X-507-66-256

NASA TM X- 55578

AIMP OPTICAL ASPECT COVERAGE — THREE DIMENSIONAL APPROXIMATION

BY

F. R. CHROMEY

JUNE 1966

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-507-66-256

AIMP OPTICAL ASPECT COVERAGE – THREE DIMENSIONAL APPROXIMATION

by

F. R. Chromey

June, 1966

Goddard Space Flight Center
Greenbelt, Maryland

AIMP OPTICAL ASPECT COVERAGE – THREE DIMENSIONAL APPROXIMATION

by

F. R. Chromey

ABSTRACT

This study derives an approximate method for determining the duration of coverage of the sunlit earth by the AIMP optical aspect system. The derivation is designed so that coverage data may be obtained through minor modifications of an existing computer program (QUIMP). Results are applicable to any spin-stabilized vehicle with a similar optical aspect system. Plots of the aspect system coverage for several AIMP launches are included.

CONTENTS

AIMP OPTICAL ASPECT COVERAGE – THREE DIMENSIONAL APPROXIMATION

INTRODUCTION

The Anchored IMP optical aspect system contains three telescopes whose optical axes are aligned at 45°, 90°, and 135° to the vehicle spin axis. Determination of vehicle orientation involves a consideration of the times during which the system can see the sunlit earth through any of these telescopes.

Reference 1 attempts to find the length of the period immediately after injection during which data may be obtained from the optical aspect system. It does this by calculating the maximum and minimum angles (measured with respect to the spin axis and known hereafter as α_{MAX} and α_{MIN}) of telescope alignment between which a portion of the sunlit earth will be viewed in the course of an instantaneous rotation of the vehicle about its spin axis. The two dimensional method used in reference 1, however, has been found to compute values of α_{MAX} and α_{MIN} which yield coverages of significantly shorter duration than would be realized in actual fact. The object of this analysis is to provide a more rigorous approximation of these angles. The results, of course, may be applied to any spin-stabilized vehicle.

Since an exact determination of the AIMP telescope viewing times is being developed, this analysis limits itself to providing optical aspect coverage data through a minimum of computer programming effort. In particular, it is intended that all data be obtained through minor modifications of the existing QUIMP computer program (QUIMP is itself a version of the Quick-Look Mission Analysis Program modified for the AIMP launch philosophy).

METHOD

Assumptions:

The following quantities are calculated by QUIMP and are considered as known at any specified time in the AIMP trajectory:

SEVA – the sun-earth-vehicle angle,

$\vec{R}$ – the position vector from the center of the earth to the vehicle,

$\vec{S}^{\circ}$ – a unit vector along the vehicle spin axis (input or computed parallel to the velocity vector at injection),

RVE – the angle between $\vec{S}^{\circ}$ and $-\vec{R}$. (Spin axis-earth angle).

$\vec{R}_s$ – the position vector from the center of the sun to the vehicle,

SDA – the earth semi-disk angle as seen from the vehicle. These quantities are illustrated in Figure 1.

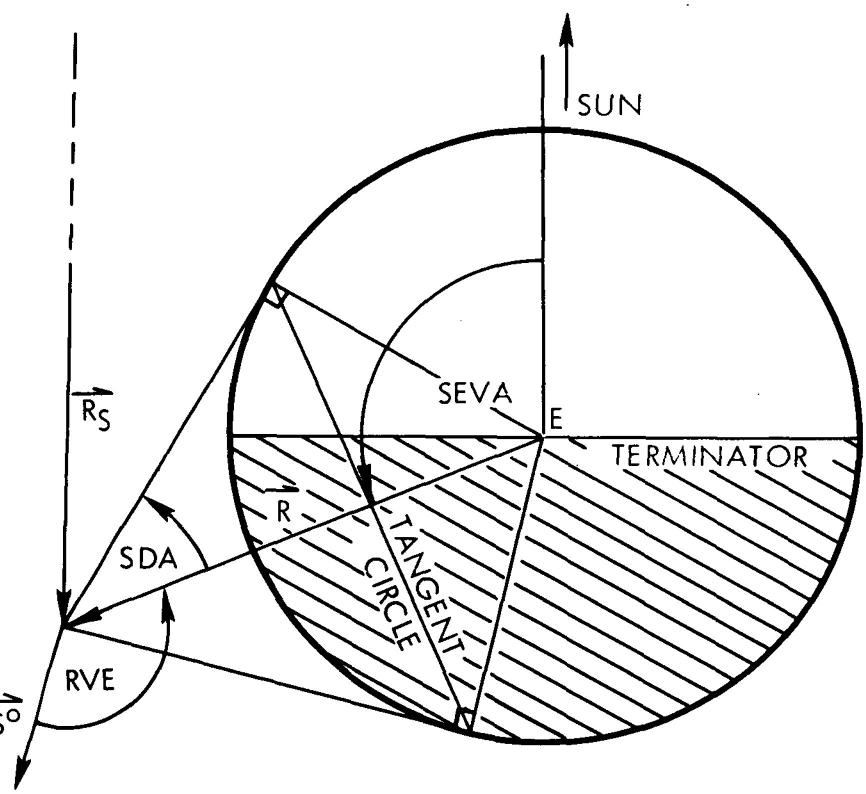


Figure 1

Now consider Figure 2, where the plane of the paper is such as to contain both $\vec{S}^{\circ}$ and $\vec{R}$. The dotted line is the tangent plane and the dashed ellipse is the terminator. The curve FGF' represents the intersection of a right circular cone of half-angle α and axis $\vec{S}^{\circ}$ with the earth. Let us take another view of this situation, this time looking antiparallel to $\vec{R}$, projecting all lines into the tangent

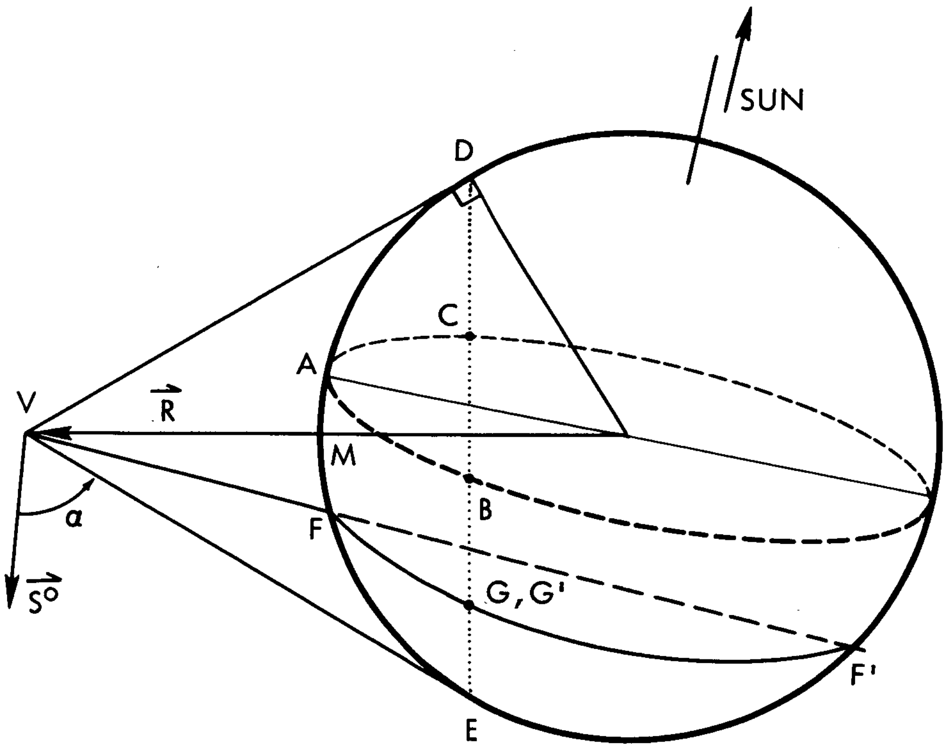


Figure 2

plane. Such a view is presented in Figure 3. In addition to curve G'FG, we have drawn in the intersections of the cones of axis $\vec{S}^{\circ}$ which contain the points A, B and C. Call the half-angles of these cones α_A, α_B, and α_C, respectively, measured from $\vec{S}^{\circ}$. Similarly, label the half-angles of the cones passing through points D and E as α_D and α_E.

We now make the following assumption:

In all cases, both α_{MAX} and α_{MIN} are equal to angles in the following list of seven; α_A, α_B, α_C, α_D, α_E, 0°, and 180°.

Note that there are cases where this assumption might be justified only as an approximation, as in Figure 4 (i.e. where it is possible to draw a cone-earth intersection tangent to the terminator on the dark side at a point other than A, B, or C).

Note also that the assumptions made in reference 1 (two-dimensional geometry) are equivalent to restricting the terminator plane perpendicular to the plane of Figure 2, and to eliminating α_B and α_C as possible values for α_{MAX} and α_{MIN}.

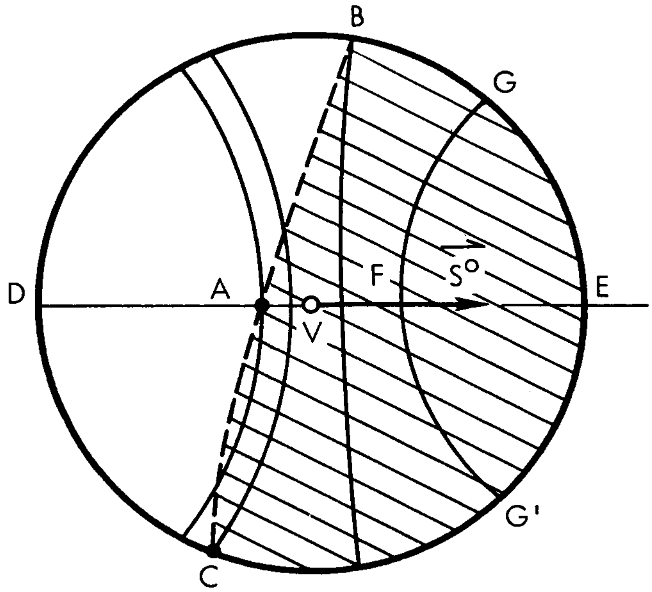


Figure 3

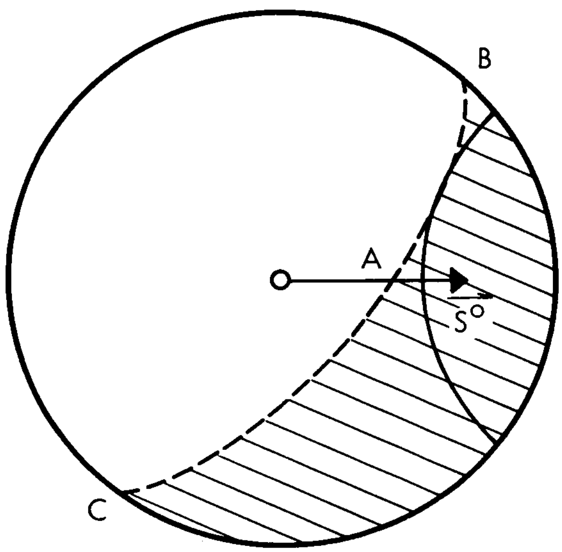


Figure 4

We must now develop both a method for calculating α_A, α_B, α_C, α_D, and α_E, and a method for determining which of the seven angles in our list to select as α_{MAX} and α_{MIN}.

Mathematics

Consider Figure 5. We define a coordinate system centered at the vehicle as follows:

$$\hat{X}^0 \equiv \frac{\vec{R}}{|\vec{R}|}, \quad \hat{Z}^0 \equiv \frac{\vec{R} \times \vec{R}_s}{|\vec{R} \times \vec{R}_s|}, \quad \hat{Y}^0 \equiv \hat{Z}^0 \times \hat{X}^0$$

We write $\vec{S}$, the representation of $\vec{S}^0$ in this new system, as

$$\vec{S} = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

where

$$S_1 = \vec{S}^0 \cdot \hat{X}^0, \; S_2 = \vec{S}^0 \cdot \hat{Y}^0, \; S_3 = \vec{S}^0 \cdot \hat{Z}^0.$$

We now draw vectors $\vec{D}_A$, $\vec{D}_B$ and $\vec{D}_C$ from the vehicle to points A , B , and C respectively, and define

$$\text{VET} = 90° \cdot - \text{SEVA}$$

Now, to find $\vec{D}_B$ and $\vec{D}_C$, we consider spherical triangle NMB and write, from the law of sines:

$$\frac{\sin (180 - \beta)}{\sin (90)} = \frac{\sin (\text{VET})}{\sin (90 - \text{SDA})}$$

so that

$$\beta = \text{Sin}^{-1} \left[\frac{\sin (\text{VET})}{\cos (\text{SDA})} \right] :$$

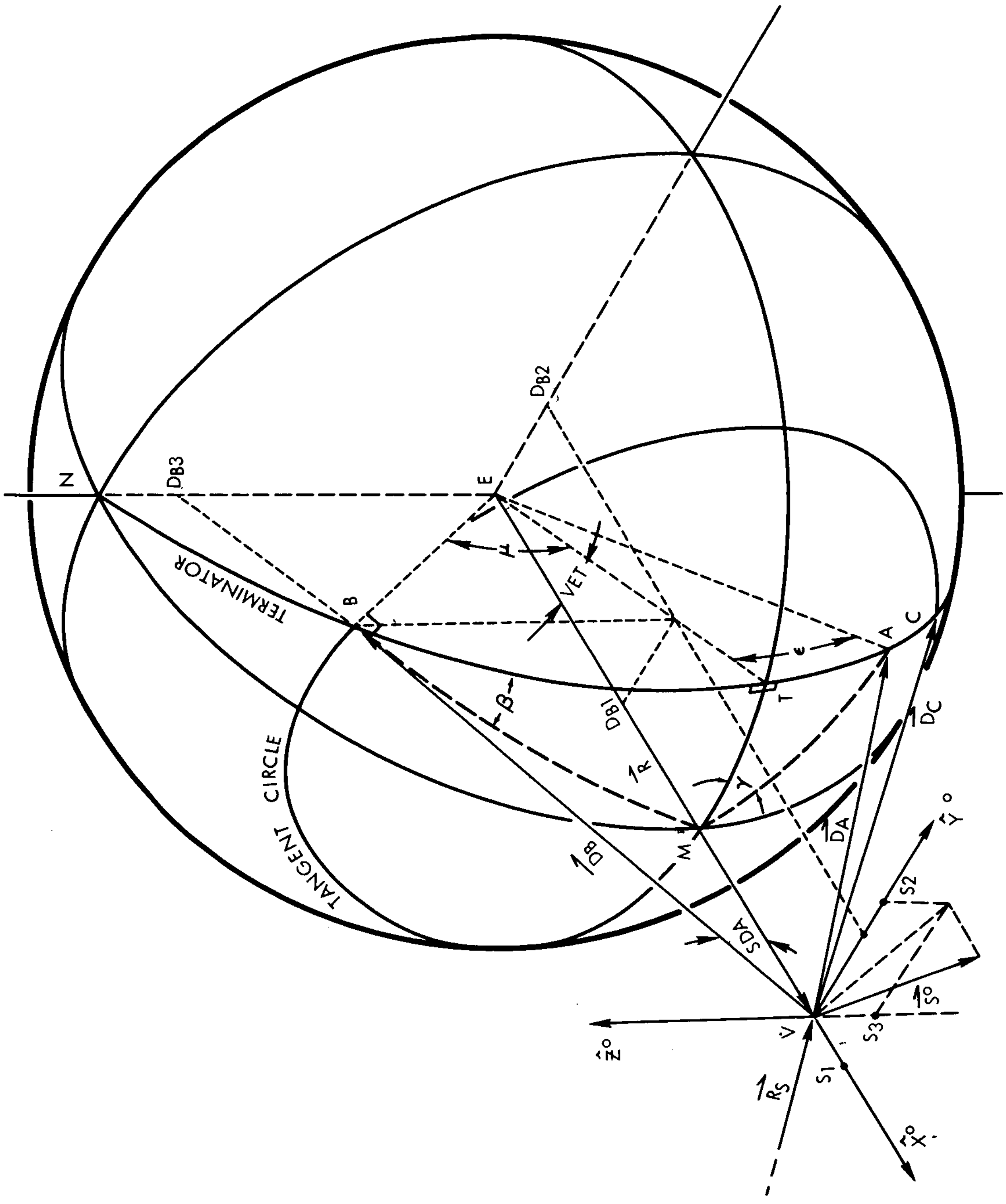

N
E
B
M
A
C
T
V
DB3
DB2
DB1
TERMINATOR
TANGENT CIRCLE
VET
SDA
S1
S2
S3

Figure 5

If we consider right spherical triangle MBT, we may write

$$\sin \mu = \tan (\mathrm{VET}) \operatorname{ctn} \beta$$

$$\mu = \mathrm{Sin}^{-1} [\tan(\mathrm{VET}) \operatorname{ctn} \beta]$$

Now, we let

$$\vec{D}_B = \begin{pmatrix} D_{B1} \\ D_{B2} \\ D_{B3} \end{pmatrix}, \text{and } \vec{D}_C = \begin{pmatrix} D_{B1} \\ D_{B2} \\ -D_{B3} \end{pmatrix}$$

in our coordinate system and write, from Figure 5,

$$D_{B1} = [R_e \cos(\mu) \cos(\mathrm{VET})] - |\vec{R}|,$$

where R_e is the radius of the earth,

$$D_{B2} = R_e \cos(\mu) \sin(\mathrm{VET}),$$

and

$$D_{B3} = |R_e \sin(\mu)|$$

We find $\vec{D}_A$ through similar procedures: We note

$$\gamma = \mathrm{Tan}^{-1} \frac{S_3}{S_2}$$

and, considering right spherical triangle MTC, write

$$\sin \mathrm{VET} = \tan \epsilon \cot \gamma$$

so that

$$\epsilon = \mathrm{Tan}^{-1}\left[\frac{S_3 \cdot \sin(\mathrm{VET})}{S_2}\right]$$

thus, if

$$\vec{D}_A = \begin{pmatrix} D_{A1} \\ D_{A2} \\ D_{A3} \end{pmatrix}$$

in our coordinate system, we may write from Figure 5

$$D_{A1} = [R_e \cos(\epsilon) \cos(\mathrm{VET})] - |\vec{R}|$$

$$D_{A2} = R_e \cos(\epsilon) \sin(\mathrm{VET})$$

$$D_{A3} = R_e \sin(\epsilon)$$

We are now in a position to compute

$$\alpha_A = \cos^{-1}\left(\frac{\vec{S} \cdot \vec{D}_A}{|\vec{S}|\ |\vec{D}_A|}\right)$$

$$\alpha_B = \cos^{-1}\left(\frac{\vec{S} \cdot \vec{D}_B}{|\vec{S}|\ |\vec{D}_B|}\right)$$

$$\alpha_C = \cos^{-1}\left(\frac{\vec{S} \cdot \vec{D}_C}{|\vec{S}|\ |\vec{D}_C|}\right)$$

Note also that α_D or α_E may be equal to either

$$\mathrm{RVE} + \mathrm{SDA},$$

$$\mathrm{RVE} - \mathrm{SDA},$$

$$SDA - RVE,$$

or

$$360° - RVE - SDA$$

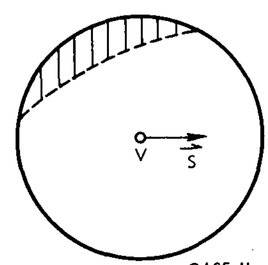
Logic

Thus, α_{MIN} and α_{MAX} may each be calculated in nine different ways (α_A, α_B, α_C, 0°, 180°, and the four values of α_D and α_E).

Of the 81 possible combinations, 51 may be eliminated as having no physical significance. The remaining thirty combinations may be conveniently distributed in nine cases arising from distinct geometric situations. Once the case is ascertained, α_{MIN} and α_{MAX} may be chosen through simple magnitude tests. The nine cases follow:

Case I: The sunlit portion of the earth is not visible from the vehicle. Set $\alpha_{MIN} = \alpha_{MAX} = 0°$

Case II: The disk either is completely illuminated or the situation is as shown at the right. In either event we may have either



CASE II

Case IIb: where $-\vec{S}$ extended will intersect the disk. Set

$$\alpha_{MIN} = RVE - SDA$$

$$\alpha_{MAX} = 180°$$

Case IIc: where $\vec{S}$ will intersect the disk if extended. Set

$$\alpha_{MIN} = 0°$$

$$\alpha_{MAX} = RVE + SDA$$

Case IIa: All other case II orientation of $\vec{S}$. Set

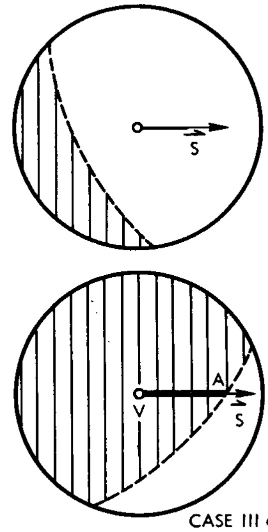
$$\alpha_{MIN} = RVE - SDA$$

$$\alpha_{MAX} = RVE + SDA$$

Case IIIa: Situation as shown at right where $\vec{S}$ will not intersect the disk if extended and $-\vec{S}$ will intersect neither the sunlit portion nor the portion between V and A if extended. Set

$$\alpha_{MIN} = RVE - SDA$$

$$\alpha_{MAX} = \text{the largest of } (\alpha_A\ \alpha_B \text{ and } \alpha_C)$$

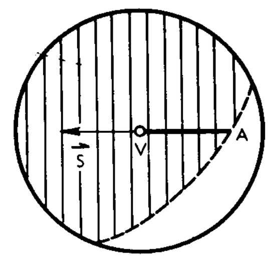


CASE III a

Case IIIb: Situation as shown at right where $-\vec{S}$ extended must intersect the dark portion of the disk between V and A. Set

α_{MIN} = the smallest of (α_B, α_C, and 360 - RVE - SDA)

α_{MAX} = the largest of (α_A, α_B, and α_C)



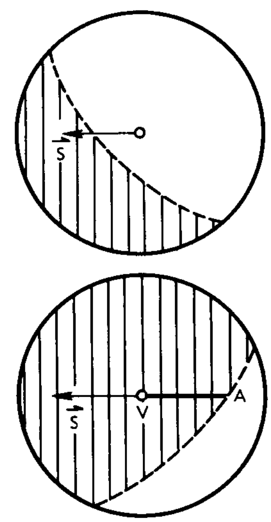
CASE III b

Case IVa: Situation illustrated at right. $-\vec{S}$ extended will not intersect the disk.

$\vec{S}$ extend will intersect neither the sunlit portion of the disk nor the darkend portion between V and A. Set

α_{MIN} = the smallest of (α_A, α_B, and α_C)

$$\alpha_{MAX} = RVE + SDA$$

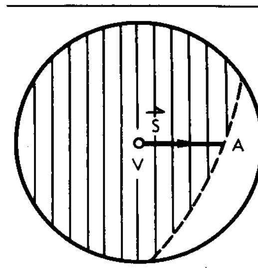


CASE IV a

Case IVb: Situation illustrated at right. $\vec{S}$ extended must intersect the dark portion of the desk between V and A. Set

α_{MIN} = the smallest of (α_A, α_B and α_C)

α_{MAX} = the largest of (α_B, α_C and SDA- RVE)

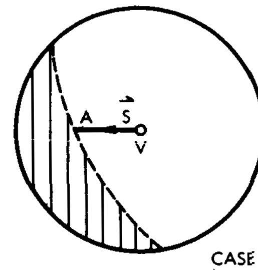


CASE IV b

CASE V: Situation illustrated at right. $-\vec{S}$ extended must intersect the sunlit portion of the disk between V and E and not between V and A. Set

$$\alpha_{MIN} = \text{the smallest of } (\alpha_A, \alpha_B, \alpha_C, 360-\text{RVE}-\text{SDA})$$

$$\alpha_{MAX} = 180°$$

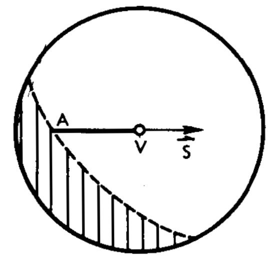


CASE V

Case VI: Situation illustrated at right, $-\vec{S}$ must intersect the sunlit portion of the disk between V and A. Set

$$\alpha_{MIN} = \text{the smallest of } (\alpha_A, \alpha_B, \alpha_C, \text{ and RVE}-\text{SDA})$$

$$\alpha_{MAX} = 180°$$

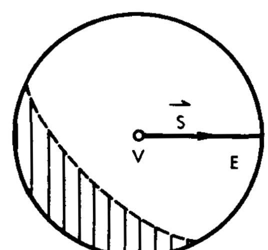


CASE VI

Case VII: Situation illustrated at right. $\vec{S}$ must intersect the sunlit portion of the disk between V and E. Set

$$\alpha_{MIN} = 0°$$

$$\alpha_{MAX} = \text{The largest of } (\alpha_A, \alpha_B, \alpha_C, \text{SDA} - \text{RVE})$$

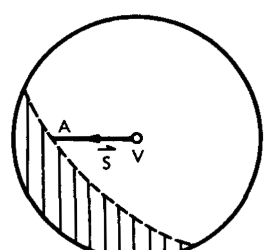


CASE VII

Case VIII: Situation illustrated at right. $\vec{S}$ must intersect the sunlit portion of the disk between V and A. Set

$$\alpha_{MIN} = 0$$

$$\alpha_{MAX} = \text{the largest of } (\alpha_A, \alpha_B, \alpha_C, \text{SDA} - \text{RVE})$$

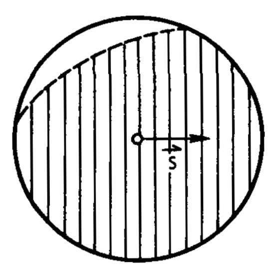


CASE VIII

Case IX: Situation illustrated at right. Either $\vec{S}$ and $-\vec{S}$ may or may not intersect the disk. Set

$$\alpha_{MIN} = \text{the smaller of } (\alpha_A \text{ and } \alpha_B)$$

$$\alpha_{MAX} = \text{the larger of } (\alpha_A \text{ and } \alpha_B).$$



CASE IX

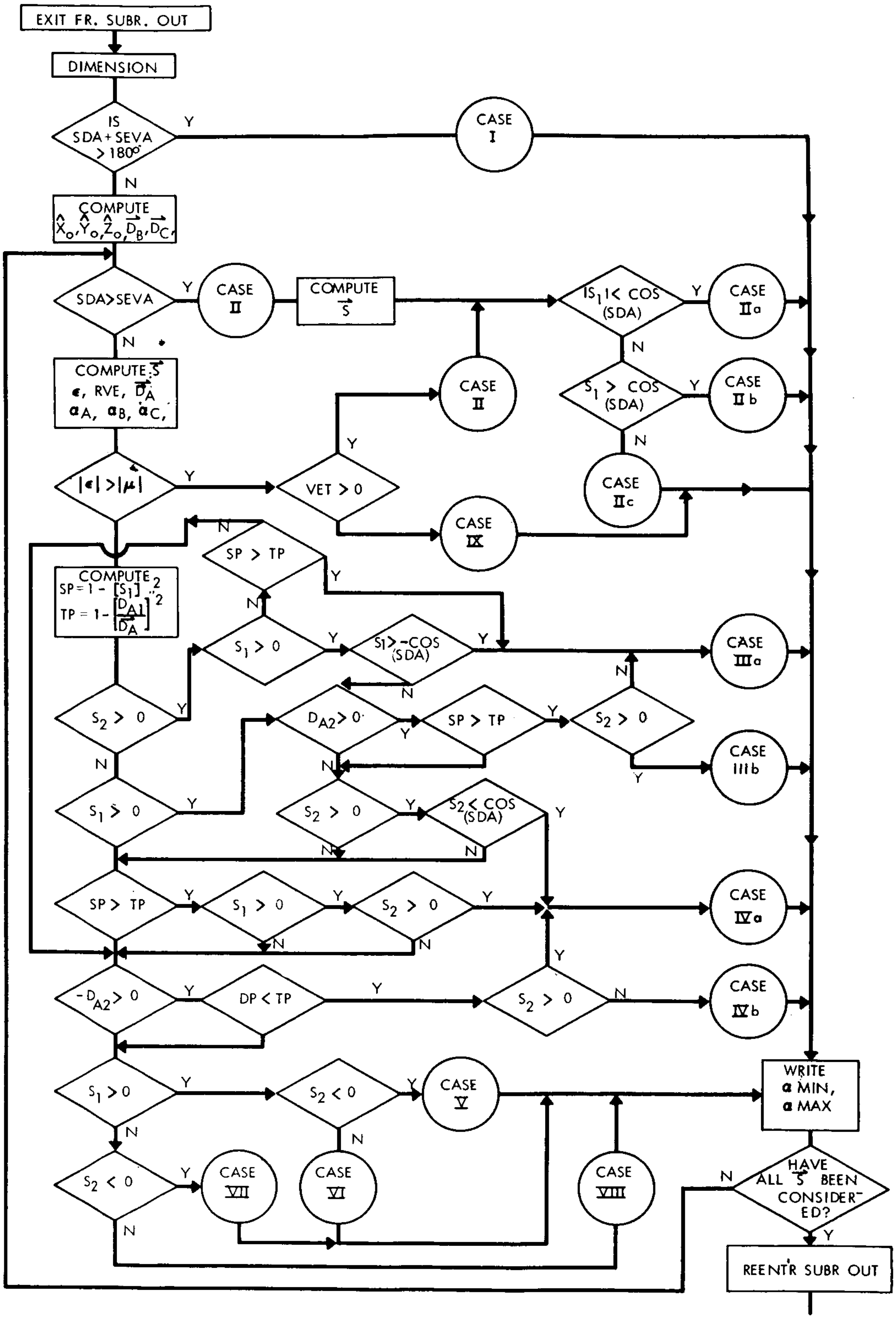

EXIT FR. SUBR. OUT
DIMENSION
IS SDA + SEVA > 180°
CASE I
COMPUTE
SDA>SEVA
CASE II
COMPUTE S
CASE IIa
CASE IIb
CASE IIc
COMPUTE S ϵ, RVE, DA αA, αB, αC,
CASE II
VET > 0
CASE IX
SP > TP
COMPUTE
CASE IIIa
CASE IIIb
CASE IVa
CASE IVb
DP < TP
WRITE α MIN, α MAX
CASE V
CASE VII
CASE VI
CASE VIII
HAVE ALL S BEEN CONSIDER-ED?
REENTR SUBR OUT

Figure 6

The logic used in determining the case of a particular situation in the modified version of QUIMP is flowcharted in Figure 6.

RESULTS

Plots of α_{MIN} and α_{MAX} versus time for a few sample AIMP launches, similar to those listed in Reference 3, are included in the appendix. In these cases $\vec{S}$ was taken parallel to the velocity vector at injection.

CONCLUSION

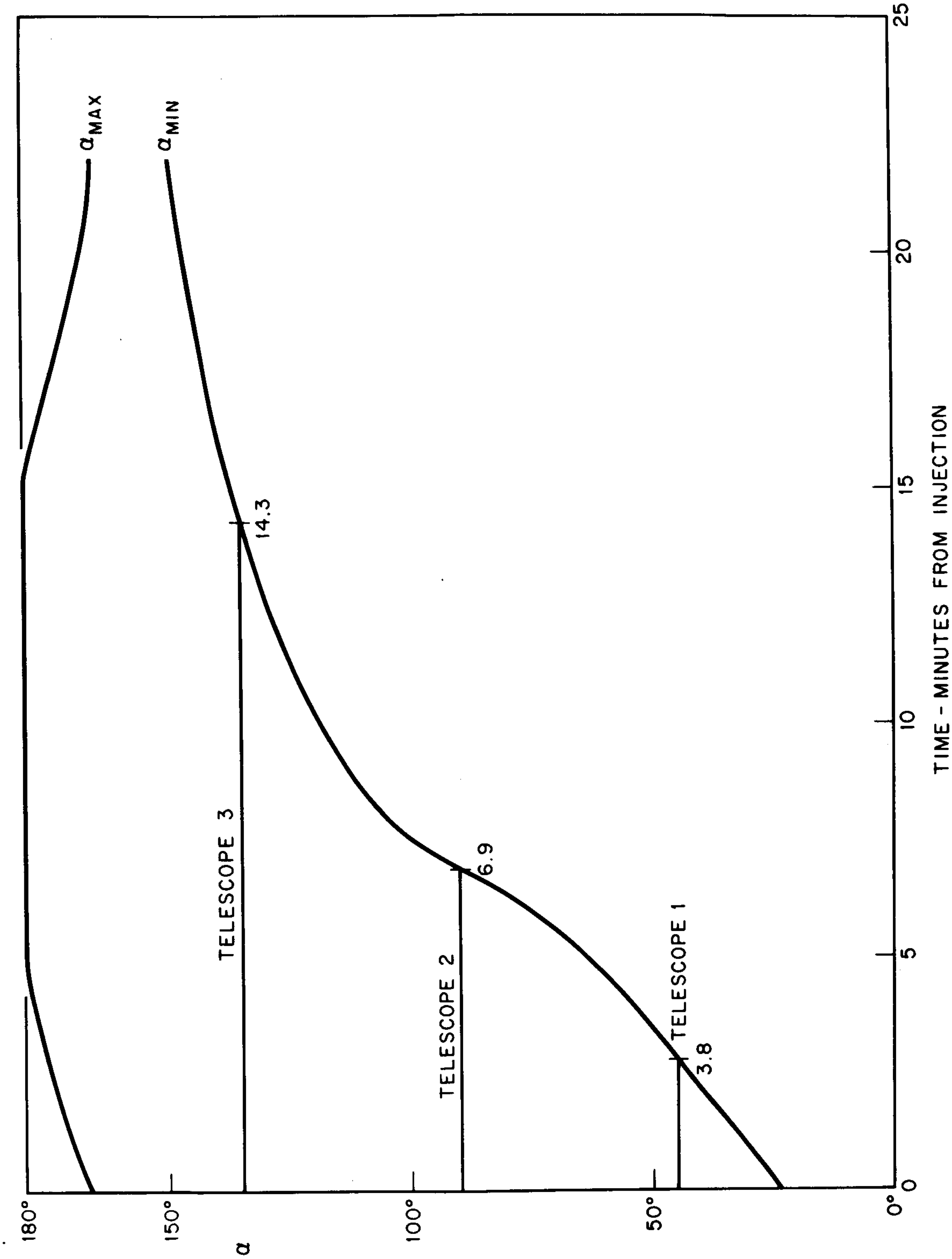
Preliminary checks with more exact determination of optical aspect coverage indicate the assumptions made in this study to be excellent for the cases considered.

REFERENCES

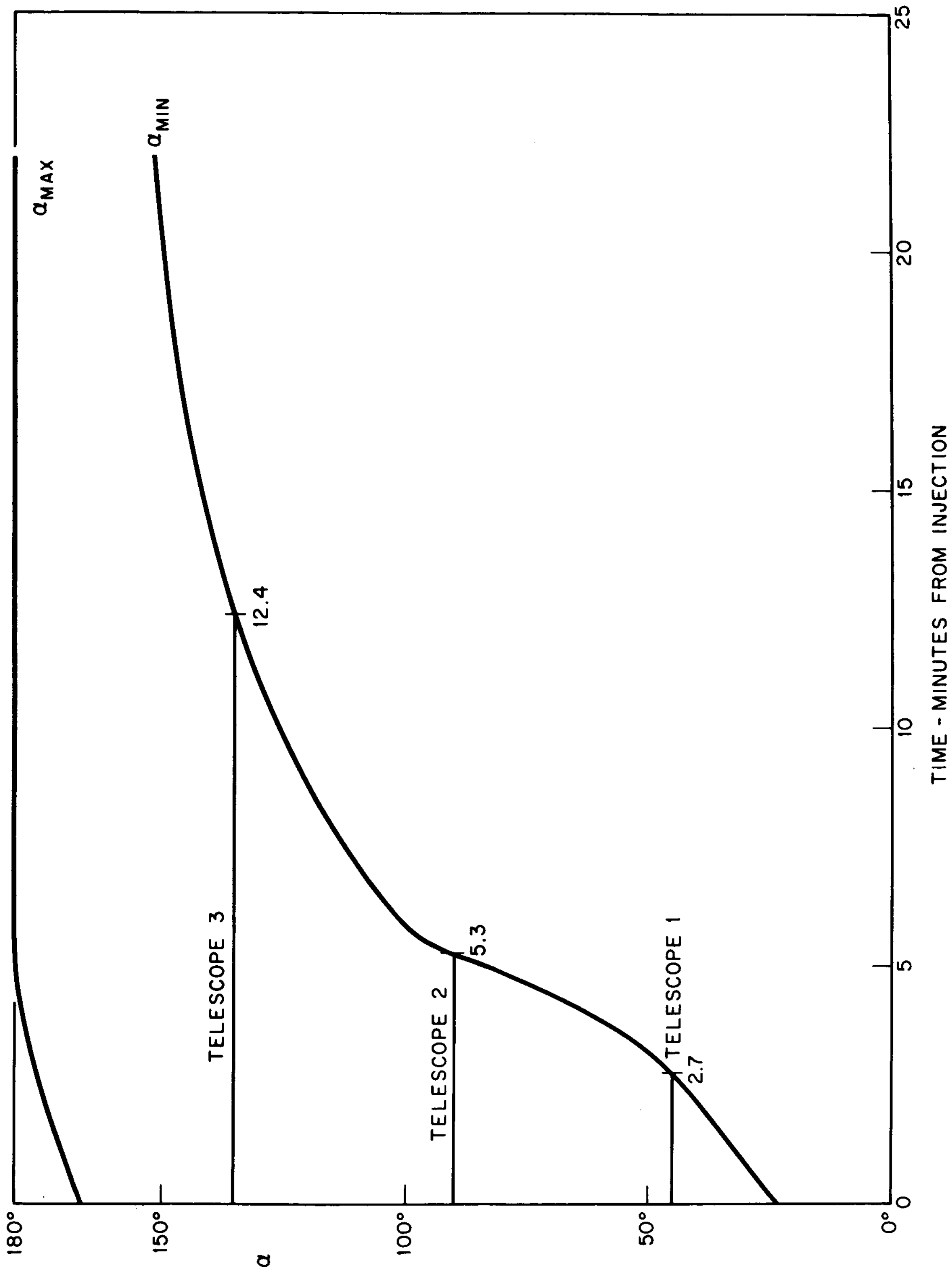
1. Groves, R. T., "AIMP Optical Aspect System Coverage for Early Post-Injection Attitude Determination," Systems Analysis Office Technical Brief, June 3, 1965.

2. "Programmer's Manual for Quick-Look Mission Analysis Program," prepared for Goddard Space Flight Center by Philco Corporation, WDL Division.

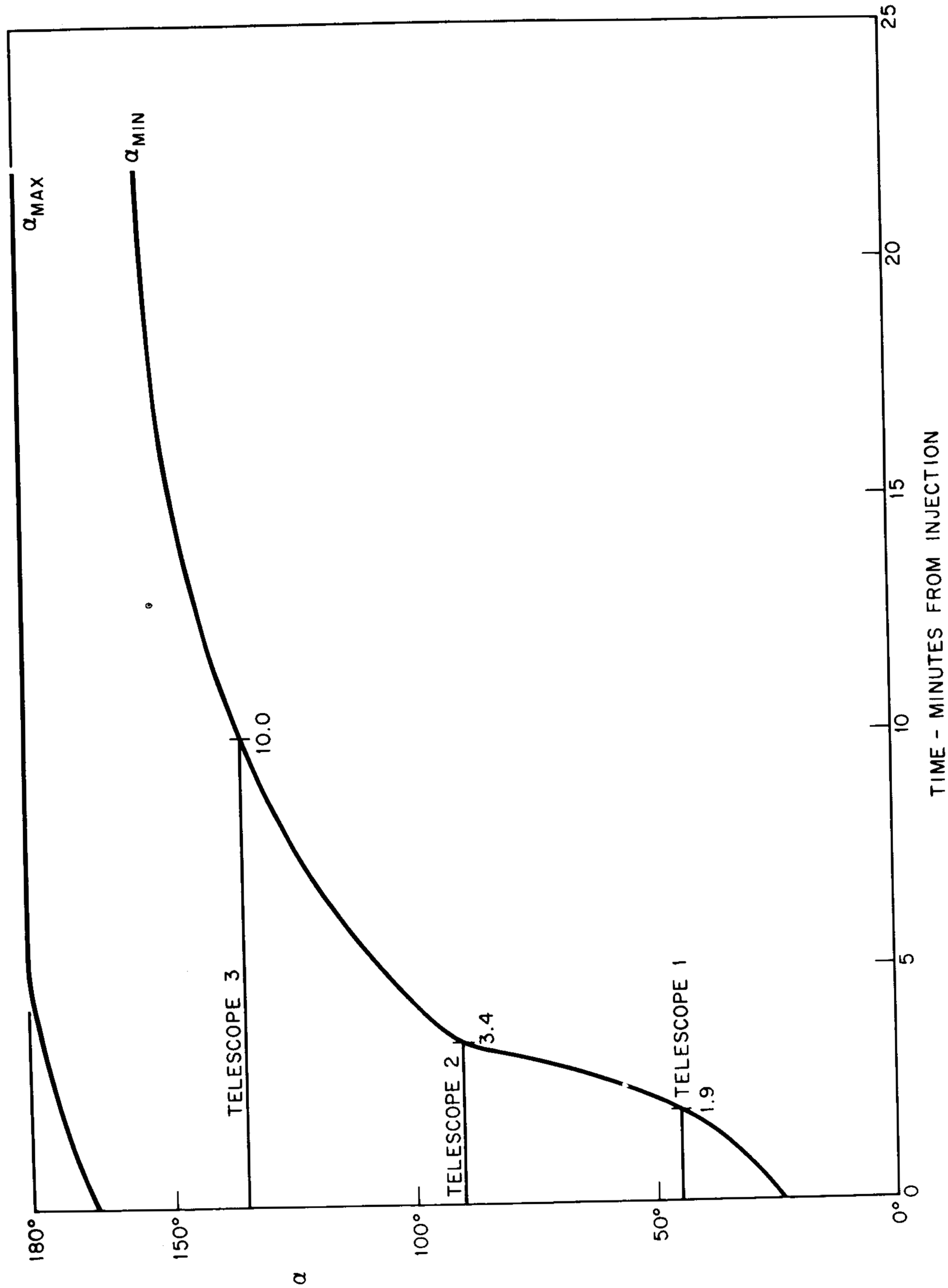
3. Groves, Fuchs, and Sandifer, "AIMP Mission Profile and Real Time Computing System," GSFC X-500-65-255, June 22, 1965.



AIMP – Injection at 1 July, 1966 $16^h21^m25.2^s$ GMT



AIMP – Injection at 2 July, 1966 $17^h16^m17.3^s$ GMT



AIMP – Injection at 3 July 1966 $18^h 9^m 51.9^s$ GMT