REMOTE DETECTION OF CLEAR AIR TURBULENCE:
PART I—PULSED MICROWAVE RADARS

by

B. M. Fannin

ANTENNAS AND PROPAGATION DIVISION
ELECTRICAL ENGINEERING RESEARCH LABORATORY
THE UNIVERSITY OF TEXAS
Austin, Texas

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Prepared Under
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PREFACE

This is the second technical report prepared under NASA Grant NGR 44-012-048. The first report was


This present report uses available models of the clear air turbulence including that proposed in Report P-12 to estimate the backscattering of microwaves from such turbulence.

Other techniques for clear air turbulence detection will be considered in a later report.

A third report which is in the process of preparation will be concerned with equipment for direct measurement of refractive index differences and the examination of the initial data taken on a 270 foot tower. The results of these measurements should shed considerable light on the nature of refractive index anomalies associated with refractive index variation of the atmosphere.

A fourth aspect of the research is concerned with the use of radar for measuring the return from the refractive index variation in the atmosphere. Analysis of experimental programs is currently under way. Preliminary proposals have been made to the Electronics Research Center for unique tests which would measure scattering from radar beams and the associated refractive index anomalies. It is hoped that this program can be continued in an extension of the grant period.

A. W. Straiton
Principal Investigator
CONCLUSIONS

Calculations are reported for the expected mean returned power from clear air turbulence, relative to the minimum detectable level, with the chosen values of the many contributing parameters carefully noted and discussed. It is found that operational systems for normal jet flights, constrained by reasonable limits on weight, size, power, cost, etc, are not feasible. On the other hand, research-type ground-based systems should be capable of reliable detection of regions of CAT for wavelengths from a few centimeters to a few meters. However, not only was near-the-state-of-the-art performance assumed for the ground-based systems, but favorable conditions generally were assumed, including pointing at the zenith, a cloudless sky, pointing away from the galactic plane, etc.
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INTRODUCTION

A wide variety of methods have been postulated as having some promise of being effective remote sensors of the presence of CAT. In the last few years, research groups with various affiliations and support have been engaged in trying to advance the state of the art in the instrumentation and to evaluate the future potential of a number of these proposed methods. Though it is human nature for researchers to be optimistic concerning the prospect of the eventual usefulness of their own area of endeavor, a searching review of the published papers in the field reveals that no method (with the present instrumentation) comes very close to providing a tool that would furnish the pilot of a jet aircraft with sufficient advanced notice and information to allow him to alter the flight path so as to avoid regions of CAT.

The property of CAT of primary concern is the wind-gust or turbulent-velocity field but most of the remote detection methods rely on sensing some other physical parameter (such as refractive index, temperature, particulate matter, ozone, etc.) which it is hoped can be correlated with CAT. Any effort to arrive at a qualitative evaluation of the potential of the different proposed schemes is critically limited by a lack of reliable knowledge of the overall physical properties of CAT. Nevertheless, the objective of this report is to present a critical review of the different methods proposed for the remote detection of CAT that appear in the
published literature and to delineate, to the extent that it is possible to do so, their potentialities for the foreseeable future.

The majority of the papers that contain the most up-to-date published information on detection of CAT are included in the ION-SAE Conference Proceedings of the National Air Meeting on Clear Air Turbulence, Washington, D.C., February 23-24, 1966. This collection includes a contribution from a major investigator in almost all of the more actively researched areas. Thus, in a very real sense, this report constitutes a summarization and mild critique of the papers in the middle section, "Detection of Clear Air Turbulence," of that compilation.¹

Most of the emphasis has been concentrated on projects whose ultimate goal is to devise instrumentation to be placed on jet passenger transport planes to give warning of, and detailed information about, regions of CAT in the line of flight. To provide a net of ground-based sensors that would provide anything near complete coverage (assuming satisfactory sensors can be developed) would be prohibitively expensive in equipment costs and operator manpower. However, so little is known about the basic physics and meteorology of CAT that probably the most pressing need is to accumulate detailed data covering all the physical parameters associated with this phenomenon and ground-based sensors may serve a very important function as a research tool to supplement our basic knowledge of the topic. Therefore, an effort will be made to evaluate the potential of each scheme as a ground-based device as well as an "on board" instrument.
There are so many ill-defined constraints imposed in a given situation by such considerations as economics, operational conditions, fundamental objectives, etc., that it is not feasible to try to give unqualified answers to questions concerning optimum frequency, maximum range of detection, etc. Therefore, the assumptions made concerning each factor which contributes to the end result will be clearly noted so that when a specific problem is uniquely defined a minimum of effort will be required to extend the information presented here to form the basis of reasonable conclusions for the case at hand.

The approaches to the detection of CAT will be grouped under the following headings according to the type of technique central to the method:

(a) Back Scatter - including sonic, VLF, microwave, infrared and optical devices.

(b) Forward Scatter - again considering a wide range of possible frequencies.

(c) Radiometry - particularly at microwave and infrared frequencies.

(d) Direct Measurements - properties of the medium such as temperature, electrostatic field, ozone content, etc. being measured at the position of the instrument (usually on an aircraft).

Discussions of these four categories of approaches to the detection of CAT will now be presented in two parts; namely, Part I - Pulsed Microwave Radars, and Part II - Other Sensing Methods.
EQUATIONS FOR BACK-SCATTER MEASUREMENTS

The standard radar equation is well known but the symbols will be carefully defined since, for the problem at hand, one is forced to select representative values of each for each system analyzed. Considering a back-scattering "object" to be at the point Q in space, the following symbols are defined for the general prototype of systems in this category:

\[ P_t, P_r = \text{the peak power radiated by the transmitting antenna and intercepted by the receiving antenna. The transmitted wave may be either pulse modulated or \( CW \), the rate of energy radiation being constant \( (P_t) \) during the "on" period} \]

\[ G_t, G_r = \text{the power gains of the antennas (relative to an isotropic radiator) for propagation in the directions to and from Q.} \]

\[ A_t, A_r = \text{the effective antenna areas for transmission to and reception from Q.} \]

\[ R_t, R_r = \text{the distances from the antennas to Q.} \]

\[ \lambda = \text{the wavelength of the wave.} \]

\[ \sigma = \text{the scattering cross section per unit scattering volume; i.e., the ratio of \( 4\pi \times \text{the energy per unit solid angle scattered into the direction of the receiver per unit volume} \) to \( \text{the energy per unit area in the wave incident upon the scattering volume}. \) The first portion of this ratio is seen to equal the total energy that would be scattered if it was scattered in all directions at the same intensity as in the receiver direction.} \]

From the last definition it is apparent the scattering is assumed to result from random-type anomalies dispersed throughout the region of CAT. The case of reflections from surface discontinuities in the refractive index will be considered later.
The fundamental radar equation for a single target is

\[ P_r = \left( \frac{P_t G_t}{4\pi R^2} \right) \left( \frac{\rho}{4\pi R^2} \right) (A_r), \]  

(1)

\( \rho \) being the radar cross section for the target. The first factor gives the power per unit area of the incident wavefront at the target, the first two factors then representing the power per unit area in the wavefront of the scattered wave at the receiver. For a distributed scattering region, the mean received power is given by considering each \( dv \) of the region to have a scattering cross section of \( \sigma \). That is, \( \rho \) in (1) is replaced by \( \sigma \ dv \) and the expression integrated over the region of anomalous refractive index, \( G_t, A_r, \sigma \), as well as \( R \) being functions of the position of \( dv \). However, if \( \sigma \) is essentially uniform over the effective scattering volume (dependent on the region "illuminated" by the transmitted wave as well as the extent of the refractive index anomalies), and the radial dimension of this volume is relatively small compared with \( R \), it is convenient to write

\[ \overline{P_r} = \frac{P_t G_t A_r \sigma V}{(4\pi R^2)^2}, \]  

(2)

in which \( G_t, A_r \) are assigned the "center-of-the-beam" values, \( V \) is the volume of the "effective" scattering region and \( R \) is the range to the center of this volume. If the "effective" scattering volume is beam limited, Probert-Jones\(^{19} \) and Battan\(^{4} \) have each deduced that the "effective" extent of the beam is somewhat less than that bounded by the "half-power" directions, a correction factor of roughly \( 4/9 \) being in order if the half-power
solid angle is taken as the effective extent of the beam. Delving into the fine points associated with eq. (2) is hardly warranted here since uncertainties of considerably greater import will be interjected through the estimates of values for the basic physical parameters of CAT upon which estimates of $\sigma$ are based; nevertheless, eq. (2) will be supplanted by

$$\overline{P_r} = \frac{P_t G A_r \sigma V}{\frac{24}{36\pi^2} R}$$

in order to be in agreement with the expressions employed in the papers by Smith & Rogers and by Atlas, Hardy & Naito.

The relation between the gain, beam angle, and effective area of an antenna

$$G = \frac{4\pi}{\Omega} = \frac{4\pi A}{\lambda^2},$$

$\Omega$ denoting the solid angle "filled" by the beam, allows one to convert eq. (3) into a number of equivalent forms.

Since the back-scattered signal fluctuates rather randomly with time, it has the same general character as noise. Thus the output back-scattered signal is recognizable in the additive noise only if the ratio of signal power to noise power exceeds a minimum limit. The equivalent average noise power at the receiving antenna terminals is

$$\overline{P_n} = k T_o B F' = k T_e B$$

in which $k$ is Boltzmann's constant $= 1.38(10)^{-23}$ joules/deg, $k$, $T_o = 290^\circ K$ B is the receiver bandwidth and $F'$ is the noise figure of the receiving
system embodying the internal receiver noise, the noise from external sources picked up by the antenna and the effects of losses in the antenna-to-receiver transmission system. \( T_e \), the effective noise temperature at the antenna terminals, is equal to

\[
T_e = T_o F' = T_a + (L - 1)T_t + LT_r (G)
\]

in which \( T_a \) is the apparent antenna noise temperature (the integrated equivalent noise temperature of the region "viewed" by the antenna), \( L \) is the loss factor for the transmission line from antenna to receiver, \( T_t \) is the temperature of this transmission system, and \( T_r \) is the effective receiver temperature = \((F - 1)T_o\), \( F \) being the noise figure for the receiver alone. \( L = G^{-1} \), \( G \) being the "gain" for the transmission system which is less than unity.

Letting \( \gamma \) denote the ratio of minimum detectable average signal (backscattered) power to noise power; i.e.,

\[
\overline{P}_{\text{min}} = \gamma \overline{P}_n;
\]

then

\[
\frac{\overline{P}_r}{\overline{P}_{\text{min}}} = \frac{P_t G A_t \sigma V}{36\pi^2 R^4 k B T_e \gamma}
\]

becomes the basic expression for which estimates of the constituent factors must be obtained in order to predict the observability of the back-scattered signal from regions of CAT. A brief discussion of typical values for these parameters will follow for several basic classes of systems.
PARAMETER VALUES FOR MICROWAVE RADARS

The teams of Smith & Rogers\textsuperscript{25} and Atlas, Hardy & Naito\textsuperscript{2} have each published the results of their analyses of this problem, their results being essentially in agreement with each other and with the presentation to follow.

**Bandwidth and pulse duration.** If the system bandwidth is appreciably less than $1/\tau$ ($\tau$ denoting the pulse duration) the receiver response is too sluggish to effectively respond to the pulse and if it is appreciably greater than $1/\tau$ it allows through an excessive amount of noise. Thus it is well known [see Lawson & Uhlenbeck, Section 8.6] that for optimum detection the bandwidth should be such that

$$B \tau \approx 1, \quad (9)$$

this condition therefore being assumed.

Since $B$ appears in the denominator of eq. (8), it would appear desirable to make $B$ small and $\tau$ large, maintaining the relation of eq. (9).

However, the radial resolution of the radar varies inversely as $\tau$, the region of scatter for energy reaching the receiver at a given time having a radial dimension of $\tau c/2$, $c$ denoting the wave velocity $= 3(10)^8$ meters/sec.

Rather arbitrarily setting the upper limit on this range resolution at 150 meters converts to

$$\tau = 1 \mu \text{sec}, \text{ and } B = 1 \text{ MHz}. \quad (10)$$
Scattering volume. The scattering volume is confined by the antenna beam, half the space pulse length and the region occupied by CAT.

\[ V = K(R^2 \Omega) \left( \frac{\tau_c}{2} \right) = K \left( \frac{R^2 \lambda^2}{A_t} \right) (150 \text{ m}), \]  

(11)
in which \( K \) denotes the portion of the beam-pulse defined volume occupied by CAT. For ground-based radars \( K \) can reasonably be set equal to unity but for plane-borne sets CAT may not fill the vertical extent of the beam. Denoting the thickness of the CAT layer by \( \Delta H \), then \( K \) for the latter can be crudely estimated by

\[ K = \frac{\Delta H}{R(\lambda / D)} = \frac{(\Delta H)D}{R\lambda} \quad \text{(for } K^1 \text{s} < 1) \]  

(12)
in which \( D \) denotes the diameter (vertical dimension) of the transmitting antenna.

With the exception of the beam filling factor, \( K \), the above parameter values appear to be equally appropriate for the airborne and ground-based systems, the remaining factors depending critically on either the choice between these two types of operation or on the wavelength. Tentatively assuming the same or equivalent antennas for transmission and reception and incorporating eqs. (4), (10) and (11) into eq. (8) gives

\[ \frac{P_r}{P_{\min}} \approx \frac{P_t A \sigma K (l. 32)(10)^{15}}{R^2 (T_e / T_o) \gamma}. \]  

(13)
**Range.** CAT has been observed to occur over a wide range of altitudes but most frequently at roughly 9 km [Stephens and Reiter], this being selected as the value of \( R \) for the ground-based system. Atlas, Hardy & Naito and Rosenberg have selected 10 and 30 nautical miles, respectively, as the desired minimum range for pilot warning systems, these corresponding to 1 and 3 minute warning times for conventional jet aircraft. Anticipating inability to achieve even the shorter range, it is selected for the present consideration. That is, the ranges

\[
R_a \approx 18 \text{ km} \quad \text{and} \quad R_g \approx 9 \text{ km}
\]

will be assumed, subscripts "a" and "g" being introduced to distinguish "airborne" and "ground-based" systems.

**Antenna size.** Both Smith and Rogers and Atlas, Hardy & Naito address themselves to the consideration of radars to possibly become standard equipment on commercial jet aircraft and select 1.0 meter in diameter as the maximum feasible antenna size for such an application, the effective area being taken as 67 per cent of the actual area. For an especially equipped research aircraft somewhat larger antennas could be envisioned while for ground-based operation very extensive structures become a possibility. Diameters of 120 ft. are considered in the realm of feasibility for the long wavelengths while for the other end of the range a 30 ft. dish for \( \lambda = 1 \text{ cm} \) has been chosen as reasonable. Larger antennas are possible but a "feasible limit" has been rather arbitrarily imposed.
Using an effective to actual area ratio of 2/3 a diameter of 1.0 m corrects to an effective area of approximately 0.5 m$^2$, so for the evaluations to follow the airborne and ground-based effective antenna areas are taken to be as indicated in Fig. 1.

**Transmitter peak power.** State-of-the-art but commercially available tube output powers, as indicated in Fig. 1 [see Barton$^3$, Hull$^{12}$] are assumed for the ground stations. Parallel operation would make larger radiated powers possible but is not considered here. For the operational airborne systems, somewhat more conventional values are indicated.

**Beam filling factor.** As already indicated, values for $K$ -- the portion of the beam and pulse limited volume filled by the CAT region -- are taken as

$$K_g = 1 \quad \text{and} \quad K_a = \frac{(AH)D}{R\lambda} \quad (\text{for } K < 1). \quad (12')$$

A rather nebulous situation exists from which to select a representative value for $\Delta H$. The average vertical thicknesses of the turbulent layers appear to lie between 500 to 3000 ft. [Stephens and Reiter, 1966$^{26}$] so a midrange value of 500 meters seems a reasonable choice for $\Delta H$. For this $\Delta H$ and the choices of $D_a = 1$ m and $R_a = 19$ km indicated above in eq. (12') leads to

$$K_a = \frac{25}{9\lambda} \quad \text{for } \lambda \text{ in cm and } \lambda > \frac{25}{9}, \quad (15)$$

this factor being shown in Fig. 1; an additional rounding of the corner is introduced for aesthetic reasons.
FIGURE 1
Scattering cross-section or reflectivity. The refractive-index structure function, $D_n(r)$, is defined such that for a locally homogeneous and isotropic region

$$D_n(r) = 2(\Delta n)^2 [1 - \rho(r)]$$

(16)

in which $(\Delta n)^2$ is the mean square of the refractive-index deviations from a mean distribution and $\rho(r)$ is the normalized space autocorrelation of these refractive-index deviations [see Tatarski]. If the region is turbulent and the scale lengths of interest fall in the inertial subrange [inherent in Kolmogoroff's universal equilibrium theory of homogeneous turbulence] and the refractive index is taken to be a conservative passive additive, then dimensional considerations indicate that [see Obukhov$^{18}$, Tatarski$^{27}$, and Stephens and Reiter$^{26}$]

$$D_n(r) = C_n^2 r^{2/3}$$

(17)

in which $C_n^2$ is a parameter indicative of the strength of the turbulence and depending upon the physical properties of the medium and its state.

For the form of $D_n(r)$ indicated in eq. (17), single-scattering theory [see Booker and Gordon$^6$, Silverman$^{22}$ and Tatarski$^{27}$] leads to the radar scattering cross-section being given by

$$\sigma = \frac{\Gamma(\frac{8}{3}) \sin(\frac{\pi}{3})}{8} C_n^2 (2k)^{1/3}$$

(18)

in which $k = 2\pi/\lambda$, or

$$\sigma = 0.39 C_n^2 \lambda^{-1/3}$$

(19)
Thus, assuming the validity of the inertial subrange model, only an appropriate value for the parameter $C_n^2$ for regions of CAT needs to be determined to complete our estimate of the radar cross-section as a function of wavelength. Tatarski deduced that

$$C_n^2 = a^2 L_o^{4/3} M^2$$

(20)

in which $a^2$ is a parameter which, for stable stratification, depends on the Richardson number; $L_o$ is the outer scale (largest sized eddies) and $M$ is the mean vertical gradient of (potential) refractive index. Atlas, Hardy & Naito estimated typical values of $C_n^2$ by assigning values to the parameters in eq. (20) as deduced from an analysis of data reported by various investigators indicative of magnitudes of related physical quantities. They deduced that $C_n^2$ ranges roughly from $10^{-16}$ to $10^{-14}$ cm$^{-2/3}$ for weak to severe CAT. Subsequently, Stephens and Reiter have analyzed two vertical profiles through regions of moderate CAT from data reported by Endlich and arrive at values of $C_n^2$ for the turbulent zones that in the neighborhood of $10^{-17}$ cm$^{-2/3}$, roughly two orders of magnitude below the Atlas, Hardy & Naito estimates. Stephens and Reiter attribute this variance to the contention that Atlas, et al. overestimated $a^2$ by a factor of approximately 24.

Though eq. (19) is appropriate for the inertial subrange portion of the turbulence spectrum, for $(\lambda/2)$ of the order of magnitude of $\ell_i$ ($\ell_i$ denoting the inner scale for temperature inhomogeneities, smaller eddies being rapidly dissipated due to viscous forces) or less the result needs to be
modified to reflect the dissipation effect. Theoretical expressions for the variation with wave number of the spectral density functions (and thus the radar cross-section also) for wavelengths in the neighborhood of $\lambda_1$ are lacking but Gurvitch, Tsvang and Yaglom have reported a form for the one-dimensional spectrum obtained empirically by Gorshkov. Atlas, Hardy & Naito use this form of wave number dependence to extend their reflectivity estimates into the dissipation subrange. Their lead will be followed here, their selection of $\lambda_1 = 0.82$ cm for CAT of moderate intensity also being adopted since it seems in general agreement with other estimates.

Taking $C_n^2 = 10^{-17}$ cm$^{-2/3}$ (in accordance with Reiter and Stephens' result) then results in the radar cross-section curve in Fig. 1 as being representative of moderately-intense CAT.

**Effective noise temperature.** The effective noise temperature is here defined relative to the antenna (rather than the receiver proper). The attenuation due to losses in the transmitter-to-antenna and antenna-to-receiver transmission networks and due to atmospheric absorption was not included in eq. (2) because, for the frequencies for which it is significant, the corresponding "black body" radiation noise is the dominant effect.

Equation (6) expresses the effective noise temperature as the sum of contributions from the region of space viewed by the antenna, from the areas of energy absorption (joule heat losses) in the antenna and transmission line system, and from the receiver. $T_a$ encompasses all sources radiating noise energy that is picked up by the antenna, including atmospheric noise
(lightning, etc.), cosmic noise (extraterrestrial sources), atmospheric absorption noise (black body radiation from atmospheric constituents), man-made noise (ignition systems, neon signs, fluorescent lights, etc.) and thermal radiation from surrounding objects. Of these the atmospheric and man-made sources are not of concern for the range of frequencies being considered, though they both play significant roles at lower frequencies.

The effective temperature of cosmic noise falls off rapidly with increasing frequency [Skolinik, Kraus, Ko, Green and Benebaum] but may contribute significantly at the low-frequency end of the band being considered. The sun is a prolific producer of radio-frequency energy but it is here assumed to not be in the main beam of the antenna receiving pattern. Our galaxy (the Milky Way) also contributes strong noise signals, the effective noise temperature in the direction of the galactic center being roughly 70 times that in the vicinity of the galactic poles. Values roughly twice the minimum, as indicated in Fig. 2, are assumed for the present calculations.

The atmospheric absorption noise (so called because absorbers of energy are equally prolific radiators of energy at the same frequency so absorption and emission properties can be equated) is given by

$$T_{ab} = T_m \frac{(L - 1)}{L}$$

in which \( L \) is the loss (ratio of the energies in the wave upon entering and upon leaving the absorbing medium) and \( T_m \) denotes the temperature of the
\( T_e \) - Total effective noise temperature
\( T_r \) - Receiver noise temperature
\( T_{ab} \) - Contribution by absorbing atmospheric gases
\( T_g, T_s \) - Ground and sky temperatures
\( K \) - Portion of pattern impinging on ground

**Figure 2**
absorbing medium. [See Lawson and Uhlenbeck, Skolnik, Hogg and Mumford, Greene and Lebenbaum]. According to eq. (21), if $T_m = 260^\circ K$ and $L = 0.1$ db, then $T_{ab} = 5.9^\circ$ so that relatively small amounts of absorption can correspond to significant noise in low-noise systems.

The atmospheric absorption effect is naturally much more severe when the antenna is pointed toward the horizon than when it is pointed toward the zenith. For the ground-based system, it is assumed the antenna will always be directed vertically.

Bean and Dutton give curves of the gaseous atmospheric absorption (in db/km) as a function of height above the surface for mean profiles at Bismarck, N. D., and Washington, D. C., for February and August (4 cases in all) for seven frequencies between 100 MHz and 50 GHz. They also give curves of net atmospheric thermal noise versus frequency for a narrow beam antenna pointed at six different angles from the zenith for the Bismarck station in February. Unfortunately this site-season combination is one for which the atmosphere is quite cold and low in moisture but the family of curves clearly illustrates the severe dependence of the total path absorption on the elevation angle. The atmospheric absorption equivalent temperatures chosen for the calculations here are shown in Fig. 2 and were obtained by crudely integrating the absorption versus altitude curves given by Bean and Dutton as representative of Washington, D. C., in August along a vertical path from ground level and along a horizontally directed path from 9 km altitude.
The noise power discussed in the last two paragraphs enters the system through the main lobe of the antenna receiving pattern. Energy entering through the side lobes (including back lobes) also contributes to the equivalent noise temperature of the antenna. One can write

\[ T_a = (1-k)T_s + kT_g = T_s + k(T_g - T_s) \]  

(22)

in which \( T_s \) and \( T_g \) are the temperature equivalents of the noise power entering from the direction of the sky and ground, respectively, and \( k \) is the fraction of the total power which is radiated in the direction of the "ground" (when the antenna is considered as transmitting, this being appropriate because of the equivalence of the radiation and reception patterns). If a given region is a good reflector for the wavelength being used, the power density impinging upon the antenna from that direction is determined by the temperature of the image rather than the temperature of the reflector.

For the airborne antenna \( k = 0.1 \) is a reasonable value and \( k = 0.01 \) is chosen for the ground-based system, it being assumed a Cassegrain feed and/or side-lobe-reducing shields are used in the latter case. A Cassegrain feed would probably also be used in the airborne system but the size of the antenna, the rapid variation of apparent temperature with aspect in the forward direction and radome losses (lumped here for convenience) lead to the choice of \( k = 0.1 \). The value \( T_g = 290^\circ \) is assumed, the hypothetical contributions of the "ground-effect" term \( k(T_g - T_s) \) being
shown in Fig. 2, it being taken as zero when \( T_s > T_g \).

If a Cassegrain feed is used, the RF plumbing can be kept at a minimum, and thus also the RF losses. Here the RF losses are assumed to be negligible though this is admittedly overly optimistic at the low cm and mm wavelengths. Though these losses are neglected here, it is important to leave the RF plumbing loss terms in eq. (6) because if special care is not taken to keep this loss at a minimum it may contribute a very significant effect.

It has been seen that for a research-type, ground-based system the effective sky temperature can be kept appreciably less than 10°K in the wavelength range 3 - 30 cm. Thus a maser preamplifier would certainly be chosen for such an operation in the indicated wavelength range. [See, for example, Matthei\textsuperscript{16}.] At the present, 30 GHz may be taken as the feasible upper frequency limit for maser operation so it is assumed a maser would be used up to this frequency and a helium-cooled parametric amplifier beyond this point (for ground-based operation). Optimum equivalent noise temperature for such devices are shown in Fig. 2, a smooth transition being shown for the maser-parametric amplifier transition. For an operational airborne system cost, reliability and maintenance must be considered. Since little is to be gained by reducing the receiver noise temperature below the effective sky temperature, a parametric amplifier employing a mechanically simple, oil-lubricated refrigeration system (such as a Gifford-McMahon closed-cycle unit) would be the logical choice.
[See Matthei\textsuperscript{16} or Slaughter, Cone, Miller\textsuperscript{24}.] The effective noise temperature curve representative of such devices (with state-of-the-art engineering) is shown in Fig. 2.

The net effective noise temperature, $T_e$ of eq. (6), for the two systems is shown in Fig. 2.

**Minimum detectable signal-to-noise ratio.** For a single pulse, a signal power-to-noise power ratio, $(S/N)$ of the order of 10 is needed for reliable detection (90%). [See Lawson and Uhlenbeck\textsuperscript{15}, North\textsuperscript{17}.] With more than one pulse to base the decision upon, the required signal-to-noise ratio can be reduced. With ideal coherent (predetector) integration, the value of $(S/N)$ for 90% detection would decrease roughly as $n^{-1}$, $n$ denoting the number of pulses integrated. For ideal incoherent (postdetector) integration, then $(S/N)$ varies approximately as $n^{-1/2}$. The predetector integration obviously gives superior results but, since it requires phase coherence between pulses, requires a more complex transmitter as well as receiver. Also, the maximum effective integration time would be set by the coherence time of the reflected signal, this being of the order of the wavelength divided by the gust velocity for this application. Colson\textsuperscript{7} lists 20 - 35 ft/sec. as the range of gust velocities for moderate CAT. Selecting a velocity of 10 meters/sec. and a wavelength of 10 cm, the maximum effective integration time would be of the order of $10^{-2}$ sec. Since the values of transmitted energy per pulse have been chosen to represent near maximum values this preempts the use of unusually large pulse
repetition rates. Therefore, since only a relatively small number of
pulses could be coherently integrated, one would undoubtedly choose a
system employing some type of postdetection integration but not predetec-
tion integration.

The simplest and most frequently used postdetection integration
deVICES are the cathode-ray tube and film, both in conjunction with the
human eye. However, for operational use on a jet airliner it would not be
desirable to have an operator sit with an eye constantly on a cathode-ray
scope so an electronic warning device would most likely be devised. In
any case, the maximum integration time could not be very long. The
problem of estimating the minimum signal-to-noise ratio for detection is
further complicated by the extended nature of the target. For the airborne
case a unity (S/N) for detection seems a reasonable choice [see Lawson
and Uhlenbeck, Skolnik, North, etc.] while for the ground-based
system it is anticipated that the use of film integration with side-by-side
display of successive traces (or more sophisticated schemes) would make
an added improvement of -10 db possible. [See Skolnik, Tucker, Saxton et al., Watkins and Sutchiffe.] That is, for use in eq. (13) the
values
\[ \gamma_a = 1.0 \quad \text{and} \quad \gamma_g = 0.1 \]  
have been selected.
PREDICTION OF DETECTABILITY OF CAT

Using the choices indicated in the last few paragraphs for the parameters involved, eq. (13) gives values of the received power relative to the minimum detectable level as plotted in Fig. 3. Since in this figure 0 db corresponds to the received signal being equal to the minimum detectable value, it is seen that if the parameter values selected are indeed reasonably representative, then moderate CAT can be detected (with some margin) by good research-class ground-based systems but not by systems presently within the realm of practicality for operation on conventional jet airline flights. The more pessimistic result ($\approx -24$ db) for the airborne case than that obtained by Atlas, Hardy and Naito is principally due to the smaller estimate of the scattering cross-section. The more rapid falloff at longer wavelengths is due to the emergence of galactic noise as the dominant noise term, a term evidently not included in the earlier evaluation.
REFERENCES


14 Kraus, J. D., 1964: Recent Advances in Radio Astronomy, IEEE Spectrum, 1(9), 78-95.


