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SIMULATION OF AN OPTICAL ASPECT SYSTEM

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ABSTRACT

A method is described to calculate the angle through which a telescope mounted on a spinning spacecraft "sees" the sunlit portion of a specified central body. The central body is assumed to be a sphere and the spacecraft is assumed to make a full revolution at the given time. The equations of the terminator plane (the plane separating the sunlit and dark portions of the central body) and the tangent plane (the plane separating the portions of the central body that are visible and non-visible from the spacecraft) are derived.

Author

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SIMULATION OF AN OPTICAL ASPECT SYSTEM

I. INTRODUCTION

The Optical Aspect System for the AIMP vehicle has three telescopes mounted at 45, 90, and 135 degrees to the spin axis. These telescopes are connected to photosensors which are activated when the sunlit portion of the earth or moon is "seen" by the telescope. This paper will describe a method for calculating the angles through which each of the telescopes "sees" the sunlit portion of the central body. This information, together with knowledge of the spin axis-sun angle and spacecraft position is sufficient to determine the inertial spin axis orientation. This method requires the vehicle's position, $\vec{\rho}$, the sun's position, $\vec{\rho}_s$, the unit spin axis vector, $\vec{S}^\circ$, and the radius of the central body. The method described assumes an instantaneous rotation of 360 degrees about the spin axis at the given time. The central body is considered to be a sphere. The coordinate system uses the center of the central body as the origin.

II. TANGENT AND TERMINATOR PLANES

A. Tangent Plane Equation

The tangent plane is the plane of the intersection of the cone from the vehicle, tangent to the central body, and the surface of the central body (see Figure 1).

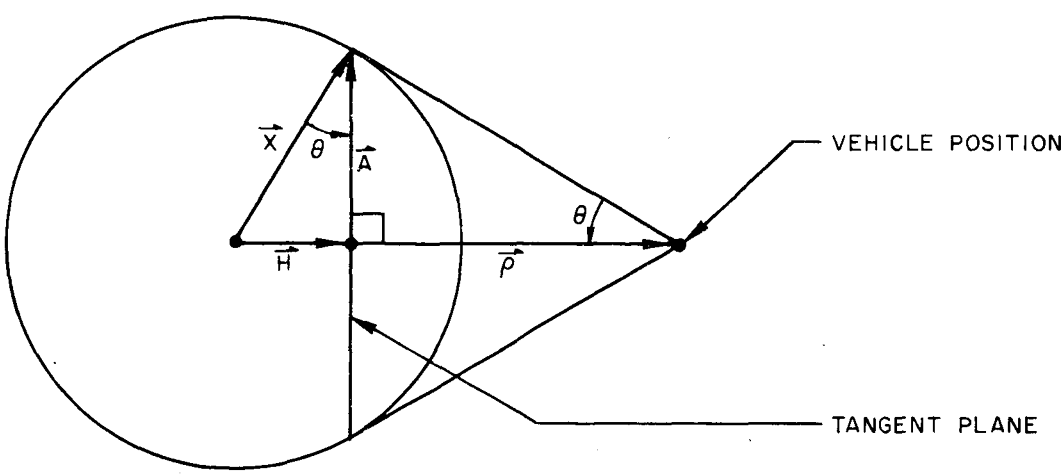

Figure 1—Tangent Plane

Let $\vec{x}$ be a vector from the center of the central body to a point on its surface which lies in the tangent plane. Then $\vec{A} = \vec{x} - \vec{H}$ is a vector in the tangent plane (see Figure 1) and $\vec{\rho} \cdot \vec{A} = 0$.

$$|\vec{H}| = |\vec{x}| \sin\theta = R \sin\theta = \frac{R^2}{|\vec{\rho}|}$$

where $R = |\vec{x}|$, the radius of the central body.

$$\vec{\rho} \cdot \vec{A} = \vec{\rho} \cdot \left(\vec{x} - \frac{R^2}{|\vec{\rho}|} \frac{\vec{\rho}}{|\vec{\rho}|} \right) = 0.$$

Let $\vec{\rho} = (a, b, c)$, and $\vec{x} = (x, y, z)$. Then

$$\vec{\rho} \cdot \vec{x} - \vec{\rho} \cdot \left(R^2 \frac{\vec{\rho}}{|\vec{\rho}|^2} \right) = \vec{\rho} \cdot \vec{x} - R^2 = 0$$

or $ax + by + cz - R^2 = 0$, which is the equation of the tangent plane.

B. Terminator Plane Equation

The terminator plane is the plane which divides the sunlit and dark portions of the central body. It is assumed that this plane passes through the center of the central body.

Let $\vec{\rho}_s = (u, v, w)$ be the vector from the center of the central body to the sun, and $\vec{x} = (x, y, z)$ be a vector from the center to a point in the terminator plane. Then $\vec{x} \cdot \vec{\rho}_s = 0$ and the equation of the terminator plane is $ux + vy + wz = 0$.

III. TELESCOPE PERPENDICULAR TO SPIN AXIS

A. Equation of Telescope Plane

The telescope plane is the plane through the vehicle, perpendicular to the spin axis vector $\vec{S}^\circ$. Let $\vec{x} = (x, y, z)$ be a vector to a point in the telescope plane, and let $\vec{S}^\circ = (\ell, m, n)$. Then $\vec{x} - \vec{\rho}$ lies in the telescope plane and $\vec{S}^\circ \cdot (\vec{x} - \vec{\rho}) = 0$. Letting $\vec{\rho} = (a, b, c)$, the equation of the telescope plane becomes $\ell x + my + nz - (\ell a + mb + nc) = 0$.

B. Calculations of Intersections

In the 90 degree case, the points of intersection of the tangent plane, telescope plane, and the central body are found. If two such intersections are not found, the viewing angle is 0. If the intersections are found, they are tested to see if they are sunlit and then the intersections (if any) with the terminator plane are calculated. (For mathematical solution, see Appendix).

The tangent plane solutions are tested to see if they are sunlit. The dark solutions are discarded. A solution will be sunlit if, any only if, the vector, $\vec{R}_S$ from the center of the central body to the solution and the sun vector $\vec{\rho}_s$ form an acute angle. That is $\vec{R}_S \cdot \vec{\rho}_s > 0$.

Terminator plane solutions must be tested for visibility from the vehicle. The solution is visible if its distance from the vehicle is less than the length of the tangent from the vehicle to the central body.

C. Viewing Angle

If $\vec{R}_{S_1}$ and $\vec{R}_{S_2}$ are two sunlit, visible solution vectors, the viewing angle θ is the angle between the two vectors from the vehicle to the solutions,

$$\theta = \cos^{-1} \frac{(\vec{\rho} - \vec{R}_{S_1}) \cdot (\vec{\rho} - \vec{R}_{S_2})}{|\vec{\rho} - \vec{R}_{S_1}| \; |\vec{\rho} - \vec{R}_{S_2}|}$$

IV. TELESCOPE NOT PERPENDICULAR TO SPIN AXIS

If the telescope-spin axis angle is not 90 degrees, the calculations are more complicated. First the equation of the telescope cone is calculated, and a test is made to see if the telescope cone lies outside the tangent cone. Then the intersections of the tangent plane, telescope cone, and central body are computed from a quartic equation. Next a test is made to eliminate spurious solutions introduced by squaring. The good solutions are then tested to see if they are sunlit. If there are two solutions, the tangent plane viewing angle is calculated. If there are fewer than two solutions, the solutions are saved, and the above calculations are repeated with the terminator plane replacing the tangent plane. If there are two terminator solutions, the terminator plane angle is calculated. If there is one terminator and one tangent solution, the viewing angle is calculated from these. If there is one terminator and no tangent solutions or no terminator solutions, the cone is tested to see if it is sunlit and the viewing angle is set to 0 or 360 degrees accordingly.

A. Equation of Telescope Cone

The telescope cone is the locus of points $\vec{x} = (x, y, z)$ such that $\vec{x} - \vec{\rho}$ and $\vec{S}^\circ$ make an angle equal to the telescope-spin axis angle α. Thus

$$(\vec{x} - \vec{\rho}) \cdot \vec{S}^\circ = |\vec{x} - \vec{\rho}| \, |\vec{S}^\circ| \cos \alpha.$$

Again let $\vec{\rho} = (a, b, c)$, $\vec{S}^\circ = (\ell, m, n)$. Then

$$\ell (x - a) + m (y - b) + n (z - c) = \sqrt{(x - a)^2 + (y - b)^2 + (z - c)^2} \, \sqrt{\ell^2 + m^2 + n^2} \, \cos \alpha.$$

Squaring both sides, and letting the equation of the cone be

$$a_1 x^2 + a_2 y^2 + a_3 z^2 + a_4 xy + a_5 xz + a_6 yz + a_7 x + a_8 y + a_9 z + a_{10} = 0,$$

we obtain by collecting terms

$$a_1 = \ell^2 - (\ell^2 + m^2 + n^2) \cos^2 \alpha, \; a_2 = m^2 - (\ell^2 + m^2 + n^2) \cos^2 \alpha,$$

$$a_3 = n^2 - (\ell^2 + m^2 + n^2) \cos^2 \alpha, \; a_4 = 2 \ell m, \; a_5 = 2 \ell n, \; a_6 = 2mn,$$

$$a_7 = - 2 \ell (a\ell + bm + cn) + 2a (\ell^2 + m^2 + n^2) \cos^2 \alpha$$

$$a_8 = - 2m (a\ell + bm + cn) + 2b (\ell^2 + m^2 + n^2) \cos^2 \alpha$$

$$a_9 = - 2n (a\ell + bm + cn) + 2c (\ell^2 + m^2 + n^2) \cos^2 \alpha$$

$$a_{10} = (a\ell + bm + cn)^2 - (a^2 + b^2 + c^2) (\ell^2 + m^2 + n^2) \cos^2 \alpha.$$

B. Calculations of Intersections

The points desired are the points of intersection of the telescope cone, tangent or terminator plane as the case may be, and the central body. The equations of these surfaces have been obtained above. For the mathematical solution, see the appendix.

C. Tests on Solutions

For a correct solution the vector from the vehicle to the solution must make the specified angle, α , with the spin axis,

$$\frac{(\vec{R}_s - \vec{\rho}) \cdot \vec{S}^\circ}{|\vec{R}_s - \vec{\rho}|} = \cos \alpha$$

The sunlit and terminator plane solution visibility tests are the same as in the perpendicular case.

D. Viewing Angle (Figure 2)

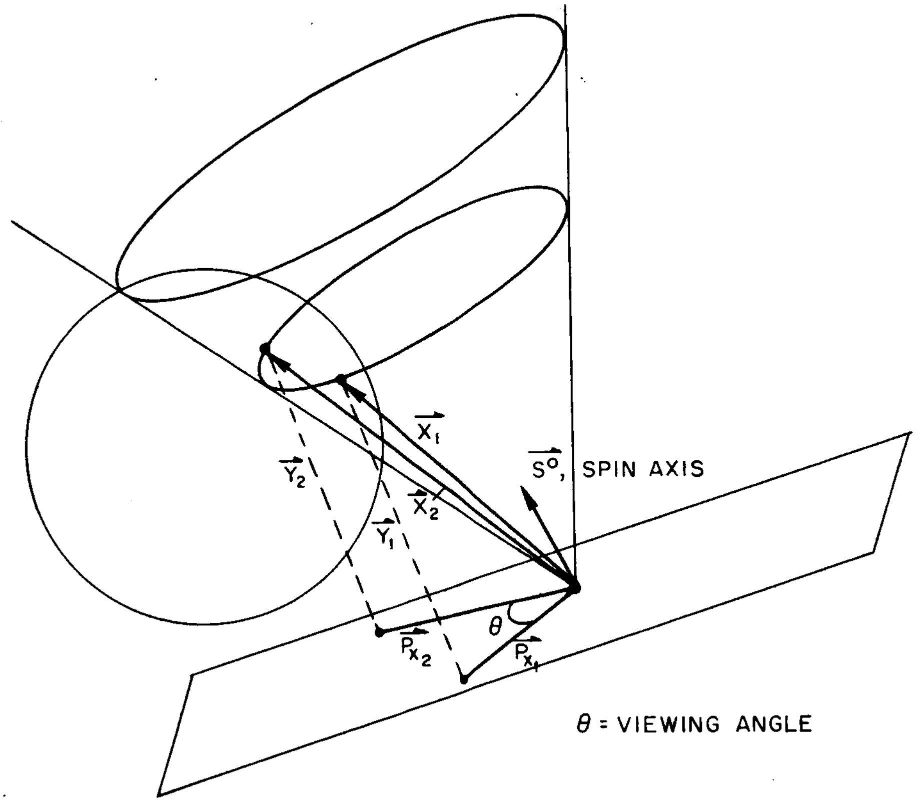

Figure 2–Viewing Angle

If there are two sunlit, visible solutions, take the vectors $\vec{x}_1$, and $\vec{x}_2$ from the vehicle to the solutions and project them parallel to the spin axis into a plane perpendicular to the spin axis. The angle between the two projections, Px_1 and Px_2, is the true rotation angle of the vehicle and hence the viewing angle (or, 360 - the angle). If $\beta = \pi/2 - \alpha$, and considering subscripts fixed, $|\vec{y}| = |\vec{x}| \sin \beta$ and $\vec{P}x = \vec{x} - |\vec{y}|\ \vec{S}^\circ$ or $\vec{P}x = \vec{x} - |\vec{x}| \sin\beta\ \vec{S}^\circ$. The viewing angle is either $\cos^{-1}(\vec{P}x_1 \cdot \vec{P}x_2)$ or $360 - \cos^{-1}(\vec{P}x_1 \cdot \vec{P}x_2)$. To decide which, calculate the midpoint of the line joining the endpoints of $\vec{P}x_1$ and $\vec{P}x_2$, and project it parallel to the spin axis onto the telescope cone. The angle is $\vec{P}x_1 \cdot \vec{P}x_2$ if the vector from the vehicle through the point just found is sunlit (see Figure 3). That is

$$\vec{Q}_1 = \frac{\vec{P}x_1 + \vec{P}x_2}{2}$$

$$|\vec{Q}_3| = |\vec{Q}_1| \tan\beta$$

$$\vec{Q}_2 = \vec{Q}_1 \pm |\vec{Q}_3|\ \vec{S}^\circ$$

$\vec{Q}_2$ is tested the same as in testing for a sunlit cone in (E) below.

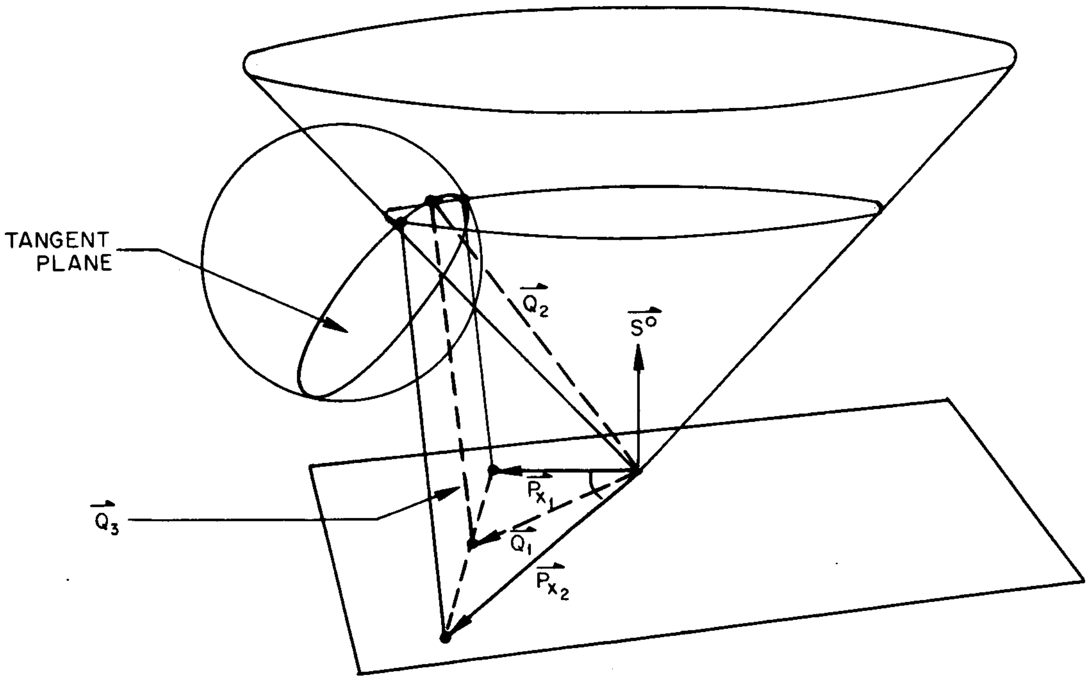

Figure 3–Midpoint Test

The case of two sunlit tangent and two terminator solutions is an exception to the above test. There will be two segments sunlit separated by a dark segment. In most of these cases, Q_2 will fall in the dark segment. Then the total sunlit portion would still be the calculated terminator angle, θ^1, minus the calculated tangent angle, θ, that is $(360 - \theta^1) - (360 - \theta) = \theta - \theta^1$.

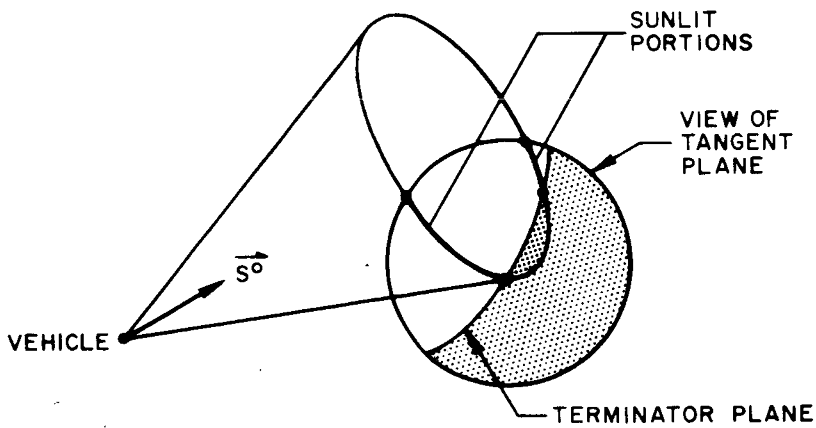

Figure 4–Two Tangent and Two Terminator Plane Solutions

E. Special Cases

If two solutions are not found, the telescope cone is either completely sunlit or completely dark. In either case the complete cone is the same. Thus for some number k, $|\vec{\rho} + k\vec{S}^\circ| = R$ and the vector $\rho + k\vec{S}^\circ$ will be sunlit if the cone is completely sunlit. Let $\vec{\rho} = (a, b, c)$, $\vec{S}^\circ = (\ell, m, n)$, then $(a + k\ell)^2 + (b + km)^2 + (c + kn)^2 = R^2$ which is a quadratic in k. Solve for the smallest k (in absolute value) and test if sunlit. If k is imaginary, $\vec{S}^\circ$ does not intersect the central body and the cone is dark. (See Figure 5.)

V. COMPUTER PROGRAM

The method described has been programmed as a subroutine to the IMP Mission Analysis Program, QUIMP, which is a modification of the Quick Look Mission Analysis Program, reference 1. The calling sequence is CALL TELANG (NOR) where NOR is the number of the central body (see QUIMP writeup). The following quantities are required from common, C(1000):

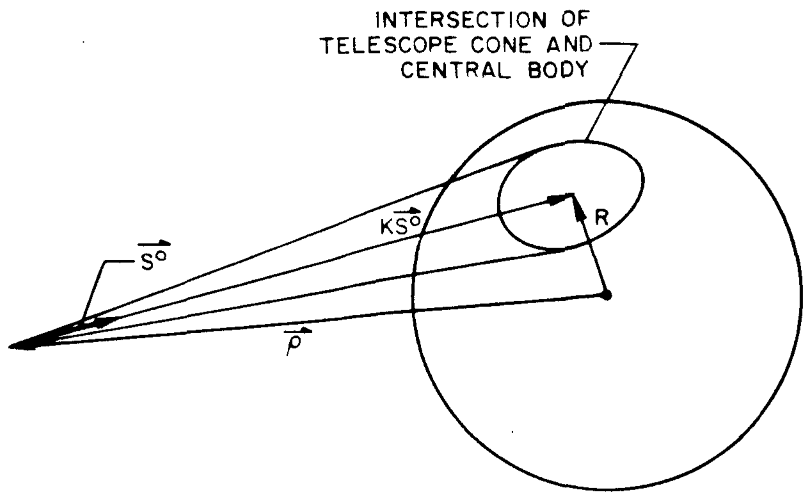

Figure 5–Cone Sunlit Test

C(58), C(59), C(60)	The telescope-spin axis angles
	These must be input to QUIMP in OVRLAY:
C(21), . . . , C(28)	The radii of the ephemeris bodies.
C(721), . . . , C(744)	Position of the vehicle with respect to the ephemeris bodies.
C(138), . . . , C(140)	Spin axis vector.

TELANG uses FNORM, DOT, and ADOT from the QUIMP program and subroutine QUART for solving the quartic equations.

If two tangent solutions exist, TELANG computes the angle between them and prints this as the tangent solution. TELANG then tries to find two terminator solutions and prints their angle if it exists. If only one sunlit tangent solution exists, the tangent solution angle is printed as zero and the tangent solution is used with a terminator solution if it exists.

VI. RESULTS

Figures 6, 7, and 8 show some calculations done for the AIMP vehicle. Most cases were encountered, that is two tangent solutions, two terminator solutions, two of each, and one of each. The entire cone being sunlit was not encountered.

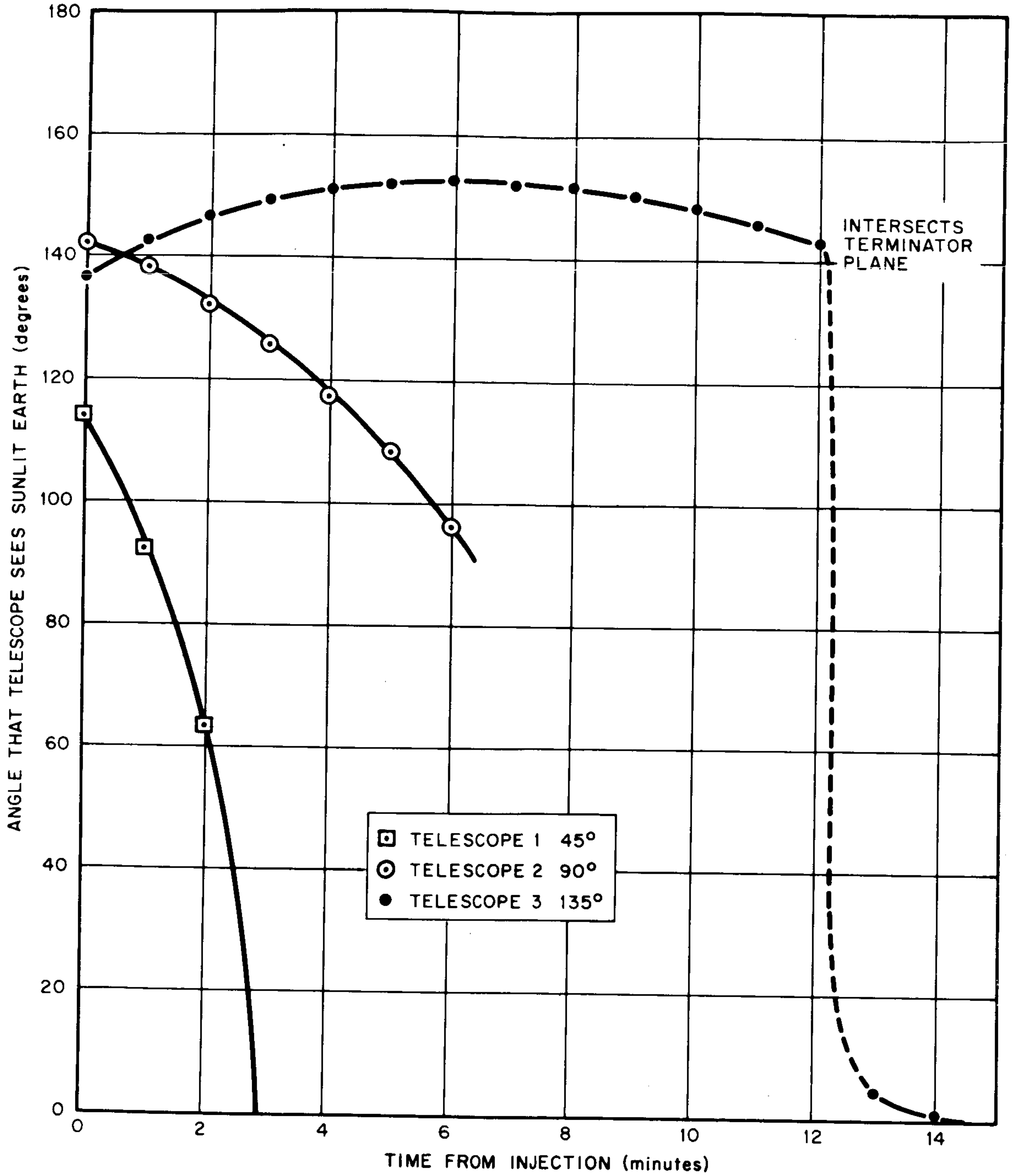

Figure 6—Nominal Spin Axis

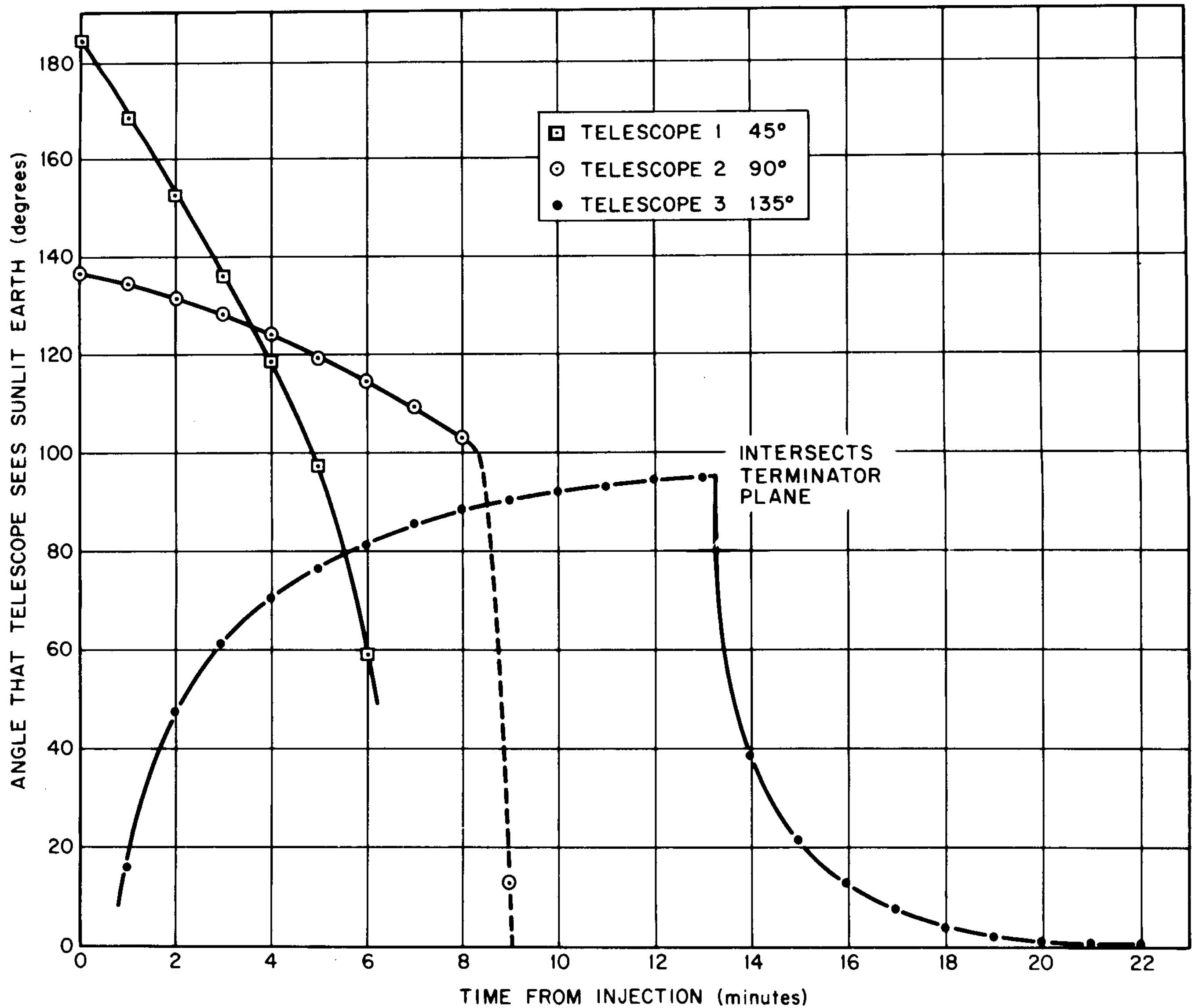

Figure 7–Spin Axis Pitched Down 34° From Nominal

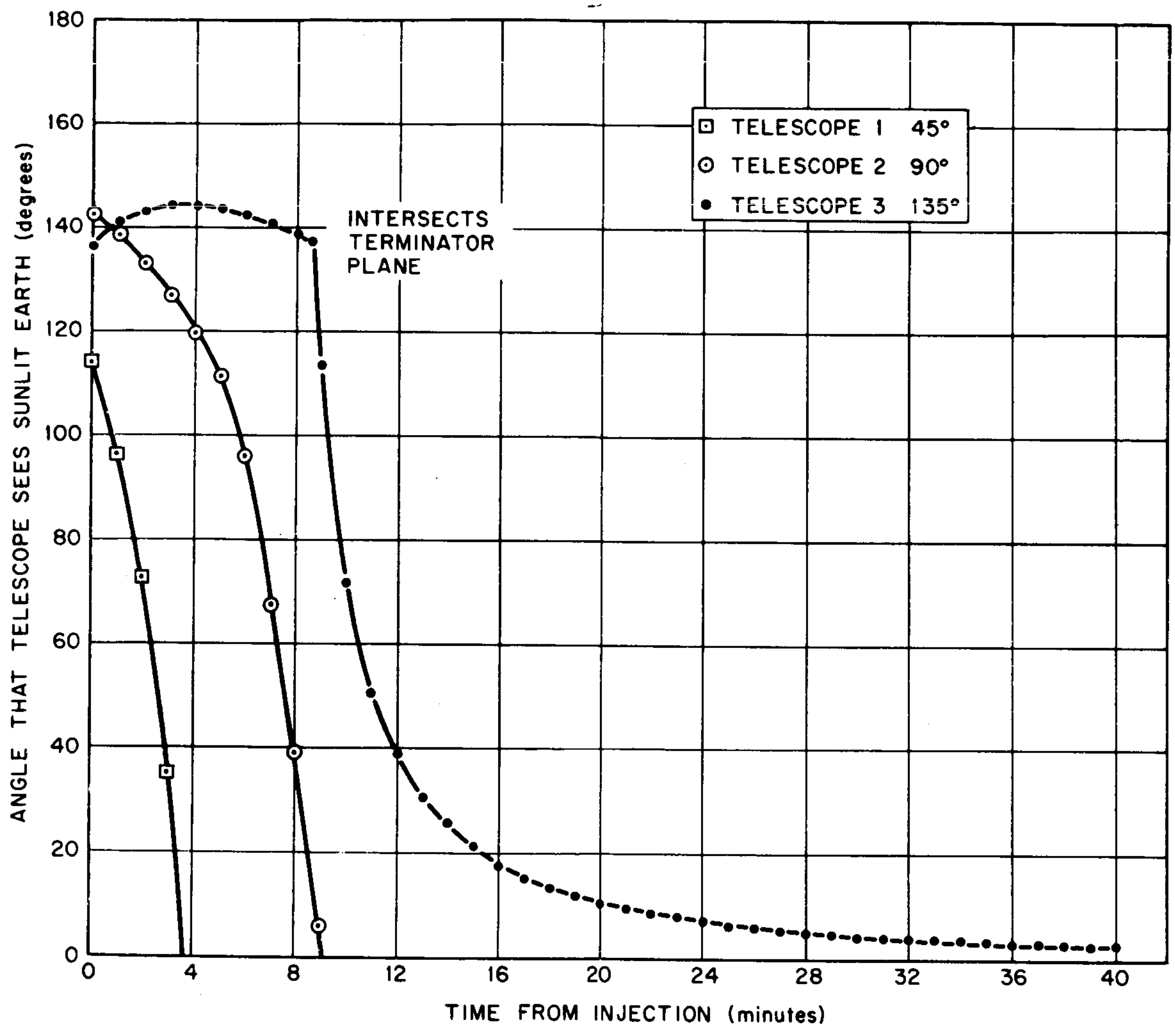

Figure 8–Spin Axis Yawed 34° From Nominal

VII. CONCLUSIONS

The described method has been successfully used as a subroutine in the QUIMP Program, and it could be used with any program which provides the necessary position and orientation information.

VIII. ACKNOWLEDGMENT

The authors wish to acknowledge helpful discussions with R. T. Groves and J. S. Linnekin and the work of J. P. Walsh in checking and debugging the program.

IX. REFERENCES

1. "Quick Look Mission Analysis Program" Prepared for Goddard Space Flight Center by Philco Corporation, WDL Division (Contract Number NAS5-3342).

X. APPENDIX

A. Calculations of Intersections (90 Degree Case)

Let $a_1x + a_2y + a_3z + a_4$ be the equation of the tangent or terminator plane as the case may be; $b_1x + b_2y + b_3z + b_4 = 0$ the equation of the telescope plane; and $x^2 + y^2 + z^2 = R^2$ the equation of the central body. Assume below that the coefficients divided by are not zero. If they are, solve for x or y instead of z. Then

$$z = -\frac{1}{a_3}(a_1x + a_2y + a_4). \tag{1}$$

Substituting (1) into the equation of the telescope plane, we get

$$b_1x + b_2y - \frac{b_3}{a_3}(a_1x + a_2y + a_4) + b_4 = 0, \tag{2}$$

or

$$c_1x + c_2y + c_4 = 0 \tag{3}$$

where the c's are obtained by collecting terms in (2). Solving for y, we have

$$y = -\frac{1}{c_2}(c_1x + c_4) \tag{4}$$

and substituting this back into (1), we obtain

$$z = -\frac{1}{a_3}\left[a_1x - \frac{a_2}{c_2}(c_1x + c_4) + a_4\right] \tag{5}$$

or

$$z = d_1x + d_2, \tag{6}$$

where the d's are obtained by collecting terms in (5). Substituting for y and z from (4) and (6) into the equation of the central body, a quadratic equation in x is obtained. If there are any real solutions, use (4) and (6) to solve for y and z.

B. Calculations of Intersections (non-90 degree case)

Given the equation of the telescope cone (see p. 4) the equation of the tangent or terminator plane, i.e.,

$$b_1 x + b_2 y + b_3 z + b_4 = 0, \tag{7}$$

and the equation of the central body

$$x^2 + y^2 + z^2 = R^2, \tag{8}$$

we solve simultaneously. Again assume the coefficients divided by are not zero. Solving for z from (7),

$$z = -\frac{1}{b_3}(b_1 x + b_2 y + b_4),$$

and substituting in (8), we obtain

$$c_1 x^2 + c_2 y^2 + c_3 xy + c_4 x + c_5 y + c_6 = 0 \tag{9}$$

where

$$c_1 = 1 + \frac{b_1^2}{b_3^2}, \quad c_2 = 1 + \frac{b_2^2}{b_3^2}, \quad c_3 = \frac{2b_1 b_2}{b_3^2}, \quad c_4 = \frac{2b_1 b_4}{b_3^2}, \quad c_5 = \frac{2b_2 b_4}{b_3^2}, \quad c_6 = \frac{b_4^2}{b_3^2} - R^2.$$

Consider (9) as a quadratic equation in y. Solving for y in terms of x;

$$y = \frac{-(c_3 x + c_5) \pm \sqrt{(c_3 x + c_5)^2 - 4c_2(c_1 x^2 + c_4 x + c_6)}}{2c_2} \tag{10}$$

The - sign before the radical is superfluous in obtaining the final equation, but is necessary, once having solved for x, to obtain all intersections. Solving for z in terms of x,

$$z = -\frac{1}{b_3}\left(b_1 x + \frac{b_2}{2c_2}\left(-(c_3 x + c_5) \pm \sqrt{(c_3 x + c_5)^2 - 4c_2(c_1 x^2 + c_4 x + c_6)}\right) + b_4\right) \tag{11}$$

Letting

$$y = d_1 x + d_2 + d_3 \sqrt{d_4 x^2 + d_5 x + d_6}$$

and

$$z = e_1 x + e_2 + e_3 \sqrt{e_4 x^2 + e_5 x + e_6},$$

$$d_1 = -\frac{c_3}{2c_2} \qquad d_2 = \frac{c_5}{2c_2} \qquad d_3 = \frac{1}{2c_2} \qquad d_4 = e_4 = c_3^2 - 4c_2 c_1$$

$$d_5 = e_5 = 2c_3 c_5 - 4c_2 c_4 \qquad d_6 = e_6 = c_5^2 - 4c_2 c_6$$

$$e_1 = -\frac{1}{b_3}\left(b_1 - \frac{b_2 c_3}{2c_2}\right) \qquad e_2 = -\frac{1}{b_3}\left(b_4 - \frac{b_2 c_5}{2c_2}\right) \qquad e_3 = -\frac{b_2}{2b_3 c_2}$$

Expressions for y^2 and z^2 are needed for substituting in the telescope cone equation.

$$y^2 = f_1 x^2 + f_2 x + f_3 + (f_4 x + f_5) \sqrt{d_4 x^2 + d_5 x + d_6}$$

$$z^2 = g_1 x^2 + g_2 x + g_3 + (g_4 x + g_5) \sqrt{e_4 x^2 + e_5 x + e_6}$$

where

$$f_1 = d_1^2 + d_3^2 d_4 \qquad f_2 = 2d_1 d_2 + d_3^2 d_5 \qquad f_3 = d_2^2 + d_3^2 d_6$$

$$f_4 = 2d_1 d_3 \qquad f_5 = 2d_2 d_3$$

$$g_1 = e_1^2 + e_3^2 e_4 \qquad g_2 = 2e_1 e_2 + e_3^2 e_5 \qquad g_3 = e_2^2 + e_3^2 e_6$$

$$g_4 = 2e_1 e_3 \qquad g_5 = 2e_2 e_3$$

Now substituting in the equation for telescope cone,

$$h_1 x^2 + h_2 x + h_3 = -(h_4 x + h_5) \sqrt{d_4 x^2 + d_5 x + d_6}$$

where

$$h_1 = a_1 + a_2 f_1 + a_3 g_1 + a_4 d_1 + a_5 e_1 + a_6 (d_1 e_1 + d_3 d_4 e_3)$$

$$h_2 = a_2 f_2 + a_3 g_2 + a_4 d_2 + a_5 e_2 + a_6 (d_1 e_2 + d_2 e_1 + d_3 e_3 d_5) + a_7 + a_8 d_1 + a_9 e_1$$

$$h_3 = a_2 f_3 + a_3 g_3 + a_6 (d_2 e_2 + d_3 e_3 d_6) + a_8 d_2 + a_9 e_2 + a_{10}$$

$$h_4 = a_2 f_4 + a_3 g_4 + a_4 d_3 + a_5 e_3 + a_6 (d_1 e_3 + d_3 e_1)$$

$$h_5 = a_2 f_5 + a_3 g_5 + a_6 (d_2 e_3 + d_3 e_2) + a_8 d_3 + a_9 e_3$$

Squaring both sides above results in a quartic equation in x

$$p_1 x^4 + p_2 x^3 + p_3 x^2 + p_4 x + p_5 = 0$$

where,

$$p_1 = h_1^2 - h_4^2 d_4 \qquad p_2 = 2h_2 h_1 - (h_4^2 d_5 + 2h_4 h_5 d_4)$$

$$p_3 = 2h_1 h_3 + h_2^2 - (h_4^2 d_6 + 2h_4 h_5 d_5 + h_5^2 d_4) \qquad p_4 = 2h_2 h_3 - (2h_4 h_5 d_6 + h_5^2 d_5)$$

$$p_5 = h_3^2 - h_5^2 d_6$$

The roots of this quartic, and equations (10) and (11) give the solutions.