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EFFECT OF RADIATION ATTENTUATION UPON THE MOTION OF A RELATIVISTIC PARTICLE IN A UNIFORM FIELD

by

G. E. Gernet

(USSR)

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SUMMARY

The effect of radiation attenuation upon the motion of a relativistic particles in a uniform magnetic field is evaluated by the behavior of the transverse velocity components, for which the approximate equations are derived.

* * *

When a charged particle moves in a magnetic field, its energy decreases because of magnetic bremmstrahlung (synchrotron) radiation, and this leads to trajectory change.

In the cases, when the particle's sojourn in the magnetic field is comparable with the characteristic time of energy decrease

$$T \sim m^3c^5 / e^4H^2$$

(as, for example, for electrons in cosmic fields), one must take into account the radiation attenuation when determining the trajectory.

The particle's equation of motion has the form

$$\frac{d\mathbf{p}}{dt} = \frac{e}{c}[\mathbf{v}, \mathbf{H}] + \mathbf{f}_T, \tag{1}$$

where the attenuation force $\mathbf{f}_T$ is [1]

$$\mathbf{f}_T = \frac{2e^4}{3m^2c^5}\left\{[\mathbf{H}[\mathbf{H}, \mathbf{v}]] - \frac{\mathbf{v}}{1 - v^2/c^2}\,\frac{1}{c^2}([\mathbf{v}, \mathbf{H}])^2\right\}. \tag{2}$$

Let us direct the axiz z of the Descartes system of coordinates

* VLIYANIYE RADIATSIONNOGO TORMOZHENIYA NA DVIZHENIYE RELYATIVISTKOY CHASTITSY V ODNORODNOM MAGNITNOM POLE

along the field and denote

$$\omega = eH / mc, \quad \delta = {}^2/_3 e^4 H^2 / m^3 c^5. \tag{3}$$

We shall express v in fractions of c and the energy E in fractions of mc^2:

$$u = v / c, \quad w = E / mc^2 = 1 / \sqrt{1 - u^2}.$$

Passing then to components and taking into account that $\mathbf{p} = E\mathbf{v} / c^2$, we shall obtain

$$\begin{aligned} du_x w / dt &= \omega u_y - \delta u_x (1 - u_z^2) w^2, \\ du_y w / dt &= -\omega u_x - \delta u_y (1 - u_z^2) w^2, \\ du_z w / dt &= -\delta u_z w^2 (u_x^2 + u_y^2). \end{aligned} \tag{4}$$

It follows from (4), first of all, that

$$u_z = \text{const.} \tag{5}$$

Taking into account (5) we obtain from (4) the equation for energy variation

$$dw / dt = -\delta [(w / w_\infty)^2 - 1], \tag{6}$$

where

$$w_\infty = 1 / \sqrt{1 - u_z^2} \tag{7}$$

is the limit value of w at $t \to \infty$.

The integral of (6) is

$$w = w_\infty \operatorname{cth} (\delta t / w_\infty + C_0). \tag{8}$$

The constant C_0 is determined from the condition $w = w_0$ at $t = 0$.

It is appropriate to express the quantity w_∞ by the angle θ between the initial velocity and the direction of the field and by the initial value of energy. Assuming in (7) $u_z = u_0 \cos\theta$, and taking into account that $w_0 = 1 / \sqrt{1 - u_0^2}$, we have

$$w_\infty = \frac{w_0}{\sqrt{\cos^2\theta + w_0^2 \sin^2\theta}}. \tag{9}$$

Taking advantage of (8), we may integrate the equations for the transverse components u_x and u_z

$$\begin{aligned} u_x &= u_\perp(0) e^{-\delta\tau} \sin (\omega\tau + \varphi_0), \\ u_y &= u_\perp(0) e^{-\delta\tau} \cos (\omega\tau + \varphi_0), \end{aligned} \tag{10}$$

where

$$u_{\perp}(0) = \sqrt{u_x^2(0) + u_y^2(0)};$$

φ_0 is the initial phase; τ is the proper time

$$\tau = \int_0^t \frac{dt}{w} = \frac{1}{\delta} \ln \frac{\mathrm{ch}\,(\delta t/w_\infty + C_0)}{\mathrm{ch}\, C_0}. \qquad (11)$$

It may be seen from (10) that the transverse velocity components damp and become zero at $t \to \infty$.

The proper time is a complex function of t, as distinct from the motion without taking into account the radiation friction, when $\tau = t / w_0$. As a consequence of that the variation of u_x and u_y with time will no longer be harmonic, which should exert an influence on the emission spectrum.

The equations obtained are substantially simplified when passing to the extreme relativistic case characterized by the correlation $w_0 \gg 1$.

Outside a narrow cone around the direction with an angle $\theta_0 \sim 1 / w_0^2$ we have from (9) $w_\infty \approx 1 / \sin\theta$. Then (8) passes into

$$\frac{1}{w} = \frac{1}{w_0} + \sin\theta\,\mathrm{th}\,(\delta t \sin\theta). \qquad (12)$$

Inasmuch as th x practically attains its limit value at a value of x slightly greater than the unity, the main energy loss will take place in a time

$$t_0 \sim 1 / \delta \sin\theta. \qquad (13)$$

In the interval $0 \leqslant t \leqslant t_0$ it is sufficient to limit oneself to the first term of the expansion of th x in series, so that

$$\frac{1}{w} = \frac{1}{w_0} + (\sin^2\theta)\,\delta t \quad (t\delta \sin\theta \ll 1). \qquad (14)$$

Hence it may be seen that the energy decreases by a factor of two in a time $\sim 1 / w_0 \delta \sin^2\theta$.

For the proper time we have in the same approximation

$$\tau = \frac{t}{w_0} + t^2 \frac{\delta}{2} \sin^2\theta. \qquad (15)$$

Substituting (15) into (10), we shall obtain the approximate expressions for the transverse velocity components.

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*** THE END ***

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