Ionospheric Research

NASA Grant NsG-114-61

Scientific Report

on

A Method for the Determination of Mean Ionospheric Height from Satellite Signals

by

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August 30, 1966

Scientific Report No. 278

Ionosphere Research Laboratory

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ABSTRACT

N66-39695

A method is developed for the determination of mean ionospheric height by analysing radio signals from a beacon satellite. Two major effects of the ionosphere on the received signals, Faraday rotation and doppler dispersion, are measured. By searching for their consistency and using height dependence of the earth's magnetic field, it becomes possible to calculate mean ionospheric height along the focus of satellite directions from the observing station. The method is applied to a number of satellite passes involving both normal and unusual ionospheric conditions and height profiles are developed over a range of about five degrees of latitude.

Author

CHAPTER 1

INTRODUCTION

1. Methods of Studying Ionization Profiles in the Ionosphere

One of the most widely used methods of studying the height distribution of free electrons in the ionosphere has been the pulse sounding method which is based on measuring the time taken for a pulse of radio waves to travel to the ionosphere and back as a function of frequency. These measurements can be used to deduce electron density as a function of height up to the height of the electron density maximum. By making use of both satellite and ground sounders in recent years, it has become possible to obtain complete electron density profiles with this method.[10] Although the satellite has continuous spatial coverage, on the ground this can only be achieved with closely spaced stations.

The incoherent scatter technique[2] also provides a means of determining the electron density profile by transmitting high frequency radar pulses and measuring the amount of power scattered back by free electrons in the ionosphere. This method requires very sensitive equipment, but its coverage is not limited to altitudes lower than the maximum electron density height, and it has some ability for spatial coverage around the station.

There are many other methods for accurate determination of electron density profile using rockets. Because of their costly operations rocket measurements are limited to single profiles at a time without any time or spatial variations.

2. Satellite Beacon Experiments

The beacon satellites, usually used exclusively for ionospheric research, transmit linearly polarized unmodulated waves of frequencies somewhat greater than the critical frequency of the ionosphere. Ground stations deduce information about the ionosphere by analyzing received signal from the satellites.

One of the major methods of investigation involves measurements of the dispersive doppler frequency shift[8] resulting from the change of phase path length between the satellite and the receiver due to the ionosphere. This can be done by measuring the departure from a harmonic relationship of the received signals from harmonically related satellite transmitters. The reduction of phase path length is dependent upon the integrated electron density along the wave path.

A second technique for the determination of this integrated electron density, total electron content, involves observation of Faraday rotation[1]. Travelling through a magneto-ionic medium like the ionosphere, a linearly polarized electro-magnetic wave will experience a rotation of its plane of polarization about the axis of propagation. The amount of rotation depends upon the integrated electron density and also on the longitudinal component of the earth's magnetic field which is a function of height. Because of its magnetic field and height dependency, any deduction of the total electron content from Faraday rotation requires a knowledge of the electron density profile; however the received signals from the satellite yield no information about the ionization distribution.

Usually, total electron calculations have been done by assuming an ionospheric height and evaluating the magnetic field at that altitude.

3. Statement of the Problem

A method is to be developed to determine mean ionospheric height by comparing Faraday rotation and dispersive doppler frequency shift, and searching for consistency between these two effects of the ionosphere. The magnetic field dependency of Faraday rotation makes this ionospheric height calculation possible. By determining the mean ionospheric height at a number of points along the locus of the satellite's transit, it will be possible to study spatial variations in height as well.

CHAPTER 2

FIRST-ORDER THEORY

1. Faraday Rotation

In propagating through a magneto-ionic medium, an electromagnetic wave travels in two different characteristic modes. These characteristic waves, called ordinary and extraordinary, generally are elliptically polarized with opposite senses of rotation, and their complex refractive indices are given by the well-known Appleton-Hartree equation.[7]

$$n^2_{o,e} = 1 - \frac{X}{1 - iZ - \frac{Y_L^2}{2(1-X-iZ)} \pm \left[\frac{Y_T^4}{4(1-X-iZ)} + Y_L^2\right]^{1/2}} \qquad (1)$$

where

$X = f_N^2/f^2$

$Y_L = f_L/f$

$Y_T = f_T/f$

$Z = \nu/2\pi f$

$f_N^2 = Ne^2/4\pi^2\varepsilon_o m$

$f_L = (Be/2\pi m)\text{Cos}\theta$

$f_T = (Be/2\pi m)\text{Sin}\theta$

f = wave frequency

ν = frequency of collisions of electrons with heavy particles

N = number density of electrons

e = the charge of an electron

m = mass of an electron

ε_o = electric permittivity of free space

B = induction of the imposed magnetic field

θ = the angle between the magnetic field and the wave normal

All quantities are in rationalized MKS units. Subscripts "e" and "o" denote extra-ordinary and ordinary modes respectively.

For wave frequencies used in satellite beacon experiments, the refractive index is close to unity and the quantity Z, which is small, appears significantly only in the imaginary part of the refractive index, i.e. in absorptive effects. Therefore in the present study where refractive effects alone are involved, it is possible to simplify the expression for the refractive index by assuming $Z = 0$.

Further simplification is possible by making use of the quasi-longitudinal (QL) approximation,

$$\frac{Y_T^4}{4Y_L^2} \ll (1-X)^2 \tag{2}$$

The QL approximation is valid for a wave frequency of 40MHz provided that the wave normal is not within about three degrees of being perpendicular to the geomagnetic field. For a mid-latitude receiving station like University Park, Pennsylvania (40.8°N, 77.9°W), the propagation of satellite signals is always quasi-longitudinal. This means that the index of refraction assumes the form

$$n_{e,o} = 1 - \left(\frac{X}{1 \pm |Y_L|} \right)^{1/2} \qquad (3)$$

Also, under these assumptions the two characteristic waves can be approximated as circularly polarized with opposite senses of rotation. Due to the difference between the phase velocities of the two modes, the resulted received signal from a linearly polarized source becomes a linearly polarized wave with a direction of polarization rotated from the original around the axis of propagation. This rotation, Ω, known as Faraday rotation, can be expressed in terms of the difference between the refractive indices of the two modes,[1]

$$\Omega_o = \frac{\pi}{\lambda} \int_{s_o} \mu_o ds - \frac{\pi}{\lambda} \int_{s_e} \mu_e ds \doteq \frac{\pi}{\lambda} \int_s (\mu_o - \mu_e) ds$$

where ds is the incremental ray path and the integrals are taken over the entire ray path.

This expression is valid for frequencies much higher than both the plasma frequency (f_N) and the electron gyro-frequency (f_L), because it is assumed that the ray path for both modes are common. With these high frequency approximations,

$$|X| << 1 \qquad \text{and} \qquad |Y_L| << 1$$

the following equation can be written for the Faraday rotation:

$$\Omega_o = \frac{e^3 c}{2\pi^2 m^2 f^2} 10^{-7} \int_s B_L N \, ds \qquad (4)$$

where Ω_o is expressed in half-rotations.

2. Doppler Effect

Besides the Faraday rotation, another effect of the ionosphere on the signals received from a satellite is a reduction of the phase path length, ΔP_o, between the source and the observer.

$$\Delta P_o = P_o - P$$

where

P_o = the phase path length for a free-space medium

P = the actual phase path length

In terms of the refractive index, P can be expressed as

$$P = \int_s n ds$$

and with the high frequency approximation the expression for ΔP_o can be reduced to (8)

$$\Delta P_o = \frac{e^2 c}{2\pi m f} 10^{-7} \int_s N \, ds \qquad (5)$$

where ΔP_o is expressed in free space wave lengths at frequency f.

The rate of change of this phase path length reduction will be observed at a ground-base receiving station as a dispersive doppler frequency shift in addition to the non-dispersive doppler shift which is due solely to the movement of the satellite. Direct evaluation of the ionospheric effect on the doppler frequency is often not possible as this requires very accurate knowledge of the position of the satellite with time, and of the frequency of its radiated signal. However, the frequency dependent part of the ionospheric doppler effect can be measured by analyzing signals from harmonically related satellite transmitters. The departure of the received signals from such harmonic relationship can be measured and attributed to the dispersive properties of the changing ionospheric path of propagation.

3. Total Electron Content[4]

Total electron content is defined as the total ionization in a vertical column between the satellite and the ground, and can be expressed with the following equation:

$$N_T = \int_o^{h_s} N dh$$

where h_s is the satellite height.

With an assumption of a horizontally stratified ionosphere, equation 4 can be rewritten as

$$\Omega_o = \frac{e^3 c}{2\pi^2 m^2 f^2} 10^{-7} \int_o^{h_s} B_L \operatorname{Sec}\theta \, N \, dh \qquad (6)$$

where θ is the zenith angle at height h.

By defining $\overline{B_L \mathrm{Sec}\theta}$ as the weighted mean value of $B_L \mathrm{Sec}\theta$ along the straight line path between the satellite and the observation point, total electron content can be calculated from the Faraday rotation using the following equation

$$N_T = \frac{2\pi^2 m^2 f^2}{e^3 c} 10^7 \frac{\Omega_o}{\overline{B_L \mathrm{Sec}\theta}} \qquad (7)$$

4. Mean Ionospheric Height

Equation 4 may be rewritten as

$$\Omega_o = \frac{e^3 c}{2\pi^2 m^2 f^2} 10^{-7} \overline{B}_L \int_s N\, ds \qquad (8)$$

where $\overline{B}_L$ is the mean value of the longitudinal field component along the ray path weighted by the electron density distribution along this slant path. The height at which B_L assumes its mean value, $\overline{B}_L$, as given by equation 8 will be defined as the "mean ionospheric height".

By seeking consistency between the doppler shift and the Faraday rotation from equations 5 and 8 we obtain

$$\overline{B}_L = K \frac{\Omega_o}{\Delta P_o} \qquad (9)$$

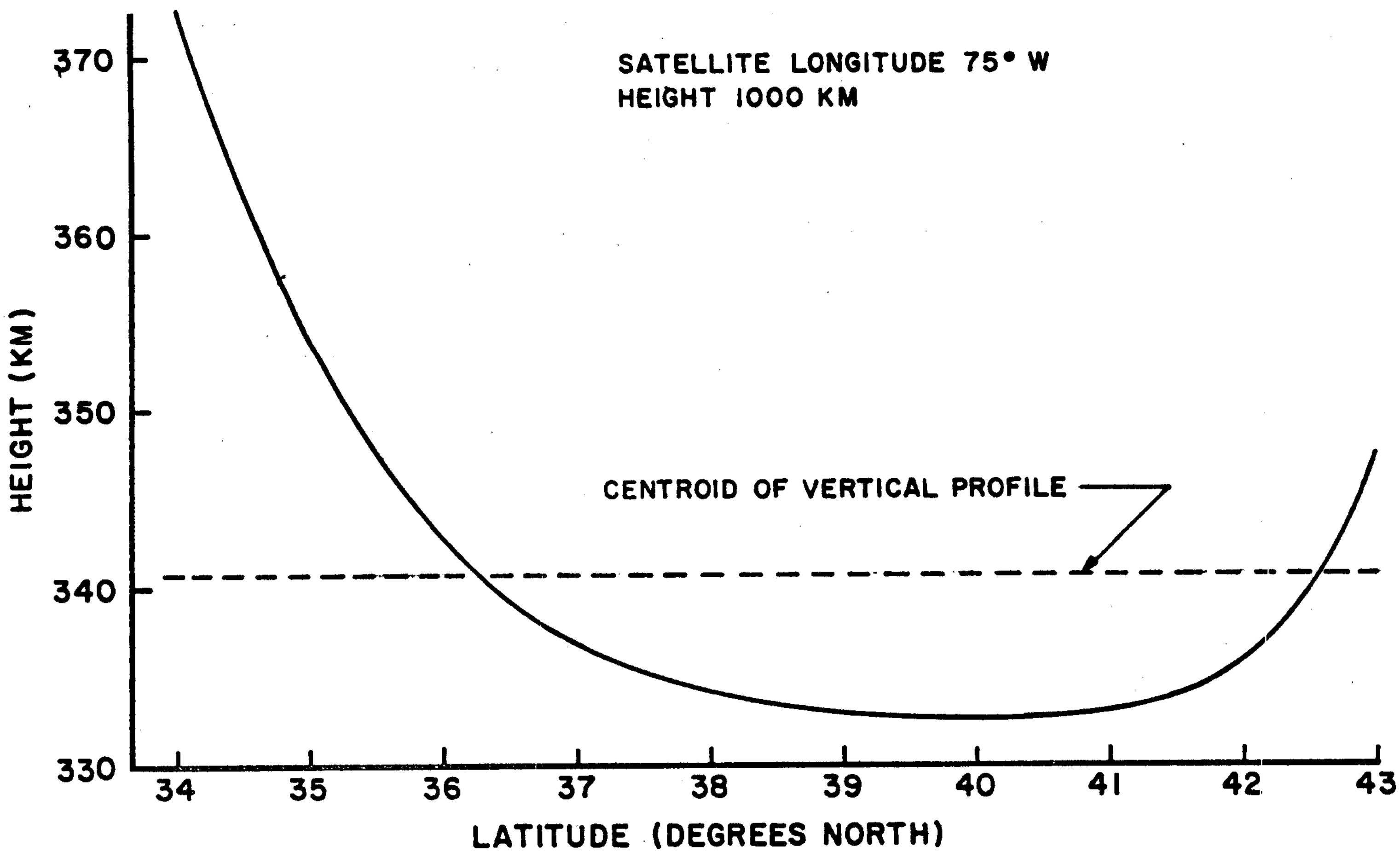
where K is a constant equal to $\pi mf/e$.

Thus if it is possible to determine Ω_o, and ΔP_o absolutely then the value of $\bar{B}_L$ can be computed and the mean ionospheric height found by comparison of this computed value with a magnetic field model.

Since B_L is usually a slowly varying function of height, it will assume its mean value, $\bar{B}_L$, at a point somewhere near the center of the ionization distribution, e.g. if B_L has a linear variation with distance then its weighted mean value will be at the centroid of the ionization distribution along the slant path.

Figure 1 shows a plot of mean ionospheric height as a function of ionospheric point latitude for a fictitious satellite pass along the 75^oW geographic meridian at 1000km altitude. A Chapman-α model with the scale height of 75km and 250km as the height of the maximum ionization density is used in this model calculation. The vertical centroid of such a distribution is found by numerical integration to be at 340.5 km. As it can be seen from the graph, the calculated mean ionospheric height does not differ from the centroid of the ionization by more than 8 km between 36^oN and 42^oN latitudes.

Reasons for larger deviations outside this region will be discussed later.



MEAN IONOSPHERIC HEIGHT vs LATITUDE
FOR A CHAPMAN-α MODEL

FIGURE 1

CHAPTER 3

INSTRUMENTATION

1. Satellite

Data for this study have been obtained by making use of signals transmitted by the S-66 Ionospheric Beacon Satellite, Beacon B which has an almost polar and circular orbit with a nominal altitude of 1000km.

Orbit parameters are as follows:

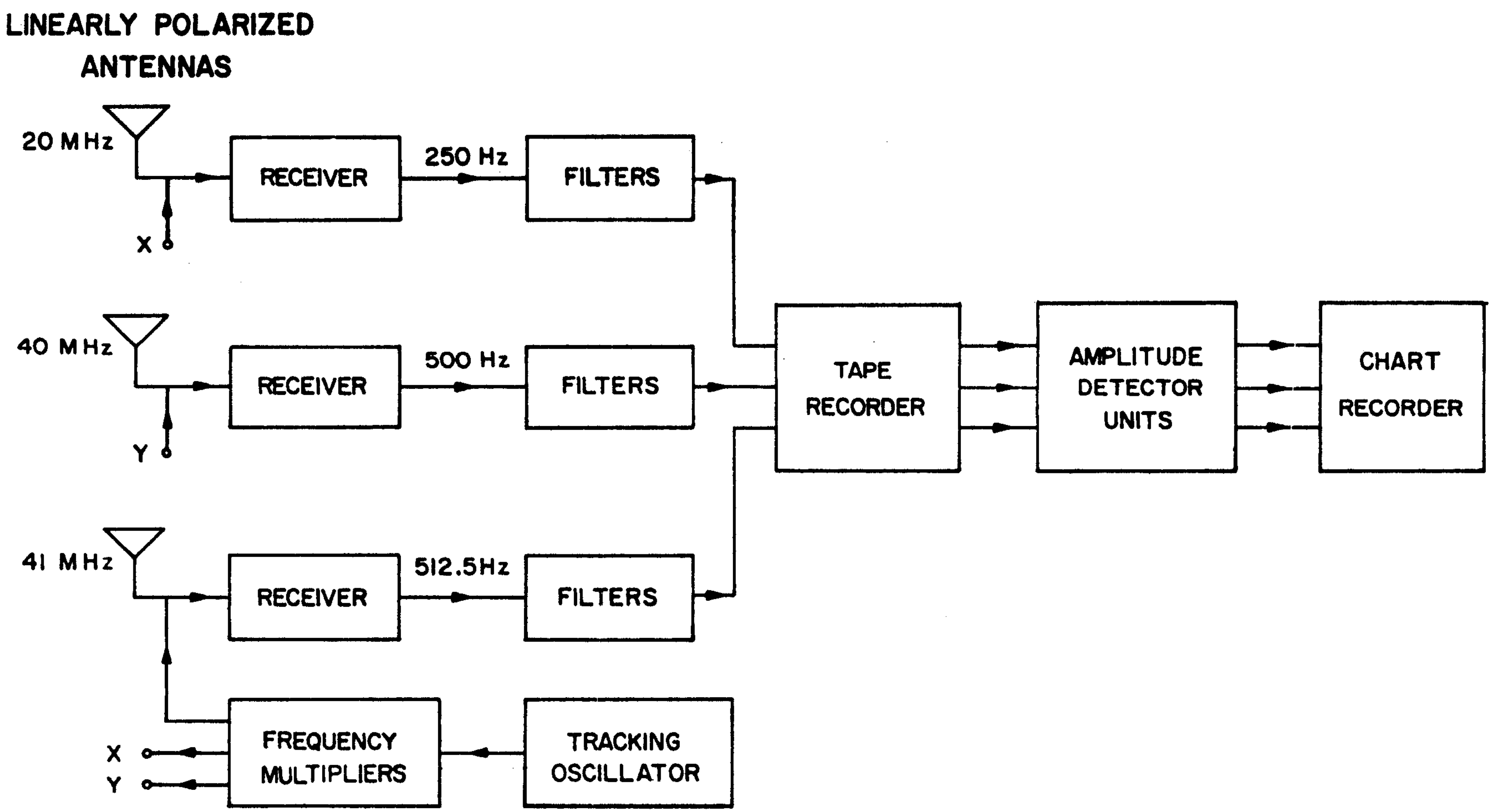
Nodal Period	104.8 minutes
Inclination	79.7 degrees
Perigee	890 km.
Apogee	1070 km.

The satellite transmits continuous, unmodulated, linearly polarized waves at 20MHz, 40MHz, and 41MHz with power outputs of 250 mW. Harmonic relationships between the signals are achieved by operating the transmitters from a single ultrastable crystal oscillator.

The satellite antennas at these frequencies are colinear dipoles placed normal to the satellite axis which is magnetically stabilized to be oriented along the direction of the local magnetic field. The spin about the axis is damped mechanically to a rate designed to be less than 0.02 rotations per minute.(12)

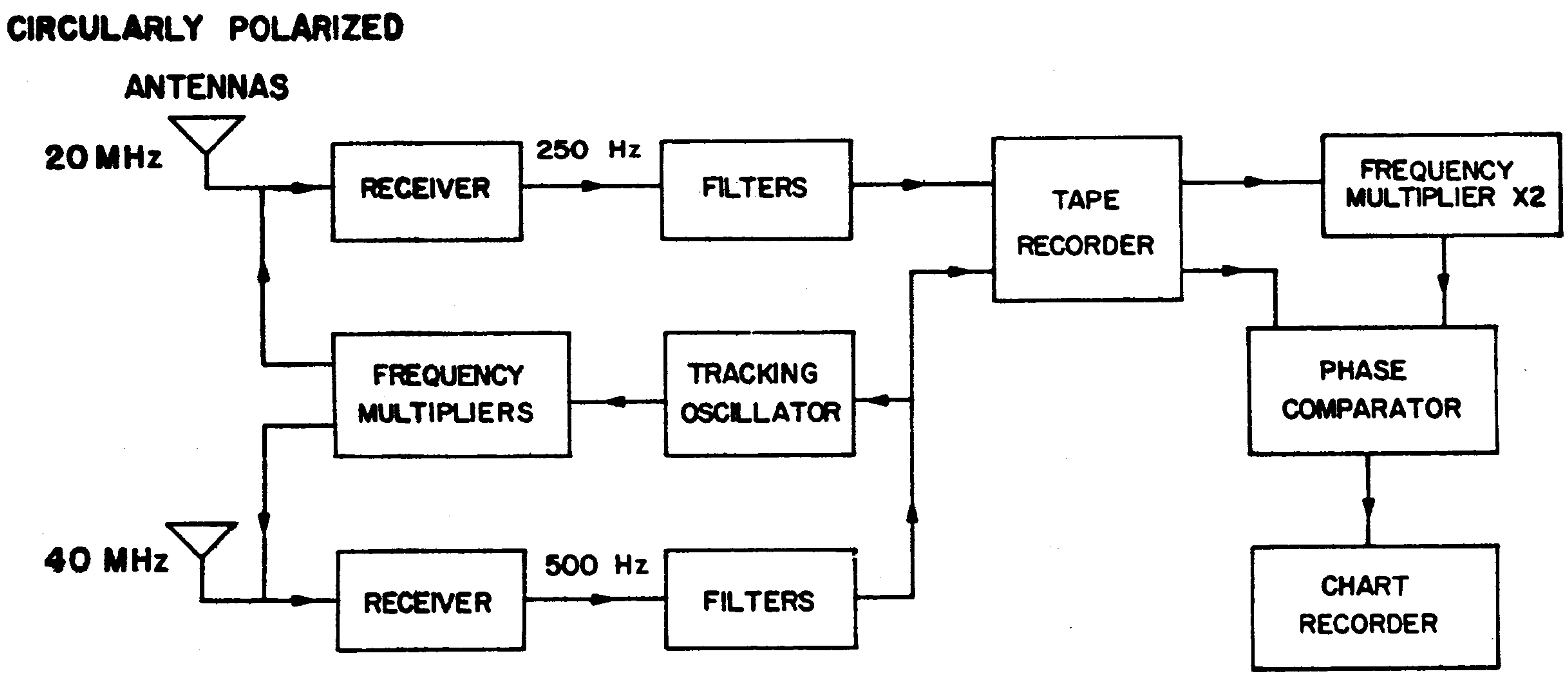
2. Receiving and Analysis Equipment

Figures 2 and 3 show block diagrams for the equipment designed to record Faraday rotation and doppler shift data.



BLOCK DIAGRAM OF RECEIVING AND RECORDING SYSTEM FOR FARADAY ROTATION

FIGURE 2



BLOCK DIAGRAM OF RECEIVING AND RECORDING SYSTEM FOR DOPPLER SHIFT

FIGURE 3

Signals from the satellite are received by two linearly and two circularly polarized antennas. The 40MHz and 41MHz linear channels use the same horizontally polarized linear dipole so that for these frequencies both the satellite source antenna and the receiving antenna are common to both frequencies. At the input to each receiver a signal from a tracking oscillator is added to the satellite signal picked up on the antenna, and both are amplified together. As they pass through the detector those two signals beat together to produce an audio beat frequency which is further amplified and filtered before being recorded on the magnetic tape recorder.

The same tracking oscillator provides reference signals for each satellite frequency by means of a frequency multiplying chain from a 1 MHz base oscillator. A discriminator loop causes the beat frequency of each receiver to be almost constant by changing the base oscillator frequency to maintain a constant difference frequency above the satellite frequency for a selected channel. For this experiment an offset of 500Hz above the frequency of the 40 MHz circularly polarized channel was used.

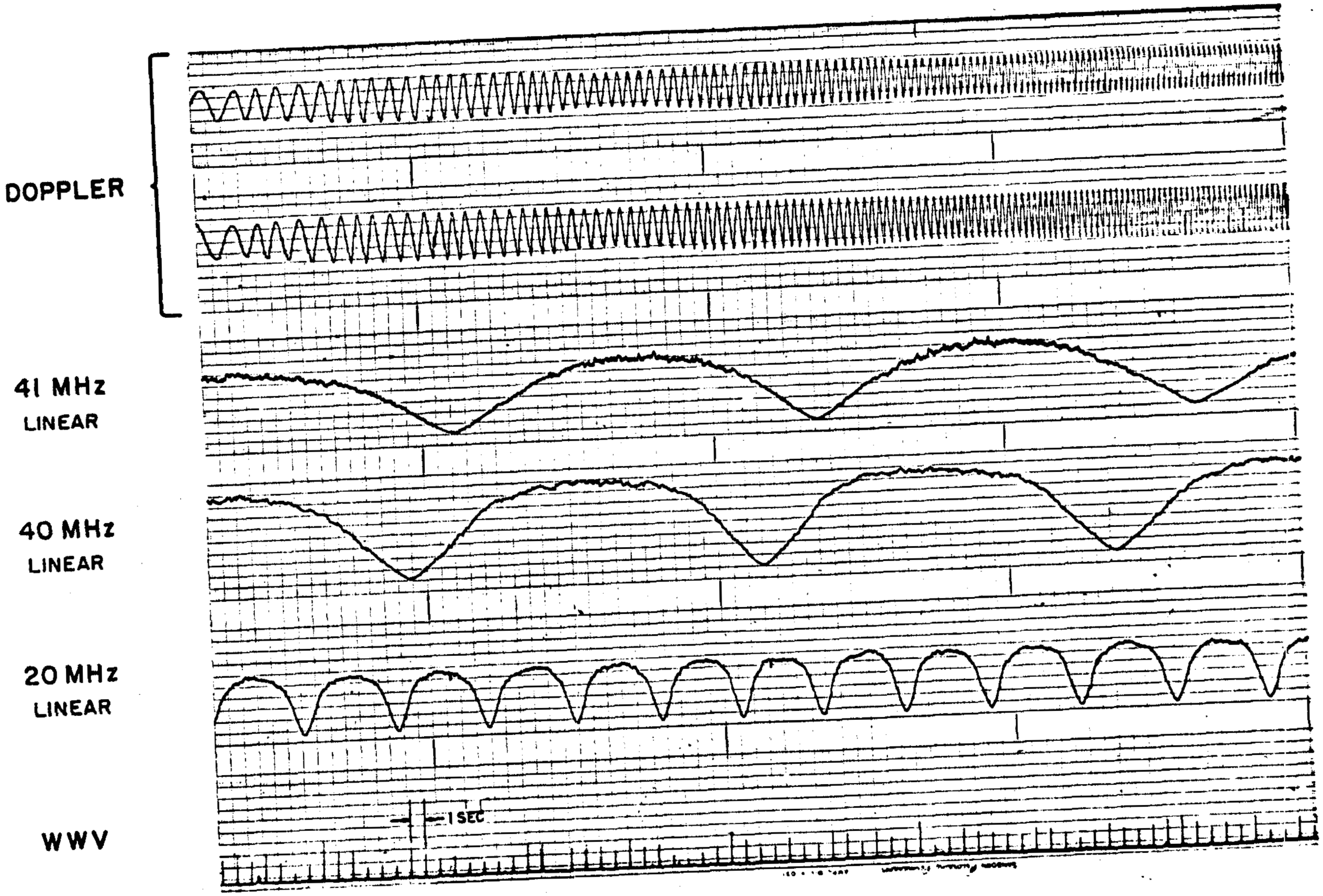
By referring all frequencies to harmonics of the reference oscillator, phase relationships between the received signals are preserved, while the amplitude of the audio beat frequency is proportional to the satellite signal amplitude. All signals are recorded on magnetic tape together with the standard time signal from WWV.

When the tapes are played back, the signals from the linear antennas are envelope detected to produce Faraday rotation data.

Dispersive doppler shift data are obtained from the phase relationship between the signals from 20MHz and 40MHz circularly polarized antennas. All data are recorded on a multichannel chart recorder. Figure 4 shows a portion of a typical record.

The plots marked "linear" on figure 4 display the amplitude of the signals received on the linear antennas and are used for measurement of Faraday rotation. Every time the direction of the polarization of a received signal passes through the plane perpendicular to the linearly polarized antennas, which are oriented in the east-west direction, the amplitude of the signal reaches a minimum level. Thus for a continuous rate of Faraday rotation, each successive null seen on figure 4 corresponds to a half-rotation of the plane of polarization of the received signal.

The plots marked "doppler" show the rate of change for the difference between phase path reductions of 20 MHz and 40 MHz signals measured at a comparison frequency of 40 MHz . The two plots are from quadrature phase comparators and are used to detect any reversals of the rate of change of differential phase path reduction.



SAMPLE RECORDING OF SATELLITE SIGNALS
FIGURE 4

CHAPTER 4

METHOD OF ANALYSIS

1. Determination of Absolute Faraday Rotation

As can be seen from figure 4, only differential Faraday rotation and doppler shift can be measured directly from the data. Because of amplitude fluctuations in the received signals and non-linearity of the Faraday rotation rate, the most accurate determination of the rotation angle can be made at the nulls of the signals and interpolation between nulls may give erroneous results. (Each successive null represents a half-rotation of the plane of polarization of the propagated signal.) For a pass of the Beacon-B satellite, the rotation always increases uniformly toward the south, except for few passes for which the rotation may change direction for an extremely disturbed ionosphere.

For a mid-latitude station like University Park, Pennsylvania, since the satellite does not pass through the transverse condition while above the radio horizon, it is not known how many rotations are present at the northernmost null recorded. This problem is solved by making use of two closely-spaced frequencies.[11]

If there is a difference of $\Delta\Omega$ between the absolute rotations, Ω_1 and Ω_2, of two signals with frequencies f_1 and f_2, then

$$\Delta\Omega = \Omega_1 - \Omega_2$$

and

$$\frac{\Delta\Omega}{\Omega_1} = 1 - \frac{\Omega_2}{\Omega_1}$$

Since in the first order theory the Faraday rotation is inversely proportional to the square of the frequency of a signal,

$$\frac{\Delta\Omega}{\Omega_1} = 1 - \frac{f_1^2}{f_2^2} \qquad (10)$$

40MHz and 41MHz signals are used to determine the absolute value of the Faraday rotation. Equation 10 reduces the problem to finding the fractional difference of rotation between these two signals. This is done most accurately when a 40MHz null coincides with a 41MHz null so that $\Delta\Omega$ becomes an integer. For such case, from equation 10,

$$\Omega_o = 20.75\Delta\Omega = 20.75n \qquad (11)$$

The ambiguity of finding the value of the integer n is resolved without any difficulty virtually for all instances. n is usually equal to either one or two, and only the correct value gives reasonable results for electron content, especially with regard to magnitude, gradient, and time of day.

2. Determination of Absolute Phase Path Reduction

There is no simple method for the determination of absolute phase path reduction, since the satellite contains no provision for such purpose. Only changes of this quantity with satellite position can be found from the phase path dispersion between harmonically related transmitters. Because of experimental complexities, only 20MHz and 40MHz signals are used for data reduction.

The problem of evaluating absolute integrated doppler shift can be solved by making use of the equation 9 which can be rewritten as

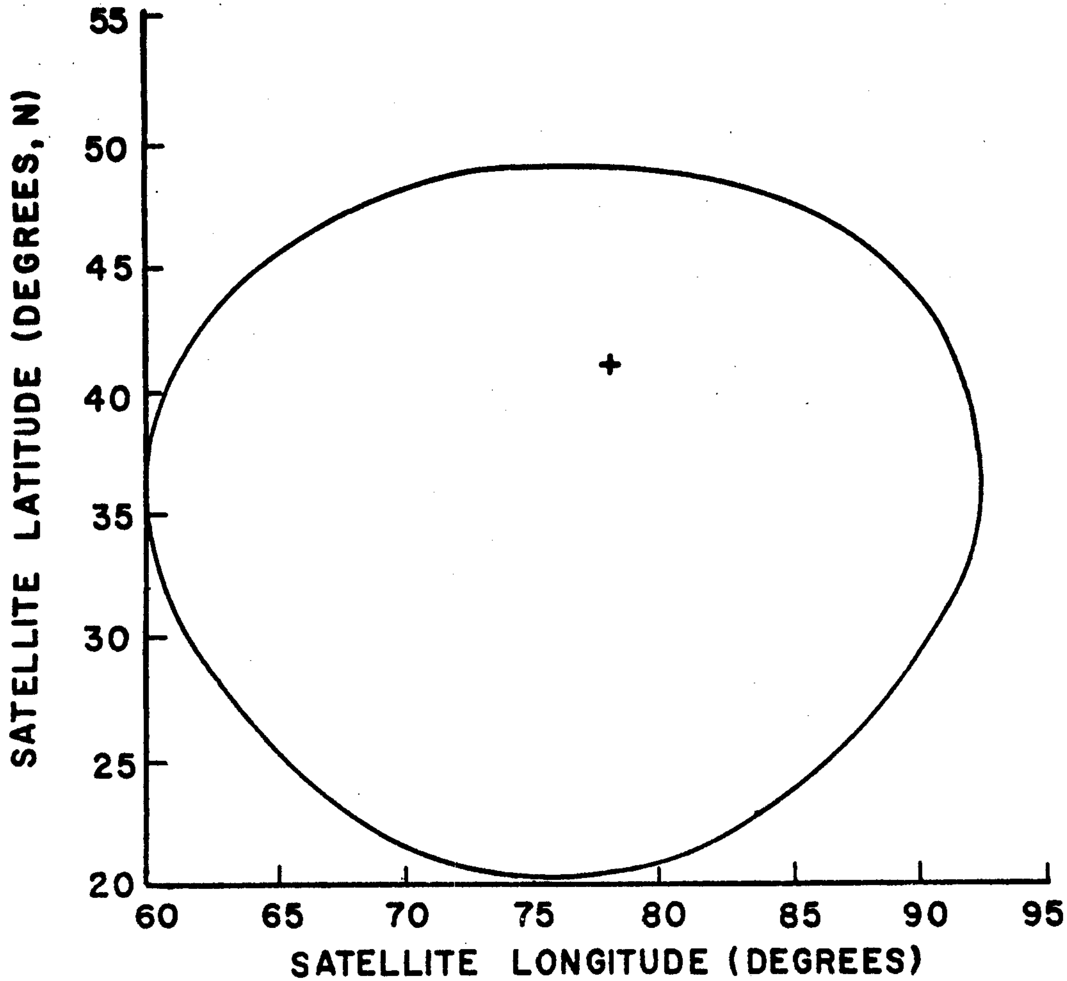
$$\Delta P_o = K \frac{\Omega_o}{\overline{B}_L} \qquad (12)$$

Since Ω_o can be calculated, a knowledge of the value of $\overline{B}_L$ enables us to evaluate ΔP_o. Figure 5 shows a plot of the longitudinal magnetic field component against height along the ray path for a specially chosen satellite position. The dotted curve presents the ionization density profile for a normalized Chapman-α layer which may be considered as a typical profile. It can be observed that B_L has almost no gradient with height between 200km and 500km altitudes. The centroid of any normal electron density profile will be in this range; therefore, for this satellite position the measured value of Ω can be converted into a corresponding value of ΔP with almost no dependence on the assumed mean ionospheric height. For a mid-latitude observing station there is a locus of directions in which the longitudinal component of the earth's magnetic field has the above property. Figure 6 shows a locus of satellite positions for which B_L has zero gradient at 300km altitude taking satellite height at 1000km. The cross mark at the middle of the figure indicates the position of the recording station at University Park, Pennsylvania for which the locus of positions is calculated.

3. Calculation of Mean Ionospheric Height

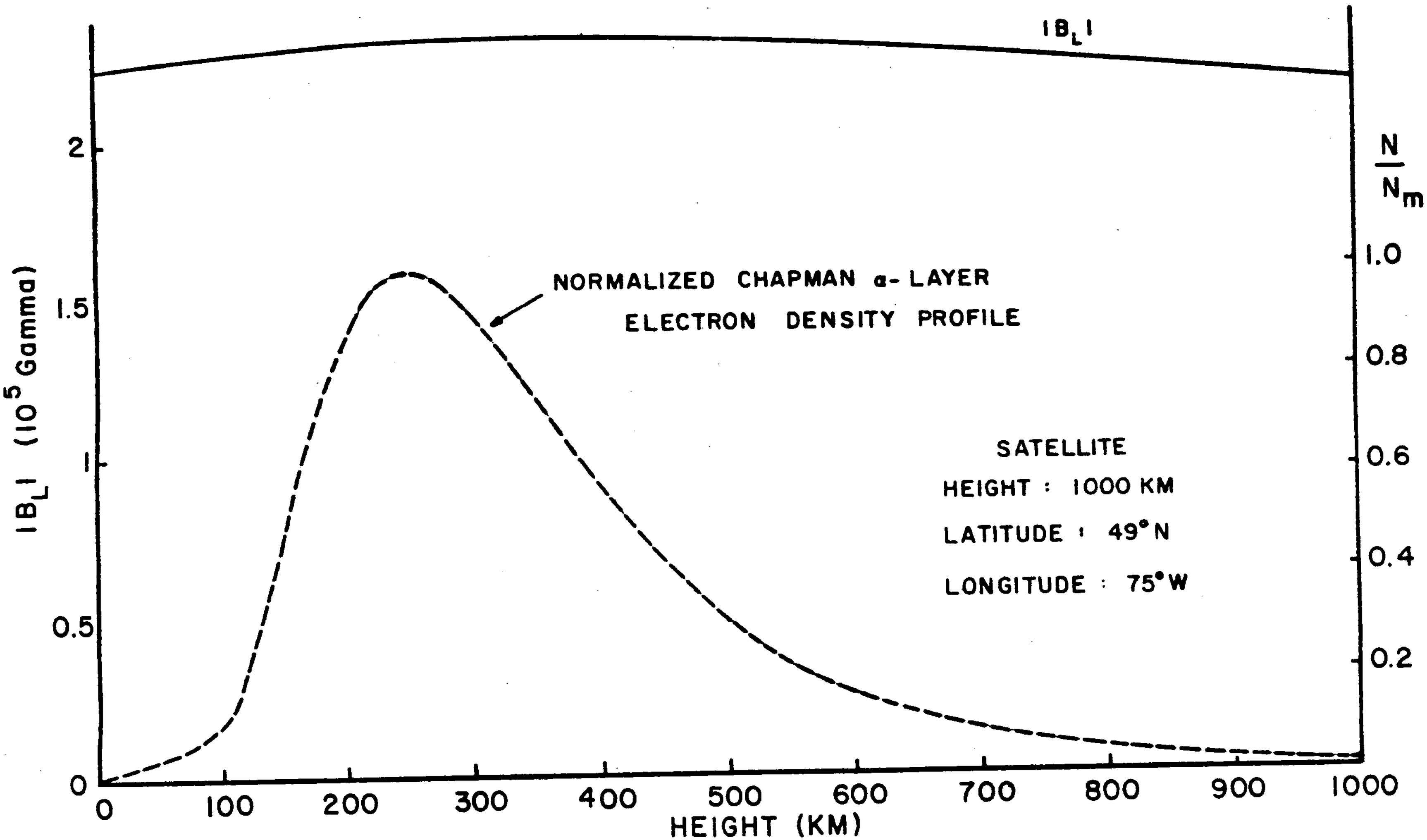
The preceding sections describe some of the theory and techniques used for the calculation of mean ionospheric height. Actual data reduction has been on IBM 7074 computer following the procedure to be explained below.

At first the fractional rotation difference, $\Delta\Omega$, between 40MHz and 41MHz signals is measured usually with linear



LOCUS OF SATELLITE POSITIONS FOR WHICH $\frac{\partial B_L}{\partial h} = 0$ AT $h = 300$ km, SATELLITE HEIGHT 1000 km

FIGURE 5



PLOT OF $|B_L|$ VS HEIGHT ALONG THE RAY PATH

FIGURE 6

interpolation between two 41MHz nulls. Although the Faraday rotation is seldom linear with time, the $\Delta\Omega$ measurement can be done at a place where a 40MHz null almost coincides with a 41MHz null, and the uncertainty due to this nonlinearity can be lowered to be less than 0.5 per cent. Starting with this initial null and adding or subtracting, depending on the direction which the satellite is travelling, a half-rotation for each successive null, Ω at every 40MHz null is calculated for the entire pass.

In order to evaluate the absolute doppler shift, a program was written to find the satellite position for which B_L has zero gradient at 300km altitude along the ray path. This height was chosen arbitrarily as a reasonable ionospheric height and its particular value introduces very little uncertainty in the results. Ω for this position, which is usually to the north of the station, is evaluated by interpolating between the two adjacent 40MHz nulls. Uncertainties due to non-linearities of the Faraday rotation are reduced with a non-linear interpolation using positions of the 20MHz nulls as reference. Knowing Ω_o and using the value of B_L at 300km as $\overline{B}_L$, ΔP_o is calculated at this point from equation 12.

By counting changes of ΔP_o from this point to the positions of 40MHz nulls, it is possible to determine ΔP_o, to calculate $\overline{B}_L$ at these points from equation 9, and hence to find the mean ionospheric height.

For satellite positions, orbital ephemerides supplied by the Goddard Space Flight Center have been used. All magnetic field calculations have been made using GSFC(9/65) field model. (see later discussion).

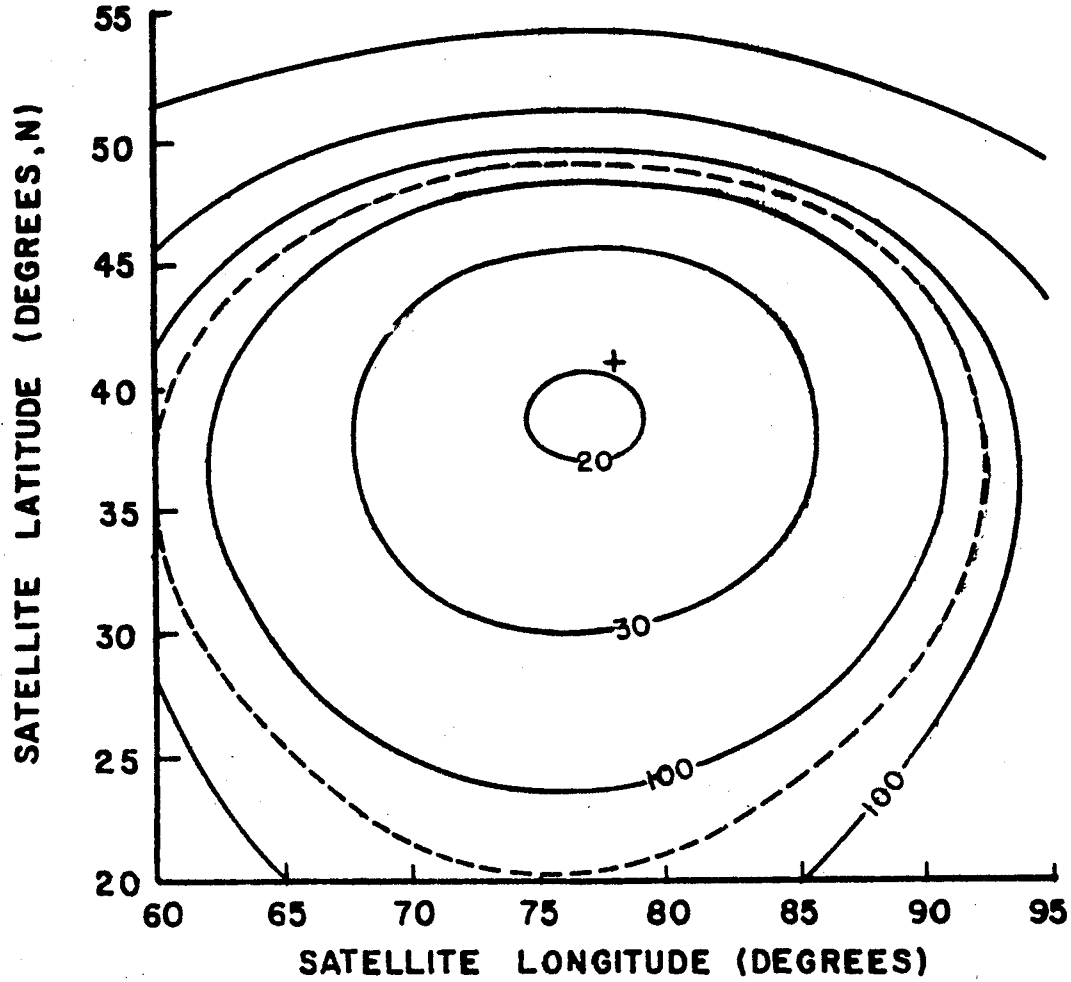
4. Resolution Power of the Method

The assignment of an absolute value to the phase path reduction can only be done if there is a direction in which the longitudinal component of the magnetic field is independent of height. However, the subsequent calculation of mean ionospheric height using equation 9 requires that a substantial gradient of the field component with height must be present if good height resolution is to be obtained. These conflicting requirements can be met in different directions within the field of view of the observing station.

Figure 7 shows a contour map of the height resolution in kilometers which can be obtained in different directions for an arbitrary tolerance of one per cent in the value of $\overline{B}_L$ computed by equation 9. In this figure the dashed curve corresponds to the locus of zero longitudinal field variation with height. It can be seen that there is a significant range of directions within which the mean ionospheric height may be determined reasonably accurately. With an assumption that experimental uncertainties can be kept within this one per cent tolerance, any height calculation inside the contour of 30km will be considered significant. This area is about 15^o across for the satellite latitude positions and 5^o for the ionospheric points-ionospheric point is defined as the point which corresponds to the ionospheric height along the ray path.

5. Second-Order Corrections

As figure 7 indicates, in order to obtain significant results,



CONTOUR MAP OF HEIGHT UNCERTAINTY CORRESPONDING TO 1% VARIATION OF B_L AT 300km, SATELLITE HEIGHT 1000km

FIGURE 7

the tolerance of $\overline{B}_L$ must be at the most 1 per cent. Such restriction makes this technique a precision method, and for such purpose the first-order theory described in chapter 2 becomes insufficient. The first-order theory is based on the assumption of straight line propagation which is equivalent to the medium being of uniform ionization density over the entire propagation path and having a uniform magnetic field. In order to relax these simplifying approximations second-order corrections were derived by Ross.[9] The second-order equations increase the accuracy of the method to a desirable level, however they are still based on the assumption of horizontally stratified ionosphere. With these correction terms the following equation can be written for Faraday rotation:

$$\Omega = \Omega_o \left[1 + \frac{1}{2} \beta \overline{X} + \frac{1}{2}(\beta - 1) G \overline{X} \right] \qquad (13)$$

where

Ω = second-order polarization rotation angle.

Ω_o = first-order polarization rotation angle given by equation 4.

$\overline{X}$ = mean value of X over the slant path.

$\beta = \dfrac{\overline{X^2}}{\overline{X}^2}$, a measure of the non-uniformity of the ionization distribution along the ray path.

G = a geometrical parameter involving the direction of straight line propagation, the magnetic field, and the vertical at the ionosphere point near which the bulk of ionization lies.

For phase path reduction

$$\Delta P = \Delta P_o \left[1 \mp Y_L + \frac{1}{4}\beta\overline{X} + \frac{1}{4}(\beta-1)\overline{X}\tan^2\theta \right] \qquad (14)$$

where

ΔP_o = the first order theory value of phase path reduction given by equation 5.

θ = the zenith angle at the ionosphere point

$\mp$ signs in front of the term Y_L correspond to extraordinary and ordinary modes of propagation. The proper sign must be picked according to the mode of received signal.

For this study the phase path reduction is measured by taking phase difference between 20MHz and 40MHz signals, and the final equation for the doppler shift effect can be written as

$$\Delta P_D = \frac{3e^2 c10^{-7}}{8\pi mf} \int_s Nds \left[1 + \frac{5}{4}\overline{X} + \frac{5}{4}(\beta-1)\overline{X}\,\mathrm{Sec}^2\theta \right] + \frac{7}{2}\Omega_o \quad (15)$$

where

ΔP_D is in cycles measured at 40MHz

$\overline{X}$ = the average of X for 40MHz

Ω_o = the first-order Faraday rotation for 40MHz expressed in half-rotations.

Equation 9 which is used to calculate $\overline{B}_L$ and determine mean ionospheric height requires first-order values of phase path reduction and Faraday rotation. These quantities, ΔP_o and Ω_o, are calculated from data which contain values of ΔP_D and Ω by making use of equations 13 and 15. For the second-order corrections β is taken to be 2.5 which is the value for a Chapman layer with a scale height of 67km and satellite height of 1000km,[9] and $\overline{X}$ is calculated from the following equation,

$$\overline{X} = \frac{e^2}{4\pi^2 \varepsilon_o m f^2} \quad \frac{N_T}{h}$$

where h is the vertical separation between the satellite and the receiver. N_T is determined from equation 7 using the measured value of Faraday rotation as Ω_o. Since the difference between Ω and Ω_o is just a few per cent, any iteration for a more exact calculation of $\overline{X}$ becomes unnecessary.

CHAPTER 5

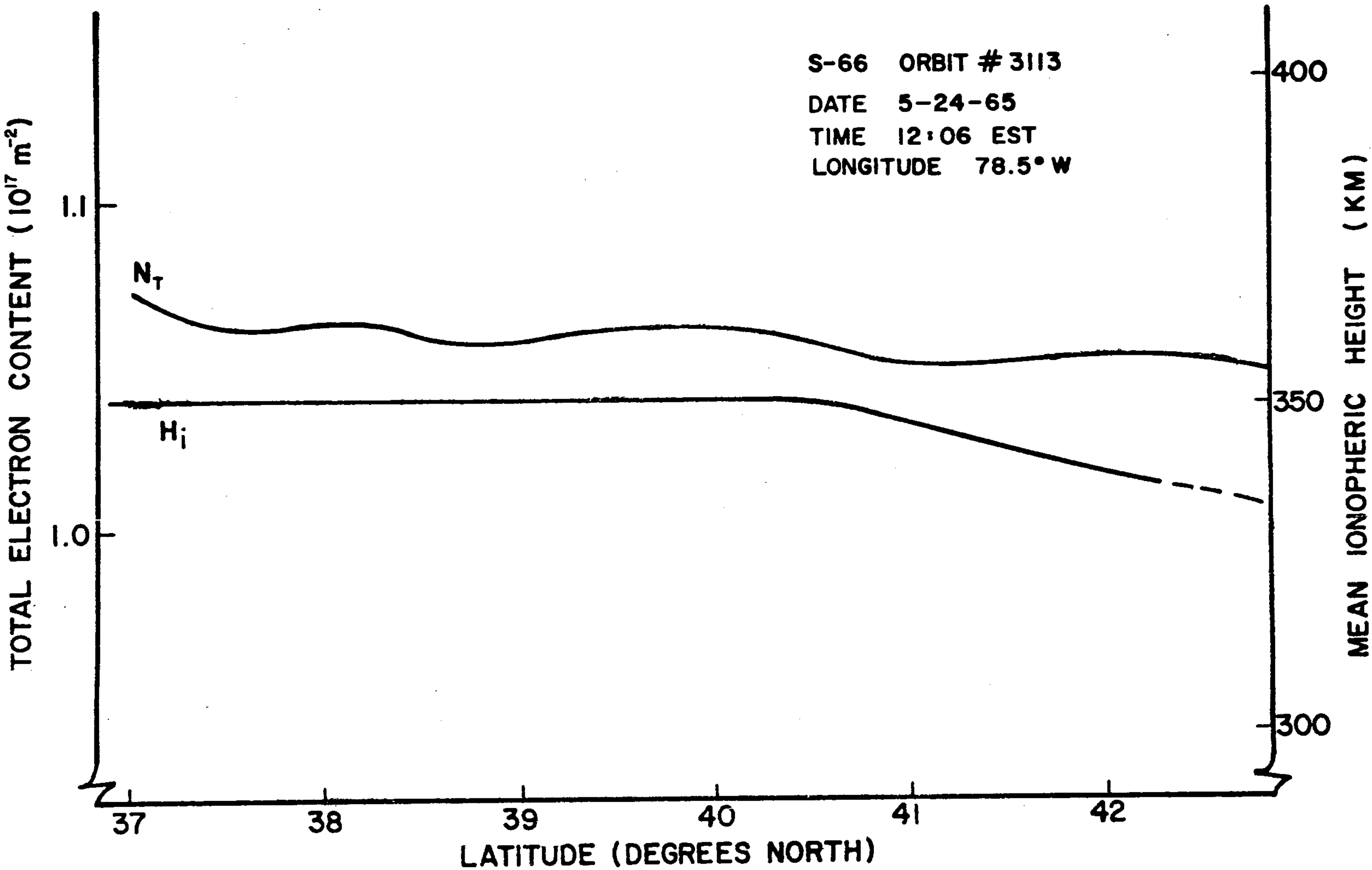
RESULTS

1. Results for Selected Passes

This method for calculating mean ionospheric height has been applied to several passes of the Beacon B satellite. Most of the data reductions were done for day-time passes, because of difficulties involved determining $\Delta\Omega$ for night-time passes. At night the electron content decreases to about one-fifth of its day-time value and Faraday rotation becomes too slow to obtain any degree of accuracy in interpolating between nulls. Also only the passes for which the satellite is between 70° and 83°W longitudes have been analyzed, since outside this range the portion of the pass that lies within 30km uncertainty region becomes too short to do any useful analysis of the height variation with latitude.

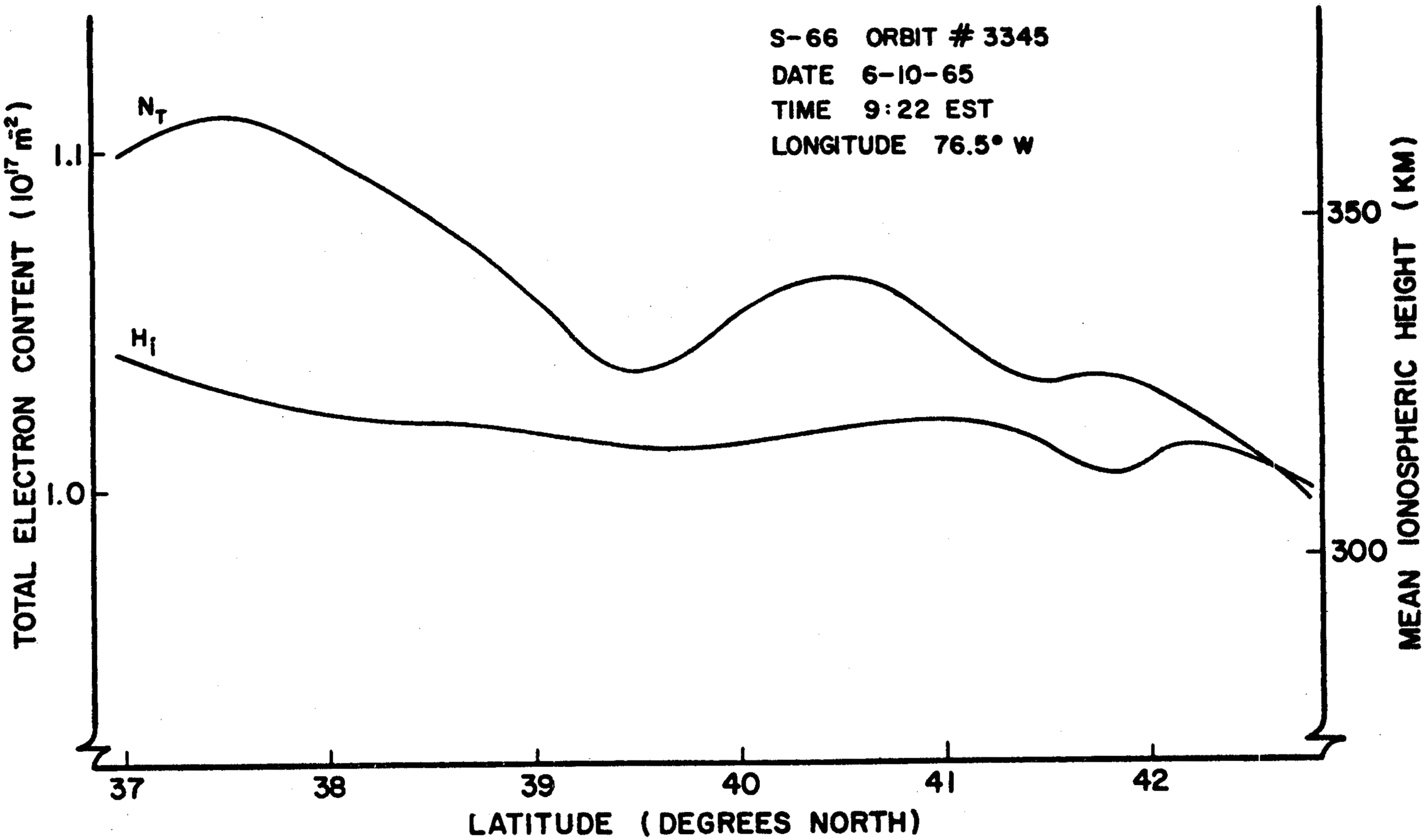
Figures 8 thru 11 show results for few selected data. Both mean ionospheric heights and total electron contents, which are calculated with the assumption of horizontally stratified ionosphere, are plotted against the latitude of ionospheric point. Since the Beacon-B travels almost in the north-south direction at a constant height, a mean satellite height and a mean longitude for ionospheric points are included for each graph.

Figure 8 shows a very commonly encountered mid-afternoon condition in which both the ionospheric height and the electron content have no significant variation with latitude. Dashed parts of the ionospheric height curves indicate the regions where the resolution is worse than 30km for 1 per cent uncertainty of B_L (figure 7).



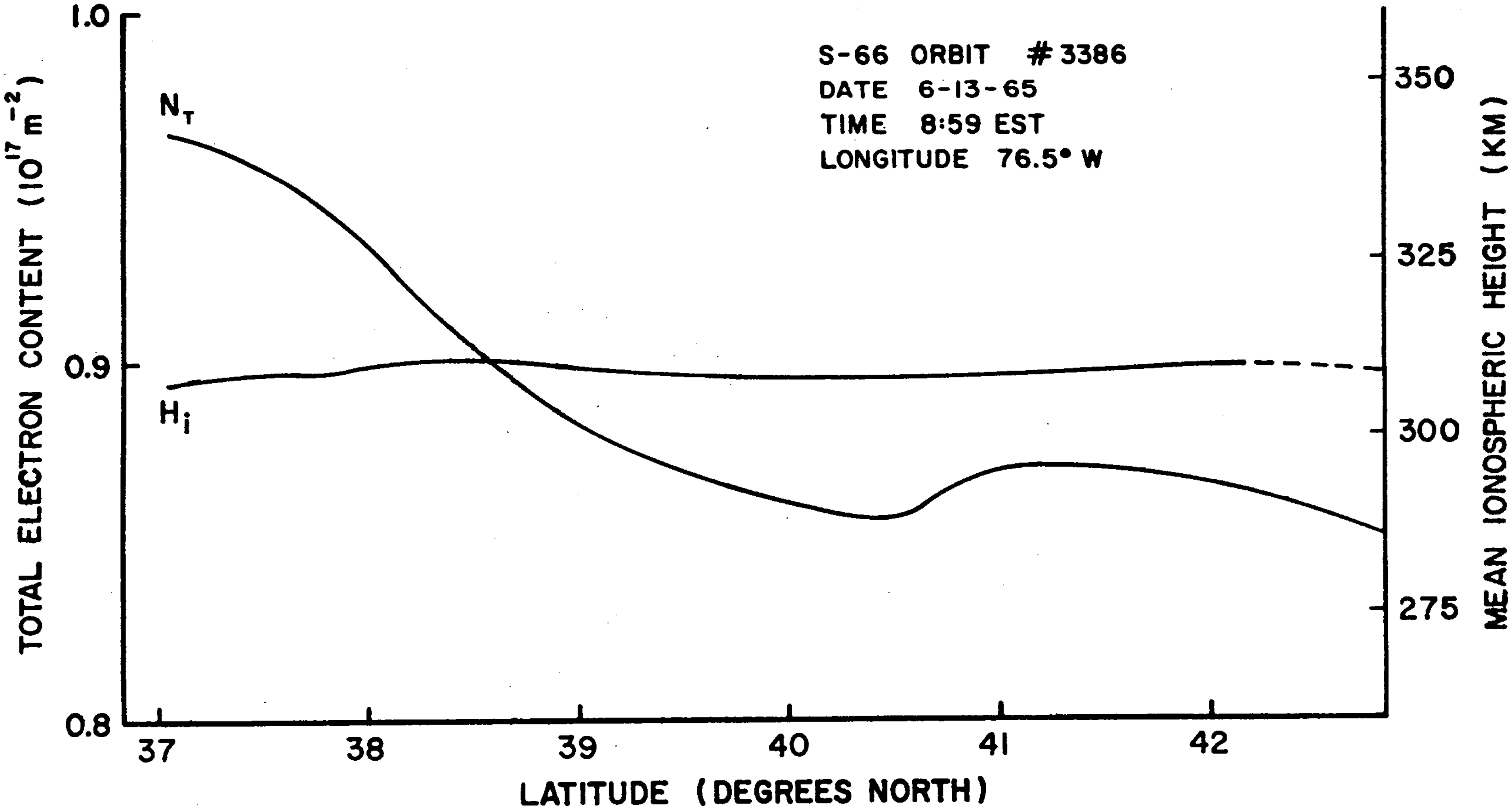
ELECTRON CONTENT AND MEAN HEIGHT VS LATITUDE

FIGURE 8



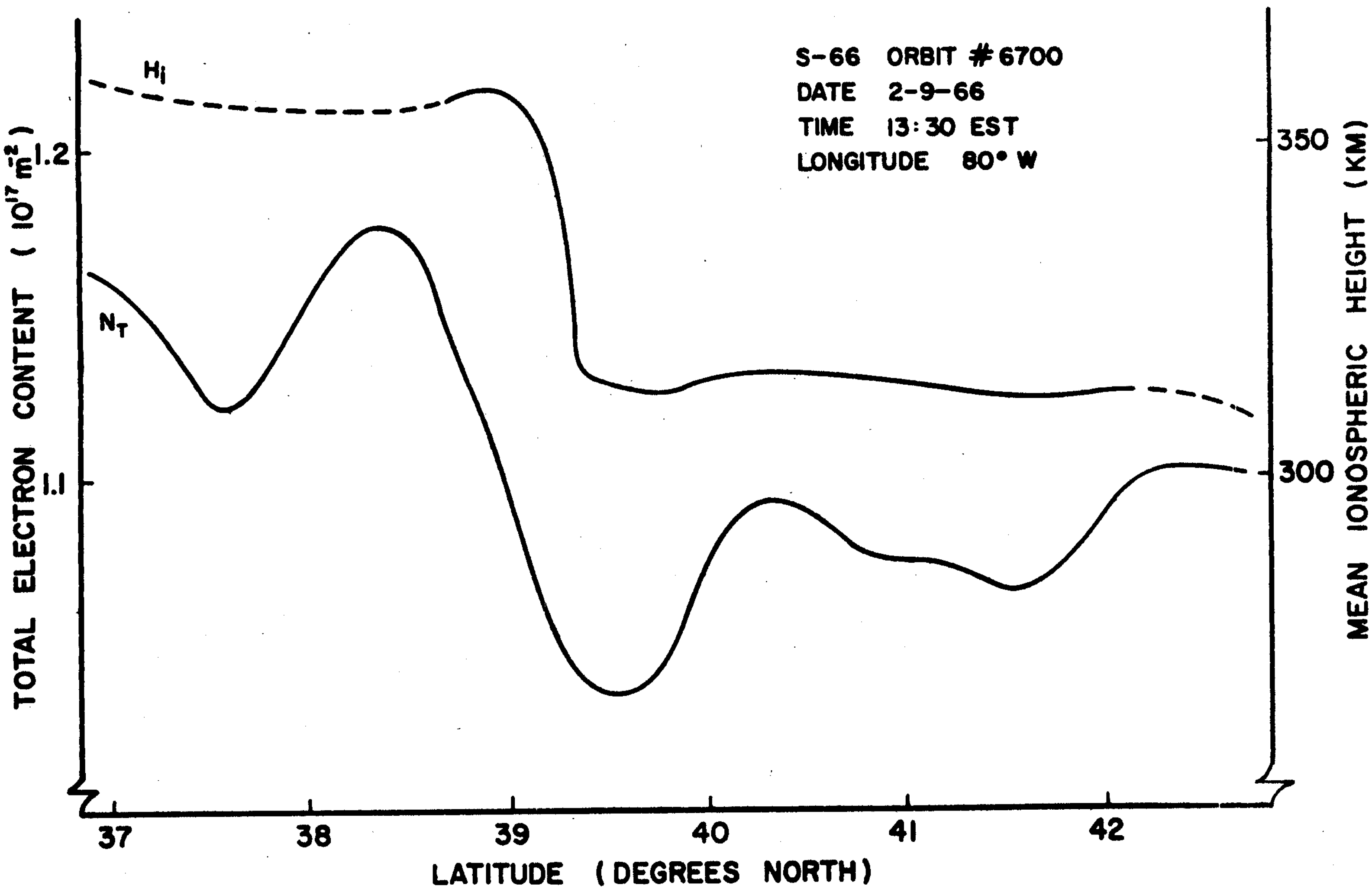
ELECTRON CONTENT AND MEAN HEIGHT VS LATITUDE

FIGURE 9



ELECTRON CONTENT AND MEAN HEIGHT VS LATITUDE

FIGURE 10



ELECTRON CONTENT AND MEAN HEIGHT VS LATITUDE

FIGURE 11

2. Physical Interpretation of Mean Ionospheric Height

Although the mean ionospheric height is defined as the height at which B_L assumes the value of $\overline{B}_L$, it has a very close relationship to the centroid of the ionization profile. This matter was discussed at the end of chapter 2 with figure 1 showing that for latitudes between 36^o and 42^o the ionospheric height does not differ appreciably from the centroid of the vertical ionization density profile. A study of figure 7 indicates that this portion is in the region where the height resolution is 30km or better. If the longitudinal component of the earth's magnetic field is a linear function of altitude, then the calculated ionospheric height should become equal to the centroid of the ionization distribution along the slant path between the satellite and the receiver. For a flat-earth approximation the centroids of the slant path and the vertical column coincide, however for a spherical earth the lower parts of the ionosphere are weighted more while calculating $\overline{B}_L$. Therefore, as the zenith angle of the satellite increases, theoretically for linear B_L the centroid of the slant path assumes lower values than the vertical centroid. The difference is about 4km at 45^o zenith angle for a Chapman-α layer.

When the value of B_L does not vary linearly with height, a further displacement of the mean ionospheric height from the centroid will result. In figure 1 the mean ionospheric height is lowered in the central region because B_L decreases with height at a rate greater than linear, while near the edges the reverse is the case.

Considering above discussions, it can be said that, within the tolerances of the method, the calculated mean ionospheric height represents the height of the centroid of the ionization profile.

3. Discussion of Abnormal Passes

With above physical interpretation of mean ionospheric height, an attempt can be made for the explanation of abnormal passes shown on figures 9, 10 and 11.

Figures 9 and 10 show cases for which ionospheric height remains almost constant all throughout the pass while electron density shows some increase toward the south. This phenomenon is almost as common as the case shown on figure 8. An increasing electron content gradient with latitude to the south has also been found by others working on the mid-latitude ionosphere.[11]

Figure 11 introduces a more complicated case where both the height and electron content have an abnormal variation with latitude. The wave-shaped appearance of the electron content and the sudden change of the ionospheric height might be an indication of a traveling disturbance. This almost sine-wave ripple of the electron content is observed occasionally, however lack of data from near-by stations which have ionospheric sounding facilities prevented any study that might show a correlation between these irregularities and traveling disturbances.

All these abnormal passes coincide with quiet days of the sun, but relationships between these and other geophysical events like solar flare were not one of the objects of this study and no conclusion has been drawn on the subject.

CHAPTER 6

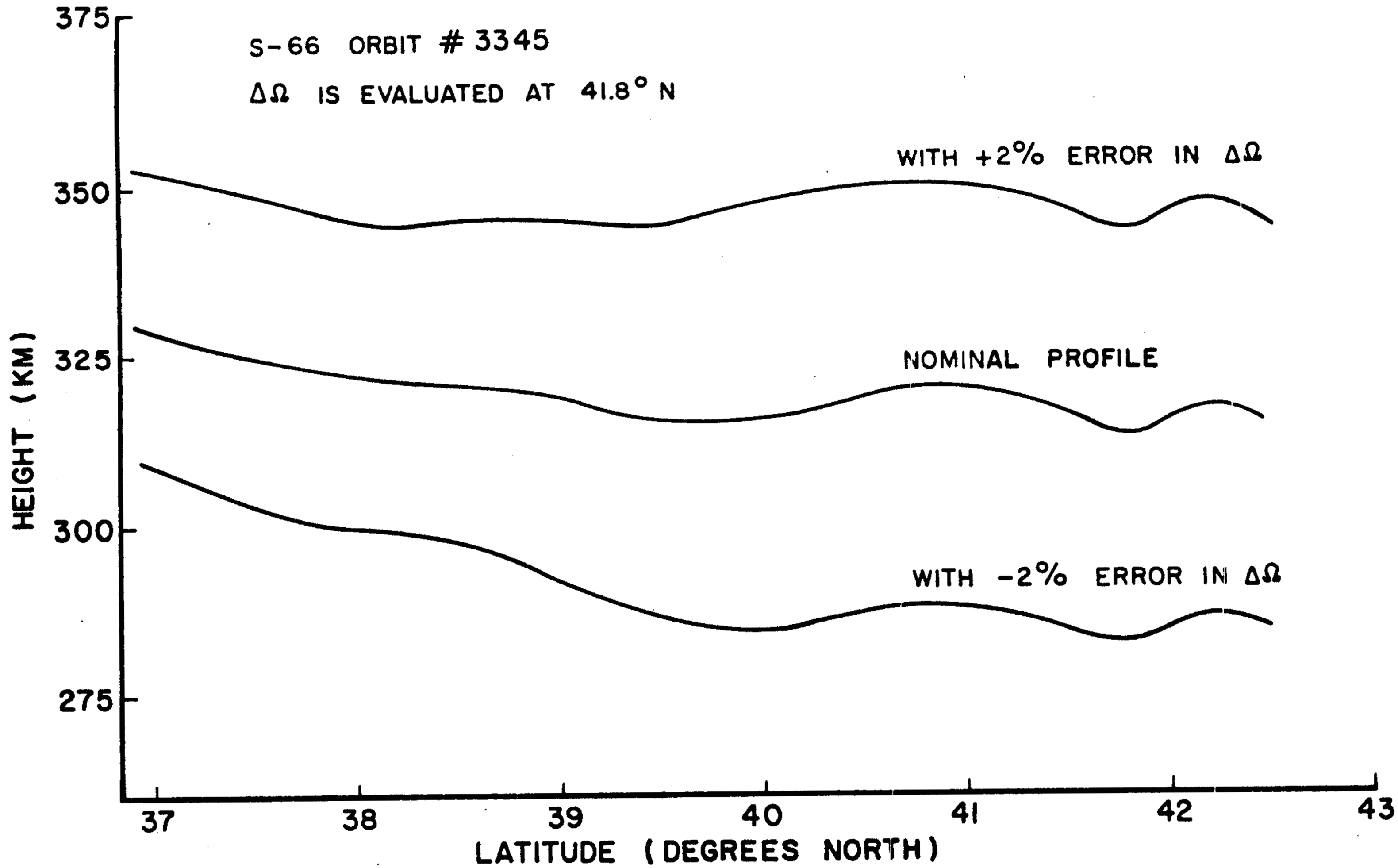
UNCERTAINTIES

1. Measurement Uncertainties

Because of the fact that the height resolution of this method is limited, its application requires high quality data, and uncertainties must be kept at a low level so that they will not exceed the arbitrary one per cent tolerance for the calculated B_L. Some of those uncertainties are due to data reduction errors; others occur because of the approximations that were made to obtain simpler propagation equations or are due to assumptions about the satellite and the ionosphere, e.g. it is assumed that the satellite has no spin around its magnetic axis.

As stated before, much care has been taken measuring the fractional difference of rotation between 40MHz and 41MHz nulls, since any measurement errors on this quantity affect the results directly. With careful selection of the place where $\Delta\Omega$ is determined, this major source of error can be kept well-below 0.5 per cent tolerance.

Figure 12 shows effect of an error in the initial value of $\Delta\Omega$ on the mean ionospheric height profile. These curves were obtained by adding a ∓ 2 per cent error to the measured value of $\Delta\Omega$ which was evaluated at a time corresponding to 47.8°N ionospheric latitude. The curved marked "nominal profile" shows the results derived from the actual measured value of $\Delta\Omega$. Differences between the profiles are almost simple displacements in height with very small gradients with latitude. Since the actual



EFFECT OF AN ERROR IN ΔΩ ON IONOSPHERIC HEIGHT

FIGURE 12

uncertainty of $\Delta\Omega$ is about 0.5 per cent only instead of 2 per cent, the uncertainty introduced by an erroneous measurement of $\Delta\Omega$ may cause about 8km displacement into height profile.

Another source of error is in the scaling process; i.e. the reading of the times of the observed nulls. Most nulls are of sufficient sharpness to be read to the nearest tenth of a second, and this will cause a deviation of much less than 0.1 per cent in $\overline{B}_L$ which is very small compared with other uncertainty terms. Also the determination of the differential doppler cycles between two points can be made without introducing any significant areas as the counting procedure is almost exact.

2. Field Model

The accuracy of the magnetic field model is also of vital concern. The calculations in this study have been made using the GSFC (9/65) field model for epoch 1960 with 99 harmonic coefficients and 35 time derivatives. Although discrepancies in the order of few hundred gammas between this field model and actual surface measurements have been found, comparisons of the magnitude of the magnetic field from the model with results from satellite magnetometers yield a rms error of only 20 gammas over the United States sector. (Private communication with S. Hendricks, Goddard Space Flight Center) An analysis based on this figure indicates that it should provide values of B_L which are not in error by more than about 0.1 per cent.

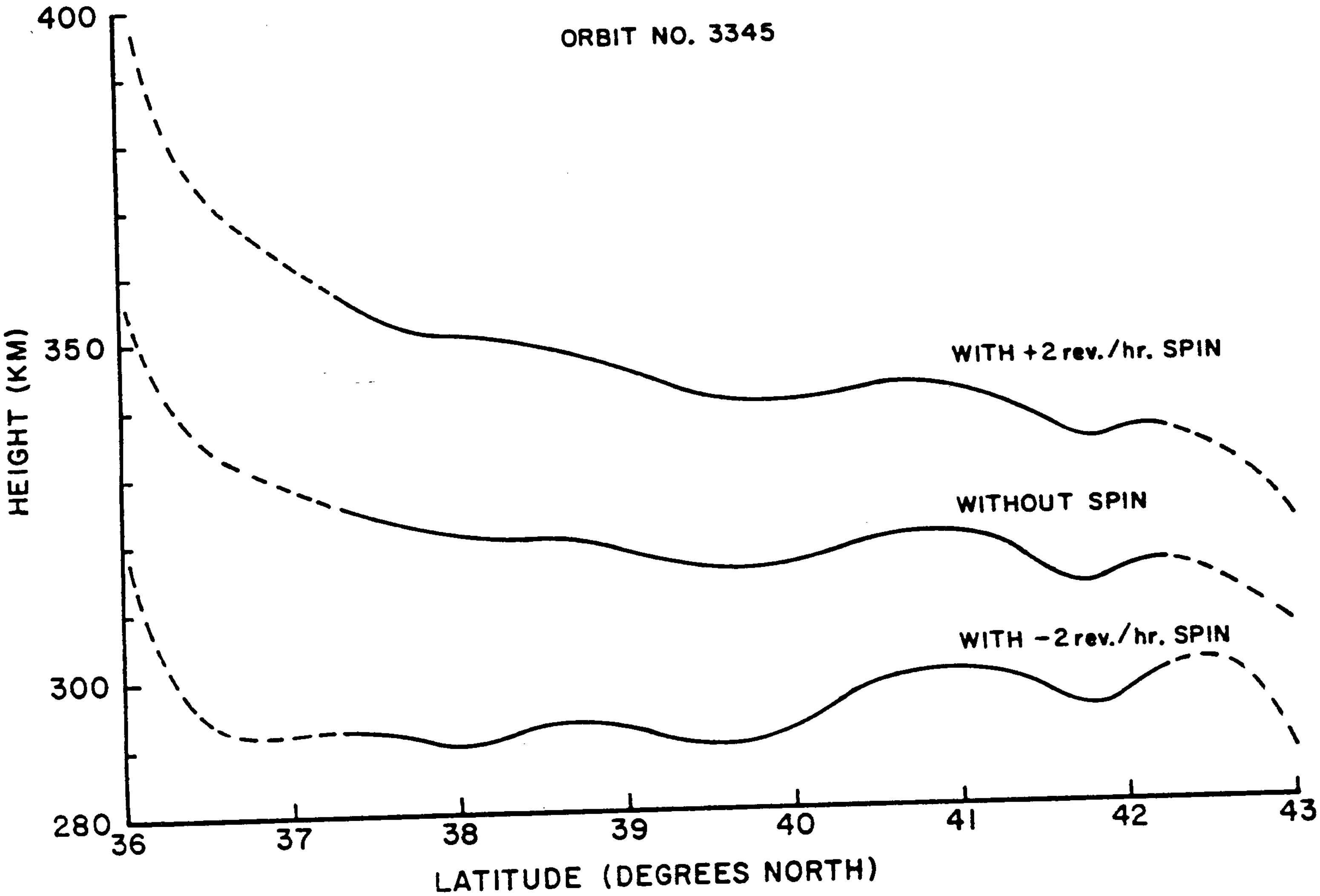
3. Satellite Spin

An error can be caused in the observed rotation of the plane of polarization due to the spinning of the satellite about its magnetic axis. In the original specifications published, the satellite is said to spin less than 1.2 revolutions every hour. Figure 13. shows effects of satellite spin on the ionospheric height. for a pass of the Beacon B satellite. These curves were obtained by assuming that satellite had a 2 revolution-per-hour spin around its magnetic axis, and the actual Faraday rotation data were modified accordingly. The middle curve shows the results for assumed zero spin; the others indicate effects of the satellite spin as it is assumed in the clockwise or the counter-clockwise directions.

As it can be seen a constant satellite spin introduces an almost constant displacement in the ionospheric heights over the range of calculations. Because of magnetic damping devices on the satellite, any irregular spin patterns are not expected, and during a time of an observation which is usually less than 10 minutes any residual spin should be of a systematic nature. Since the actual spin-rate is expected to be less than 1.2 revolution per hour, the maximum height uncertainty is about 10km from this cause, and is almost uniform over the useful range of directions.

4. Unstratified Ionosphere

The second-order correction terms that were discussed in chapter 4 were derived with the assumption of a horizontally stratified ionosphere. Naturally any disturbed ionosphere will not be



EFFECT OF SATELLITE SPIN ON IONOSPHERIC HEIGHT

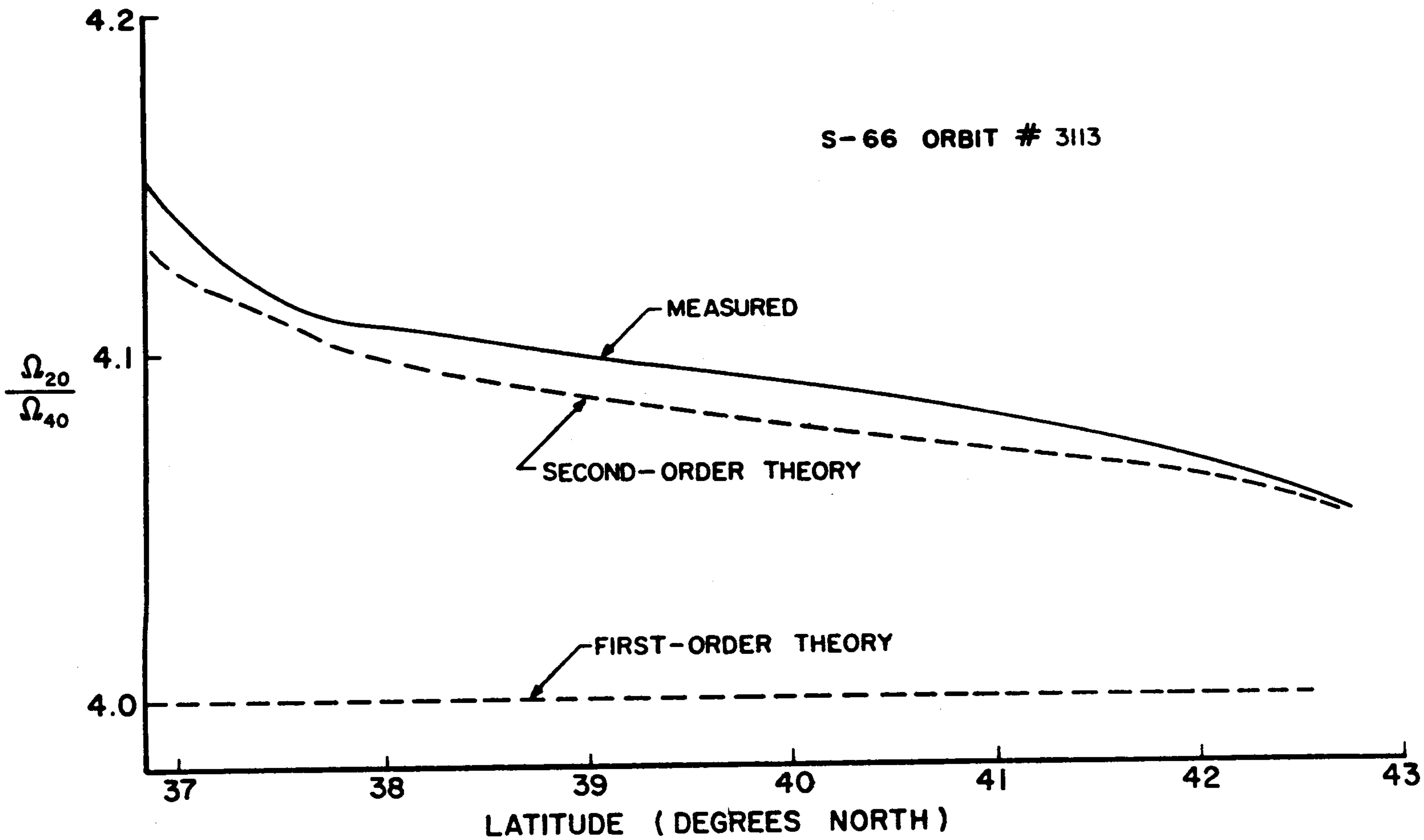
FIGURE 13

horizontally stratified, and these correction factors may not be adequate. Most cases of disturbances can be studied with two simple models. One of them is a model with a "tilted" ionosphere. A qualitative study of such case indicates that the additional correction terms might become negative or positive depending on the tilt angle of the stratification. If the geometry that was used deriving the second-order equations[9] is studied, it can be seen that such tilting of the ionosphere will be equivalent to tilting the horizontal plane at the observation station.

Another deviation from the horizontally-stratified ionosphere is the case for which the total electron content has a horizontal gradient. If the ionosphere can be simplified to be in the form of a constant density slab, then this effect can be studied, using a model with the ionosphere in the form of a prism.

Although no complete quantitative analysis of these effects has been attempted, it is to be expected that their inclusion in the propagation equations will result in correction terms similar to those derived from the analysis of a stratified ionosphere, and this effect should vary as the inverse frequency squared. Their effect has been studied experimentally by comparing Faraday rotations of harmonically related signals.

Figure 14 shows a plot of the observed ratio of Faraday rotations for 20MHz and 40MHz. The dotted line shows the same ratio calculated using the second-order correction terms (equation 11). An analysis of this pass indicates some horizontal gradient of the ionospheric content (figure 9). The differences



A COMPARISON OF MEASURED AND THEORETICAL RATIOS OF FARADAY ROTATIONS

FIGURE 14

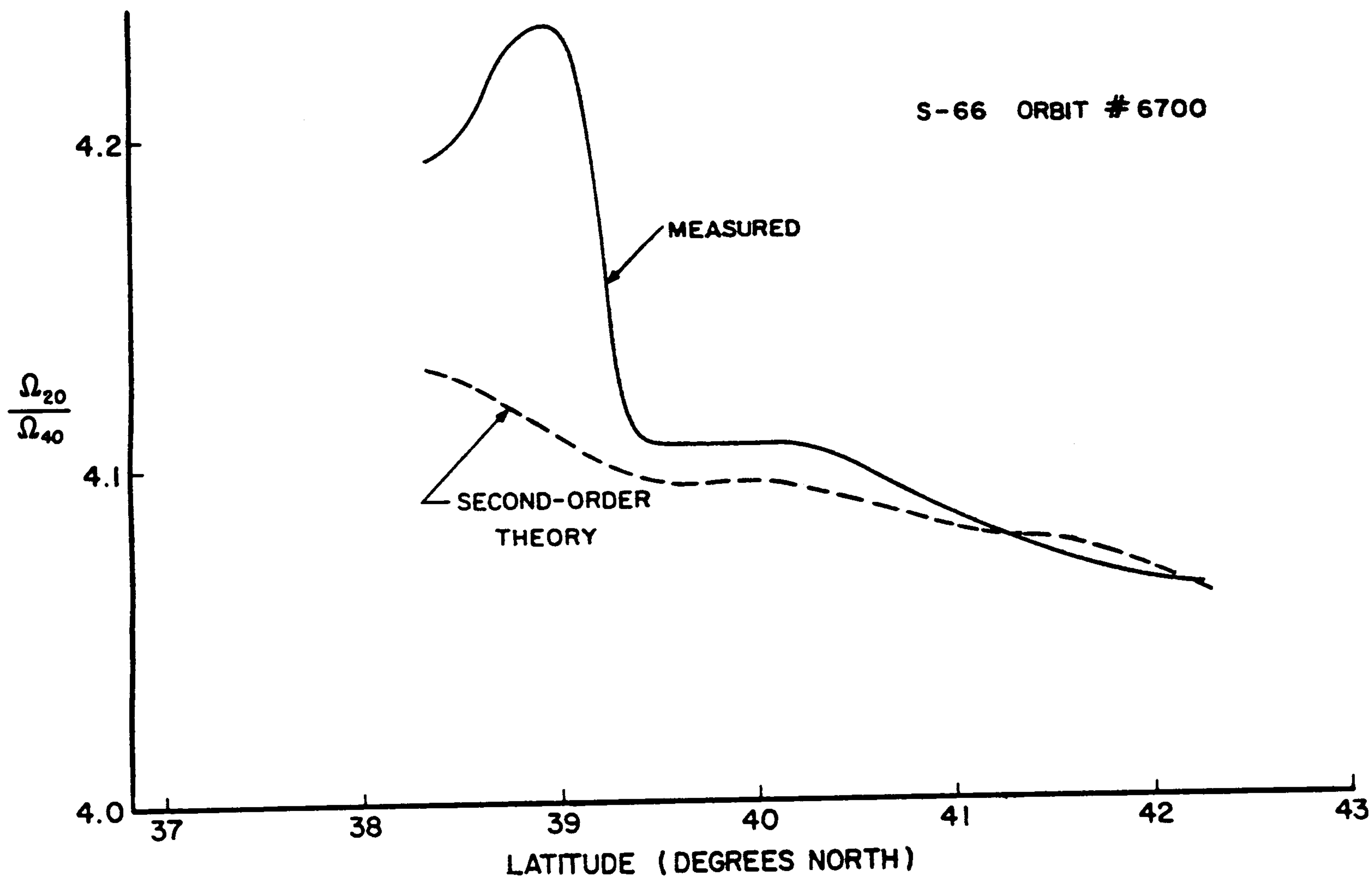
between the actual and the calculated ratios are in the order of 0.5 per cent which corresponds to an uncertainty of less than 0.2 per cent for the rotation of the polarization plane for the 40MHz signal.

Figure 15 shows a plot of the same ratio for another pass which was discussed before (figure 10). This pass is the most irregular one that has been found among the data that covers a period of one and a half years. Even at the worst case the difference between the two curves is about 2.5 per cent which corresponds to 0.75 per cent uncertainty for the Faraday rotation of the 40MHz signal.

Lack of an additional harmonically related signal prevented any study of the prism and the tilt effects for the doppler shift. It is believed that these uncertainties will not cause errors over 1 per cent of the calculated $\overline{B}_L$, except perhaps in the most severe cases.

In any case the errors introduced by inadequate correction for non-stratified medium conditions should be of a transient nature and should be restricted to the interval where the changes are occurring. Emergence of the satellite locus into a stratified region should restore the accuracy of the method. Therefore while the detailed form of the changes may be imperfectly described, their magnitudes should be quite accurately determined.

In summary, it may be concluded that height variations which exceed about 20km in magnitude are always significant, even



A COMPARISON OF MEASURED AND THEORETICAL RATIOS OF FARADAY ROTATIONS

FIGURE 15

if the absolute height values are possibly displaced by certain of the systematic errors described here.

CHAPTER 7

SUMMARY

1. Summary and Conclusions

A method has been developed for the determination of the mean ionospheric height from analysis of unmodulated constant frequency signals from ionospheric beacon satellites. Major effects of the ionosphere that can be readily observable on the received signals are a rotation of the plane of polarization and a reduction of phase path length. By looking for consistency between these two measurable quantities, it becomes possible to determine a height which has a very close relationship to the centroid of the ionization profile.

Calculation of such height requires knowledge of the absolute values for the rotation angle of the plane of polarization, and the reduction of phase path length; however only changes in these quantities with satellite position can be measured. The absolute value of Faraday rotation is determined by making use of the relationship between two closely-spaced frequencies. The problem of determining the absolute value of the reduction of phase path length is solved in a direction where the longitudinal component of the earth's magnetic field has zero gradient with height. A locus of such directions exists for a mid-latitude station.

Based on an analysis of the uncertainties in the system, it can be concluded that with this method variations of mean ionospheric with latitude can be determined with a tolerance of $\mp$ 20km, although absolute height calculations might carry larger uncertainties.

2. Advantages of the Method

Although with the method developed, only the height of the centroid of the electron distribution can be determined, coupled with total electron content measurements and because of its continuous coverage it carries some advantages for studying ionospheric variations with location and large size irregularities in the F-region, especially traveling disturbances.

Some of the other methods that are presently used for determination of electron density profiles are ionospheric soundings, topside and bottomside, and incoherent scatter methods. Although topside sounders have the same continuity advantage as the beacon satellite experiments, they have to be complemented with results from bottom side sounders which have very little spatial coverage. Also the accuracy of height determination of sounding methods becomes quite low near the maximum ionization level.

3. Topics That Need Further Research

As discussed in chapter 6, a main contribution to the uncertainties of the method comes from the insufficiency of the second-order theory to approximate irregular cases where the assumption of a horizontally startified ionosphere cannot be applied. Only some qualitative analyses and some experimental measurements of these cases have been carried out for this study.

Such irregularities which can be simplified into two different models, prism and tilted ionosphere, need further quantitative and analytical research. With solutions of this uncertainty the accuracy of the method can be improved considerably.

The major purpose of this study has been development of the method, therefore there has not been any systematic data reduction. Further work on this subject will yield determination of diurnal and seasonal variations of the ionospheric height.

This method can be applied only at mid-latitude observation stations where there can be a locus of the directions for which the longitudinal component of the earth's magnetic field along the ray path has no height dependence. Although there is no such locus for an equatorial receiving station for which the height resolution is much greater, the method still can be applied to equatorial station. In this case the absolute value of the reduction of the phase path has to be determined at a location where the electron density profile can be obtained from other sources, such as the incoherent back scatter method.

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