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# EFFECTS OF PRECESSION AND NUTATION

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## **SUMMARY**

Transformations to account for the precession and nutation in the equator and equinox of the earth are given in units of degrees with time measured in Julian days and initial epoch of 2436099.5 J.D., the Julian Date for Space.

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# EFFECTS OF PRECESSION AND NUTATION

by

James P. Murphy

## INTRODUCTION

The true equator and equinox of the earth are in continuous motion in space. The motion consists of two parts, general precession and nutation. General precession carries the mean pole of the equator around the pole of the ecliptic in about 26,000 years and nutation carries the true pole around the mean pole in about 18.6 years. Thus the effect of precession is secular and the effect of nutation is periodic.

Three separate coordinate systems are to be considered. The  $Z$ -system is moving under the influence of precession and nutation; the  $\tilde{Z}$ -system is moving under the influence of nutation; and the  $W$ -system is fixed in space. These coordinate systems are now defined.

In the  $Z$ -system the  $Z_3$  axis points toward the instantaneous direction with respect to the celestial sphere of the mean polar axis, and  $Z$  points toward the instantaneous vernal equinox. At the same time in the  $\tilde{Z}$ -system, the  $\tilde{Z}_3$  axis points toward the mean direction of the mean polar axis, and  $\tilde{Z}$  points toward the mean vernal equinox. At some specified epoch in the  $W$ -system, the  $W_3$  axis points toward the mean direction of the mean polar axis and  $W_1$  points toward the mean vernal equinox.

## PRECESSION

Let  $\kappa$  and  $\omega$  be the angle between the line of intersection of the mean equator at  $T_0$  and the mean equator at  $T$  with the  $W_2$  and  $\tilde{Z}_2$  axes, respectively (See Figure 1). Let  $\nu$  be the angle between the planes of the two mean equators.

The relationship between the  $\tilde{Z}$  and  $W$  coordinate systems then is

$$\tilde{Z} = R_3(-\omega) R_2(\nu) R_3(-\kappa) W$$

and

$$W = R_3(\kappa) R_2(-\nu) R_3(\omega) \tilde{Z}$$

(1)

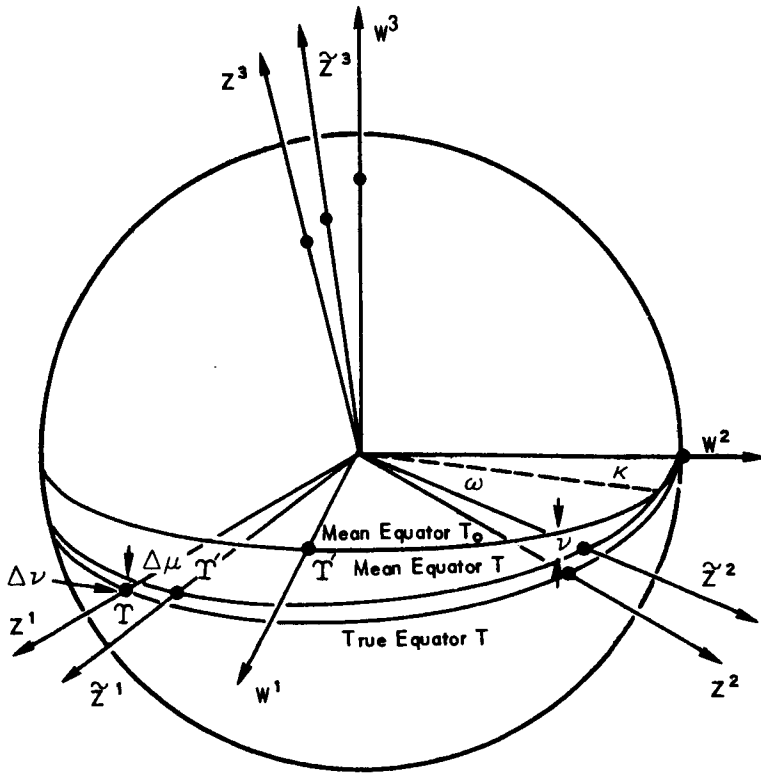


Figure 1

where  $R_n(\alpha)$  indicates a rotation matrix with the rotation about the  $n^{\text{th}}$  axis by an angle  $\alpha$ .

The explicit relationships indicated by Equations (1) are

$$\begin{bmatrix} \tilde{Z}_1 \\ \tilde{Z}_2 \\ \tilde{Z}_3 \end{bmatrix} = \begin{bmatrix} -\sin \kappa \sin \omega + \cos \kappa \cos \omega \cos \nu & -\cos \kappa \sin \omega - \sin \kappa \cos \omega \cos \nu & -\cos \omega \sin \nu \\ \sin \kappa \cos \omega + \cos \kappa \sin \omega \cos \nu & \cos \kappa \cos \omega - \sin \kappa \sin \omega \cos \nu & -\sin \omega \sin \nu \\ \cos \kappa \sin \nu & -\sin \kappa \sin \nu & \cos \nu \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}$$

and

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} = \begin{bmatrix} \cos \nu \cos \omega \cos \kappa - \sin \kappa \sin \omega & \cos \nu \cos \kappa \sin \omega + \cos \omega \sin \kappa & \cos \kappa \sin \nu \\ -\cos \nu \cos \omega \sin \kappa - \cos \kappa \sin \omega & -\cos \nu \sin \kappa \sin \omega + \cos \omega \cos \kappa & -\sin \kappa \sin \nu \\ -\sin \nu \cos \omega & -\sin \nu \sin \omega & \cos \nu \end{bmatrix} \begin{bmatrix} \tilde{Z}_1 \\ \tilde{Z}_2 \\ \tilde{Z}_3 \end{bmatrix}$$

The values of  $\kappa$ ,  $\omega$ , and  $\nu$  are

$$\begin{aligned} \kappa = & \left[ 23042''.53 + 139''.73 (T_0 - 1.900) + 0''.06 (T_0 - 1.900)^2 \right] (T - T_0) \\ & + \left[ 30''.23 - 0''.27 (T_0 - 1.900) \right] (T - T_0)^2 + 18''.00 (T - T_0)^3 \end{aligned}$$

$$\begin{aligned} \omega = & \left[ 23042''.53 + 139''.73 (T_0 - 1.900) + 0''.06 (T_0 - 1.900)^2 \right] (T - T_0) \\ & + \left[ 109''.50 + 0''.39 (T_0 - 1.900) \right] (T - T_0)^2 + 18''.32 (T - T_0)^3 \end{aligned}$$

$$\begin{aligned} \nu = & \left[ 20046''.85 - 85''.33 (T_0 - 1.900) - 0''.37 (T - 1.900)^2 \right] (T - T_0) \\ & + \left[ -42''.67 - 0''.37 (T_0 - 1.900) \right] (T - T_0)^2 - 41''.80 (T - T_0)^3 \end{aligned}$$

where  $T_0$  and  $T$  are measured in thousand tropical years and where  $T_0$  refers to the epoch time of the fixed mean equator and  $T$  refers to the time of the moving equator.

#### NUTATION

Let  $\Delta\mu$ ,  $\Delta\nu$ , and  $\Delta\epsilon$  be the nutation in right ascension, declination, and obliquity, respectively. The relationship between the  $Z$  and  $\tilde{Z}$  systems is then

$$Z = R_3(-\Delta\mu) R_2(\Delta\nu) R_1(-\Delta\epsilon) \tilde{Z}$$

and

(2)

$$\tilde{Z} = R_1(\Delta\epsilon) R_2(-\Delta\nu) R_3(\Delta\mu) Z$$

The explicit relationships indicated by Equations (2) are then

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} \cos \Delta\nu \cos \Delta\mu & -\cos \Delta\mu \sin \Delta\nu \sin \Delta\epsilon - \cos \Delta\epsilon \sin \Delta\mu & -\cos \Delta\mu \sin \Delta\nu \cos \Delta\epsilon + \sin \Delta\epsilon \sin \Delta\mu \\ \cos \Delta\nu \sin \Delta\mu & -\sin \Delta\mu \sin \Delta\nu \sin \Delta\epsilon + \cos \Delta\epsilon \cos \Delta\mu & -\sin \Delta\mu \sin \Delta\nu \cos \Delta\epsilon - \sin \Delta\epsilon \cos \Delta\mu \\ \sin \Delta\nu & \sin \Delta\epsilon \cos \Delta\nu & \cos \Delta\epsilon \cos \Delta\nu \end{bmatrix} \begin{bmatrix} \tilde{Z}_1 \\ \tilde{Z}_2 \\ \tilde{Z}_3 \end{bmatrix}$$

and

$$\begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \\ \tilde{z}_3 \end{bmatrix} = \begin{bmatrix} \cos \Delta \nu \cos \Delta \mu & \cos \Delta \nu \sin \Delta \mu & \sin \Delta \nu \\ -\cos \Delta \mu \sin \Delta \nu \sin \Delta \epsilon - \cos \Delta \epsilon \sin \Delta \mu & -\sin \Delta \mu \sin \Delta \nu \sin \Delta \epsilon + \cos \Delta \epsilon \cos \Delta \mu & \sin \Delta \epsilon \cos \Delta \nu \\ -\cos \Delta \mu \sin \Delta \nu \cos \Delta \epsilon + \sin \Delta \epsilon \sin \Delta \mu & -\sin \Delta \mu \sin \Delta \nu \cos \Delta \epsilon - \sin \Delta \epsilon \cos \Delta \mu & \cos \Delta \epsilon \cos \Delta \nu \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

The expressions for  $\Delta \mu$ ,  $\Delta \nu$ , and  $\Delta \epsilon$  may be obtained from expressions for the nutation in longitude  $\Delta \psi$  and in obliquity  $\Delta \epsilon$  appearing on pages 44 and 45 of Reference 3.

This relationship is illustrated in Figure 2.

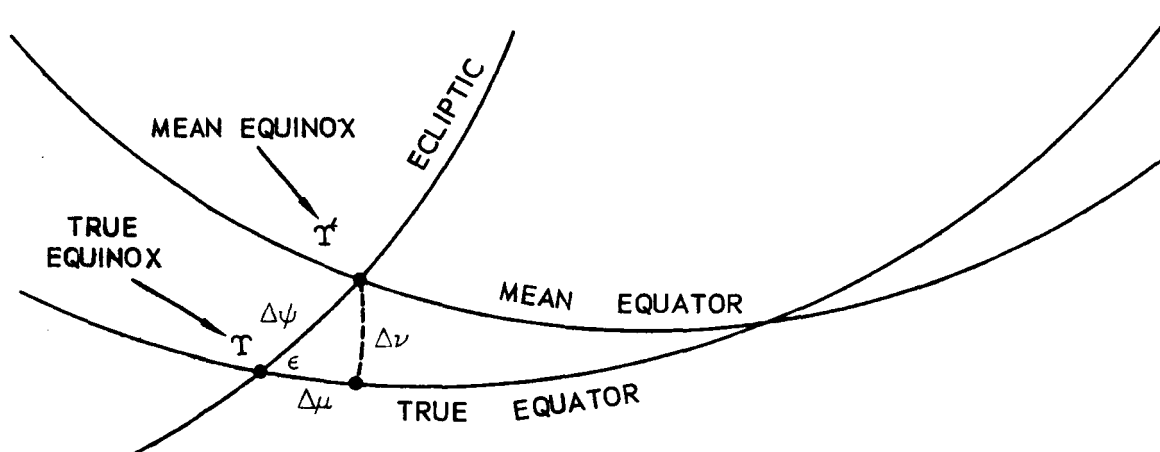


Figure 2

Thus,

$$\begin{aligned} \Delta \mu &= \Delta \psi \cos \epsilon \\ \Delta \nu &= \Delta \psi \sin \epsilon \end{aligned} \quad (3)$$

The "equation of the equinoxes" or nutation in right ascension is the right ascension of the mean equinox referred to the true equator and equinox and represents the difference between the mean and true right ascensions for a body on the equator. It is thus the difference between mean and apparent sidereal time.



## FORMULAS FOR EPOCH OF THE JULIAN DATE FOR SPACE

If we wish to use the Julian date for space as the initial epoch, the day as the time unit and the degree as the unit of angular measure, then some formulas of the previous sections undergo changes in the coefficients. That is, the precession formulas for  $\kappa$ ,  $\omega$ , and  $\nu$  are adjusted for the new units. An adjustment must be made in the coefficients in the series for  $\Delta\psi$  and  $\Delta\epsilon$  as well as the motion of the arguments of the various terms.

The formulas for precession are then given by

$$\begin{aligned}
 \kappa &= \left[ 1^{\circ}753067 \times 10^{-5} + 2^{\circ}9097 \times 10^{-13} (t_o - t_{J.D.S.}) \right. \\
 &\quad \left. + 3^{\circ}4 \times 10^{-22} (t_o - t_{J.D.S.})^2 \right] (t - t_o) \\
 &+ \left[ 6^{\circ}291 \times 10^{-14} - 1^{\circ}5 \times 10^{-21} (t_o - t_{J.D.S.}) \right] (t - t_o)^2 \\
 &\quad + 1^{\circ}026 \times 10^{-19} (t - t_o)^3 \\
 \\
 \omega &= \left[ 1^{\circ}753067 \times 10^{-5} + 2^{\circ}9097 \times 10^{-13} (t_o - t_{J.D.S.}) \right. \\
 &\quad \left. + 3^{\circ}4 \times 10^{-22} (t_o - t_{J.D.S.})^2 \right] (t - t_o) \\
 &+ \left[ 2^{\circ}2805 \times 10^{-13} + 2^{\circ}3 \times 10^{-21} (t_o - t_{J.D.S.}) \right] (t - t_o)^2 \\
 &\quad + 1^{\circ}044 \times 10^{-19} (t - t_o)^3 \\
 \\
 \nu &= \left[ 1.524289 \times 10^{-5} - 1^{\circ}7777 \times 10^{-13} (t_o - t_{J.D.S.}) \right. \\
 &\quad \left. - 2^{\circ}1 \times 10^{-21} (t_o - t_{J.D.S.})^2 \right] (t - t_o) \\
 &+ \left[ - 8^{\circ}889 \times 10^{-14} - 2^{\circ}1 \times 10^{-21} (t_o - t_{J.D.S.}) \right] (t - t_o)^2 \\
 &\quad - 2^{\circ}383 \times 10^{-19} (t - t_o)^3
 \end{aligned} \tag{4}$$

where  $t_{J.D.S.} = 2436099.5$  J.D. and where  $(t_o - t_{J.D.S.})$  and  $(t - t_o)$  are measured in Julian days.

In the series for nutation in longitude and obliquity terms of the order of a hundredth of a second of arc or larger are kept. This amounts to about one quarter of the terms listed in Reference 3 for these series. Thus

$$\begin{aligned}
\Delta\psi = & \quad (-4.78965 \times 10^{-3} - 1.323 \times 10^{-10}t) \sin \Omega \\
& + (5.800 + 10^{-5} + 1. \times 10^{-13}t) \sin 2\Omega \\
& + (-3.5361 \times 10^{-4} - 1. \times 10^{-12}t) \sin 2(F - D + \Omega) \\
& + (3.497 \times 10^{-5} - 2.4 \times 10^{-12}t) \sin \ell' \\
& + (-1.38 \times 10^{-5} + 9. \times 10^{-13}t) \sin (\ell' + 2F - 2D + 2\Omega) \\
& + (-5.94 \times 10^{-6} + 4. \times 10^{-13}t) \sin (\ell' - 2F + 2D - 2\Omega) \\
& + (3.44 \times 10^{-6} + 8. \times 10^{-14}t) \sin (2F - 2D + \Omega) \\
& + (-5.659 \times 10^{-5} - 1. \times 10^{-13}t) \sin 2(F + \Omega) \\
& + (1.88 \times 10^{-5} + 8. \times 10^{-14}t) \sin \ell \\
& + (-9.50 \times 10^{-6} - 3. \times 10^{-13}t) \sin (2F + \Omega) \\
& - 7.25 \times 10^{-6} \sin (\ell' + 2F + 2\Omega) \\
& - 4.14 \times 10^{-6} \sin (\ell' - 2D) \\
& - 3.17 \times 10^{-6} \sin (\ell' - 2F - 2\Omega) \\
& + 1.7 \times 10^{-6} \sin 2D \\
& + 1.6 \times 10^{-6} \sin (\ell' + \Omega) \\
& + 1.6 \times 10^{-6} \sin (\ell' - \Omega) \\
& + 1.4 \times 10^{-6} \sin (\ell' - 2F - 2D - 2\Omega)
\end{aligned}$$

and

$$\begin{aligned}
\Delta \epsilon = & \quad (2.5585 \times 10^{-3} + 6.9 \times 10^{-12} t) \cos \Omega \\
& + (-2.51 \times 10^{-5} + 3. \times 10^{-13} t) \cos 2 \Omega \\
& + (1.533 \times 10^{-4} - 2.2 \times 10^{-12} t) \cos 2 (F - D + \Omega) \\
& + (5.99 \times 10^{-6} - 4.4 \times 10^{-13} t) \cos (\ell' + 2F - 2D + 2\Omega) \\
& + (-2.58 \times 10^{-6} + 2. \times 10^{-13} t) \cos (\ell - 2F + 2D - 2\Omega) \\
& - 1.8 \times 10^{-6} \cos (2F - 2D + \Omega) \\
& + (2.46 \times 10^{-5} - 3.9 \times 10^{-13} t) \cos 2 (F + \Omega) \\
& + 5.08 \times 10^{-6} \cos (2F + \Omega) \\
& + (3.14 \times 10^{-6} - 8. \times 10^{-14} t) \cos (\ell + 2F + 2\Omega) \\
& - 1.4 \times 10^{-6} \cos (\ell - 2F - 2\Omega)
\end{aligned}$$

where

$$\begin{aligned}
\ell & = 299^{\circ}615949 + 13^{\circ}0649927374d + 6^{\circ}909 \times 10^{-12}d^2 + 2^{\circ}95 \times 10^{-19}d^3 \\
\ell' & = 254^{\circ}436609 + 0^{\circ}9856002621d - 0.116 \times 10^{-12}d^2 - 0^{\circ}68 \times 10^{-19}d^3 \\
F & = 239^{\circ}342609 + 13^{\circ}2293503475d - 2^{\circ}407 \times 10^{-12}d^2 - 0^{\circ}07 \times 10^{-19}d^3 \\
D & = 285^{\circ}634588 + 12^{\circ}1907491461d - 1^{\circ}074 \times 10^{-12}d^2 + 0^{\circ}39 \times 10^{-19}d^3 \\
\Omega & = 222.941764 - 0^{\circ}0529538565d + 1^{\circ}560 \times 10^{-12}d^2 + 0^{\circ}46 \times 10^{-19}d^3
\end{aligned} \tag{5}$$

and where  $d$  is measured in Julian Ephemeris Days since the Epoch Date of the Julian Day for Space, 2436099.5 J.D.

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## APPENDIX A

### LIST OF SYMBOLS

Z-system	coordinate system moving under the influence of precession and nutation
$\tilde{Z}$ -system	coordinate system moving under the influence of nutation
W-system	coordinate system fixed in space
$\kappa, \omega, \nu$	precession angles given by Equations (4)
$\Delta\mu$	nutation in right ascension
$\Delta\nu$	nutation in declination
$\Delta\epsilon$	nutation in obliquity
$\Delta\psi$	nutation in longitude
$T$	true equinox
$T'$	mean equinox
$T_0$	epoch time of fixed mean equator in tropical centuries
$T$	epoch time of moving equator in tropical centuries
$t_0$	$T_0$ expressed in Julian days
$t$	$T$ expressed in Julian days
$t_{J.D.S.}$	Julian Date for Space = 2436099.5 J.D.
$R_n(\alpha)$	rotation about the $i^{th}$ axis by an angle $\alpha$
$\ell, \ell', F, D, \Omega$	angular variables defined in Equations (5)