in supersonic diffuser example at diffuser exit, at 3 feet, and at 5 feet.

Figure 7. - Supersonic diffuser example.

$$\frac{F_{ss}}{F_{in}} = \frac{D_{in}}{D_{ss}} \tag{102}$$

where

$$D_{ss} = \frac{M_{ss}}{\left(1 + \frac{\gamma - 1}{2} M_{ss}^2\right)^{(\gamma+1)/2(\gamma-1)}} \tag{103}$$

The initial ratio of shock location area to inlet area for the previously specified conditions is 1.034. The initial subsonic Mach number just downstream of the shock M_{sub} may be found by the use of Prandtl's equation

$$M_{sub}^* = \frac{1}{M_{ss}^*} \tag{104a}$$

(cf. eq. (5.17a) of ref. 6) where the dimensionless velocity M^* is determined by

$$M^{*2} = \frac{(\gamma + 1)M^2}{2 + (\gamma - 1)M^2} \tag{104b}$$

For this example, the value of the subsonic Mach number M_{sub} is 0.654.

To determine conditions in the diffuser, an iterative procedure is necessary. Initially the nondimensional stagnation acoustic velocity $\mathscr{A}_T$, which it will be noted is a constant, may be computed from

$$\mathscr{A}_T = \mathscr{A}_{ex}\left(1 + \frac{\gamma - 1}{2} M_{ex}^2\right)^{1/2} \tag{105}$$

using the initial values of $\mathscr{A}_{ex}$ and M_{ex}. Next, an estimate of $\mathscr{Q}_{ex}$ is used to determine the acoustic velocity at the diffuser exit $\mathscr{A}_{ex}$ at any later time from (ref. 5, p. 64)

$$\mathscr{A}_{ex} = \frac{\mathscr{Q}_{ex} + \sqrt{\frac{\gamma + 1}{\gamma - 1}\mathscr{A}_T^2 - \frac{\gamma - 1}{2}\mathscr{Q}_{ex}^2}}{\frac{\gamma + 1}{\gamma - 1}} \tag{106}$$

which was derived by conservation of energy and the $\mathcal{Q}$ compatibility relation. The nondimensional velocity is then given by

$$\mathcal{U}_{ex} = \frac{2}{\gamma - 1} \mathcal{A}_{ex} - \mathcal{Q}_{ex} \tag{107}$$

The end point of the $\mathcal{Q}$ characteristic and the corresponding parameters $\mathcal{U}_2$, $\mathcal{A}_2$, and S_2 can be ascertained by methods discussed in the left boundary procedures. At this point the nondimensional entropy increase may be calculated by equation (100), and the entropy at the diffuser exit S_{ex} may be deduced. The Riemann variable at the diffuser exit $\mathcal{Q}_{ex}$ may then be calculated by

$$\mathcal{Q}_{ex} = \mathcal{Q}_2 + \mathcal{A}_{2,ex}\left(S_{ex} - S_2\right) \tag{108a}$$

where $\mathcal{A}_{2,ex} = (\mathcal{A}_2 + \mathcal{A}_{ex})/2$, and

$$\mathcal{Q}_2 = \frac{2}{\gamma - 1} \mathcal{A}_2 - \mathcal{U}_2 \tag{108b}$$

Equation (108a) is a finite difference form of equation (59) since in this example $\frac{DS}{D\tau} = 0$. The Riemann variable $\mathcal{Q}_2$ is defined by the interpolated values of $\mathcal{A}_2$ and $\mathcal{U}_2$ at the end point ξ_2 of the $\mathcal{Q}$ characteristic. The Riemann variable $\mathcal{Q}_{ex}$ computed from equation (108a) may be compared to the estimated value, and unless there is agreement between the initial guess and final calculation of $\mathcal{Q}_{ex}$ within a reasonable tolerance, additional guesses will be necessary. After these iterations have been completed, the shock Mach number M_{ss} and the shock location represented by F_{ss}/F_{in} may be computed from equations (101) and (102), respectively. Moreover, the pressure loss $\Delta P_T/P_T$ is known from

$$\frac{\Delta P_T}{P_T} = \frac{F_{in}}{F_{ex}} \frac{D_{in}}{D_{ex}} - 1 = \frac{P_{T,ex}}{P_{T,in}} - 1 \tag{109}$$

At the right boundary, the system, it will be remembered, is perturbed by varying the choking throat area with time. Clearly as the duct to throat area ratio at the right boundary decreases (i.e., the throat area is being enlarged) the Mach number at the right boundary increases or an expansion wave is initiated at this boundary. When this wave finally arrives at the left boundary, the shock moves forward as seen in figure 7(b) increasing the supersonic Mach number ahead of the shock and the pressure losses (see fig. 7(c)). As the choking orifice at the right boundary returns to its former position, compression waves resulting from this motion cause the shock to return nearly to its former position. Had the compression waves been too strong, the shock would have been blown out of the diffuser altogether. Pressure oscillations at specified locations along the duct due to this motion are shown in figure 7(d).

CONCLUDING REMARKS

A theory, numerical methods, and computer programs for one-dimensional, nonsteady, nonhomentropic fluid flow have been presented; and it has been demonstrated that from the same general compatibility relations, nonsteady liquid flow and nonsteady gas flow with heat addition and shock perturbations may be analyzed. Moreover, in two of the examples selected, verification of the computations by alternate methods has been shown.

The theory, presented herein, does not preclude the possibility of analyzing a fluid in which the properties (e.g., molecular weight) of the fluid on each side of a defined interface are different. Such an instance may arise in a combustion chamber due to the initial injection of fuel, or during the expulsion of air from an open-ended duct as, say, nitrogen gas issues into the duct from a high pressure bottle at the opposite end.

In the formulation of the general compatibility relations, body and dissipative forces were omitted together with the variation of cross-sectional area in a conduit. The fundamental equations (i.e., conservation of energy, momentum, and continuity) may be extended to include these effects, and new compatibility relations may be derived which will have a broader application in nonsteady liquid and gas flow problems. Further, since no equation of state has been specified, the theory is not limited only to the dynamic analysis of liquids and gases. The state of the fluid may be described by a table of values such as a steam table. The theory, thus, may be applied to the dynamics of homogeneous two phase flow with heat addition.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 8, 1966,
120-27-04-27-22.

APPENDIX A

SYMBOLS

$\mathscr{A}$ nondimensional acoustic velocity defined in eq. (57)

a acoustic velocity, ft/sec

C_a water-hammer variable (see eq. (61)), ft

C_b water-hammer variable (see eq. (63)), ft

C_p specific heat at constant pressure, Btu/(lb)(^{O}R)

D parameter (see eqs. (99a) and (99b))

F cross-sectional area, sq ft

f frequency, cps

g_c acceleration due to Earth's gravitational field, 32.2 ft/sec^2

H head, ft

h enthalpy, Btu/lb

J mechanical equivalent of heat, 778.26 ft-lb/Btu

K_ν orifice coefficient (see eq. (68)), ft$^{5/2}$/sec

M Mach number

M_{ss} shock Mach number

P pressure, lb/sq ft or lb/sq in.

$\mathscr{P}$ Riemann variable (left running)

Q heat per pound of fluid, Btu/lb

$\mathscr{Q}$ Riemann variable (right running)

q volume flow, cu ft/sec

R gas constant, ft-lb/(slug)(^{O}R)

$\mathscr{R}$ orifice coefficient (see eq. (96)), ft$^{1/2}$/sec

S nondimensional entropy defined in eq. (56)

$\mathscr{S}_a$ water-hammer parameter (see eq. (72)), sec/sq ft

$\mathscr{S}_b$ water-hammer parameter (see eq. (73)), sec/sq ft

s specific entropy, Btu/(lb)(^{O}R)

T temperature, ^{O}R

t time, sec

$\mathscr{U}$ nondimensional velocity defined in eq. (58)

v velocity, ft/sec

W specific weight, lb/cu ft

y conduit length, ft

γ ratio of specific heats

ζ nondimensional distance defined in eq. (54a)

η parameter (see eq. (10))

ξ parameter (see eq. (9))

ρ density, slug/cu ft

τ nondimensional time defined in eq. (54b)

ψ parameter (see eq. (3))

ω frequency, radians/sec

Subscripts:

A	base point (see fig. 1)
a	refers to left running wave in liquid dynamics (see fig. 1)
amp	amplitude (see eq. (97))
B	base point (see fig. 1)
b	refers to right running wave in liquid dynamics (see fig. 1)
C	base point (see fig. 1)
ex	diffuser exit
in	diffuser inlet
lim	limit
ℓ	left or upstream of fluid particle
m	refers to fluid particle
o	reference
r	right or downstream of fluid particle
ss	supersonic
sub	subsonic
T	stagnation
0	see eq. (97)
1	end point of left running wave (see fig. 1)
2	end point of right running wave (see fig. 1)
3	intersection of left running wave, right running wave, and particle path (see fig. 1)
4	end point of particle path
I, II, III, IV, V	see fig. 4(a)

Superscripts:

1	alternate base point (see fig. 2)
2	alternate base point (see fig. 2)
'	refers to free characteristic net point (see fig. 1)
*	signifies state at which $M = 1$ (see ref. 6)

APPENDIX B

DERIVATION OF GENERAL COMPATIBILITY RELATIONS

First, equations (11), (12), and (13) must be solved for $\frac{\partial P}{\partial \xi}$ and $\frac{\partial s}{\partial \xi}$ according to the methods of reference 4. The other unknown $\frac{\partial v}{\partial \xi}$ is omitted since it will yield no additional compatibility relations (cf. ref. 6, pp. 974-977). The denominator in every case is

$$\begin{vmatrix} \frac{1}{\rho}\frac{\partial \xi}{\partial y} & v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t} & 0 \\ \frac{1}{a^2}\left(v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right) & \rho\frac{\partial \xi}{\partial y} & \frac{\partial \rho}{\partial s}\left(v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right) \\ 0 & 0 & v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t} \end{vmatrix} = 0 \tag{B1}$$

Hence,

$$\left(v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right)\left[\left(\frac{\partial \xi}{\partial y}\right)^2 - \frac{1}{a^2}\left(v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right)^2\right] = 0 \tag{B2}$$

One solution is

$$v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t} = 0 \tag{B3a}$$

Hence,

$$\frac{dy}{dt} = v \tag{B3b}$$

and the second solution is

$$a^2\left(\frac{\partial \xi}{\partial y}\right)^2 = \left(v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right)^2 \tag{B4a}$$

$$\pm a\frac{\partial \xi}{\partial y} = v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t} \tag{B4b}$$

Hence,

$$\frac{dy}{dt} = v + a \tag{B4c}$$

$$\frac{dy}{dt} = v - a \tag{B4d}$$

Now if

$$X = -\frac{\partial P}{\partial \eta}\frac{1}{\rho}\frac{\partial \eta}{\partial y} - \frac{\partial v}{\partial \eta}\left(v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) \tag{B5a}$$

$$Y = -\frac{\partial P}{\partial \eta}\frac{1}{a^2}\left(v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) - \frac{\partial v}{\partial \eta}\rho\frac{\partial \eta}{\partial y} - \frac{\partial s}{\partial \eta}\left(v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right)\frac{\partial \rho}{\partial s} \tag{B5b}$$

$$Z = -\frac{\partial s}{\partial \eta}\left(v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) + \psi \tag{B5c}$$

solve for $\frac{\partial P}{\partial \xi}$ by

$$\begin{vmatrix} X & v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t} & 0 \\ Y & \rho\frac{\partial \xi}{\partial y} & \frac{\partial \rho}{\partial s}\left(v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right) \\ Z & 0 & v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t} \end{vmatrix} = 0 \tag{B6}$$

If $\frac{\partial \xi}{\partial t} = -v\frac{\partial \xi}{\partial y}$, this determinant vanishes. If, as in reference 4, the following is assumed:

$$\frac{\partial \xi}{\partial t} = -(v + a)\frac{\partial \xi}{\partial y} \tag{B7}$$

which is implicit in equation (B4b), then

$$\begin{vmatrix} X & -a\dfrac{\partial \xi}{\partial y} & 0 \\ Y & \rho\dfrac{\partial \xi}{\partial y} & -\dfrac{\partial \rho}{\partial s}\, a\dfrac{\partial \xi}{\partial y} \\ Z & 0 & -a\dfrac{\partial \xi}{\partial y} \end{vmatrix} = 0 \tag{B8}$$

or

$$\left(\frac{\partial \xi}{\partial y}\right)^2 \left(-\rho aX - a^2 Y + a^2 \frac{\partial \rho}{\partial s} Z\right) = 0 \tag{B9}$$

Dividing by a results in one possible solution to equation (B8), that is,

$$-\rho X - aY + a\frac{\partial \rho}{\partial s} Z = 0 \tag{B10}$$

along a line on the y, t plane having the slope given by equation (B4c). If the following is assumed:

$$\frac{\partial \xi}{\partial t} = -(v - a)\frac{\partial \xi}{\partial y} \tag{B11}$$

instead of equation (B10)

$$-\rho X + aY - a\frac{\partial \rho}{\partial s} Z = 0 \tag{B12}$$

is obtained along a line on the y, t plane having the slope given by equation (B4d). By solving equations (11), (12), and (13) for $\frac{\partial s}{\partial \xi}$ the compatibility relation along the particle path is found from

$$\begin{vmatrix} \frac{1}{\rho}\frac{\partial \xi}{\partial y} & v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t} & X \\ \frac{1}{a^2}\left(v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t}\right) & \rho\frac{\partial \xi}{\partial y} & Y \\ 0 & 0 & Z \end{vmatrix} = 0 \tag{B13}$$

or expanding along the third row

$$Z\left(\frac{\partial \xi}{\partial y}\right)^2 = 0 \tag{B14}$$

since along the particle path

$$v\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial t} = 0 \tag{B15}$$

A possible solution to equation (B14) is

$$Z = 0 \tag{B16}$$

Therefore, equation (B5c) results in

$$\frac{\partial s}{\partial \eta}\left(v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) = \psi \tag{B17}$$

but

$$v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t} = \frac{D\eta}{Dt} \tag{B18}$$

or

$$\frac{\partial s}{\partial \eta}\frac{D\eta}{Dt} = \frac{Ds}{Dt} = \psi \tag{B19}$$

If there is no heat addition to the system, then

$$\frac{Ds}{Dt} = 0 \tag{B20}$$

or

$$s = \text{const} \tag{B21}$$

as in equation (15) of reference 4.

At this point, the left and right running compatibility equations of reference 4 are extended to accommodate nonhomentropic flow in which there may be not only shock waves but also heat addition. From equations (B5a), (B5b), and (B5c), the left running compatibility relation (eq. (B10)) may be presented in the following form:

$$\frac{1}{a}\frac{\partial P}{\partial \eta}\left(a\frac{\partial \eta}{\partial y} + v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right) + \rho\frac{\partial v}{\partial \eta}\left(a\frac{\partial \eta}{\partial y} + v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}\right)$$

$$+a\frac{\partial s}{\partial \eta}\frac{\partial \rho}{\partial s}v\frac{\partial \eta}{\partial y} + a\frac{\partial s}{\partial \eta}\frac{\partial \eta}{\partial t}\frac{\partial \rho}{\partial s} - a\frac{\partial \rho}{\partial s}\frac{\partial s}{\partial \eta}v\frac{\partial \eta}{\partial y} - a\frac{\partial \rho}{\partial s}\frac{\partial s}{\partial \eta}\frac{\partial \eta}{\partial t} + a\psi\frac{\partial \rho}{\partial s} = 0 \tag{B22}$$

Now if

$$\frac{\delta^+}{\delta t} = \frac{\partial}{\partial t} + (v + a)\frac{\partial}{\partial y} \tag{B23a}$$

and

$$\frac{\delta^-}{\delta t} = \frac{\partial}{\partial t} + (v - a)\frac{\partial}{\partial y} \tag{B23b}$$

then equation (B22) may be expressed by

$$\frac{1}{\rho a}\frac{\partial P}{\partial \eta}\frac{\delta^+ \eta}{\delta t} + \frac{\partial v}{\partial \eta}\frac{\delta^+ \eta}{\delta t} = -a\psi\frac{\partial(\ln \rho)}{\partial s} \tag{B24}$$

or

$$\frac{1}{\rho a}\frac{\delta^+ P}{\delta t} + \frac{\delta^+ v}{\delta t} = -a\psi\frac{\partial(\ln \rho)}{\partial s} \tag{B25}$$

which is the compatibility equation for a fluid (gas or liquid) including heat addition in the left running direction (cf. fig. 1). A similar development may be made for the right running direction. Such a development yields

$$\frac{1}{\rho a}\frac{\delta^{-}P}{\delta t} - \frac{\delta^{-}v}{\delta t} = -a\psi\frac{\partial(\ln \rho)}{\partial s} \tag{B26}$$

which is the compatibility equation for a fluid (gas or liquid) including heat addition in the right running direction (cf. fig. 1).

APPENDIX C

COMPUTER PROGRAM FOR NONHOMENTROPIC NONSTEADY GAS FLOW

In the following, the input or required data will be presented for this program so that it may be readily employed. The input of this computer program contains several options not utilized in either the example for nonsteady gas flow with cooling or the supersonic diffuser example. This will be made clear in the following outline of the input required for this program.

Input Variables and Explanations

First input card (corresponding to read statement 200170)

NL number of points to be computed on each line

NTIM when multiplied by time increment DELT represents the value of time on initial line

NTOT number of time intervals to be computed

KTPT if 1, results will not be punched on cards; if 2, results will be punched on cards for possible use in plotting

Second input card (corresponding to read statement 200180)

GAM ratio of specific heats

GG acceleration due to gravity

RR gas constant

AJJ mechanical equivalent of heat

PRS1 reference pressure

RHO1 reference density

AL0 reference length

Third input card (corresponding to read statement 200190)

AKDS amplitude for $\frac{DS}{D\tau}$ oscillation (cf. card 700220)

AKMK amplitude for Mach number oscillation at right boundary (cf. card 700210)

AKPS amplitude for pressure oscillation at left boundary (cf. card 500140)

AKS amplitude for entropy oscillation at left boundary (cf. card 500170)

DSDTI initial value of $\frac{DS}{D\tau}$

Fourth input card (corresponding to read statement 200200)

FFDS frequency for $\frac{DS}{D\tau}$ oscillation (cf. card 200350)

FFMK frequency for Mach number oscillation at right boundary (cf. card 200840)

FFPR frequency for pressure oscillation at left boundary (cf. card 200340)

FFS frequency for entropy oscillation at left boundary (cf. card 200360)

DELT nondimensional time increment

DELL distance between any two points on any base line; if properties on initial line are not constant, DELL = 0.0

EM1 shock Mach number; if diffuser problem is not being run, EM1 = 0.0

A21 diffuser exit to throat area ratio

After the fourth input card, there are two options which are selected by setting DELL equal to either zero or to its value, the distance between two points on a base line. If DELL is zero, parameters at each location on the initial line are different. One card is required for each point. Hence, if the number of points on a base line (NL) is five, for example, five additional cards are required.
So, for the first option:

Input cards (corresponding to card 200230)

Z(L) location of L^{th} point on initial line

R(L, 1) nondimensional acoustic velocity at L^{th} point on initial line

U(L, 1) nondimensional velocity at L^{th} point on initial line

S(L, 1) nondimensional entropy at L^{th} point on initial line

TIM(1) nondimensional time on initial line and for the second option:

Fifth input card (corresponding to card 200260)

AZ	location of left boundary point, usually zero
AR	constant value of nondimensional acoustic velocity along initial line
AU	constant value of nondimensional velocity along initial line
AS	constant value of nondimensional entropy along initial line
TIM(1)	nondimensional time on initial line

Fortran Program Listing

```
C     MAIN PROGRAM
      COMMON /CONST/ A0, AJJ, AKDS, AKPS, AKS, AMOK, CON1, CON2, CON3,
     1   CX1, DSDT, DSDTI, GAM, GG, GMM1, GMP1, OMGD, OMGP, OMGS,
     2   PRS1, PRSIL, RR, RHO1, S3I, TIMO
      COMMON /PROP/ NIT(20,10), R(20,10), S(20,10), TIM(10), TIMR(10),
     1   U(20,10), Z(20)
      COMMON /RUN/ DELL, DELT, FFDS, FFPR, FFS, KTPT, NL, NTIM, NTOT
      COMMON /SUPSON/ AAST(10), AKMK, AMKI, AX1(10), A21,
     X   DI, DLS, DPP(10),
     1   EM1, EM2, EMX(10), EMY(10), FFMK, OMGM, RT, SPR
      DIMENSION  PRS(20), RHO(20), TEMP(20), ENT(10), DST(10), VEL(10)
      DIMENSION AMK(10), PLTD(840)
    1 CALL REDY
      KDSTP=1
      IBEG=1
      CALL BNET(NL,1)
      AAST(1)=AMKI
   21 ILST=IBEG+7
      DO 24 I=2,ILST
      TIM(I)=NTOT
      TIM(I)=DELT*TIM(I)
      TIMR(I)=TIMO*TIM(I)
      NTOT=NTOT+1
      IF(NTOT-NTIM)23,23,22
   22 ILST=I-1
      KDSTP=2
      GO TO 28
   23 CONTINUE
      CALL BNET(NL,I)
      DST(I)=DSDT
      AAST(I)=AMOK
   24 CONTINUE
   28 IF(ILST-1)29,87,29
C     PRINTED AND PUNCHED OUTPUT
   29 WRITE(6,30) (TIM(I),I=IBEG,ILST)
   30 FORMAT(10H1    TIM = 8G15.6)
      WRITE(6,330) (TIMR(I),I=IBEG,ILST)
  330 FORMAT(10HJREL TIM= 8G15.6)
      WRITE(6,340) (DST(I),I=IBEG,ILST)
  340 FORMAT(10HJ  DSDT = 8G15.6)
      IF(EM1)32,36,32
```

```
   32 DO 34 I=IBEG,ILST
   34 AAST(I)=AST(AAST(I),GMM1,GMP1)
      WRITE(6,350) (AAST(I),I=IBEG,ILST)
  350 FORMAT(10HJ A/AST = 8G15.6)
      WRITE(6,360) (AX1(I),I=IBEG,ILST)
  360 FORMAT(10HJ AX/A1 = 8G15.6)
      WRITE(6,370) (DPP(I),I=IBEG,ILST)
  370 FORMAT(10HJ  DP/P = 8G15.6)
      WRITE(6,380) (EMX(I),I=IBEG,ILST)
  380 FORMAT(10HJ    MX = 8G15.6)
      WRITE(6,390) (EMY(I),I=IBEG,ILST)
  390 FORMAT(10HJ    MU = 8G15.6)
   36 GO TO (56,38),KTPT
   38 K=0
      DO 39 I=IBEG,ILST
      K=K+1
   39 PLTD(K)=TIM(I)
      IF(EM1)41,56,41
   41 DO 43 L=1,NL
      DO 43 I=IBEG,ILST
      K=K+1
   43 PLTD(K)=S(L,I)
      DO 45 I=IBEG,ILST
      K=K+1
      PLTD(K)=AAST(I)
      K=K+1
      PLTD(K)=AX1(I)
      K=K+1
      PLTD(K)=DPP(I)
      K=K+1
   45 PLTD(K)=EMX(I)
   56 DO 73  L=1,NL
      WRITE (6,40) Z(L), (R(L,I),I=IBEG,ILST)
   40 FORMAT(1HKF7.4,2HR=8G15.6)
      WRITE (6,50) (U(L,I),I=IBEG,ILST)
   50 FORMAT(10HJ        U=8G15.6)
      WRITE (6,60) (S(L,I),I=IBEG,ILST)
   60 FORMAT(10HJ        S=8G15.6)
      WRITE(6,75) (NIT(L,I),I=IBEG,ILST)
   75 FORMAT(10HJ      NIT=8G15.6)
      DO 63 I=IBEG,ILST
      CN1=EXP(-GAM*S(L,I))
      RHO(I)=CN1*RHO1*(R(L,I)**CON1)
      PRS(I)=CN1*PRS1*(R(L,I)**CON2)
      AA=A0*R(L,I)
      TEMP(I)=AA*AA/(GAM*GG*RR)
      ENT(I)=S(L,I)*GAM*RR/AJJ
      VEL(I)=A0*U(L,I)
      AMK(I)=U(L,I)/R(L,I)
   63 CONTINUE
      WRITE(6,65) (ENT(I),I=IBEG,ILST)
   65 FORMAT(10HJ      ENT=8G15.6)
      WRITE(6,70) (RHO(I),I=IBEG,ILST)
   70 FORMAT(10HJ      RHO=8G15.6)
      WRITE(6,80) (PRS(I),I=IBEG,ILST)
   80 FORMAT(10HJ      PRS=8G15.6)
      WRITE(6,90) (TEMP(I),I=IBEG,ILST)
   90 FORMAT(10HJ     TEMP=8G15.6)
      WRITE(6,100) (VEL(I),I=IBEG,ILST)
  100 FORMAT(10HJ      VEL=8G15.6)
      WRITE(6,110) (AMK(I),I=IBEG,ILST)
```

```
  110 FORMAT(10HJ      MOK=8G15.6)
      R(L,1)=R(L,ILST)
      U(L,1)=U(L,ILST)
      S(L,1)=S(L,ILST)
      GO TO (73,67),KTPT
   67 DO 69 I=IBEG,ILST
      K=K+1
      PLTD(K)=TEMP(I)
      K=K+1
      PLTD(K)=RHO(I)
      K=K+1
      PLTD(K)=PRS(I)
      K=K+1
   69 PLTD(K)=AMK(I)
   73 CONTINUE
      TIM(1)=TIM(ILST)
      GO TO (85,78),KTPT
   78 IF(IBEG-1)83,81,83
   81 WRITE(6,130) NL, K
  130 FORMAT(2H$ 2I5)
   83 CALL BCDUMP(PLTD(1),PLTD(K),1)
   85 IBEG=2
   87 GO TO (21,88),KDSTP
   88 GO TO (93,91),KTPT
   91 WRITE(6,130) K
   93 GO TO 1
      END
```

```
      SUBROUTINE REDY
      COMMON /CONST/ AO, AJJ, AKDS, AKPS, AKS, AMOK, CON1, CON2, CON3,
     1   CX1, DSDT, DSDTI, GAM, GG, GMM1, GMP1, OMGD, OMGP, OMGS,
     2   PRS1, PRSIL, RR, RHO1, S3I, TIMO
      COMMON /PROP/ NIT(20,10), R(20,10), S(20,10), TIM(10), TIMR(10),
     1   U(20,10), Z(20)
      COMMON /RUN/ DELL, DELT, FFDS, FFPR, FFS, KTPT, NL, NTIM, NTOT
      COMMON/ALNE/ALIM,CA2,CA3,DIST,SLP,ZLIM,
     1   CU(19),CR(19),CS(19),CURP(19),CURM(19),
     2   ZL(20),RL(20),UL(20),SL(20),TL(20)
      COMMON /SUPSON/ AAST(10), AKMK, AMKI, AX1(10), A21,
     X   D1, DLS, DPP(10),
     1   EM1, EM2, EMX(10), EMY(10), FFMK, OMGM, RT, SPR
      PI=3.1415927
C     INITIAL INPUT AND OUTPUT
    1 READ(5,10) NL,NTIM,NTOT,KTPT
      READ(5,20) GAM, GG, RR, AJJ, PRS1, RHO1, AL
      READ(5,20) AKDS, AKMK, AKPS, AKS, DSDTI
      READ(5,20) FFDS, FFMK, FFPR, FFS, DELT, DELL, EM1, A21
      IF(DELL)5,3,5
    3 DO 4 L=1,NL
    4 READ(5,20)  Z(L), R(L,1), U(L,1), S(L,1), TIM(1)
      DELL=Z(2)-Z(1)
      GO TO 8
    5 READ(5,20) AZ, AR, AU, AS, TIM(1)
C     CALCULATE INITIAL CONSTANTS
      DO 6 L=1,NL
      CN1=L-1
      Z(L)=AZ+CN1*DELL
      R(L,1)=AR
      U(L,1)=AU
```

```
  6 S(L,1)=AS
  8 OMGP=2.*PI*FFPR
    OMGD=2.*PI*FFDS
    OMGS=2.*PI*FFS
    R3IL=R(1,1)
    S3I=S(1,1)
    GMM1=GAM-1.
    GMP1=GAM+1.
    CON1=2./GMM1
    CON2=GAM*CON1
    RHO0=RHO1/GG
    A0=SQRT(GAM*PRS1*144./RHO0)
    TIMO=ALO/A0
    AMKI=U(NL,1)/R(NL,1)
    AAST(1)=AST(AMKI,GMM1,GMP1)
    WRITE(6,100) NL, NTIM, NTOT, KTPT
100 FORMAT(8H1  NL = I3,5X,7HNTIM = I3,5X,7HNTOT = I3,5X,7HKTPT = I3)
    WRITE(6,110) DELL, DELT, PRS1, RHO1, ALO
110 FORMAT(8HKDELL = G16.8,3X,7HDELT = G16.8,3X,7HPRS1 = G16.8,3X,
   17HRHO1 = G16.8,3X,7H  LO = G16.8)
    WRITE(6,120) GAM, GG, RR, AJJ
120 FORMAT(8HK GAM = G16.8,3X,7H   G = G16.8,3X,7H  RR = G16.8,3X,
   17H   J = G16.8)
    WRITE(6,130) DSDTI, AMKI, R3IL, S3I
130 FORMAT(8HKDSDTI= G16.8,3X,7HAMKI = G16.8,3X,7HR3IL = G16.8,3X,
   17H S3I = G16.8)
    WRITE(6,140) AKDS, AKMK, AKPS, AKS
140 FORMAT(8HKAKDS = G16.8,3X,7HAKMK = G16.8,3X,7HAKPS = G16.8,3X,
   17H AKS = G16.8)
    WRITE(6,150) FFDS, FFMK, FFPR, FFS
150 FORMAT(8HKFFDS = G16.8,3X,7HFFMK = G16.8,3X,7HFFPR = G16.8,3X,
   17H FFS = G16.8)
    WRITE(6,160) AAST(1), A0, EM1, A21
160 FORMAT(8HKA/A* = G16.8,3X,7H  A0 = G16.8,/,
   18HK  M1 = G16.8,3X,7HA2/A1= G16.8)
    CN1=EXP(-GAM*S3I)
    PRSIL=CN1*PRS1*(R3IL**CON2)
    CN1=.5*GMM1
    CON3=EXP(CN1)
    CX1=CN1/GAM
C      DIST = DISTANCE BETWEEN 2 POINTS IN THE GAS CIRCUIT
    DIST=DELL
C      SLP = THE SLOPE OF THE BASE LINE IN THE GAS CIRCUIT
    SLP=0.
C      CA2 = SQRT(1.+SLP**2)
    CA2=1.
    CA3=CA2/DIST
    ALIM=Z(1)
    ZLIM=Z(NL)
    IF(EM1)21,25,21
 21 CALL SSBZ(GAM,GMM1,R(1,1),U(1,1),S(1,1))
    OMGM=2.*PI*FFMK
    DSDT=DSDTI
 25 CONTINUE
 10 FORMAT(16I5)
 20 FORMAT(8E10.0)
    RETURN
    END
```

```
      SUBROUTINE BNET (NL,I)
C     ORGANIZATION OF CALCULATIONS ON LINE CORRESPONDING TO TIME I
      COMMON /PROP/ NIT(20,10), R(20,10), S(20,10), TIM(10), TIMR(10),
     1   U(20,10), Z(20)
      COMMON/ALNE/ALIM,CA2,CA3,DIST,SLP,ZLIM,
     1   CU(19),CR(19),CS(19),CURP(19),CURM(19),
     2   ZL(20),RL(20),UL(20),SL(20),TL(20)
      LF=NL-1
      IF(I-1)10,10,1
    1 J=I-1
C     SET UP NET FOR LOCATIONS 1 TO NL(FINAL LOCATION)
      CALL GASPZ (Z(NL), TIM(I), R(NL,I), U(NL,I), S(NL,I),
     1   NIT(NL,I), TIMR(I), NL, I)
      DO 8 M=2,LF
      L=LF+2-M
      CALL GASP (Z(L), TIM(I), R(L,I), U(L,I), S(L,I), NIT(L,I), L)
    8 CONTINUE
      CALL GASPA (Z(1), TIM(I), R(1,I), U(1,I), S(1,I),
     1   NIT(1,I), TIMR(I), 1,I)
C     SET UP BASE LINE CONSTANTS TO BE USED IN CALCULATING THE NEXT LINE
   10 DO 28 LL=1,LF
      LR=LL+1
      ZL(LL)=Z(LL)
      RL(LL)=R(LL,I)
      UL(LL)=U(LL,I)
      SL(LL)=S(LL,I)
      TL(LL)=TIM(I)
      CU(LL)=U(LR,I)-UL(LL)
   14 CU(LL)=CA3*CU(LL)
   16 CR(LL)=R(LR,I)-RL(LL)
   22 CR(LL)=CA3*CR(LL)
   25 CURP(LL)=CU(LL)+CR(LL)
      CURM(LL)=CU(LL)-CR(LL)
      CS(LL)=CA3*(S(LR,I)-SL(LL))
   28 CONTINUE
      ZL(NL)=Z(NL)
      RL(NL)=R(NL,I)
      UL(NL)=U(NL,I)
      SL(NL)=S(NL,I)
      TL(NL)=TIM(I)
      RETURN
      END
```

```
      SUBROUTINE GASP(Z3,T3,R3,U3,S3,IT,L)
C     BASIC NET POINT CALCULATIONS
      COMMON /CONST/ A0, AJJ, AKDS, AKPS, AKS, AMOK, CON1, CON2, CON3,
     1   CX1, DSDT, DSDTI, GAM, GG, GMM1, GMP1, OMGD, OMGP, OMGS,
     2   PRS1, PRSIL, RR, RHO1, S3I, TIMO
      COMMON/ALNE/ALIM,CA2,CA3,DIST,SLP,ZLIM,
     1   CU(19),CR(19),CS(19),CURP(19),CURM(19),
     2   ZL(20),RL(20),UL(20),SL(20),TL(20)
      CRIT=.001
C     INITIAL CALCULATIONS
      KDLN=1
      IT=0
      KDSTP=1
      KLOP=1
      UP3=1000.
      RP3=1000.
      TST2U=1.E+20
      TST2R=1.E+20
      U3=UL(L)
      R3=RL(L)
C      BEGINNING OF LOOP
    5 IT=IT+1
C     CALCULATIONS FOR POINT 1
    7 LB=L
    8 LA=LB-1
      CN1=T3-TL(LA)+SLP*ZL(LA)
      CN2=U3+R3+UL(LA)+RL(LA)-CURP(LA)*ZL(LA)
      AA=SLP*CURP(LA)
      BB=SLP*CN2-CN1*CURP(LA)-2.
      CC=2.*Z3-CN1*CN2
      Z1=QUAD(AA,BB,CC,-1.,Z3)
      CN1=Z1-ZL(LA)
      T1=TL(LA)+SLP*CN1
      IF(CN1)12,19,19
   12 IF(ALIM-Z1)14,14,20
   14 LB=LB-1
      GO TO 8
   19 U1=UL(LA)+CN1*CU(LA)
      R1=RL(LA)+CN1*CR(LA)
      S1=SL(LA)+CN1*CS(LA)
      GO TO 22
   20 T1=T3-(Z3-ALIM)*(T3-T1)/(Z3-Z1)
      Z1=ALIM
      RELTM=TIMO*T1
      CALL GASPA(Z1,T1,R1,U1,S1,NDM,RELTM,LA)
      KDLN=2
C     CALCULATIONS FOR POINT 2
   22 LB=L
   25 LC=LB+1
      CN1=T3-TL(LB)+SLP*ZL(LB)
      CN2=U3-R3+UL(LB)-RL(LB)-CURM(LB)*ZL(LB)
      AA=SLP*CURM(LB)
      BB=SLP*CN2-CN1*CURM(LB)-2.
      CC=2.*Z3-CN1*CN2
      Z2=QUAD(AA,BB,CC,1.,Z3)
      CN1=Z2-ZL(LB)
      T2=TL(LB)+SLP*CN1
      IF(Z2-ZL(LC))32,32,27
   27 IF(Z2-ZLIM)28,28,34
   28 LB=LB+1
```

```
      GO TO 25
   32 U2=UL(LB)+CN1*CU(LB)
      R2=RL(LB)+CN1*CR(LB)
      S2=SL(LB)+CN1*CS(LB)
      GO TO 37
   34 T2=T3-(Z3-ZLIM)*(T3-T2)/(Z3-Z2)
      Z2=ZLIM
      CALL GASPZ(Z2,T2,R2,U2,S2,NDM,LC)
      KDLN=2
C     CALCULATIONS FOR POINT 4
   37 GO TO (39,49),KDLN
   39 LB=L
   41 LA=LB-1
      CN1=U3+UL(LA)-CU(LA)*ZL(LA)
      CN2=T3-TL(LA)+SLP*ZL(LA)
      AA=CU(LA)*SLP
      BB=CN1*SLP-CU(LA)*CN2-2.
      CC=2.*Z3-CN1*CN2
      Z4=QUAD(AA,BB,CC,-1.,Z3)
      CN1=Z4-ZL(LA)
      IF(CN1)45,47,47
   45 LB=LB-1
      GO TO 41
   47 T4=TL(LA)+SLP*CN1
      U4=UL(LA)+CU(LA)*CN1
      S4=SL(LA)+CS(LA)*CN1
      GO TO 51
C     ALTERNATE CALCULATIONS FOR POINT 4 WHEN BASE SLOPE DOES NOT = 0.
   49 D12=SQRT((T1-T2)**2+(Z1-Z2)**2)
      SL12=(T2-T1)/(Z2-Z1)
      CN1=SQRT(1.+SL12*SL12)*(U2-U1)/D12
      CN2=T3-T2+SL12*Z2
      CN3=U3+U2-CN1*Z2
      AA=CN1*SL12
      BB=SL12*CN3-CN1*CN2-2.
      CC=2.*Z3-CN2*CN3
      Z4=QUAD(AA,BB,CC,-1.,Z3)
      CN2=Z2-Z4
      T4=T2-SL12*CN2
      U4=U2-CN1*CN2
      S4=S2-CN2*SQRT(1.+SL12*SL12)*(S2-S1)/D12
      KDLN=1
C     CALCULATE S3
   51 S3=S4+DSDT*(T3-T4)
      GO TO (54,57),KLOP
C     CALCULATE U3
   54 P1=R1*2./GMM1+U1
      Q2=R2*2./GMM1-U2
      R13=.5*(R1+R3)
      P3=P1+R13*(GMM1*DSDT*(T3-T1)+(S3-S1))
      R23=.5*(R2+R3)
      Q3=Q2+R23*(GMM1*DSDT*(T3-T2)+(S3-S2))
      U3=.5*(P3-Q3)
      KLOP=2
      GO TO 7
C     CALCULATE R3
   57 Q2=R2*2./GMM1-U2
      R23=.5*(R2+R3)
      Q3=Q2+R23*(GMM1*DSDT*(T3-T2)+(S3-S2))
      KLOP=1
      R3=.5*(Q3+U3)*GMM1
```

```
C     TEST FOR CONVERGENCE OF U3 AND R3
      TST1U=ABS(U3-UP3)
      TST1R=ABS(R3-RP3)
      IF(TST1U-CRIT)62,65,65
   62 IF(TST1R-CRIT)95,65,65
   65 IF(TST2U-TST1U)71,71,67
   67 IF(TST2R-TST1R)73,73,84
   71 WRITE(6,600) Z3, T3
  600 FORMAT(28HK* U IS NOT CONVERGING AT Z=G16.8,5X,2HT=G16.8)
      GO TO 75
   73 WRITE(6,610) Z3, T3
  610 FORMAT(28HK* R IS NOT CONVERGING AT Z=G16.8,5X,2HT=G16.8)
   75 CONTINUE
   76 WRITE(6,50) Z1, R1, U1, S1, T1
      WRITE(5,60) Z2, R2, U2, S2, T2
      WRITE(6,60) IT, Z4, U4, S4, T4
      WRITE(6,60) P1, Q2, P3, Q3
      WRITE(6,60) Z3, R3, U3, S3, T3
   50 FORMAT(4HL** 5G20.8)
   60 FORMAT(4HJ   5G20.8)
   82 GO TO (84,96),KDSTP
   84 UP3=U3
      RP3=R3
      TST2U=TST1U
      TST2R=TST1R
      GO TO 5
   95 CONTINUE
   96 IF(U3-R3)101,101,98
   98 WRITE(6,70)U3,R3,Z3,T3
   70 FORMAT(30HL** STOP   SUPERSONIC FLOW  U=G16.6,5X,2HR=G16.6,
     15X,6HAT Z3=G16.6,5X,7HAND T3=G16.6)
      STOP
  101 RETURN
      END
```

```
      SUBROUTINE GASPA (Z3,T3,R3,U3,S3,IT,RELTM,L,I)
C     LEFT BOUNDARY CALCULATIONS
      COMMON /CONST/ AO, AJJ, AKDS, AKPS, AKS, AMOK, CON1, CON2, CON3,
     1   CX1, DSDT, DSDTI, GAM, GG, GMM1, GMP1, OMGD, OMGP, OMGS,
     2   PRS1, PRSIL, RR, RHO1, S3I, TIMO
      COMMON/ALNE/ALIM,CA2,CA3,DIST,SLP,ZLIM,
     1   CU(19),CR(19),CS(19),CURP(19),CURM(19),
     2   ZL(20),RL(20),UL(20),SL(20),TL(20)
      CRIT=.001
C     INITIAL CALCULATIONS
C     CALCULATIONS FOR S3 AND R3
      CN1=SIN(OMGP*RELTM)
      PRS3=PRSIL+CN1*AKPS*PRSIL
      CN2=(PRS3/PRS1)**CX1
      CN1=SIN(OMGS*RELTM)
      S3=S3I+AKS*CN1*S3I
      R3=CN2*(CON3**S3)
      IF(UL(1)-RL(1))4,2,4
```

```
    2 Z2=ZL(1)
      T2=TL(1)
      U2=UL(1)
      R2=RL(1)
      S2=SL(1)
      Q2=R2*2./GMM1-U2
      R23=.5*(R2+R3)
      Q3=Q2+R23*(S3-S2)
      U3=2.*R3/GMM1-Q3
      GO TO 25
    4 IT=0
      UP3=1000.
      KDSTP=1
      TST2=1.E+20
      U3=UL(L)
C      BEGINNING OF LOOP
    5 IT=IT+1
C     CALCULATIONS FOR POINT 2
      LB=L
    8 LC=LB+1
      CN1=T3-TL(LB)+SLP*ZL(LB)
      CN2=U3-R3+UL(LB)-RL(LB)-CURM(LB)*ZL(LB;
      AA=SLP*CURM(LB)
      BB=SLP*CN2-CN1*CURM(LB)-2.
      CC=2.*Z3-CN1*CN2
      Z2=QUAD(AA,BB,CC,1.,Z3)
      IF(Z2-ZL(LC))12,12,10
   10 LB=LB+1
      GO TO 8
   12 CN1=Z2-ZL(LB)
      T2=TL(LB)+SLP*CN1
      U2=UL(LB)+CN1*CU(LB)
      R2=RL(LB)+CN1*CR(LB)
      S2=SL(LB)+CN1*CS(LB)
      Q2=R2*2./GMM1-U2
C     CALCULATE U3
      R23=.5*(R2+R3)
      Q3=Q2+R23*(GMM1*DSDT*(T3-T2)+(S3-S2))
      U3=2.*R3/GMM1-Q3
C     TEST FOR CONVERGENCE OF U3
      TST1=ABS(U3-UP3)
      IF(TST1-CRIT)25,15,15
   15 IF(TST2-TST1)16,17,17
   16 WRITE(6,50) Z2, R2, U2, S2, T2
      WRITE(6,60) IT, Q2, Q3
      WRITE(6,60) Z3, R3, U3, S3, T3
   50 FORMAT(4HL** 5G20.8)
   60 FORMAT(4HJ   5G20.8)
      GO TO (17,26),KDSTP
   17 UP3=U3
      TST2=TST1
      GO TO 5
   25 IF(U3-R3)34,34,33
   33 U3=R3
   34 CONTINUE
   26 RETURN
      END
```

```
      SUBROUTINE GASPA (Z3,T3,R3,U3,S3,IT,RELTM,L,I)
C     LEFT BOUNDARY SUBROUTINE FOR THE SUPERSONIC BUZZ PROGRAM
      COMMON /CONST/ A0, AJJ, AKDS, AKPS, AKS, AMOK, CON1, CON2, CON3,
     1   CX1, DSDT, DSDTI, GAM, GG, GMM1, GMP1, OMGD, OMGP, OMGS,
     2   PRS1, PRSIL, RR, RHO1, S3I, TIMO
      COMMON/ALNE/ALIM,CA2,CA3,DIST,SLP,ZLIM,
     1   CU(19),CR(19),CS(19),CURP(19),CURM(19),
     2   ZL(20),RL(20),UL(20),SL(20),TL(20)
      COMMON /SUPSON/ AAST(10), AKMK, AMKI, AX1(10), A21,
     X   D1, DLS, DPP(10),
     1   EM1, EM2, EMX(10), EMY(10), FFMK, OMGM, RT, SPR
      CRIT=.001
C     INITIAL CALCULATIONS
      CNN=GMP1/GMM1
      IF(UL(1)-RL(1))4,2,4
    2 Z2=ZL(1)
      T2=TL(1)
      U2=UL(1)
      R2=RL(1)
      S2=SL(1)
      Q2=R2*2./GMM1-U2
      R23=.5*(R2+R3)
      Q3=Q2+R23*(S3-S2)
      U3=2.*R3/GMM1-Q3
      GO TO 25
    4 IT=0
      KDSTP=1
      TST2=1.E+20
      Q3=2.*RL(L)/GMM1-UL(L)
      YY1=0.
      XX1=.99*Q3
      QP3=Q3
C      BEGINNING OF LOOP
    5 IT=IT+1
      XX2=Q3
C     CALCULATIONS FOR R,U, AND S AT DIFFUSER EXIT
      R3=Q3+SQRT(CNN*RT*RT-.5*GMM1*Q3*Q3)
      R3=R3/CNN
      U3=2.*R3/GMM1-Q3
      EM3=U3/R3
      D3=1.+.5*GMM1*EM3*EM3
      D3=EM3/(D3**(.5*CNN))
      DSR=ALOG(A21*D3/D1)
      DLS=DSR/GAM
      S3=SPR+DLS
C     CALCULATIONS FOR POINT 2
      LB=L
    8 LC=LB+1
      CN1=T3-TL(LB)+SLP*ZL(LB)
      CN2=U3-R3+UL(LB)-RL(LB)-CURM(LB)*ZL(LB)
      AA=SLP*CURM(LB)
      BB=SLP*CN2-CN1*CURM(LB)-2.
      CC=2.*Z3-CN1*CN2
      Z2=QUAD(AA,BB,CC,1.,Z3)
      IF(Z2-ZL(LC))12,12,10
   10 LB=LB+1
      GO TO 8
   12 CN1=Z2-ZL(LB)
      T2=TL(LB)+SLP*CN1
      U2=UL(LB)+CN1*CU(LB)
      R2=RL(LB)+CN1*CR(LB)
      S2=SL(LB)+CN1*CS(LB)
      Q2=R2*2./GMM1-U2
```

```
      R23=.5*(R2+R3)
      Q3=Q2+R23*(GMM1*DSDT*(T3-T2)+(S3-S2))
C     TEST ON CONVERGENCE OF THE Q COMPATIBILITY RELATION
      YY2=Q3-QP3
      TST1=ABS(YY2/Q3)
      IF(TST1-CRIT)25,17,17
   17 QP3=Q3
      Q3=(XX1*YY2-XX2*YY1)/(YY2-YY1)
      YY1=YY2
      XX1=XX2
      TST2=TST1
      GO TO 5
   25 IF(U3-R3)34,34,33
   33 U3=R3
   34 CONTINUE
   26 EMX(I)=FMX(EM1,DSR,GAM,GMM1,GMP1)
      CN1=EMX(I)*EMX(I)
      DX=1.+.5*GMM1*CN1
      DX=EMX(I)/(DX**(.5*CNN))
C     CALCULATION FOR SHOCK LOCATION
      AX1(I)=D1/DX
      IF(1.-AX1(I))46,46,42
   42 WRITE(6,70)
   70 FORMAT(26HL**  AX/A1 IS LESS THAN 1.)
   44 STOP
   46 IF(AX1(I)-A21)49,49,47
   47 WRITE(6,80)
   80 FORMAT(32HL**  AX/A1 IS GREATER THAN A2/A1)
      GO TO 44
   49 EMXST=SQRT(GMP1*CN1/(2.+GMM1*CN1))
      EMYST=1./EMXST
      CN1=EMYST*EMYST
      EMY(I)=SQRT(2.*CN1/(GMP1-GMM1*CN1))
C     CALCULATION FOR PRESSURE RECOVERY
      DPP(I)=A21*D3/D1-1.
      RETURN
      END
```

```
      SUBROUTINE GASPZ (Z3,T3,R3,U3,S3,IT,RELTM,L,I)
C     RIGHT BOUNDARY CALCULATIONS
      COMMON /CONST/ A0, AJJ, AKDS, AKPS, AKS, AMOK, CON1, CON2, CON3,
     1   CX1, DSDT, DSDTI, GAM, GG, GMM1, GMP1, OMGD, OMGP, OMGS,
     2   PRS1, PRSIL, RR, RHO1, S3I, TIMO
      COMMON/ALNE/ALIM,CA2,CA3,DIST,SLP,ZLIM,
     1   CU(19),CR(19),CS(19),CURP(19),CURM(19),
     2   ZL(20),RL(20),UL(20),SL(20),TL(20)
      COMMON /SUPSON/ AAST(10), AKMK, AMKI, AX1(10), A21,
     X   D1, DLS, DPP(10),
     1   EM1, EM2, EMX(10), EMY(10), FFMK, OMGM, RT, SPR
C     II U3 IS NOT KNOWN,    FIND U3, R3, AND S3
      CRIT=.001
C     INITIAL CALCULATIONS
      IT=0
      RP3=1000.
      KDSTP=1
      LM=L-1
      TST2=1.E+20
      AMOK=AMKI+AKMK*SIN(OMGM*RELTM)
      DSDT=DSDTI+AKDS*SIN(OMGD*RELTM)
```

```
      R3=RL(L)
      U3=AMOK*R3
C      BEGINNING OF LOOP
    5 IT=IT+1
C     CALCULATIONS FOR POINT 4
      LB=L
    7 LA=LB-1
      CN1=U3+UL(LA)-CU(LA)*ZL(LA)
      CN2=T3-TL(LA)+SLP*ZL(LA)
      AA=CU(LA)*SLP
      BB=CN1*SLP-CU(LA)*CN2-2.
      CC=2.*Z3-CN1*CN2
      Z4=QUAD(AA,BB,CC,-1.,Z3)
      IF(ZL(LA)-Z4)9,9,8
    8 LB=LB-1
      GO TO 7
    9 CN1=Z4-ZL(LA)
      T4=TL(LA)+SLP*CN1
      U4=UL(LA)+CU(LA)*CN1
      S4=SL(LA)+CS(LA)*CN1
      S3=S4+DSDT*(T3-T4)
C     CALCULATIONS FOR POINT 1
      LB=L
   11 LA=LB-1
      CN1=T3-TL(LA)+SLP*ZL(LA)
      CN2=U3+R3+UL(LA)+RL(LA)-CURP(LA)*ZL(LA)
      AA=SLP*CURP(LA)
      BB=SLP*CN2-CN1*CURP(LA)-2.
      CC=2.*Z3-CN1*CN2
      Z1=QUAD(AA,BB,CC,-1.,Z3)
      CN1=Z1-ZL(LA)
      IF(CN1)12,13,13
   12 LB=LB-1
      GO TO 11
   13 T1=TL(LA)+SLP*CN1
      U1=UL(LA)+CN1*CU(LA)
      R1=RL(LA)+CN1*CR(LA)
      S1=SL(LA)+CN1*CS(LA)
      P1=R1*2./GMM1+U1
C     CALCULATIONS FOR U3 AND R3
      R13=.5*(R1+R3)
      P3=P1+R13*(GMM1*DSDT*(T3-T1)+(S3-S1))
      U3=2./(GMM1*AMOK)
      U3=P3/(1.+U3)
      R3=U3/AMOK
      TEST FOR CONVERGENCE OF R3
      TST1=ABS(R3-RP3)
      IF(TST1-CRIT)25,15,15
   15 IF(TST2-TST1)16,17,17
   16 WRITE(6,50) Z1, R1, U1, S1, T1
      WRITE(6,60) IT, Z4, U4, S4, T4
      WRITE(6,60) P1, P3
      WRITE(6,60) Z3, R3, U3, S3, T3
   50 FORMAT(4HL** 5G20.8)
   60 FORMAT(4HJ    5G20.8)
      GO TO (17,26),KDSTP
   17 RP3=R3
      TST2=TST1
      GO TO 5
   25 CONTINUE
   26 RETURN
      END
```

```
      SUBROUTINE SSBZ (GAM,GMM1,R2,U2,S2)
C     INITIAL CALCULATIONS FOR THE SUPERSONIC DIFFUSER PROBLEM
      COMMON /SUPSON/ AAST(10), AKMK, AMKI, AX1(10), A21,
     X   D1, DLS, DPP(10),
     1   EM1, EM2, EMX(10), EMY(10), FFMK, OMGM, RT, SPR
      GMP1=GAM+1.
      EM2=U2/R2
      PHE=R2*R2*(2./GMM1+EM2*EM2)
      RT=SQRT(.5*GMM1*PHE)
      CN1=.5*GMP1/GMM1
      D1=1.+.5*GMM1*EM1*EM1
      D1=EM1/(D1**CN1)
      D2=1.+.5*GMM1*EM2*EM2
      D2=EM2/(D2**CN1)
      DSR=ALOG(A21*D2/D1)
      DLS=DSR/GAM
      SPR=S2-DLS
      GMX=.5*(1.+EM1)
      EMX(1)=FMX(GMX,DSR,GAM,GMM1,GMP1)
      CN2=EMX(1)*EMX(1)
      DX=1.+.5*GMM1*CN2
      DX=EMX(1)/(DX**CN1)
      AX1(1)=D1/DX
      EMXST=SQRT(GMP1*CN2/(2.+GMM1*CN2))
      EMYST=1./EMXST
      CN2=EMYST*EMYST
      EMY(1)=SQRT(2.*CN2/(GMP1-GMM1*CN2))
      DPP(1)=A21*D2/D1-1.
      WRITE(6,10)
   10 FORMAT(48HL INITIAL OUTPUT FOR THE SUPERSONIC BUZZ PROGRAM)
      WRITE(6,20) AX1(1), EM2, EMX(1), EMY(1)
   20 FORMAT(8HJAX/A1= G16.8,3X,7H  M2 = G16.8,3X,7H  MX = G16.8,3X,
     17H  MY = G16.8)
      WRITE(6,30) DLS, RT, SPR, DPP(1)
   30 FORMAT(8HK DLS = G16.8,3X,7H  RT = G16.8,3X,7H SPR = G16.8,3X,
     17HDP/P = G16.8)
      RETURN
      END
```

```
      FUNCTION FMX(GMX,DSR,GAM,GMM1,GMP1)
C     CALCULATION OF SHOCK MACH NUMBER FROM A KNOWN ENTROPY RISE (DSR)
      CRIT=.0001
      IT=0
      EMXPR=1.E+20
      EMX=GMX
      CN1=GAM/GMM1
      CN2=GMM1/GMP1
    1 IT=IT+1
      CN3=EMX*EMX
      F=ALOG(2./(GMP1*CN3)+CN2)
      F=ALOG(2.*GAM*CN3/GMP1-CN2)/GMM1+CN1*F-DSR
      FPR=EMX/(2.*GAM*CN3-GMM1)
      FPR=FPR-1./(EMX*(2.+GMM1*CN3))
      FPR=4.*CN1*FPR
      TEST=ABS(EMX-EMXPR)
      IF(TEST-CRIT)5,5,4
    4 EMXPR=EMX
      EMX=EMX-F/FPR
      GO TO 1
    5 FMX=EMX
      RETURN
      END
```

```
      FUNCTION AST(AMK,GMM1,GMP1)
C     CALCULATION FOR CRITICAL AREA RATIO
      AST=.5*AMK*AMK*GMM1+1.
      AST=2.*AST/GMP1
      CN1=.5*GMP1/GMM1
      AST=(AST**CN1)/AMK
      RETURN
      END
```

```
      FUNCTION QUAD(AA,BB,CC,SGN,CLOS)
C     SOLUTION FOR QUADRATIC EQUATION
      IF(AA)8,5,8
    5 QUAD=-CC/BB
      GO TO 20
    8 WRK=2.*AA
      DISC=SQRT(BB*BB-4.*AA*CC)/WRK
      WRK=-BB/WRK
      X1=WRK+DISC
      X2=WRK-DISC
      TST1=SGN*(X1-CLOS)
      IF(TST1)16,10,10
   10 TST2=SGN*(X2-CLOS)
      IF(TST2)14,12,12
   12 IF(TST1-TST2)14,14,16
   14 QUAD=X1
      GO TO 20
   16 QUAD=X2
   20 RETURN
      END
```

APPENDIX D

COMPUTER PROGRAM FOR NONSTEADY LIQUID FLOW

The following is the computer program used for the example for liquid dynamics. It is given for reference purposes only. This program may be compared to the program in reference 3.

```
      COMMON /CHIN/ AK, AKK(11), CNKH, LBEG, LLST, LL2, INC(10),
     1   KDSTP, KTQH, NSAV, NTOT, TIM(10), TMPD
      COMMON /RUNWH/ AKH(20), AKAPT(20), AREA(10), DELT,
     1   HL(11,20), HR(11,20), KST(11), KTRL(10), NINT, NPTS,
     2   QQ(11,20), SX(10), SY(10), TME(20)
      EQUIVALENCE (KT1,KTRL(1)), (KT2,KTRL(2)), (KT3,KTRL(3)),
     1   (KT4,KTRL(4)), (KT5,KTRL(5))
    5 CALL WHIN
C     CALCULATE CONTROLS TO DETERMINE WHEN INITIAL WAVE ARRIVES AT EACH
C      LOCATION
      DO 14 I=1,5
      KST(I)=1
      IF(KT5 -I)11,8,10
    8 KST(I)=2
      GO TO 14
   10 KIN=I
      KFIN=KT5-1
      GO TO 12
   11 KIN=KT5
      KFIN=I-1
   12 DO 13 K=KIN,KFIN
   13 KST(I)=KST(I)+INC(K)
   14 CONTINUE
      KDSTP=1
      NTOT=0
      LBEG=1
   18 LLST=LBEG+7
      CALL WHAM
      GO TO (38,38,48),KDSTP
   38 CALL WHOUT(KDSTP)
      GO TO (45,48),KDSTP
   45 LBEG=LL2
      GO TO 18
   48 GO TO 5
      END
```

```
      SUBROUTINE WHIN
      COMMON /CNRD/ ALEN(10), ANP, BA, BO, CON1,
     1   FF, GG, PI, QGG, TAUN
      COMMON /CHIN/ AK, AKK(11), CNKH, LBEG, LLST, LL2, INC(10),
     1   KDSTP, KTQH, NSAV, NTOT, TIM(10), TMPD
      COMMON /RUNWH/ AKH(20), AKAPT(20), AREA(10), DELT,
     1   HL(11,20), HR(11,20), KST(11), KTRL(10), NINT, NPTS,
     2   QQ(11,20), SX(10), SY(10), TME(20)
      DIMENSION DIA(10), TH(10), VCW(10), VW(10)
      EQUIVALENCE (KT1,KTRL(1)), (KT2,KTRL(2)), (KT3,KTRL(3)),
     1   (KT4,KTRL(4)), (KT5,KTRL(5))
      EQUIVALENCE (HA,HTANK)
C     READ INPUT
 1000 READ(5,100) NINT, NPTS, KTRL
  100 FORMAT(16I5)
      NI=NINT+1
      READ(5,200) DELT, BO, BA, FF, CON1, TAUN, ANP
      READ(5,200) GG, QGG, HTANK, E1, RO
  200 FORMAT(8E10.0)
      READ(5,200) AK, (AKK(I),I=1,NI)
      READ(5,200) (ALEN(I),I=1,NINT)
      PI=3.1415927
      KTQH=1
      IF(ANP )1008,1005,1005
 1005 PERD=1./FF
      TMPD=ANP*PERD
      GO TO 1009
 1008 TMPD=NPTS+1
      TMPD=TMPD*DELT
 1009 CNKH=AKK(NI)
      GO TO (1,3),KT3
    1 READ(5,200) (DIA(I),I=1,NINT)
      READ(5,200) (TH(I),I=1,NINT)
C     INITIAL CALCULATIONS
      VO=SQRT(GG*AK*144./RO)
      DO 2 I=1,NINT
      AREA(I)=PI*DIA(I)*DIA(I)/576.
      VCW(I)=SQRT(1.+DIA(I)*AK/(TH(I)*E1))
    2 VW(I)=VO/VCW(I)
      GO TO 4
    3 READ(5,200) (VW(I),I=1,NINT)
      READ(5,200) (AREA(I),I=1,NINT)
    4 NSAV=0
      DO 7 I=1,NINT
      SX(I)=VW(I)/(GG*AREA(I))
      SY(I)=-SX(I)
      TIM(I)=ALEN(I)/VW(I)
      INC(I)=TIM(I)/DELT+.5
      IF(NSAV-INC(I))6,7,7
    6 NSAV=INC(I)
    7 CONTINUE
      LL2=NSAV+2
      GO TO (17,20,20),KT4
   17 AKAPT(1)=BO
      AKK(1)=CON1*AKAPT(1)
   20 HR(1,1)=HA
      IM=1
      DO 23 I=1,NI
      HL(I,1)=HR(IM,1)
      IM=I
```

```
      HR(I,1)=HL(IM,1)-(QGG/AKK(I))**2
   23 CONTINUE
C     INITIAL OUTPUT
      WRITE(6,210) NINT, NPTS, (KTRL(I),I=1,5)
      WRITE(6,220) DELT, BO, BA, FF, CON1, TAUN, ANP
      WRITE(6,230) GG, QGG, HA, HR(NI,1), E1, RO, AK
      WRITE(6,240) (AKK(I),I=1,NI)
  210 FORMAT(8H1NINT = I3,5X,7HNPTS = I3,/,
     18HKKTRL = I3,9I7)
  220 FORMAT(8HKDELT = G13.6,5X,7H  BO = G13.6,5X,7H  BA = G13.6,5X,
     17H  FF = G13.6,/,
     28HKCON1 = G13.6,5X,7HTAUN = G13.6,5X,7H ANP = G13.6)
  230 FORMAT(8HK    G = G13.6,5X,7H QGG = G13.6,5X,7HHTANK= G13.6,5X,
     17H  HC = G13.6,/,
     28HK   E1 = G13.6,5X,7H  RO = G13.6,5X,7H   K = G13.6)
  240 FORMAT(8HK   KA = G13.6,5X,7H  KB = G13.6,5X,7H KDE = G13.6,5X,
     17H  KG = G13.6,5X,7H  KH = G13.6)
      GO TO (43,48),KT3
   43 WRITE(6,235)VO
      WRITE(6,245) (DIA(I),I=1,NINT)
      WRITE(6,260) (TH(I),I=1,NINT)
      WRITE(6,280) (VCW(I),I=1,NINT)
   48 WRITE(6,250) (ALEN(I),I=1,NINT)
      WRITE(6,270) (AREA(I),I=1,NINT)
      WRITE(6,290) (VW(I),I=1,NINT)
      WRITE(6,300) (SX(I),I=1,NINT)
      WRITE(6,310) (SY(I),I=1,NINT)
      WRITE(6,320)  (TIM(I),I=1,NINT)
      WRITE(6,330) (INC(I),I=1,NINT)
  235 FORMAT(8HK   VO = G16.8,//,
     22HK ,14X,1H1,21X,1H2,21X,1H3,21X,1H4)
  245 FORMAT(8HJ    D = G16.8,4G22.8)
  260 FORMAT(8HK   TH = G16.8,4G22.8)
  280 FORMAT(8HK VCW = G16.8,4G22.8)
  250 FORMAT(8HK    L = G16.8,4G22.8)
  270 FORMAT(8HK    A = G16.8,4G22.8)
  290 FORMAT(8HK   VW = G16.8,4G22.8)
  300 FORMAT(8HK   SX = G16.8,4G22.8)
  310 FORMAT(8HK   SY = G16.8,4G22.8)
  320 FORMAT(8HK TIM = G16.8,4G22.8)
  330 FORMAT(8HK INC = G16.8,4G22.8)
      RETURN
      END
```

```
      SUBROUTINE WHAM
C      CHARACTERISTIC SOLUTION OF WATER HAMMER BY LATTICE POINT METHOD
      COMMON /CNRD/ ALEN(10), ANP, BA, BO, CON1,
     1   FF, GG, PI, QGG, TAUN
      COMMON /CHIN/ AK, AKK(11), CNKH, LBEG, LLST, LL2, INC(10),
     1   KDSTP, KTQH, NSAV, NTOT, TIM(10), TMPD
      COMMON /RUNWH/ AKH(20), AKAPT(20), AREA(10), DELT,
     1   HL(11,20), HR(11,20), KST(11), KTRL(10), NINT, NPTS,
     2   QQ(11,20), SX(10), SY(10), TME(20)
      EQUIVALENCE (AKA,AKK(1))
      EQUIVALENCE (KT1,KTRL(1)), (KT2,KTRL(2)), (KT3,KTRL(3)),
     1   (KT4,KTRL(4)), (KT5,KTRL(5))
      DIMENSION CX(11), CY(11)
      NI=NINT+1
```

```
C     BEGINNING OF LOOP ON TIME
      DO 90 LT=LBEG,LLST
      NTOT=NTOT+1
      IF(NTOT-NPTS)4,4,2
    2 LLST=LT-1
      KDSTP=2
      IF(LLST-LBEG)3,95,95
    3 KDSTP=3
      GO TO 95
    4 AT=NTOT-1
      TIME=AT*DELT
      TME(LT)=TIME
      AKH(LT)=AKK(NI)
      AKAPT(LT)=AKA/CON1
      IF(TIME-TMPD) 6,20,20
    6 GO TO( 7, 8,20),KT4
    7 WRK=2.*PI*FF*TIME
      AKAPT(LT)=BO+BA*SIN(WRK)
      AKA=CON1*AKAPT(LT)
      GO TO 20
    8 AKH(LT)=CNKH*(1.-AT/TAUN)
      IF(AKH(LT))9,9,12
    9 AKH(LT)=0.
      KTQH=2
   12 AKK(NI)=AKH(LT)
C     BEGINNING OF LOOP TO CALCULATE EACH LOCATION AT A GIVEN TIME
C     (TME(LT))
   20 DO 85 I=1,NI
      IF(I-NINT)21,21,23
   21 LPST=MAX0(LT-INC(I),1)
      CY(I)=HL(I+1,LPST)+SY(I)*QQ(I+1,LPST)
      CX(I+1)=HR(I,LPST)+SX(I)*QQ(I,LPST)
   23 IF(KST(I)-NTOT)27,27,24
   24 QQ(I,LT)=QGG
      HL(I,LT)=HL(I,1)
      HR(I,LT)=HR(I,1)
      GO TO 85
   27 GO TO (32,42,52,62,72),I
   32 TRM=AKK(1)*SY(1)
      WRK=TRM*TRM-4.*(CY(1)-HL(1,1))
      QQ(1,LT)=.5*AKK(1)*(TRM+SQRT(WRK))
      HR(1,LT)=CY(1)-QQ(1,LT)*SY(1)
      GO TO 85
   42 XX=CX(2)-CY(2)
      YY=SX(1)-SY(2)
      QQ(2,LT)=QCALC(AKK(2),XX,YY)
      HL(2,LT)=CX(2)-SX(1)*QQ(2,LT)
      HR(2,LT)=CY(2)-SY(2)*QQ(2,LT)
      GO TO 85
   52 XX=CX(3)-CY(3)
      YY=SX(2)-SY(3)
      QQ(3,LT)=QCALC(AKK(3),XX,YY)
      HL(3,LT)=CX(3)-SX(2)*QQ(3,LT)
      HR(3,LT)=CY(3)-SY(3)*QQ(3,LT)
      GO TO 85
   62 XX=CX(4)-CY(4)
      YY=SX(3)-SY(4)
      QQ(4,LT)=QCALC(AKK(4),XX,YY)
      HL(4,LT)=CX(4)-SX(3)*QQ(4,LT)
      HR(4,LT)=CY(4)-SY(4)*QQ(4,LT)
      GO TO 85
   72 GO TO (78,76),KTQH
   76 QQ(5,LT)=0.
      GO TO 79
```

```
 78 XX=CX(5)-HR(5,1)
    QQ(5,LT)=QCALC(AKK(5),XX,SX(4))
 79 HL(5,LT)=CX(5)-SX(4)*QQ(5,LT)
 85 CONTINUE
 90 CONTINUE
 95 RETURN
    END
```

```
      SUBROUTINE WHOUT(KDSTP)
C     BASIC OUTPUT AND INITIALIZATION
      COMMON /CHIN/ AK, AKK(11), CNKH, LBEG, LLST, LL2, INC(10),
     1   KDSTP, KTQH, NSAV, NTOT, TIM(10), TMPD
      COMMON /RUNWH/ AKH(20), AKAPT(20), AREA(10), DELT,
     1   HL(11,20), HR(11,20), KST(11), KTRL(10), NINT, NPTS,
     2   QQ(11,20), SX(10), SY(10), TME(20)
      DIMENSION   VEL(20)
      WRITE(6,30) (TME(L),L=LBEG,LLST)
      WRITE(6,40) (AKAPT(L),L=LBEG,LLST)
      WRITE(6,50) (AKH(L),L=LBEG,LLST)
      IL=NINT+1
      DO 28 I=1,IL
      WRITE(6,60) I, (QQ(I,L),L=LBEG,LLST)
      DO 8 L=LBEG,LLST
    8 VEL(L)=QQ(I,L)/AREA(1)
      WRITE(6,70) (VEL(L),L=LBEG,LLST)
      IF(I-1)12,14,12
   12 WRITE(6,80) (HL(I,L),L=LBEG,LLST)
   14 IF(I-IL)16,18,16
   16 WRITE(6,90) (HR(I,L),L=LBEG,LLST)
   18 GO TO (22,28),KDSTP
   22 LA=NSAV+1
      LB=LLST-NSAV
      DO 24 L=2,LA
      LB=LB+1
      QQ(I,L)=QQ(I,LB)
      HL(I,L)=HL(I,LB)
      HR(I,L)=HR(I,LB)
   24 CONTINUE
   28 CONTINUE
   30 FORMAT(10H1    TIM = 8G15.6)
   40 FORMAT(10HJ AKAPT = 8G15.6)
   50 FORMAT(10HJ    AKH = 8G15.6)
   60 FORMAT(2HK ,I2,6H  Q = 8G15.5)
   70 FORMAT(10HJ    VEL = 8G15.6)
   80 FORMAT(10HJ     HL = 8G15.6)
   90 FORMAT(10HJ     HR = 8G15.6)
      RETURN
      END
```

```
      FUNCTION QCALC(AKK,XX,YY)
      WRK=4.*ABS(XX)/(AKK*AKK*YY*YY)
      WRK=SQRT(1.+WRK)-1.
      WRK=AKK*AKK*YY*WRK/2.
      QCALC=SIGN(WRK,XX)
      RETURN
      END
```

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