A STATISTICAL INVESTIGATION OF THE EFFECTS CONTRIBUTED BY TAPE RECORDERS AND BY WOW AND FLUTTER OF MAGNETIC TAPE ON THE ACCURACY OF A TELEMETRY SYSTEM

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ABSTRACT

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The purpose of this report is to describe an experiment in which the effects contributed by analog tape recorders were investigated. Also, the noise contributed by the wow and flutter effect of magnetic tape was studied in relation to the accuracy of a telemetry system.

Since the data collected for this experiment did not conform to a normal distribution, a non-parametric test was employed in testing for significant difference between the tape tracks and tape recorders, in regard to their noise indexes.

The ratio of the standard deviation over the range was used as a relative measure of the error or noise effect in the system and provided the best information for ranking tape recorders and tape tracks in terms of system noise.

A secondary experiment in noise analysis was performed in order to corroborate the results obtained in the original experiment.

Author

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SECTION I. THE EXPERIMENT

In late August, 1965, an experiment to determine the effects contributed by analog tape recorders and by wow and flutter of magnetic tapes on the accuracy of a telemetry system was performed in the Ground Station of the Telemetry Branch, Astrionics Laboratory, George C. Marshall Space Flight Center, Huntsville, Alabama. The experiment was conducted by several members of the Systems Engineering Group of the University of Alabama, assisted by the Ground Station personnel.

A. PURPOSE

The purpose of this report is to describe an experiment involving the investigation of the effects contributed by analog tape recorders and by wow and flutter of magnetic tape on the accuracy of a telemetry system. The specific purpose of the experiment was to determine:

(1) if there are significant differences among the tape tracks on the 14-track Mincom Analog Tape Recorders;

(2) if there is a significant difference between the tape recorders themselves;

(3) if there is a significant noise or error effect contributed by the tape recorders as a link in a telemetry system.

Only two Mincom Analog Tape units were available; therefore, two 14-track magnetic tapes were randomly selected for the experiment. However, when the two tapes were recorded simultaneously, one of the tapes did not record correctly and only garbled information could be distinguished on this tape. The remaining tape was played back on both recorders. Therefore, purpose (2) may be difficult to ascertain directly.

However, purpose (1) and purpose (3) may still be determined directly.

B. SYSTEM UNDER TEST

The system under test consisted of the output of a Boonton FM-AM Signal Generator being sequenced through an eleven-step calibration sequence simultaneously into two Mincom Analog Tape Recorders and through GFD-5 discriminators into the SEL system for comparative real time analyses. The analog tapes were replayed by tracks at a later time through the discriminators and into the SEL system for analysis. (See Figure 1 for a representation of the system.)

Only 13 of the tape tracks were available on each analog tape recorder due to a malfunction in the preamplifier associated with Track 14 on Recorder 1. The data was recorded on 12 of the available 13 tracks, since one track was reserved for a voice identification of each input level on each recorder.

IRIG FM/FM channel 12 was selected at random to be the channel under test. The input levels for the signal generator are listed below in Table 1.

Table 1. Test Input Level

Input Level (% of full scale)	Frequency (CPS)
0	9,712
10	9,870
20	10,027
30	10,185
40	10,344
50	10,500
60	10,658
70	10,815
80	10,973
90	11,130
100	11,288

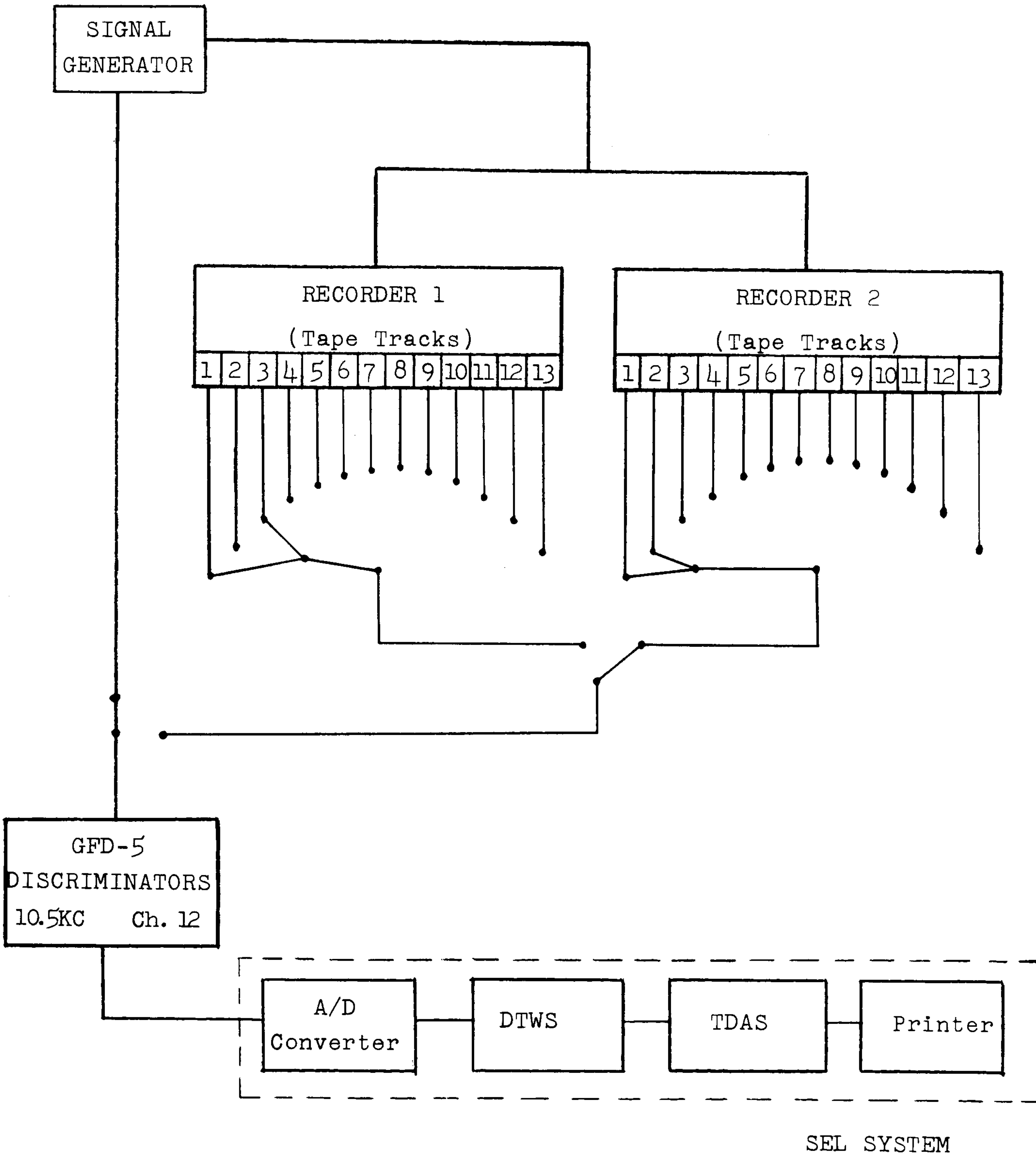

Figure 1. The System Under Test

C. EXPERIMENTAL DESIGN

1. Input. The test procedure was to input the signal from the signal generator simultaneously to each of the two analog tape recorders and through the discriminator to the SEL system for real time analysis. The signals were randomly stepped through an eleven-step calibration sequence. Each track on each of the analog tapes was to be played back through the discriminator to the SEL system; however, as has already been noted, tape 2 on Recorder 2 recorded only unintelligible information. Therefore, only tape 1 was available for analysis. Each of the two SEL programs used in the experiment, i.e., (1) the Quick Look Program, and (2) the Mean, Difference, and Variance Program, can process up to 20 information channels from the SEL digitizer. Since only two channel 12 discriminator units were available, only two tracks were processed at one time, and thus only two information channels were used for data.

2. Sample Rate. Only one information channel can be digitized at each instant; therefore, the sampling rate for each channel was equal to the word rate divided by the number of channels used. The word rate was set at 1.25 KC. Thus, the sampling rate for each channel was 1250/2 = 625 samples per second. The sampling time was set at about 8 seconds; therefore, approximately 5000 samples were obtained for each tape track on each recorder and for real time analysis.

3. Randomization. Since one track was to be reserved for a voice indentification of the input level, the selection of which track to place the voice I.D. on was randomized. In addition, the calibration sequence was randomized. Each recorder received identical voice identification patterns. The calibration sequence and voice identification pattern for each recorder are listed in Table 2.

Table 2. Calibration Sequence and Voice Identification Pattern: Recorder I and Recorder II

TAPE TRACK

INPUT LEVEL	1	2	3	4	5	6	7	8	9	10	11	12	13
60%	-	-	-	-	-	-	-	-	-	Voice Iden.	-	-	-
20%	Voice Iden.	-	-	-	-	-	-	-	-	-	-	-	-
0%	-	-	Voice Iden.	-	-	-	-	-	-	-	-	-	-
90%	-	-	-	-	-	-	-	-	-	-	-	Voice Iden.	-
80%	-	-	-	-	-	-	-	-	-	-	-	-	Voice Iden.
10%	-	-	-	-	-	Voice Iden.	-	-	-	-	-	-	-
30%	-	-	-	-	-	-	Voice Iden.	-	-	-	-	-	-
40%	-	-	Voice Iden.	-	-	-	-	-	-	-	-	-	-
100%	-	Voice Iden.	-	-	-	-	-	-	-	-	-	-	-
50%	-	-	Voice Iden.	-	-	-	-	-	-	-	-	-	-
70%	-	-	-	-	-	-	-	-	-	-	-	Voice Iden.	-

4. Output. In addition to obtaining the individual values via the Quick Look Program, means and variances were obtained from use of the SEL Mean, Variance, and Difference program for all tracks of both recorders and real time. A sample size of 128 was used for this program.

D. ORGANIZATION OF REPORT

Section I of this report describes the experiment that was conducted to investigate the desired characteristics. Section II is concerned with a description of the statistical analyses performed on the data. Section III deals with an investigation of the noise effects of the tape recorders. Section IV contains a secondary experiment in noise analysis. Section V presents a summary of the conclusions and recommendations.

SECTION II. THE ANALYSIS

In investigating the noise effects contributed by the tape recorders and by the wow and flutter of the magnetic tape, the questions of primary interest are: (1) are there significant differences among the tape tracks due to the amount of noise contributed by wow and flutter? (2) is there a significant difference between the tape recorders? (3) is there a significant difference between the tape recorders and real time analysis due to the noise effect of the tape recorders?

Several statistical tests which might be used to answer the above questions require the testing of two important assumptions. These assumptions are (1) the values which are being tested are distributed normally, and (2) the variances of these distributions are homogeneous (they come from the same universe). The validity of these assumptions must be tested prior to deciding which test will be used to examine the difference between tape tracks and recorders.

A. PRESENTATION OF DATA

Table 3 contains the summary of means and variances for this experiment. Figures 2 and 3 represent sample distributions from randomly selected input levels, recorders and track. The data contained in these figures and tables are a representative sample of the data collected from this experiment. They will be used to illustrate the testing of the assumptions of normality and homogeneity of variances.

B. TEST FOR NORMALITY

The method employed to test the normality of the experi-

Table 3. Means and Variances, All Levels, All Tracks

Input Levels			Tape Tracks 1	2	3	4	5
0%	Recorder 1	$\overline{X}$	62.632	65.476		59.383	64.407
		σ^2	5.216	5.628		3.099	3.153
	Recorder 2	$\overline{X}$	(Lost	76.086		76.375	78.398
		σ^2	Data)	4.256		3.156	2.509
10%	Recorder 1	$\overline{X}$	147.601	152.914	149.164	154.446	151.719
		σ^2	3.556	3.879	3.079	4.140	4.250
	Recorder 2	$\overline{X}$	166.546	165.804	167.281	168.071	168.141
		σ^2	3.953	4.094	3.280	3.542	3.755
20%	Recorder 1	$\overline{X}$		239.445	240.570	238.633	243.657
		σ^2		2.622	2.354	4.482	4.774
	Recorder 2	$\overline{X}$		255.546	254.851	255.688	258.866
		σ^2		3.734	4.130	3.915	3.662
30%	Recorder 1	$\overline{X}$	328.125	333.118	328.680	334.102	329.164
		σ^2	4.578	4.187	3.714	2.990	4.704
	Recorder 2	$\overline{X}$	346.812	347.132	347.460	349.039	348.171
		σ^2	5.199	5.208	4.065	4.713	3.675
40%	Recorder 1	$\overline{X}$	417.804	422.578		420.047	424.859
		σ^2	4.875	3.917		4.020	3.654
	Recorder 2	$\overline{X}$	437.937	438.078		437.602	440.937
		σ^2	3.652	4.386		3.943	3.089
50%	Recorder 1	$\overline{X}$	509.571	514.180	509.804		513.929
		σ^2	6.011	4.917	5.047		4.877
	Recorder 2	$\overline{X}$	528.438	527.664	527.907		531.024
		σ^2	4.309	3.883	2.911		3.967
60%	Recorder 1	$\overline{X}$	601.352	603.415	603.008	606.469	604.071
		σ^2	5.419	5.871	5.751	5.503	8.058
	Recorder 2	$\overline{X}$	617.554	618.742	620.125	621.492	621.218
		σ^2	4.778	5.003	4.812	5.328	4.592
70%	Recorder 1	$\overline{X}$	691.297	694.946	692.679	695.563	691.860
		σ^2	4.180	4.234	5.530	4.698	4.521
	Recorder 2	$\overline{X}$	710.399	710.804	710.047	711.180	711.821
		σ^2	4.741	5.141	5.004	5.527	4.378
80%	Recorder 1	$\overline{X}$	781.437	785.156	782.625	785.593	783.633
		σ^2	3.761	4.084	5.078	6.397	2.435
	Recorder 2	$\overline{X}$	797.992	799.507	798.734	799.899	800.008
		σ^2	2.398	3.906	4.056	3.241	4.501
90%	Recorder 1	$\overline{X}$	871.523	873.835	873.601	875.921	876.336
		σ^2	7.535	6.204	6.666	7.261	5.147
	Recorder 2	$\overline{X}$	889.375	891.335	889.204	890.477	889.531
		σ^2	7.526	6.336	4.170	4.642	5.842
100%	Recorder 1	$\overline{X}$	963.766		965.313	964.211	966.008
		σ^2	6.773		7.217	5.874	5.564
	Recorder 2	$\overline{X}$	981.179		981.968	981.109	983.585
		σ^2	5.616		6.014	5.865	4.184

Table 3. (cont'd) Means and Variances, All Levels, All Tracks

Input Levels			Tape Tracks				
			6	7	8	9	10
0%	Recorder 1	$\overline{X}$	59.486	64.351	60.375	66.109	62.235
		σ^2	3.186	3.701	4.095	4.208	4.808
	Recorder 2	$\overline{X}$	78.133	76.766	76.859	79.726	78.554
		σ^2	3.435	3.710	3.685	3.125	2.653
10%	Recorder 1	$\overline{X}$		156.125	150.516	155.796	151.500
		σ^2		4.021	2.972	3.203	3.500
	Recorder 2	$\overline{X}$		167.219	166.968	170.524	168.258
		σ^2		4.601	3.561	4.045	4.916
20%	Recorder 1	$\overline{X}$	239.571	243.968	240.485	254.516	241.922
		σ^2	2.730	3.483	3.187	3.847	4.674
	Recorder 2	$\overline{X}$	257.226	257.304	257.383	261.355	259.500
		σ^2	4.132	4.196	4.427	4.399	3.078
30%	Recorder 1	$\overline{X}$	333.968		330.976	335.594	333.218
		σ^2	3.714		4.167	2.587	4.842
	Recorder 2	$\overline{X}$	348.664		347.367	351.015	348.024
		σ^2	3.930		5.060	3.533	4.717
40%	Recorder 1	$\overline{X}$	419.829	424.407	421.000	425.540	423.477
		σ^2	4.672	4.887	3.992	3.718	3.486
	Recorder 2	$\overline{X}$	437.726	439.085	436.859	440.859	436.516
		σ^2	4.421	3.395	4.638	4.185	3.988
50%	Recorder 1	$\overline{X}$	510.593	515.086	511.812	515.680	514.531
		σ^2	5.209	4.819	5.621	6.183	3.342
	Recorder 2	$\overline{X}$	529.086	529.477	528.430	532.485	529.875
		σ^2	3.475	3.376	3.074	4.328	4.203
60%	Recorder 1	$\overline{X}$	607.399	605.187	606.579	605.274	
		σ^2	6.428	4.730	5.012	4.186	
	Recorder 2	$\overline{X}$	622.141	620.421	622.125	622.758	
		σ^2	3.958	4.308	3.468	6.041	
70%	Recorder 1	$\overline{X}$	694.782	696.601	697.507	696.625	698.936
		σ^2	4.967	3.853	4.109	6.140	4.386
	Recorder 2	$\overline{X}$	712.500	713.008	713.781	712.157	713.852
		σ^2	3.906	6.361	5.702	6.140	5.435
80%	Recorder 1	$\overline{X}$	785.976	786.820	787.093	785.296	785.976
		σ^2	3.324	5.788	6.569	8.382	7.199
	Recorder 2	$\overline{X}$	801.188	801.954	803.735	800.796	802.796
		σ^2	4.579	3.526	4.582	4.132	4.320
90%	Recorder 1	$\overline{X}$	878.093	877.617	877.796	876.718	877.804
		σ^2	3.803	4.767	5.351	7.624	6.688
	Recorder 2	$\overline{X}$	890.695	892.219	893.883	891.313	893.079
		σ^2	5.149	5.054	5.146	5.389	5.801
100%	Recorder 1	$\overline{X}$	966.140	967.656	967.016	967.461	969.789
		σ^2	4.826	4.600	6.371	5.499	7.420
	Recorder 2	$\overline{X}$	982.266	983.820	983.257	986.758	984.773
		σ^2	4.859	4.256	5.332	4.212	5.117

Table 3. (cont'd) Means and Variances, All Levels, All Tracks

Input Levels			Tape Tracks 11	12	13	Real Time 1	2
0%	Recorder 1	$\overline{X}$	66.469	62.539	68.023	$\overline{X}$=65.922	$\overline{X}$=66.023
		σ^2	6.894	2.893	4.089		
	Recorder 2	$\overline{X}$	77.757	76.961	78.383		
		σ^2	3.683	3.264	2.974	σ^2=1.897	σ^2=1.934
10%	Recorder 1	$\overline{X}$	155.390	152.844	157.672	$\overline{X}$=155.594	$\overline{X}$=155.618
		σ^2	3.662	3.724	4.202		
	Recorder 2	$\overline{X}$	167.930	167.188	169.476		
		σ^2	3.933	4,204	3.957	σ^2=2.087	σ^2=1.890
20%	Recorder 1	$\overline{X}$	246.040	242.656	247.773	$\overline{X}$=244.813	$\overline{X}$=244.718
		σ^2	4.796	3.834	4.726		
	Recorder 2	$\overline{X}$	259.835	256.602	258.477		
		σ^2	3.782	3.208	3.642	σ^2=3.428	σ^2=2.670
30%	Recorder 1	$\overline{X}$	336.421	331.882	336.188	$\overline{X}$=334.570	$\overline{X}$=334.656
		σ^2	5.464	5.853	5.688		
	Recorder 2	$\overline{X}$	348.633	348.407	350.641		
		σ^2	4.248	3.700	3.169	σ^2=1.745	σ^2=1.725
40%	Recorder 1	$\overline{X}$	426.703	442.172	426.118	$\overline{X}$=424.032	$\overline{X}$=424.087
		σ^2	4.117	3.741	7.718		
	Recorder 2	$\overline{X}$	440.390	438.204	440.133		
		σ^2	3.943	4.108	4.091	σ^2=1.651	σ^2=1.879
50%	Recorder 1	$\overline{X}$	516.742	513.711	517.696	$\overline{X}$=513.547	$\overline{X}$=513.657
		σ^2	4.707	3.624	4.320		
	Recorder 2	$\overline{X}$	530.204	529.125	532.195		
		σ^2	4.374	4.521	5.219	σ^2=2.872	σ^2=2.836
60%	Recorder 1	$\overline{X}$	607.086	606.445	609.672	$\overline{X}$=602.696	$\overline{X}$=602.727
		σ^2	3.975	5.512	7.952		
	Recorder 2	$\overline{X}$	624.094	620.594	623.110		
		σ^2	5.314	4.291	4.916	σ^2=2.976	σ^2=2.947
70%	Recorder 1	$\overline{X}$	697.414		699.726	$\overline{X}$=692.875	$\overline{X}$=692.796
		σ^2	6.481		7.125		
	Recorder 2	$\overline{X}$	712.290		715.297		
		σ^2	4.835		4.461	σ^2=2.328	σ^2=2.179
80%	Recorder 1	$\overline{X}$	786.867	789.031		$\overline{X}$=783.094	$\overline{X}$=783.055
		σ^2	4.912	5.202			
	Recorder 2	$\overline{X}$	798.391	802.180			
		σ^2	3.337	3.980		σ^2=2.455	σ^2=2.292
90%	Recorder 1	$\overline{X}$	878.016		878.946	$\overline{X}$=873.383	$\overline{X}$=873.383
		σ^2	5.152		5.031		
	Recorder 2	$\overline{X}$	891.062		892.937		
		σ^2	6.074		6.136	σ^2=3.787	σ^2=4.333
100%	Recorder 1	$\overline{X}$	968.625	968.796	969.563	$\overline{X}$=963.063	$\overline{X}$=963.171
		σ^2	7.546	6.257	6.760		
	Recorder 2	$\overline{X}$	985.601	982.687	985.531		
		σ^2	4.072	4.917	4.436	σ^2=2.034	σ^2=2.296

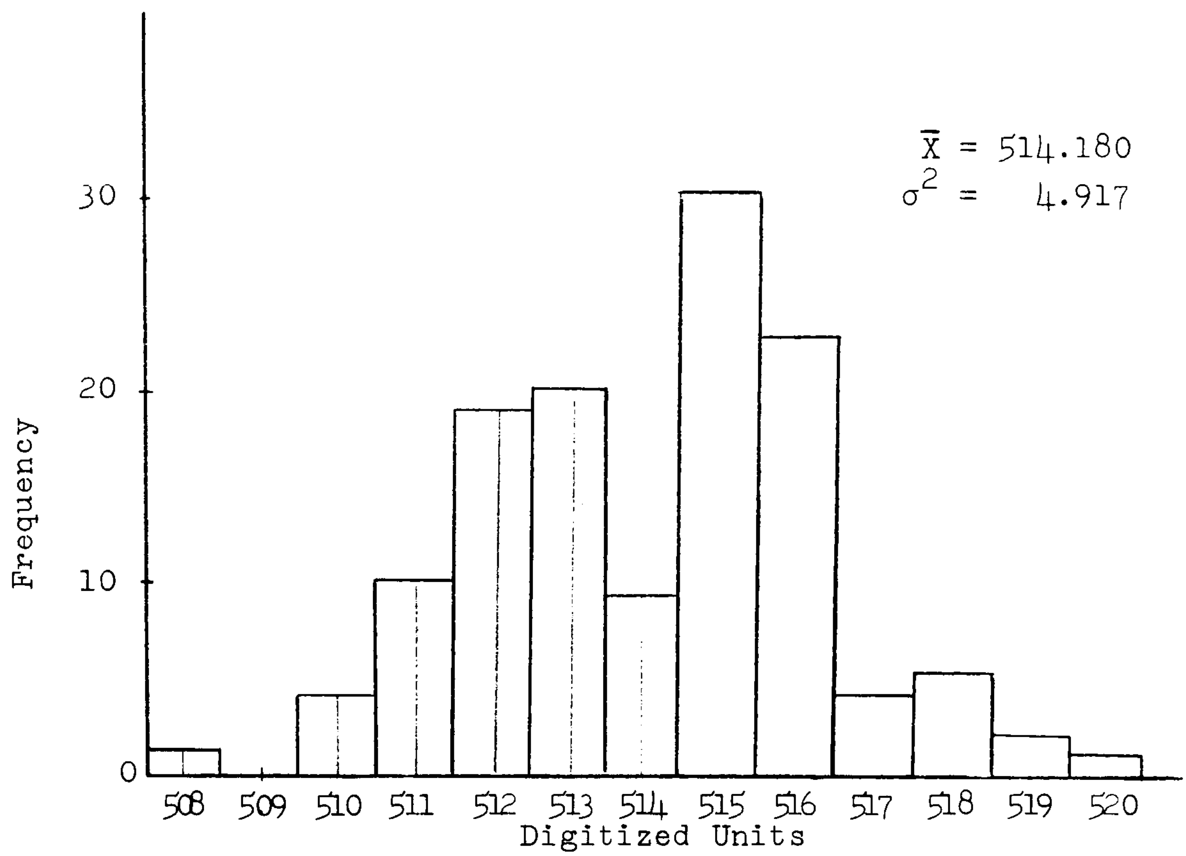

Figure 2. Graph of the Frequency Distribution of the Individual Values for Recorder 1, Track 2, 50% Input Level.

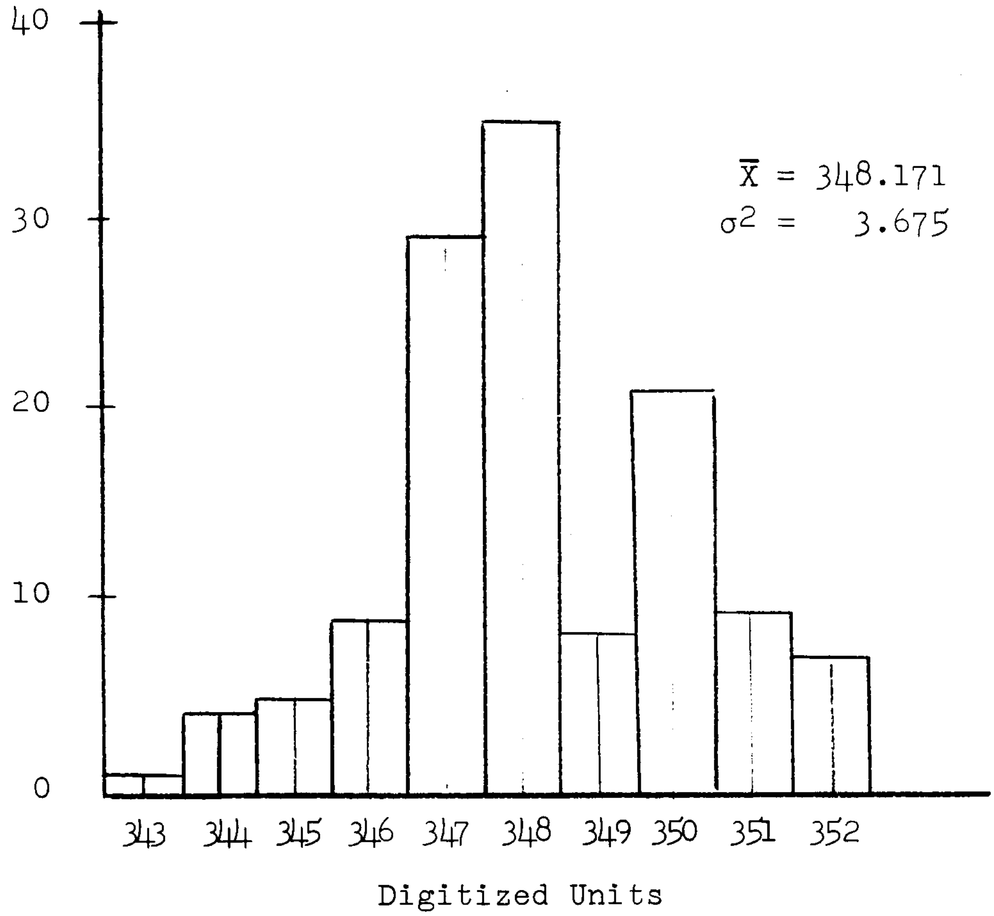

Figure 3. Graph of the Frequency Distribution of the Individual Values for Recorder 2, Tracks, 30% Input Level.

mental data was a χ^2 test of goodness of fit.

The theory underlying a χ^2 test is as follows: Let $F_1, F_2, \ldots, F_k$ be sample frequencies of k classes, and let $f_1, f_2, \ldots, f_k$ be the theoretical frequencies of a normal distribution.

If the sample in question follows a normal distribution, then sample values of the quantity

$$\sum_{i=1}^{k} \frac{1}{f_i} (F_i - f_i)^2 \qquad [1]$$

will follow a χ^2 distribution with d degrees of freedom. The degrees of freedom, d, equals the number of classes, k, minus the number of relations between the F_i and the f_i, that are used to determine the f_i. In this particular case, when the normality of the data in question is being tested, the fitting process involves three restrictions:

$$(1) \quad \sum f_i = \sum F_i$$

$$(2) \quad \overline{X}' = \frac{1}{N} \sum F_i X_i$$

$$(3) \quad \sigma'^2 = \frac{1}{N} \sum F_i X_i^2$$

These three restrictions cause a loss of three degrees of freedom. Consequently, the degrees of freedom, d, are equal to the number of classes minus three:

$$d = k - 3$$

For the test of normality of the experimental data, the following terms are defined:

X = Cell upper limit.

$Z = (X_i - \bar{X})/\sigma$

$F(Z)$ = area under the normal curve from $-\infty$ to Z.

R.F. Cell = relative frequency of cell.

F_i = actual cell frequency.

f_i = theoretical absolute frequency of the normal distribution.

The assumption that the individual values were normally distributed was tested by the χ^2 test. Sample tests and corresponding results are reported in Tables 4 and 5. Note that frequencies are grouped when the theoretical absolute frequency is less than five.

Fifty samples were randomly selected as to input level, recorder number, and tape track number. The data was then grouped and the tests performed. At the .05 confidence level, only 6% of the fifty samples tested were accepted as normal. Increasing the confidence level to .01, only 18% of the samples were accepted as normal. We cannot, therefore, conclude that the data is normally distributed. Table 6 shows a summary of all of the χ^2 tests performed.

C. TEST FOR HOMOGENEITY OF VARIANCES

To determine whether a set of variances could all have come from the same universe, "Bartlett's Test" is employed. In this test the ratio M/C is computed from the following formulae:

$$M = 2.3026 \left[m \log \left(\frac{\Sigma n_i V_i}{n}\right) - \Sigma(m_i \log V_i)\right]$$

$$C = 1 + \frac{1}{3(g - 1)} \left[\Sigma \frac{1}{n_i} - \frac{1}{n}\right]$$

g = number of variances

Table 4

Chi-Square Normality Test, Recorder 1, Track 2, 50% Input Level

$\overline{X} = 514.180, \ \sigma^2 = 4.917, \ \sigma = 2.217$

1		2	3	4	5	6	7	8	9
X	$\chi = X - \overline{X}$	$Z = \frac{X-\overline{X}}{\sigma}$	Relative Frequency $-\infty$ to Z	Relative Frequency of cell	Absolute Theoretical Frequency f	Actual Frequency F	(F-f)	$(F-f)^2$	$\frac{(F-f)^2}{f}$
508.000	-6.180	-2.79	.0026	.0026	.333	1			
509.000	-5.180	-2.33	.0099	.0073	.934	0			
510.000	-4.180	-1.89	.0294	.0195	2.496 } 9.779	4 } 15			
511.000	-3.180	-1.43	.0764	.0470	6.016	10	5.221	27.259	2.788
512.000	-2.180	-0.98	.1635	.0871	11.149	19	7.851	61.638	5.529
513.000	-1.180	-0.53	.2981	.1346	17.229	20	2.771	7.678	.446
514.000	-0.180	-0.08	.4681	.1700	21.760	9	-12.760	162.818	7.482
515.000	0.820	0.37	.6443	.1762	22.554	30	7.446	55.443	2.459
516.000	1.820	0.82	.7939	.1496	19.149	23	3.851	14.830	.774
517.000	2.820	1.27	.8980	.1041	13.325	4	-9.325	86.956	6.526
518.000	3.820	1.72	.9573	.0593	7.590	5	-2.590	6.708	.884
519.000	4.820	2.17	.9850	.0277	3.545 } 5.465	2 } 3	-2.465	6.076	1.112
∞	∞	∞	1.0000	.0150	1.920	1			
					128.000	128			27.999

for d = 9 - 3 = 6 d.f. $\begin{cases} \chi^2_{.05} = 12.6 \\ \chi^2_{.01} = 16.8 \end{cases}$

$\chi^2_{computed} = 27.999$

The hypothesis that the data is from a normal distribution is rejected since the computed χ^2 value is significant at both the 95% and 99% confidence levels.

Table 5

Chi-Square Normality Test, Recorder 2, Track 5, 30% Input Level

$\overline{X} = 348.171,\ \sigma^2 = 3.675,\ \sigma = 1.917$

1 X	 $\chi = X - \overline{X}$	2 $Z = \frac{X-\overline{X}}{\sigma}$	3 Relative Frequency $-\infty$ to Z	4 Relative Frequency of cell	5 Absolute Theoretical Frequency f	6 Actual Frequency F	7 (F-f)	8 $(F-f)^2$	9 $\frac{(F-f)^2}{f}$
343.000	-5.171	-2.697	.0035	.0035	.448 } 6.335	1 } 10			
344.000	-4.171	-2.176	.0146	.0111	1.421	4			
345.000	-3.171	-1.654	.0495	.0349	4.466	5	3.66	13.40	2.114
346.000	-2.171	-1.132	.1292	.0797	10.202	9	1.20	1.44	.141
347.000	-1.171	-0.611	.2709	.1417	18.138	29	10.86	117.94	6.501
348.000	-0.171	-0.089	.4641	.1932	24.730	35	10.27	105.47	4.264
349.000	0.829	0.432	.6664	.2023	25.894	8	17.89	320.05	12.361
350.000	1.829	0.954	.8289	.1625	20.800	21	.20	.04	.000
351.000	2.829	1.476	.9306	.1017	13.018	9	4.02	16.16	1.241
∞	∞	∞	1.0000	.0694	8.883	7	1.88	3.53	.398
					128.000	128			27.020

for d = 8 - 3 = 5d.f. $\begin{cases} \chi^2_{.05} = 11.1 \\ \chi^2_{.01} = 15.1 \end{cases}$

$\chi^2_{computed} = 27.020$

The hypothesis that the data is from a normal distribution is rejected since the computed χ^2 value is significant at both the 95% and 99% confidence levels.

Table 6. Summary of Normality Tests

Sample	Recorder	Level	Track	$\chi^2_{comp.}$	$\chi^2_{.05}$	$\chi^2_{.01}$	Conclusion
1	1	80%	10	20.94	15.51	20.09	Bimodal
2	2	50%	2	20.07	9.49	13.28	High Kurtosis
3	2	80%	1	23.20	5.99	9.21	Bimodal
4	2	60%	9	37.81	12.59	16.81	Bimodal
5	2	10%	2	18.99	9.49	13.28	Skewed
6	1	10%	10	19.10	12.59	16.81	High Kurtosis
7	1	100%	7	25.73	12.59	16.81	Skewed
8	1	30%	2	20.81	12.59	16.81	Bimodal
9	2	20%	9	18.28	11.07	15.09	Bimodal
10	2	90%	4	30.91	11.07	15.09	Bimodal
11	2	50%	10	30.78	11.07	15.09	Bimodal
12	2	70%	11	25.41	12.59	16.81	High Kurtosis
13	2	50%	13	25.09	12.59	16.81	Multi-Modal
14	1	10%	2	5.65	9.49	13.28	Accept at .05
15	1	80%	9	21.73	12.59	16.81	Bimodal
16	1	0%	12	37.40	9.49	13.28	High Kurtosis
17	2	20%	6	36.13	11.07	15.09	Bimodal
18	1	40%	8	29.87	9.49	13.28	Bimodal
19	2	100%	3	17.65	12.59	16.81	Bimodal
20	1	20%	7	20.29	11.07	15.09	Skewed
21	2	70%	5	24.10	11.07	15.09	High Kurtosis
22	2	20%	4	7.97	11.07	15.09	Accept at .05
23	1	100%	5	29.01	11.07	15.09	Skewed
24	2	90%	2	17.78	14.07	18.48	Accept at .01
25	1	80%	5	37.85	0.49	13.28	High Kurtosis

Table 6. (Cont'd) Summary of Normality Tests

Sample	Recorder	Level	Track	$\chi^2_{comp.}$	$\chi^2_{.05}$	$\chi^2_{.01}$	Conclusion
26	1	60%	9	24.93	9.49	13.28	Bimodal
27	2	60%	11	33.44	12.59	16.81	High Kurtosis
28	2	50%	1	19.87	5.99	9.21	Skewed
29	1	70%	3	15.89	11.07	15.09	Bimodal
30	1	30%	9	26.11	11.07	15.09	High Kurtosis
31	1	20%	4	18.14	12.59	16.81	Skewed
32	2	80%	9	41.94	9.49	13.28	Skewed
33	1	100%	8	12.43	14.07	18.48	Accept at .05
34	2	100%	13	29.99	12.59	16.81	Bimodal
35	1	90%	7	24.76	12.59	16.81	Skewed
36	1	0%	6	12.25	11.07	15.09	Accept at .01
37	1	90%	4	16.15	14.07	18.48	Accept at .01
38	1	70%	1	27.22	11.07	15.09	Bimodal
39	1	40%	2	19.85	11.07	15.09	High Kurtosis
40	1	50%	2	27.99	12.59	16.81	Bimodal
41	2	30%	5	27.02	11.07	15.09	Skewed
42	2	10%	1	14.41	12.59	16.81	Accept at .01
43	2	60%	5	13.49	11.07	15.09	Accept at .01
44	1	10%	11	25.64	11.07	15.09	High Kurtosis
45	2	0%	9	23.87	11.07	15.09	High Kurtosis
46	2	90%	5	24.15	12.59	16.81	Bimodal
47	1	80%	2	35.86	11.07	15.09	Bimodal
48	1	70%	9	11.26	7.82	11.34	Accept at .01
49	1	20%	3	25.52	7.82	11.34	Skewed
50	1	70%	8	29.13	11.07	15.09	Skewed

V_i = an individual variance

n_i = sample size of an individual variance minus one

$n = \Sigma n_i$

It can be shown that the ratio M/C is approximated by a χ^2 distribution with g - 1 degrees of freedom. To test the hypothesis that the variances are all homogeneous, we need only to calculate the quantity M/C and compare it with the value of the theoretical χ^2 for g - 1 degrees of freedom from a table of values of the χ^2 distribution.

The assumption was made that the variances computed for each calibration level were homogeneous. It was tested by applying Bartlett's test at each level.

Sample calculations of the results of the Bartlett's Test for homogeneity of variances at the 0%, 50% and 100% levels are presented in tables 7, 8, and 9, respectively. Also, table 10 contains a general summary of the results of Bartlett's tests for homogeneity of variances at each input level.

It can be inferred from the results of these tests that in general the variances of the various input levels are not homogeneous, and therefore the assumption of homogeneity for the performance of the analysis of variance is not satisfied.

It must be remembered, however, that Bartlett's test is based on the assumption that the random variation within each of the groups follows the normal law. If this is not true, the ratio M/C may indicate departure from normality rather than heterogeneity of variance. Consequently, even if the variances are homogeneous, the assumption of normality is not met, and other methods of analysis need to be investigated.

Table 7
Bartlett's Test, 0% Level

Recorder	Track	n_i	V_i	log V_i	n_i log V_i	$n_i V_i$
I	1	127	5.216	0.71734	91.102	662.432
	2	127	5.628	0.75035	95.294	714.756
	3	127	3.099	0.49122	62.385	393.573
	4	127	3.153	0.49872	63.337	400.431
	5	127	3.186	0.50325	63.913	404.622
	6	127	3.701	0.56820	72.161	470.027
	7	127	4.095	0.61225	77.756	520.065
	8	127	4.208	0.62408	79.258	534.416
	9	127	4.808	0.68196	86.609	610.616
	10	127	6.894	0.83847	106.486	875.538
	11	127	2.893	0.46135	58.591	367.411
	12	127	4.089	0.61162	77.675	519.303
II	1	127	4.256	0.62900	79.883	540.512
	2	127	3.156	0.49914	63.908	400.812
	3	127	2.509	0.39950	50.737	318.643
	4	127	3.435	0.53593	68.063	436.245
	5	127	3.710	0.56937	72.310	471.170
	6	127	3.685	0.56644	71.938	467.995
	7	127	3.125	0.49485	62.846	396.875
	8	127	2.653	0.42374	53.815	336.931
	9	127	3.683	0.56620	71.907	467.741
	10	127	3.264	0.51375	65.246	414.528
	11	127	2.974	0.47334	60.114	377.698
		2921			1,655.334	11,102.340

$$M = 2.3026 \left[2921 \log \left(\frac{11{,}102.340}{2921} \right) - 1{,}655.334 \right]$$

$$= 2.3026 \left[2921 \log 3.800 - 1{,}655.334 \right] = 88.102$$

$$C = 1 + \frac{1}{3(23-1)} \left[\frac{23}{127} - \frac{1}{2921} \right]$$

$$= 1 + .01515 \left[0.1811 - .0003 \right] = 1.003$$

$$M/C = \frac{88.102}{1.003} = 87.838$$

$$\chi^2_{(.05)(d=22)} = 33.924$$

The hypothesis that the variances for the 0% level are homogeneous is rejected.

Table 8

Bartlett's Test, 50% Level

Recorder	Track	n_i	V_i	log V_i	n_i log V_i	$n_i V_i$
I	1	127	6.011	0.77895	98.927	763.397
	2	127	4.917	0.69170	87.846	624.459
	3	127	5.047	0.70303	89.285	640.969
	5	127	4.877	0.68815	87.395	619.379
	6	127	5.209	0.71675	91.027	661.543
	7	127	4.819	0.68296	86.736	612.013
	8	127	5.621	0.74981	95.226	713.867
	9	127	6.183	0.79120	100.482	785.241
	10	127	3.342	0.52401	66.549	424.434
	11	127	4.707	0.67274	85.438	597.789
	12	127	3.624	0.55919	71.017	460.248
	13	127	4.320	0.63548	80.706	548.640
II	1	127	4.309	0.63438	80.566	547.243
	2	127	3.883	0.58917	74.825	493.141
	3	127	2.911	0.46404	58.933	369.697
	5	127	3.967	0.59846	76.004	503.809
	6	127	3.475	0.54096	68.702	441.325
	7	127	3.376	0.52840	67.107	428.752
	8	127	3.074	0.48770	61.938	390.398
	9	127	4.328	0.63629	80.809	549.656
	10	127	4.203	0.62356	79.192	533.781
	11	127	4.374	0.64088	81.392	555.498
	12	127	4.521	0.65524	83.215	574.167
	13	127	5.219	0.71759	91.134	662.813
		3048			1,944.451	13,502.259

M = 7018.325 log 4.430 - 4,477.293

M = 7018.325 (0.64640) - 4,477.293

M = 4,536.645 - 4,477.293

M = 59.352

$$M/C = \frac{59.352}{1.003} = 59.174$$

$$\chi^2_{.05} \text{ w/23df} = 35.2$$

Reject hypothesis; variances are not homogeneous.

Table 9

Bartlett's Test, 100% Level

Recorder	Track	n_i	V_i	$\log V_i$	$n_i \log V_i$	$n_i V_i$
I	1	127	6.773	0.83078	102.186	860.171
	3	127	7.217	0.85836	109.012	916.559
	4	127	5.874	0.76893	97.654	745.998
	5	127	5.564	0.74539	94.665	706.638
	6	127	4.826	0.68359	86.816	612.902
	7	127	4.600	0.66276	84.171	584.200
	8	127	6.371	0.80421	102.135	809.117
	9	127	5.499	0.74028	94.016	698.373
	10	127	7.420	0.87040	110.541	942.340
	11	127	7.546	0.87772	111.470	958.342
	12	127	6.257	0.79637	101.139	794.639
	13	127	6.760	0.82995	105.404	858.520
II	1	127	5.616	0.74943	95.178	713.232
	3	127	6.014	0.77916	98.953	763.778
	4	127	5.865	0.76827	97.570	744.855
	5	127	4.184	0.62159	78.942	531.368
	6	127	4.859	0.68655	87.192	617.093
	7	127	4.256	0.62900	79.833	540.512
	8	127	5.332	0.72689	92.315	677.164
	9	127	4.212	0.62449	79.310	534.924
	10	127	5.117	0.70902	90.046	649.859
	11	127	4.072	0.60981	77.446	517.144
	12	127	4.917	0.69170	87.846	624.459
	13	127	4.436	0.64699	82.168	563.372
		3048			2,246.008	16,965.549

$$M = 7018.325 \log \frac{16,965.549}{3048} - 5,171.658$$

$$M = 7018.325 \log 5.566 - 5,171.658$$

$$M = (7018.325)(.74554) - 5,171.658$$

$$M = 5,232.442 - 5,171.658 = 60.784$$

$$M/C = \frac{60.784}{1.003} = 60.602$$

$$\chi^2_{.05} \text{ w/23df} = 35.2$$

Reject hypothesis; variances are not homogeneous.

Table 10

A General Summary of the Results of Bartlett's Test

Input Level	α	Are Variances Homogeneous?
0%	.05	No
10%	.05	Yes
20%	.05	No
30%	.05	No
40%	.05	No
50%	.05	No
60%	.05	No
70%	.05	No
80%	.05	No
90%	.05	No
100%	.05	No

D. ALTERNATIVES FOR FURTHER ANALYSIS

From the preceding tests we may conclude:

(1) Individual values for various recorder-input level-tape track distributions are not normally distributed.

(2) Variances for the various input levels are heterogeneous.

Since the two basic assumptions (normality and homogeneity) for the analysis of variance are not satisfied, there are only two alternatives for further analysis. The first is a transformation of the data in an attempt to meet the assumptions. The second is the use of some non-parametric method of analysis that does not depend on the assumptions of normality and homogeneity.

After trying several transformations without success, the second alternative for analysis was chosen.

E. THE FRIEDMAN NON-PARAMETRIC TWO-WAY ANALYSIS OF VARIANCE BY RANKS

This test, usually called the Friedman χ_T^2 test, is useful when the measurement of the variable is at least on an ordinal scale.

The Friedman χ_T^2 test is utilized in testing the null hypothesis that the k samples have been drawn from the same population.

In this particular experiment we have an ordinal scale, which consists of the level of input, from 0% to 100%, and with increments of 10%. Also, two recorders were used at each input level, as well as 13 tracks (one track was used for voice identification).

The theory upon which this test is based is as follows:

> The data are cast in a two-way table having N rows and k columns. The rows represent the responses and the columns represent the various conditions. The responses within each group are ranked in <u>each row</u>. Then, the totals for each column are obtained R_j's. Now if the null hypothesis (that all the samples - columns - came from the same population) is in fact true, then the distribution of ranks in <u>each column</u> would be a matter of chance.[1]

The Friedman test determines whether the rank totals (R_j) differ significantly. The value of the statistic χ_R^2, subject to the d.f. = k - 1, will approximate a χ^2 distribution, when

$$\chi_R^2 = \frac{12}{Nk(k+1)} \sum_{j=1}^{k} (R_j)^2 - 3N(k+1) \qquad [2]$$

where N = number of rows

k = number of columns

R_j = sum of ranks in j^{th} column

$\sum_{j=1}^{k}$ indicates a total summation of the square of the sums of ranks over all k conditions.

Considering the conservativeness of this test, and also the fact that the Friedman χ_R^2 test is one of the few tools available to analyze non-normal data, the following inferences can be drawn: a) There seems to be no significant difference between tape tracks at each input level, and b) there are reasons to suspect that there might be a difference between

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[1]Siegel, Sidney, <u>Non-Parametric Statistics for the Behavioral Sciences</u>, New York, N. Y.: McGraw-Hill Book Co., Inc., 1956, pp. 166-172.

recorders at each input level.

Sample calculations of the results of the Friedman's non-parametric tests of significant difference are presented in the next pages. Tables 11, 13, and 15 present tests for difference between tape tracks at the 0%, 50%, and 100% input levels, respectively. Tables 12, 14, and 16 present tests for difference between tape recorders at the 0%, 50%, and 100% input levels, respectively. Also, Table 17 contains a general summary of the results of Friedman's test at each level of input considered in this experiment.

Table 11

Friedman's Test for Tape Tracks, 0% Level

	Tracks 2	4	5	6	7	8	9	10	11	12	13
Recorder I	10	2	3	4	5	7	8	9	11	1	6
Recorder II	11	5	1	6	10	9	4	2	8	7	3
R_j	21	7	4	10	15	16	12	11	19	8	9
R_j^2	441	49	16	100	225	256	144	121	361	64	81

$N = 2$ $\qquad \sum_{j=1}^{k} R_j^2 = 1858$

$C = 11$

$$\chi_R^2 = \left[\frac{12}{(2)(12)(12)} (1858) \right] - [(3)(2)(12)]$$

$$\chi_R^2 = \frac{1858}{24} - 72 = \underline{5.417}$$

$$\chi^2_{df=10} = \underline{18.307}$$

$\alpha = .05$

Accept hypothesis: there is no significant difference in tape tracks due to their noise effects at the 0% level.

Table 12

Friedman's Test for Tape Recorders, 0% Level

	Tracks 2	4	5	6	7	8	9	10	11	12	13	R_j	R_j^2
Recorder I	1	2	1	2	2	1	1	1	1	2	1	15	18
Recorder II	2	1	2	1	1	2	2	2	2	1	2	225	324

N = 11

C = 2

$$\chi^2_R = \frac{12}{(11)(2)(3)} (225 + 324) - (3)(11)(3)$$

$$\chi^2_R = \frac{1098}{11} - 99$$

$$\chi^2_R = \underline{0.82}$$

$$\chi^2_{df = 1} = \underline{3.841}$$
$$\alpha = .05$$

Accept hypothesis: there is no difference in tape recorders due to their noise effects at the 0% level.

Table 13

Friedman's Test for Tape Tracks, 50% Level

	Tracks											
	1	2	3	4	5	6	7	8	9	10	11	12
Recorder I	11	7	8	6	9	5	10	12	1	4	2	3
Recorder II	8	5	1	6	4	3	2	9	7	10	11	12
R_j	19	12	9	12	13	8	12	21	8	14	13	15
R_j^2	361	144	81	144	169	64	144	441	64	196	169	225

$N = 2$

$K = 12$

$$\chi_R^2 = \frac{12}{2(12)(13)}(2202) - 3(2)(13)$$

$$\chi_R^2 = \frac{2202}{26} - 78.00$$

$$\chi_R^2 = 84.69 - 78.00 = \underline{6.69}$$

$$\chi^2_{d.f.=11} = \underline{19.675}$$

$\alpha = .05$

Accept hypothesis: there is no significant difference in tape tracks due to their noise effects at the 50% level.

Table 14

Friedman's Test for Tape Recorders, 50% Level

	Tracks 1	2	3	4	5	6	7	8	9	10	11	12	R_j	R_j^2
Recorder I	1	1	2	1	1	1	1	1	2	1	2	2	16	256
Recorder II	2	2	1	2	2	2	2	2	1	2	1	1	20	400

$K = 2$

$N = 12$

$$\chi_R^2 = \frac{12}{12(2)(3)} (256 + 400) - 3(12)(3)$$

$$\chi_R^2 = \frac{1}{6} (256 + 400) - 108.00$$

$$\chi_R^2 = \frac{656}{6} - 108.00$$

$$\chi_R^2 = 109.33 - 108.00$$

$$\chi_R^2 = \underline{1.33}$$

$$\chi^2_{d.f.=1} = \underline{3.81}$$

$\alpha = .05$

Accept hypothesis: there is no significant difference in tape recorders due to their noise effects at the 50% level.

Table 15

Friedman's Test for Tape Tracks, 100% Level

Tracks	1	2	3	4	5	6	7	8	9	10	11	12
Recorder I	9	10	5	4	2	1	7	3	11	12	6	8
Recorder II	10	12	11	2	6	4	9	3	8	1	7	5
R_j	19	22	16	6	8	5	16	6	19	13	13	13
R_j^2	361	484	256	36	64	25	256	36	361	169	169	169

$N = 2$

$K = 12$

$$\chi_R^2 = \frac{2386}{26} - 78.00$$

$$\chi_R^2 = 91.77 - 78.00$$

$$\chi_R^2 = \underline{13.77}$$

$$\chi^2_{.05} = \underline{19.675}$$
d.f. =11

Accept hypothesis: there is no significant difference in tape tracks due to their noise effects at the 100% level.

Table 16

Friedman's Test for Tape Recorders, 100% Level

	Tracks 1	2	3	4	5	6	7	8	9	10	11	12	R_j	R_j^2
Recorder I	1	1	1	1	2	1	1	1	1	1	1	1	13	169
Recorder II	2	2	2	2	1	2	2	2	2	2	2	2	23	529

$K = 2$

$N = 12$

$$\chi_R^2 = \frac{12}{12(2)(3)}(169 + 529) - 3(12)(3)$$

$$\chi_R^2 = \frac{169 + 529}{6} - 108.00$$

$$\chi_R^2 = 116.33 - 108.00$$

$$\chi_R^2 = \underline{8.33}$$

$$\chi^2_{d.f.=1} = \underline{3.81}$$

$\alpha = .05$

Reject hypothesis: there is a significant difference in tape recorders due to their noise effects at the 100% level.

Table 17

A General Summary of the Results of Friedman's Test

0%	Is there a significant difference in	tracks?	No
		recorders?	No
10%	Is there a significant difference in	tracks?	No
		recorders?	No
20%	Is there a significant difference in	tracks?	No
		recorders?	No
30%	Is there a significant difference in	tracks?	No
		recorders?	No
40%	Is there a significant difference in	tracks?	No
		recorders?	No
50%	Is there a significant difference in	tracks?	No
		recorders?	No
60%	Is there a significant difference in	tracks?	No
		recorders?	Yes
70%	Is there a significant difference in	tracks?	No
		recorders?	No
80%	Is there a significant difference in	tracks?	No
		recorders?	Yes
90%	Is there a significant difference in	tracks?	No
		recorders?	No
100%	Is there a significant difference in	tracks?	No
		recorders?	Yes

F. THE PAIRED OBSERVATIONS TEST

Since the Friedman's χ_R^2 is very conservative, there are reasons to suspect that significant differences between tape recorders went undetected. Therefore, a less conservative test will be used in this analysis: The paired observations test.

The theory upon which the test is based is as follows:

> If X_i and Y_i are two paired observations from a set of sampled data and d_i is their difference $(X_i - Y_i)$, then the distribution of d_i's is given by the "t" distribution, $u = 0$, with $N - 1$ degrees of freedom. Thus, if we wish to test the hypothesis that $u_1 = u_2$, the universe means of the two sets of sampled data are equal, we may test to see if $\bar{d}$, the average difference between X_i and Y_i, is significantly different from zero. We may do this by performing a t test with the statistic
>
> $$T = \frac{\bar{d} - 0}{S/\sqrt{N}}$$
>
> where $\bar{d} = \frac{\Sigma d_i}{N}$, S = unbiased standard deviation of d_i's, and N = the number of pairs of observations. If this statistic, t, exceeds the value of t_α with N-1 degrees of freedom from a table of the t distribution, then we reject the hypothesis that $u_1 = u_2$.

In this particular analysis the paired observations test was used to test two differences: (1) the difference between tape recorders I and II, and (2) the difference between individual tape recorders and the real time values.

Tables 18, 19, and 20 contain representative examples of the test calculations for difference between recorders at 0%, 50%, and 100% input levels, respectively. Table 21 shows the results of the test between recorder 1 and real time. Similarly, Table 22 exposes the test between recorder 2 and real time. Tables 23 and 24 contain general summaries of the

Table 18

Paired Observations Test Between Recorders, 0% Level

Tape Tracks	Recorder I	Recorder II	d_i(I-II)	d_i^2
2	65.476	76.086	-10.610	112.572
4	59.383	76.375	-16.992	288.728
5	64.407	78.398	-13.991	195.748
6	59.486	78.133	-18.647	347.711
7	64.351	76.766	-12.415	154.132
8	60.375	76.859	-16.484	271.722
9	66.109	79.726	-13.617	185.423
10	62.235	78.554	-16.319	266.309
11	66.469	77.757	-11.288	127.419
12	62.539	76.961	-14.422	207.994
13	68.023	78.383	-10.360	107.330
			-155.145	2265.043

$$\bar{d} = \frac{-155.145}{11} = \left|-14.104\right| = 14.104$$

$$s_d^2 = \frac{d_i^2 - \frac{(d_i)^2}{n}}{n - 1} = \frac{2265.043 - \frac{(-155.145)^2}{11}}{10}$$

$$s_d^2 = 226.5043 - \frac{24,069.971}{110}$$

$$s_d^2 = 226.5043 - 218.8179 = 7.6864$$

$$s_d = \underline{\underline{2.7724}}$$

$$T = \frac{\bar{d} - 0}{\frac{s_d}{\sqrt{N}}} = \frac{-14.104}{\frac{2.7724}{\sqrt{11}}} = \frac{(-14.104)(3.3166)}{2.7724} = \underline{-16.873}$$

$t_{.01}$ (N = 10) $= \pm 3.169$

$t_{.05}$ (N = 10) $= \pm 2.228$

Reject hypothesis: there is a definite difference between recorders at the 0% level.

Table 19

Paired Observations Test Between Recorders, 50% Level

Tape Track	Recorder I	Recorder II	d_i(I-II)	d_i^2
1	509.571	528.438	-18.867	355.964
2	514.180	527.664	-13.484	181.818
3	509.804	527.907	-18.103	327.719
5	513.929	531.024	-17.095	292.239
6	510.593	529.086	-18.493	341.991
7	515.086	529.477	-14.391	207.101
8	511.812	528.430	-16.618	276.158
9	515.680	532 485	-16.805	282.408
10	514.531	529.875	-15.344	235.438
11	516.742	530.204	-13.462	181.225
12	513.711	529.125	-15.414	237.591
13	517.696	532.195	-14.499	210.221
			-192.575	3129.873

$$\bar{d} = \frac{192.575}{12} = 16.048$$

$$s_d^2 = \frac{3129.873 - \frac{(192.575)^2}{12}}{11} = 3.586$$

$$s_d = \underline{\underline{1.894}}$$

$$T = \frac{(16.048)(3.4641)}{1.894} = \underline{29.352}$$

$$t_{.01(N = 11)} = \pm 3.106$$

$$t_{.05(N = 11)} = \pm 2.201$$

Reject hypothesis: there is a significant difference between recorders at 50% level.

Table 20

Paired Observations Test Between Recorders, 100% Level

Tape Tracks	Recorder I	Recorder II	d_i(I-II)	d_i^2
1	963.766	981.179	-17.413	303.213
3	965.313	981.968	-16.655	277.389
4	964.211	981.109	-16.898	285.542
5	966.008	983.585	-17.577	308.951
6	966.140	982.266	-16.126	260.048
7	967.656	983.820	-16.804	282.374
8	967.016	983.257	-15.796	249.514
9	967.461	986.758	-19.297	372.374
10	969.789	984.773	-14.984	224.520
11	968.625	985.601	-16.976	288.185
12	968.796	982.687	-13.891	192.960
13	969.563	985.531	-15.968	254.977
			-198.385	3300.047

$$\bar{d} = \left|\frac{-198.385}{12}\right| = \underline{16.532} \;; \quad S_d^2 = \frac{3300.047 - \frac{(198.385)^2}{12}}{11}$$

$$S_d^2 = \frac{3300.047 - 3279.717}{11} = \frac{20.330}{11} = 1.848$$

$$S_d = \underline{\underline{1.359}}$$

$$T = \frac{\bar{d} - 0}{\frac{S_d}{\sqrt{N}}} = \frac{(16.532)(3.4641)}{1.359} = \underline{42.141}$$

$$t_{.01(N = 11)} = 3.106$$

$$t_{.05(N = 11)} = 2.201$$

Reject hypothesis: there is a definite significant difference between recorders at the 100% level.

Table 21

Paired Observations Test Between Recorder 1 and Real Time

Input Level %	Rec. I $\bar{\bar{X}}$	Real Time $\bar{\bar{X}}$	d_i	d_i^2
0	63.456	65.973	-2.517	6.335
10	152.974	155.606	-2.632	6.927
20	242.520	244.766	-2.246	5.045
30	332.620	334.613	-1.993	3.972
40	424.545	424.040	+0.505	0.255
50	513.611	513.602	+0.009	0.008
60	605.496	602.712	+2.784	7.751
70	695.661	692.836	+2.825	7.981
80	785.459	783.075	+2.384	5.683
90	876.350	873.383	+2.967	8.803
100	967.029	963.117	+3.912	15.304
			+5.998	68.064

$$\bar{d} = \frac{5.998}{11} = 0.545$$

$$s_d^2 = \frac{68.064 - \frac{(5.998)^2}{11}}{10} = \frac{64.796}{10} = 6.479$$

$$s_d = \underline{\underline{2.545}}$$

$$T = \frac{.545}{\frac{2.545}{\sqrt{11}}} = \underline{0.710}$$

$$t_{.01(N = 10)} = 3.169$$

$$t_{.05(N = 10)} = 2.228$$

There is no difference between Recorder I readings and Real Time.

Table 22

Paired Observations Test Between Recorder 2 and Real Time

Input Level %	Rec. II $\bar{\bar{X}}$	Real Time $\bar{\bar{X}}$	d_i	d_i^2
0	77.636	65.973	11.663	136.026
10	167.784	155.606	12.178	148.304
20	257.717	244.766	12.951	167.728
30	348.447	334.613	13.834	191.380
40	438.944	424.040	14.904	222.129
50	529.659	513.602	16.057	257.827
60	621.198	602.712	18.486	341.732
70	712.261	692.836	19.425	377.331
80	800.598	783.075	17.523	307.056
90	891.259	873.383	17.876	319.551
100	983.545	963.117	20.428	417.303
			175.325	2886.367

$$\bar{d} = \frac{175.325}{11} = 15.939$$

$$s_d^2 = \frac{2,886.367 - \frac{(175.325)^2}{11}}{10} = \frac{91.926}{10} = 9.193$$

$$s_d = \underline{\underline{3.032}}$$

$$T = \frac{15.939(3.3166)}{3.032} = \underline{17.435}$$

$$t_{.01}(N = 10) = 3.169$$

$$t_{.05}(N = 10) = 2.228$$

There is a significant difference between Recorder II readings and real time.

Table 23

A General Summary of the Results of Paired Observations Test Between Recorders

Input Level	Is there a difference between recorders
0%	Yes
10%	Yes
20%	Yes
30%	Yes
40%	Yes
50%	Yes
60%	Yes
70%	Yes
80%	Yes
90%	Yes
100%	Yes

Table 24

A General Summary of the Results of Paired Observations Test Between Recorders and Real Time

Recorder / Real Time	T	$t_{(.05)}$	Hypo. tested: there is a significant difference between recorders and real time
Rec. I/ RT	0.71	2.23	No
Rec. II/RT	17.44	2.23	Yes

results of the paired observation tests between recorders, and between recorders and real time, respectively.

From the results of these tests, the following observations can be inferred:

(1) It can be concluded that there is a significant difference between recorder one and recorder two since this was shown to be true at every level in the paired observations test.

(2) The paired observations test shows no significant difference between tape recorder one and real time, while it shows a significant difference between tape recorder two and real time.

SECTION III. THE NOISE ANALYSIS

Now that the difference between tape recorders has been statistically established, the next step is to investigate the difference within tape recorders, i.e., tape tracks. Also, one of the purposes of this experiment was to determine what noise or error effect is contributed by the tape recorders as a link in the system. With these aims in mind, the section on Noise Analysis is hereby presented.

A. APPLICATION OF THE TEST

It has been noted in similar investigations that a convenient way to represent the error effect is to express the standard deviation as a percentage of the average range. This is expressed as follows:

$$\%\ \text{Error} = \frac{\sqrt{\hat{\sigma}^2(\text{response})}}{\text{mean range}} \times 100$$

B. RESULTS OF THE TEST

In this experiment it was decided to construct an index using the above model to express the noise as a percentage of range, by recorders and by tape tracks. This noise index follows in Table 25.

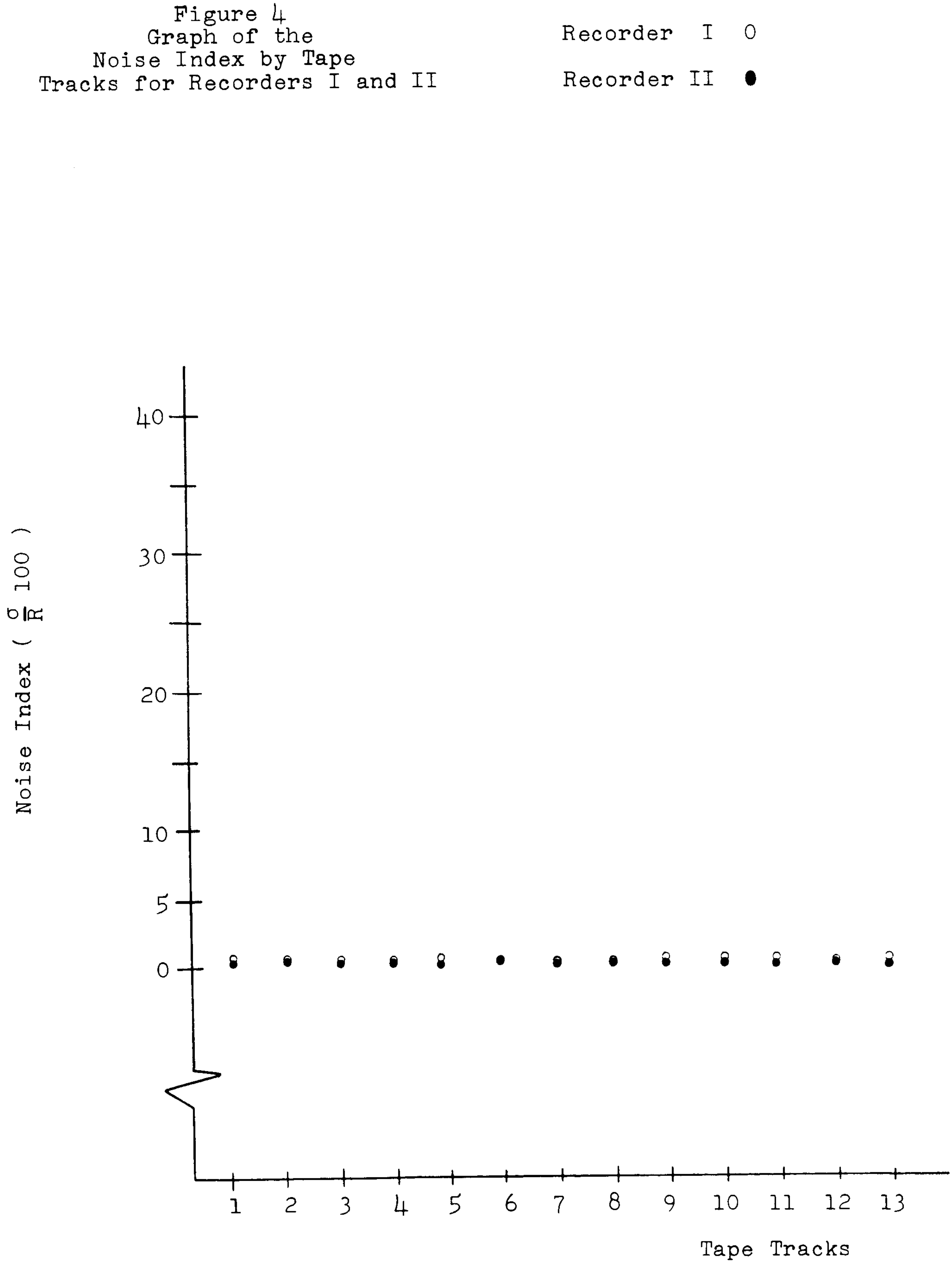
Since there appeared to be some difference in noise values for the two recorders, it was decided to perform a paired observations test to investigate this difference.

Table 25

Noise Level Index

Rec	Track	1	2	3	4	5	6	7	8	9	10	11	12	13
I	σ	2.263	2.120	2.193	2.178	2.133	2.054	2.107	2.146	2.209	2.221	2.276	2.108	2.282
	R	915	915	915	915	912	915	914	921	913	918	916	917	913
	$(\frac{\sigma}{R})100$	.2473	.2317	.2397	.2380	.2339	.2245	.2305	.2330	.2419	.2419	.2485	.2299	.2499
II	σ	2.136	2.136	2.056	2.083	1.993	2.041	2.059	2.094	2.097	2.090	2.073	2.056	2.062
	R	916	916	916	915	914	915	917	918	915	915	918	915	916
	$(\frac{\sigma}{R})100$	.2332	.2332	.2245	.2277	.2181	.2231	.2245	.2281	.2292	.2284	.2258	.2247	.2251

Figure 4
Graph of the
Noise Index by Tape
Tracks for Recorders I and II



The actual test and its results are shown in Table 26. This test shows a significant statistical difference between the noise attributed to the two recorders.

The researchers feel that the actual magnitude of the differences in noise level is of no practical concern. However, since total noise in the system amounts to approximately 1% of full range, it was decided to exhaust all possible means of testing these differences.

If the noise index values are plotted on a scale covering only the range of values, by tape tracks, there appears to be a slight quadratic tendency (correlation coefficient = 0.57) with the outer tracks having more noise than the inner tracks. However, considering the entire system, as in Figure 4, it can be seen that the values are essentially linear and any differences, either in tape recorders or tape tracks, appear rather insignificant.

A question of paramount importance to be answered in this report is: what proportion of noise is contributed by the tape recorders? The following results were obtained by comparing the values sequenced through the tape recorders with the values obtained when by-passing the recorders:

a) When signals traveled through Recorder I alone, approximately 28% of the sub-system noise may be attributed to this Recorder (see Figure 1). The total sub-system noise was .242%.

b) When signals traveled through Recorder II alone, approximately 24.5% of the sub-system noise may be attributed to Recorder II. The total sub-system noise was .229%.

c) An extension of these results shows that on an average telemetry system with 1% total noise, the recorders alone would contribute 6 or 7 percent of the total noise. Whereas, the sub-system shown in Figure 1 contributes about 24% of total noise.

Table 26

Paired Observations Test Between
Noise Levels of Recorders I and II

Tape Tracks	Recorder I	Recorder II	d_i	d_i^2
1	.2473	.2332	.0141	.00019900
2	.2317	.2332	-.0015	.00000225
3	.2397	.2245	.0152	.00023104
4	.2380	.2277	.0103	.00010609
5	.2339	.2181	.0158	.00024964
6	.2245	.2231	.0014	.00000196
7	.2305	.2245	.0060	.00003600
8	.2330	.2281	.0049	.00002401
9	.2419	.2292	.0127	.00016129
10	.2419	.2284	.0135	.00018225
11	.2485	.2258	.0227	.00051529
12	.2299	.2247	.0050	.00002500
13	.2499	.2251	.0248	.00061504
			.1449	.00234886

$$\bar{d} = \frac{.1449}{13} = .01115$$

$$s_d^2 = \frac{.00234886 - \frac{(.1449)^2}{13}}{12} = \frac{.002348 - .001615}{12}$$

$$= \frac{.000733}{12} = .00006108 = 61.08 \times 10^{-6}$$

$$s_d = 7.815 \times 10^{-3} = \underline{\underline{.007815}}$$

$$t = \frac{(.01115)(3.606)}{.007815} = \underline{5.145}$$

$$t_{(.01)} = 3.055$$

$$t_{(.05)} = 2.681$$

SECTION IV. A SECONDARY EXPERIMENT IN NOISE ANALYSIS

In Section III, it was inferred that there could be a difference within tape recorders. For instance, by averaging the four extreme outer values and the 5 inner values of $(\frac{\sigma}{R})$ 100 given in Table 25, a conceptual plot of the average noise indexes would look as shown in Figure 5.

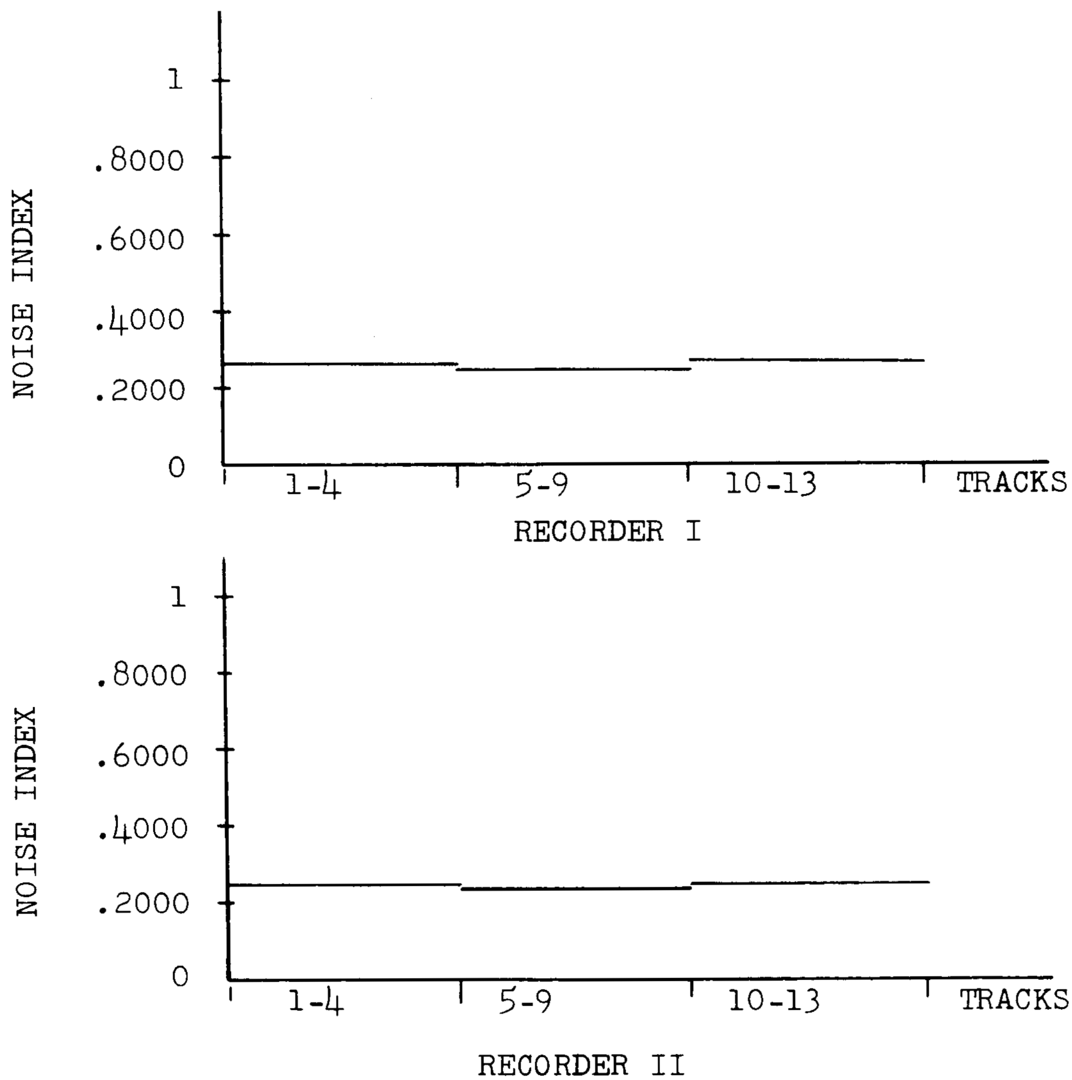


Figure 5. Average Noise Indexes

Figure 6. Block Diagram of Tape Track, Experiment Number 2

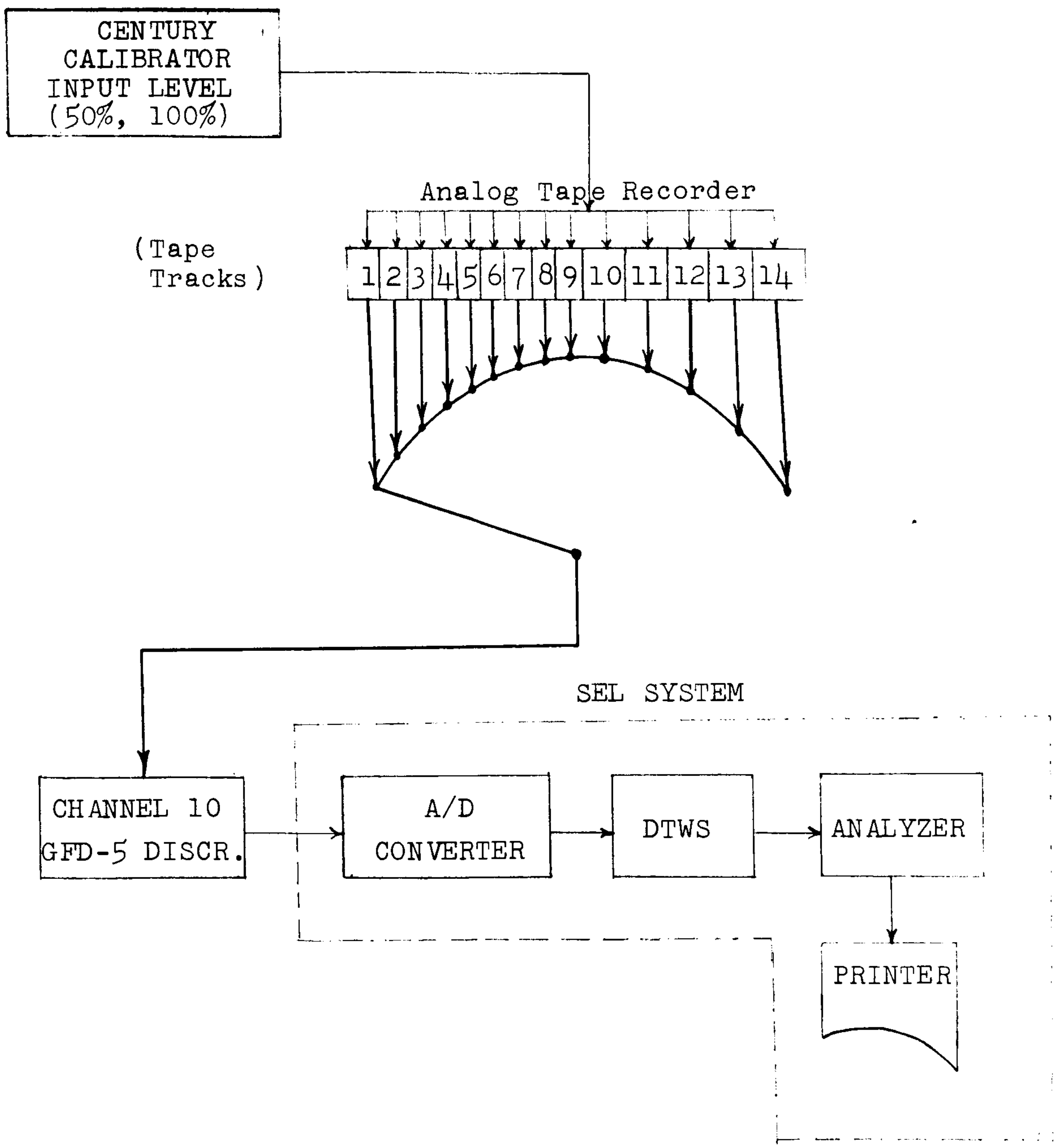


Table 27. Means, Standard Deviations, and Statistics by Levels and Tracks for Experiment No. 2

		Tape Tracks						
		1	2	3	4	5	6	7
INPUT LEVELS 50%	$\overline{X}$	509.751	511.406	510.071	509.665	510.415	512.172	509.321
	σ	1.609	1.517	1.368		1.304	1.455	1.582
INPUT LEVELS 100%	$\overline{X}$	962.031	963.930	960.079	962.283	960.773	962.609	960.336
	σ	1.571	1.638	1.342	1.612	1.379	1.246	1.349
STATISTICS	$\overline{\sigma}$	1.590	1.578	1.355	1.612	1.261	1.351	1.466
	R	461	460	458	499	458	460	460
	$(\frac{\sigma}{R})$ 100	.3449	.3430	.2959	.3230	.2753	.2937	.3187

Table 27. (Continued) Means, Standard Deviations, and Statistics by Levels and Tracks for Experiment No. 2

			Tape Tracks						
			8	9	10	11	12	13	14
INPUT LEVELS	50%	$\bar{X}$	511.352	510.844	512.633	433.677	503.507	509.984	512.258
		σ	1.535	1.354	1.509			1.516	1.472
	100%	$\bar{X}$	961.766	960.438	962.360	961.399	962.828	962.414	964.227
		σ	1.523	1.279	1.457	1.411	1.464	1.540	1.628
STATISTICS		$\bar{\sigma}$	1.529	1.317	1.483	1.411	1.464	1.528	1.550
		R	459	457	460	462	459	461	460
		$(\frac{\sigma}{\bar{R}})100$	.3331	.2882	.3224	.3054	.3190	.3315	.3370

This means that if it were possible to store information in the inner tracks, for instance, and if recorder 1 were used,

$$\frac{.2426 - .2328}{.2426} (100) = \frac{(.0098)(100)}{.2426}$$

$$= \frac{.98}{.2426} \cong 4\%$$

an improvement of 4% could be obtained. This is why it was decided to perform a refined version of the original experiment. The purpose of this secondary experiment is, then, to gather more information from which more valid conclusions can be ascertained.

A. EXPERIMENTAL DESIGN

This experiment was performed in April 1966, using the same equipment that was used in the original experiment. Care was exercised to eliminate all possible internal biases such as differences in the pre-amplifiers for the tape tracks. Since differences in tape tracks were the primary concern in this experiment, only one tape recorder was used and only two levels (50% and 100%) of input were recorded.

The output of a Century Telemetry calibrator was recorded simultaneously on all 14 tape tracks of the Mincon Tape Recorder (No. 2). The recorder information was then stripped off the analog tape by tracks and fed through the DCS GFD-5 discriminator (channel 10), into an A/D converter and onto digital tape for analysis by the SEL Telemetry Data Analysis System. A Quick Look and a Mean, Difference, Variance program were printed out. (See Figure 6 for a block diagram of the experiment.)

B. ANALYSIS OF EXPERIMENT

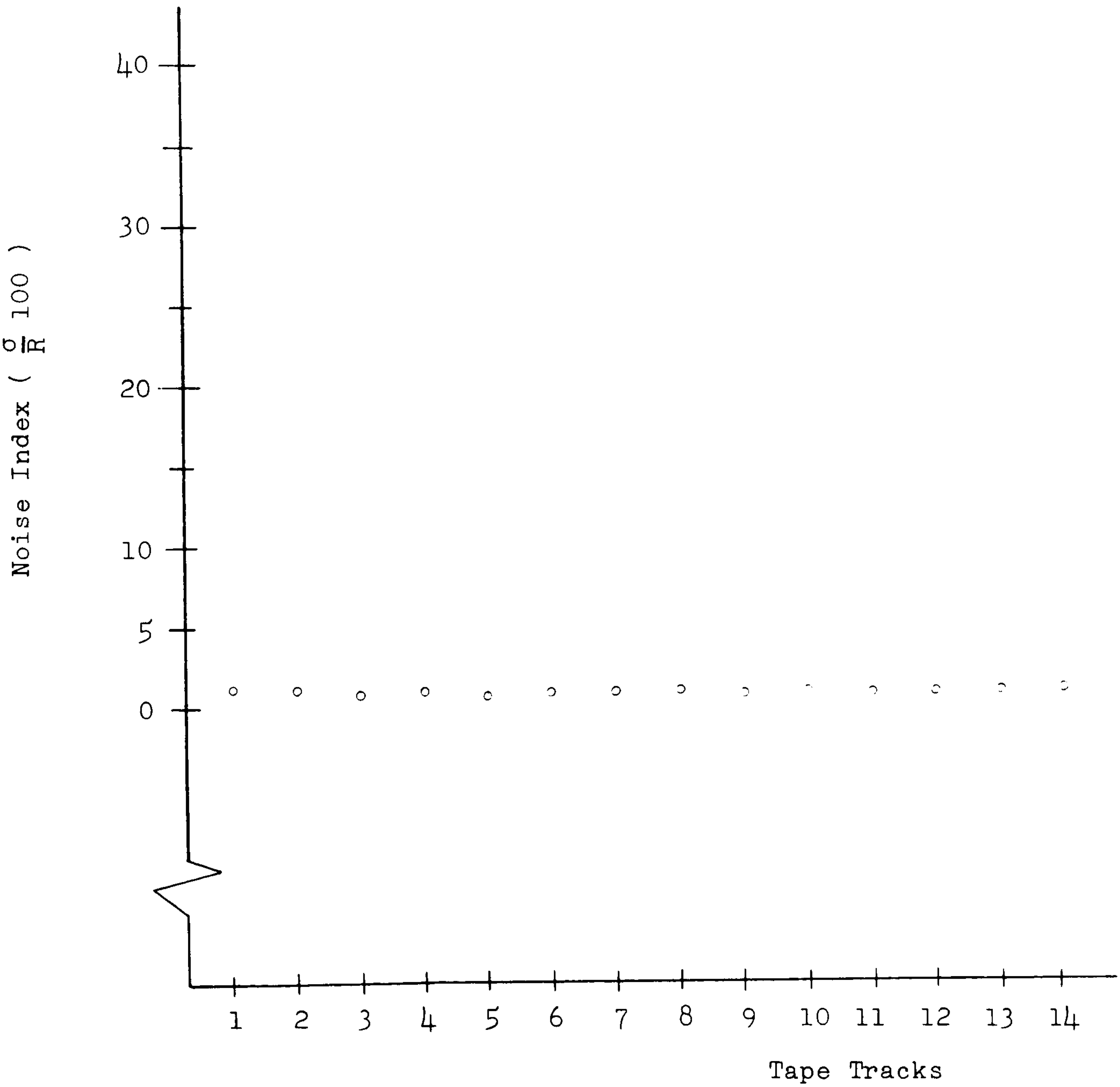
Table 27 contains a summary of the means and standard deviations by levels and tape tracks for Experiment Number 2. An average standard deviation and a noise index is also computed for each tape track. By observing the noise indexes it can be seen that there is slightly more noise on the outer tracks. In this experiment, the average noise for the six inner tracks compared to the average noise for the eight outer tracks represents a reduction in noise of about 6%.

C. RESULTS OF EXPERIMENT

The first question to be answered at this time is: is there a practical difference in tape tracks? Although it has been shown that if by some means the most critical information could be stored in the inner tape tracks, some reduction of noise could be obtained, the researchers feel that the small magnitude of the gain does not warrant such a careful pre-programming. True, a reduction of 6%, or 4% of any unwanted element is always welcomed. In this case the unwanted element constitutes approximately 1% of full range. Therefore, it is very unlikely that this difference is large enough to be of any practical concern.

Figure 7 is a plot of the noise levels as a function of the fourteen tape tracks. By thinking of this plot as a representation of the system as a whole, it can be inferred that differences in tape tracks are immaterial in regard to a telemetry system.

Figure 7. Graph of the Noise Index by Tape Tracks for Experiment No. 2



SECTION V. CONCLUSIONS AND RECOMMENDATIONS

In the early part of Section II of this report, it has been shown that the data collected for this experiment failed to meet with two important properties which are essential for most statistical testing; i.e., normality and homogeneity of variances. On account of this fact, non-parametric tests were used to compare both the tape recorders and the tape tracks. It should be realized that these non-parametric methods are much more conservative than parametric tests.

However, some important conclusions can be drawn from this study. Among these conclusions are:

1. There is a statistically significant difference between the noise characteristics of the two recorders tested.

2. Recorder I produces significantly more noise than does Recorder II.

3. Recorder II differs significantly from real time values while Recorder I does not.

4. Although the noise analysis seems to indicate that there is some statistical difference between tape tracks, it is very doubtful that this difference within recorders is large enough to be truly significant in a practical sense. In light of this, it may be concluded that the noise effect in tape tracks produced by the wow and flutter within the recorder does not warrant pre-programming regarding the importance of the information.

5. It has been shown that about one-fourth of the sub-system noise could be attributed to the tape recorders. Therefore, it may be concluded that a reduction of the noise of the tape recorders will significantly diminish the noise in the sub-system as well as total noise of a telemetry system.

As a result of this study, it is recommended that additional research be performed in the area of noise analysis in a telemetry system. This could be accomplished by designing experiments similar to this, in which other parts of the system are isolated and the outcome is analyzed. This will indicate where the largest portion of the noise is generated and, consequently, correcting efforts can be directed in the right direction.

An area of particular interest is the study of the quality characteristics of magnetic tape. The researchers feel that a large portion of the noise generated in the tape recorders may be attributed to surface defects in the tape.

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