COVARIANT NOETHER IDENTITIES IN COVARIANT FIELD THEORIES

John R. Ray

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Abstract

In covariant field theories such as general relativity the application of Noether's theorem results in a set of identities which are satisfied regardless of whether the field equations are satisfied or not (so-called strong conservation laws). In the conventional formulation these identities contain non-tensorial quantities and non-tensorial operations. Thus, for example, the energy-momentum-pseudo-tensor of general relativity is obtained from such formulations. In this paper the identities arising from Noether's theorem are written in an obviously covariant form containing only tensors and covariant derivatives. As an example we give these identities for general relativity.

I. Introduction

Consider a field ψ_A [1] whose field equations may be derived from the principle of stationary action. If $\mathcal{L}$ is the Lagrangian density for the field ψ_A then the action I is defined by

$$I = \int_{R(x^\ell)} \mathcal{L}(x^\ell | \psi_A | \psi_{A,r})\, d^4x \ , \qquad (1.1)$$

where $R(x^\ell)$ is the integration region and $\psi_{A,r}$ denotes the derivative of ψ_A with respect to x^r. $\mathcal{L}$ can always be written in the form

$$\mathcal{L} = \sqrt{-g}\ L \qquad (1.2)$$

where L is a scalar function. In order to introduce our notation we shall consider the variation of I in detail.

Perform the variations

$$\psi_A \rightarrow \psi_A + kv_A = \psi'_A \ , \ \bar{\delta}\psi_A = \psi'_A - \psi_A$$

$$\psi_{A,r} \rightarrow \psi_{A,r} + kv_{A,r} = \psi'_{A,r} \ , \ (\bar{\delta}\psi_A)_{,r} = \bar{\delta}(\psi_{A,r}) \tag{1.3}$$

$$R(x^\ell) \rightarrow R(x^\ell + ku^\ell) = R'(x^\ell) \ , \ \delta x^\ell = ku^\ell$$

where k is an infinitesimal parameter, $u^\ell(x)$ is an arbitrary vector field and v_A is an arbitrary tensor field. The variation $\bar{\delta}\psi_A$ is the Lie differential of ψ_A. The variation of the limits of integration is defined by the third equation in (1.3). Under these variations the action becomes a function of k, expanding about k = 0 and integrating by parts yields the varied action δI

$$\delta I = \int_R \left[\mathcal{L}^A \bar{\delta}\psi_A + \left(\frac{\partial \mathcal{L}}{\partial \psi_{A,r}} \bar{\delta}\psi_A + \mathcal{L}\delta x^r \right)_{,r} \right] d^4x = 0 \ , \tag{1.4}$$

where

$$\mathcal{L}^A = \frac{\partial \mathcal{L}}{\partial \psi_A} - \left(\frac{\partial \mathcal{L}}{\partial \psi_{A,r}} \right)_{,r} \ . \tag{1.5}$$

For all the following analysis we specialize the variations to infinitesimal coordinate transformations. Following Bergmann[2] the field variation $\bar{\delta}\psi_A$ can be written

$$\bar{\delta}\psi_A = F^{Br}_{As}\, \delta x^s{}_{,r}\, \psi_B - \psi_{A,s}\, \delta x^s \ . \tag{1.6}$$

Since $\bar{\delta}\psi_A$ is the Lie differential of a tensor the ordinary derivatives on the right hand side of (1.6) can be replaced by covariant derivatives (which will be denoted by a semicolon)

$$\bar{\delta}\psi_A = F^{Br}_{As}\, \delta x^s{}_{;r}\, \psi_B - \psi_{A;s}\, \delta x^s \ . \tag{1.7}$$

From (1.6) and (1.7) follows

$$\psi_{A;r} = \psi_{A,r} + F^{Bp}_{As}\,\Gamma_{r\ p}^{\ s}\,\psi_B \tag{1.8}$$

where $\Gamma_{r\ p}^{\ s}$ is the Christoffel symbol constructed from the metric tensor $g_{\ell k}$. Equation (1.8) holds for all tensors including $g_{\ell k}$ for which

$$g_{\ell k;r} = \psi_{A;r} = 0 \ . \tag{1.9}$$

There are some results in this paper which are not valid for the $g_{\ell k}$ field because of (1.9). When a formula contains $\psi_{A;r}$ this formula may not hold for the metric field $g_{\ell k}$. When these cases arise they will either be explicitly pointed out or obvious from the context. For variations which vanish on the boundary we can use Gauss's law and (1.4) to deduce Lagrange's equations

$$\mathfrak{L}^A = 0 \ . \tag{1.10}$$

In order to derive the identities implied by Noether's theorem, however, we assume the field equations are not satisfied.

II. Conventional Formulation

In order to derive Noether's identities in the conventional formalism one uses (1.6) and (1.4) and obtains

$$\delta I = \int_R \Big[\Big(- (\mathcal{L}^A F^{Br}_{As} \psi_B)_{,r} - \mathcal{L}^A \psi_{A,s} \Big) \delta x^s$$

$$+ \Big((\mathcal{L} \delta^r_s + \mathcal{L}^A F^{Br}_{As} \psi_B) \delta x^s$$

$$+ \frac{\partial \mathcal{L}}{\partial \psi_{A,r}} (F^{Bp}_{As} \psi_B \delta x^s{}_{,p} - \psi_{A,s} \delta x^s) \Big)_{,r} \Big] d^4x = 0 \qquad (2.1)$$

If Gauss's law is used in (2.1) and variations which vanish on the boundary are considered, one obtains the identities

$$(\mathcal{L}^A F^{Br}_{As} \psi_B)_{,r} + \mathcal{L}^A \psi_{A,s} = 0 \qquad (2.2)$$

which are called the Bianchi identities by Bergmann.[2] Using the Bianchi identities in (2.1) one obtains

$$\delta I = \int_R \Big[- \Big(\frac{\partial \mathcal{L}}{\partial \psi_{A,r}} \psi_{A,s} - \delta^r_s \mathcal{L} - \mathcal{L}^A F^{Br}_{As} \psi_B \Big)_{,r} \delta x^s$$

$$- \Big(\frac{\partial \mathcal{L}}{\partial \psi_{A,r}} \psi_{A,s} - \delta^r_s \mathcal{L} - \mathcal{L}^A F^{Br}_{As} \psi_B - \big(F^{Br}_{As} \frac{\partial \mathcal{L}}{\partial \psi_{A,p}} \psi_B\big)_{,p} \Big) \delta x^s{}_{,r}$$

$$+ \frac{\partial \mathcal{L}}{\partial \psi_{A,\ell}} F^{Br}_{As} \psi_B \, \delta x^s{}_{,r,\ell} \Big] d^4x = 0 \, . \qquad (2.3)$$

At any point δx^s, $\delta x^s{}_{,r}$ and $\delta x^s{}_{,r,\ell}$ are independent and their coefficients must vanish separately. One must use the symmetry property

$$\delta x^s{}_{,r,\ell} = \delta x^s{}_{,\ell,r}$$

before setting the coefficients of $\delta x^s{}_{,r,\ell}$ equal to zero. Define

$$t^r_s = \frac{\partial \mathcal{L}}{\partial \psi_{A,r}} \psi_{A,s} - \delta^r_s \mathcal{L} \ , \tag{2.4a}$$

$$T^r_s = t^r_s - \mathcal{L}^A F^{Br}_{As} \psi_B \ , \tag{2.4b}$$

$$b_s{}^{r\ell} = F^{Br}_{As} \frac{\partial \mathcal{L}}{\partial \psi_{A,\ell}} \psi_B \ , \tag{2.4c}$$

where t^r_s will be called the canonical momentum complex, T^r_s will be called the momentum complex and $b_s{}^{r\ell}$ will be called the spin complex. The term complex means that these quantities are only tensor densities under linear transformations. For example in special relativity t^r_s is called the canonical momentum "tensor" while in general relativity t^r_s is the energy momentum pseudo-tensor. In terms of these non-tensorial quantities, the varied action becomes

$$\delta I = \int_R \Big[- T^r_{s,r} \, \delta x^s - (T^r_s - b_s{}^{r\ell}{}_{,\ell}) \, \delta x^s{}_{,r} + b_s{}^{r\ell} \, \delta x^s{}_{,r,\ell} \Big] d^4x = 0 \ . \tag{2.5}$$

The identities which follow are (Noether's theorem[3])

$$(\mathcal{L}^A F^{Br}_{As} \psi_B)_{,r} + \mathcal{L}^A \psi_{A,s} = 0 \tag{2.5a}$$

$$T^r_{s,r} = 0 \tag{2.5b}$$

$$T^r_s - b_s{}^{r\ell}{}_{,\ell} = 0 \tag{2.5c}$$

$$b_s{}^{r\ell} + b_s{}^{\ell r} = 0 \ . \tag{2.5d}$$

III. Covariant Formulation

In this section we shall rewrite equations (2.5) so that only tensors and tensorial operations (covariant differentiation) appear. To do this we use (1.8) and write $\mathcal{L}$ in terms of $\psi_{A;r}$ instead of $\psi_{A,r}$

$$\mathcal{L}(x^\ell|\psi_A|\psi_{A,r}) = \mathcal{L}(x^\ell|\psi_A|\psi_{A;r} - \bar{F}^{Bp}_{A\ell}\,\Gamma^\ell_{pr}\,\psi_B) = \mathcal{L}_c(x^\ell|\psi_A|\psi_{A;r}) \ . \tag{3.1}$$

Of course this substitution cannot be made for the gravitational field $g_{\ell k}$ because $g_{\ell k}$ satisfies (1.9). The same type of relation as (1.2) connects $\mathcal{L}_c$ with L_c

$$\mathcal{L}_c = \sqrt{-g}\ L_c \ . \tag{3.2}$$

The variation of the action associated with $\mathcal{L}_c$ leads to

$$\delta I = \int_R \left[L^A_c\,\bar{\delta}\,\psi_A + \left(\frac{\partial L_c}{\partial \psi_{A;r}}\,\bar{\delta}\,\psi_A + L_c\,\delta x^r\right)_{;r}\right]\sqrt{-g}\ d^4x = 0 \tag{3.3}$$

$$L^A_c = \frac{\partial L_c}{\partial \psi_A} - \left(\frac{\partial L_c}{\partial \psi_{A;r}}\right)_{;r} \ . \tag{3.4}$$

Lagrange's equations are

$$L^A_c = 0 \ . \tag{3.5}$$

One can easily prove the following relation

$$\sqrt{-g}\ L^A_c = \mathcal{L}^A \ , \tag{3.6}$$

and, therefore, the two forms of Lagrange's equations are equivalent. Using (1.7) in (3.3) one obtains

$$\delta I = \int_R \Big[- \Big((L_c^A F_{As}^{Br} \psi_B)_{;r} + L_c^A \psi_{A;s} \Big) \delta x^s$$

$$+ \Big(L_c^A F_{As}^{B\ell} \psi_B \delta x^s + \frac{\partial L_c}{\partial \psi_{A;r}} F_{As}^{Br} \psi_B \delta x^s{}_{;r}$$

$$- \frac{\partial L_c}{\partial \psi_{A;\ell}} \psi_{A;s} \delta x^s + L_c \delta x^\ell \Big)_{;\ell} \Big] \sqrt{-g}\, d^4x = 0 \,. \quad (3.7)$$

From (3.7) one derives the covariant Noether identities in the same way as the non-covariant identities (2.5) were derived. Using Gauss's law and (3.7) we can deduce the Bianchi identities

$$\Big(L_c^A F_{As}^{Br} \psi_B \Big)_{;r} + L_c^A \psi_{A;s} = 0 \,. \quad (3.8)$$

Using the Bianchi identities in (3.7) one obtains

$$\delta I = \int_R \Big[- S^\ell_{r;\ell} \delta x^r - \Big(S_r^\ell - B_r^{\ell k}{}_{;k} \Big) \delta x^r{}_{;\ell}$$

$$+ B_r^{\ell k} \delta x^r{}_{;\ell;k} \Big] \sqrt{-g}\, d^4x = 0 \,, \quad (3.9)$$

where

$$u_r^\ell = \frac{\partial L_c}{\partial \psi_{A;\ell}} \psi_{A;r} - \delta_r^\ell L_c \quad (3.10a)$$

$$S_r^\ell = u_r^\ell - L_c^A F_{Ar}^{B\ell} \psi_B \quad (3.10b)$$

$$B_r^{\ell k} = F_{Ar}^{B\ell} \frac{\partial L_c}{\partial \psi_{A;k}} \psi_B \,. \quad (3.10c)$$

In (3.9) the coefficients of δx^r, $\delta x^r{}_{;\ell}$ and $1/2(\delta x^r{}_{;\ell;k} + \delta x^r{}_{;k;\ell})$ must vanish separately. This leads to the following set of identities

$$\Big(L_c^A F_{As}^{Br} \psi_B \Big)_{;r} + L_c^A \psi_{A;s} = 0 \,, \quad (3.11a)$$

$$S^\ell_{r;\ell} - 1/2\, R^\ell_{rsp} B_\ell^{ps} = 0 \,, \quad (3.11b)$$

$$S^{\ell}_{r} - B^{\ell k}_{r\ ;k} = 0 \ , \qquad (3.11c)$$

$$B^{\ell k}_{r} + B^{k\ell}_{r} = 0 \ . \qquad (3.11d)$$

where R^{ℓ}_{rsp} is the Riemann tensor which enters by way of the Ricci identity

$$(\delta x^{r}{}_{;\ell;k} - \delta x^{r}{}_{;k;\ell}) = - \delta x^{p} R^{r}_{p\ell k} \ . \qquad (3.12)$$

S^{ℓ}_{r}, u^{ℓ}_{r} and $B^{\ell k}_{r}$ are the tensors which replace T^{ℓ}_{r}, t^{ℓ}_{r} and $b_{r}^{\ell k}$ of (2.5). The identities (3.11) along with the field equations $L^{A}_{c} = 0$ represent a completely covariant field theory. All the quantities appearing in (3.11) are tensors and only covariant derivatives appear.

The existence of the completely covariant identities (3.8) is not due to the fact that $\psi_{A;r}$ was used in the Lagrangian density instead of $\psi_{A,r}$, but to the way the terms were collected together in (3.9). In (3.9) we have used covariant derivatives $\delta x^{r}{}_{;\ell}$ and $\delta x^{r}{}_{;\ell;k}$ whereas in the conventional formulation $\delta x^{r}{}_{,\ell}$ and $\delta x^{r}{}_{,\ell,k}$ were used. Even the theory of gravitation for which

$$g_{rs;\ell} = \psi_{A;\ell} = 0$$

can be written in a completely covariant form.

The two sets of identities (2.5) and (3.11) must be equivalent because they arise from the same variational principle. This equivalence may be verified directly by writing out (3.11) in terms of ordinary quantities and operations and noting that (2.5) follows.

IV. Covariant Formulation of General Relativity

In this section we write the identities corresponding to (3.8) for the Lagrangian of general relativity. Writing this Lagrangian in the form

$$\mathcal{L}_c = \sqrt{-g}\, L_c = \mathcal{L}_c(g_{\ell k}|g_{\ell k,r}|g_{\ell k,r,s}|\psi_A|\psi_{A;r}) = c_1 \sqrt{-g}\, R + \mathcal{L}_M \tag{4.1}$$

where $c_1 = \frac{c^4}{16\pi G}$; c is the speed of light and G is the Newtonian gravitational constant. In (4.1) R is the usual Lagrangian for the gravitational field $g_{\ell k}$ and $\mathcal{L}_M = \sqrt{-g}\, L_M$ is the Lagrangian for the source fields ψ_A. We shall assume that second derivatives of the metric tensor occur only in R since this is the case for all practical situations. Define

$$S^r_s = \frac{\partial L_c}{\partial \psi_{A;r}} \psi_{A;s} - \delta^r_s L_c + \frac{2}{\sqrt{-g}} E^r_{cs} - L^A_c F^{Br}_{As} \psi_B \tag{4.2a}$$

$$B^{kr}_s = -\frac{2g_{p\ell}}{\sqrt{-g}} \frac{d\mathcal{L}_M}{dg_{pk,r}} + \frac{\partial L_M}{\partial \psi_{A;r}} F^{Bk}_{A\ell} \psi_B \tag{4.2b}$$

$$C^{knr}_s = -\frac{2g_{s\ell}}{\sqrt{-g}} \frac{d\mathcal{L}_c}{dg_{\ell k,n,r}} , \tag{4.2c}$$

where

$$\frac{d\mathcal{L}_c}{dg_{\ell k\ldots}} = \frac{\partial \mathcal{L}_c}{\partial g_{\ell k\ldots}} + \frac{\partial \mathcal{L}_c}{\partial \psi_{A;r}} \frac{\partial \psi_{A;r}}{\partial g_{\ell k\ldots}} , \tag{4.3}$$

$$E^{rs}_c = \frac{d\mathcal{L}_c}{dg_{rs}} - \left(\frac{d\mathcal{L}_c}{dg_{rs,p}}\right)_{,p} + \left(\frac{d\mathcal{L}_c}{dg_{rs,p,q}}\right)_{,q,p} , \tag{4.4}$$

and

$$E^r_{cs} = g_{sk} E^{kr}_c . \tag{4.5}$$

One now proceeds in exactly the same way as when deriving (3.11) and finds the following identities for general relativity

$$\left(L^A_c F^{Br}_{As} \psi_B\right)_{;r} + L^A_c \psi_{A;s} + \left(\frac{2}{\sqrt{-g}} E^k_{cs}\right)_{;k} = 0 \tag{4.6a}$$

$$S^r_{s;r} - 1/2\, R^p_{skr} \left(B^{kr}_p + C^{krn}_p{}_{;n}\right) - 1/3 \left(R^p_{srk;n} + R^p_{snk;r}\right) C^{knr}_p = 0 \tag{4.6b}$$

$$S^r_s - B^{kr}_s{}_{;k} - 1/2\, R^p_{sn\ell} C^{rn\ell}_p - 1/2\, R^r_{kn\ell} C^{kn\ell}_s - 1/2\, R^p_{snk} C^{knr}_p - 1/2\, R^p_{snk} C^{knr}_p + 1/6\, R^r_{pnk} C^{knp}_s + 1/6\, R^r_{npk} C^{knp}_s = 0 \,, \tag{4.6c}$$

$$B^{kr}_s + B^{rk}_s + C^{krn}{}_{;n} + C^{rkn}{}_{;n} = 0 \,, \tag{4.6d}$$

$$C^{knr}_s + C^{krn}_s + C^{nkr}_s + C^{nrk}_s + C^{rnk}_s + C^{rkn}_s = 0 \,. \tag{4.6e}$$

Equations (4.6) along with the field equations

$$E^{rs}_c = 0$$
$$L^A_M = 0 \tag{4.7}$$

represents a completely covariant theory of gravitation. All quantities appearing in (4.6) and (4.7) are tensors and all derivatives are covariant. In order to reduce B^{kr}_s to the form (4.2b) one must use the following interesting identity

$$\frac{\partial\sqrt{-g}\,R}{\partial g_{\ell k,r}} - \left(\frac{\partial\sqrt{-g}\,R}{\partial g_{\ell k,r,n}}\right)_{,n} + \Gamma^{\ell}_{pn} \frac{\partial\sqrt{-g}\,R}{\partial g_{pk,n,r}} + \Gamma^{\ell}_{pn} \frac{\partial\sqrt{-g}\,R}{\partial g_{pk,n,r}} = 0 \,, \tag{4.8}$$

in the expression

$$B_s^{kr} = \frac{-2}{\sqrt{-g}} g_{s\ell} \left(\frac{d\mathcal{L}_c}{dg_{\ell k,r}} - \left(\frac{d\mathcal{L}_c}{dg_{\ell k,r,n}}\right)_{,n} + \Gamma^{\ell}_{pn} \frac{d\mathcal{L}_c}{dg_{pk,n,r}} + \Gamma^{\ell}_{pn} \frac{d\mathcal{L}_c}{dg_{kp,n,r}} \right) + \frac{\partial L_c}{\partial \psi_{A;r}} F^{Bk}_{As} \psi_B \qquad (4.9)$$

which appears in the varied action associated with (4.1).

We shall delay the interpretation of the various terms in (4.6) until a later publication.

References

1. For simplicity all fields in this paper will be assumed to be defined in a hyperbolic Riemannian space of signature (+ + + -), however, many of the results are valid in a general differentiable manifold. The capital Latin indices A, B, C... stand for a set of tensor indices while small Latin indices i, j, k, ℓ... run over the set (1,2,3,4). The summation convention is employed for both sets.

2. Bergmann, P., Phys. Rev. 75, 680 (1949).

3. Trautman, A., Gravitation: An Introduction to Current Research (A book edited by L. Witten, Wiley, New York, 1962). Many relevant references may be found in this article.

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In covariant field theories such as general relativity the application of Noether's theorem results in a set of identities which are satisfied regardless of whether the field equations are satisfied or not (so-called strong conservation laws). In the conventional formulation these identities contain non-tensorial quantities and non-tensorial operations. Thus, for example, the energy-momentum-pseudo-tensor of general relativity is obtained from such formulations. In this paper the identities arising from Noether's theorem are written in an obviously covariant form containing only tensors and covariant derivatives. As an example we give these identities for general relativity. (Author)

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