FIRST-ORDER ANALYSIS OF SUNBLAZER ORBITS

by

John V. Harrington

Reissued as
CSR TR-66-10 October 1966

FIRST-ORDER ANALYSIS OF SUNBLAZER ORBITS

by

John V. Harrington

A vehicle launched from the earth and accelerated to a velocity slightly greater than the terrestrial escape velocity will be injected into a heliocentric orbit, the orbital parameters being determined by the vector relationship between the earth's orbital velocity and the excess velocity of the vehicle after it has achieved terrestrial escape. In the simplest situation and, for purposes of a solar probe the most useful situation, the excess velocity is caused to be exactly opposite to the earth's velocity. The vehicle will then be injected into an orbit with aphelion at 1 A.U. and perehelion less than this by an amount depending upon the magnitude of the excess velocity. The relationship between these is given by:

$$[1] \qquad \Delta v = v_{c1} \left[\sqrt{\frac{2r_p}{1 + r_p}} - 1\right]$$

where v_{c1} is the circular orbital velocity for a 1 A.U. orbit and r_p is the perehelion distance measured in A.U.

To achieve the excess velocity Δv, the launch vehicle must have attained a velocity v_b at burnout of:

$$[2] \qquad v_b = \sqrt{v_{esc}^2 + \Delta v^2}$$

where v_{esc} is the escape velocity at the burnout altitude. The sidereal period of a satellite in such a heliocentric orbit follows directly from Keppler's third law. When the units of time are chosen to be years and distances are in A.U., the expression for the period has the relatively simple form:

$$[3] \qquad T = \left(\frac{1 + r_p}{2}\right)^{3/2}$$

From these fundamental relationships the data in Tables I and II are computed as an indication of the orbits achievable when small payloads are boosted to velocities slightly in excess of escape velocity. It would seem at first glance that almost any of these orbits would be useful in the sense that they all will allow a transmission path to be set up between the satellite and the earth which eventually must penetrate the inner corona. Some further analysis, however, indicates that there is a preferred family of orbits leading to the most desirable experimental situation.

Retrograde Injection

If superior conjunction is made to occur on the line of apses of the satellite orbit, a symmetry in the transmission path geometry before and after conjunction occurs. Since the most important measurements to be made by the Sunblazer vehicle are the transmission path delay measurements as the path penetrates the corona, the opportunity to repeat the measurements with the same path geometry after conjunction seems most attractive. The transmission path length and offset have been calculated (Figures 1, 2, 3) for satellites launched in a retro-

TABLE I. Orbital Parameters for Solar Satellites

Retrograde Launching

Launch Velocity v_b	Excess Velocity Δv	Perihelion r_p	Eccentricity e	Sidereal Period T_{sid}	Synodic Period T_{syn}	Time to Conjunction T_{180}
36.7 K ft/sec	0 K ft/sec	1.00 A.U.	0	1.00 yr	∞ yrs	∞ yrs
37.7	8.7	0.71	0.17	0.79	3.75	1.87
38.5	12.4	0.63	0.23	0.73	2.70	1.35
39.6	15.9	0.55	0.29	0.68	2.12	1.06
40.6	17.7	0.52	0.32	0.66	1.94	0.97
41.5	19.9	0.47	0.36	0.63	1.70	0.85

TABLE II. Orbital Parameters for Solar Satellites

Direct Launching

Launch Velocity v_b	Excess Velocity Δv	Aphelion r_a	Eccentricity e	Sidereal Period T_{sid}	Synodic Period T_{syn}	Time to Conjunction T_{180}
36.7 K ft/sec	0 K ft/sec	1.0 A.U.	0	1.00 yr	∞ yrs	∞ yrs
37.7	8.7	1.47	0.19	1.36	3.78	1.89
38.5	12.4	1.67	0.25	1.55	2.82	1.41
39.6	15.9	2.05	0.34	1.86	2.16	1.08
40.6	17.7	2.23	0.38	2.06	1.95	0.98
41.5	19.9	2.58	0.44	2.40	1.71	0.86

grade direction, using three possible values for launch velocities. Direct inspection of the three sets of curves will show that the path symmetry is best achieved for Case 2 in which the launch velocity is of the order of 40,000 feet per second. It is instructive to examine further the conditions under which this symmetry is achieved to determine what other orbits may also be acceptable.

If $\omega_o = 2\pi$ radian per year is the mean angular velocity of the earth about the sun and $\omega_s = \frac{2\pi}{t_s}$ is the mean angular velocity of the satellite, then the relative angular velocity $\omega_r = (\omega_s - \omega_o)$ multiplied by t_π, the time to conjunction, must satisfy the relation:

[4] $$(\omega_s - \omega_o)t_\pi = n\pi; \quad n = 1, 3, 5, \ldots$$

where n is an odd integer and is a measure of the number of synodic half-periods between launch and conjunction. The condition that conjunction occur on the line of apses requires that the earth be on this line at that time so that:

[5] $$t_\pi = m \cdot \frac{1}{2}$$

where m is any integer and is a measure of the number of half-earth rotations (half-years) between launch and conjunction. By eliminating t_π in [4] and [5] and solving for t_s, we find that the family of satellite periods which will satisfy the desired symmetry conditions is given simply by:

[6] $$t_s = \frac{m}{n + m} \text{ yrs}$$

In Table III are listed the preferred satellite periods for the first few values of n and m. Periods below 0.353 years are not achievable since this corresponds to $r_p = o$.

Of the first three of these possible orbits, the one for which n = 1, m = 2 appears the most attractive since the necessary launch velocity (39.4 K ft/sec) seems practically achievable and the time to conjunction (1.0 yr) is not too long (Table IV).

Direct Injection

When the satellite is launched so that its excess velocity adds to the earth's orbital velocity, it is injected into an orbit with perehelion at 1 A.U. and aphelion several times greater than this. The conditions for symmetry may be set down in much the same manner as before requiring $(\omega_o - \omega_s)t_\pi = n\pi$, but now the time to conjunction t_π must equal k half periods of the slowest orbiter which is now the satellite. The resultant expression for permissible satellite periods is then:

$$[7] \qquad t_s = \frac{n + k}{k} \text{ years}$$

From the tabulation of preferred satellite periods in Table V, it is evident that the earliest conjunction time is obtained for n = k = 1, and this is also achievable by a reasonable launch velocity (Table VI).

Effect of Injection Errors

If we adopt as an objective the retrograde injection of the satellite into a heliocentric orbit having a perehelion at

TABLE III. Orbital Periods for Superior Conjunction on Line of Apses

Earth Half Periods	Satellite Period Years $Z_s = \frac{m}{n+m}$			Time to Conjunction $t_\pi = \frac{m}{2}$
m	n = 1	n = 3	n = 5	
1	0.50	(0.25)	(0.167)	0.5 yrs.
2	0.667	0.40	(0.286)	1.0
3	0.75	0.50	0.375	1.5
4	0.80	0.555	0.444	2.0
5	0.833	0.625	0.50	2.5
6	0.855	0.667	0.555	3.0
7	0.875	0.700	0.583	3.5

TABLE IV. Characteristics of First Few (n = 1) Orbits Producing Desired Symmetry

Retrograde Injection

m	Satellite Period t_s	Time to Conjunction t	Perihelion r_p	Excess Velocity Δv	Burnout Velocity at 10^6 Ft. v_b
1	0.50 yrs	0.5 yrs	0.26 A.U.	35.0 K ft/sec	49.7 K ft/sec
2*	0.67	1.0	0.53	16.4	39.4
3	0.75	1.5	0.648	11.1	37.4
4	0.80	2.0	0.724	8.3	36.8

* Recommended for Sunblazer

TABLE V. Orbital Periods Giving Superior Conjunction on Line of Apses

Half Periods	Satellite Period t_s			Time to Opposition t_π		
k	n = 1	3	5	n = 1	3	5
1	2.0 yrs	4.0 yrs	6.0 yrs	1.0 yrs	2.0 yrs	3.0 yrs
2	1.5	2.5	3.5	1.5	2.5	3.5
3	1.33	2.0	7.67	2.0	3.0	4.0
4	1.25	1.75	2.25	2.5	3.5	4.5
5	1.20	1.60	2.0	3.0	4.0	5.0

TABLE VI. Characteristics of a Few ($n = 1$) Orbits Producing Desired Symmetry

Direct Injection

k	Satellite Period t_s	Time to Conjunction t_π	Aphelion v	Excess Velocity Δv	Burnout Velocity at 10^6 Ft. v_b
0	∞	0.5 yrs	∞	40.5 K ft/sec	54.0 K ft/sec
1	2.0 yrs	1.0	2.18 A.U.	16.6	39.4
2	1.5	1.5	1.62	11.2	37.4
3	1.33	2.0	1.43	8.7	36.8

0.53 A.U., then the sensitivity of the desired orbit to orientation errors in the excess velocity vector may be studied. An elliptical orbit is characterized by the following equations where θ is the argument of perehelion, β is the angle between the injection velocity vector and the direction orthogonal to the radius vector; $k = \frac{v}{v_c}$ is the ratio of the injection velocity to the circular velocity at that radius; e is the eccentricity of the orbit, and a is the semi-major axis.

[8] $$\tan \theta = \frac{k^2 \sin \beta \cos \beta}{(k^2-1) \cos^2\beta - 1}$$

[9] $$e^2 = (k^2 - 1)^2 \cdot \cos^2\beta + \sin^2\beta$$

[10] $$\frac{a}{r} = \frac{1}{2 - k^2}$$

[11] $$t = \frac{2\pi}{\sqrt{(g_o r_o^2)}} \cdot a^{3/2}$$

Since the injection velocity v is obtained by adding the very small excess velocity Δv to the relatively large earth's orbital velocity v_c, it is evident that the angle β cannot be very large. In fact, its maximum value in the unlikely case that the excess velocity misses the preferred direction by 90° is only 0.15 radians or 9°. Thus tan θ for all cases of interest to us is given by:

[12] $$\tan \theta \sim \frac{k^2 \sin \beta}{k^2 - 1}$$

and

[13] $$e^2 \sim (k^2 - 1)^2 + \sin^2 \beta$$

Consider the case where Δv lies in the ecliptic but is displaced from its intended orientation by an angle α. Then

[14] $$v = v_c - \Delta v \cdot \cos \alpha$$

[15] $$k^2 \sim 1 - 2 \frac{\Delta v}{v_c} \cos \alpha$$

and

[16] $$\beta \sim \frac{\Delta v \sin \alpha}{v_c - \Delta v \cos \alpha} \quad \text{for } \frac{\Delta v}{v_c} < 1$$

Using these approximations in [12], we find

[17] $$\tan \theta \sim \frac{\tan \alpha}{2} (1 - 2 \frac{\Delta v}{v_c})$$

or for $\frac{\Delta v}{v_c} = 0.15$

[18] $$\theta \sim 0.35\alpha$$

A 30° error in launch angle will then produce a rotation in the argument of perehelion of 9° and will not be serious.

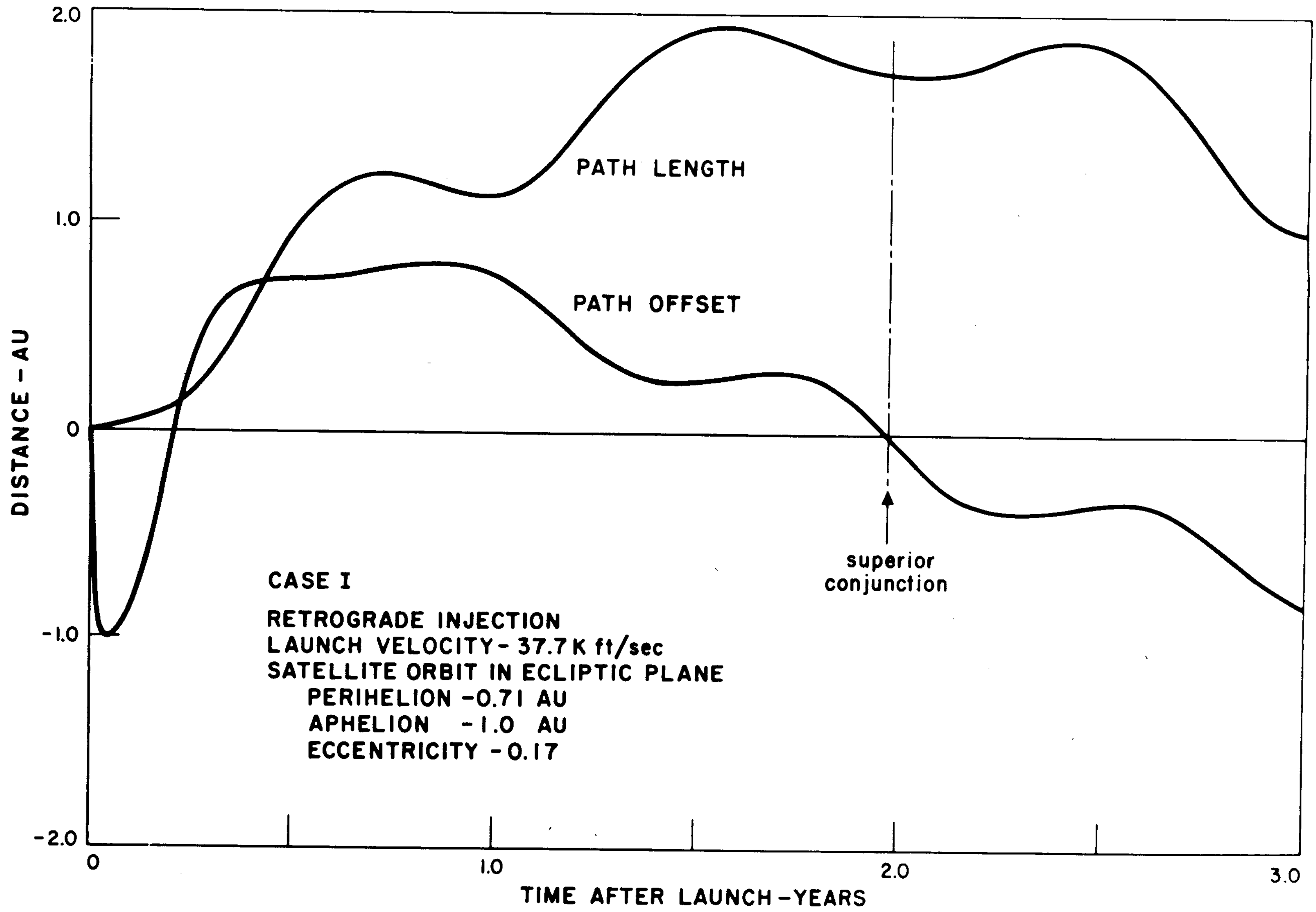
If the excess velocity contains a component orthogonal to the plane of the ecliptic, an inclination of the satellite's orbital plane relative to the ecliptic will result. The magnitude of this inclination is small and for the 30° misalignment used earlier would be $\frac{7.5}{84}$ or 5° at most. This will not be serious provided superior conjunction still occurs on or close to the line of apses. At conjunction then both bodies lie in the ecliptic, and the transmission path at the time of conjunction must still penetrate the solar disk. Thus a second advantage of causing conjunction to occur on the line of apses is that the effect of any inclination of the satellite's orbital plane on establishing some minimum path offset is minimized.

Conclusions

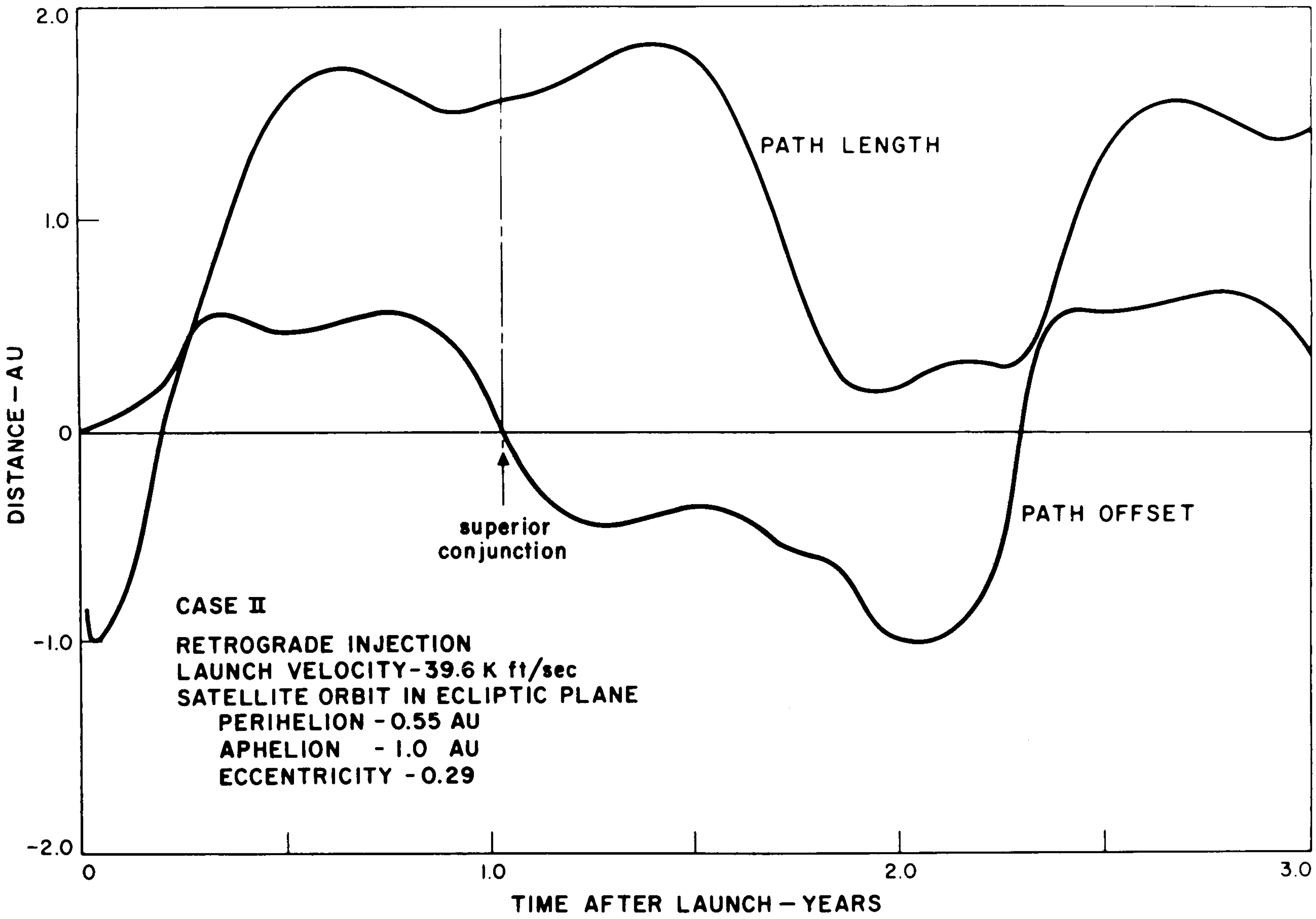
The most desirable orbit for the Sunblazer heliocentric satellite has the following characteristics:

Sidereal Period	0.667 years
Perehelion	0.53 A.U.
Aphelion	1.0 A.U.
Eccentricity	0.31
Time to Superior Conjunction	1.0 years

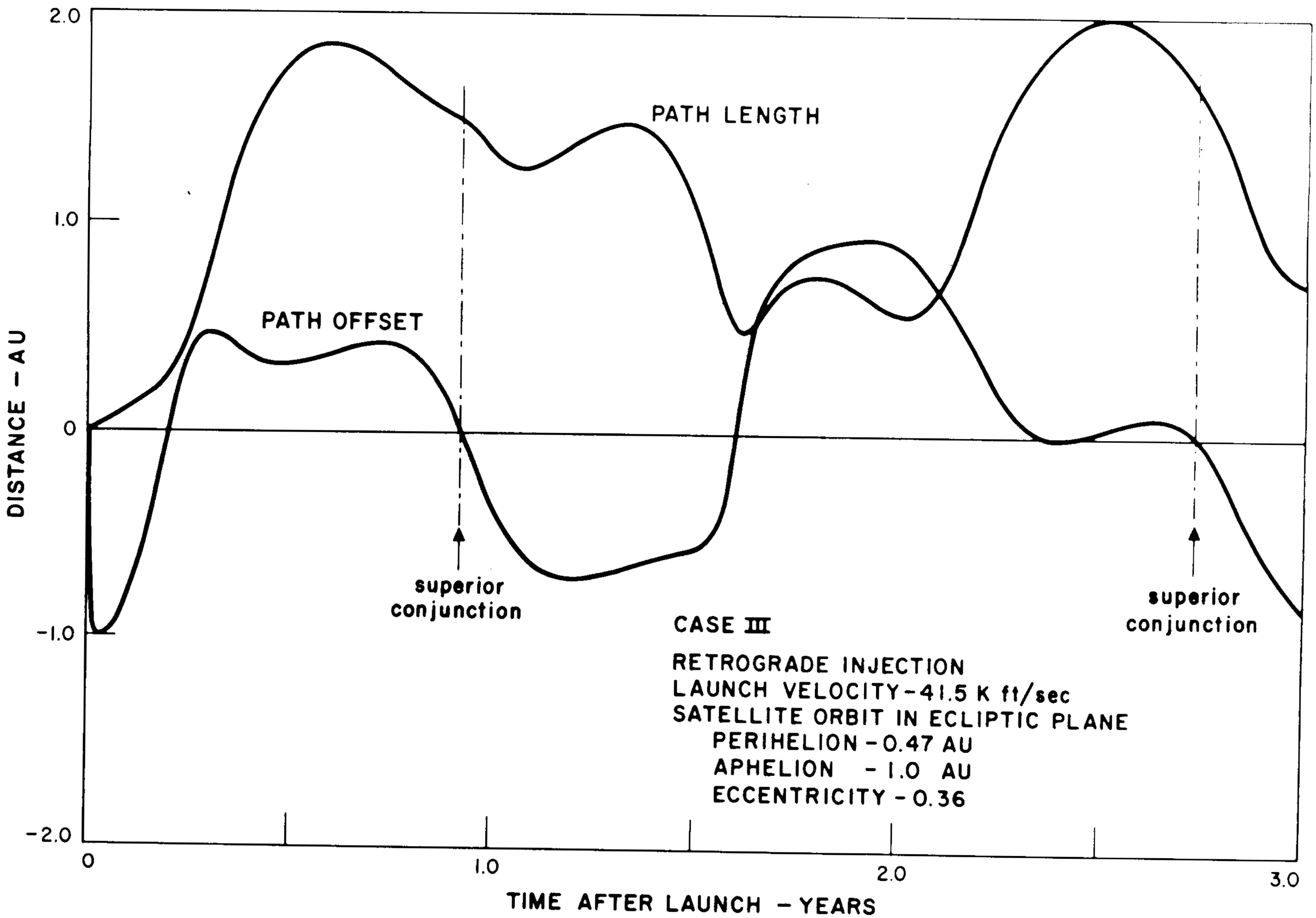
Launch Velocity for Burnout at 10^6 ft	39.4 K ft/sec
Excess Velocity	16.4 K ft/sec in a direction opposite the earth's orbital velocity



TRANSMISSION PATH BETWEEN HELIOCENTRIC SATELLITE AND EARTH
(path length and solar offset as functions of time after launch)



TRANSMISSION PATH BETWEEN HELIOCENTRIC SATELLITE AND EARTH
(path length and solar offset as functions of time after launch)



TRANSMISSION PATH BETWEEN HELIOCENTRIC SATELLITE AND EARTH
(path length and solar offset as functions of time after launch)