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EQUATIONS FOR THE APPLICATION OF WIND-TUNNEL WALL CORRECTIONS TO PITCHING MOMENTS CAUSED BY THE TAIL OF AN AIRCRAFT MODEL

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SUMMARY

Equations are derived for the application of wall corrections to pitching moments due to the tail in two different manners. The first system requires only an alteration in the observed pitching moment; however, its application requires a knowledge of a number of quantities not measured in the usual wind-tunnel tests, as well as assumptions of incompressible flow, linear lift curves, and no stall. The second method requires a change in the tailplane incidence and, in general, a smaller change in the observed pitching moment. The latter system appears preferable even though it may require tests with several tailplane settings. In any event, the use of a separate tail balance is recommended whenever the corrections are expected to be large.

INTRODUCTION

The necessity of correcting wind-tunnel measurements for the presence of the wind-tunnel walls has been recognized for many years. (See, for example, refs. 1 to 3.) The theory generally provides information on either the wall-induced interference angles (for example, refs. 3 and 4) or interference velocities (ref. 5) in the region in which the tail of an aircraft model is likely to be located. If the possibility of differing corrections to the dynamic pressures at the lifting element and at the tail is ignored, it has been observed (ref. 6) that the required correction is essentially equivalent to an altered tail setting on the model. Despite the fact that very limited experimental confirmation of the calculated corrections exists, such studies (refs. 6 to 8) do indicate substantial improvement in correlating pitching moments obtained in different wind tunnels when corrections are properly applied.

The present emphasis on testing large models representing aircraft having extreme lift coefficients (such as those representing V/STOL aircraft) puts particular emphasis on the corrected values of pitching moment, largely because of the powerful devices which may be required for trim. In particular, the theoretical treatment most appropriate to V/STOL models (ref. 5) indicates not only that the corrections are large, but also that there may be a significant difference in the corrections which must be applied to the dynamic pressures at the lifting system and at the tail as a result of the presence of the wind-tunnel boundaries. The problem is further complicated by the fact that, in order to avoid severe nonuniformities caused by close proximity to the wake, V/STOL aircraft often have horizontal tails mounted well above the plane of the lifting system. In the present analysis vertical tail height is shown to contribute to the corrected moment.

Considering the aforementioned features, comparatively simple expressions are developed in order to correct the tail contribution to pitching moment. These expressions are derived herein for the case in which the correction is applied as a direct alteration in the measured pitching moment, as well as for the case in which the correction is considered as a change in tail incidence.

No attempt is made in this paper to provide the appropriate correction factors at any point in the wind tunnel. The analysis is confined to the correction of pitching moment when the correction factors have already been determined from other sources (for example, refs. 3 to 5).

SYMBOLS

a	slope of lift curve for horizontal tail, per radian
A	effective aspect ratio of tail
A_m	momentum area of lifting system
$\bar{c}_t$	mean aerodynamic chord of horizontal tail
$\bar{c}_w$	mean aerodynamic chord of wing, or reference length used in nondimensionalizing pitching moment of other types of lifting systems
$C_{A,meas}$	measured axial-force coefficient of horizontal tail (with respect to fuselage reference line), positive rearward
C_D	drag coefficient of horizontal tail
$C_{D,i}$	induced-drag coefficient of horizontal tail, $C_L^2/\pi A$
$C_{D,o}$	profile-drag coefficient of horizontal tail
C_L	lift coefficient of horizontal tail, L/qS_t
C_M	pitching-moment coefficient of complete configuration, $M/qS_w\bar{c}_w$, positive nose up
$C_{M,c}$	corrected pitching-moment coefficient of complete configuration, $M_c/qS_w\bar{c}_w$, positive nose up
$C_{M,o,t}$	pitching-moment coefficient of horizontal tail at zero lift, positive nose up

$C_{M,t}$ pitching-moment coefficient of horizontal tail, positive nose up

$C_{N,meas}$ measured normal-force coefficient of horizontal tail (with respect to fuselage reference line)

$C_q = 1 - \dfrac{(q_c/q)_t}{(q_c/q)_w}$

C_R resultant-force coefficient of horizontal tail

$C_\alpha = \Delta\alpha_w - \Delta\alpha_t \dfrac{(q_c/q)_t}{(q_c/q)_w}$

$D(\Delta\alpha,q)$ drag of horizontal tail when under the influence of the specified $\Delta\alpha$ and q

h vertical distance of aerodynamic center of horizontal tail above the aircraft center of moment, perpendicular to aircraft reference line

$h' = h \cos \alpha - l \sin \alpha$

i_t angle between fuselage reference line and zero-lift line of horizontal tail

$i_{t,c}$ corrected angle between fuselage reference line and zero-lift line of horizontal tail

i_w angle between fuselage reference line and chord line of wing (or equivalent angle for other types of lifting system)

l horizontal distance of horizontal tail behind the aircraft center of moment

$l' = l \cos \alpha + h \sin \alpha$

$L(\Delta\alpha,q)$ lift of horizontal tail when under the influence of the specified $\Delta\alpha$ and q

M aircraft pitching moment, positive nose up

M_c corrected aircraft pitching moment, positive nose up

M_t pitching moment of horizontal tail about its own aerodynamic center, positive nose up

$M_t(\Delta\alpha,q)$ pitching moment of horizontal tail about its own aerodynamic center when under the influence of the specified $\Delta\alpha$ and q, positive nose up

n ratio of induced velocity in the far wake to the induced velocity at the lifting system

q dynamic pressure, $\frac{1}{2}\rho V^2$

q_c corrected dynamic pressure

q_t dynamic pressure at tail

S_t area of horizontal tail

S_w area of wing, or appropriate equivalent area for other lifting systems

T_s static thrust

V free-stream velocity

w_h induced velocity in hovering

α angle between fuselage reference line and free-stream direction, positive nose up

α_g geometric angle of attack of tail, measured from zero lift, $\alpha_w + i_t - i_w$, radians

α_t angle of attack at tail, angle between relative wind and zero-lift line of tail, radians

α_w angle of attack of wing, measured from zero lift, or equivalent angle for other types of lifting system, radians

ΔC_M additive correction to aircraft pitching-moment coefficient due to effect of wind-tunnel boundaries on horizontal tail

Δi correction to tailplane incidence due to effect of wind-tunnel boundaries on horizontal tail, radians

ΔM additive correction to aircraft pitching moment due to effect of wind-tunnel boundaries on horizontal tail

Δu horizontal (parallel to free-stream direction) interference velocity due to wind-tunnel boundaries, positive rearward

Δw vertical (perpendicular to free-stream direction) interference velocity due to wind-tunnel boundaries, positive upward

$\Delta\alpha$ interference angle due to wind-tunnel boundaries, positive upward, radians

$\Delta\alpha_t$ interference angle at horizontal tail due to wind-tunnel boundaries, positive upward, radians

$\Delta\alpha_w$ interference angle at wing (or lifting system) due to wind-tunnel boundaries, positive upward, radians

ϵ downwash angle at tail caused by wing (or lifting system), positive downward, radians

$\theta = \tan^{-1} \dfrac{C_{A,meas}}{C_{N,meas}}$, radians

ρ mass density of air or test medium

Subscripts:

meas measured value

t horizontal tail

w wing or lifting system

RESULTS AND DISCUSSION

Initial Considerations

It is assumed that the model lift, drag, and pitching moments (other than those moments due to the tail) have already been corrected for wall interference. It is further assumed that the appropriate transferal, if required, of the lifting-system forces and moments from the balance center to the desired center of moment has already been made. The terms $\Delta\alpha_w$ and $\Delta\alpha_t$ are understood to mean the wall-induced interference angles at the wing (or lifting system) and tail, respectively. These angles must include all appropriate corrections for wake inclination, tail location, and so forth. (See, for example, ref. 4.) It is thus inherently assumed that any changes in ϵ and q at the tail are included in the appropriate values of wind-tunnel interference. In cases where the theory gives the corrections in terms of the horizontal and vertical interference velocities Δu and Δw, it is understood that at the appropriate points in space, these interference velocities have been converted into the equivalent angles and dynamic pressures; that is (see fig. 1):

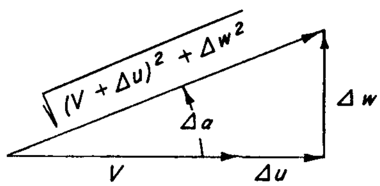

Figure 1.- Equivalent velocity and interference angle when corrections are stated in terms of interference velocities.

$$\Delta\alpha = \tan^{-1}\left(\frac{\frac{\Delta w}{V}}{1 + \frac{\Delta u}{V}}\right) \tag{1}$$

$$\frac{q_c}{q} = \left(1 + \frac{\Delta u}{V}\right)^2 + \left(\frac{\Delta w}{V}\right)^2 \tag{2}$$

Correction to Pitching Moment Only

One approach to correcting the tail contribution to pitching moment requires correction of the measured pitching moment only. It is assumed that the lift and drag of the lifting system, but not the overall pitching moment, have already been corrected to free-air conditions. The effect of the boundaries at the tail is different from the effect at the lifting system in terms of both $\Delta\alpha$ and q_c. Thus, the problem is one of applying a pitching-moment correction to the data that will account for the difference between the model moments caused by the forces and moments on the tail in the wind tunnel and the model moments caused by the forces and moments that would exist at the tail if the wall interference were the same as at the wing.

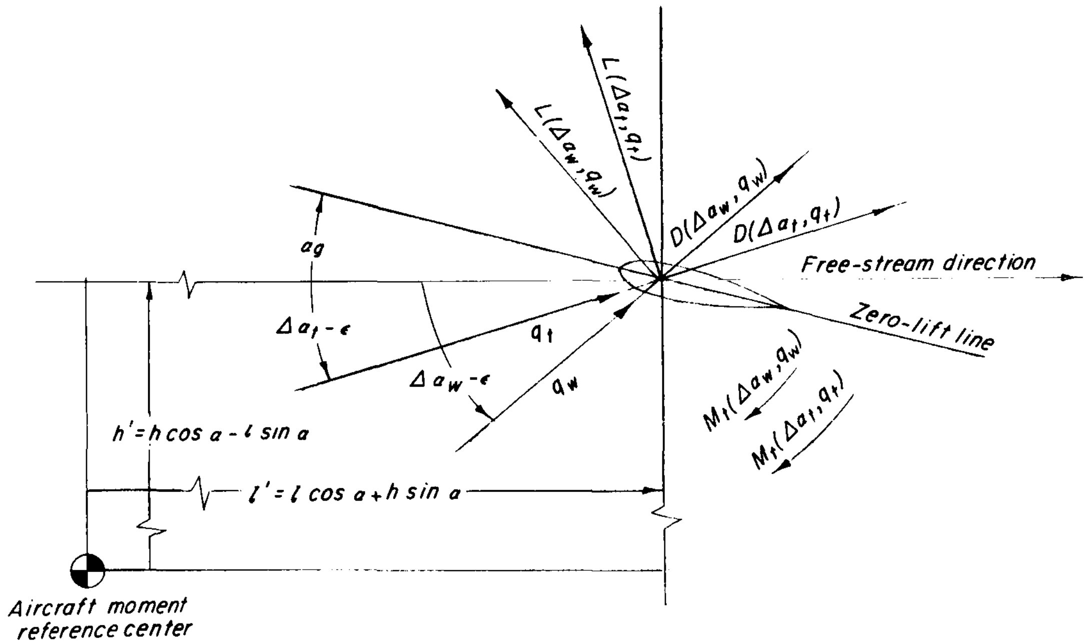

Figure 2.- Forces, moments, and angles at tail for the case of correction to pitching moment only.

If the interference conditions at the tail were the same as those at the wing, the total moment about the model center of moment would be (see fig. 2 and note that lift and drag are taken as perpendicular and parallel, respectively, to the relative airflow):

$$\begin{aligned}\text{Moment} = {} & h'\left[D\left(\Delta\alpha_w, q_w\right)\cos\left(\Delta\alpha_w - \epsilon\right) - L\left(\Delta\alpha_w, q_w\right)\sin\left(\Delta\alpha_w - \epsilon\right)\right] \\ & - l'\left[L\left(\Delta\alpha_w, q_w\right)\cos\left(\Delta\alpha_w - \epsilon\right) + D\left(\Delta\alpha_w, q_w\right)\sin\left(\Delta\alpha_w - \epsilon\right)\right] \\ & + M_t\left(\Delta\alpha_w, q_w\right) \qquad (3)\end{aligned}$$

However, the model tail is actually in a different flow so that the measured moment is

$$\begin{aligned}\text{Moment} = h'&\Big[D(\Delta\alpha_t, q_t)\cos(\Delta\alpha_t - \epsilon) - L(\Delta\alpha_t, q_t)\sin(\Delta\alpha_t - \epsilon)\Big] \\ &- l'\Big[L(\Delta\alpha_t, q_t)\cos(\Delta\alpha_t - \epsilon) + D(\Delta\alpha_t, q_t)\sin(\Delta\alpha_t - \epsilon)\Big] \\ &+ M_t(\Delta\alpha_t, q_t)\end{aligned} \tag{4}$$

The desired correction moment is then the difference between the pitching moments as given by equations (3) and (4), or

$$\begin{aligned}\Delta M = h'&\Big[D(\Delta\alpha_w, q_w)\cos(\Delta\alpha_w - \epsilon) - D(\Delta\alpha_t, q_t)\cos(\Delta\alpha_t - \epsilon) \\ &- L(\Delta\alpha_w, q_w)\sin(\Delta\alpha_w - \epsilon) + L(\Delta\alpha_t, q_t)\sin(\Delta\alpha_t - \epsilon)\Big] \\ &- l'\Big[L(\Delta\alpha_w, q_w)\cos(\Delta\alpha_w - \epsilon) - L(\Delta\alpha_t, q_t)\cos(\Delta\alpha_t - \epsilon) \\ &+ D(\Delta\alpha_w, q_w)\sin(\Delta\alpha_w - \epsilon) - D(\Delta\alpha_t, q_t)\sin(\Delta\alpha_t - \epsilon)\Big] \\ &+ M_t(\Delta\alpha_w, q_w) - M_t(\Delta\alpha_t, q_t)\end{aligned} \tag{5}$$

but, in general,

$$L = C_L q_t S_t = a(\alpha_g - \epsilon)q\left(\frac{q_t}{q}\right)S_t \tag{6}$$

$$D = \left(C_{D,o} + \frac{C_L{}^2}{\pi A}\right)q_t S_t = \left[C_{D,o} + \frac{a^2(\alpha_g - \epsilon)^2}{\pi A}\right]q\left(\frac{q_t}{q}\right)S_t \tag{7}$$

$$M_t = \left[C_{M,o,t} + \frac{dC_{M,t}}{d\alpha}(\alpha_g - \epsilon)\right]q\left(\frac{q_t}{q}\right)S_t\bar{c}_t \tag{8}$$

It will be observed that equation (7) actually represents a linearization of the induced-drag relationship. The effect of this linearization on induced drag and lift is negligible for wings under most conditions (ref. 9). It is further noted that the analysis is limited to the linear portion of the tail lift curve and is <u>not</u> valid if the tail is stalled. In addition, equations (6) and (7) are correct only for incompressible flow. Furthermore, the analysis is limited to devices such as normal tails which approximately obey the relations displayed above. For less conventional tails, such as rotors or tail propellers that might be used on V/STOL configurations, equations (6) to (8) may be severely violated.

If equations (6), (7), and (8) are substituted into equation (5) and the additional wall interference effects on α and q are accounted for, the result is

$$\begin{aligned}
\Delta M = h'\Bigg\{&\left[C_{D,o} + \frac{a^2(\alpha_g + \Delta\alpha_w - \epsilon)^2}{\pi A}\right]\left(\frac{q_c}{q}\right)_w\left(\frac{q_t}{q}\right)qS_t \cos\left(\Delta\alpha_w - \epsilon\right) \\
&- \left[C_{D,o} + \frac{a^2(\alpha_g + \Delta\alpha_t - \epsilon)^2}{\pi A}\right]\left(\frac{q_c}{q}\right)_t\left(\frac{q_t}{q}\right)qS_t \cos\left(\Delta\alpha_t - \epsilon\right) \\
&- a\left(\alpha_g + \Delta\alpha_w - \epsilon\right)\left(\frac{q_c}{q}\right)_w\left(\frac{q_t}{q}\right)qS_t \sin\left(\Delta\alpha_w - \epsilon\right) \\
&+ a\left(\alpha_g + \Delta\alpha_t - \epsilon\right)\left(\frac{q_c}{q}\right)_t\left(\frac{q_t}{q}\right)qS_t \sin\left(\Delta\alpha_t - \epsilon\right)\Bigg\} \\
&+ l'\Bigg\{-a\left(\alpha_g + \Delta\alpha_w - \epsilon\right)\left(\frac{q_c}{q}\right)_w\left(\frac{q_t}{q}\right)qS_t \cos\left(\Delta\alpha_w - \epsilon\right) \\
&+ a\left(\alpha_g + \Delta\alpha_t - \epsilon\right)\left(\frac{q_c}{q}\right)_t\left(\frac{q_t}{q}\right)qS_t \cos\left(\Delta\alpha_t - \epsilon\right) \\
&- \left[C_{D,o} + \frac{a^2(\alpha_g + \Delta\alpha_w - \epsilon)^2}{\pi A}\right]\left(\frac{q_c}{q}\right)_w\left(\frac{q_t}{q}\right)qS_t \sin\left(\Delta\alpha_w - \epsilon\right) \\
&+ \left[C_{D,o} + \frac{a^2(\alpha_g + \Delta\alpha_t - \epsilon)^2}{\pi A}\right]\left(\frac{q_c}{q}\right)_t\left(\frac{q_t}{q}\right)qS_t \sin\left(\Delta\alpha_t - \epsilon\right)\Bigg\} \\
&+ \left[C_{M,o,t} + \frac{dC_{M,t}}{d\alpha}\left(\alpha_g + \Delta\alpha_w - \epsilon\right)\right]\left(\frac{q_c}{q}\right)_w\left(\frac{q_t}{q}\right)qS_t\bar{c}_t \\
&- \left[C_{M,o,t} + \frac{dC_{M,t}}{d\alpha}\left(\alpha_g + \Delta\alpha_t - \epsilon\right)\right]\left(\frac{q_c}{q}\right)_t\left(\frac{q_t}{q}\right)qS_t\bar{c}_t
\end{aligned} \tag{9}$$

The squared terms and trigonometric terms are then expanded, the second-order terms in $\Delta\alpha_w$ and $\Delta\alpha_t$ are dropped, and both sides of equation (9) are divided by

$$q\left(\frac{q_t}{q}\right)\left(\frac{q_c}{q}\right)_w l'S_t$$

By definition,

$$1 - \frac{(q_c/q)_t}{(q_c/q)_w} = C_q \qquad\qquad \Delta\alpha_w - \Delta\alpha_t \frac{(q_c/q)_t}{(q_c/q)_w} = C_\alpha$$

Thus,

$$\begin{aligned}
\frac{\Delta M}{q\left(\frac{q_t}{q}\right)\left(\frac{q_c}{q}\right)_w l' S_t} = {} & C_q C_{D,o} \frac{h'}{l'} \cos\epsilon + C_\alpha C_{D,o} \frac{h'}{l'} \sin\epsilon + C_q \frac{h'}{l'} \frac{a^2}{\pi A}(\alpha_g - \epsilon)^2 \cos\epsilon \\
& + C_\alpha \frac{h'}{l'} \frac{a^2}{\pi A}(\alpha_g - \epsilon)^2 \sin\epsilon + C_\alpha \frac{h'}{l'} \frac{2a^2}{\pi A}(\alpha_g - \epsilon)\cos\epsilon \\
& + C_q \frac{h'}{l'} a(\alpha_g - \epsilon)\sin\epsilon - C_\alpha \frac{h'}{l'} a(\alpha_g - \epsilon)\cos\epsilon \\
& + C_\alpha \frac{h'}{l'} a \sin\epsilon - C_q a(\alpha_g - \epsilon)\cos\epsilon - C_\alpha a(\alpha_g - \epsilon)\sin\epsilon \\
& - C_\alpha a \cos\epsilon - C_\alpha C_{D,o} \cos\epsilon + C_q C_{D,o} \sin\epsilon \\
& - C_\alpha \frac{a^2}{\pi A}(\alpha_g - \epsilon)^2 \cos\epsilon + C_q \frac{a^2}{\pi A}(\alpha_g - \epsilon)^2 \sin\epsilon \\
& + C_\alpha \frac{2a^2}{\pi A}(\alpha_g - \epsilon)\sin\epsilon + C_q C_{M,o,t} \frac{\bar{c}_t}{l'} + C_q \frac{dC_{M,t}}{d\alpha}(\alpha_g - \epsilon)\frac{\bar{c}_t}{l'} + C_\alpha \frac{dC_{M,t}}{d\alpha} \frac{\bar{c}_t}{l'}
\end{aligned} \tag{10}$$

Collecting like terms reduces equation (10) to

$$\begin{aligned}
\frac{\Delta M}{q\left(\frac{q_t}{q}\right)\left(\frac{q_c}{q}\right)_w l' S_t} = {} & C_q \Bigg\{ C_{M,o,t} \frac{\bar{c}_t}{l'} + \frac{dC_{M,t}}{d\alpha}(\alpha_g - \epsilon)\frac{\bar{c}_t}{l'} \\
& + \left[\frac{h'}{l'} C_{D,o} + \frac{h'}{l'} \frac{a^2}{\pi A}(\alpha_g - \epsilon)^2 - a(\alpha_g - \epsilon)\right] \cos\epsilon \\
& + \left[\frac{h'}{l'} a(\alpha_g - \epsilon) + C_{D,o} + \frac{a^2}{\pi A}(\alpha_g - \epsilon)^2\right] \sin\epsilon \Bigg\} \\
& + C_\alpha \Bigg\{ \left[2\frac{h'}{l'} \frac{a^2}{\pi A}(\alpha_g - \epsilon) - \frac{h'}{l'} a(\alpha_g - \epsilon) - a - C_{D,o} - \frac{a^2}{\pi A}(\alpha_g - \epsilon)^2\right] \cos\epsilon \\
& + \left[\frac{h'}{l'} C_{D,o} + \frac{h'}{l'} \frac{a^2}{\pi A}(\alpha_g - \epsilon)^2 + \frac{h'}{l'} a - a(\alpha_g - \epsilon) + \frac{2a^2}{\pi A}(\alpha_g - \epsilon)\right] \sin\epsilon \\
& + \frac{dC_{M,t}}{d\alpha} \frac{\bar{c}_t}{l'} \Bigg\}
\end{aligned} \tag{11}$$

Now

$$C_M = \frac{M}{q_w S_w \bar{c}_w} = \frac{M}{q\left(\frac{q_c}{q}\right)_w S_w \bar{c}_w} \tag{12}$$

Simplifying equation (11) by the use of equations (6), (7), (8), and (12) yields the total corrected pitching moment of the complete configuration as

$$C_{M,c} = \frac{C_{M,meas}}{\left(\frac{q_c}{q}\right)_w} + \Delta C_M \tag{13a}$$

where

$$\Delta C_M = \frac{q_t}{q}\frac{S_t}{S_w}\frac{l'}{\bar{c}_w}\left\{C_q\left[C_{M,o,t}\frac{\bar{c}_t}{l'} + \frac{dC_{M,t}}{d\alpha}\frac{C_L}{a}\frac{\bar{c}_t}{l'} + \left(\frac{h'}{l'}C_D - C_L\right)\cos\epsilon + \left(\frac{h'}{l'}C_L + C_D\right)\sin\epsilon\right]\right.$$

$$\left. + C_\alpha\left[\frac{dC_{M,t}}{d\alpha}\frac{\bar{c}_t}{l'} + \left(2\frac{h'}{l'}a\frac{C_{D,i}}{C_L} - \frac{h'}{l'}C_L - a - C_D\right)\cos\epsilon + \left(\frac{h'}{l'}C_D + \frac{h'}{l'}a - C_L + 2a\frac{C_{D,i}}{C_L}\right)\sin\epsilon\right]\right\} \tag{13b}$$

Examination of equations (13) indicates that, in order to correct rigorously the tail contribution to pitching moment, it is necessary to know not only the values of C_L, C_D, and C_M for the tail in the absence of corrections, but also the values of ϵ and q_t/q (that is, the downwash field of the lifting system). Unfortunately, it is the very need to know these quantities that requires tail-on tests.

On the other hand, provided that the corrections at the tail are not more than moderately large, a separate tail balance installed in the model will yield values of the tail forces accurate enough for insertion in equations (13). Of course, it will be necessary to correct the tail forces themselves for wall effects before using the values in equations (13) if the corrections are large. A few survey measurements in the tail region with a pitot-static-pitch head will yield sufficiently accurate values of ϵ and q_t/q. In the absence of a tail balance and surveys, estimates can be made of these quantities by various means (refs. 10 to 14); however, the accuracy of correction will probably suffer if the corrections are large.

Simpler expressions than equation (13b) may be derived under certain restrictive assumptions. For example, if the corrections to the dynamic pressures at the lifting system and at the tail are approximately equal $\left(C_q \approx 0\right)$, and if the downwash angle is relatively small (that is, $\cos\epsilon \approx 1$ and $\sin\epsilon \approx \epsilon$), equation (13b) reduces to

$$\Delta C_M = \frac{q_t}{q}\frac{S_t}{S_w}\frac{l'}{\bar{c}_w}\left(\Delta\alpha_w - \Delta\alpha_t\right)\left[\frac{h'}{l'}\left(C_D\epsilon + 2a\frac{C_{D,i}}{C_L} - C_L + a\epsilon\right)\right.$$

$$\left. - \left(C_D + a + C_L\epsilon - 2a\frac{C_{D,i}}{C_L}\epsilon\right) + \frac{dC_{M,t}}{d\alpha}\frac{\bar{c}_t}{l'}\right] \tag{14}$$

If, in addition, it is assumed that the tail lies approximately in line with the lifting system ($h' \approx 0$ and $l' \approx l$), and it is noted that the contribution of the tail's own pitching moment will be small compared with the contribution of the tail lift and drag, equation (14) reduces to

$$\Delta C_M = \frac{q_t}{q}\frac{S_t}{S_w}\frac{l}{\bar{c}_w}\left(\Delta\alpha_t - \Delta\alpha_w\right)\left(C_D + a + C_L\epsilon - 2a\frac{C_{D,i}}{C_L}\epsilon\right) \tag{15}$$

It has already been assumed that ϵ is small. Furthermore, under the assumptions of the analysis, which is restricted to linear lift curves and no stall, $C_D < a$ and $C_{D,i} < C_L < a$. Thus, equation (15) can be further reduced to the approximation

$$\Delta C_M = \frac{q_t}{q}\frac{S_t}{S_w}\frac{l}{\bar{c}_w}a\left(\Delta\alpha_t - \Delta\alpha_w\right) \tag{16}$$

Equation (16) is useful for preliminary estimates of the order of magnitude of the required corrections to the pitching moments of the complete configuration.

Alternate Form

Equation (13b) presents the tail pitching-moment correction in terms of C_M. For a number of reasons, other forms are occasionally useful. One such form, often used in V/STOL testing, nondimensionalizes pitching moments with respect to the product of static thrust and the mean aerodynamic chord of the wing. In this case, the corrected nondimensional pitching moment is

$$\frac{M_c}{T_s\bar{c}_w} = \frac{M_{meas}}{T_s\bar{c}_w} + \frac{\Delta M}{T_s\bar{c}_w} \tag{17}$$

where, from equation (13b) and the definition of C_M,

$$\frac{\Delta M}{T_s\bar{c}_w} = \frac{q\left(\frac{q_t}{q}\right)\left(\frac{q_c}{q}\right)_w l'S_t}{T_s\bar{c}_w}\left\{C_q\left[C_{M,o,t}\frac{\bar{c}}{l'} + \frac{dC_{M,t}}{d\alpha}\frac{C_L}{a}\frac{\bar{c}_t}{l'} + \left(\frac{h'}{l'}C_D - C_L\right)\cos\epsilon + \left(\frac{h'}{l'}C_L + C_D\right)\sin\epsilon\right]\right.$$

$$\left. + C_\alpha\left[\frac{dC_{M,t}}{d\alpha}\frac{\bar{c}_t}{l'} + \left(2\frac{h'}{l'}a\frac{C_{D,i}}{C_L} - \frac{h'}{l'}C_L - a - C_D\right)\cos\epsilon + \left(\frac{h'}{l'}C_D + \frac{h'}{l'}a - C_L + 2a\frac{C_{D,i}}{C_L}\right)\sin\epsilon\right]\right\}$$

(18)

However, from reference 15,

$$T_s = n\rho A_m w_h^{\,2} \tag{19}$$

Substitution of equation (19) into equation (18) and simplifying yields

$$\begin{aligned}\frac{\Delta M}{T_s\bar{c}_w} = \frac{1}{2n}\left(\frac{V}{w_h}\right)^2\left(\frac{q_t}{q}\right)\left(\frac{q_c}{q}\right)_w \frac{l'}{\bar{c}_w}\frac{S_t}{A_m}\Bigg\{C_q\Bigg[C_{M,o,t}\frac{\bar{c}_t}{l'} + \frac{dC_{M,t}}{d\alpha}\frac{C_L}{a}\frac{\bar{c}_t}{l'} + \left(\frac{h'}{l'}C_D - C_L\right)\cos\epsilon \\ + \left(\frac{h'}{l'}C_L + C_D\right)\sin\epsilon\Bigg] + C_\alpha\Bigg[\frac{dC_{M,t}}{d\alpha}\frac{\bar{c}_t}{l'} + \left(2\frac{h'}{l'}a\frac{C_{D,i}}{C_L} - \frac{h'}{l'}C_L - a - C_D\right)\cos\epsilon \\ + \left(\frac{h'}{l'}C_D + \frac{h'}{l'}a - C_L + 2a\frac{C_{D,i}}{C_L}\right)\sin\epsilon\Bigg]\Bigg\}\end{aligned} \tag{20}$$

Correction as a Change in Tail Incidence

As pointed out in reference 6, a simple way to apply tail corrections is to view the correction as a rotation of the flow at the tail through an angle equal to the difference between the wall-induced interference angles at the lifting system and the tail. The first result of this rotation is that the tail incidence is altered by the rotation angle (fig. 3). In other words, the corrected tail incidence is

$$i_{t,c} = i_t + \Delta i \tag{21}$$

where

$$\Delta i = \Delta\alpha_t - \Delta\alpha_w \tag{22}$$

It will be observed that although figure 3 defines incidence from zero lift in order to maintain consistency of definitions throughout the paper, equations 21 and 22 are correct for incidence angles measured from any convenient chord line. The tail resultant-force coefficient and the tail's own pitching-moment coefficient are unaltered in magnitude by the correction to the tail incidence. The resultant-force vector is, however, rotated through the same angle Δi. As a result of the rotation of the resultant-force vector, a small correction must be made to the measured pitching moment due to the tail.

The measured moment due to the tail (fig. 3) is

$$\text{Moment} = hC_Rq_tS_t\cos\theta - lC_Rq_tS_t\sin\theta + C_{M,t}q_tS_t\bar{c}_t \tag{23}$$

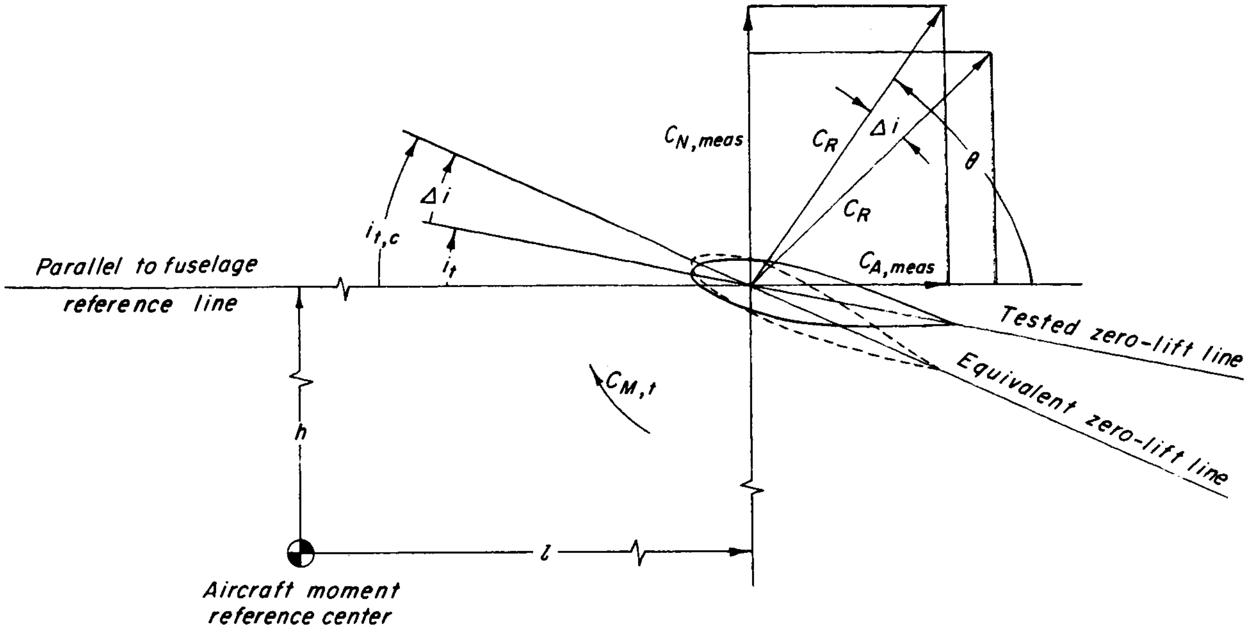

Figure 3.- Force and moment coefficients and angles at tail for the case of correction to tailplane incidence.

The desired moment, after rotation, is

$$\text{Moment} = hC_Rq_tS_t\cos(\theta - \Delta i) - lC_Rq_tS_t\sin(\theta - \Delta i) + C_{M,t}q_tS_t\bar{c}_t \tag{24}$$

The correction to the observed moment is the difference between these moments, or

$$\Delta M = hC_Rq_tS_t\left[\cos(\theta - \Delta i) - \cos\theta\right] - lC_Rq_tS_t\left[\sin(\theta - \Delta i) - \sin\theta\right] \tag{25}$$

Expanding the trigonometric terms and assuming that Δi is reasonably small (that is, $\cos\Delta i \approx 1$ and $\sin\Delta i \approx \Delta i$) yields

$$\Delta M = hC_Rq_tS_t\ \Delta i\sin\theta + lC_Rq_tS_t\ \Delta i\cos\theta \tag{26}$$

But

$$C_R\sin\theta = C_{N,meas} \tag{27a}$$

$$C_R\cos\theta = C_{A,meas} \tag{27b}$$

so that

$$\Delta M = \Delta i \; q_t S_t \left(hC_{N,meas} + lC_{A,meas} \right) \tag{28}$$

and, in consequence of the definition of C_M,

$$\Delta C_M = \Delta i \frac{S_t}{S_w} \frac{1}{(q_c/q)_t} \frac{l}{\bar{c}_w} \left(\frac{h}{l} C_{N,meas} + C_{A,meas} \right) \tag{29}$$

Recognizing that corrections to the dynamic pressures may be different at the tail and at the lifting system leads to the final expression for the corrected pitching-moment coefficient when the tail incidence is corrected according to equation (21):

$$C_{M,c} = \frac{(C_M)_w}{\left(\frac{q_c}{q}\right)_w} + \frac{(C_M)_t}{\left(\frac{q_c}{q}\right)_t} + \Delta i \frac{S_t}{S_w} \frac{1}{\left(\frac{q_c}{q}\right)_t} \frac{l}{\bar{c}_w} \left(\frac{h}{l} C_{N,meas} + C_{A,meas} \right) \tag{30}$$

For conventional models, where Δi is held small by proper sizing of the model and where the tail height is small, the last term of equation (30) is generally small and can be safely neglected. If, in addition, the corrections to the dynamic pressures at the wing (or lifting system) and tail are approximately equal, then essentially no correction to the tail moment is required and the total correction is the change in tail incidence. Thus, under these stated conditions, the procedure of reference 6 is completely correct. On the other hand, if Δi, h/l, and $C_{N,meas}$ are large, as may be the case in tests of large V/STOL models, a significant correction to the pitching-moment coefficient may be required in addition to the correction to tail incidence.

Note that the form of equation (30) is particularly convenient if a tail balance is installed in the model to measure the normal and axial forces on the tail. Such a balance is recommended if large corrections are likely to be necessary. If these tail forces must be estimated instead of measured, the accuracy of correction can be substantially lessened.

It will be observed in equation (30) that rigorous correction of the pitching-moment coefficient by change in tailplane incidence requires a knowledge of the division of pitching moment between the lifting system and the tail. This division can be obtained with sufficient accuracy by running one set of tests with the tail removed from the model and taking these results as indicative of the pitching moment of the lifting system. If the corrections to the dynamic pressures at the lifting system and tail are essentially equal, equation (30) indicates that this step will not be required.

This system for correcting tail moments possesses one major disadvantage. Since $\Delta\alpha_t$, $\Delta\alpha_w$, and Δi all vary with the forces on the model, it follows that the corrected tailplane incidence will vary throughout any given test run in the wind tunnel. If large corrections are necessary, this variance in tailplane incidence may seriously affect the apparent stability level of the model. Thus, if the corrections are large, it may prove necessary to conduct tests with several values of tailplane incidence and to cross plot the data to obtain test results representative of a single tailplane incidence.

Aerodynamic Distortion of Model

In general, some wall-induced curvature of flow exists over the extent of a model in the wind tunnel. This curvature is evidenced by the varying values of $\Delta\alpha$ and q_c/q computed over the extent of the model. If the wall-induced flow varies rapidly in the spanwise direction the tested model will be aerodynamically equivalent to an actual aircraft having wing twist other than that built into the model. The forces at the tail will thus be measured in the presence of a wing which may have a decidedly different spanwise load distribution than the wing in actual flight. Furthermore, if the wall-induced flow varies rapidly in the streamwise direction, the model wing will be aerodynamically equivalent to a wing with a camber line different from the physical camber of the model. This effect is most noticeable in two-dimensional testing. On the other hand, this same streamwise gradient also produces, aerodynamically speaking, an altered vertical location of the tail. The latter effect may be extremely important for three-dimensional models if the nonuniformity of wall-induced flow is large in the longitudinal direction.

It is noted in particular that the longitudinal gradient of vertical interference velocity can produce large changes in the pitching moment of many lifting systems (for example, lifting propellers). This source of pitching moment is additive to the effect of the walls on the tail which is discussed herein.

None of the aerodynamic distortions discussed in this section are considered in the preceding analysis. Inclusion of these effects, particularly for V/STOL models, is beyond the scope of the present analysis. The surest way to avoid large errors due to these causes is to design the model to such a size that the overall corrections, and consequently the nonuniformities of the corrections, are small.

Comparison of Correction Procedures

There is a profound difference in philosophy between the two correction procedures outlined in the present paper. The first method attempts to alter the measured data to correspond to a new condition at which the tail did not actually operate during the test, and large corrections may be encountered. In contrast, the second method merely finds the free-air condition which is equivalent to the condition at which the tail was actually tested, and then applies a minor correction to the observed moment.

Aside from the large magnitude of the corrections encountered when corrections are applied to the pitching moment only, there remains the problems of correctly estimating the many terms in equation (13b), as well as the restrictions introduced by the assumptions of linear lift curves, no stall, and incompressible flow. The necessity for these assumptions is avoided when corrections are applied to the tail incidence. In view of the fact that corrections to tail-plane incidence are not only more convenient, but also more rigorous, it would appear that this method is preferable under most conditions.

CONCLUDING REMARKS

Expressions have been derived for the application of wall-effect corrections to the pitching moments due to the tail as measured in wind-tunnel tests of a complete model. Two systems of correction are presented. The first system requires a change in the observed pitching moment; however, the use of this system requires a knowledge of many quantities not normally measured as well as flow surveys in the region of the tail. The second system requires a change in the geometric angle of incidence of the tail, as well as a small correction to the pitching moment. The latter system appears preferable under most circumstances since it is not only more convenient, but also more rigorously correct; however, tests at several values of tail incidence may be necessary. In either case, if large corrections are expected, the use of an independent tail balance is recommended.

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