NASA TECH BRIEF

Equation Relates Flow at Free Jet to Flow Downstream

$$\dot{W}_{\ell}(t) = \frac{\dot{W}_o(t-\tau)}{1 - \dfrac{\bar{\tau}(\bar{\dot{W}})(\ddot{W}_o)(t-\tau)}{\left[\dot{W}_o(t-\tau)\right]^2}}$$

Where

$\ddot{W}_o$	=	injector flow acceleration
$\bar{\dot{W}}$	=	nominal weight flowrate
$\bar{\tau}$	=	nominal time delay
ℓ	=	distance from injector to measurement station
$\dot{W}_{\ell}(t)$	=	weight flowrate at station ℓ at time t
$\dot{W}_o(t-\tau)$	=	weight flowrate at orifice at time $t-\tau$

The problem:

To derive a mathematical expression to relate the flowrate at an orifice to that at a station downstream from the orifice. Current equations do not correlate well with actual data and are therefore of little use in injector design.

The solution:

A nonlinear equation which relates the flows involved for a free jet. This equation, which has been verified experimentally, should aid in understanding combustion instabilities and should not be subject to the substantial errors of prior analytical methods.

How it's done:

A detailed examination was made of the transport of fluid from an injector to a station downstream. A finite difference approach to a description of the phenomenon mathematically indicated the potential for a large amplification. In addition, it appeared that sinusoidal injector flow variations could be converted to a sharp-spiked (periodic pulse) waveform at stations downstream.

Subsequent testing with sinusoidally perturbed water jets verified the prediction. Although considerable spiking occurred with the amplification, the gain and phase correlated well with the analytical formulation.

The source of the amplification appears to involve the coherent phase deviation associated with velocity variations. It is proportional to the wave number ($W\ \tau$) for those in excess of one radian. High speed viewing with a strobe unit indicated that the amplification was caused by high speed drops overtaking slower ones, and therefore was due to the velocity gradient. As the peak-to-peak velocity variation was increased, the gain was decreased through nonlinear limiting. The limiting was primarily a result of fast

(continued overleaf)

and slow droplets combining to form a drop flowing at an intermediate velocity, which reduced the local velocity gradient and prevented further amplification.

Notes:

1. The equations are equally applicable to single or impinging jets.
2. This information may be useful in the understanding of the combustion process of sprayed fuel, particularly in the analysis of chug and buzz combustion instability problems where there is a well-defined flame front.
3. A mathematical derivation of the free-jet equation is available. Inquiries concerning this innovation may be directed to:

   Technology Utilization Officer
   Marshall Space Flight Center
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   Reference: B67-10612

Patent status:

No patent action is contemplated by NASA.

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