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"The Temperature Coupling of Ions in the Ionosphere"

by

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ABSTRACT

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An investigation has been made of the extent to which O^+, He^+, and H^+ ions can be considered as having a common temperature in the ionosphere. By assuming that each ion gas has its own Maxwellian velocity distribution, energy balance equations including the effects of heating by the electron gas, cooling by the neutral atmosphere, and energy coupling between the ion species have been obtained. Solutions to the energy equations for three models of the neutral atmosphere indicate that the H^+ and He^+ ion temperatures are higher than the O^+ temperature in the regions between 250 and 650 km. A peak temperature difference greater than two hundred degrees occurs for H^+ and O^+ ions, while values which may reach one hundred degrees are noted for the He^+ - O^+ separation. An explanation of the temperature inequality has been made in terms of the atmospheric and ionospheric conditions, showing that the altitude extent of ion temperature separation increases for rising thermospheric and electron temperatures and decreases for progressively larger electron densities.

Author

I.- INTRODUCTION

The thermal balance of atomic oxygen ions in the upper atmosphere was first analyzed by Hanson (1963). By equating the electron-ion heating rate with the loss of ion thermal energy to the neutral atmosphere, he was able to show that the ion temperature should have a pronounced altitude dependence, increasing gradually from the neutral gas temperature near 300 km towards the electron temperature above 800 km. Although this model did not take into account the presence of He^+ or H^+ ions or include the effects of thermal conduction, experimental confirmation for the thermal decoupling of the ion gas from the neutral atmosphere can be inferred from recent incoherent radio backscatter and satellite measurements (Evans, 1965 ; Boyd and Raitt, 1965).

Theoretical calculations of ion temperature must be based upon the heat balance equations for O^+, He^+, and H^+ ions, including the influences of differing rates of energy production, loss, and, for altitudes above 400 km, the effect of heat conduction along the lines of geomagnetic force. In previous studies of charged particle concentrations and even in the interpretation of experimental data, it has been tacitly assumed that the ion gases of the ionosphere can be regarded as having a single temperature. When the energy balance equation for each ion gas is examined, however, it becomes evident that the condition of a common ion temperature must be established by inter-species energy transfer and, under certain conditions, it is possible that competing processes such as ion-neutral energy loss may be sufficiently large to create an appreciable separation between the temperatures of the different ions. Since such differences can be expected to have direct consequences upon both theoretical and experimental analyses, it is necessary to determine the aeronomic conditions under which inter-ion temperature inequality is possible.

This paper is directed towards the theoretical analysis of the problems involved in the establishment of a common ion temperature in the ionosphere under the influence of heating by the ambient electron gas. Hence, the basic questions to be investigated revolve about the nature of energy coupling between ion species, the degree to which the ion velocity distributions can be considered as independently Maxwellian, and the possible effects of changing aeronomic parameters, including both charged and neutral particle densities. The actual application of the ion energy balance equations to the parameters of the upper atmosphere for the purposes of determining useful profiles of ion temperature is made in another paper (Banks, 1966a) where a discussion is made of ion energy sources and the influence of thermal conduction.

In brief outline, Section 2 of this paper contains an analysis of the ion energy balance equations which do not assume the existence of a common ion temperature. To include the mutual effects of energy exchange between ion species, a series of energy coupling terms are introduced. The problems of electron-ion, ion-neutral, and ion-ion collisions are discussed in terms of their effects upon ion Maxwellian velocity distribution. In Section 3, solutions to the energy balance equations are given for different values of the atmospheric parameters under the assumption that thermal conduction is not acting. It is shown that while the O^+ and He^+ ion gases are coupled to within 100°K, the H^+ ion gas can have a temperature greater than 200°K above that of the O^+ ions. In Section 4 a discussion and summary of the results is given.

II.- ION ENERGY BALANCE EQUATIONS

2.1. Introduction

The energy balance for the ion gases of the upper ionosphere (O^+, He^+, H^+) must take into account a wide variation in aeronomic parameters which affect both the rate at which energy is given to the ion gases by electrons and the rate at which ion thermal energy is lost to the neutral atmosphere. Further, although previously unnoted, the thermal conductivity of the ion gases is important in determining the calculated profiles of ion temperature and should be included in the formulation of the energy balance equations. Although it is convenient, in most instances, to assume that the three ion gases have a single temperature, in fact, the energy production and loss processes vary greatly in magnitude for the different species, and it is only by means of completely dominating ion inter-species energy transfer rates that such a state can be achieved. To investigate the possible temperature conditions when the ion energy couplings are not dominating, it is necessary to consider separately the heat budget for each ion gas and to introduce suitable Coulomb energy coupling terms to represent the cooling or heating effects of other ion gases which may be at different temperatures.

In the absence of large scale vertical motions, the time dependent energy balance equation for the j-th ion gas in a mixture of N ion gases, each having a separate Maxwellian velocity distribution, is

$$\frac{\partial U_j}{\partial t} = P_j - L_j + \sin^2 I \frac{d}{dz} \left[K_j \frac{dT_i(j)}{dz}\right] + \sum_k^N C_{jk} \tag{1}$$

where $U_j = 3/2\, n(j)kT_i(j)$ and $C_{jj} = 0$. The quantity U_j is the ion kinetic energy per unit volume, $n(j)$ is the ion number density, P_j is the heating rate due to external sources, L_j is the energy loss rate to the neutral gases, I is the magnetic dip angle, K_j is the ion thermal conductivity, and $T_i(j)$ is the temperature of the j-th ion gas. The ion energy coupling terms, C_{jk}, are defined such that $C_{jk} = -C_{kj}$.

For many aeronomic processes it is not convenient to use (1) directly since changes can occur in the total thermal energy without affecting the temperature of the gas. The temperature rate, $\partial T_i(j)/\partial t$, does not have this disadvantage and a transformation of (1) using the relation

$$\frac{\partial U_j}{\partial t} = 3/2k \left\{ n(j) \frac{\partial T_i(j)}{\partial t} + T_i(j) \frac{\partial n(j)}{\partial t} \right\} \tag{2}$$

is useful once the details of the rate $\partial n(j)/\partial t$ are known.

The complexity of the relations implied by (1) is shown in Figure 1 for the electron, ion, and neutral gases of the upper atmosphere with the electron gas assumed to be the principal source of ion heating. Not only do the ion-neutral energy transfer rates differ for each neutral gas, but also the electron-ion and ion-ion energy rates vary as much as a factor of 16 for the different species. As shown in Section 3, these effects are of primary importance in raising the H^+ temperature significantly above the O^+ and He^+ temperatures.

2.2. Energy Production and Loss for the Ion Gases

For the purposes of this study it is adequate to assume the existence of an electron gas heat source of temperature T_e and density n_e which is able to transfer energy to the different ion gases are the rates (Banks, 1966b) :

$$P(O^+) = 4.8 \times 10^{-7}\, n_e n(O^+)[T_e - T_i(O^+)]\, T_e^{-3/2} \tag{3a}$$
$$P(He^+) = 1.9 \times 10^{-6}\, n_e n(He^+)\, [T_e - T_i(He^+)]\, T_e^{-3/2} \tag{3b}$$
$$P(H^+) = 7.7 \times 10^{-6}\, n_e n(H^+)[T_e - T_i(H^+)]\, T_e^{-3/2} \text{ ev cm}^{-3}\text{sec}^{-1}, \tag{3c}$$

which are in the ratio 1:4:16. Following the results of another analysis (Banks, 1966a), it is possible to ignore in most circumstances the contributions made by other energy sources in heating the ion gases.

The rates at which the atmospheric ions lose energy by means of elastic collisions in neutral gases have been recently discussed by Banks (1966c). For the regions above 250 km the energy effects of chemical reactions which lead to ion species other than O^+, He^+, and H^+ are not important. The ion energy loss rates for all processes other than those between

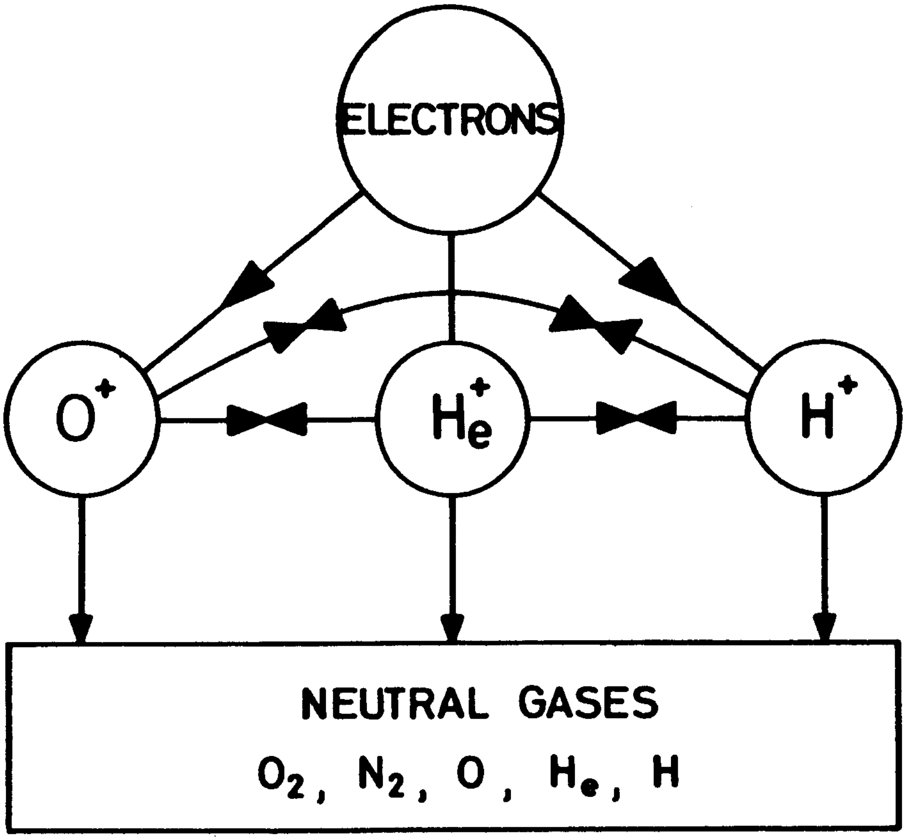

Fig. 1.- Energy couplings for the ionospheric ions. The arrows indicate the flow of thermal energy between the different gases of the ionosphere. The electron gas acts as a heat source for the ions, while the neutral atmosphere, with its efficient thermal conduction, acts as a heat sink.

oxygen and hydrogen ions and atoms are listed in Table 1.

TABLE I : Ion energy loss rates

Ion Mixture	Energy Loss rate (10^{-14} ev cm^{-3} sec^{-1})	Comment
$O^+ - N_2$	$6.6\ n(O^+)n(N_2)(T_i - T)$	Polarization Interaction
$O^+ - O_2$	$5.8\ n(O^+)n(O_2)(T_i - T)$	Polarization Interaction
$O^+ - He$	$2.8\ n(O^+)n(He)(T_i - T)$	Polarization Interaction
$O^+ - O$	$0.21\ n(O^+)n(O)(T_i + T)^{1/2}(T_i - T)$	Resonance Charge Exchange
$He^+ - N_2$	$5.3\ n(He^+)n(N_2)(T_i - T)$	Polarization Interaction
$He^+ - O_2$	$4.5\ n(He^+)n(O_2)(T_i - T)$	Polarization Interaction
$He^+ - O$	$5.8\ n(He^+)n(O)\ (T_i - T)$	Polarization Interaction
$He^+ - H$	$10.\ n(He^+)n(H)(T_i - T)$	Polarization Interaction
$He^+ - He$	$0.4\ n(He^+)n(He)(T_i + T)^{1/2}(T_i - T)$	Resonance Charge Exchange
$H^+ - N_2$	$3.1\ n(H^+)n(N_2)(T_i - T)$	Polarization Interaction
$H^+ - O_2$	$2.8\ n(H^+)n(O_2)(T_i - T)$	Polarization Interaction
$H^+ - He$	$5.5\ n(H^+)n(He)(T_i - T)$	Polarization Interaction
$H^+ - H$	$1.4\ n(H^+)n(H)(T_i + T)^{1/2}(T_i - T)$	Resonance Charge Exchange

Rates taken from Banks (1966c). T_i is the ion temperature for the particular ion under consideration.

For O^+ and H^+ ions there exists the possibility of energy loss by means of accidentally resonant charge exchange with neutral atomic hydrogen and oxygen, respectively. By taking account of ion energy production and loss in the reaction

$$O^+ + H \rightleftharpoons H^+ + O \tag{4}$$

it is possible to derive the ion energy loss rates as

$$L(O^+) = 3/2k \left\{ n(O^+)\ T_i(O^+)\ \bar{\nu}_E\ (O^+, H) - n(H^+)T\ \bar{\nu}_E\ (H^+, O) \right\} \quad (5a)$$

$$L(H^+) = 3/2k \left\{ n(H^+)\ T_i(H^+)\ \bar{\nu}_E\ (H^+, O) - n(O^+)T\ \bar{\nu}_E\ (O^+, H) \right\} \quad (5b)$$

In these equations T is the neutral gas temperature and $\bar{\nu}_E$ is the accidentally resonant charge exchange collision frequency between the quantities indicated within the brackets. As discussed previously, it is convenient to work with the temperature rate rather than the energy rate when the problem of charge exchange is introduced. Through manipulation of equations (5) using the definition of the ion thermal energy and neglecting ion diffusion, it is possible to derive the relations

$$\frac{\partial T_i(O^+)}{\partial t} = - \frac{n(H^+)}{n(O^+)}\ \bar{\nu}_E\ (H^+, O) \left\{ T_i(O^+) - T \right\} \quad (6a)$$

$$\frac{\partial T_i(H^+)}{\partial t} = - \frac{n(O^+)}{n(H^+)}\ \bar{\nu}_E\ (O^+, H) \left\{ T_i(H^+) - T \right\} , \quad (6b)$$

showing that when the ion and neutral gas temperatures are equal the temperature rates are zero. It is noted that the condition of no change in the energy loss rate is more stringent, requiring both a chemical equilibrium of reactants and the equality of the ion and neutral gas temperatures.

Equations (5) and (6) may be reduced by expressing the charge exchange collision frequencies as (Banks, 1966c),

$$\bar{\nu}_E(O^+, H) = 4/3\ n(H)\ T^{1/2}\ \bar{Q}_E(O^+, H) \quad (7a)$$

$$\bar{\nu}_E(H^+, O) = 4/3\ n(O)\ T_i^{1/2}(H^+)\ \bar{Q}_E(H^+, O) , \quad (7b)$$

where $\bar{Q}_E$ is the charge exchange collision cross section for the process indicated within the brackets. The differing temperature dependences of the collision frequencies arise from terms involving the relative velocities of motion between oxygen and hydrogen atoms and ions.

These factors, having the forms $[T_i(O^+)/16 + T]^{1/2}$ and $[T/16 + T_i(H^+)]^{1/2}$ have been reduced to $T^{1/2}$ and $T_i^{1/2}(H^+)$, respectively.

From the study of Rapp (1963) and the experimental work of Stebbings, et al. (1964), the charge exchange cross sections needed for the evaluation of (7) can be extrapolated to thermal energies as

$$\bar{Q}_E(O^+, H) = [4.5 - 0.21 \text{ Log}_{10} T]^2 \times 10^{-16} \text{ cm}^2 \tag{8a}$$

$$\bar{Q}_E(H^+, O) = [4.3 - 0.20 \text{ Log}_{10} T_i(H_i^+)]^2 \times 10^{-16} \text{ cm}^2 \tag*{(8a)}$$

For thermal equilibrium between ion and neutral gases, it is also known that (Stebbings, et al., 1964)

$$\bar{Q}_E(O^+, H) = 9/8\ \bar{Q}_E(H^+, O). \tag{9}$$

With (8), equations (7) become

$$\bar{\nu}_E(O^+, H) = 2.9 \times 10^{-11}\ n(H) T^{1/2} \text{ sec}^{-1} \tag{10a}$$

$$\bar{\nu}_E(H^+, O) = 2.6 \times 10^{-11}\ n(O)\ T_i^{1/2}(H^+) \text{ sec}^{-1} \tag{10b}$$

where it is assumed that $T_i(H^+) = T = 1000°K$ in the arguments of the logarithms of equations (8). With (9) and (10), the oxygen and hydrogen ion energy loss rates due the charge exchange become

$$L(O^+) = 3.8 \times 10^{-15}\ n(O^+)\ n(H)\ T^{1/2} \left[T_i(O^+) - \frac{8n(O)\ n(H^+)}{9n(H)\ n(O^+)}\ T^{1/2}\ T_i^{1/2}(H^+)\right] \tag{11a}$$

$$L(H^+) = 3.4 \times 10^{-15} n(H^+)\ n(O)\ T_i^{1/2}(H^+) \left[T_i(O^+) - \frac{9n(H)\ n(O^+)\ T^{3/2}}{8n(O)\ n(H^+)\ T_i^{1/2}(H^+)}\right] \text{ev cm}^{-3} \text{ sec}^{-1}, \tag{11b}$$

while the ion temperature rates are

$$\frac{\partial T_i(O^+)}{\partial t} = -2.6 \times 10^{-11} \frac{n(O)\ n(H^+)}{n(O^+)} T_i^{1/2}(H^+)\ [T_i(O^+) - T]\ °K\ sec^{-1} \tag{12a}$$

$$\frac{\partial T_i(H^+)}{\partial t} = -2.9 \times 10^{-11} \frac{n(H)\ n(O^+)}{n(H^+)} T^{1/2}\ [T_i(H^+) - T]\ °K\ sec^{-1}. \tag{12b}$$

2.3. Energy coupling between ion gases

With the assumption that each ion gas has its own Maxwellian temperature, the rate of energy transfer between two mixed ion gases is determined by the characteristics of Coulomb collisions. By identifying the two ion gases by the subscripts j and k, the energy transfer rate from the j-th to the k-th gas is (Banks, 1966c),

$$\frac{\partial U(j,k)}{\partial t} = C_{jk} = 3.3 \times 10^{-4} \frac{n(j)\ n(k)[T_i(j) - T_i(k)]}{A_j A_k \left[\frac{T_i(j)}{A_j} + \frac{T_i(k)}{A_k}\right]^{3/2}}\ ev\ cm^{-3}\ sec^{-1}, \tag{13}$$

where $A_{j,k}$ are the respective ion masses in atomic units. The interchange of indices shows that $C_{jk} = -C_{kj}$ as required for the ion-ion energy coupling terms. Hence, a given coupling term can act as either a source or loss of thermal energy, depending upon the difference in the temperatures.

For the problem of energy transfer between O^+, He^+, and H^+, it is found that, with identical particle densities and temperature differences, the He^+- H^+ rate is twice as fast as the O^+ - He^+ rate, while the O^+ - H^+ rate is a factor of 0.32 slower. The differences in the inter-ion energy transfer rates act to accentuate the effects of electron gas energy production which, for equal ion densities, greatly favors the H^+ ions.

2.4 Maxwellian velocity distributions

The energy coupling terms of the preceding section are valid only if the individual ion velocity distribution functions are Maxwellian.

In fact, the processes of ion-neutral and inter-ion energy exchange are sufficiently rapid in many circumstances to strongly distort the ion velocity distributions when large sources of ion heating are applied. For the atmospheric ions above 300 km, however, the effects of energy production and loss are not large and deviations from the equilibrium Maxwellian velocity distribution as a result of electron and neutral particle collisions are small.

To show this directly is difficult and requires an actual solution to the velocity distribution equations. A simpler, but less exact, method involves the comparison of the rate of change of ion thermal energy with the total ion thermal energy, U_j. This ratio can be used to define an effective time for energy transfer, τ_h, given by

$$\tau_h = U_j / [dU_j/dt]. \tag{14}$$

By comparing this effective time for energy transfer with the mean collision time, τ_{ii}, of an ion in its own gas, one can make a qualitative estimate of the influence of the energy exchange process in distorting the ion velocity distribution (Delcroix, 1960). For small values of the ratio, τ_{ii}/τ_h, it is assumed that the collisions between the gas ions will be adequate to establish most of the characteristics of the Maxwellian velocity distribution. In the following paragraphs the conditions appropriate to collisions between electrons and ions, ions and neutral particles, and between different ions are discussed.

Because the electron-ion mass ratio is relatively unfavorable for rapid energy transfer, the effective time for energy transfer under ionospheric conditions is large, being given by

$$\tau_h = 16.9\, A_j \frac{T_e^{3/2}}{n(j)[T_e/T_i(j) - 1]} \quad \text{sec}, \tag{15}$$

where T_e is the temperature of the electron gas which is assumed to have a Maxwellian velocity distribution, A_j is the ion mass in atomic units, and $T_i(j)$ is the ion temperature. Using the collision frequencies of Banks (1966c), with the relation $\tau_{ii} = 1/\nu_{ii}$, the collision time of an ion in its own gas is

$$\tau_{ii} = 0.56\ A_j^{1/2}\ T_i^{3/2}(j)/n(j) \quad \text{sec} \ , \tag{16}$$

which leads to the ratio

$$\tau_{ii}/\tau_h = 3.3 \times 10^{-2}\ [T_e/T_i(j) - 1][T_e/T_i(j)]^{-3/2}\ A_j^{-1/2} . \tag{17}$$

A maximum value for (17) is found at $T_e/T_i(j) = 3$, giving

$$(\tau_{ii}/\tau_h)_{\text{Maximum}} = 1.3 \times 10^{-2}\ A_j^{1/2} . \tag{18}$$

Thus, even for H^+ ions, the ion-ion collision time is about 76 times shorter than the time required for electron-ion energy transfer to accumulate an amount of energy equal to the ion gas thermal energy. On this basis it appears that electron-ion energy transfer in the ionosphere does not significantly distort the ion Maxwellian velocity distribution.

For altitudes below 300 km collisions between ions and neutral gas particles are frequent and a rapid exchange of energy is possible for even minor separations of the ion and neutral gas temperatures. To analyze the effects of ion-neutral collisions upon ion velocity distributions, the aeronomically important example of the energy loss of O^+ ions in atomic oxygen has been chosen.

From Table 1 the O^+ - O charge exchange energy transfer rate is

$$L(O^+) = 2.1 \times 10^{-15}\ n(0)\ n(O^+)\ [T_i(O^+) + T]^{1/2}[T_i(O^+) - T]\ \text{ev cm}^{-3}\ \text{sec}^{-1} . \tag{19}$$

Using (14), the effective time for energy transfer is

$$\tau_h = 5.0 \times 10^{10} \frac{T_i(0^+)}{n(0)n(0^+)[T_i(0^+) + T]^{1/2}\,[T_i(0^+) - T]} \qquad (20)$$

which, by applying (16), leads to the ratio

$$\tau_{ii}/\tau_h = 3.6 \times 10^{-11}\left(\frac{n(0)}{n(0^+)}\right)T_i^{\;1/2}(0^+)[T_i(0^+) + T]^{1/2}[T_i(0^+) - T]. \qquad (21)$$

To evaluate this expression it is necessary to consider the aeronomic parameters appropriate to an altitude of 300 km. Thus, from Nicolet (1967), we adopt $n(0) = 4 \times 10^8$ cm^{-3}, $n(0^+) = 7 \times 10^5$ cm^{-3}, $T_i(0^+) = 1500$°K, and $T = 1000$°K, giving $\tau_{ii}/\tau_h = 2 \times 10^{-2}$. This indicates that even at the altitude of 300 km the ion-neutral energy transfer time is about 50 times longer that the oxygen ion collision time. Hence, for the regions above 300 km it appears correct to assume that neutral particle collisions do not significantly affect the 0^+ ion Maxwellian velocity distribution. Below 300 km there will be an increasing amount of distortion as the ion-neutral energy transfer time becomes comparable with the ion-ion collision time. However, as shown by Hanson(1963) the electron-ion heating rate in these regions is not large enough to create any substantial difference in the ion and neutral gas temperatures. Thus, the problem of neutral particle collisions distorting the ionospheric 0^+ velocity distribution can be neglected.

Analyses similar to the above have been made for He^+ and H^+ ions. In each case the conclusion is the same : Above 300 km the ion-neutral energy transfer times are much larger than the ion-ion collision times, leaving sufficient opportunity for the ion Maxwellian velocity distributions to be established.

When the effects of energy coupling between the different ion species are considered, it is much more difficult to justify the assumption that each ion gas has its own, independent Maxwellian velocity distribution.

Clearly, an exact approach requires an extensive computational analysis based upon the coupled velocity distribution equations. In fact, it is more expedient to make the prior assumption of separate Maxwellian temperatures which permits the use of simple inter-species ion energy coupling equations and a direct solution of (1). For situations where the coupling between ion species is strong, the three ion temperatures will vary by only a few degrees, indicating that the use of a single ion temperature (T_i) is justified.

In some cases, however, the ion coupling terms are not adequate to yield a close temperature correspondence. Although these temperature separations are described here as actual differences in Maxwellian temperatures, the true implication which must be understood is that the velocity distributions of the minor ions perhaps deviate significantly from the Maxwellian form. Hence, although the results of Section 3 are given in terms of temperatures based upon Maxwellian velocity distributions, a better interpretation is that the ion velocity distributions should be regarded as being significantly different from those Maxwellian distributions which would be calculated using the oxygen ion temperature.

2.5. Charged particle concentrations

The solution of (1) requires a knowledge of the electron and ion densities under conditions where the electron and ion temperatures are free to vary with altitude in an arbitrary manner. A recent discussion of the problems involved in calculating ion densities under the mutual effects of diffusion and chemical reaction has been given by Bauer(1966). Briefly, it is found that the assumption of O^+ ion diffusive equilibrium is adequate for altitudes above 300 km. For He^+ and H^+ ions, it is not yet certain where the dividing point between chemical and diffusive equilibrium occurs, although altitudes above 500 km are indicated. While it appears that the reaction given by (4) controls the H^+ ion densities in the regions of chemical equilibrium, there presently exists considerable doubt about the aeronomic loss processes for He^+.

Preliminary solutions to the three ion energy balance equations given by (1) have indicated that the problem of ion temperature separation is largely confined to the atmospheric regions below 650 km where large variations occur in the ion-neutral energy loss processes. Thus, the problem of determining the proper electron and ion densities for use in (1) can be approached by first calculating the O^+ ion density using (Bauer, 1966),

$$\frac{d}{dz} \ln \left\{ n(O^+) \; [T_e + T_i(O^+)] \right\} = - \frac{mg}{k[T_e + T_i(O^+)]} \tag{22}$$

which is valid for diffusive equilibrium ignoring the effects of the minor ions. Here, m is the O^+ ion mass and g is the earth's gravitational acceleration.

The appropriate densities for the He^+ and H^+ ions are found by assuming that there exists chemical equilibrium in the height range of interest. Thus, for He^+ ions we take, following Bauer (1966),

$$n(He^+) = \frac{I_{He} n(He)}{k_1 n(N_2) + k_2 n(O_2)} \tag{23}$$

where $I_{He} = (1. \pm 0.5) \times 10^{-7}$ sec^{-1} is the photoionization rate for He atoms (Nicolet, 1967) and $k_{1,2}$ are the respective rate coefficients for the loss of He ions through chemical processes involving N_2 and O_2. Bauer has pointed out that there exists a large discrepency between the experimental measurements of $k_{1,2}$ in the laboratory and the results of aeronomic observations. For this study it is adequate to adopt Bauer's upper limit of $k_1 \simeq k_2 \simeq 10^{-11}$ cm^3 sec^{-1}.

The chemical equilibrium for H^+ ions in reaction (4) is found by aquating the total rates of reaction, giving

$$n(H^+) \;/\; n(O^+) = \bar{\nu}_E(O^+, H)/\bar{\nu}_E(H^+, O) \tag{24}$$

where $\bar{\nu}_E$ is the appropriate charge exchange collision frequency introduced in (7). On expanding (24) we have

$$\frac{n(H^+)}{n(O^+)} = \frac{n(H)}{n(O)} \left[\frac{T_i(O^+)/16 + T}{T_i(H^+) + T/16} \right]^{1/2} \frac{\bar{Q}_E\,(O^+, H)}{\bar{Q}_E\,(H^+, O)} \tag{25}$$

where $\bar{Q}_E$ is again the charge exchange cross section for the indicated processes. Equation (9) can be used directly in (25) to give,

$$\frac{n(H^+)}{n(O^+)} = \frac{9}{8}\,\frac{n(H)}{n(O)} \left[\frac{T_i(O^+)/16 + T}{T_i(H^+) + T/16} \right]^{1/2} , \tag{26}$$

fixing the ratio of H^+ and O^+ ions for the regions of chemical equilibrium.

III.- RESULTS

Using the energy production and loss rates of Section 2 in conjunction with the coupled ion energy balance equations, it has been possible to obtain the O^+, He^+, and H^+ ion temperatures as a function of altitude. To take account of the differing neutral particle densities which are characteristic of different thermospheric temperatures, three model atmospheres derived by Nicolet (1967) have been used. With thermospheric temperatures of 600°, 1000°, and 1394°K, these models span a considerable range of conditions with respect to the densities of atomic hydrogen and helium.

Throughout all calculations the electron temperature has been treated as a parameter, constant in altitude, which is given a set of values for each neutral atmospheric model.

The boundary condition for the electron density at the starting point for diffusive equilibrium of O^+ (460 km) has been chosen as 5×10^5 cm^{-3} for each of the atmospheric models. The electron density profiles below the region of diffusion have been taken from the Flight 6.07 rocket measurements made by Spencer, et al. (1965) and remain unchanged both for the variations in electron temperature and for the different atmospheric models. In the region of O^+ ion diffusion, however, the electron density at a given altitude increases slightly with increasing electron temperature. The values of the electron and neutral particle densities used in this study are listed in Table 2.

The solutions to the ion energy balance equations have been obtained by ignoring the complicating effect of thermal conduction and by assuming a steady state ($\partial U_j/\partial t = 0$) for each ion gas. The ion temperatures satisfying the resulting nonlinear, algebraic, equations have been found numerically by means of the Newton difference method.

The results of these calculations for each of the atmospheric models are shown in Figures 2 through 10. For each atmospheric model it has been convenient to plot first the O^+ temperature as a function of altitude and electron temperature. To show the effects of the temperature decoupling, the associated figures for each atmospheric model indicate the profiles of the temperature differences between the minor ions and O^+.

It is evident that there exist significant temperature separations between the ionospheric ions over a wide range of altitudes for all atmospheric conditions. The altitude limits for the decoupling are seen to depend strongly upon the particular model atmosphere. The general behavior for the $H^+ - O^+$ temperature difference consists of a moderate rise from small values near 200 km to a peak value of 100°-250°K which generally occurs in the range 300 - 500 km.

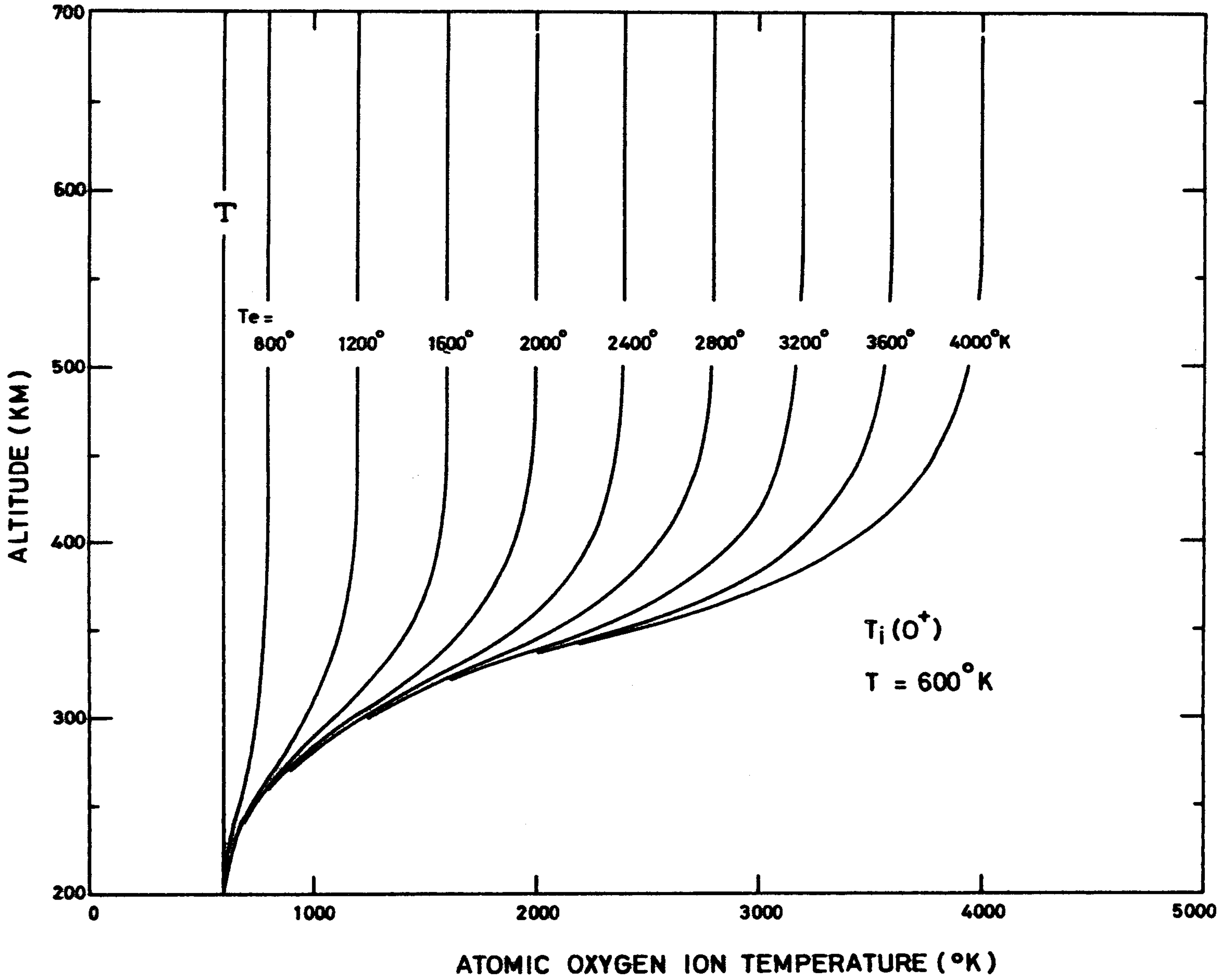

Fig. 2.- Temperature of atomic oxygen ions for different electron temperatures and a neutral atmosphere temperature of 600°K. The thermal decoupling of the oxygen ions and the neutral gas begins near 225 km for this model. Figures 2, 3 and 4 form a sequence showing the ion temperature separations for the 600°K atmospheric model.

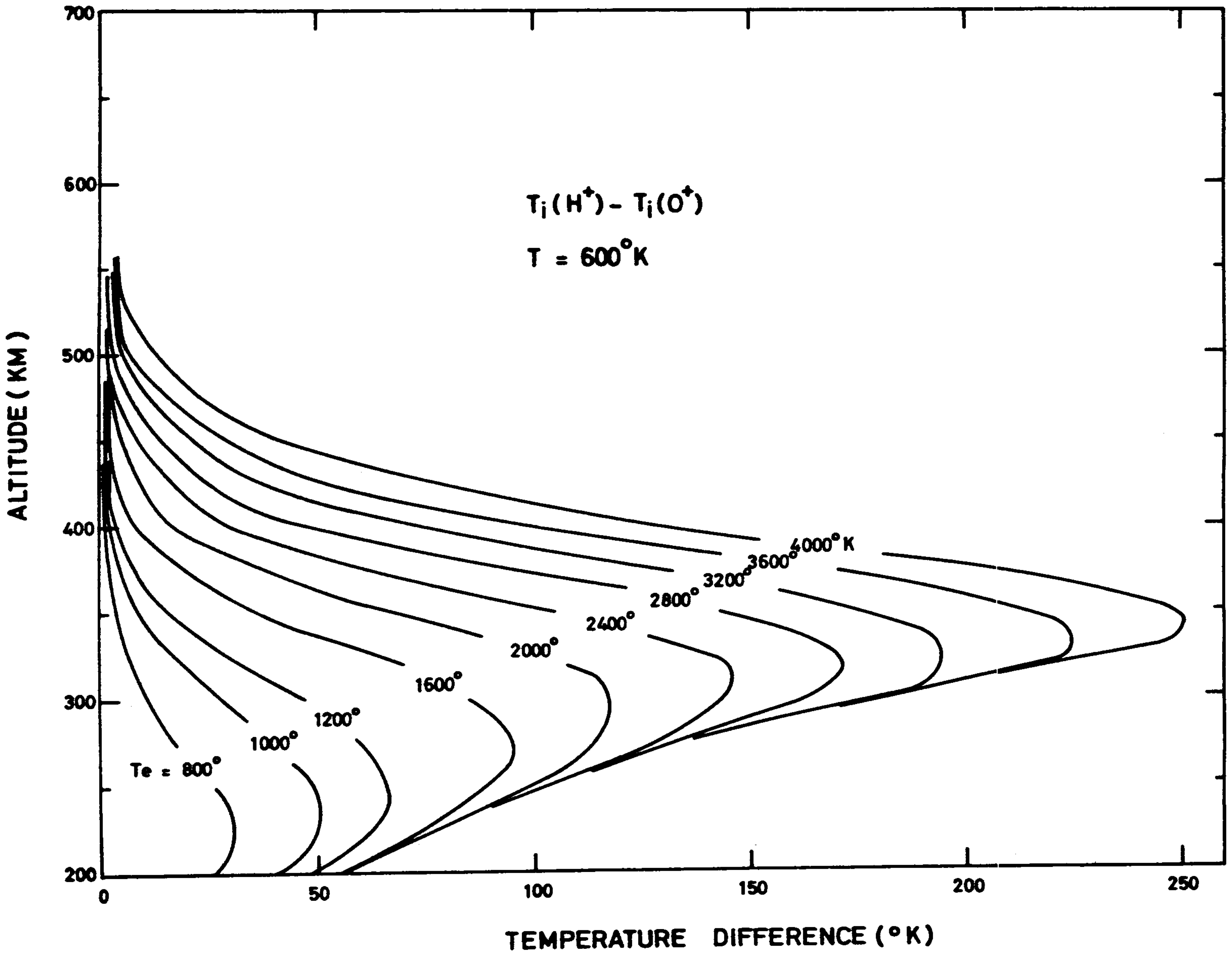

Fig. 3.- Temperature difference between atomic hydrogen and oxygen ions for different electron temperatures and a neutral atmosphere temperature of 600°K.

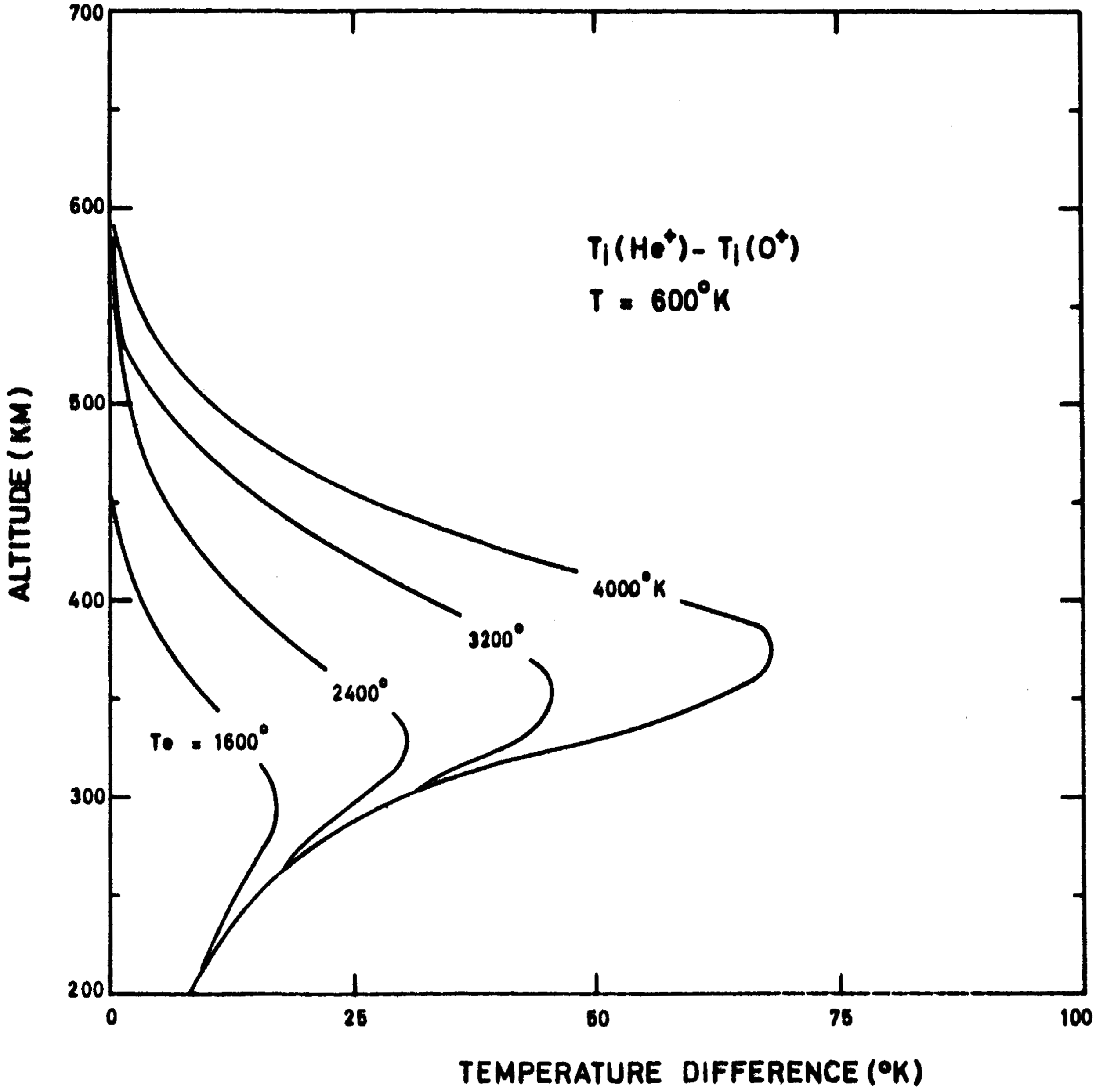

Fig. 4.- Temperature difference between helium and atomic oxygen ions for different electron temperatures and a neutral atmosphere temperature of 600°K.

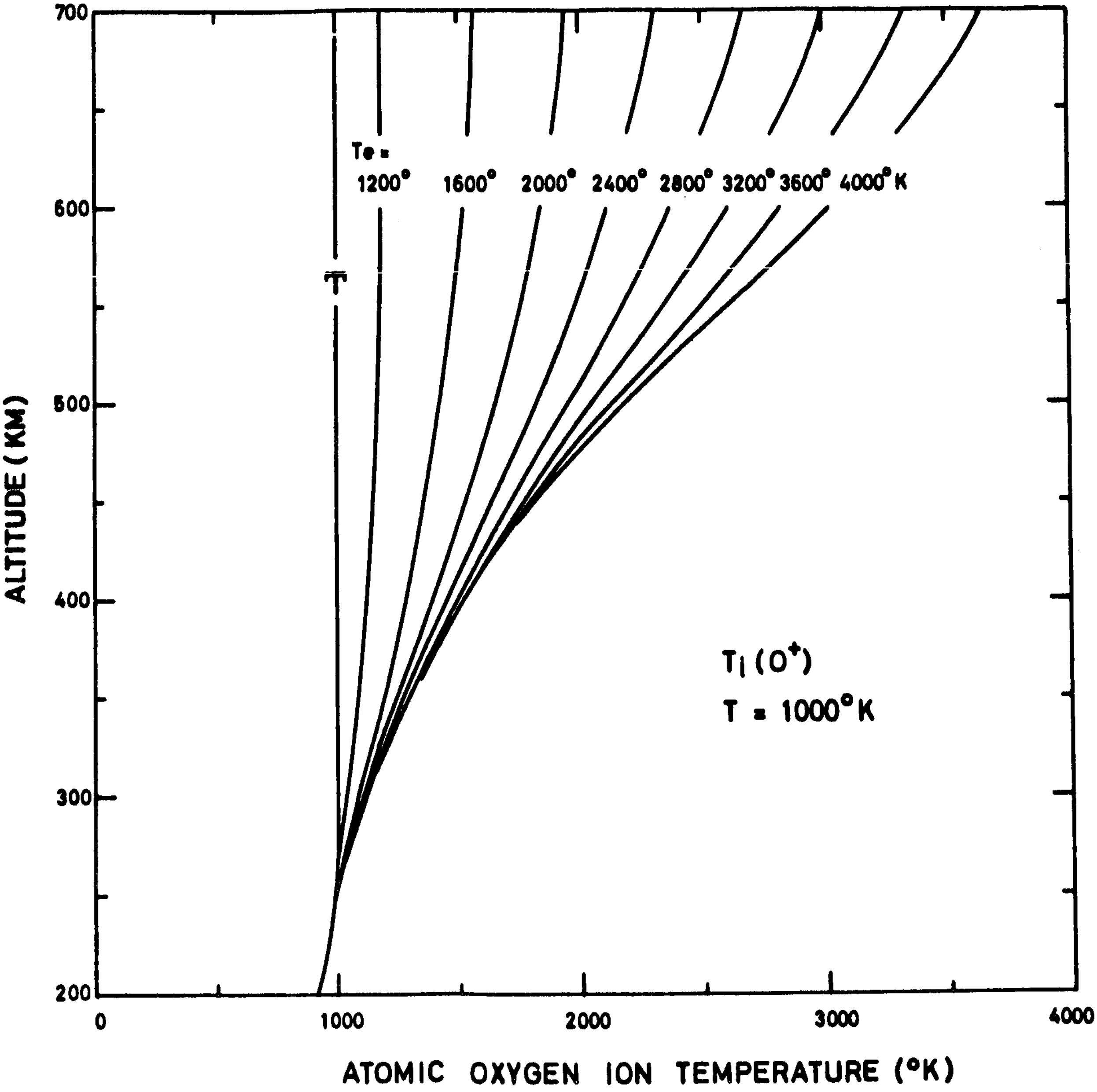

Fig. 5.- Temperature of atomic oxygen ions for different electron temperatures and a neutral atmosphere temperature of 1000°K. The larger neutral particle densities of this atmospheric model increase the altitude for ion-neutral thermal decoupling to 250 km. Comparison with Figure 2 shows this model has a much more gradual oxygen ion temperature transition zone. Figures 5, 6, and 7 form a sequence for the 1000°K model atmosphere.

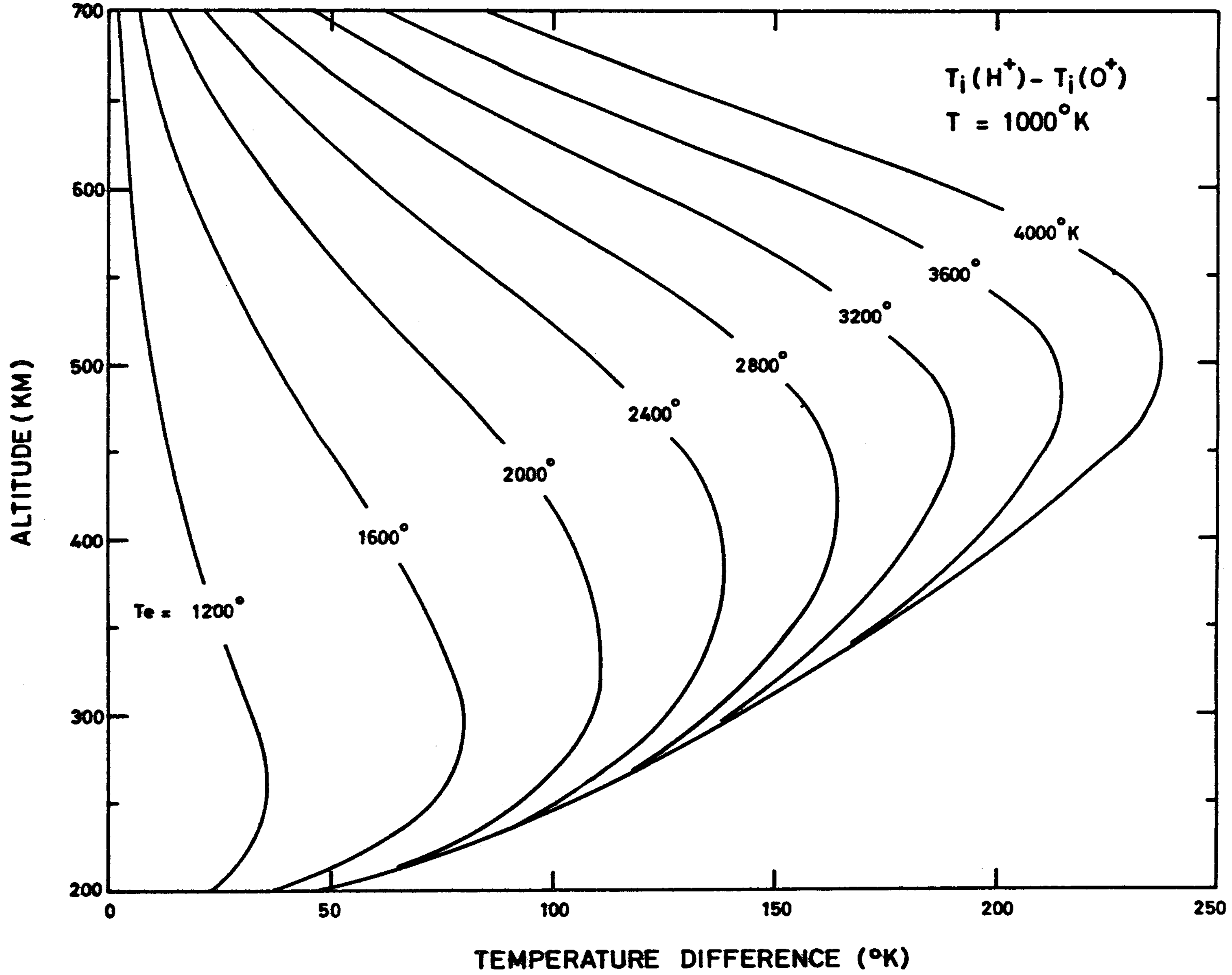

Fig. 6.- Temperature difference between atomic hydrogen and oxygen ions for different electron temperatures and a neutral atmosphere temperature of 1000°K.

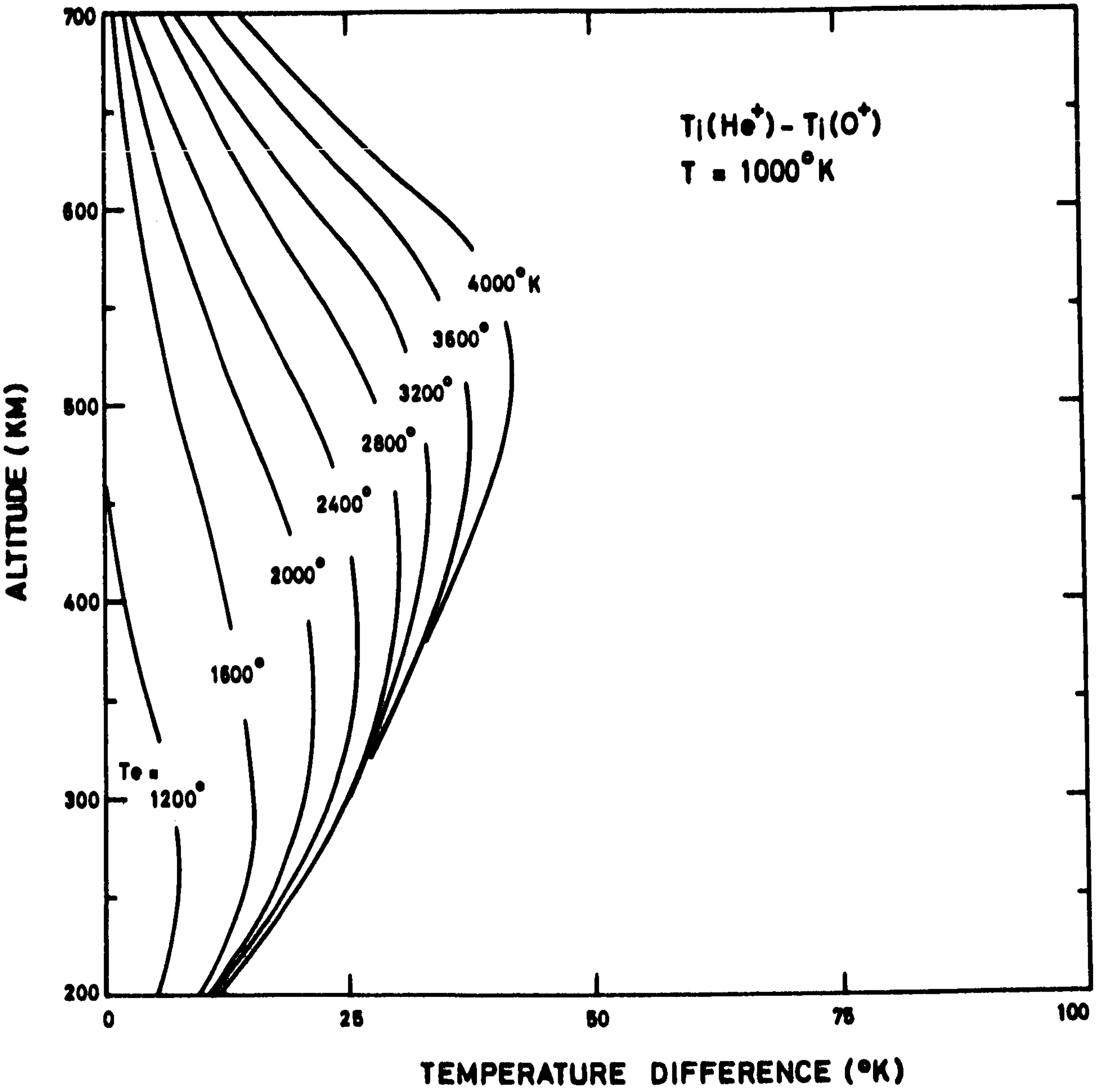

Fig. 7.- Temperature difference between helium and atomic oxygen ions for different electron temperatures and a neutral atmosphere of 1000°K.

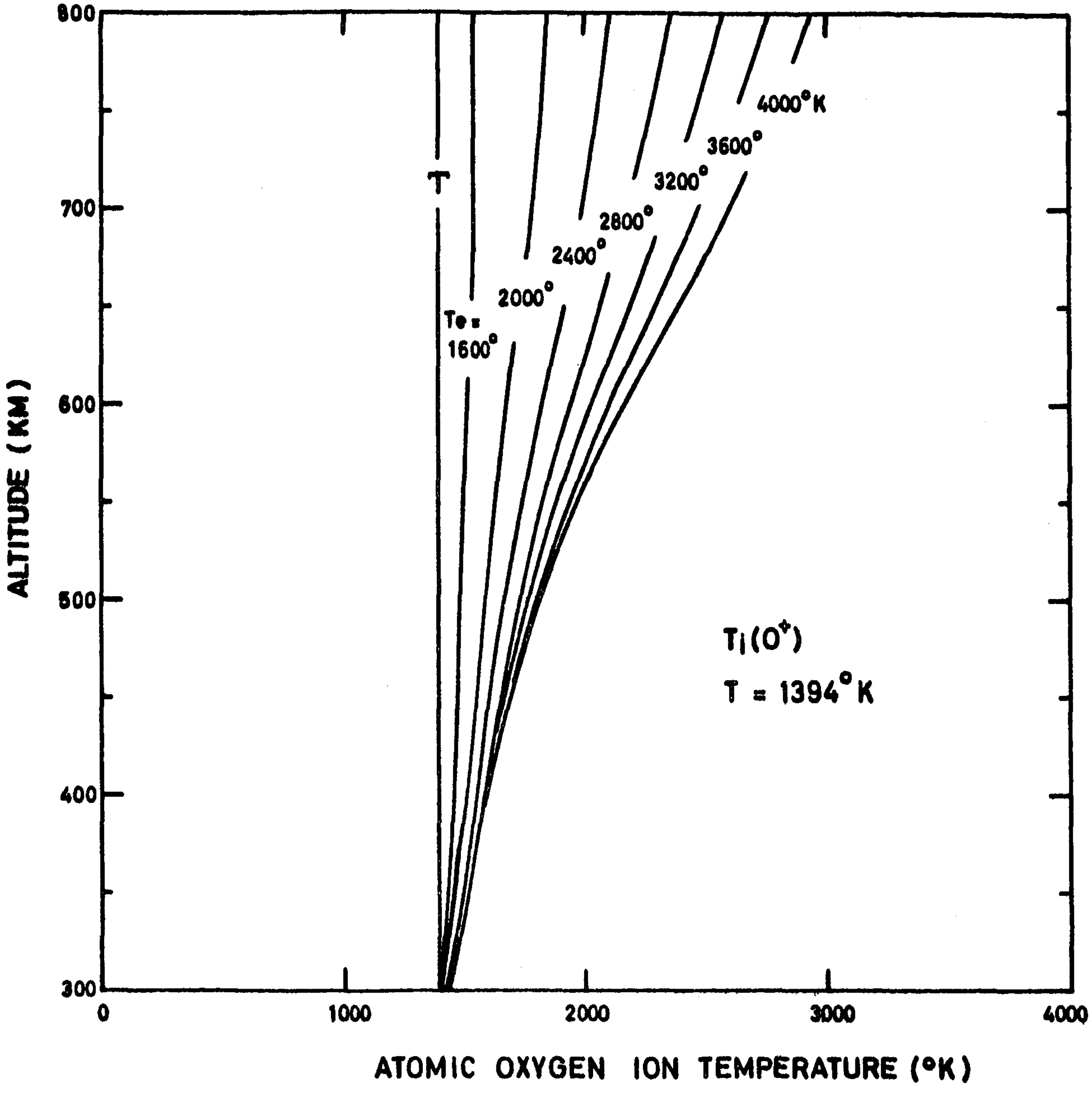

Fig. 8.- Temperature of atomic oxygen ions for different electron temperatures and a neutral atmosphere temperature of 1394°K. The atomic oxygen ion temperature is strongly coupled to the neutral gas temperature in this model to an altitude of 325 km. To show the important regions in the temperature profiles the vertical axes of Figures 8, 9, and 10 have been shifted upwards by 100 km.

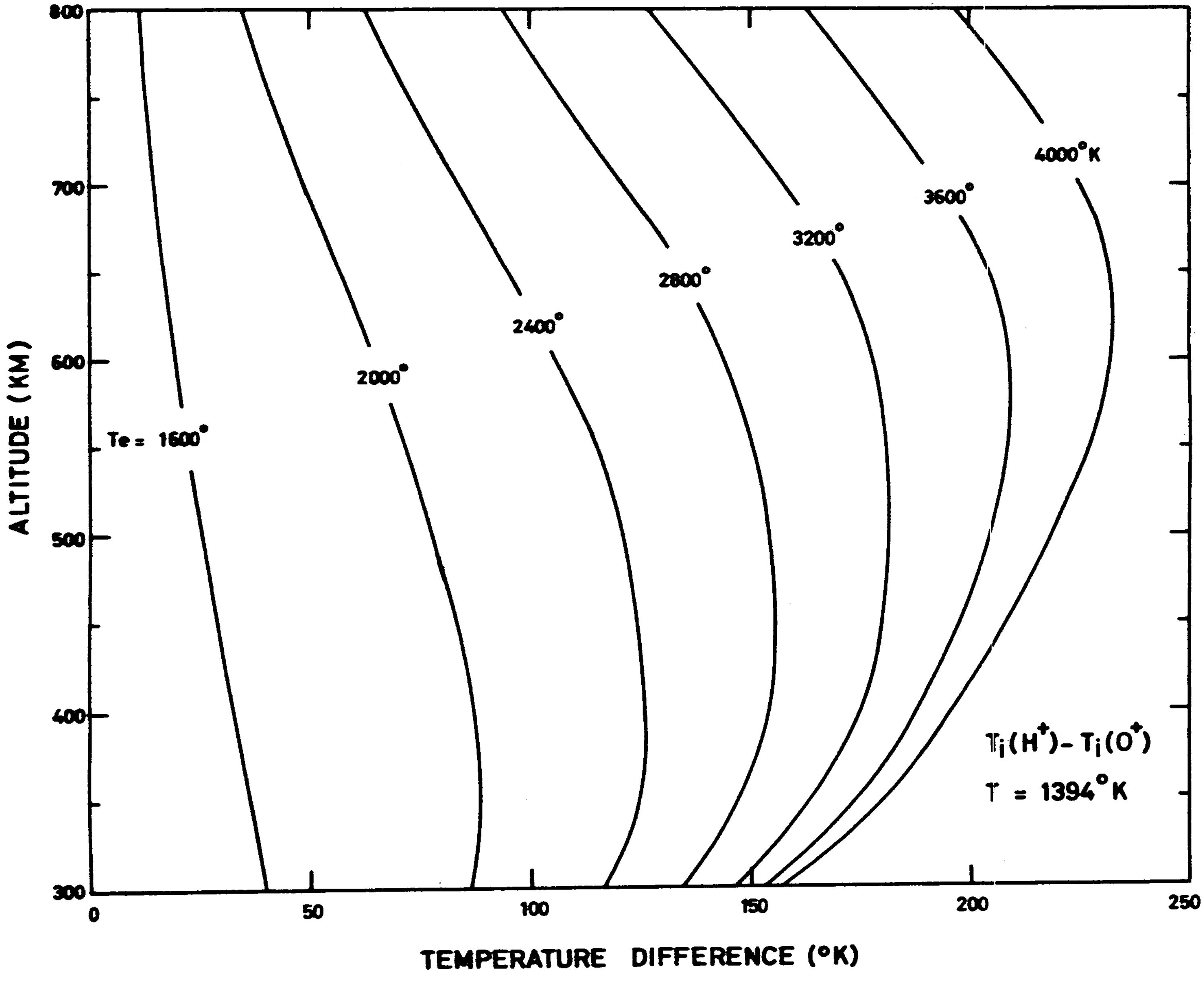

Fig. 9.- Temperature difference between atomic hydrogen and oxygen ions for different electron temperatures and a neutral atmosphere temperature of 1394°K.

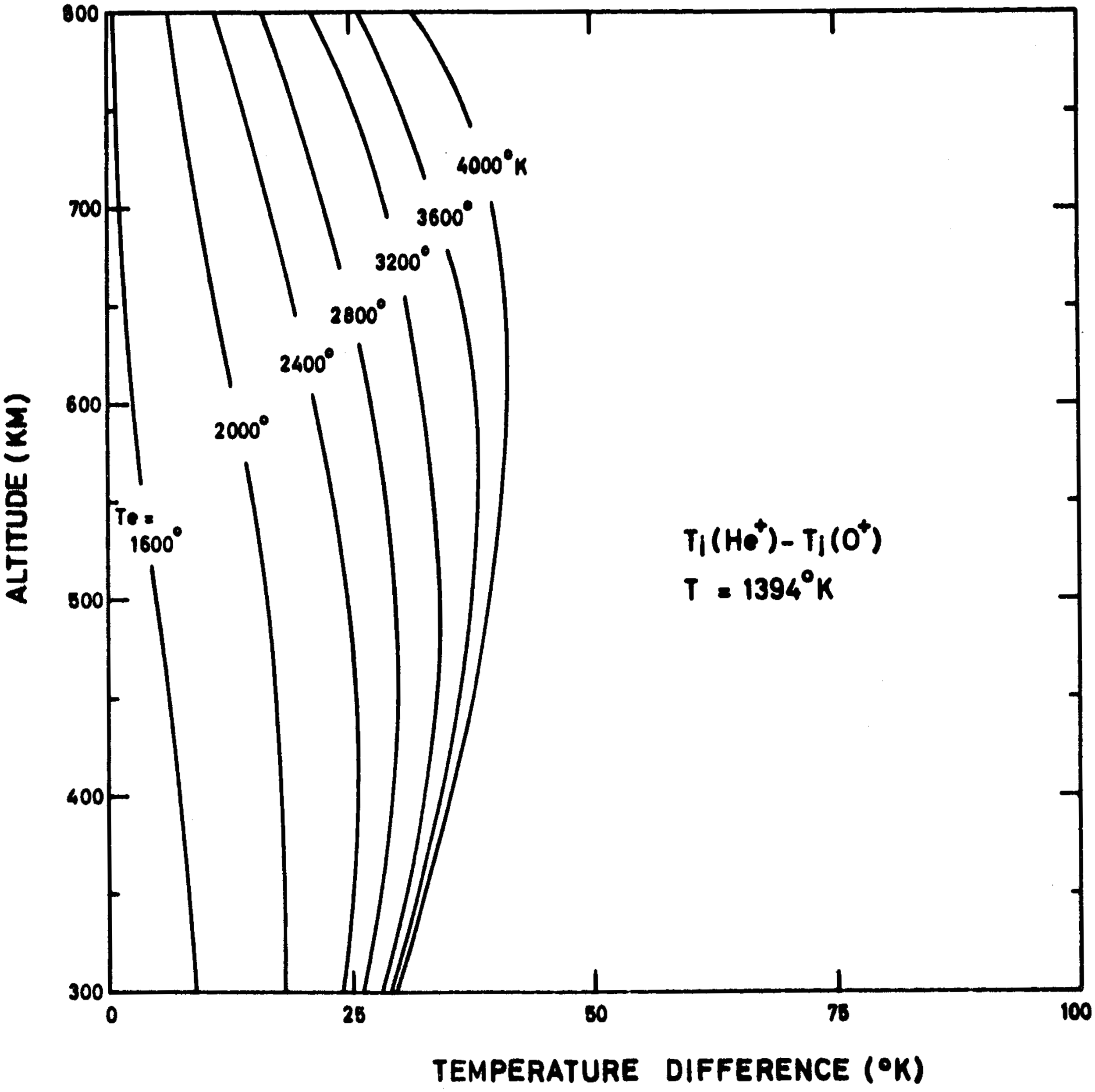

Fig. 10.- Temperature difference between helium and atomic oxygen ions for different electron temperatures and a neutral atmosphere temperature of 1394°K.

TABLE II : Atmospheric parameters

Model		Altitude (km) 200	300	400	500	600	700
600°K	n(O)	2.0(9)	1.1(8)	6.2(6)	4.0(5)	2.8(4)	2.1(3)
	$n(O_2)$	1.7(8)	4.7(5)	1.6(3)	6.7(0)	-	-
	$n(N_2)$	1.7(9)	1.0(6)	7.1(4)	5.8(2)	5.5(0)	-
	n(He)	5.9(6)	2.8(6)	1.4(6)	7.0(5)	3.6(5)	1.9(5)
	n(H)	1.3(6)	1.0(6)	8.7(5)	7.3(5)	6.2(5)	5.3(5)
1000°K	n(O)	2.7(9)	4.1(8)	7.3(7)	1.4(7)	2.9(6)	6.0(5)
	$n(O_2)$	4.6(8)	1.2(7)	3.8(5)	1.4(4)	5.7(2)	2.6(1)
	$n(N_2)$	4.0(9)	1.5(8)	7.8(6)	4.4(5)	2.7(4)	1.8(3)
	n(He)	5.4(6)	3.3(6)	2.1(6)	1.4(6)	9.5(5)	6.4(5)
	n(H)	5.7(4)	3.9(4)	3.4(4)	3.0(4)	2.8(4)	2.5(4)
1394°K	n(O)	2.5(9)	5.5(8)	1.6(8)	4.8(7)	1.5(7)	5.0(6)
	$n(O_2)$	5.0(8)	2.9(7)	2.4(6)	2.3(5)	2.3(4)	2.5(3)
	$n(N_2)$	4.1(9)	3.4(8)	3.8(7)	4.8(6)	6.4(5)	9.2(4)
	n(He)	4.8(6)	3.2(8)	2.3(6)	1.7(6)	1.3(6)	9.6(5)
	n(H)	2.1(4)	8.0(3)	5.8(3)	5.0(3)	4.5(3)	4.2(3)
	n_e	3.2(5)	9.0(5)	7.2(5)	3.7(5)	1.8(5)	9.6(4)

The neutral atmosphere models are from Nicolet (1967). Bracketed quantities following each entry are exponent powers of 10.

At altitudes above the peak there is a lessening of the temperature difference , but the actual slope of the curves depends upon the particular atmospheric models involved.

Similar results are shown in each group of figures for the temperature separation between He^+ and O^+ ions. In this case, however, the effects of ion energy coupling are relatively stronger and the peak temperature separations are less than 100°K.

An analysis can be made of the calculated ion temperature differences by considering the effects of changing aeronomic parameters upon the dominant terms in the individual ion energy balance equations. To explain the large H^+- O^+ temperature separation and its altitude dependence, we note from Section 2.3 that for equivalent conditions the rate of energy production in the H^+ ion gas is 16 times that for the O^+ ions. Further, for H^+ in chemical equilibrium and for O^+, the principal energy losses are to atomic oxygen by means of charge exchange reactions, the specific rates having similar numerical factors and the same dependence upon the atomic oxygen density. For these circumstances it follows that at a given altitude the H^+ temperature decoupling from the neutral gas temperature will be proportionately larger than for O^+. In order for the O^+ ions to reach the same temperature as a given H^+ temperature, it is necessary to have a lower density of atomic oxygen, thus implying that there is an effective upwards displacement of the entire O^+ temperature profile with respect to that of H^+. Although the effects of H^+- O^+ ion energy coupling complicate the actual temperature values, essentially it is the relative displacement of the individual temperature profiles which is responsible for the calculated differences in the ion temperatures. Thus, at low altitudes the H^+ ion temperature attempts to rise rapidly towards the electron temperature in a region where the oxygen ions are firmly coupled to the neutral gas temperature, thus creating a temperature separation which increases with altitude.

This process cannot continue upwards indefinitely, however, since the atomic oxygen density decreases much more rapidly with altitude than the electron density. Thus, the O^+ temperature begins at a higher altitude the decoupling process from the neutral gas temperature already started by the H^+ ions at a lower altitude. The net result is than under the influence of H^+ - O^+ energy transfer the difference between the H^+ and O^+ temperatures progressively diminishes, as shown in Figures 3, 6 and 9. At high altitudes, where the O^+ and H^+ energy losses are small, the inter-species ion energy transfer rates are large for even small temperature separations, thus assuring the existence of a single ion temperature.

The comparison of the H^+- O^+ temperature separations for the three model atmospheres indicates the importance of changing neutral particle densities. For the 600°K model the atomic oxygen density decreases rapidly with altitude, leading to rapid rises of $T_i(H^+)$ and $T_i(O^+)$ and a relatively small zone of ion temperature inequality. In the 1000° and the 1394°K models, the atomic oxygen densities are progressively larger at each altitude and the O^+ ion temperature is strongly coupled to the neutral gas temperature to higher altitudes, creating a relatively gradual transition of $T_i(O^+)$ towards the electron temperature. Hence, the displacement of the H^+ and O^+ temperature profiles is large and there is a considerable region of temperature inequality.

Changing values of the electron density do not directly affect the magnitude of the temperature decoupling for the minor ions. In fact, the principal alteration occurs again as an upward or downward displacement of the individual ion temperature profiles. This effect is shown in Figure 11 where the profile of the H^+-O^+ temperature separation is shown for the 1000°K model atmosphere with an electron temperature of 2800°K. The solid curve has been taken directly from Figure 6, while the broken curve has been calculated using an electron density which has been increased everywhere by a factor of two. While the magnitude of the peak temperature separation changes by only a few degrees, the peak moves downward by 50 km.

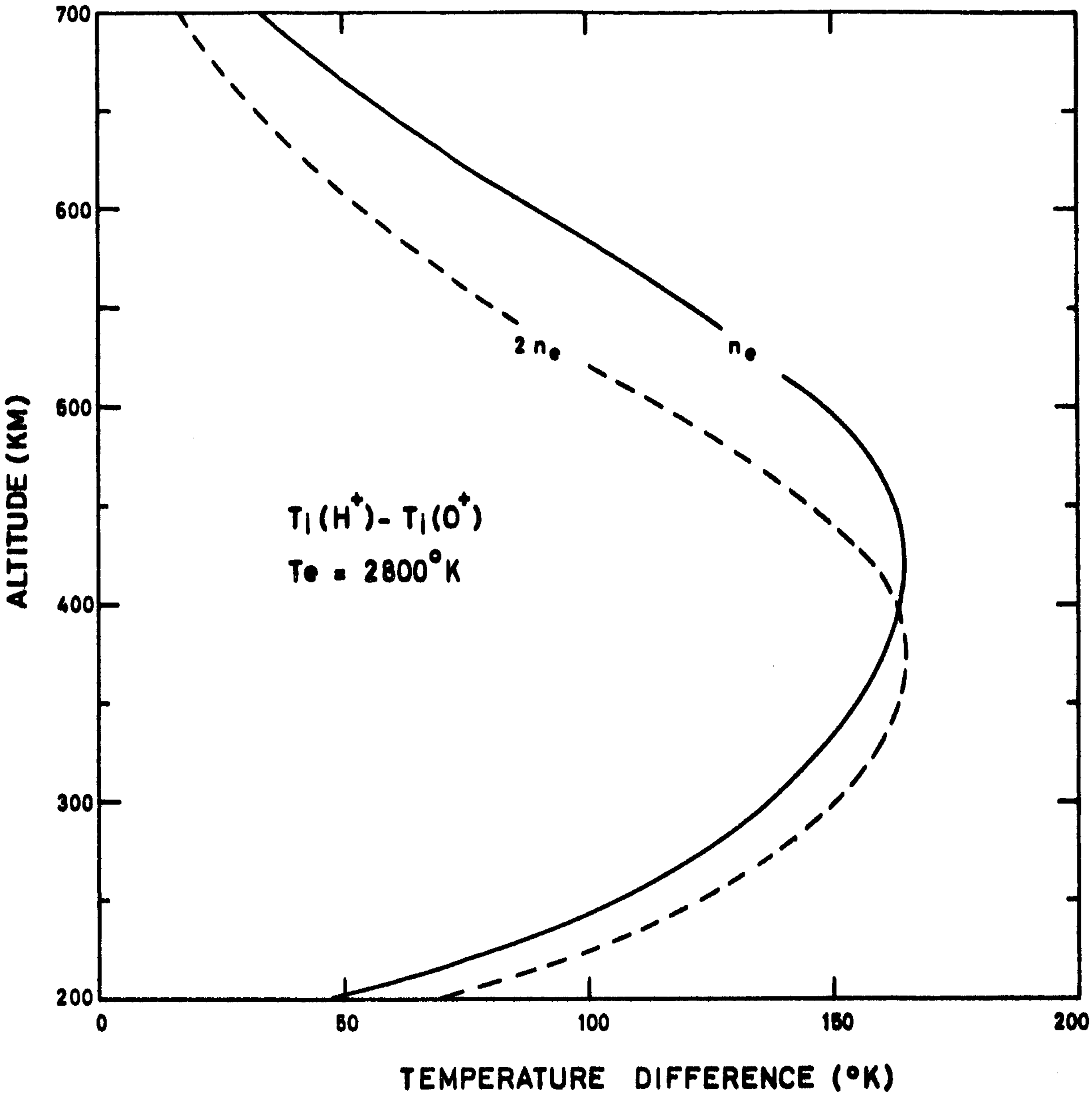

Fig. 11.- Effects of changing the electron density model upon the atomic hydrogen and oxygen ion temperature separation. The 1000°K model atmosphere has been used with an electron temperature of 2800°K. The solid curve, marked as n_e, is taken from Figure 6, while the broken curve, indicated as 2 n_e, results when the original electron density is multiplied by a factor of two. The net downward shift in the profile is consistent with the interpretation of the temperature difference being the result of a relative shift in the atomic hydrogen and oxygen ion temperature profiles.

The problem of the He^+ - O^+ temperature coupling is similar to that of H^+ - O^+ except that the inter-ion energy coupling terms are more important and the ratio between the two ion gas heating rates is a factor of 4 smaller. Thus, with the primary energy loss of He^+ being atomic oxygen, the He^+ - O^+ temperature difference is substantially smaller than for H^+ - O^+.

IV.- DISCUSSION AND SUMMARY

The results of the preceding section indicate a considerable degree of temperature inequality between the O^+, He^+, and H^+ ion gases of the upper atmosphere. The principal cause of this state of thermal non-equilibrium must be attributed to the relatively large rates of electron gas heating for He^+ and H^+ ions in comparison with O^+. The magnitude and extent of the temperature differences are strongly dependent upon the electron temperature and the parameters of the neutral atmosphere, the density of atomic oxygen being especially important. By describing the numerical solutions to the energy balance equations in terms of vertically displaced profiles of the individual ion temperatures, the essential features of the calculated temperature differences can be explained. Thus, increases in the thermospheric temperature of the neutral atmosphere lead to enhanced atomic oxygen densities, displacing the O^+ temperature profile upwards and increasing the vertical region of temperature separation for the ions. Likewise, for a given atmospheric model, an overall increase in the electron density will lower the O^+ temperature profile with respect to that of H^+ and He^+, resulting in a smaller vertical extent of the ion temperature separations.

Because the He^+ ions are less than 20%, and the H^+ ions less than 2%, of the total ion number density for the regions discussed here, the calculated temperature differences of the ions do not significantly affect the full ion energy balance.

Hence, the assumption that the O^+ temperature is "the" ion temperature, T_i, is certainly adequate for describing the transfer of ion thermal energy to the neutral atmosphere and the cooling action of the ions upon the ionospheric electron gas.

With regard to the distribution of charged particles above the F-2 region, the differing ion temperatures do alter the respective ion scale heights to some extent. However, because the differences in the ion temperatures are less than 15% of the O^+ temperature, the uncertainty introduced by neglecting the ion temperature inequality is much less than that found in our current knowledge of ion production, diffusion, and reaction.

The interpretation of the ion temperatures presented here has been based upon the prior assumption that each ion gas is characterized by a Maxwellian velocity distribution. In fact, the true extent of the degrading effects of inter-species ion collisions upon the minor ion velocity distributions is not known. In any case, this analysis indicates, at the very least, that the velocity distributions for He^+ and H^+ should be significantly different than those Maxwellian distributions which would be calculated for a temperature corresponding to that of the O^+ ions. It would appear that the deviations indicated here could be experimentally verified by direct satellite or rocket measurements of the velocity spectrums of the individual ion gases.

Finally, it has been noted that small concentrations of He^+ and H^+ ions can significantly alter the interpretation of incoherent radio backscatter frequency spectra (Evans, and Loewenthal, 1964). It is possible that the process of temperature decoupling of the ion gases, especially with respect to H^+, may lead, at times, to a basic uncertainty in the accuracy of the deduced ion temperature profiles.

V.- ACKNOWLEDGEMENTS

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