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HYDRODYNAMIC JOURNAL BEARING PROGRAM

QUARTERLY PROGRESS REPORT NO. 2

For Period : July 29, 1965 Thru October 29, 1965

By

J. D. McHUGH, H. E. NICHOLS,
W. D. C. RICHARDS, and H. C. LEE

prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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SPACE POWER AND PROPULSION SECTION
MISSILE AND SPACE DIVISION
GENERAL ELECTRIC
CINCINNATI, OHIO 45215

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Covering the Period
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J. D. McHugh and H. E. Nichols
W. D. C. Richards and H. C. Lee

Approved by
E. Schnetzer, Manager
Development Engineering

Prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Contract NAS 3-6479

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Technical Management
NASA - Lewis Research Center
Nuclear Power Technology Branch
Joseph P. Joyce, Technical Manager

RESEARCH AND DEVELOPMENT CENTER
SPACE POWER AND PROPULSION SECTION
MISSILE AND SPACE DIVISION
CINCINNATI, OHIO 45215

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

ABSTRACT

A computer program is presented which predicts rotor response and threshold of instability for a symmetrical rotor-bearing configuration and given bearing spring and damping coefficients. The program also calculates the bearing spring and damping coefficient from experimentally obtained fluid film forces and rotor displacements. These coefficients apply to bearings of similar geometry and are independent of the test rotor configuration.

The test rig for the experimental part of this program has been designed and is in manufacture. Also, bench testing of the Bently proximity probes for improved accuracy is under way.

SUMMARY

During the present Quarterly reporting period, work has progressed in the design and procurement of the new test rig components, in the check-out testing of Bently gages, and in rotor-bearing response analysis.

All detailed manufacturing drawings of the new test rig, instrumentation, and support structure have been completed, and parts are presently being manufactured. In addition, instrumentation has been assembled for check-out testing of Bently gages using the presently existing test rig, and this testing is underway.

To guide test planning and for the purpose of generalizing experimentally obtained bearing dynamic characterisitics, a rotor-response computer program has been written and checked-out. This program makes it possible to predict rotor-bearing response for arbitrary rotors if the bearing dynamic characteristics and rotor configuration are known. The experimental rotor response data can be used to obtain bearing dynamic characteristics through use of the program. The latter data will be general, applying to any rotor-bearing system having dynamically similar bearings to those tested. The computer program is completely described, including input and output data and program listing. Several examples are worked out demonstrating the use of the different program options.

The program schedule is shown in Figure 1.

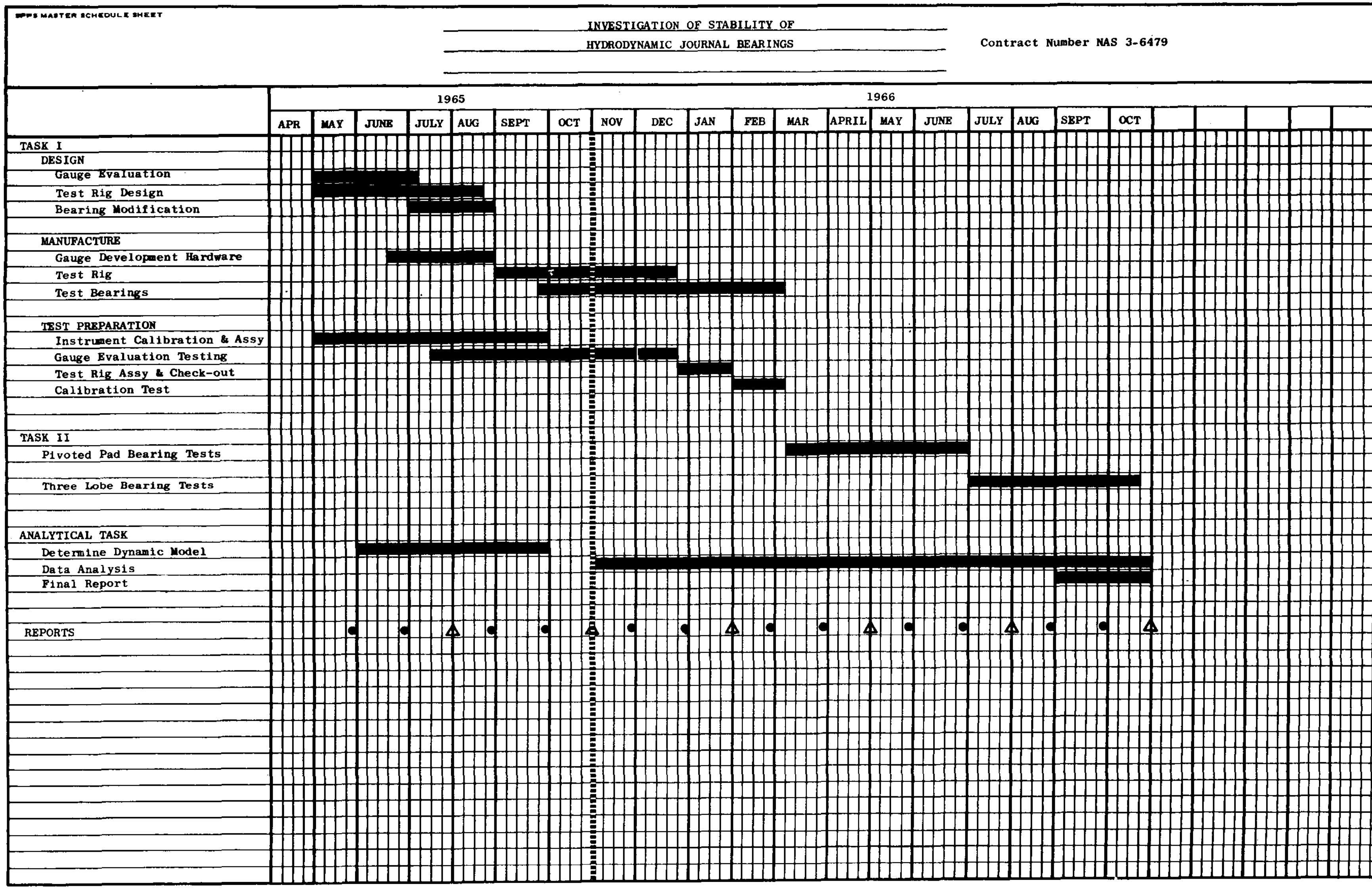


Figure 1. Program Schedule.

Forecast

During the next Quarterly reporting period, the gage evaluation testing will continue to completion, along with check-out and calibration of force gages and the determination of loader-bearing torque by experimental testing. All test hardware will be procured.

INTRODUCTION

The Space Power and Propulsion Section, in cooperation with the Research and Development Center of the General Electric Company, has been under contract since April 29, 1965 to the National Aeronautics and Space Administration for the design, fabrication, and testing of journal bearings which possess characteristics, e.g. stability under zero radial load, required for use in space power systems. Requirements include long term unattended operation under zero "g" conditions using low kinematic viscosity lubricants such as potassium at temperatures from 600°F to 1200°F.

The program represents a continuation of work carried out under contract NAS 3-211 (Reported in report NASA-CR-54039), and involves the testing and evaluation of two bearings, the four pivoted-pad and the three-lobe bearings, under conditions of angular and transverse linear misalignment, and non-rigid bearing supports. Bearing testing shall begin after the bearing test assembly, including instrumentation, has demonstrated the ability to obtain the required data with acceptable accuracy.

The program is primarily experimental, and is paralleled by analytical studies. These analytical investigations will compare the physical testing of bearing parameters with results based on theoretical assumptions. The goal of such experiments is to generalize the various bearing parameters thereby extending the usefulness of the results as design tools. The experimental tool of this program is a rotational speed test assembly comprised of a rotor and two test bearings which permits interchangeability of bearings and rotor. The lubricant will be distilled water, temperaturecontrolled to simulate the kinematic viscosity of potassium. The stability behavior of the rotating shaft will be measured with non-contacting Bently inductance gages.

The specific requirements of the system are:

1.	Shaft speed	3600 to 30,000 rpm.
2.	Inlet lubricant temperature	70 to 150°F
3.	Inlet lubricant supply pressure	0 to 150 psia
4.	Bearing linear misalignment	0 to 0.004 $\pm$ 0.0005 in.
5.	Bearing angular misalignment	0 to 400 $\pm$ 12 sec.
6.	Nominal bearing diameter	1.25 in.
7.	Bearing L/D ratio	1
8.	Diametral clearance	0.005 in.

The program will be performed in two tasks, the first of which will be the modification of the existing bearing test assembly and instrumentation and a demonstration of the ability to obtain accurate data. Task II will involve testing and analysis of the 4 pad pivot-pad and 3-lobed bearings. Data shall be presented in a way to permit application to bearings of similar design but of different dimensions.

The present report covers progress during the quarter ending October 29, 1965.

I. MECHANICAL DESIGN AND TESTING

During this quarterly reporting period, detailed manufacturing drawings of all test rig components, according to the configuration shown in Figure 2, have been completed, and parts manufacture is presently underway. This includes manufacture of the major test rig parts and fittings, proximity gage holder assemblies, test shaft, assembly tooling, and test rig support and environmental structure. The delivery of completed parts is scheduled for mid-December.

Test Rig Design and Procurement

The variable frequency motor and quill shaft arrangement to be employed on this test are the same as that which was purchased (from the Standard Electrical Tool Company, Cincinnati, Ohio) for use on the previous Bearing Stability Investigation Program (contract NAS 3-2111). The spindle of the rotor is hollow, through which is fitted a cylindrical quill shaft, and held concentric to the drive spindle by Teflon bushings. The quill shaft is attached to the drive spindle and the test shaft by use of locking collets. The quill shaft twist is sensed by electromagnetic pickups off two 18 tooth serrated disks, thereby indicating shaft torque during operating. Quill shafts of various diameters will be used for different ranges of torque.

The test shaft assembly comprises the shaft with an unbalance disk at each end. The test shaft is being manufactured from 420 stainless steel, through-hardened to a hardness of RC 50 to 53. A total of 6 different

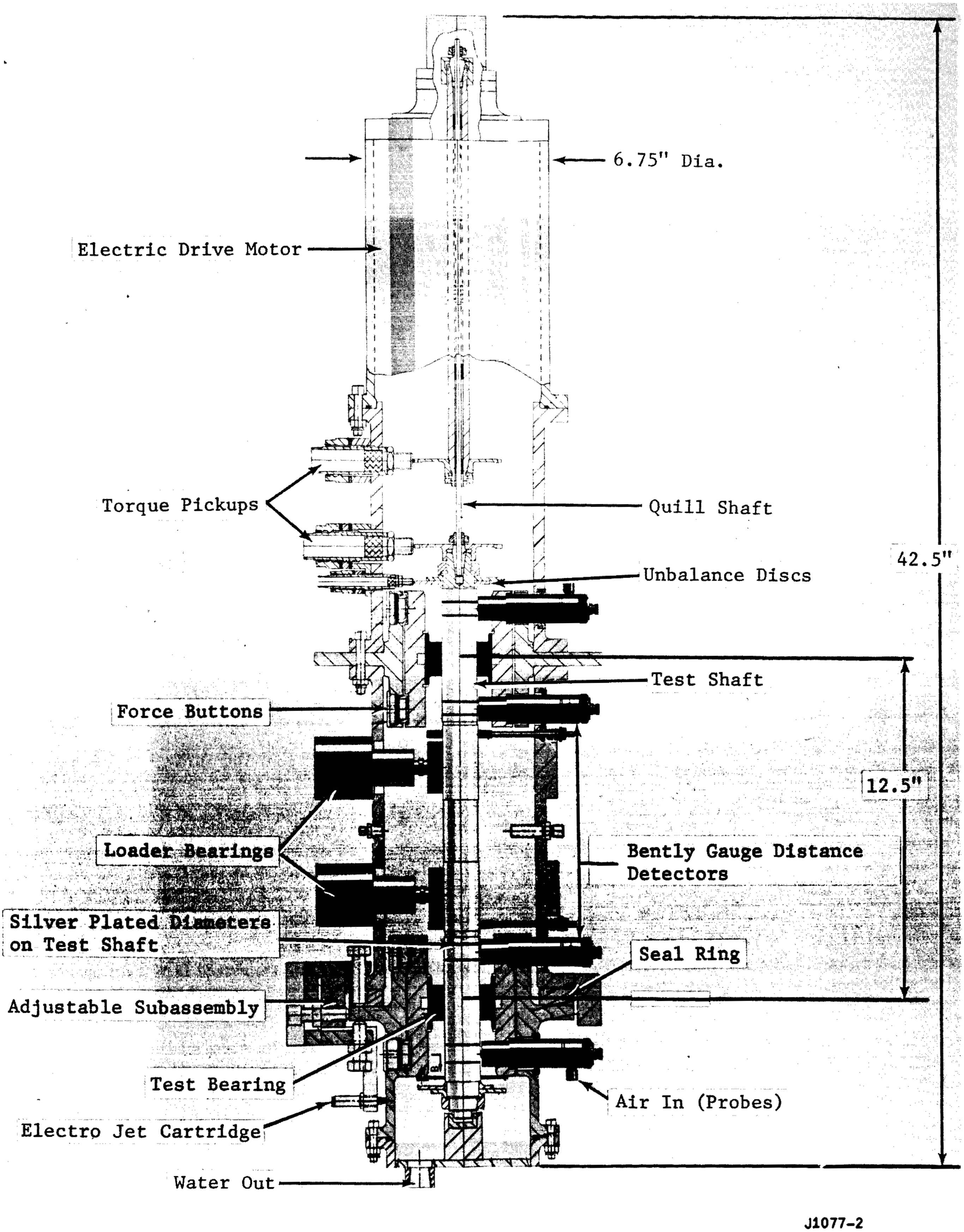
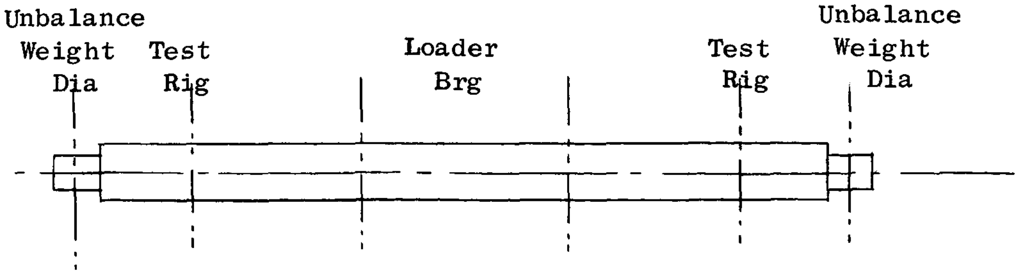


Figure 2. Bearing Stability Test Rig.

diameters are being machined round within 0.000050 inches and concentric within 0.0003 inches in the locations shown below.



During initial testing, the shaft will be balanced to a 0.01 gram-inch (or better) condition of residual unbalance will subsequently be established by inserting prescribed weights in the unbalance disks.*

A magnetic pickup will sense a notch in the upper unbalance disk, the signal of which will be fed to the Z-axis of the oscilloscope monitoring shaft orbit, thereby producing an intensified dot on the orbit. The position of this slot identifies the angular position of the out-of-balance load which can be compared to the angular position of the shaft minimum film thickness to give phase angle. When the force buttons are used, the attitude angle can be obtained.

The housing below the drive motor is the instrumentation section, which houses the electromagnetic torque pickups. This housing was used during the previous program, and has been modified to accommodate four new Bently gage holder assemblies. Large openings have been provided in the side of this housing to facilitate shaft-motor assembly and disassembly.

*Dynamic Balancer Model MU-6, Micro Balancing Inc., Farmingdale, N. Y.

The main test rig assembly, supporting the test bearings, shaft, loader bearings, and shaft position sensing gages is fabricated entirely of 316 stainless steel, and is mounted in the test rig support structure from its upper flange. Both test bearings are supported in the test rig as follows. The test bearing is mounted in a close-fitting sleeve (inner bearing housing) and is tightly secured against rotation or axial motion by a set-screw. This inner bearing housing is equipped with water lubricant feed ducting, and an annulus to distribute the lubricant around the bearing (Figure 2). Also, the housing provides for three internal thermocouples and a lubricant pressure tap. This bearing-sleeve assembly is in turn, supported in an outer housing(which is bolted to the casing) by eight"force button" assemblies, four assemblies in each of two planes. The force buttons comprise disks with strain gages attached to their back face which measure deflection of the disk, and thereby indicate force transmitted to the button from the bearing subassembly. The force buttons are initially loaded by a belleville spring and locknut assembly mounted in the outer housing as shown in Figure 3. The function of these gages has been described in detail in Quarterly Progress Report #1 [1]. As seen in Figure 3, the force button assemblies and Bently gage holder assemblies are space alternately at 45^{o} intervals and in four planes, as shown in Figure 2.

The major difference between upper and lower bearing assemblies is that the lower assembly is adjustable both angularly and transversely

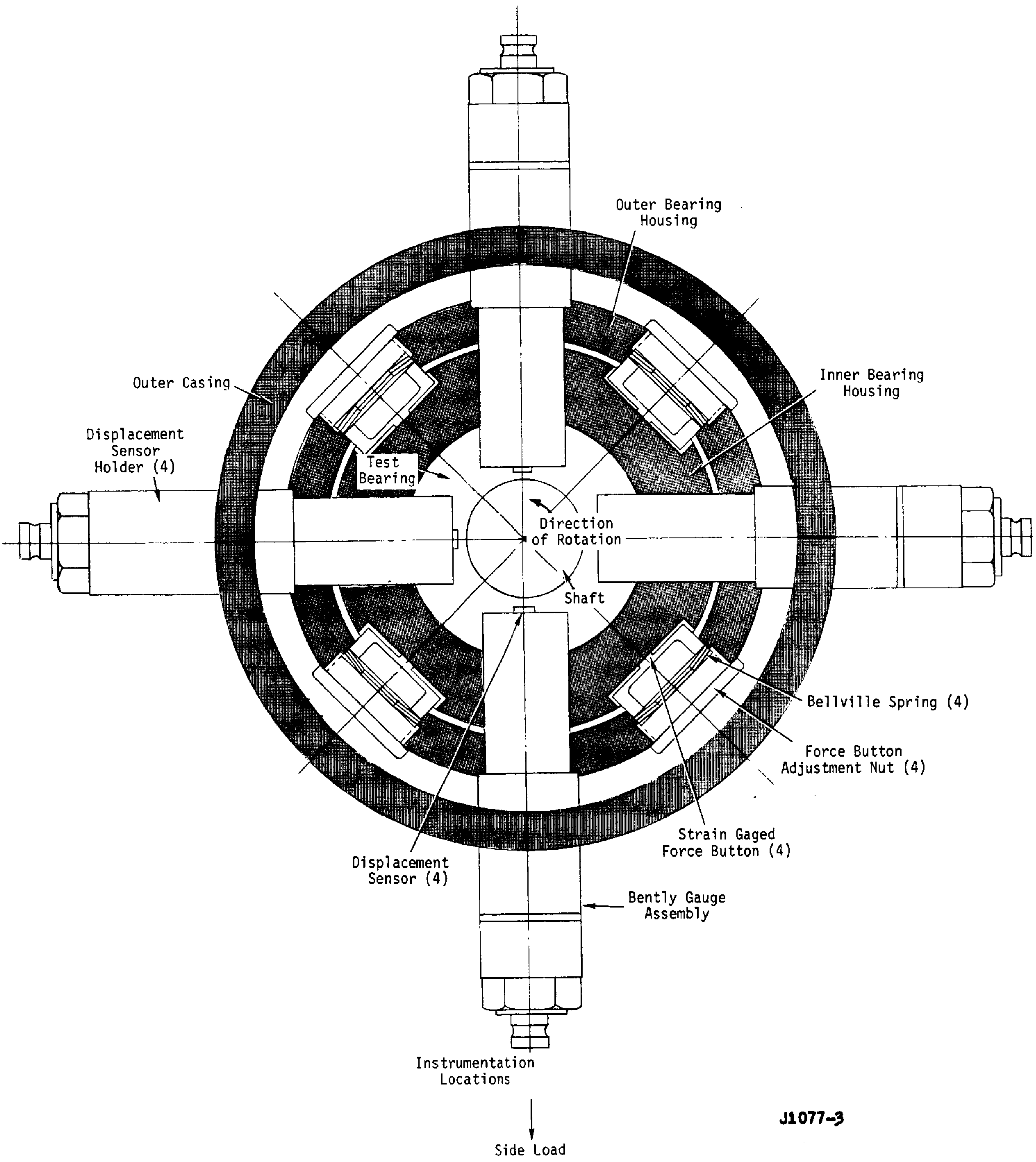


Figure 3. Force Button and Proximity Gauge Installation.

by adjustment of the several holding clamps shown in Figure 2. Four flats are machined on the O.D. of the lower housing to accommodate four contacting-type position gages*.

Several flow ports have been provided in the wall of the bearing housings inside the test rig to accommodate the flow and collection of water in the lower sump region of the test rig. The overall assembly is being manufactured with standard tolerances on all rabbet diameters ($\pm$ 0.001 inch) since the high precision alignment is obtained after assembly of the test rig. Side loads are imposed on the test shaft by use of two piston actuated partial-arc loader bearings which were used on the previous test program (NAS 3-2111).

In addition to the above hardware manufacture, a critical speed analysis is being performed to aid in establishing the detailed test plan (avoiding critical speed operating regions) employing several assumed constant values of bearing stiffness and force gage stiffness.

Testing Sub-Tasks

The major effort here continues in carrying out the various sub-tasks described in detail in Reference 1.

Investigation has continued on potential methods to reduce the sensitivity of the Bently gages to minute flaws or inhomogeneities in the stainless

*Electrojet Gage Cartridge - Model #59-230-113, Sheffield Corp, Dayton 1, Ohio.

steel test shaft. Various materials have been plated on the shaft surface in the vicinity of the Bently gages, with the result that a 0.005 inch thick silver plate has been selected for the final test shaft. The plating thickness is uniform to a deviation of less that 1% of nominal plating thickness. This testing of various platings is being done in a bench set-up using an existing shaft from the previous Bearing Stability Program.

Bently gages are being calibrated against the silver plated shaft. These gages will be used in a push-pull or opposed arrangement such that symmetrical effects, such as those due to uniform temperature expansion or centrifugal growth of the test shaft, will be cancelled. Increased sensitivity is also obtained without further signal amplification. A simple form of this push-pull arrangement is shown schematically in Figure 4.

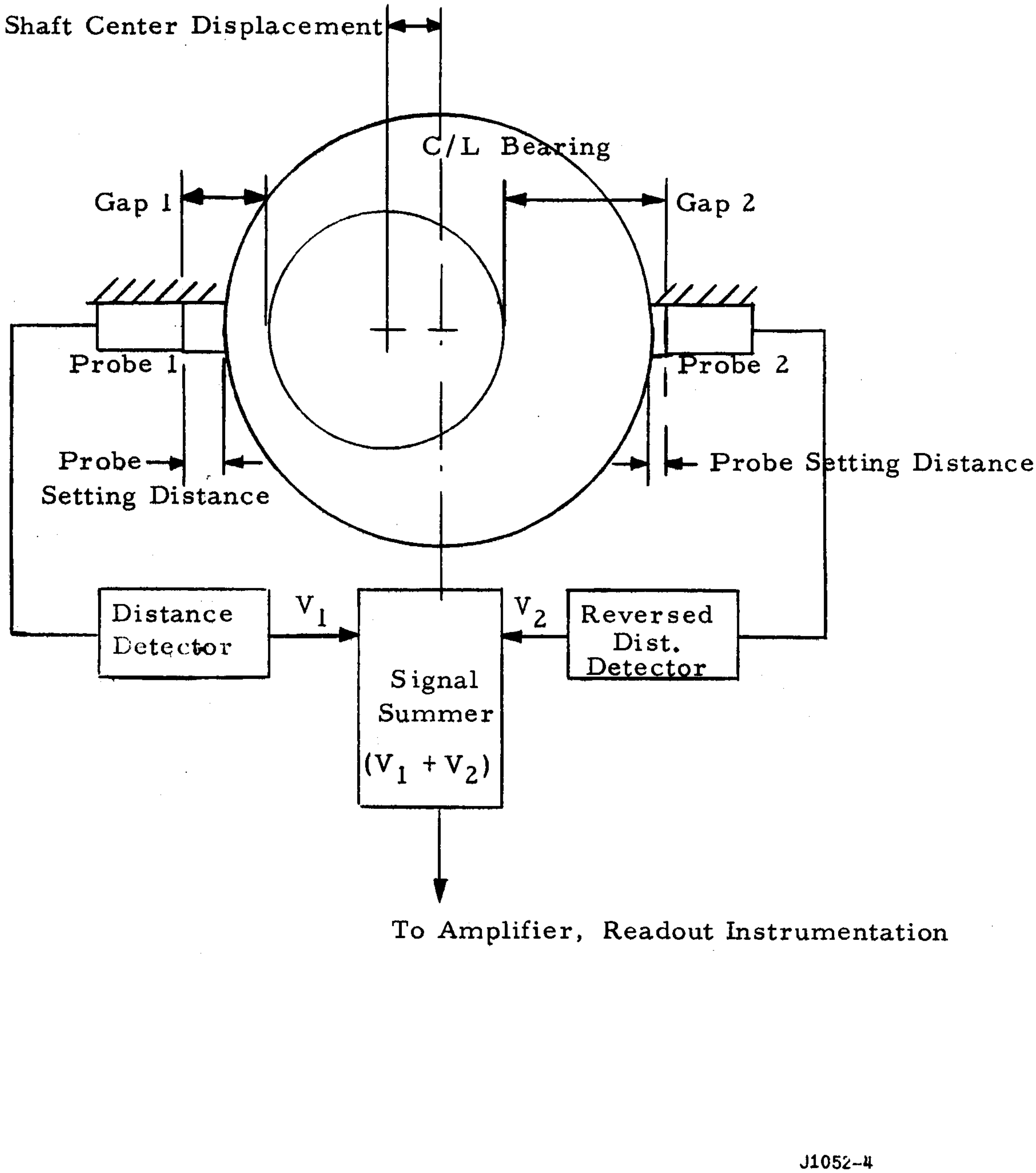


Figure 4. Schematic Diagram - Bently Displacement Probes in Simple Push-Pull Arrangement.

II. ANALYSIS OF ROTOR - BEARING RESPONSE

The present contract has as its objective to provide experimental data leading to the proper selection and sizing of bearing type for application to Rankine cycle power systems for space applications using liquid potassium as the lubricant. The data must be generalized so as to apply to dynamically similar bearings of different dimensions. Besides providing eccentricity ratio and non-dimensional torque variation with Sommerfeld and Reynolds numbers, which comes directly from the measurements, non-dimensional bearing dynamic characteristics and fractional frequency whirl (stability) thresholds must be provided. In planning tests predictions of expected rotor response and the stability threshold are required. In reduction of the test data means for generalizing the less direct information, namely, the dynamic characteristic of a given bearing, must be established. A computer program has been written and checked out which accomplishes these two tasks. The bearing-rotor response analysis which follows is the basis for this computer program.

Shown in Figure 5 is a sketch indicating the method of representing a fluid film bearing by an eight-parameter linear model. This model is widely used in bearing literature, e.g., reference 2.

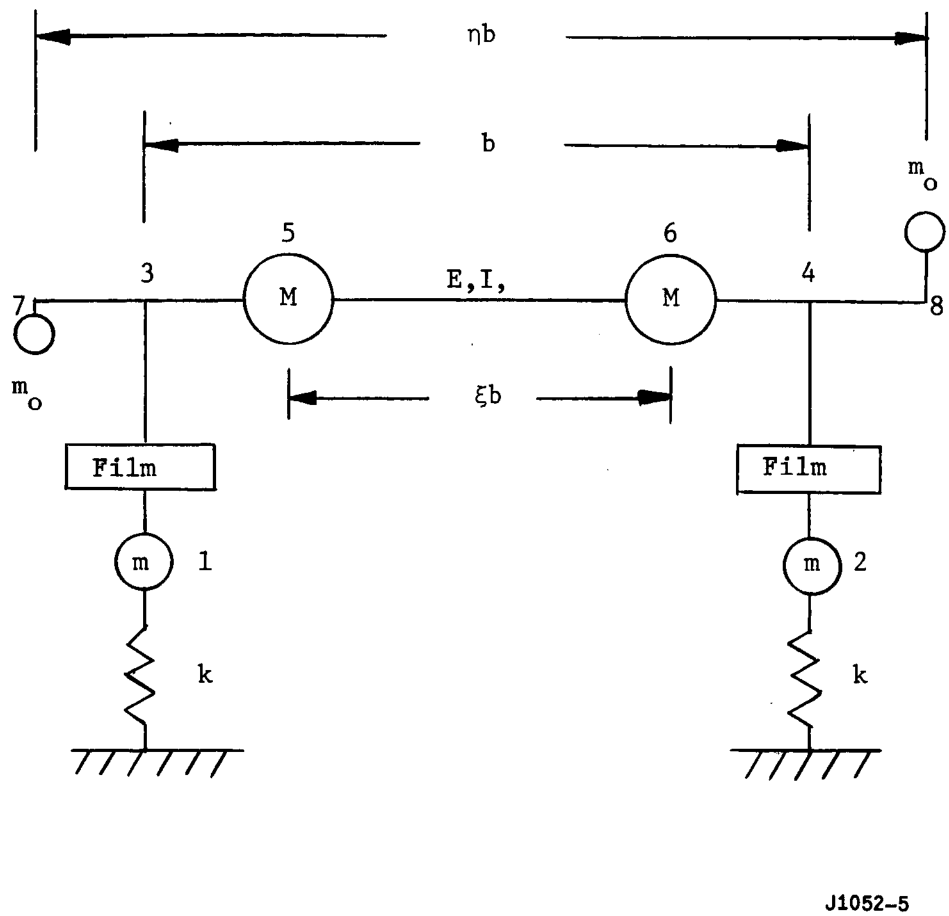


Figure 5. System Model.

This eight parameter model (of bearing film) is characterized by the following equations:

$$-F_x = K_{xx}x + C_{xx}\dot{x} + K_{xy}y + C_{xy}\dot{y}$$

$$-F_y = K_{yx}x + C_{yx}\dot{x} + K_{yy}y + C_{yy}\dot{y} \qquad (1)$$

All symbols are defined in Appendix A.

where F's are dynamic film forces and (x,y) are journal displacements (with respect to bearings) from an initial steady-state position produced by a steady, unidirectional load. The constants K_{xx}, C_{xx}, etc. are obtained from the first order Taylor expansion of the bearing force change with respect to displacement and velocity. The more realistic values may be obtained by an experiment using the equations given in the following section. When the bearing constants K_{xx}, C_{xx}, etc. are known for a given set of hydrodynamic journal bearings, one proceeds for the rotor dynamic analysis as discussed in the reference (3) to (5). In this report, the method similar to Ref. (5) will be used. For the calculation or instability conditions, a symmetric rotor with two masses and flexible pedestals are used. In the unbalance response calculations, one may wish to place the unbalances at positions different from the rotor masses. Hence, two additional masses as unbalances are to be symmetrically placed somewhere along the rotor, making the total number of masses to be four. The unbalance masses may have different eccentricities at different directions. If more than two unbalances are desired, one should super-impose two calculations each with one or two unbalances. The pedestals are assumed to be flexible in these analyses. The computer code is described in appendix B and examples of its use are given in appendices C and D.

BEARING CONSTANTS

When equations (1) are assumed to describe the bearing characteristics, the steady-state response of the rotor bearing system is of harmonic nature, and, therefore, one may write:

$$x = x_e \cos \omega t + x_s \sin \omega t \qquad y = y_c \cos \omega t + y_s \sin \omega t$$

$$F_x = F_{xc} \cos \omega t + F_{xs} \sin \omega t \qquad F_y = F_{yc} \cos \omega t + F_{ys} \sin \omega t$$

The more compact and convenient form is:

$$x = Xe^{-i\omega t},\ y = Ye^{-i\omega t},\ F_x = F_X e^{-i\omega t},\ F_y = F_Y e^{-i\omega t}$$

where $X = x_c + i\, x_s,\ Y = y_c + i\, y_s,\ F_X = F_{xc} + i\, F_{xs},\ F_Y = F_{yc} + i\, F_{ys}$

The quantity ω is the steady state load frequency.

Then in dimensionless terms

$$-\bar{F}_x = (\bar{K}_{xx} - i\, \bar{C}_{xx})\, \bar{X} + (\bar{K}_{xy} - i\, \bar{C}_{xy})\, \bar{Y} = \bar{A}\bar{X} + \bar{B}\bar{Y}$$

$$-F_Y = (\bar{K}_{yx} - i\, \bar{C}_{yx})\, \bar{X} + (\bar{K}_{yy} - i\, \bar{C}_{yy})\, \bar{Y} = \bar{C}\bar{X} + \bar{D}\bar{Y}$$

where $\bar{A} = \bar{K}_{xx} - i\, \bar{C}_{xx} \qquad \bar{B} = \bar{K}_{xy} - i\, \bar{C}xy$ etc.

$$\bar{F}_{x,y} = \frac{1}{W} F_{x,y}, \qquad \bar{K}_{xx} = \frac{C}{W} K_{xx}, \qquad \bar{C}_{xx} = \frac{C}{W}\, \omega\, C_{xx} \text{ etc.}$$

The factors C and W have dimensions, length and force respectively. A common practice is to use the bearing clearance for C and the bearing load for W.

Since the model has eight parameters, and the measurements of displacements and forces in x - y directions give only four quantities, two independent sets are required to determine the eight parameters. The set obtained with symmetric unbalances and the set with anti-symmetric will be independent. Or, with non-symmetric loads, two bearings will give results independent to each other. If these independent sets are denoted by $(\bar{X}_1, \bar{Y}_1, \bar{F}_{x1}, \bar{F}_{y1})$ and $(\bar{X}_2, \bar{Y}_2, \bar{F}_{x2}, \bar{F}_{y2})$ one has

$$\begin{aligned} \bar{A}\bar{X}_1 + \bar{B}\bar{Y}_1 &= -\bar{F}_{x1} \\ \bar{A}\bar{X}_2 + \bar{B}\bar{Y}_2 &= -\bar{F}_{x2} \\ C\bar{X}_1 + \bar{D}\bar{Y}_1 &= -\bar{F}_{y1} \\ \bar{C}\bar{X}_2 + \bar{D}\bar{Y}_2 &= -\bar{F}_{y2} \end{aligned} \qquad (2)$$

One determines $\bar{A}$, $\bar{B}$, etc. from Equations (2) and then $\bar{K}_{xx}$, $\bar{C}_{xx}$ etc. are obtained.

It can be shown mathematically that equations (2) result in an indeterminant form for the special case when the orbit is circular. Thus, for four pivoted pad bearings with negligible pad mass and loaded symmetrically between pivots a separate mathematical analysis must be prepared. The measurement of the forces may be eliminated because the solution is defined if the dynamic behavior of the rotor alone is known along with the vectorial displacements at the bearings. The usefulness of results depends upon how closely the assumed rotor dynamic characteristics are represented in the computer program. In this program the rotor with a distributed mass is approximated by a four-mass symmetric rotor.

DYNAMIC ANALYSIS

Figure 5 shows the rotor-bearing-pedestal system. A flexible shaft is supported on fluid-film bearings possessing stiffness and damping. The bearing housings also possess mass and elastic support. The symmetric rotor has two concentrated masses, each with one-half the total mass and at a distance such that the moment of mass inertia about the mass center is equal to the transverse moment of inertia of the rotor, that is, (see the remark in Example 4)

$$(1/2)\ M\ \xi^2 b^2 = I_T$$

At the equal distances from the mass center, external forces are applied. The magnitude and direction may be different. The equations of motion are obtained by using influence coefficients to express relative displacements in terms of inertia forces and external forces. For instance, from Figure 6, one can write for a simply supported beam,

$$\bar{\delta}_5 = \alpha_{55}\ \bar{P}_5 + \alpha_{56}\ \bar{P}_6 + \alpha_{57}\ \bar{P}_7 + \alpha_{58}\ \bar{P}_8$$

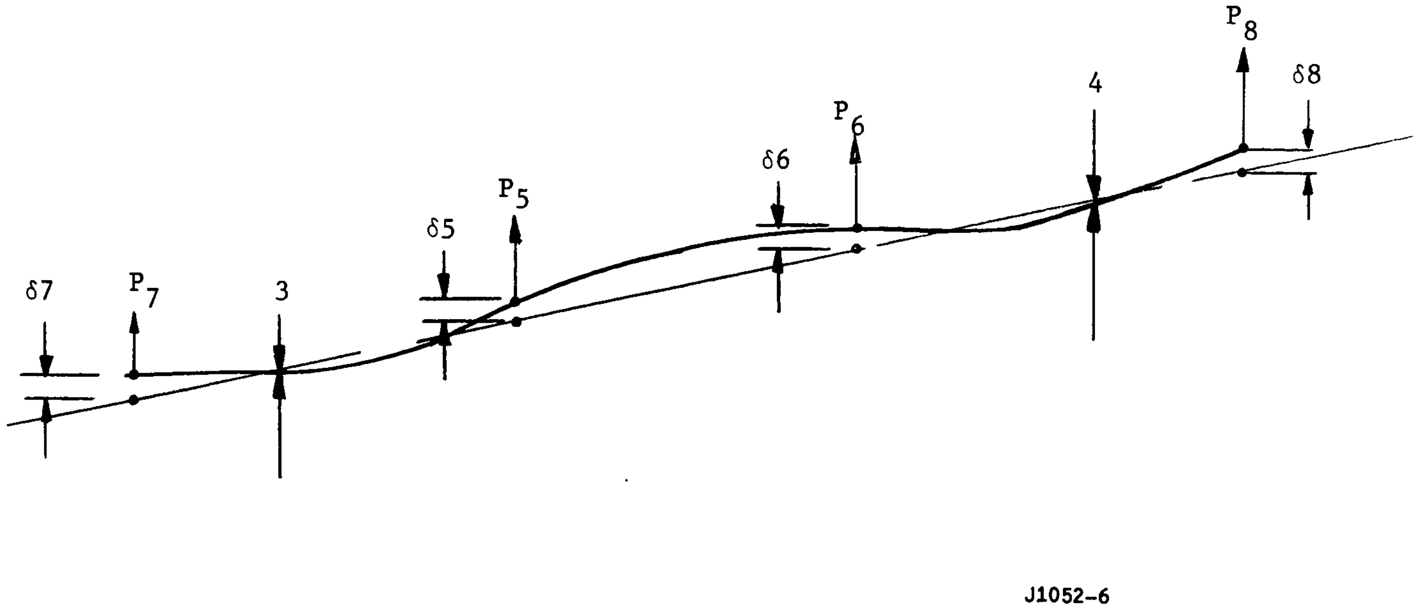


Figure 6. Relative Displacements of Rotor.

Similarly the expressions for $\bar{\delta}_6$, $\bar{\delta}_7$ and $\bar{\delta}_7$ are obtained. Getting $\bar{\delta}$'s in terms absolute displacements $\bar{R}$'s and replacing $\bar{P}$'s by inertia forces and external forces, one obtains,

$$
\begin{aligned}
&\bar{R}_5 - \bar{R}_3 - (1/2)(1-\xi)(\bar{R}_4-\bar{R}_3) = -\alpha_{55}M\ddot{\bar{R}}_5 - \alpha_{56}M\ddot{\bar{R}}_6 + \alpha_{57}\bar{Q}_1 + \alpha_{58}\bar{Q}_2 \\
&\bar{R}_6 - \bar{R}_3 - (1/2)(1+\xi)(\bar{R}_4-\bar{R}_3) = -\alpha_{65}M\ddot{\bar{R}}_5 - \alpha_{66}M\ddot{\bar{R}}_6 + \alpha_{67}\bar{Q}_1 + \alpha_{68}\bar{Q}_2 \\
&\frac{b}{2}(\eta-1)\bar{Q}_1 + \frac{b}{2}(1-\xi)\, M\ddot{\bar{R}}_5 + \frac{b}{2}(1+\xi)\, M\ddot{\bar{R}}_6 - b\bar{F}_2 - \frac{b}{2}(1+\eta)\bar{Q}_2 = 0 \\
&\frac{b}{2}(\eta-1)\bar{Q}_2 + \frac{b}{2}(1-\xi)\, M\ddot{\bar{R}}_6 + \frac{b}{2}(1+\xi)\, M\ddot{\bar{R}}_5 - b\bar{F}_1 - \frac{b}{2}(1+\eta) = 0 \\
&m\ddot{\bar{R}}_1 + k\bar{R}_1 - \bar{F}_1 = 0 \\
&m\ddot{\bar{R}}_1 + k\bar{R}_2 - \bar{F}_2 = 0
\end{aligned}
\tag{3A}
$$

where: $\bar{R}_j = x_j\bar{i} + y_j\bar{j} \qquad j = 1, 2, \ldots\ldots\ldots 6$

$\bar{F}_r = \bar{i}\, F_{xr} + \bar{j}\, F_{yr} \qquad r = 1, 2$

The vectors $\bar{i}$ and $\bar{j}$ are unit vectors along x and y axis. The influence coefficients α_{ab} represents deflection at a due to unit force at b. The external forces are denoted by Q's.

The film forces F's are:

$$
\begin{aligned}
-F_{x1} &= K_{xx}\,(x_3-x_1) + C_{xx}\,(\dot{x}_3-\dot{x}_1) + K_{xy}\,(y_3-y_1) + C_{xy}\,(\dot{y}_3-\dot{y}_1) \\
-F_{x2} &= K_{xx}\,(x_4-x_2) + C_{xx}\,(\dot{x}_4-\dot{x}_2) + K_{xy}\,(y_4-y_2) + C_{xy}\,(\dot{y}_4-\dot{y}_2)
\end{aligned}
\tag{3B}
$$

$$-F_{y1} = K_{yx}\ (x_3-x_1) + C_{yx}\ (\dot{x}_3-\dot{x}_1) + K_{yy}\ (y_3-y_1) + C_{yy}\ (\dot{y}_3-\dot{y}_1)$$

$$-F_{y2} = K_{yx}\ (x_4-x_2) + C_{yx}\ (\dot{x}_4-\dot{x}_2) + K_{yy}\ (y_4-y_2) + C_{yy}\ (\dot{y}_4-\dot{y}_2)$$

The subscripts are station numbers as shown in Figure 5. The use of symmetry reduced Equations 3A and 3B to:

$$\alpha_1 M\ddot{\overline{R}}_o + \overline{R}_o - \overline{R}_s = \alpha_2 \overline{Q}_R$$

$$M\ddot{\overline{R}}_o - \overline{F}_R = \overline{Q}_R \qquad (4A)$$

$$m\ddot{\overline{R}}_b + k\overline{R}_b + \overline{F}_R = 0$$

$$\alpha_3 M\ddot{\overline{W}}_o + \overline{W}_o - \xi\overline{W}_s = \alpha_4 \overline{Q}_W$$

$$\xi M\ddot{\overline{W}}_o + \overline{F}_W = \eta\ \overline{Q}_w \qquad (4B)$$

$$m\ddot{\overline{W}}_b + k\overline{W}_b + \overline{F}_w = 0$$

Where: $\alpha_1 = \alpha_{55} + \alpha_{56}$, $\alpha_2 = \alpha_{57} + \alpha_{58}$, $\alpha_3 = \alpha_{55} - \alpha_{56}$, $\alpha_4 = \alpha_{57} - \alpha_{58}$

$\overline{R}_o = \overline{i}\ (x_5+x_6) + \overline{j}(y_5+y_6)$ $\qquad$ $\overline{R}_s = \overline{i}(x_3+x_4) + \overline{j}(y_3+y_4)$

$\overline{R}_b = \overline{i}\ (x_1+x_2) + \overline{j}(y_1+y_2)$ $\qquad$ $\overline{W}_o = \overline{i}(x_6-x_5) + \overline{j}(y_6-y_5)$

$\bar{W}_s = \bar{i}\ (x_4 - x_3) + \bar{j}(y_4 - y_3)$ $\qquad$ $\bar{W}_b = \bar{i}\ (x_2 - x_1) + \bar{j}(y_2 - y_1)$

$\bar{Q}_R = \bar{i}\ (Q_{x1} + Q_{x2}) + \bar{j}(Q_{x1} + Q_{x2})$ $\qquad$ $\bar{Q}_w = \bar{i}(Q_{x2} - Q_{x1}) + \bar{j}(Q_{y2} - Q_{y1})$

$\bar{F}_R = \bar{i}(F_{x1} + F_{x2}) + \bar{j}\ (F_{y1} + F_{y2})$ $\qquad$ $\bar{F}_w = \bar{i}\ (F_{x2} - F_{x1}) + \bar{j}\ (F_{y2} - F_{y1})$

Since equations (4A) can be obtained from (4B) by replacing α_3, α_4, ξ, η, and $\bar{Q}_w$ by α_1, α_2, 1, 1, and $\bar{Q}_R$ respectively, one only needs to solve equations (4B).

Stability Analysis

The condition at threshold of instability can be found by taking homogeneous equations of (4B) and letting $\bar{R}(t) = \bar{R}e^{-i\nu t}$. The elimination of $\bar{W}_o$ and $\bar{W}_b$, and the vanishing determinant of scaler equations gives (in dimensionless quantities)

$$\bar{\mathcal{R}} = \frac{\bar{K}_{xx}\bar{C}_{yy} + \bar{C}_{xx}\bar{K}_{yy} - \bar{C}_{xy}\bar{K}_{yx} - \bar{C}_{yx}\bar{K}_{xy}}{\bar{C}_{xx} + \bar{C}_{yy}} \tag{5A}$$

$$\gamma^2 = \frac{(\bar{K}_{xx} - \bar{\mathcal{R}})\ (\bar{K}_{yy} - \bar{\mathcal{R}}) - \bar{K}_{xy}\bar{K}_{yx}}{\bar{C}_{xx}\bar{C}_{yy} - \bar{C}_{yx}\bar{C}_{xy}} \tag{5B}$$

$$\bar{\mathcal{R}} = \frac{\bar{\mathcal{R}}_s}{1 - \bar{\mathcal{R}}_b \bar{\mathcal{R}}_s} \tag{5C}$$

$$\bar{\mathcal{R}}_b = \frac{1/\bar{k}}{1 - \gamma^2 S^2 Sn^2} \tag{5D}$$

$$\bar{\mathcal{R}}_s = \frac{\gamma^2 S^2 \xi^2 / \bar{\alpha}_3}{1 - \gamma^2 S^2} \tag{5E}$$

Where: $\gamma = \dfrac{\nu}{\omega}$, $S = \dfrac{\omega}{\omega_{ns}}$, $\omega_{np}^{\ 2} = \dfrac{k}{m}$, $\omega_{ns}^{2} = \dfrac{1}{\alpha_3 M}$, $S_n^2 = \dfrac{k\alpha_3 M}{m}$, $\bar{\alpha}_3 = \dfrac{W}{C\alpha_3}$,

$\bar{k} = \dfrac{C}{W}k$, $\bar{K}_{xx} = \dfrac{C}{W} K_{xx}$, $\bar{C}_{xx} = \dfrac{C}{W}\ \omega C_{xx}$ $\qquad$ etc.

The quantity ω is the rotating speed at the threshold of instability. Equations (5A-B) give values of $\bar{\mathcal{R}}$ and γ and then (5C-E) determines the value of S at the threshold of instability.

Response Calculations

If the external forces, $\bar{Q}_1$ (t) and $\bar{Q}_2$ (t), are unbalance forces, one may write

$$\bar{Q}_1 (t) = -m_o \ddot{\bar{R}}_7 + \bar{q}_7 (t)$$
$$\bar{Q}_2 (t) = -m_o \ddot{\bar{R}}_8 + \bar{q}_8 (t) \tag{6}$$

Where m_o is unbalance mass, $\bar{R}_7 = x_7\bar{i} + y_7\bar{j}$, $\bar{R}_8 = x_8\bar{i} + y_8\bar{j}$, and denoting the unbalance eccentricities by δ_o and δ_1

$$\bar{q}_7 (t) = m_o\delta_o\omega^2 (\bar{i} \cos \omega t + \bar{j} \sin \omega t)$$
$$\bar{q}_8 (t) = m_o\delta_1\omega^2 \left[\bar{i} \cos (\omega t-\psi) + \bar{j} \sin (\omega t-\psi)\right] \tag{7}$$

The angle ψ is the phase lag of $\bar{q}_8$ from $\bar{q}_7$. Here it is noted that

Total rotor mass = $2(M + m_o)$

$$I_T = (1/2)\ b^2\ (M\ \xi^2 + m_o\eta^2)$$

In addition to equations (3), one has

$$\bar{R}_7 - \bar{R}_3 - 1/2(1-\eta)(\bar{R}_4-\bar{R}_3) = -\alpha_{75}M\ddot{\bar{R}}_5 - \alpha_{76}M\ddot{\bar{R}}_6 + \alpha_{77}\bar{Q}_1 + \alpha_{78}\bar{Q}_2$$
$$\bar{R}_8 - \bar{R}_3 - 1/2(1+\eta)(\bar{R}_4-\bar{R}_3) = -\alpha_{85}\ M\ddot{\bar{R}}_5 - \alpha_{86}M\ddot{\bar{R}}_6 + \alpha_{87}\bar{Q}_1 + \alpha_{88}\bar{Q}_2 \tag{8}$$

Noting the symmetry, $\alpha_{75} = \alpha_{57}$ etc., one obtains

$$\alpha_2 M\ddot{\bar{R}}_o + \bar{R}_q - \bar{R}_s = \alpha_5\bar{Q}_R \tag{9A}$$

$$\alpha_4 M\ddot{\bar{W}}_o + \bar{W}_q - \eta W_s = \alpha_6\bar{Q}_W \tag{9B}$$

Where $\alpha_5 = \alpha_{77} + \alpha_{78}$

$\alpha_6 = \alpha_{77} - \alpha_{78}$

$$\bar{R}_q = (x_7+x_8)\ \bar{i} + (y_7+y_8)\ \bar{j}$$

$$\bar{W}_q = (x_8-x_7)\ \bar{i} + (y_8-y_7)\ \bar{j}$$

Again, equation (9B) reduces to equation (9A) when α_4, α_6, $\bar{Q}_W$ and η are replaced by α_2, α_5, $\bar{Q}_R$ and 1 respectively. The substitution of equations (6) into equations (4B) and (9B) gives

$$\alpha_3 M\ddot{\bar{W}}_o + \alpha_4 m_o \ddot{\bar{W}}_q + W_o - \xi \bar{W}_s = \alpha_4 \bar{q}_w$$

$$\alpha_4 M\bar{W}_o + \alpha_6 \dot{m}_o \ddot{\bar{W}}_q + \bar{W}_q - \eta \bar{W}_s = \alpha_6 \bar{q}_w \tag{10B}$$

$$\xi M\ddot{\bar{W}}_o' + \eta m_o \ddot{\bar{W}}_q - \bar{F}_w = \eta \bar{q}_w$$

$$m\ddot{\bar{W}}_b + k\bar{W}_b + \bar{F}_w = 0$$

Where: $\bar{q}_w = \bar{q}_8 - \bar{q}_7$

Similarly four more equations are obtained from equations (4A) and (9A) with $\bar{q}_R = \bar{q}_7 + \bar{q}_8$.

The unbalance forces may be expressed as real parts of:

$$\bar{q}_R/W = (\bar{Q}_x \bar{i} + \bar{Q}_y \bar{j})\ e^{-i\omega t}$$

$$\bar{q}_W/W = (\bar{Q}_u \bar{i} + \bar{Q}_v \bar{j})\ e^{-i\omega t} \tag{11}$$

Where:

$$\bar{Q}_x = (m_o \omega^2/W) \quad [(\delta_o + \delta_1) \cos \psi + i\ \delta_1 \cos \psi]$$

$$\bar{Q}_y = i\ \bar{Q}_x$$

$$\bar{Q}_u = (m_o \omega^2/W) \quad [\delta_o \cos \psi - \delta_1 + i\ \delta_1 \sin \psi]$$

$$\bar{Q}_v = i\ \bar{Q}_u$$

Therefore, the solutions take the form,

$$\bar{R}\ (t) = \bar{\bar{R}}\ e^{-i\omega t}$$

$$\bar{W}\ (t) = \bar{\bar{W}}\ e^{-i\omega t} \tag{12}$$

Where

$$\bar{\bar{R}} = X\bar{i} + Y\bar{j}$$

$$\bar{\bar{W}} = U\bar{i} + V\bar{j}$$

$$X = x_c + x_s$$

$$Y = y_c + i\ y_s$$

$$U = u_c + i\ u_s$$

$$V = v_c + i\ v_s$$

The substitutions of equations (11) and (12) into equations (10B) and the elimination process gives, in complex terms,

$$\begin{bmatrix} (\bar{A} - \bar{\Re}) & \bar{B} \\ \bar{C} & (\bar{D} - \bar{\Re}) \end{bmatrix} \begin{bmatrix} (U_s - U_b) \\ (V_s - V_b) \end{bmatrix} = \left(\frac{G_4 \bar{\Re}}{G_1}\right) \begin{bmatrix} \bar{Q}_u \\ \bar{Q}_v \end{bmatrix} \tag{13}$$

and therefore

$$\begin{aligned} \frac{1}{C} (U_s - U_b) &= \frac{\Delta_1}{\Delta} \left[(\bar{D}-\bar{\Re})\, \bar{Q}_u - \bar{B}\, Q_v \right] \\ \frac{1}{C} (V_s - V_b) &= \frac{\Delta_1}{\Delta} \left[\bar{C}\, \bar{Q}_u - (A-\bar{\Re})\, \bar{Q}_v \right] \end{aligned} \tag{13A}$$

$$\begin{aligned} U_s/C &= \frac{R_1 R_4 G_2}{G_3} (U_s - U_b)/C + \bar{Q}_u\, G_4/G_3 \\ V_s/C &= \frac{R_1 R_4 G_2}{G_3} (V_s - V_b)/C + \bar{Q}_v\, G_4/G_3 \end{aligned} \tag{14}$$

$$\begin{aligned} U_q/C &= \frac{1}{R_2} \left[(\xi - \alpha_4 \eta/\alpha_6)\, \bar{\alpha}_6 \bar{Q}_u + \eta\xi\, U_s/C - \bar{\alpha}_4 \bar{F}_u \right] \\ V_q/C &= \frac{1}{R_2} \left[(\xi - \alpha_4 \eta/\alpha_6)\, \bar{\alpha}_6 \bar{Q}_v + \eta\xi\, V_s/C - \bar{\alpha}_4 \bar{F}_v \right] \end{aligned} \tag{15}$$

$$\begin{aligned} U_o/C &= - \frac{\bar{\alpha}_3}{\xi\, S_n^2} \left[\eta \bar{Q}_v + \bar{F}_v + S_m^2\, \eta V_q/(C\, \bar{\alpha}_6) \right] \\ V_o/C &= - \frac{\bar{\alpha}_3}{\xi\, S_n^2} \left[\eta\, \bar{Q}_v + \bar{F}_v + S_m^2\, \eta\, V_q/(C\bar{\alpha}_6) \right] \end{aligned} \tag{16}$$

Where

$$\Delta = (\bar{A}-\bar{\Re})\ (\bar{D}-\bar{\Re}) - \bar{B}\ \bar{C}$$

$$\bar{\Re} = G_1 G_2/G_3$$

$$\Delta_1 = G_2 G_4/G_3$$

$$G_1 = R_1R_5 + R_2R_3$$

$$G_2 = \bar{k}\ (1-S_p^2)$$

$$G_3 = -G_1 + R_1R_4G_2$$

$$G_4 = \eta R_1 + R_3\bar{\alpha}_4$$

$$R_1 = 1 - S_n^2 - S_m^2[(1-S_n^2) + S_n^2\ \bar{\alpha}_4^2/(\bar{\alpha}_3\bar{\alpha}_6)]$$

$$R_2 = \xi - S_m^2\ (\eta\bar{\alpha}_4/\bar{\alpha}_6 - \xi)$$

$$R_3 = \xi\ S_n^2/\bar{\alpha}_3 + S_n^2\ S_m^2\ (\eta\bar{\alpha}_4/\bar{\alpha}_6 - 1)/\bar{\alpha}_3$$

$$R_4 = 1 - S_m^2$$

$$R_5 = \eta^2\ S_m^2/\bar{\alpha}_6$$

$$S_n^2 = M\ \alpha_3\omega^2$$

$$S_m^2 = m_o\alpha_6\omega^2$$

$$S_p^2 = m\omega^2/k$$

$$\bar{\alpha}_4 = W/C\alpha_4$$

$$\bar{\alpha}_6 = W/C\alpha_6$$

$$\bar{F}_R = \bar{i}\ (F_{x1} + F_{x2}) + \bar{j}\ (F_{y1} + F_{y2}) = F_x\ \bar{i} + F_y\ \bar{j}$$

$$\bar{F}_w = \bar{i}\ (F_{x2} - F_{x1}) + \bar{j}\ (F_{y2} - F_{y1}) = F_u\bar{i} + F_v\bar{j}$$

When the solution is in the form $x(t) = Xe^{-i\omega t}$ and $y(t) = Ye^{-i\omega t}$ where $X = x_c + i\ x_s$ $Y = y_c + y_s i$, the path of the journal center is an ellipse, and, therefore, there exists major axis making angle α with X axis such that (Fig. 7)

$$x'(t) = a\cos(\omega t - \phi)$$
$$y'(t) = b\sin(\omega t - \phi) \qquad (17)$$

$$x' = x\cos\alpha + y\sin\alpha$$
$$y' = y\cos\alpha - x\sin\alpha \qquad (18)$$

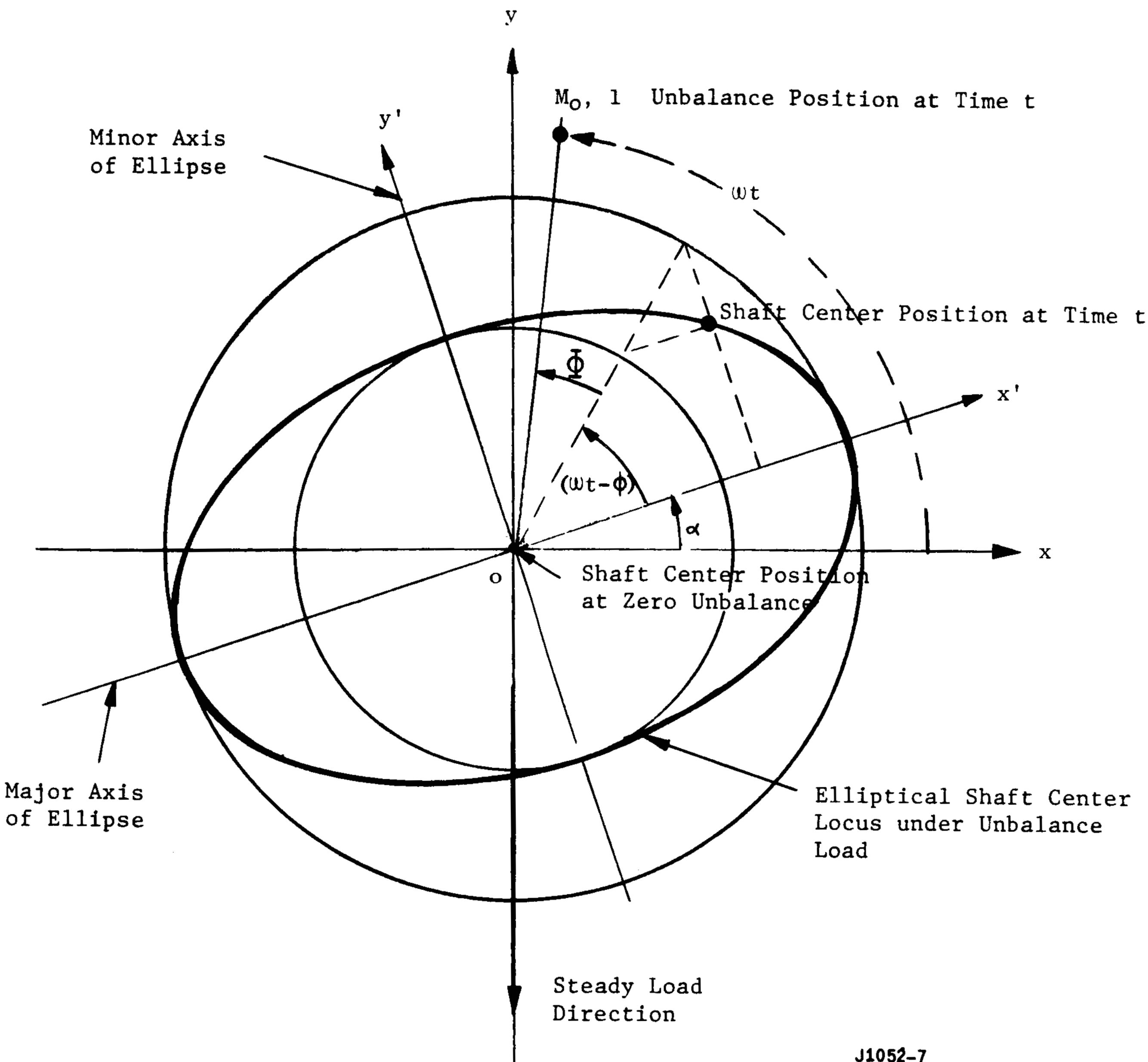


Figure 7. Elliptical Orbit.

$$x = \sqrt{x_c^2 + x_s^2} \quad \cos(\omega t - \phi_x)$$

$$y = \sqrt{y_c^2 + y_s^2} \quad \sin(\omega t - \phi_y) \qquad (19)$$

$$\Phi = \phi - \alpha$$

$$\text{where } \tan\phi_x = \frac{x_s}{x_c} \quad \text{and} \quad \tan\phi_y = -\frac{y_c}{y_s}$$

This establishes the relationship between (a, b, Φ, α) and (x_c, x_s, y_c, y_s).

COMPUTER PROGRAM

The computer program performs the following calculations:

- Computation of K_{xx}, C_{xx}, etc. from experimental data.
- Computation of conditions at the threshold of instability (for rigid or flexible pedestals).
- Unbalance response calculations (for rigid or flexible pedestals).

Bearing Constants

Let j = 1 represent displacements in measurement No. 1

j = 2 represent forces in measurement No. 1

j = 3 represent displacements in measurement No. 2

j = 4 represent forces in measurement No. 2

The input data may be prepared in one of the two forms:

a) Ellipse: a_j, b_j, α_j, Φ_j $\quad j = 1, 2, 3, 4$

Where a's and b's are major and minor axis respectively, α's are angles between major axis and x axis, and Φ's are phase angles, the angle lag from unbalance mass (see Figure 7)

b) Amplitude and phase angles: (X_j), ϕ_{xj}, (Y_j), ϕ_{yj}, $j = 1, 2, 3, 4$

Where $X_j(t) = |X_j| \cos(\omega t - \phi_{xj})$

$y_j(t) = |Y_j| \sin(\omega t - \phi_{yj})$

The angles α, $\bar{\phi}$ and ϕ should be given in degrees. The computer program converts the input to x_{cj}, y_{cj}, x_{sj} and y_{sj} and obtains K_{xx}, C_{xx}, etc. from the solution of Equation (2), which are:

$$K_{xx} = \frac{1}{\Delta}(pA_1 + qA_2) \qquad \omega C_{xx} = \frac{1}{\Delta}(qA_1 - pA_2)$$

$$K_{xy} = \frac{1}{\Delta}(pB_1 + qB_2) \qquad \omega C_{xy} = \frac{1}{\Delta}(qB_1 - pB_2)$$

$$K_{yx} = \frac{1}{\Delta}(pC_1 + qC_2) \qquad \omega C_{yx} = \frac{1}{\Delta}(qC_1 - pC_2)$$

$$K_{yy} = \frac{1}{\Delta}(pD_1 + qD_2) \qquad \omega C_{yy} = \frac{1}{\Delta}(qD_1 - pD_2)$$

where: $\Delta = p^2 + q^2$

$$p = x_{c1}\, y_{c3} - y_{s3}\, x_{s1} - x_{c3}\, y_{c1} + x_{s3}\, y_{s1}$$

$$q = x_{s1}\, y_{c3} + y_{s3}\, x_{c1} - x_{s3}\, y_{c1} - y_{s1}\, x_{c3}$$

$$A_1 = y_{c1}\, x_{c4} - y_{s1}\, x_{s4} - x_{c2}\, y_{c3} + x_{s2}\, y_{s3}$$

$$A_2 = y_{s1}\, x_{c4} + y_{c1}\, x_{s4} - x_{s2}\, y_{c3} - y_{s3}\, x_{c2}$$

$$B_1 = x_{c3}\, x_{c2} - x_{s3}\, x_{s2} - x_{c1}\, x_{c4} + x_{s1}\, x_{s4}$$

$$B_2 = x_{s3}\, x_{c2} + x_{c3}\, x_{s2} - x_{s1}\, x_{c4} - x_{c1}\, x_{s4}$$

$$C_1 = y_{c1}\, y_{c4} - y_{s1}\, y_{s4} - y_{c3}\, y_{c2} + y_{s3}\, y_{s2}$$

$$C_2 = y_{s1}\, y_{c4} + y_{c1}\, y_{s4} - y_{c3}\, y_{s2} - y_{s3}\, y_{c2}$$

$$D_1 = x_{c3}\, y_{c2} - x_{s3}\, y_{s2} - x_{c1}\, y_{c4} + x_{s1}\, y_{s4}$$

$$D_2 = x_{s3}\, y_{c2} + x_{c3}\, y_{s2} - x_{s1}\, y_{c4} - x_{c1}\, y_{s4}$$

For input data, either dimensionless values or actual values may be used. The factors which make dimensionless should be consistent as follows:

dimensionless displacement

$$\left.\begin{aligned} \bar{a}_j &= \frac{a_j}{C}, & \bar{b}_j &= \frac{b_j}{C} \\ |\bar{X}_j| &= \left|\frac{X_j}{C}\right|, & |\bar{Y}_j| &= \left|\frac{Y_j}{C}\right| \end{aligned}\right\} \quad j = 1,3$$

dimensionless forces

$$\left.\begin{aligned} \bar{a}_j &= \frac{a_j}{W}, & \bar{b}_j &= \frac{b_j}{W} \\ |\bar{X}_j| &= \left|\frac{X_j}{W}\right|, & |\bar{Y}_j| &= \left|\frac{Y_j}{W}\right| \end{aligned}\right\} \quad j = 2,4$$

Then the dimensionless K_{xx}, C_{xx}, etc. are:

$$\bar{K}_{xx} = \frac{C}{W} K_{xx}, \quad \bar{C}_{xx} = \frac{C}{W} C_{xx}, \quad \bar{K}_{xy} = \frac{C}{W} K_{xy}, \quad \bar{C}_{xy} = \frac{C}{W} C_{xy} \quad \text{etc.}$$

Stability Analysis

The computer may retain the previous result of bearing constant calculations or read in new data on K_{xx}, C_{xx}, etc. The additional data on the rotor and pedestals are ξ, α_{55}, α_{56}, $\omega_{ns1} = (1/\alpha_1 M)^{1/2}$, ω_{np}, $\bar{k}$. Then Equations (5A) - (5E) give the conditions at the threshold of instability. Again α_{55}, α_{56}, k may be either actual values or dimensionless as $\bar{\alpha}_{55} = \frac{W}{C}\alpha_{55}$, $\bar{\alpha}_{56} = \frac{W}{C}\alpha_{56}$ and $\bar{k} = \frac{C}{W}k$. The first natural frequency of the rotor ω_{ns1}, the natural frequency of pedestals ω_{np} and the span ratio ξ are actual values. Equations (5A) - (5E) gives the following as used in the program.

Case I:Flexible Pedestals

$$(S_2)_{1,2} = \frac{1 + (S_{n2})_1 + K_1 (1 + J) \mp \sqrt{[1 + (S_{n2})_1 + K_1 (1 + J)]^2 - 4 (S_{n2})_1 (1+K_1)}}{2 \gamma_2 (S_{n2})_1 (1 + K_1)}$$

$$(S_2)_{3,4} = \frac{1 + (S_{n2})_2 + K_2(1 + J) \mp \sqrt{[1 + (S_{n2})_2 + K_2 (1 + J)]^2 - 4 (S_{n2})_2 (1 + K_2)}}{2 \gamma_2 (S_{n2})_2 (1 + K_2)}$$

Where

$$(S_{n2})_1 = \left(\frac{\omega_{ns1}}{\omega_{np}}\right)^2 \qquad (S_{n2})_2 = \frac{\bar{\alpha}_1}{\bar{\alpha}_2} (S_{n2})_1 \qquad J = \frac{K}{\bar{K}}$$

$$\gamma_2 = \frac{(\bar{K}_{xx} - K)(\bar{K}_{yy} - K) - \bar{K}_{xy}\bar{K}_{yx}}{\bar{C}_{xx}\bar{C}_{yy} - \bar{C}_{xy}\bar{C}_{yx}}$$

$$K = \frac{\bar{K}_{xx}\bar{C}_{yy} + \bar{K}_{yy}\bar{C}_{yy} - \bar{K}_{xy}\bar{C}_{yx} - \bar{K}_{yx}\bar{C}_{xy}}{\bar{C}_{xx} + \bar{C}_{yy}}$$

$$K_1 = \frac{1}{\bar{\alpha}_1 K}$$

$$K_2 = \frac{\xi^2}{\bar{\alpha}_2 K}$$

$$\bar{\alpha}_1 = \alpha_{55} + \bar{\alpha}_{56}, \ \bar{\alpha}_2 = \bar{\alpha}_{55} - \bar{\alpha}_{56}$$

The subscripts 5 and 6 designate the mass stations of the rotor.

Case II : Flexible supports with zero bearing masses.

In other words, $\bar{k} \neq \infty$ but $\omega_{np} = \infty$. The indication of this case is done by supplying in input $\omega_{np} = 0$ and $\bar{k} \neq 0$. When such indication is given the computer uses the following equations:

$$(S_2)_{1,2} = \frac{1}{\gamma_2 \left[1+K_{1,2}(1+J)\right]} \qquad (S_2)_2 = (S_2)_4 = 0$$

One may also feed in a large quantity for ω_{np}, but it should not exceed 2^{32}.

Case III : Rigid pedestals.

This means $\bar{k} = \infty$, or $\omega_{np} = 0$ with $\bar{k}$ finite. The indication for this case is done by supplying $\omega_{np} = \bar{k} = 0$. With these values read in the computer will use:

$$(S_2)_{1,2} = \frac{1}{\gamma_2} \frac{1}{1+K_{1,2}} \qquad (S_2)_2 = (S_2)_4 = 0$$

One may also use a larger value for $\bar{k}$ less than 2^{128} and any finite value for ω_{np} The program also has a built-in loop to change ω_{nsl}. One simply specifies increment $\Delta\omega_{nsl}$ and the number of increments.

Rotor Response Calculations

After additional input data on the unbalances are read in, the computer calculates the unbalance response using equations (13) - (16). This part also has a built-up loop for ω so that many different ω may be used. The quantity ω_{ns1} will not be used (only the initial value will be used). The unbalance data are; η, $\bar{\alpha}_{77}$, $\bar{\alpha}_{78}$, $\bar{\alpha}_{57}$, $\bar{\alpha}_{58}$, ω_{ms1}, $\bar{\delta}_o$, $\bar{\delta}_1$, ψ, ω, $\Delta\omega$ and N_J where $\omega_{ms1} = \sqrt{1/(m_o \alpha_5)}$, m_o is the unbalance mass (one of the two), $\bar{\delta}$'s the unbalance eccentricities, ϕ the phase angle lag in degree of $\bar{\delta}_1$ from $\bar{\delta}_o$, $\Delta\omega$ the increment of the driving frequency, N_J the number of steps, and α's the influence coefficient. If the rotor is uniform in cross section, the α's may be obtained from the equations given in Appendix E

The computational results are converted into elliptical terms using,

$$a_j = \sqrt{\frac{1}{2}\left(d_j + \sqrt{e_j^2 + g_j^2}\right)}$$

$$b_j = \sqrt{\frac{1}{2}\left(d_j - \sqrt{e_j^2 + g_j^2}\right)}$$

$$\alpha_j = \frac{1}{2}\tan^{-1}\left(\frac{g_j}{e_j}\right)$$

$$\emptyset_j = \tan^{-1}\frac{x_{s_j}\cos\alpha_j + y_{s_j}\sin\alpha_j}{x_{c_j}\cos\alpha_j + y_{c_j}\sin\alpha_j}$$

$$\Phi_j = \emptyset_j - \alpha_j$$

$$d_j = x_{c_j}^2 + x_{s_j}^2 + y_{c_j}^2 + y_{s_j}^2$$

$$e_j = d_j - 2y_{c_j}^2 - 2y_{s_j}^2$$

$$g_j = 2(x_{c_j} y_{c_j} + x_{s_j} y_{s_j})$$

$$j = 1, 2, 3 \ldots .8$$

Here the subscripts denote the station number as shown in Figure 5.

The displacements x_c's and y_c's etc. are obtained from the equations following equations (4B). For example,

$$\overline{W}_o = \overline{i}\,(x_6 - x_5) + \overline{j}(y_6 - y_5) = U_o i + V_o j$$

$$R_o = \overline{i}\,(x_5 + x_6) + \overline{j}(y_5 + y_6) = X_o i + Y_o j$$

Here X_o and Y_o are obtained as U_o and V_o are computed except $\xi = \eta = 1$, $\alpha_3 = \alpha_1$, $\alpha_4 = \alpha_2, \alpha_6 = \alpha_5$. Hence

$$x_5 = x_{c_5} + iy_{s_5} = \frac{1}{2}(X_o - U_o) + i\,\frac{1}{2}(Y_o - V_o)$$

$$x_6 = x_{c_6} + iy_{s_6} = \frac{1}{2}(X_o + U_o) + i\,\frac{1}{2}\,(Y_o - V_o)$$

or

$$X_{c_5} = \frac{1}{2}(X_o - U_o), \quad y_{s_5} = \frac{1}{2}(Y_o - V_o), \qquad \text{etc.}$$

The output will be dimensionless quantities if inputs are dimensionless. The dimensionless displacement is U/C. If U/δ_o instead of U/C is desired, one should supply $\overline{\delta}_o = 1$ and $\overline{\delta}_1 = \dfrac{\delta_1}{\delta_o}$. The forces remain the same, i.e., $\overline{F} = F/W$.

APPENDIX A

NOMENCLATURE

NOMENCLATURE

a_j	Major axis of ellipse	in.
A	$K_{xx} - i\omega C_{xx}$	lbs/in.
b	Distance between bearings	in.
b_j	Minor axis of ellipse	in.
B	$K_{xy} - i\omega C_{yx}$	lbs/in.
C	$K_{yx} - i\omega Cyx$	lbs/in.
C	Radial Clearance	in.
D	$K_{yy} - i\omega C_{yy}$	lbs/in.
E	Elasticity Modulus	in-lbs/in^3
F	Force	lbs.
i	$\sqrt{-1.0}$	dimensionless
$\bar{i}$	Unit vector	dimensionless
I	Area moment of inertia	in^4
I_T	Transverse moment of inertia	in-lbs-sec^2
$\bar{j}$	Unit vector	dimensionless
k	Pedestal stiffness	lbs/in
K_{xx}, K_{xy}, etc.	Bearing stiffness coefficient*	lbs/in
M	One-half the rotor mass	lbs-sec^2/in.
m	Bearing mass	lbs-sec^2/in.
m_o	Mass at unbalance	lbs-sec^2/in.
$\bar{p}$	Force vector	lbs.
$\bar{q}$	Unbalance force vector (equation 7)	lbs.

*First subscript indicates direction of force
Second subscript indicates direction of motion

NOMENCLATURE (Continued)

$\overline{Q}$	Unbalance force vector (equation 6)	lbs.
$\overline{R}$	Displacement vector (translational mode, equation 4)	in.
t	Time	sec.
U	Displacement amplitude (rotational mode, x-component)	in.
V	Displacement amplitude (rotational mode, y-component)	in.
W	Bearing Load	lbs.
$\overline{W}$	Displacement vector (rotational mode, equation 4)	in.
x	Displacement, x-component	in.
X	Displacement amplitude (translational mode, x-component)	in.
y	Displacement, y-component	in.
Y	Displacement amplitude (translational mode, y-component)	in.
α	Angle between x and x' (Figure 7)	degrees
α_{ab}	Influence coefficient, displacement at b due to force at a	in/lb.
α_1, α_2, etc.	Influence coefficients (see Page 16)	in/lb.
γ	Frequency ratio at threshold of instability	dimensionless
δ_o, δ_1	Unbalance eccentricity (Figure 5)	in.
$\overline{\delta}$	Statical displacement vector	in.
ξ, η	See Figure 5	dimensionless
ν	Self-sustained vibration frequency at threshold of instability	dimensionless
ϕ	Phase angles (see Figure 7)	degrees
Φ	$\phi - \alpha$	degrees
ψ	Angle between unbalance vectors	degrees

NOMENCLATURE (Continued)

ω	Rotational speed	radians/sec.
$\omega\, C_{xx}$, $\omega\, C_{xy}$, etc.	Product of rotational speed and bearing damping coefficient	in/lb.

Superscripts

$(\bar{\ })$	Dimensionless quantity of (nonvectorial)
$(\dot{\ })$	Time derivative
$(\ )'$	Coordinates along axes of ellipse (Figure 7)

Subscripts

x, y	x- and y-component
c, s	Cosine - and sine - component
o	Rotor mass station
s	Journal station
b	Bearing station
q	Unbalance station
a, b	1, 2, ------, 8 station number as shown in Figure 5
n	1, 2 Refers to the two independent sets of measurements which are used in Equation (2)

APPENDIX B

ROTOR RESPONSE COMPUTER PROGRAM

INPUT INFORMATION

I. Determination of Bearing Constants: In the following j = 1 and j = 2 stand for displacements and forces in measurement set up No. 1 respectively and j = 3 and j = 4 for displacements and forces in measurement set No. 2 respectively.

a) Calculation of the constants only.

Card 1: Identification card. Punch any text within column 72.

Card 2a:

Column 1: Blank

Column 2: 4 or 6 if elliptical data; a_j, b_j, α_j, Φ_j are to be read in.

5 or 7 if harmonic data; x_j , ϕ_{x_j}, y_j , ϕ_{y_j} , are to be read in.

4 and 5 indicate more sets of input data are to be followed, 6 and 7 indicate this set to be the last and direct to exit after the current calculation has been performed.

Column	3 - 15	a_1 or x_1	(E 13.4)
	16 - 30	b_1 or ϕ_{x_1}	(E 15.4)
	31 - 45	α_1 or y_1	(E 15.4)
	46 - 60	Φ_1 or ϕ_{y_1}	(E 15.4)

Card 2b:

Column	1 - 15	a_2 or x_2	(E 15.4)
	16 - 30	b_2 or ϕ_{x_2}	(E 15.4)
	31 - 45	α_2 or y_2	(E 15.4)
	46 - 60	Φ_2 or ϕ_{y_2}	(E 15.4)

Card 2c and 2d: The same as Card 3 but for j = 3 and j = 4 respectively. Displacements are all in inches or dimensionless, angles in degrees and forces in pounds or in dimensionless quantity. (See page 24)

b) Calculation of bearing constants followed by dynamic analysis: In other words the computer is to calculate the bearing constants and then proceed to dynamic analysis using the calculated bearing constants.

Card 1: Identification Card

Card 2a: Exp. Data

Column 1: Blank

Column 2: 1 for elliptical data a_j, b_j, α_j, Φ_j

2 for harmonic data x_j , ϕ_{x_j} , y_j , ϕ_{y_j} .

Column 3 - 60: the same as case (a)

Card 2b, 2c, 2d: the same as case (a)

Card 3a: Rotor Data

Column 1 - 15 (E 15.4); ξ (see Fig. 5 and pages 15 and 18)

16 - 30 (E 15.4); α_{55} in/lb. or dimensionless

31 - 45 (E 15.4); α_{56} in/lb. or dimensionless

46 - 60 (E 15.4); $\omega_{ns1} = \sqrt{\frac{1}{M(\alpha_{55} + \alpha_{56})}}$, rad/sec

Card 3b: Pedestal Data and Control

Column 1 - 15 (E 15.4); $\omega_{np} = \sqrt{k/m}$, rad/sec

16 - 30 (E 15.4); k (pedestal stiffness), lb/in or dimensionless

35 (termed lane) ; Integer 1 Causes to return to the beginning of the program after the end of present computation.

2 Causes to return to reading in new rotor data. (Used when the bearing constants remain the same).

3 Directs to exit at the end of the computation (no more input to follow).

Column 40 (termed IFF); The number 0 Instructs to read in unbalance data and perform response calc. only.

Integer 1 Means to perform stability analysis only.

2 Perform both stability analysis and response calc. after reading in unbalance data.

Column 44 - 45 (I2): number of increments of $\Delta\omega_{nsl}$. The last digit should be on col. 45

46 - 60(E 15.4): $\Delta\omega_{nsl}$, rad/sec.

If no step up is desired, leave col. 41-60 blank.

If IFF $\neq$ 1, there must follow two more cards.

Card 4a (Iff $\neq$ 1): Unbalance data

Column 1 - 12 (E 12.4); η

13 - 24 (E 12.4); α_{57}

25 - 36 (E 12.4); α_{58}

37 - 48 (E 12.4); α_{77}

49 - 60 (E 12.4); α_{78}

$\left.\begin{array}{l}\alpha_{57}\\ \alpha_{58}\\ \alpha_{77}\\ \alpha_{78}\end{array}\right\}$ in/lb or dimensionless

61 - 72 (E 12.4); $\omega_{msl} = \sqrt{\dfrac{1}{m_o(\alpha_{77} + \alpha_{78})}}$, rad/sec

Card 4b (IFF $\neq$ 1);

Column 1 - 12 (E 12.4); δ_o, in.or dimensionless

13 - 24 (E 12.4); δ_1, in.or dimensionless

25 - 36 (E 12.4); ψ (phase lag of δ_1 from δ_o), degrees

37 - 48 (E 12.4); ω (initial driving freq.) , rad/sec

49 - 60 (E 12.4); $\Delta\omega$, rad/sec

64 - 65 (E 12.4); number of steps. The last digit should be on col. 65

II. Dynamic Analysis with Given Bearing Constants

Card 1: Identification Card

Card 2a: Bearing Constants

Column 1; Blank

2; 3

Column 3 - 15 (E 15.4); K_{xx} (lb/in) or $\bar{K}_{xx}$ (dimensionless)

16 - 30 (E 15.4); ωC_{xx}(lb/in) or $\bar{C}_{xx}$ (dimensionless)

31 - 45 (E 15.4); K_{xy} (lb/in) or $\bar{K}_{xy}$ (dimensionless)

46 - 60 (E 15.4); ωC_{xy}(lb/in) or $\bar{C}_{xy}$ (dimensionless)

Card 2b:

Column 1 - 15 (E 15.4); K_{yx} (lb/in) or $\bar{K}_{yx}$ (dimensionless)

16 - 30 (E 15.4); ωC_{yx}(lb/in) or $\bar{C}_{yx}$ (dimensionless)

31 - 45 (E 15.4); K_{yy} (lb/in) or $\bar{K}_{yy}$ (dimensionless)

46 - 60 (E 15.4); ωC_{yy}(lb/in) or $\bar{C}_{yy}$ (dimensionless)

Card 3a, 3b, 4a, 4b; the same as case Ib.

Note that cards 2c and 2d are not needed in this case.

OUTPUT FORMAT

The output format is almost self-explanatory. The terminology used in output is as follows:

XI = ξ

ETA = η

ALFAA = ALF 55 = α_{55}

ALFAB = ALF 56 = α_{56} etc.

BRG. N. FREQ = ω_{np} = $(k/m)^{1/2}$

PED. spring = k

SHFT. N. FRQ 1 = ONS 1 = ω_{ns1} = $\left[M\ (\alpha_{55} + \alpha_{56})\right]^{-1/2}$

OMSI = $\omega_{ms1} = [M_o \, (\alpha_{77} + \alpha_{78})]^{-1/2}$

DEL SHFT. FR = $\Delta\, \omega_{ns1}$

THRES. FREQ. RATIO = γ

GAMMA SQUARED = $\gamma^2 = \gamma_2$

KAPA = $\bar{\kappa}$

SPEED RATIO = S = ω/ω_{ns} at the threshold of instability

RPM SPEED = 30 ω/π at the threshold of instability

OMEGA = ω

$$\left.\begin{array}{l} \text{DELTA ZERO} = \delta_o \\ \text{DELTA ONE} = \delta_1 \end{array}\right\} \text{unbalance eccentricities}$$

DEL. FIE = ψ = angle between δ_o and δ_1

DEL OMEGA = $\Delta\omega$

DISPLMT RATIO = displacement/c

FORCE RATIO = transmitted force/w

MAX (A) = Major axis of ellipse

MIN (B) = minor axis of ellipse

AXIS ANGLE = α = angle between major axis and x axis

```
              OLE G KRISTENSEN-HC LEE RM 37-1020 PH 54907 REV.8-8-1966
      DIMENSION XC(12),XS(12),YC(12),YS(12),AX(12),BX(12),ALP(12)
      DIMENSION FIJ(12),ALPP(12)
      COMMON NN,A(4),B(4),ALFA(4),FI(4)
      WRITE(6,1)
    1 FORMAT(1H1,46X,19HGENERAL ELECTRIC CO//33X, 44HBEARING PROPERTY AN
     1D ROTOR DYNAMICS ANALYSIS,1H //)
  111 READ(5,2)
    2 FORMAT(72H
     1
      WRITE(6,2)
  211 READ(5,6)NN,A(1),B(1),ALFA(1),FI(1)
    6 FORMAT(I2,E13.4,3E15.4)
      NC=4
      IF(NN.EQ.3) NC=2
      READ(5,66)(A(J),B(J),ALFA(J),FI(J), J=2,NC)
   66 FORMAT(4E15.4)
      IF(3.EQ.NN) GO TO 9
      CALL CONST(XXK,XXC,XYK,XYC,YXK,YXC,YYK,YYC)
      IF (NN.EQ.1.OR.NN.EQ.4) GO TO 992
      IF(NN.EQ.6) GO TO 992
      WRITE(6,991)
  991 FORMAT(1H //25X,1HX,18X,3HFIX,18X,1HY,18X,3HFIY/)
      GO TO 993
  992 WRITE(6,7)
    7 FORMAT(1H //25X,1HA,20X,1HB,18X,4HALFA,18X,2HFI,1H /)
  993 WRITE(6,8)(A(J),B(J),ALFA(J),FI(J),J=1,4)
    8 FORMAT(11X,4E20.4)
   10 WRITE(6,11)
   11 FORMAT(2H   //6X,3HKXX,12X,3HCXX,12X,3HKXY,12X,3HCXY,12X,3HKYX,12X,
     13HCYX,12X,3HKYY,12X,3HCYY,1H /)
      WRITE(6,12)XXK,XXC,XYK,XYC,YXK,YXC,YYK,YYC
   12 FORMAT(8E15.4///)
      IF(NN.EQ.6.OR.NN.EQ.7) GO TO 113
      IF(3-NN) 115,112,112
    9 XXK=A(1)
      XXC=B(1)
      XYK=ALFA(1)
      XYC=FI(1)
      YXK=A(2)
      YXC=B(2)
      YYK=ALFA(2)
      YYC=FI(2)
      GO TO 10
C
C     ROTOR INFORMATION
C
  112 READ(5,13)XI,ALFAA,ALFAB,ONS1,NNX,ONP,BK,LANE,IFF,NSTEP,DONS1
   13 FORMAT(4E15.4,I12/2E15.4,3I5,E15.4)
   25 FORMAT(1H //51X,18HSTABILITY ANALYSIS///)
   26 FORMAT(1H //47X,26HFLEXIBLE PEDESTAL ANALYSIS///)
   27 FORMAT(1H //47X,26HZERO BEARING MASS ANALYSIS///)
   28 FORMAT(1H //49X,23HRIGID PEDESTAL ANALYSIS///)
   29 FORMAT(      45X,30HUNBALANCE RESPONSE CALCULATION///)
      IF(XI.NE..0) GO TO 986
```

```
      WRITE(6,989)
  989 FORMAT(1H///10X,22HXI=0 IS NOT ACCEPTABLE//)
      GO TO 987
  986 ALF1=ALFAA+ALFAB
      ALF2=ALFAA-ALFAB
      IF(IFF.EQ.0) GO TO 18
C
C     STABILITY      B = 0
C
      WRITE(6,25)
      WRITE(6,73)
   73 FORMAT(7X,2HXI,9X,5HALFAA,8X,5HALFAB,6X,10HBRG.N.FREQ,4X,9HPED.SPR
     1G.,3X,11HSHFT.N.FRQ1,3X,12HDEL SHFT.FRQ,3X,7HN.STEPS,3X,3HIFF,4X,4
     2HLANE)
      WRITE(6,74)XI,ALFAA,ALFAB,ONP,BK,ONS1,DONS1,NSTEP,IFF,LANE
   74 FORMAT(5E13.4,2E14.4,2I8,I7)
      CAPA= (XXK*YYC+YYK*XXC-XYK*YXC-YXK*XYC)/(XXC+YYC)
      GAM2= ((XXK-CAPA)*(YYK-CAPA)-XYK*YXK)/(XXC*YYC-XYC*YXC)
      CAP1= 1./(ALF1*CAPA)
      CAP2= XI**2/(ALF2*CAPA)
      NSTEPP=NSTEP+1
      DO 190 KC=1,NSTEPP
      OONS2=ALF1*ONS1**2/ALF2
      IF (OONS2 .LT.0.) WRITE(6,1001)
 1001 FORMAT(58HALFAA SHOULD BE LARGER THAN ALFBB.IF NOT, SQRT(-B) RESUL
     1TS  )
      ONS2=SQRT(OONS2)
C
C     CHECH VALUE OF ONP AND BK
C
      ONPK= ONP*BK
      IF(ONPK.EQ..0) GO TO 15
C
C     CASE I
C
      IF(KC    .EQ.1) WRITE(6,26)
      XJ= CAPA/BK
C ERRORS ON FLEXIBLE PEDASTAL CORRECTED JULY 14,1966
      XJ=-CAPA/BK
      SN21= (ONS1/ONP)**2
      SN22= OONS2/(ONP**2)
      DENOM = 2.*GAM2*SN21*(1.+CAP1)
      PT1= (1.+SN21+CAP1*(1.-XJ))/DENOM
      SQ1=      ((1.+SN21+CAP1*(1.-XJ))**2-4.*SN21*(1.+CAP1))
      IF(SQ1.GE. 0.) GO TO 1002
      LL=1002
      WRITE (6,1000) LL
 1000 FORMAT( 26HSQRT(-B) IN STATEMENT NO.=, I5,I5)
 1002 SQ1=SQRT(SQ1)
      S21= PT1-SQ1/DENOM
      S22= PT1+SQ1/DENOM
      DENOMI = 2.*GAM2*SN22*(1.+CAP2)
      PT2= (1.+SN22+CAP2*(1.-XJ))/DENOMI
      SQ2=      ((1.+SN22+CAP2*(1.-XJ))**2-4.*SN22*(1.+CAP2))
      IF(SQ2.GE.0.) GO TO 1003
```

```
      LL=1003
      WRITE (6,1000) LL
 1003 SQ2=SQRT(SQ2)
      S23= PT2-SQ2/DENOMI
      S24= PT2+SQ2/DENOMI
      GO TO 17
   15 IF(BK.EQ..0) GO TO 16
C
C     CASE II
C
      IF(KC     .EQ.1) WRITE(6,27)
      XJ=CAPA/BK
C  NEXT EQUATION CORRECTS ERRORS
      XJ=-CAPA/BK
      S21= 1./(GAM2*(1.+CAP1*(1.-XJ)))
      S23= 1./(GAM2*(1.+CAP2*(1.-XJ)))
      S22=0.
      S24=0.
      GO TO 17
C
C     CASE III
C
   16 S21= 1./(GAM2*(1.+CAP1))
      S23= 1./(GAM2*(1.+CAP2))
      S22 =0.
      S24 =0.
      IF(KC     .EQ.1) WRITE(6,28)
C
   17 CONTINUE
      IF (S21.GE.0.) GO TO 1004
      LL=1004
      WRITE(6,1000) LL
 1004 R1 = SQRT(S21)
      IF (S22.GE.0.) GO TO 1005
      LL=1005
      WRITE(6,1000) LL
 1005 R2 = SQRT(S22)
      IF (S23 .GE.0.) GO TO 1006
      LL=1006
      WRITE (6,1000) LL
 1006 R3 = SQRT(S23)
      IF(S24 .GE.0.) GO TO 1007
      LL=1007
      WRITE (6,1000) LL
 1007 R4 = SQRT(S24)
      SPE1 = ONS1*R1/0.1047198
      SPE2 = ONS1*R2/0.1047198
      SPE3 = ONS2*R3/0.1047198
      SPE4 = ONS2*R4/0.1047198
      IF (GAM2.GE.0.) GO TO 1008
      LL=1008
      WRITE (6,1000) LL
 1008 FRQRT = SQRT(GAM2)
      WRITE(6,188)
  188 FORMAT(2X,16HTHRES.FREQ.RATIO,2X,13HSPEED RATIO 1,4X,13HSPEED RATI
```

```
     10 2,4X,13HSPEED RATIO 3,4X,13HSPEED RATIO 4,4X,13HGAMMA SQUARED,8X
     2,4HKAPA)
      WRITE(6,189)FRQRT,R1,R2,R3,R4,GAM2,CAPA
  189 FORMAT(E15.4,6E17.4//)
      WRITE(6,223)
  223 FORMAT(9X,4HONS1,7X,13HRPM  SPEED 1 ,4X,13HRPM  SPEED 2 ,4X,13HRPM
     1  SPEED 3 ,4X,13HRPM  SPEED 4 ,7X,6HALFA 1,10X,6HALFA 2)
      WRITE(6,224)ONS1,SPE1,SPE2,SPE3,SPE4,ALF1,ALF2
  224 FORMAT(E15.4,6E17.4///)
      ONS1=ONS1+DONS1
  190 CONTINUE
      STEP=NSTEPP
      ONS1=ONS1-DONS1*STEP
      IF(IFF.EQ.2) GO TO 18
C
C     GO BACK OR NOT
      GO TO (115,112,113),LANE
  115 WRITE(6,116)
  116 FORMAT(1H1)
      GO TO 111
C
C     B NOT ZERO     RESPONSE
C
   18 READ(5,20)ETA,ALFAC,ALFAD,ALFCC,ALFCD,OMS1,DZE,DONE,FIE,OMG,DMG,NJ
     1UMP
   20 FORMAT(6E12.4/5E12.4,I5)
      WRITE(6,29)
      WRITE(6,711)
  711 FORMAT(7X,2HXI,10X,3HETA,10X,5HALF55,8X,5HALF56,8X,5HALF57,8X,5HAL
     1F58,8X,5HALF77,8X,5HALF78,9X,4HONS1)
      WRITE(6,712)XI,ETA,ALFAA,ALFAB,ALFAC,ALFAD,ALFCC,ALFCD,ONS1
  712 FORMAT(9E13.4)
      WRITE(6,713)
  713 FORMAT(1H //6X,4HOMS1,6X,10HBRG.N.FREQ,4X,9HPED.SPRG.,3X,10HDELTA
     1ZERO,4X,9HDELTA ONE,5X,7HDEL.FIL,6X,5HOMEGA,6X,9HDEL.OMEGA,2X,5HNS
     2TEP,1X,3HIFF,1X,4HLANE)
      WRITE(6,714)OMS1,ONP,BK,DZE,DONE,FIE,OMG,DMG,NJUMP,IFF,LANE
  714 FORMAT(8E13.4,I4,2I5)
      OONS2=ALF1*ONS1**2/ALF2
      IF (OONS2 .LT.0.) WRITE(6,1001)
      ONS2=SQRT(OONS2)
      ALF3= ALFAC+ALFAD
      ALF4= ALFAC-ALFAD
      ALF5=ALFCC+ALFCD
      ALF6=ALFCC-ALFCD
      FIEE=FIE*0.017453
      SS1=DZE+DONE*COS(FIEE)
      SS2= DONE*SIN(FIEE)
      SS3= SS1-2.*DZE
      NJUMPP=NJUMP+1
      DO 3 KKK=1,NJUMPP
      DZERO= 1./(ALF5*OMS1**2)
      DZERO=DZERO*OMG**2
      S1= OMG/ONS1
      S2= OMG/ONS2
```

```
      OOMS2=ALF5*OMS1**2/ALF6
      IF (OOMS2 .GE.0.) GO TO 1009
      LL=1009
      WRITE(6,1000) LL
 1009 OMS2=SQRT(OOMS2)
      SQ1= S1**2
      SQ2= S2**2
      SQ3=(OMG/OMS1)**2
      SQ4=(OMG/OMS2)**2
      WS1 = 1.- SQ1
      WS2 = 1.- SQ2
      WW1=(ALF3/ALF5)-1.
      WW2=(ETA*ALF4/ALF6)-XI
      HT1=WS1-SQ3*(WS1+SQ1 *ALF3**2/(ALF1*ALF5))
      HR1=WS2-SQ4*(WS2+SQ2 *ALF4**2/(ALF2*ALF6) )
      HT2=1.+ SQ3*WW1
      HR2=XI+ SQ4*WW2
      HT3=SQ1*(1.+SQ3*WW1)/ALF1
      HR3=SQ2*(XI+SQ4*WW2)/ALF2
      HT4=1.- SQ3
      HR4=1.- SQ4
      HT5=SQ3/ALF5
      HR5=SQ4*ETA**2/ALF6
      DW1=HT1*HT5+HT2*HT3
      DW2=HR1*HR5+HR2*HR3
      PD1=HT1+HT3*ALF3
      PD2=ETA*HR1+HR3*ALF4
      RT1= DZERO*SS1
      RT2= DZERO*SS2
      RT3= DZERO*SS3
      RR2 = RT2
      RR3 = DZERO*SS3
C
C
      IF(ONP.NE..0) GO TO 21
      IF(BK.EQ..0) GO TO 24
      KASE = 2
      BRG=BK
      IF(KKK.EQ.1) WRITE(6,27)
      GO TO 23
   24 KASE = 3
      IF(KKK.EQ.1) WRITE(6,28)
      IF (HR1.NE..0) H2 = DW2/(HR1*HR4)
      IF (HR1.NE..0) T2=DZERO*PD2/(HR1*HR4)
      IF(HT1.EQ..0) GO TO 998
      H1=DW1/(HT1*HT4)
      T1=DZERO*PD1/(HT1*HT4)
      GO TO 22
C
   21 KASE = 1
      BRG=BK*(1.- OMG**2/ONP**2)
      IF(KKK.EQ.1) WRITE(6,26)
   23 PW1=HT1*HT4*BRG
      PW2=HR1*HR4*BRG
      H1=DW1*BRG/(PW1+DW1)
```

```
C   NEXT EQUATION CORRECTS ERRORS
      H1=DW1*BRG/(PW1-DW1)
      T1=DZERO* PD1*H1/DW1
      H2=DW2*BRG/(PW2+DW2)
C   NEXT EQUATION CORRECTS ERRORS
      H2=DW2*BRG/(PW2-DW2)
      T2=DZERO*PD2*H2/DW2
   22 ST1 = SS1*T1
      ST2 = SS2*T1
      PT =(XXK-H1)*(YYK-H1)-XXC*YYC-XYK*YXK+XYC*YXC
      QT =-(XXC*(YYK-H1)+YYC*(XXK-H1)-XYC*YXK-YXC*XYK)
      DTDT = PT**2 + QT**2
      TT1= ST1*(YYK-H1-XYC)+ST2*(YYC+XYK)
      TT2=-(ST1*(YYC+XYK)+ST2*(XYC-YYK+H1))
      TT3= ST2*(H1-XXK-YXC)+ST1*(XXC-YXK)
      TT4=-(ST2*(YXK-XXC)-ST1*(XXK-H1+YXC))
C
C
      V=1./DTDT
      XSBTC = V*(TT1*PT+TT2*QT)
      XSBTS = V*(TT2*PT-TT1*QT)
      YSBTC = V*(TT3*PT+TT4*QT)
      YSBTS = V*(TT4*PT-TT3*QT)
      IF (KASE.EQ.3) GO TO 985
      HHS1=PW1/(PW1+DW1)
C   NEXT EQUATION CORRECTS ERRORS
      HHS1=PW1/(PW1-DW1)
      XJ1=PD1/(PW1+DW1)
C   NEXT EQUATION CORRECTS ERRORS
      XJ1=-PD1/(PW1-DW1)
      XSTC = HHS1  *XSBTC - XJ1*RT1
      XSTS = HHS1  *XSBTS - XJ1*RT2
      YSTC = HHS1  *YSBTC + XJ1*RT2
      YSTS = HHS1  *YSBTS - XJ1*RT1
      GO TO 994
  998 DD3=DZERO*ALF3/HT2
      XSBTC=-DD3*SS1
      XSBTS=-DD3*SS2
      YSBTC=DD3*SS2
      YSBTS=-DD3*SS1
  985 XSTC=XSBTC
      XSTS=XSBTS
      YSTC=YSBTC
      YSTS=YSBTS
  994 FXTC  =     (XXK*XSBTC+XYK*YSBTC+XXC*XSBTS+XYC*YSBTS)
      FXTS  =     (XXK*XSBTS-XXC*XSBTC+XYK*YSBTS-XYC*YSBTC)
      FYTC  =       (YXK*XSBTC+YXC*XSBTS+YYK*YSBTC+YYC*YSBTS)
      FYTS  =       (YXK*XSBTS-YXC*XSBTC+YYK*YSBTS-YYC*YSBTC)
      CQ1=1./(1.+ SQ3*WW1)
      CQ2=1./(XI+SQ4*WW2)
      XQTC=CQ1*(XSTC+ALF3*FXTC-ALF5*WW1*RT1)
      XQTS=CQ1*(XSTS+ALF3*FXTS-ALF5*WW1*RT2)
      YQTC=CQ1*(YSTC+ALF3*FYTC+ALF5*WW1*RT2)
      YQTS=CQ1*(YSTS+ALF3*FYTS-ALF5*WW1*RT1)
      CO1=-ALF1/SQ1
```

```
      SQZ1= SQ3/ALF5
      XOTC=CO1*(RT1-FXTC+SQZ1*XQTC)
      XOTS=CO1*(RT2-FXTS+SQZ1*XQTS)
      YOTC=CO1*(-RT2-FYTC+SQZ1*YQTC)
      YOTS=CO1*(RT1-FYTS+SQZ1*YQTS)
      IF ((HR1+BK).EQ..0) GO TO 997
      SR2 = SS2*T2
      SR3 = SS3*T2
      PR =(XXK-H2)*(YYK-H2)-XXC*YYC-XYK*YXK+XYC*YXC
      QR =-(XXC*(YYK-H2)+YYC*(XXK-H2)-XYC*YXK-YXC*XYK)
      DRDR = PR**2 + QR**2
      TR1 =  SR3*(YYK-H2-XYC)+SR2*(YYC+XYK)
      TR2 =-(SR3*(YYC+XYK)+SR2*(XYC-YYK+H2))
      TR3 =  SR2*(H2-XXK-YXC)+SR3*(XXC-YXK)
      TR4 =-(SR2*(YXK-XXC)-SR3*(XXK-H2+YXC))
      R=1./DRDR
      XSBRC = R*(TR1*PR+TR2*QR)
      XSBRS = R*(TR2*PR-TR1*QR)
      YSBRC = R*(TR3*PR+TR4*QR)
      YSBRS = R*(TR4*PR-TR3*QR)
      IF (KASE.EQ.3) GO TO 984
      HHS2=PW2/(PW2+DW2)
C  NEXT EQUATION CORRECTS ERRORS
      HHS2=PW2/(PW2-DW2)
      XJ2=PD2/(PW2+DW2)
C  NEXT EQUATION CORRECTS ERRORS
      XJ2=-PD2/(PW2-DW2)
      XSRC = HHS2  *XSBRC-XJ2*RR3
      XSRS = HHS2  *XSBRS-XJ2*RR2
      YSRC = HHS2  *YSBRC+XJ2*RR2
      YSRS = HHS2  *YSBRS-XJ2*RR3
      GO TO 996
  997 DD4=DZERO*ALF4/HR2
      XSBRC=-DD4*SS3
      XSBRS=-DD4*SS2
      YSBRC=DD4*SS2
      YSBRS=-DD4*SS3
  984 XSRC=XSBRC
      XSRS=XSBRS
      YSRC=YSBRC
      YSRS=YSBRS
  996 XBRC = XSRC-XSBRC
      XBRS = XSRS-XSBRS
      YBRC = YSRC-YSBRC
      YBRS = YSRS-YSBRS
      XBTC = XSTC - XSBTC
      XBTS = XSTS - XSBTS
      YBTC = YSTC - YSBTC
      YBTS = YSTS - YSBTS
      FXRC   =            (XXK*XSBRC+XYK*YSBRC+XXC*XSBRS+XYC*YSBRS)
      FXRS   =            (XXK*XSBRS-XXC*XSBRC+XYK*YSBRS-XYC*YSBRC)
      FYRC   =            (YXK*XSBRC+YXC*XSBRS+YYK*YSBRC+YYC*YSBRS)
      FYRS   =            (YXK*XSBRS-YXC*XSBRC+YYK*YSBRS-YYC*YSBRC)
      EAXI=ETA*XI
      XQRC= CQ2*(EAXI*XSRC+ALF4*FXRC-ALF6*WW2*RR3)
```

```
      XQRS= CQ2*(EAXI*XSRS+ALF4*FXRS-ALF6*WW2*RR2)
      YQRC= CQ2*(EAXI*YSRC+ALF4*FYRC+ALF6*WW2*RR2)
      YQRS= CQ2*(EAXI*YSRS+ALF4*FYRS-ALF6*WW2*RR3)
      CO2=-ALF2/(XI*SQ2)
      SQZ2= ETA*SQ4/ALF6
      XORC=CO2*(ETA*RR3-FXRC+SQZ2*XQRC)
      XORS=CO2*(ETA*RR2-FXRS+SQZ2*XQRS)
      YORC=CO2*(-ETA*RR2-FYRC+SQZ2*YQRC)
      YORS=CO2*(ETA*RR3- FYRS+SQZ2*YQRS)
C
C
      XC(1)=.5*(XBTC-XBRC)
      YC(1)=.5*(YBTC-YBRC)
      XS(1)=.5*(XBTS-XBRS)
      YS(1)=.5*(YBTS-YBRS)
      XC(2)=.5*(XBTC+XBRC)
      YC(2)=.5*(YBTC+YBRC)
      XS(2)=.5*(XBTS+XBRS)
      YS(2)=.5*(YBTS+YBRS)
      XC(3)= .5*(XSTC-XSRC)
      YC(3)= .5*(YSTC-YSRC)
      XS(3)= .5*(XSTS-XSRS)
      YS(3)= .5*(YSTS-YSRS)
      XC(4)= .5*(XSTC+XSRC)
      YC(4)= .5*(YSTC+YSRC)
      XS(4)= .5*(XSTS+XSRS)
      YS(4)= .5*(YSTS+YSRS)
      XC(5)= .5*(XOTC-XORC)
      YC(5)= .5*(YOTC-YORC)
      XS(5)= .5*(XOTS-XORS)
      YS(5)= .5*(YOTS-YORS)
      XC(6)= .5*(XOTC+XORC)
      YC(6)= .5*(YOTC+YORC)
      XS(6)= .5*(XOTS+XORS)
      YS(6)= .5*(YOTS+YORS)
      XC(7)= XC(3)-XC(1)
      YC(7)= YC(3)-YC(1)
      XS(7)= XS(3)-XS(1)
      YS(7)= YS(3)-YS(1)
      XC(8)= XC(4)-XC(2)
      YC(8)= YC(4)-YC(2)
      XS(8)= XS(4)-XS(2)
      YS(8)= YS(4)-YS(2)
      XC(9)= .5*(FXTC-FXRC)
      YC(9)= .5*(FYTC-FYRC)
      XS(9)= .5*(FXTS-FXRS)
      YS(9)= .5*(FYTS-FYRS)
      XC(10)= .5*(FXTC+FXRC)
      YC(10)= .5*(FYTC+FYRC)
      XS(10)= .5*(FXTS+FXRS)
      YS(10)= .5*(FYTS+FYRS)
      XC(11)=.5*(XQTC-XQRC)
      XS(11)=.5*(XQTS-XQRS)
      YC(11)=.5*(YQTC-YQRC)
      YS(11)=.5*(YQTS-YQRS)
```

```
      XC(12)=.5*(XQTC+XQRC)
      XS(12)=.5*(XQTS+XQRS)
      YC(12)=.5*(YQTC+YQRC)
      YS(12)=.5*(YQTS+YQRS)
      WRITE(6,79) OMG
   79 FORMAT(1H /4X,8HOMEGA = ,E11.4)
      WRITE (6,990) ONS1
  990 FORMAT( 5X,7HONS1 = ,E11.4)
      WRITE(6,716) OMS1
  716 FORMAT(5X,7HOMS1 = ,E11.4)
      WRITE(6,40)
   40 FORMAT(44X,6HMAX(A),14X,6HMIN(B),12X,10HAXIS ANGLE,10X,11HPHASE AN
     1GLE,1H //)
C
      DO 50 M= 1,12
      D    = XC(M)**2+ XS(M)**2+ YC(M)**2+ YS(M)**2
      E    = D    - 2.*YC(M)**2- 2.*YS(M)**2
      G    = 2.*(XC(M)*YC(M)+ XS(M)*YS(M))
      AX(M)= SQRT(0.5*(D    +SQRT(E    **2+G    **2)))
      BX(M)= SQRT(0.5*(D    -SQRT(E    **2+G    **2)))
      IF ((G+E).NE..0) GO TO 970
      ANGL = .0
      GO TO 983
  970  ANGL = ATAN2(G,E)
  983 ALP(M) = 28.64789*ANGL
      ALPP(M)=ALP(M)/57.29576
      ARC1    =XS(M)*COS(ALPP(M))+ YS(M)*SIN(ALPP(M))
      ARC2=XC(M)*COS(ALPP(M))+YC(M)*SIN(ALPP(M))
      IF ((ARC1+ARC2).NE..0) GO TO 982
      ANGL2 = .0
      GO TO 981
  982 ANGL2 = ATAN2(ARC1,ARC2)
  981 FIJ(M) = 57.295780*ANGL2
      FIJ(M)=FIJ(M)-ALP(M)
      IF(NNX.NE.9) GO TO 50
      WRITE(6,194)D,E,G,ARC1,ARC2,YS(10),ANGL,ANGL2
      WRITE (6,194) XC(M),XS(M),YC(M),YS(M),ALP(M), ALPP(M)   ,  FIJ (M),
     1AX(M),BX(M)
   50 CONTINUE
C
C
      FABG1=BK*AX(1)
      FBBG1=BK*BX(1)
      FABG2=BK*AX(2)
      FBBG2=BK*BX(2)
      WRITE(6,41)
   41 FORMAT(9X,18HMASS DISPLMT RATIO)
      WRITE(6,411)AX(5),BX(5),ALP(5),FIJ(5)
  411 FORMAT(14X,9HBEARING 1,18X,E11.4,3E20.4)
      WRITE(6,42)AX(6),BX(6),ALP(6),FIJ(6)
   42 FORMAT(14X,9HBEARING 2,18X,E11.4,3E20.4//)
      WRITE(6,43)
   43 FORMAT(9X,19HJOURN DISPLMT RATIO)
      WRITE(6,411)AX(3),BX(3),ALP(3),FIJ(3)
      WRITE(6,42) AX(4),BX(4),ALP(4),FIJ(4)
```

```
      WRITE(6,715)
  715 FORMAT(9X,23HUNBAL.MASS DISPMT RATIO)
      WRITE(6,411) AX(11),BX(11),ALP(11),FIJ(11)
      WRITE(6,42)  AX(12),BX(12),ALP(12),FIJ(12)
      WRITE(6,45)
   45 FORMAT(9X,17HBRG DISPLMT RATIO)
      WRITE(6,411)AX(1),BX(1),ALP(1),FIJ(1)
      WRITE(6,42) AX(2),BX(2),ALP(2),FIJ(2)
      WRITE(6,46)
   46 FORMAT(9X,23HREL JOURN DISPLMT RATIO)
      WRITE(6,411)AX(7),BX(7),ALP(7),FIJ(7)
      WRITE(6,42) AX(8),BX(8),ALP(8),FIJ(8)
      WRITE(6,47)
   47 FORMAT(9X,17HFILM FORCE RATIO )
      WRITE(6,411)AX(9),BX(9),ALP(9),FIJ(9)
      WRITE(6,42) AX(10),BX(10),ALP(10),FIJ(10)
      WRITE(6,48)
   48 FORMAT(9X,20HPEDESTAL FORCE RATIO//)
      WRITE(6,411)FABG1,FBBG1,ALP(1),FIJ(1)
      WRITE(6,42)FABG2,FBBG2,ALP(2),FIJ(2)
      OMG=OMG+DMG
      IF(NNX.NE.9) GO TO 3
      WRITE(6,194)ALF1,ALF2,ALF3,ALF4,SS1,SS2,SS3,S1,S2,ALF5
      WRITE(6,194)ALF6,  XJ1,XJ2,H1,H2,T1,RT1,RT2,RT3,DZERO
      WRITE(6,194)ST1,ST2,RR2,RR3,PT,QT,DTDT,TT1,TT2,TT3
      WRITE(6,194)TT4,V,XSBTC,XSBTS,YSBTC,YSBTS,XSTC,XSTS,YSTC,YSTS
      WRITE(6,194)DD3,    XOTC,XOTS,YOTC,YOTS,T2,SR2,SR3,PR
      WRITE(6,194)QR,DRDR,TR1,TR2,TR3,TR4,R,XSBRC,XSBRS,YSBRC
      WRITE(6,194)YSBRS,XSRC,XSRS,YSRC,YSRS,DD4,XBRC,XBRS,YBRC,YBRS
      WRITE(6,194)XBTC,XBTS,YBTC,YBTS,    XORC,XORS,YORC,YORS,FXTC
      WRITE(6,194)FXTS,FYTC,FYTS,FXRC,FXRS,FYRC,FYRS
      WRITE(6,194)SQ1,SQ2,SQ3,SQ4,WS1,WS2,HT1,HR1 ,WW1,WW2
      WRITE(6,194)HT2,HR2,HT3,HR3,HT4,HR4,HT5,HR5,DW1,DW2
      WRITE(6,194)PD1,PD2,DRG,H1,H2,T1,T2,PW1,PW2,HHS1
      WRITE(6,194)HHS2,CQ1,CQ2,CO1,CO2,XQTC,XQTS,YQTC,YQTS,XQRC
      WRITE(6,194)XQRS,YQRS,ONS1,ONS2,OHS1,OHS2,OMG
  194 FORMAT(10E12.4//)
    3 CONTINUE
  987 GO TO (115,112,113),LANE
  113 STOP
      END
$      FORTRAN DECK
$      INCODE  IBMF
      SUBROUTINE CONST(XXK,XXC,XYK,XYC,YXK,YXC,YYK,YYC)
      DIMENSION PHI(4),XC(4),XS(4),YC(4),YS(4),AXFA(4),FY(4)
      COMMON NN,A(4),B(4),ALFA(4),FI(4)
      GO TO (61,62,113,61,62,61,62),NN
  113 STOP
   61 DO 100   K=1,4
      AXFA(K)=ALFA(K)*0.017453
      FY(K)=FI(K)*0.017453
      PHI(K)=AXFA(K)+FY(K)
      XC(K)=A(K)*COS(AXFA(K))*COS(PHI(K))+B(K)*SIN(AXFA(K))*SIN(PHI(K))
      XS(K)=A(K)*COS(AXFA(K))*SIN(PHI(K))-B(K)*SIN(AXFA(K))*COS(PHI(K))
      YC(K)=A(K)*SIN(AXFA(K))*COS(PHI(K))-B(K)*COS(AXFA(K))*SIN(PHI(K))
```

```
      YS(K)=A(K)*SIN(AXFA(K))*SIN(PHI(K))+B(K)*COS(AXFA(K))*COS(PHI(K))
  100 CONTINUE
      GO TO 64
   62 DO 63 KK=1,4
      XC(KK)= A(KK)*COS(B(KK)*0.017453)
      XS(KK) =A(KK)*SIN(B(KK)*0.017453)
      YC(KK) =-ALFA(KK)*SIN(FI(KK)*0.017453)
      YS(KK) = ALFA(KK)*COS(FI(KK)*0.017453)
   63 CONTINUE
   64 P=XC(1)*YC(3)-YS(3)*XS(1)-XC(3)*YC(1)+XS(3)*YS(1)
      Q=XS(1)*YC(3)+YS(3)*XC(1)-XS(3)*YC(1)-YS(1)*XC(3)
      AA1=YC(1)*XC(4)-YS(1)*XS(4)-XC(2)*YC(3)+XS(2)*YS(3)
      AA2=YS(1)*XC(4)+YC(1)*XS(4)-XS(2)*YC(3)-YS(3)*XC(2)
      BB1=XC(3)*XC(2)-XS(3)*XS(2)-XC(1)*XC(4)+XS(1)*XS(4)
      BB2=XS(3)*XC(2)+XC(3)*XS(2)-XS(1)*XC(4)-XC(1)*XS(4)
      CC1=YC(1)*YC(4)-YS(1)*YS(4)-YC(3)*YC(2)+YS(3)*YS(2)
      CC2=YS(1)*YC(4)+YC(1)*YS(4)-YC(3)*YS(2)-YS(3)*YC(2)
      DD1=XC(3)*YC(2)-XS(3)*YS(2)-XC(1)*YC(4)+XS(1)*YS(4)
      DD2=XS(3)*YC(2)+XC(3)*YS(2)-XS(1)*YC(4)-XC(1)*YS(4)
      DELTA=P**2 +Q**2
      W=1./DELTA
      XXK = W*(P*AA1+Q*AA2)
      XXC = W*(Q*AA1-P*AA2)
      XYK = W*(P*BB1+Q*BB2)
      XYC = W*(Q*BB1-P*BB2)
      YXK = W*(P*CC1+Q*CC2)
      YXC = W*(Q*CC1-P*CC2)
      YYK = W*(P*DD1+Q*DD2)
      YYC = W*(Q*DD1-P*DD2)
      RETURN
      END
```

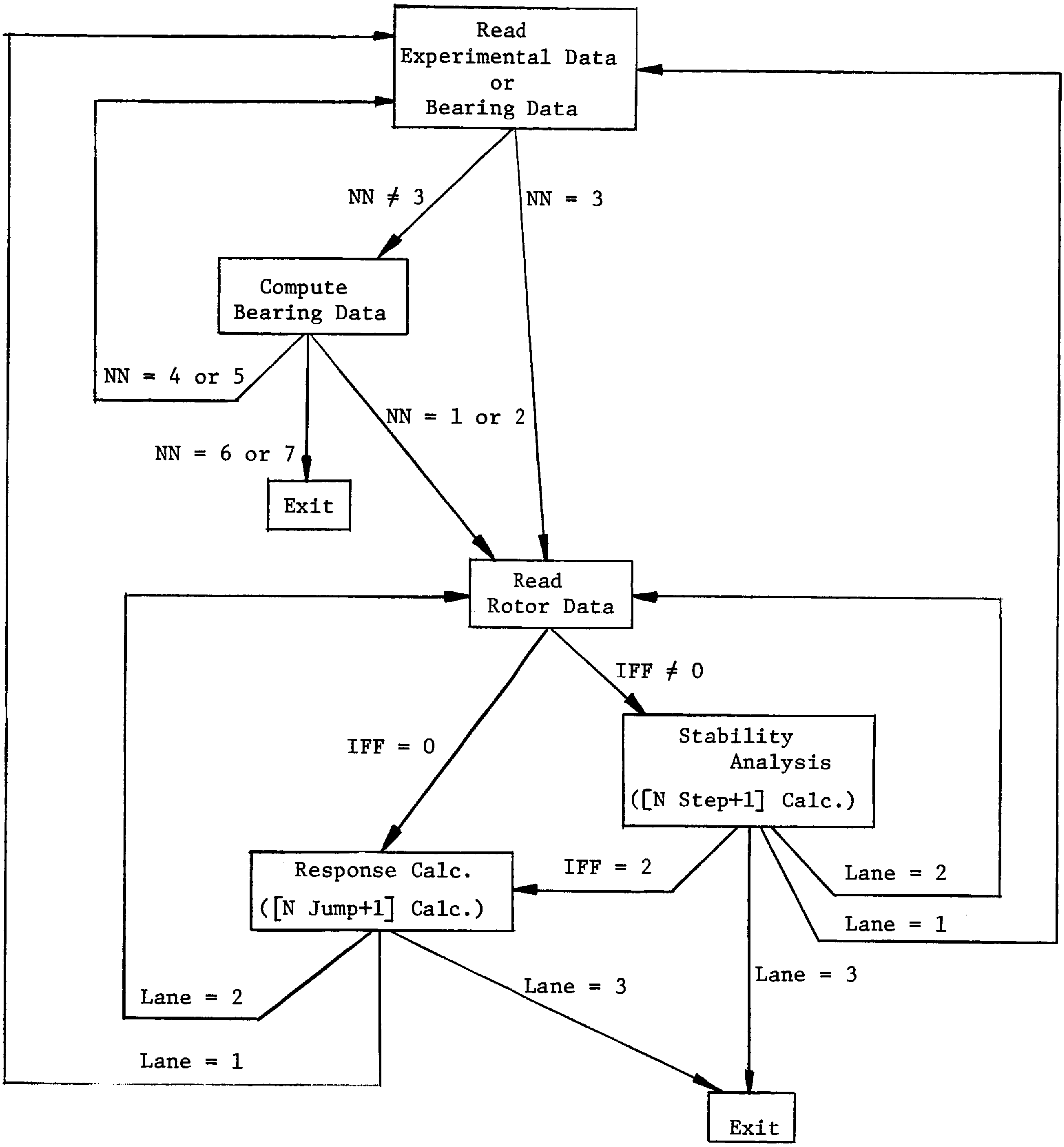


Figure 8. Flow Chart.

APPENDIX C

ROTOR RESPONSE EXAMPLES

ROTOR RESPONSE EXAMPLES

The following computations are to be made and a listing of the input cards for them is presented in Appendix D.

1. Experimental data for bearing constants

$\|x_1\| = 6.324$	$\phi_{x1} = -71.57$	$\|Y_1\| = 5.0$	$\phi_{y1} = -53.13$
$\|F_{x1}\| = 72.11$	$\phi_{fx1} = 77.82$	$\|F_{y1}\| = 154.6$	$\phi_{fy1} = 10.35$
$\|x_2\| = 3.606$	$\phi_{x2} = 33.68$	$\|Y_2\| = 6.403$	$\phi_{y2} = 231.3$
$\|F_{x2}\| = 37.35$	$\phi_{x2} = 64.6$	$\|F_{y2}\| = 97.27$	$\phi_{f2} = -25.18$

The output gives that:

$\bar{K}_{xx} = 4.051$	$\bar{C}_{xx} = 5.715$	$\bar{K}_{xy} = -2.416$	$\bar{C}_{xy} = 6.877$
$\bar{K}_{yx} = 9.667$	$\bar{C}_{yx} = 6.820$	$\bar{K}_{yy} = -1.756$	$\bar{C}_{yy} = 18.80$

2. Experimental data for bearing constants

$a_1 = .2269$	$b_1 = .1244$	$\alpha_1 = -44$	$\bar{\Phi}_1 = -68.14$
$a_2 = .3$	$b_2 = .2855$	$\alpha_2 = -75.8$	$\bar{\Phi}_2 = 175.5$
$a_3 = .5521$	$b_3 = .3019$	$\alpha_3 = -45.3$	$\bar{\Phi}_3 = -90.54$
$a_4 = .7785$	$b_4 = .6174$	$\alpha_4 = -78.97$	$\bar{\Phi}_4 = 156.8$

The output:

$\bar{K}_{xx} = 3.766$	$\bar{C}_{xx} = -6.572$	$\bar{K}_{xy} = -2.438$	$\bar{C}_{xy} = -5.748$
$\bar{K}_{yx} = 4.660$	$\bar{C}_{yx} = -5.589$	$\bar{K}_{yy} = -1.142$	$\bar{C}_{yy} = -8.290$

3. Dynamic analysis:

$\bar{K}_{xx} = 3.647$, $\bar{C}_{xx} = -6.62$, $\bar{K}_{xy} = -2.542$, $\bar{C}_{xy} = -5.674$

$\bar{K}_{xy} = 4.54$, $\bar{C}_{xy} = -5.674$, $\bar{K}_{yy} = -1.279$, $\bar{C}_{yy} = -8.234$

$\xi = .5$, $\eta = .5$, $\alpha_{55} = \alpha_{57} = \alpha_{77} = .2$, $\Delta\omega_{ns1} = 0$

$\alpha_{56} = \alpha_{58} = \alpha_{78} = .1$, $\omega_{ns1} = \omega_{ms1} = 1414.2$,

$\omega_{np} = 200$, $\bar{k} = 15$, $\delta_o = 2$, $\delta_1 = 1$,

$\omega = 100$, $\Delta\omega = 300$, $N_J = 6$

a) stability and unbalance calc. (rigid pedestals).
b) stability and unbalance calc. (zero bearing masses).
c) stability and unbalance calc. (flexible pedestals).

4. Stability and unbalance calculation

a) $\bar{K}_{xx} = 1.93$, $\bar{C}_{xx} = 16.13$, $\bar{K}_{xy} = 8.26$, $\bar{C}_{xy} = 2.93$

$\bar{K}_{yx} = -6.94$, $\bar{C}_{yx} = 3.44$, $\bar{K}_{yy} = 3.34$, $\bar{C}_{yy} = 14.7$

b) $\bar{K}_{xx} = 10.89$, $\bar{C}_{xx} = 31.88$, $\bar{K}_{xy} = 17.09$, $\bar{C}_{xy} = 10.60$

$\bar{K}_{yx} = -5.68$, $\bar{C}_{yx} = 11.49$, $\bar{K}_{yy} = 10.82$, $\bar{C}_{yy} = 18.04$

c) $\bar{K}_{xx} = 44.26$, $\bar{C}_{xx} = 76.93$, $\bar{K}_{xy} = 41.68$, $\bar{C}_{xy} = 24.02$

$\bar{K}_{yx} = .233$, $\bar{C}_{yx} = 26.28$, $\bar{K}_{yy} = 24.42$, $\bar{C}_{yy} = 24.74$

Rotor: Weight = 7.13 lb., $I_T = 746 \times 10^{-3}$ lb-in-sec^{-2}

b = 12.08, ηb = 19, dia. 1.25

Therefore, using appendix E,

$$M = \frac{7.13}{2 \times 386} = 9.25 \times 10^{-3} \qquad \xi = \left[\frac{2 \times 746 \times 10^{-3}}{9.25 \times 10^{-3} \times 146}\right]^{1/2} = 1.05$$

$$\eta = 1.57, \; I = \frac{\pi D^4}{64} = .119, \; E = 30 \times 10^6$$

$$\alpha_{55} = \frac{25 \times 10^{-4} \times 1.05 \times 1.78 \times 10^3}{24 \times 30 \times 10^6 \times .119} = .546 \times 10^{-7}$$

$$\alpha_{56} = .52 \times 10^{-7}$$

$$\alpha_{77} = \frac{.325 \times 2.57 \times 1.78 \times 10^3}{24 \times 30 \times 10^6 \times .119} = 1.74 \times 10^{-5}$$

$$\alpha_{78} = .675 \times 10^{-5}$$

$$\alpha_{57} = \frac{1.78 \times 10^3 \times .497}{48 \times 30 \times 10^6 \times .119} = .513 \times 10^{-5}$$

$$\alpha_{58} = \frac{1.78 \times 10^3 \times .57 \times .05}{12 \times 30 \times 10^6 \times .119} = .118 \times 10^{-5}$$

$$\omega_{ns1} = \left[\frac{1}{9.25 \times 10^{-3} \times 1.066 \times 10^{-7}}\right]^{1/2} = .381 \times 10^5$$

$$m_o = \frac{.378}{386} = .98 \times 10^{-3} \qquad \delta_o = \delta_1 = .019$$

$$\omega_{ms1} = \left[\frac{1}{.98 \times 10^{-3} \times 2.415 \times 10^{-5}}\right]^{1/2} = .65 \times 10^4$$

Assume $W = \lambda\omega = 100$ and $C = .005$, or $\frac{W}{C} = 2 \times 10^4$

Then,

$\bar{\alpha}_{55} = .001096$, $\bar{\alpha}_{56} = .00104$, $\bar{\alpha}_{77} = .348$, $\bar{\alpha}_{78} = .135$

$\bar{\alpha}_{57} = .1026$, $\bar{\alpha}_{58} = .0236$, $\bar{\delta}_o = 3.8 = \bar{\delta}_1$

Remark: In the example 4, the value of ξ is so close to 1 that the shaft natural frequency became unrealistically high. In such cases, it is suggested that the value of ξ be chosen so that the particular frequency, ω_{ns1} or ω_{ns2}, become a realistic value. For example, if one finds that the first natural frequency of the hinged-hinged shaft is 2,850 rad/sec., it implies that $\alpha_{55} + \alpha_{66} = .134 \times 10^{-4}$, and therefore, $\xi = .385$.

The results may be put into figures as done in Ref. (3) and (4).

5. A cylindrical bearing with L/D = 1 has the following values for the parameters:

i) $S = .1$

$\bar{K}_{xx} = 2.95$	$\bar{C}_{xx} = 5.7$	$\bar{K}_{xy} = 3.35$	$\bar{C}_{xy} = 1.66$
$\bar{K}_{yx} = -.12$	$\bar{C}_{yx} = 1.6$	$\bar{K}_{yy} = 2.0$	$\bar{C}_{yy} = 1.9$

ii) $S = .3$

$\bar{K}_{xx} = 1.5$	$\bar{C}_{xx} = 6.8$	$\bar{K}_{xy} = 3.6$	$\bar{C}_{xy} = 2.2$
$\bar{K}_{yx} = -2.2$	$\bar{C}_{yx} = 2.3$	$\bar{K}_{yy} = 2.1$	$\bar{C}_{yy} = 5.5$

iii) $S = .5$

$\bar{K}_{xx} = 1.36$ $\bar{C}_{xx} = 8.9$ $\bar{K}_{xy} = 5.0$ $\bar{C}_{xy} = 2.0$

$\bar{K}_{yx} = -3.7$ $\bar{C}_{yx} = 2.2$ $\bar{K}_{yy} = 2.18$ $\bar{C}_{yy} = 8.0$

iv) $S = .8$

$\bar{K}_{xx} = 1.22$ $\bar{C}_{xx} = 11.5$ $\bar{K}_{xy} = 7.0$ $\bar{C}_{xy} = 2.0$

$\bar{K}_{yx} = -6.0$ $\bar{C}_{yx} = 2.2$ $\bar{K}_{yy} = 2.19$ $\bar{C}_{yy} = 10.6$

a) Make a stability chart for the bearing.

b) A 7.13 lb rotor has the first natural frequency of 2,850 rad/sec with hinged-hinged ends. Find speeds at the threshold of instability. Assume $C = .0015$, $D = 1.25$, $\mu = 12 \times 10^{-8}$.

Knowing the natural frequency of the rotor, one finds $\alpha_{55} + \alpha_{66} = .134 \times 10^{-4}$ and compute $\xi = .385$ using the formulas in Appendix E.

With various values of $\bar{\alpha}$, a stability chart (Fig. 9) is obtained. Then the line for $\alpha = .135 \times 10^{-4}$ is cross plotted as shown by a dotted line (Figure 9).

6. A test rig has the following specifications:

Rotor: Weight = 7 lbs., b = 12.5, D = 1.25", first critical speed = 2,840 Rad/sec.

Bearings: 80°, 4 shoe tilting pads, L/D = 1, C'/C = 1

$C_D = .003"$, $\mu = 8.3 \times 10^{-8}$ lbs.-sec./in.^{2}

Load: 5 lbs.

Unbalance Mass: Weight = .378 lbs.

Eccentricity = .00095"

η b = 19"

Make response calculations for $500 < \omega < 4000$

Input Preparation:

$M = .00875$, $\alpha = 1/M\omega n^2 = .141 \times 10^{-4}$

$\bar{\alpha} = \frac{W}{C}\alpha = \bar{\alpha}_{55} + \bar{\alpha}_{56} = .47$ $I = \frac{\pi}{64} D^4 = .12$

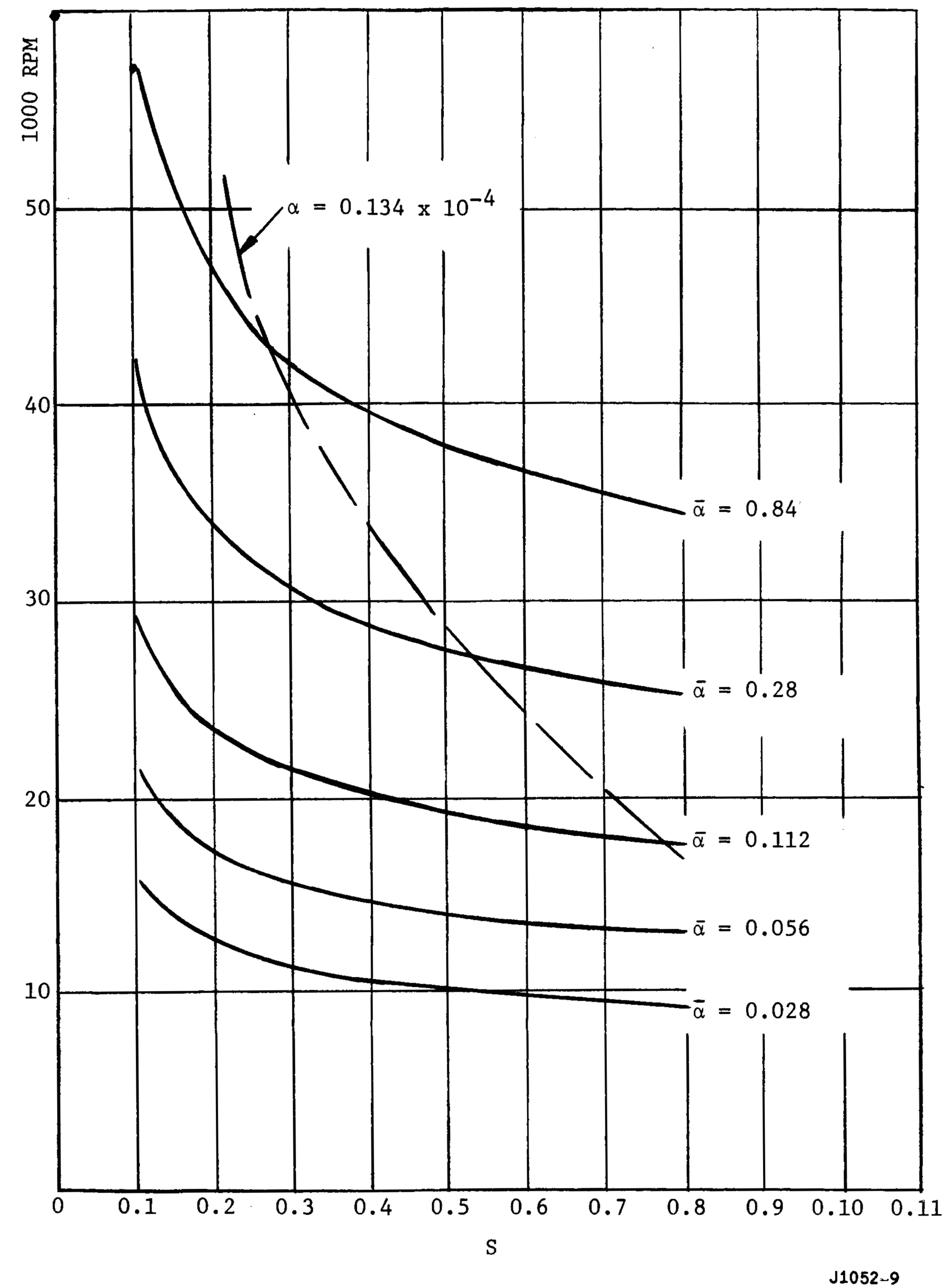


Figure 9. Stability Chart for Example 5.

Using $\alpha = \frac{b^3}{48EI} \left\{ (1 - \xi^2)^2 + (1 - \xi)^2 \left[2 - (1 - \xi)^2 \right] \right\}$

One gets $\xi = .385$

Hence

$\bar{\alpha}_{55} = .0258$ $\bar{\alpha}_{56} = .0216$

$\bar{\alpha}_{57} = -.0293$ $\bar{\alpha}_{58} = -.0226$

$\bar{\alpha}_{77} = .0593$ $\bar{\alpha}_{78} = .0231$

$\delta/C_R = .633$

For bearing parameters, one may use data given in Air Force Technical Report, AFAPL-TR-65-45. There the spring and damping coefficients are plotted as a function of Sommerfeld number. For this example,

$$S = .00713\omega$$

Thus, for a given ω, $\bar{K}$ and $\bar{C}$ are obtained.

The results may be plotted as in Figure 10.

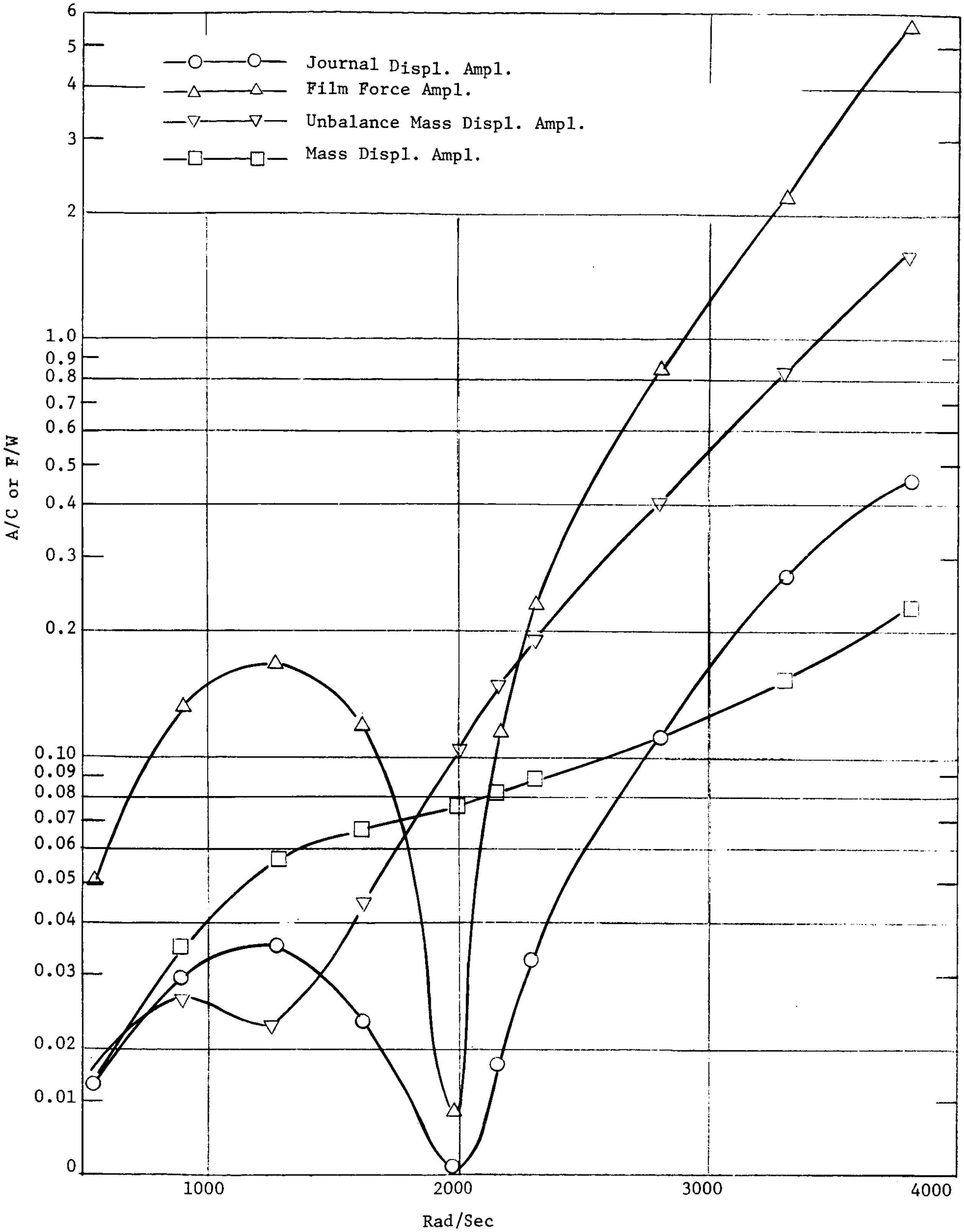


Figure 10. Example 6.

APPENDIX D

LISTING OF INPUT CARDS FOR EXAMPLES

```
 HARMONIC DATA    (EXAMPLE 1)
5 6.324          -71.57          5.          -53.13
 72.11          77.82          154.6          10.35
  3.606          33.68          6.403          231.3
 37.35          64.60          97.27          -25.18
 ELLIPTICAL DATA  (EXAMPLE  2)
4  .2269          .1244          -44.          -68.14
    .3          .2855          -75.8          175.489
  .5521          .3 19          -45.3          -90.54
 .7785          .6174          -78.97          156.79
   TEST RUN NO.3 II +  III(EXAMPLE  3)
3  3.647          -6.62          -2.542          -5.674
 4.54           5.674          -1.279          -8.234
  .5          .2          .1          1414.2
 .0          .0          2     2
  .5     .2          .1          .2          .1          1414.2
  2.          2.          .0          100.          300.          6
  .5          .2          .1          1414.2
  .0          15.          2     2
  .5     .2          .1          .2          .1          1414.2
  2.          2.          .0          100.          300.          6
  .5          .2          .1          1414.2
  200.          15.          1     2     3  500.
  .5     .2          .1          .2          .1          1414.2
  2.          2.          .0          100.          300.          6
       TEST RUN NO. 3A  (EXAMPLE  4A)
3   1.93          16.13          8.26          2.93
  -6.94          3.44          3.34          14.7
```

```
 1.05         . 1096       .00104     38100.
 .0           .0          1    2
1.57      .348      .135      .1026      .0236      6500.
 3.8       3.8       .0       5000.      4000.      9
   TEST RUN NO. 3B   (EXAMPLE  4B)
3  10.89       31.88       17.09       10.6
 -5.68       11.49        10.82       18.04
 1.05         . 1096       .00104     38100.
 .0           .0          1    2
1.57      .348      .135      .1026      .0236      6500.
 3.8       3.8       .0       5000.      4000.      12
  TEST RUN NO. 3C    (EXAMPLE  4C)
3  44.26       76.93       41.68          24.02
 .233        26.28        24.42       24.74
 1.05         . 1096       .00104     38100.
 .0           .0          1    2
1.57      .348      .135      .1026      .0236      6500.
 3.8       3.8       .0       5000.      4000.      12
  CYL  L/D = 1    S = .1  (EXAMPLE  5-1)
3   2.95       5.7         3.35         1.66
  -.12          1.6         2.          1.9
 .385         . 15         .013        2840.
.0               .0       2    1
  .385          .03          .026        2840.
    .0          .0        2    1
 .385         .06          .052       2840.
    .0          .0        2    1
 .385        .15          .13             2840.
```

```
      .0                .0             2    1
    .385              .45                 .39             2840.
      .0                .0             1    1
    CYL   L/D = 1      S = .3  (EXAMPLE   5-2)
3   1.5              6.8           3.6                2.2
   -2.2            2.3           2.1                5.5
   .385            . 15             .013          2840.
      .0                .0             2    1
     .385               .03              .026          2840.
      .0                .0             2    1
    .385              .06             .052        2840.
      .0                .0             2    1
    .385             .15              .13                2840.
      .0                .0             2    1
    .385              .45                 .39             2840.
      .0                .0             1    1
      CYL     L/D = 1      S = .5  (EXAMPLE   5-3)
3   1.36            8.9            5.                 2.02
   -3.7            2.2             2.18               8.
   .385            . 15             .013          2840.
      .0                .0             2    1
     .385               .03              .026          2840.
      .0                .0             2    1
    .385              .06             .052        2840.
      .0                .0             2    1
    .385             .15              .13                2840.
      .0                .0             2    1
    .385              .45                 .39             2840.
```

	•0	•0	1	1	
	CYL	L/D = 1	S = •8	(EXAMPLE 5-4)	
3	1•22	11•5	7•		2•0
	-6•	2•2	2•19		10•6
	•385	15	•013		2840•
	•0	•0	2	1	
	•385	•03	•026		2840•
	•0	•0	2	1	
	•385	•06	•052		2840•
	•0	•0	2	1	
	•385	•15	•13		2840•
	•0	•0	2	1	
	•385	•45	•39		2840•
	•0	•0	3	1	

(EXAMPLE 6)

```
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=530RAD/S S=0.384
 3    1.7            3.5
                                      1.7           3.5
  .385              .0258          .0216          2840.
      .0                .0            1
 1.57        -.0293       -.0226        .0593         .0231        6620.
    .633         .633          .0          530.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=898RAD/S S=0.64
 3    1.6            4.2
                                      1.6           4.2
  .385              .0258          .0216          2840.
      .0                .0            1
 1.57        -.0293       -.0226        .0593         .0231        6620.
    .633         .633          .0          898.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=1258RD/S S=0.896
 3    1.5          4.5
                                      1.5             4.5
  .385              .0258          .0216          2840.
      .0                .0            1
 1.57        -.0293       -.0226        .0593         .0231        6620.
    .633           .633        .0          1258.          800.       8
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=1611RD/S S=1.15
 3   1.4               5.
                                         1.4          5.
  .385              .0258          .0216          2840.
      .0                .0            1
 1.57        -.0293       -.0226        .0593         .0231        6620.
    .633         .633          .0          1611.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=1980RD/S S=1.41
 3   1.             6.
                                           1.          6.
  .385              .0258          .0216          2840.
      .0                .0            1
 1.57        -.0293       -.0226        .0593         .0231        6620.
    .633         .633          .0          1980.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=2160RD/S S=1.54
 3     .97            6.5
                                   .97                6.5
  .385              .0258          .0216          2840.
                                      1
 1.57        -.0293       -.0226        .0593         .0231        6620.
    .633         .633          .0          2160.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=2300RD/S S=1.64
 3      .96          7.
                                    .96              7.
  .385              .0258          .0216          2840.
      .0                .0            1
 1.57        -.0293       -.0226        .0593         .0231        6620.
    .633         .633          .0          2300.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=2800RD/S S=2.
 3    .9           7.5
                                      .9              7.5
  .385              .0258          .0216          2840.
      .0                .0            1
 1.57        -.0293       -.0226        .0593         .0231        6620.
```

```
     .633        .633            .0          2800.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=3300RD/S S=2.36
 3       .85        8.
                                       .85                 8.
   .385                .0258          .0216            2840.
        .0              .0              1
  1.57        -.0293        -.0226        .0593        .0231         6620.
     .633        .633            .0          3300.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=3800RD/S S=2.71
 3   .75             10.
                                      .75                 10.
   .385                .0258          .0216            2840.
        .0              .0              1
  1.57        -.0293        -.0226        .0593        .0231         6620.
     .633        .633            .0          3800.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=4300RD/S S=3.06
 3   .7                10.5
                                        .7               10.5
   .385                .0258          .0216            2840.
        .0              .0              1
  1.57        -.0293        -.0226        .0593        .0231         6620.
     .633        .633            .0          4300.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=4800RD/S S=3.42
 3    .67                11.
                                      .67                 11.
   .385                .0258          .0216            2840.
        .0              .0              1
  1.57        -.0293        -.0226        .0593        .0231         6620.
     .633        .633            .0          4800.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=5300RD/S S=3.78
 3     .6             13.
                                       .6                13.
   .385                .0258          .0216            2840.
        .0              .0              1
  1.57        -.0293        -.0226        .0593        .0231         6620.
     .633        .633            .0          5300.
3MIL TESTRIG4SHOE TILT C/C=L/D=1 L/B=1.432 W=5 LB OMEG=5800RD/S S=4.14
 3       .6            14.
                                        .6                14.
   .385                .0258          .0216            2840.
                                        3
  1.57        -.0293        -.0226        .0593        .0231         6620.
    .633          .633           .0          5800.        400.         6
```

APPENDIX E

INFLUENCE COEFFICIENTS FOR UNIFORM ROTOR

The influence coefficients of uniform hinged-hinged beam is readily obtained from the strength of materials. In the following E is the Young's modulus and I is the area moment of inertia.

I For $\xi < 1$ and $\eta > 1$

$$\alpha_{55} = \frac{(1-\xi^2)^2}{48EI} b^3$$

$$\alpha_{56} = \frac{(1-\xi)^2 b^3}{48EI} \left[2-(1-\xi)^2\right]$$

$$\alpha_{77} = \frac{(\eta-1)^2(\eta+1)}{24EI} b^3$$

$$\alpha_{78} = \frac{(\eta-1)^2}{24EI} b^3$$

$$\alpha_{57} = -\frac{(\eta-1)(1+\xi)}{96EI} \left[4-(1+\xi)^2\right] b^3$$

$$\alpha_{58} = -\frac{(\eta-1)(1-\xi)}{96EI} \left[4-(1-\xi)^2\right] b^3$$

II For $\xi > 1$, $\eta > 1$ and $\eta > \xi$,

$$\alpha_{55} = \frac{(\xi-1)^2(\xi+1)}{24EI} b^3$$

$$\alpha_{56} = \frac{(\xi-1)^2}{24EI} b^3$$

$$\alpha_{77} = \frac{(\eta-1)^2(\eta+1)}{24EI} \quad b^3$$

$$\alpha_{78} = \frac{(\eta-1)^2}{24EI} \quad b^3$$

$$\alpha_{57} = \frac{b^3}{48EI} \left[(\eta-\xi)^3 - (\eta-1)^3 + (\eta-1)(3\eta+1)(\xi-1) \right]$$

$$\alpha_{58} = \frac{b^3}{24EI} \quad (\eta-1)(\xi-1)$$

III For $\xi < 1, \eta < 1$ and $\eta > \xi$

$$\alpha_{55} = \frac{(1-\xi^2)^2}{48EI} \quad b^3$$

$$\alpha_{56} = \frac{(1-\xi)^2 b^3}{48EI} \left[2-(1-\xi)^2 \right]$$

$$\alpha_{77} = \frac{(1-\eta^2)^2}{48EI} \quad b^3$$

$$\alpha_{78} = \frac{(1-\eta)^2 b^3}{48EI} \left[2-(1-\eta)^2 \right]$$

$$\alpha_{57} = \frac{(1-\eta)(1+\xi)}{96EI} \quad b^3\left[4-(1+\xi)^2 - (1-\eta)^2\right]$$

$$\alpha_{58} = \frac{(1-\eta)(1-\xi)}{96EI} \quad b^3\left[4-(1-\xi)^2 - (1-\eta)^2\right]$$

For cases where $\eta < \xi$, one merely interchanges ξ and η in the above formulas.

REFERENCES

(1) E. Schnetzer: Hydrodynamic Journal Bearing Program, Quarterly #1 Contract NAS 3-6479, July 29, 1965

(2) F. K. Orcutt, C. W. Ng, J. H. Vohr, and E. B. Arwas: Lubrication Analysis in Turbulent Regime, First Quarterly Progress Report on Contract NAS W-1021, NASA CR-54195, October 1, 1964

(3) "Bearing Attenuation", General Electric Company, Report No. 61GL100, 1961, Contract no. NObs 78930

(4) "The Effect of the 150-Degree Partial Bearing on Rotor-Unbalance Vibration", P.C. Warner and R. J. Thoman, Journal of Basic Engineering, ASME Transactions, June 1964.

(5) J. W. Lund, Discussion of Reference 4.

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