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ON THE LONG-PERIOD MOTION IN THE SEMI-MAJOR AXIS OF THE ORBIT OF THE TELSTAR 2 SATELLITE

BY

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GODDARD SPACE FLIGHT CENTER
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SUMMARY

A representation for the indirect effect of solar radiation pressure on the semi-major axis of an artificial earth satellite caused by passage in and out of the earth's shadow is examined. The theory is then used to explain the appearance of periodicities in the observed semi-major axis of the orbit of Telstar 2.

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ON THE LONG-PERIOD MOTION IN THE SEMI-MAJOR AXIS OF THE ORBIT OF THE TELSTAR 2 SATELLITE

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INTRODUCTION

The Keplerian elements of a close artificial earth satellite undergo perturbations not only due to the earth's oblateness, but also lunar and solar gravitational forces, solar radiation pressure, and atmospheric drag.

In the elements published by the Goddard Space Flight Center for the Telstar 2 satellite, the effects of the oblateness have been eliminated by the Brouwer Satellite Theory (Reference 1). Since the satellite's perigee height was about 975 kilometers, it can be considered to be relatively unaffected by atmospheric drag. The influence of lunar and solar gravitation and direct solar radiation pressure has already been analyzed for long period effects in the motion of this satellite (Reference 3). However, this satellite undergoes indirect solar radiation pressure effects due to the fact that the satellite passes in and out of the earth's shadow. This effect is best observed in the semi-major axis, since this element undergoes no long-period motion from any other source.

RADIATION PRESSURE DISTURBING FUNCTION

The disturbing function may be written as

$$R = F' \frac{A}{m} r \cos s \ , \tag{1}$$

where s is the geocentric angle between the sun and the satellite. Thus,

$$
\begin{aligned}
\cos s = {} & \cos^2 \frac{i}{2} \cos^2 \frac{\epsilon}{2} \cos\left(f + \omega + \Omega - \lambda_\odot\right) \\
& + \sin^2 \frac{i}{2} \cos^2 \frac{\epsilon}{2} \cos\left(f + \omega - \Omega + \lambda_\odot\right) \\
& + \cos^2 \frac{i}{2} \sin^2 \frac{\epsilon}{2} \cos\left(f + \omega + \Omega + \lambda_\odot\right) \\
& + \sin^2 \frac{i}{2} \sin^2 \frac{\epsilon}{2} \cos\left(f + \omega - \Omega - \lambda_\odot\right) \\
& + \frac{1}{2} \sin i \sin \epsilon \left[\cos\left(f + \omega - \lambda_\odot\right) - \cos\left(f + \omega + \lambda_\odot\right)\right]
\end{aligned}
\tag{2}
$$

Also,

$$
\begin{aligned}
r \cos f &= a(\cos E - e) \\
r \sin f &= a\left(1 - e^2\right)^{1/2} \sin E .
\end{aligned}
\tag{3}
$$

Substituting Equation (2) in Equation (1) and making use of Equations (3), the disturbing function R becomes

$$
R = Fa\left[S(\cos E - e) + V\sqrt{1 - e^2} \sin E\right] ,
$$

where

$$
\begin{aligned}
S = {} & -\cos^2 \frac{i}{2} \cos^2 \frac{\epsilon}{2} \cos\left(\omega + \Omega - \lambda_\odot\right) \\
& - \sin^2 \frac{i}{2} \cos^2 \frac{\epsilon}{2} \cos\left(\omega - \Omega + \lambda_\odot\right) \\
& - \cos^2 \frac{i}{2} \sin^2 \frac{\epsilon}{2} \cos\left(\omega + \Omega + \lambda_\odot\right) \\
& - \sin^2 \frac{i}{2} \sin^2 \frac{\epsilon}{2} \cos\left(\omega - \Omega - \lambda_\odot\right) \\
& - \frac{1}{2} \sin i \sin \epsilon \left[\cos\left(\omega - \lambda_\odot\right) - \cos\left(\omega + \lambda_\odot\right)\right] ,
\end{aligned}
$$

and

$$V = +\cos^2\frac{i}{2}\cos^2\frac{\epsilon}{2}\sin(\omega+\Omega-\lambda_\odot)$$
$$+\sin^2\frac{i}{2}\cos^2\frac{\epsilon}{2}\sin(\omega-\Omega+\lambda_\odot)$$
$$+\cos^2\frac{i}{2}\sin^2\frac{\epsilon}{2}\sin(\omega+\Omega+\lambda_\odot)$$
$$+\sin^2\frac{i}{2}\sin^2\frac{\epsilon}{2}\sin(\omega-\Omega-\lambda_\odot)$$
$$+\frac{1}{2}\sin i\ \sin\epsilon\left[\sin(\omega-\lambda_\odot)-\sin(\omega+\lambda_\odot)\right] ,$$

and where

$$F = F'\frac{A}{m}$$

THEORETICAL VARIATION IN SEMI-MAJOR AXIS

The differential equation representing the variation in the semi-major axis is

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial R}{\partial M} .$$

But,

$$\frac{\partial R}{\partial M}dt \approx \frac{1}{n}\frac{\partial R}{\partial E}dE .$$

Thus,

$$\delta a = \frac{2}{n^2 a}\int\frac{\partial R}{\partial E}dE ,$$

or,

$$\delta a_R = -2 \frac{a^3}{\mu} F \left[S \cos E + V \sqrt{1 - e^2} \sin E \right]_{E_1}^{E_2} ,$$

where E_1 and E_2 are the values of the eccentric anomaly of the satellite at exit from and entrance into the shadow of the earth, and where

$$\mu = G m' .$$

Thus δa_R is the perturbation after one revolution.

SHADOW CONDITION

In order to solve for the values of the eccentric anomaly at exit from and entrance into the shadow, a variation of the method suggested by Kozai in (Reference 2) is adopted. If the unit of distance is the radius of the earth, then the boundary of the shadow is expressed by

$$r \sin s = 1 . \tag{4}$$

We also have

$$\frac{r}{a} \cos s = - S (\cos E - e) - V \sqrt{1 - e^2} \sin E \tag{5}$$

and

$$\frac{r}{a} = 1 - e \cos E \tag{6}$$

If we assume S, V, and e are constants during one revolution, then the desired values for the eccentric anomaly may be obtained. However, the relations above result in a fourth degree equation in either $\cos E$ or $\sin E$. Thus, it is possible to solve the quartic for, say, $\cos E$ to obtain four real roots between zero and one and hence have eight possibilities for the two desired values of E. The number of possibilities is immediately reduced to four by solving the quartic for sin E also and testing whether the sum of the squares of a root from each equation equals unity. There can be at most four such pairs satisfying this condition. Two of these four roots are eliminated by testing whether the condition that cos s is negative is satisfied.

The two quartics in cos E and sin E may be obtained from equations (4), (5), and (6). If we let $x = \cos E$ and $y = \sin E$, the quartics are

$$p_0 x^4 + p_1 x^3 + p_2 x^2 + p_3 x + p_4 = 0$$

and

$$q_0 y^4 + q_1 y^3 + q_2 y^2 + q_3 y + q_4 = 0$$

where

$$\begin{aligned}
p_0 &= \gamma^2 + \phi^2 \\
p_1 &= 2\left(\beta\gamma - e\phi^2\right) \\
p_2 &= \beta^2 + 2\alpha\gamma - \phi^2\left(1 - e^2\right) \\
p_3 &= 2\left(\alpha\beta + e\phi^2\right) \\
p_4 &= \alpha^2 - e^2\phi^2 \\
q_0 &= p_0 \\
q_1 &= -2\phi(\beta + e\gamma) \\
q_2 &= \beta^2 - \phi^2(1 - e^2) - 2\mu\gamma \\
q_3 &= 2\phi\,(e\mu + \beta) \\
q_4 &= \mu^2 - \beta^2
\end{aligned}$$

and where

$$\alpha = 1 + a^2 \left[-1 + e^2 S^2 + V^2 \left(1 - e^2\right)\right]$$

$$\beta = 2a^2 e \left(1 - S^2\right)$$

$$\gamma = a^2 \left[S^2 - e^2 - V^2 \left(1 - e^2\right)\right]$$

$$\phi = -2 S V a^2 \sqrt{1 - e^2}$$

$$\mu = 1 - a^2 \left(1 + e^2\right) \left(1 - S^2\right)$$

If in solving the above quartics any complex roots or real roots of magnitude greater than one appear, then these roots are discarded and the process of testing for permissible values of eccentric anomaly continues. It is possible that there are no values satisfying the conditions for shadow. This indicates that the satellite does not enter the shadow on that revolution, and hence the value of δa_R for that revolution is zero.

APPLICATION TO THE ORBIT OF TELSTAR 2

An examination of the mean orbital elements of Telstar 2 reveals long-period variations in the semi-major axis, eccentricity, inclination, argument of perigee, and longitude of the ascending node. The mean elements referred to are the "double primed" variables of Brouwer (Reference 1) in which the effects due to the zonal harmonics through J_5 have been accounted for. The periodicities present in all but the first of these elements have been explained Reference (3). The variation in the mean semi-major axis a'' of Telstar 2 over a period of about 540 days is indicated in Figure (1). This variation was explained by the indirect effect of the solar radiation pressure. A computed perturbation for each revolution, δa_R, was calculated and added to the sum of the perturbations from each of the previous revolutions. In this way an accumulative δa (since the initial epoch) was obtained. In computing the perturbations between successive epochs, the values of the angular variables ω, Ω, and $\lambda_\odot$ were updated after each revolution. The values of the other elements of the sun and satellite were updated at each epoch. Explicit relations used in updating are given in the next section.

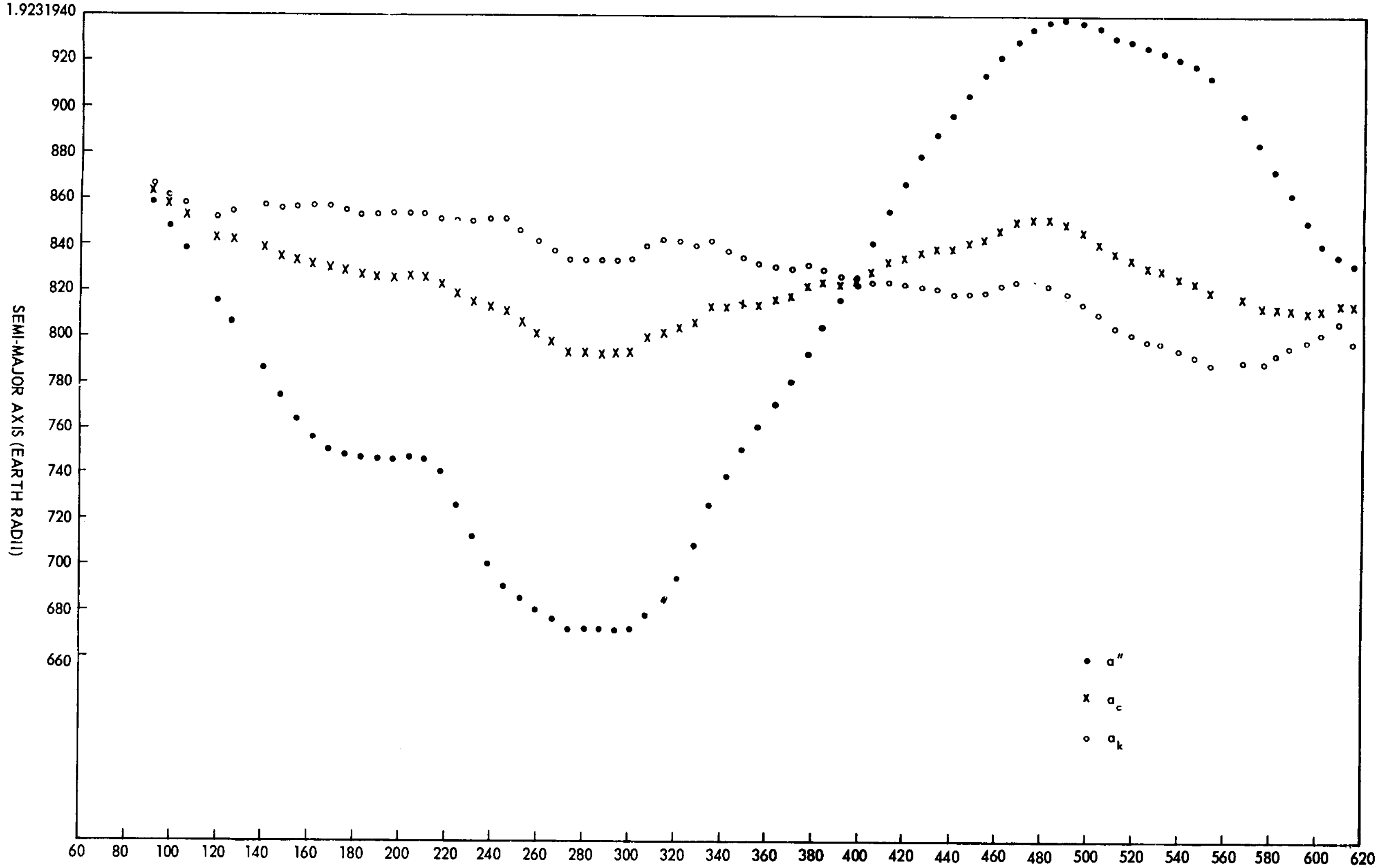

Figure 1 – Semi-major Axis Versus Time

After computing the value of δa since the initial epoch for each of the 77 epochs over almost 540 days, values of the semi-major axis constant a_c were obtained by

$$a_c = a'' - \delta a \quad .$$

It is interesting to note that, if the perturbations were multiplied by 4/3, the value of a_c would have remained much more constant than it did. Such a quantity, a_k, defined by

$$a_k = a'' - \frac{4}{3}\delta a \quad ,$$

would appear only to have a slight downward trend, perhaps due to a small amount of atmospheric drag, over the nearly 540 days of data.

Values of a'', a_c, and a_k over the period studied appears in Table (1) and graphs of a'', a_c and a_k versus time appear in Figure (1).

EPHEMERIS FOR THE SUN AND SATELLITE

The mean longitude of the sun is obtained after each revolution from (Reference 4):

$$\lambda_{\odot} = 279.^\circ 69668 + 36000.^\circ 76892T + 0^\circ.00030T^2 \quad ,$$

where T is the number of Julian Centuries since January 0.5, 1900. For January 1.0, 1960 the number of Julian days since January 0.5, 1900 is 21914.5.

Similarly, ω and Ω of the satellite are obtained after each revolution from

$$\omega = \omega_0 + \dot{\omega}\left(t - t_0\right)$$

$$\Omega = \Omega_0 + \dot{\Omega}\left(t - t_0\right) \quad ,$$

Time	a″	δa × 10⁷	a_c	a_k	Time	a″	δa × 10⁷	a_c	a_k	Time	a″	δa × 10⁷	a_c	a_k
77	1.9231871		1.9231871	1.9231871	287	1.9231672	-121	1.9231793	1.9231833	475	1.9231933	83	1.9231850	1.9231822
91	1.9231858	-6	1.9231864	1.9231866	294	1.9231672	-121	1.9231793	1.9231833	482	1.9231936	86	1.9231850	1.9231821
98	1.9231848	-10	1.9231858	1.9231861	300	1.9231672	-121	1.9231793	1.9231833	490	1.9231937	89	1.9231848	1.9231818
105	1.9231838	-15	1.9231853	1.9231858	307	1.9231678	-121	1.9231799	1.9231839	497	1.9231936	92	1.9231844	1.9231813
119	1.9231815	-28	1.9231843	1.9231852	314	1.9231683	-119	1.9231802	1.9231842	504	1.9231934	94	1.9231840	1.9231809
126	1.9231806	-36	1.9231842	1.9231854	321	1.9231694	-110	1.9231804	1.9231841	511	1.9231930	95	1.9231835	1.9231803
140	1.9231786	-53	1.9231839	1.9231857	328	1.9231708	-98	1.9231806	1.9231839	518	1.9231928	96	1.9231832	1.9231800
147	1.9231774	-61	1.9231835	1.9231855	335	1.9231726	-86	1.9231812	1.9231841	525	1.9231925	96	1.9231829	1.9231797
154	1.9231764	-69	1.9231833	1.9231856	342	1.9231738	-74	1.9231812	1.9231837	532	1.9231923	95	1.9231828	1.9231796
161	1.9231756	-76	1.9231832	1.9231857	349	1.9231750	-63	1.9231813	1.9231834	539	1.9231920	95	1.9231825	1.9231793
168	1.9231750	-80	1.9231830	1.9231857	356	1.9231760	-53	1.9231813	1.9231831	546	1.9231917	95	1.9231822	1.9231790
175	1.9231748	-80	1.9231828	1.9231855	363	1.9231770	-45	1.9231815	1.9231830	553	1.9231912	94	1.9231818	1.9231787
182	1.9231746	-80	1.9231826	1.9231853	370	1.9231780	-37	1.9231817	1.9231829	566.5	1.9231896	81	1.9231815	1.9231788
189	1.9231746	-80	1.9231826	1.9231853	377	1.9231792	-29	1.9231821	1.9231831	573.5	1.9231883	71	1.9231812	1.9231788
196	1.9231746	-80	1.9231826	1.9231853	384	1.9231804	-19	1.9231823	1.9231829	580.5	1.9231872	61	1.9231811	1.9231791
203	1.9231746	-80	1.9231826	1.9231853	391	1.9231815	-8	1.9231823	1.9231826	587.5	1.9231861	50	1.9231811	1.9231794
210	1.9231746	-80	1.9231826	1.9231853	398	1.9231825	2	1.9231823	1.9231822	594.5	1.9231849	39	1.9231810	1.9231797
217	1.9231740	-83	1.9231823	1.9231851	405	1.9231840	13	1.9231827	1.9231823	601.5	1.9231839	29	1.9231810	1.9231800
224	1.9231726	-93	1.9231819	1.9231850	412	1.9231854	23	1.9231831	1.9231823	608.5	1.9231833	21	1.9231812	1.9231805
231	1.9231712	-103	1.9231815	1.9231849	419	1.9231866	33	1.9231833	1.9231822	615.5	1.9231830	18	1.9231812	1.9231796
238	1.9231700	-113	1.9231813	1.9231851	426	1.9231878	43	1.9231835	1.9231821					
245	1.9231690	-121	1.9231811	1.9231851	433	1.9231888	51	1.9231837	1.9231820					
252	1.9231685	-121	1.9231806	1.9231846	440	1.9231897	59	1.9231838	1.9231818					
259	1.9231680	-121	1.9231801	1.9231841	447	1.9231905	65	1.9231840	1.9231818					
266	1.9231676	-121	1.9231797	1.9231837	454	1.9231913	71	1.9231842	1.9231818					
273	1.9231672	-121	1.9231793	1.9231833	461	1.9231921	75	1.9231846	1.9231821					
280	1.9231672	-121	1.9231793	1.9231833	468	1.9231928	79	1.9231849	1.9231823					

Table 1

Semi-major Axis of Telstar 2

where

$$\dot{\omega} = -\frac{3}{4} \frac{J_2 a_e^2}{a^2 \left(1 - e^2\right)^{3/2}} \left(1 - 3 \cos^2 i\right)$$

and

$$\dot{\Omega} = -\frac{3}{2} \frac{J_2 a_e^2}{a^2 \left(1 - e^2\right)^2} \cos i \quad ,$$

and where $\left(t - t_0\right)$ is measured in canonical units of 806.832 seconds. At each epoch, new values of the Keplerian elements are also used in evaluating S, V, δa, and the shadow conditions. In addition, updated values of ϵ are obtained from

$$\epsilon = 23^\circ\!.452294 - 0^\circ\!.013013T - 0^\circ\!.000002T^2 \quad .$$

CONCLUSIONS

In computing the perturbations, the multiplier $F = F' \; A/m$ must be evaluated. To calculate the presentation area A, it was assumed that the spacecraft is a sphere. However, this is only an approximation to its shape. In addition, its surface is faceted. Since at any one time only a portion of the surface of the satellite is exposed to sunlight, the presentation area is only a fraction of the total surface area. In computing the perturbations δa and the associated constant a_c, it was assumed that the presentation area is 1/4 of the total area. If we multiply δa by 4/3, which is equivalent to considering the presentation area to be 1/3 of the total area, the associated constant is a_k. If we assume that the semi-major axis decreases slightly due to drag, then inspection of the curve for a_k in Figure (1) indicates that perhaps a larger factor for F is in order.

Deviations of a_k from a straight line—especially near the end of the curve—may be due to a change in the level of solar activity. Changes in solar activity may result in the need to adjust the Radiation Pressure Constant F' from the value of -4.63×10^{-5} dynes/cm^2 which was used in computing δa.

Since similar periodicities occur in the semi-major axes of other satellites above the drag region (for example Relay 1 and Relay 2), an extended analysis of the motion of these satellites may result in an improved value of F' and of A for satellites of various shapes.

Re-radiation from the surface of the earth and the fact that the computed δa is only a first approximation to the total perturbation in semi-major axis may also aid in explaining the residual effects present in a_c or a_k.

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APPENDIX A

List of Symbols

A	Effective presentation area of the satellite
a	Semi-major axis of satellite's orbit
a_e	Mean equatorial radius of the earth (6378.388 km*)
E	Eccentric anomaly of satellite
e	Eccentricity of satellite's orbit
F'	Solar radiation pressure force constant $= -4.63 \times 10^{-5}$ dynes/cm²*
f	True anomaly of satellite
G	Gravitational constant
i	Inclination of satellite's orbit plane to earth's equatorial plane
J_2	Zonal harmonic coefficient in earth's gravitational potential (1.08219×10^{-3}*)
M	Mean anomaly of the satellite
m	Mass of the satellite
m'	Mass of the Earth
n	Mean motion of the satellite
R	Solar radiation pressure disturbing function
s	Cosine of the geocentric angle between the sun and the satellite
T	Number of Julian Centuries (of 36525 days) since January 0.5, 1900
x	$= \cos E$
y	$= \sin E$
$\lambda_\odot$	Mean longitude of the sun
Ω	Longitude of the ascending node of the satellite's orbit
$\dot{\Omega}$	Mean motion of Ω
ω	Argument of perigee of satellite's orbit
$\dot{\omega}$	Mean motion of ω
ϵ	Mean obliquity of the ecliptic

*Currently in use at Goddard Space Flight Center.