6 September 1966

STIMULUS-ORIENTED APPROACH TO DETECTION RE-EXAMINED

by

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Resulting from research done under Department of the Navy, Naval Ship Systems Command Contract NObsr-93124, Task 8103, Project Serial Number SF0010316, and under U. S. Navy Office of Naval Research Contract Nonr-3579(04), NR 124-190 on National Aeronautics and Space Administration Fund Transfer R-129

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Abstract 12233

The present paper is a re-examination of some of the conclusions of an earlier one. It is motivated by some new insights resulting from attempts to replicate experiments with human observers through the use of an electrical model of the auditory system. It is concerned primarily with the effect of signal duration on detection in the presence of a continuous masking noise. The model, of those tried, that best fits human performance consisted of a bandpass filter obtained by subtracting the output of a 500 Hz sharp-cutoff, low-pass filter from another having a cutoff of 525 Hz. The filter was followed by a linear half-wave rectifier, and it in turn by an integrator having a 100 msec decay time. The integrator can be thought of as a device which takes a running average of its input.

The probability density distributions for N and SN yielded by the model lie between the Rayleigh-Rice distributions on the one hand and a pair of normal distributions of unequal variance on the other. The exact shape of the two distributions depends upon both the bandwidth of the filter employed and the time constant of the averager.

Author

I. INTRODUCTION

In an earlier paper (Jeffress, 1964) in which I "reinvented" detection theory, I undertook to show that the probability distributions derived by Rayleigh (1878) for narrow-band noise, and by Rice (1954) for narrow-band noise plus a sinusoidal signal yielded receiver-operating-characteristics (ROC) curves appropriate to the detection performance of human observers. The development was based on the frequently made assumption of an ideal rectangular filter followed by some sort of "detector" in the radio engineer's sense of the word. The same approach had been taken earlier by Peterson, Birdsall, and Fox (1954) for the ideal detector where the phase of the signal is unspecified, and by Marill (1956) for the same case. The approach leads to the prediction that the efficiency of detection will be highest when the duration of the signal is the reciprocal of the bandwidth of the filter.

II. BANDWIDTH, DURATION, AND DETECTION

Rice employed as his parameter A/σ, where A is the amplitude of the signal ($\sqrt{2}$ times the rms signal voltage), and σ is the rms voltage of the narrow band of noise. His probability-density distributions for various values of A/σ are based on taking random samples of the envelope of a continuous noise (N) or noise plus signal (SN). Nothing is said about duration. The approach by Peterson *et al*. is based on taking 2WT samples of N and of SN, where T is the duration of the sample and W is the bandwidth of the noise. Their parameter is $(2E/N_o)^{\frac{1}{2}}$, where E is the energy in the signal (power times duration) and N_o is the energy of the noise (power per cycle). Their parameter, therefore, includes duration, but not bandwidth. Bandwidth is involved in determining the number of samples, but not in their parameter. If we equate Rice's parameter A/σ to the $(2E/N_0)^{\frac{1}{2}}$ of Peterson et al. and cancel out like quantities on the two sides of the equation, we end with the expression $1/W = T$ (Jeffress, 1965, p. 768). This is tantamount to saying, as Peterson et al. do, that the ideal detector for the phase-unknown case is a filter having a bandwidth that is the reciprocal of the signal duration.

If we attempt to apply this model (phase-unknown--narrow filter, detector) to data from psychophysical experiments involving the effect of signal duration on detection, we are immediately in trouble. Instead of achieving maximal performance at a duration that is the reciprocal of the bandwidth ("critical" bandwidth estimated from 50 Hz to more than 100 Hz for a 500 Hz signal), people show relatively uniform detection for a fairly wide range of durations (Green, Birdsall, and Tanner, 1957). They show poor performance at short durations, expecially the very short durations that would be predicted from wider estimates of the critical band (e.g., Zwicker, Flottorp, and Stevens, 1957).

III. PHASE-KNOWN CASE

Peterson et al. examined another detection situation, that for the ideal detector for the case where signal is completely specified including phase. This case led to a different family of probability-density functions--normal distributions of equal variance. The efficiency of this detector (e.g., a correlation detector) was higher than for the case where phase is unspecified. They defined a measure, d, as $(M_{sn} - M_n)^2/V_n$, where M_{sn} is the mean of the SN distribution, M_n is the mean of the

N distribution, and V_n is the variance of the N distribution (or SN since the two variances are equal). They showed that for the ideal detector, $d = 2E/N_o$. This case (signal-phase known), possibly because it is more mathematically tractable than the phase-unknown case, became the basis for much of the later application of the theory of signal detectability (TSD) to psychophysics.

In one of the earliest of such applications to human observers, Tanner and Birdsall (1958) defined two quantities, d', and η. The latter, a measure of efficiency, is the ratio of the signal energy required by the ideal detector (in a given background of noise) to the energy required by an observer (in the same noise background) to reach equal levels of detection. For the ideal detector and a completely specified signal, η is unity, and $d' = (2E/N_o)^{\frac{1}{2}}$. For the real observer, $d' = (\eta \cdot 2E/N_o)^{\frac{1}{2}}$. If the N and SN distributions employed by the observer were actually available for computation, this definition of d' would be equivalent to $(M_{sn} - M_n)/\sigma_n$. It would be $\sqrt{2}$ larger than the familiar z-score of statistics. Applied to this situation, $z = (M_{sn} - M_n)/(\sigma_{sn}^2 + \sigma_n^2)^{\frac{1}{2}}$, or, since the σ's are equal, $z = (M_{sn} - M_n)/(2\sigma_n^2)^{\frac{1}{2}}$.

The equal-variance assumption made possible the development of two convenient sets of tables relating d' to percentage of correct responses in psychophysical experiments, one for yes-no data, and one for data from n-alternative, forced-choice procedures (Elliott, 1964).

IV. RATING-SCALE EXPERIMENTS

If an observer is given the opportunity to generate more than a single point on an ROC curve, either by asking him to adopt different criteria during different sessions or to rate the stimulus according to its degree of "signal-likeness", the points often fall on a curve that could not be generated from two equal-variance normal curves with a fixed difference between the means. The subject's d' changes with his criterion. This fact implies that either the difference between the means has changed, or that the variance of one or both of the two distributions has changed. Egan, Schulman, and Greenberg (1959) interpreted the fact as an increase in the variance of the SN distribution, relative to the N distribution and found ratios of σ_n/σ_{sn} ranging from about 1.0 to about 0.7; the average of 96 ROC lines was 0.79. More recently, Watson, Rilling, and Bourbon (1964), using a 34-point rating scale found ratios from 0.92 to 0.80. In another article, Egan, Greenberg, and Schulman (1961) described the apparent differences in σ_n and σ_{sn} by the use of a power function relating the conditional probability of saying "yes" given SN to that of saying "yes" given N. The expression took the form: $P(Y|SN) = [P(Y|N)]^k$, $0 < k \leqq 1.0$.

In the earlier article, I pointed out that the phase-unknown model predicted just such a difference in variance as that found in rating-scale experiments, and presented an ROC curve obtained from the Rayleigh distribution for N, and an appropriate Rice distribution for SN which gave a satisfactory fit to rating-scale data from the experiment by Watson, *et al*. The conclusion I drew was that the Rayleigh-Rice distributions were more appropriate to human performance than the normal, equal (or even unequal) variance distributions of TSD. I also concluded that while the normal, unequal-variance distributions used to fit rating-scale data required an *ad hoc* explanation of some sort, the unequal variance followed naturally out of the phase-unknown model. But this model leaves us in the difficulty with detection *vs* duration discussed earlier, i.e., best detection should occur at short durations.

V. ELECTRICAL MODEL: EARLY EXPERIMENTS

In an attempt to determine whether, by operating on its output, a fixed-bandwidth, narrow filter could be made to replicate the results obtained with human subjects, Gaston and I (Jeffress and Gaston 1965) tried various schemes. We employed a steep-sided filter having a bandwidth of 50 Hz centered at 500 Hz. (The half-power bandwidth was 50 Hz; the equivalent rectangular width was about 55 Hz.) When followed by a precision, half-wave rectifier and an integrator with a short (1 msec) decay time, it yielded, when its envelope was sampled randomly, an almost perfect Rayleigh distribution. When sampled at the envelope peaks, it gave data which almost equally well fit the distribution function derived by Rice for envelope peaks. Figure 1 shows the two sets of data. The smooth curves are the theoretical functions, the points were measured from the envelope of the filter output. The filter showed its feet of clay, however, when we followed Green's (1966b) suggestion, employed a square-law (energy) detector and determined the mean-to-sigma ratio for it. This proved to be about $\sqrt{2}$ larger than that for an ideal rectangular filter. This disparity is in accord with the findings of Mathews and Pfafflin (1966) for the difference between an ideal filter and one of a more realizable shape.

While employing this filter in an attempt to replicate the findings of Green, Birdsall, and Tanner (1957) on the effect of signal duration on detection, we finally recognized the source of our difficulty. They used a continuous noise and a gated signal, using a four-alternative forced-choice procedure and indicating the <u>onset</u> of the signal interval by a light flash. In our experiment, we were forced either to gate both noise and signal together, or to use a continuous noise sampled randomly,

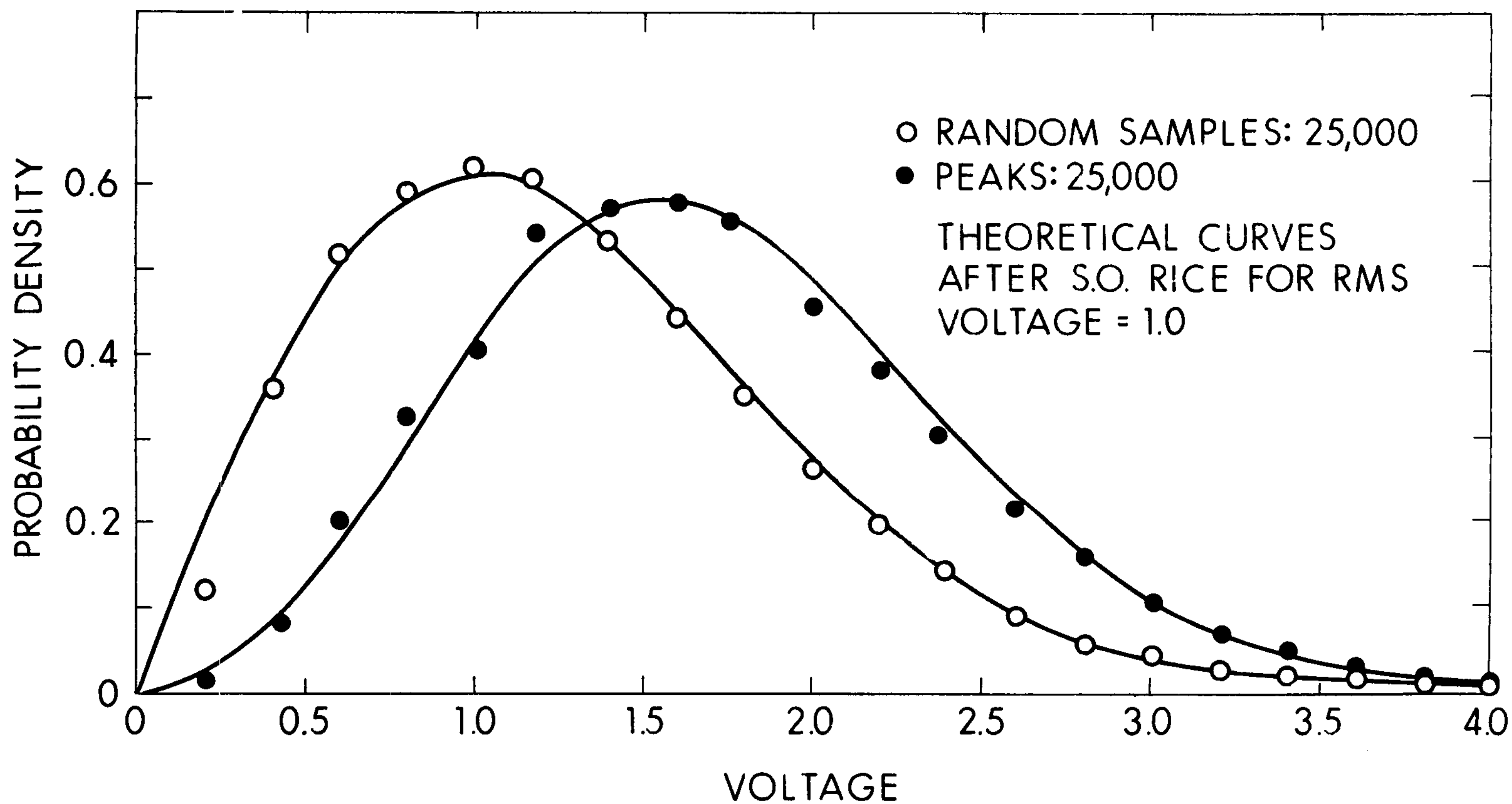

Fig. 1. Probability-density distributions for narrow-band noise.

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and a gated signal, sampled at some time during the interval. In either case we obtained our best detection when the duration was 20 msec (the reciprocal of the filter bandwidth). When both noise and signal were gated for 20 msec the efficiency of the system approached that of the ideal detector for the unknown-phase case. It was apparent that neither result resembled the psychophysical data we were attempting to fit.

Still trying to determine whether the output of a fixed-bandwidth filter could be made to yield data like those of Green et al., we followed the lead furnished by Zwislocki's work on temporal integration (1960) and increased the decay time of the integrator from the 1 msec we had been using for the envelope filter, to 200 msec. The results were promising, but appeared to indicate that our filter was too narrow. Some additional experimentation led to the procedure used in the next section.

VI. ELECTRICAL MODEL: LATER EXPERIMENTS

Deatherage (1966) reminds us that the basilar membrane serves as a series of low-pass filters, and the neurophysiologists tell us that inhibition plays as important a role in processing sensory information as excitation. Accordingly we tried combining two low-pass filters with sharp high-frequency cutoffs to form a bandpass filter. We subtracted (inhibition?) the output of one filter (500 Hz cutoff) from the output of another (525 Hz cutoff), driving the two filters in parallel. The resulting frequency response is shown in Fig. 2. Looking at the figure upside down in a mirror reminds us of the response curves for single fibers of the auditory tract reported by Galambos and Davis

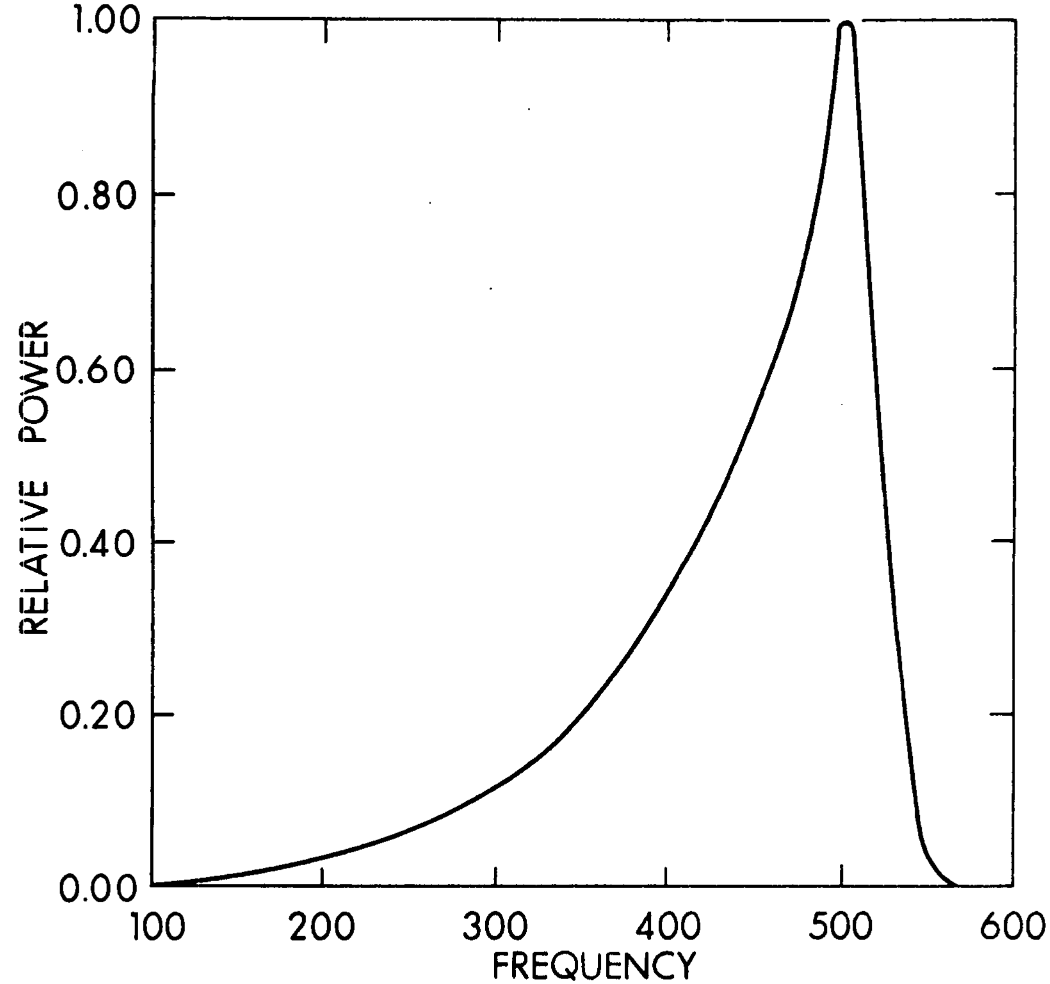

Fig. 2. Frequency response of the 525 Hz minus 500 Hz filter described in text.

(1943) and by Katsuki, Suga, and Kanno (1962). The "best frequency" for the filter was 500 Hz. Its 3 dB-down bandwidth is 78 Hz and its equivalent rectangular width is about 110 Hz.

We used this conbination (525 minus 500 Hz) filter with a linear rectifier followed by a "true" integrator with a 100 msec decay time. The integrator was an operational amplifier, and without the decay resistor gives a true integral of the input. Adding the decay resistor makes it into a running averager similar to a conventional RC integrator, except that its capacitor discharges to ground rather than back into the source. This seemed more "nervelike" since synapses leak off to their surrounding medium, not back to the axons that feed them. The choice of 100 msec was based on several preliminary experiments with other values.

Figure 3 shows an oscilloscope photograph of the noise input to the filter system (lower trace), the signal (50 msec, 500 Hz in this photograph, second trace), the filter output (third trace) and the output of the integrator or "running averager" (top trace). The moment at which the magnitude of the top trace was measured is the moment when the traces are brightened.

A quasi block diagram of the equipment is shown in Fig. 4, with the actual circuits shown where informative. The "time-constant" of the integrator, 100 msec, is the product of the decay resistance 100 kΩ and the capacitance, 1μF. The 20 kΩ input resistor functions merely as a scale factor in the "true" integrator. The actual time required by the integrator to reach a steady state with a constant voltage input is given by Exp -[t/RC], which means that in 500 msec the voltage is within 1% (0.007) of its final value. The interval between samples was therefore set at 0.5 sec.

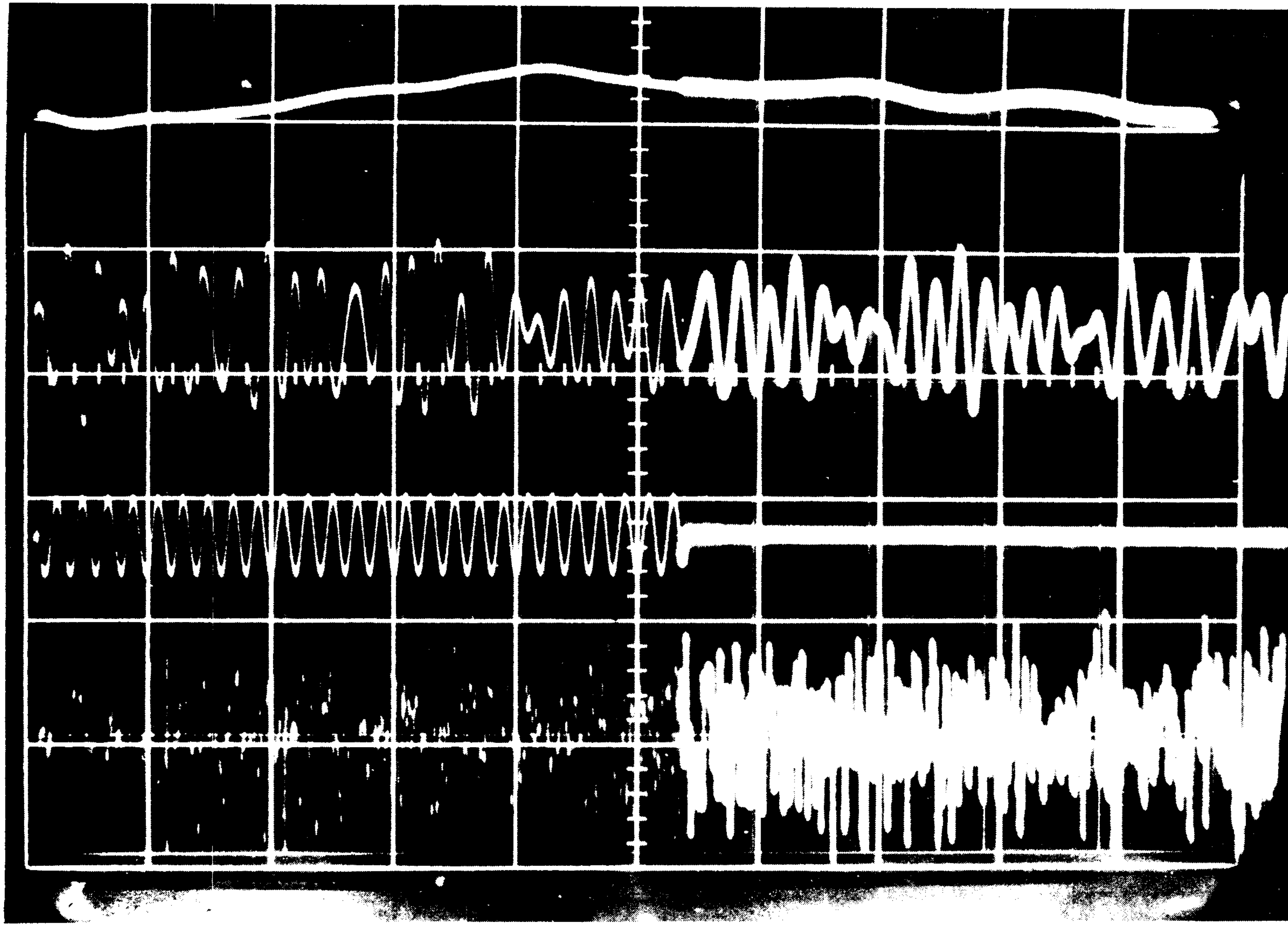

Fig. 3. Scope photograph; bottom trace is wide-band noise input to system, second trace is signal, third trace is output of narrow filter, top trace is output of running averager. Sampling is done at the moment when traces are brightened.

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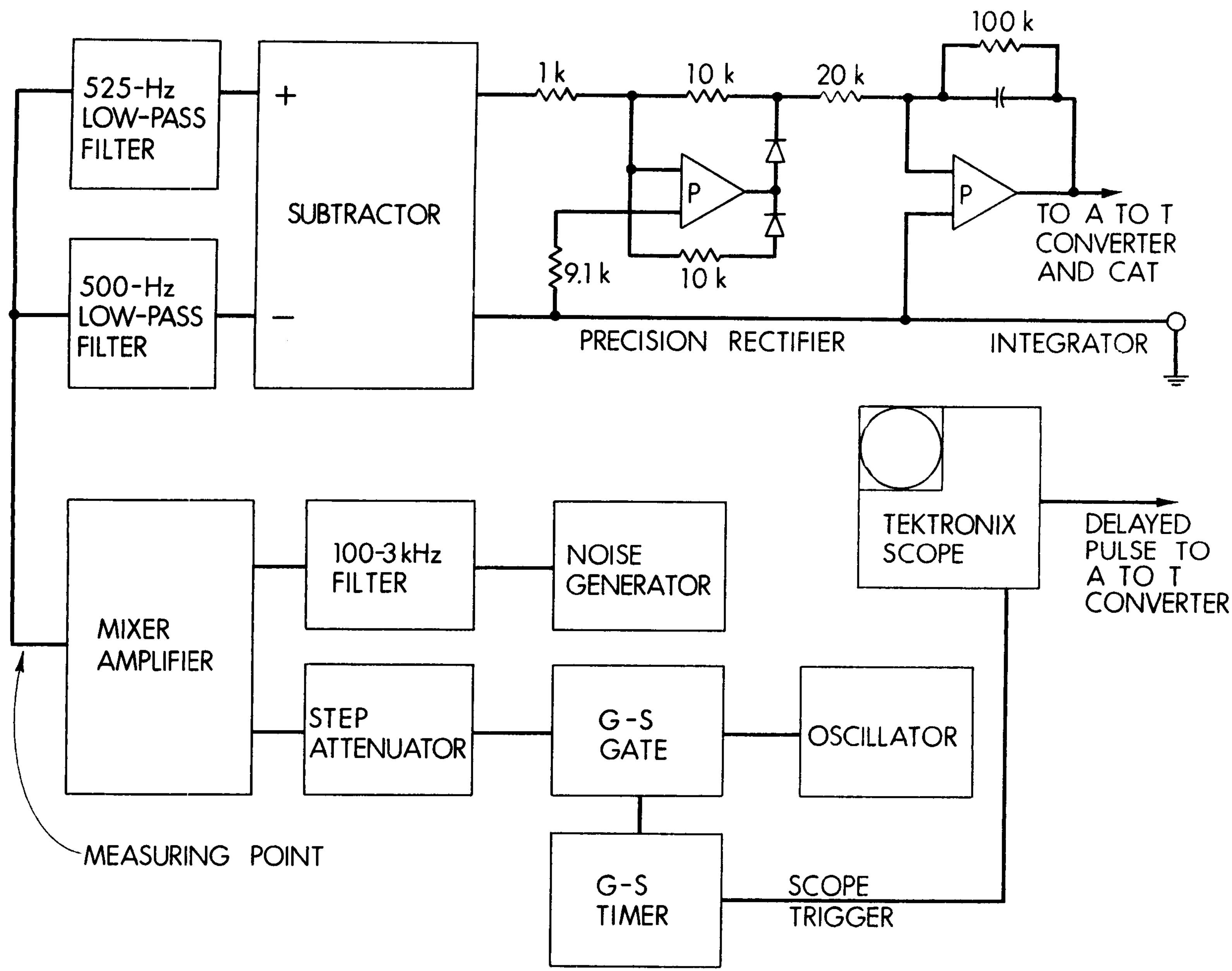

Fig. 4. Block diagram of recording equipment.

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The basic timing was done by a Grason-Stadler timer. It determined the duration of the signal (if one was present) and the interstimulus interval. A pulse from it triggered the Tektronix scope, and a pulse from the delayed-brightening circuit of the scope served to indicate to the amplitude-to-time converter of the computer of average transients (Technical Measurement Corporation CAT Model 400 C) the moment of measurement. The measurement was made at the termination of the signal.

The results of the measurements (usually 1,000 samples of N or SN) were read out of the CAT memory onto printed tape, punched on cards, and processed by a CDC 3200 computer. The data appeared in the form of probability-density distributions (more properly frequency distributions) and from them were computed means, standard deviations, measures of skewness and kurtosis, and a measure of detection based on the z-score for the particular combination of N and SN.

VII. ANOTHER DETECTION MEASURE: d_z

There are several reasons why conventional measures of detection are inappropriate in dealing with the distributions generated by the electrical model. The d_s of Egan et al. (1959) and the d_e of Clarke, Birdsall, and Tanner (1959) use only one of the points along an ROC curve. Since we have the complete distributions we would like a measure based on all of the information they contain. Green's (1964) observation that the area under the ROC curve is P(C) for a two-interval, forced-choice experiment would be appropriate except that we wish occasionally to employ signals that yield d's so high that determining the area with precision is impossible. At the same time, since we wish to fit data expressed as d', we need a d'-like measure, and for the reasons given, we need one which can be computed from the two distribution functions.

For the equal-variance normal distributions $d' = (M_{sn} - M_n)/\sigma_n = z\sqrt{2}$, as we have seen earlier. It would seem a logical step therefore to apply a similar measure to the case where the distributions are normal but unequal in variance. To avoid confusion let us call this measure d_z, since it is derived from the z-score. We will therefore define d_z as $z\sqrt{2}$. Thus $d_z = \sqrt{2}(M_{sn} - M_n)/(\sigma_{sn}^2 + \sigma_n^2)^{\frac{1}{2}}$. Let us see how closely this resembles d' under various conditions. Obviously, for the equal-variance, normal distributions the two measures are identical.

For the case where the two distributions are normal but unequal in variance, we can plot an ROC curve from the data, determine the area under the curve, and hence, as Green has shown, the equivalent P(C) for a two-alternative, forced-choice experiment. If we look up this P(C) in Elliott's table for 2AFC and find the corresponding d', we discover it to be again identical to our d_z. This is not surprising; the central limit theorum implies that since our original distributions were normal, their differences will be normally distributed.

Let us take a more extreme case, that for the Rayleigh distribution and the Rice distribution for $A/\sigma = 2.5$ and again plot an ROC curve and determine its area. We find P(C) = 89.5% which gives us a d' of 1.77. If we take the mean of the Rayleigh distribution to be 10.0, the mean of the Rice distribution proves to be (using the computer) 21.56. The corresponding standard deviations are 5.18 and 7.57. This yields $z = 1.26$ and $d_z = 1.78$.[1] Since the distributions with which we will

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[1] The value, $A/\sigma = 2.5$ was selected as the worst case for these distributions because it appeared to give about the least normal-looking ROC line and a small slope (0.85). Two, four-round-trip measurements of the area under the curve were made with a planimeter--the second because the agreement in d' appeared to be too good to be true. Both measurements yielded P(C) = 89.5%. The skewness index for the Rayleigh distribution was substantial (0.62) and for the Rice was negligible (0.10). The kurtosis in both distributions was negligible.

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be dealing here range between the Rayleigh-Rice distributions at one extreme, to the normal at the other, we may safely employ d_z as equivalent to d' and employ the d' tables with it. The computer program included the determination of d_z for the various values of N and SN employed with the electrical model.

VIII. PSYCHOMETRIC FUNCTIONS

Figure 5 presents a series of psychometric functions obtained from the electrical model, plotted as d' (d_z) against $(2E/N_o)^{\frac{1}{2}}$. All appear to be straight lines over the range shown, and all would bend into the origin at low values of the signal. The lines drawn are least-squares fits to the data.

To show that these findings are not trivial as they apply to human detection experiments, Fig. 6 presents several additional psychometric functions employing the same coordinates. The top line is for the ideal detector for the phase-known, equal-variance case. The second line is for the ideal detector, phase unknown (Rayleigh-Rice distributions). The next three lines are a replotting of data from an experiment by Green (1966a) for monaural noise and signal.[2] Three signal durations at 250 Hz were used, 1.0 sec. 0.1 sec, and 0.01 sec. The solid points are data from an experiment by Gaston (1964) who used a 500 Hz signal of 0.15 sec duration. Like the lines of Fig. 5, these appear to be straight over much of their course and bend to the origin at low signal levels.

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[2] Green's data were plotted as P(C) against 10 log E/N_o. The resulting ogives do not appear to me to be as informative as the straight lines (with a bend to the origin) of Fig. 6.

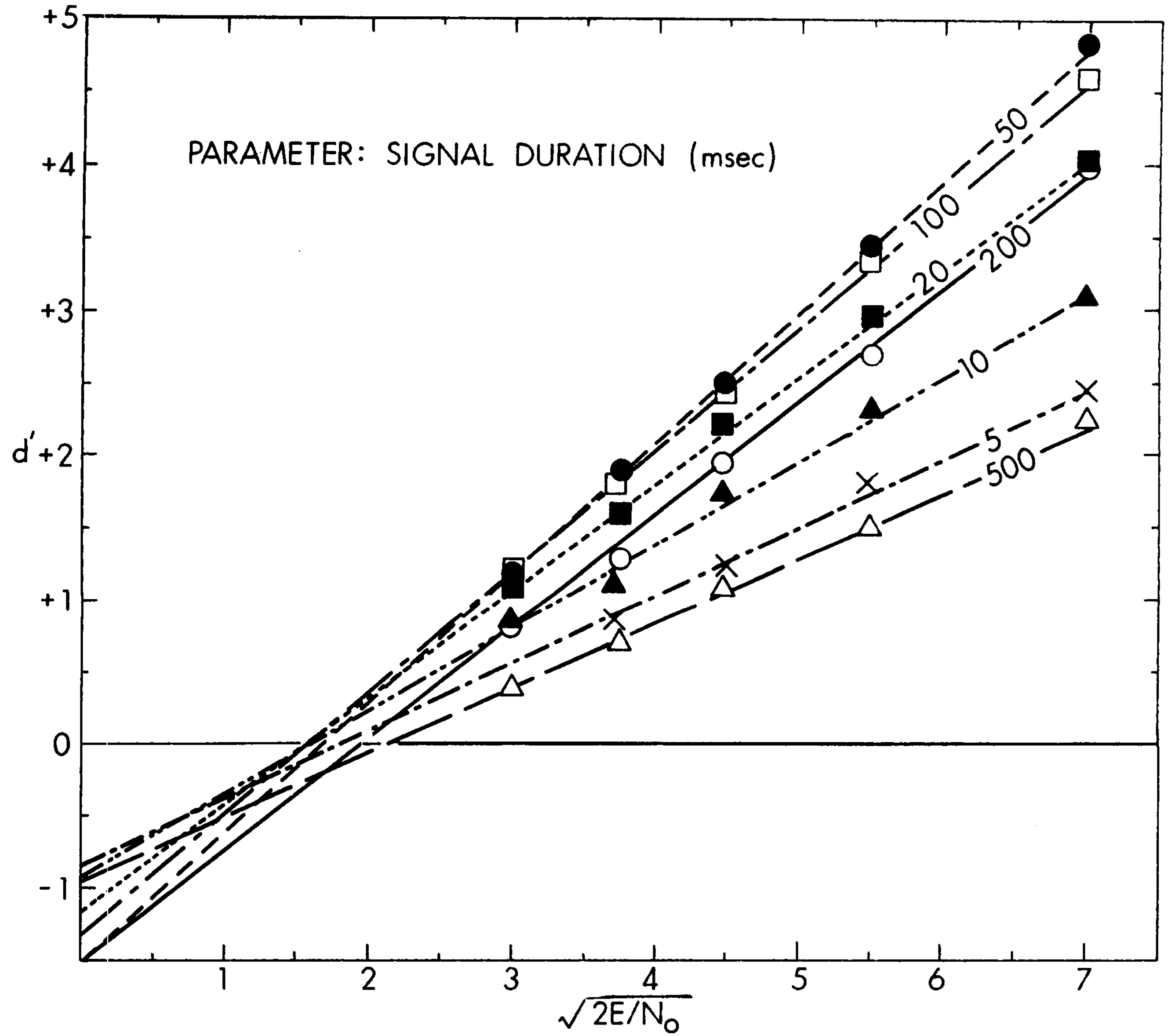

Fig. 5. Psychometric functions for ear model. The parameter is signal duration.

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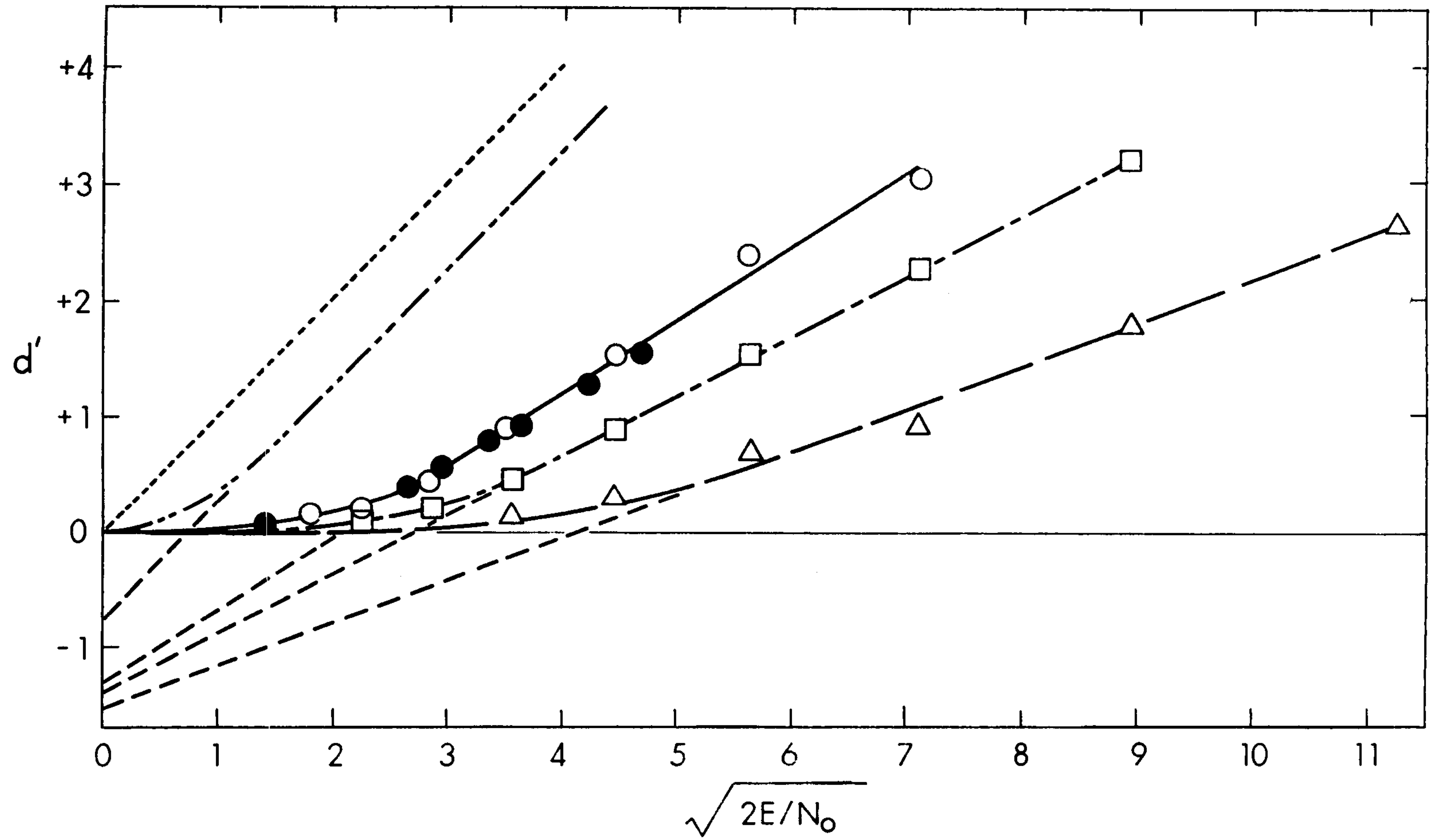

Fig. 6. Psychometric functions for ideal detectors and human observers. (See text for discussion.)

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IX. DETECTION VS DURATION: MODELS AND PEOPLE

Figure 7 presents the data from the experiment by Green et al., and data from the model for the same durations. Green et al. employed $(2E/N_o)^{\frac{1}{2}} = 5.8$; for the model we used 3.7. Our device was therefore about 2.5 times as efficient (energy ratio) as the human observers. In one series of experiments, we employed the linear, half-wave detector described earlier. In another series, the detector was a square-wave (energy) detector, obtained through the use of a "true" rms vacuum-tube voltmeter. The two detectors gave reasonably similar results, and both fit the average data of the subjects of the psychophysical study better than the subjects agreed with one another.

X. GATED NOISE AND SIGNAL

Because the mathematical foundation of TSD is based on taking similar samples of N and SN, we performed one further experiment in which we gated both the noise and the signal, using the same value of $(2E/N_o)^{\frac{1}{2}}$ as before. The results are shown as the solid circles of Fig. 7. At 500 msec, the performance was inferior to the other data, but at short durations was much better. The fact that the data do not show a single sharp peak at the reciprocal of the bandwidth of the filter is probably owing to the odd shape of the filter. In an earlier experiment, the 50 Hz filter did show a peak at 20 msec. At optimal durations both filters approached the efficiency of the ideal detector (phase unknown).

A psychophysical study comparing detection for gated N and SN, as well as for continuous noise and gated signal, has recently been completed by Tucker and Evans (1966). The results, while not showing at short durations as striking an improvement for gated noise as shown in Fig. 7,

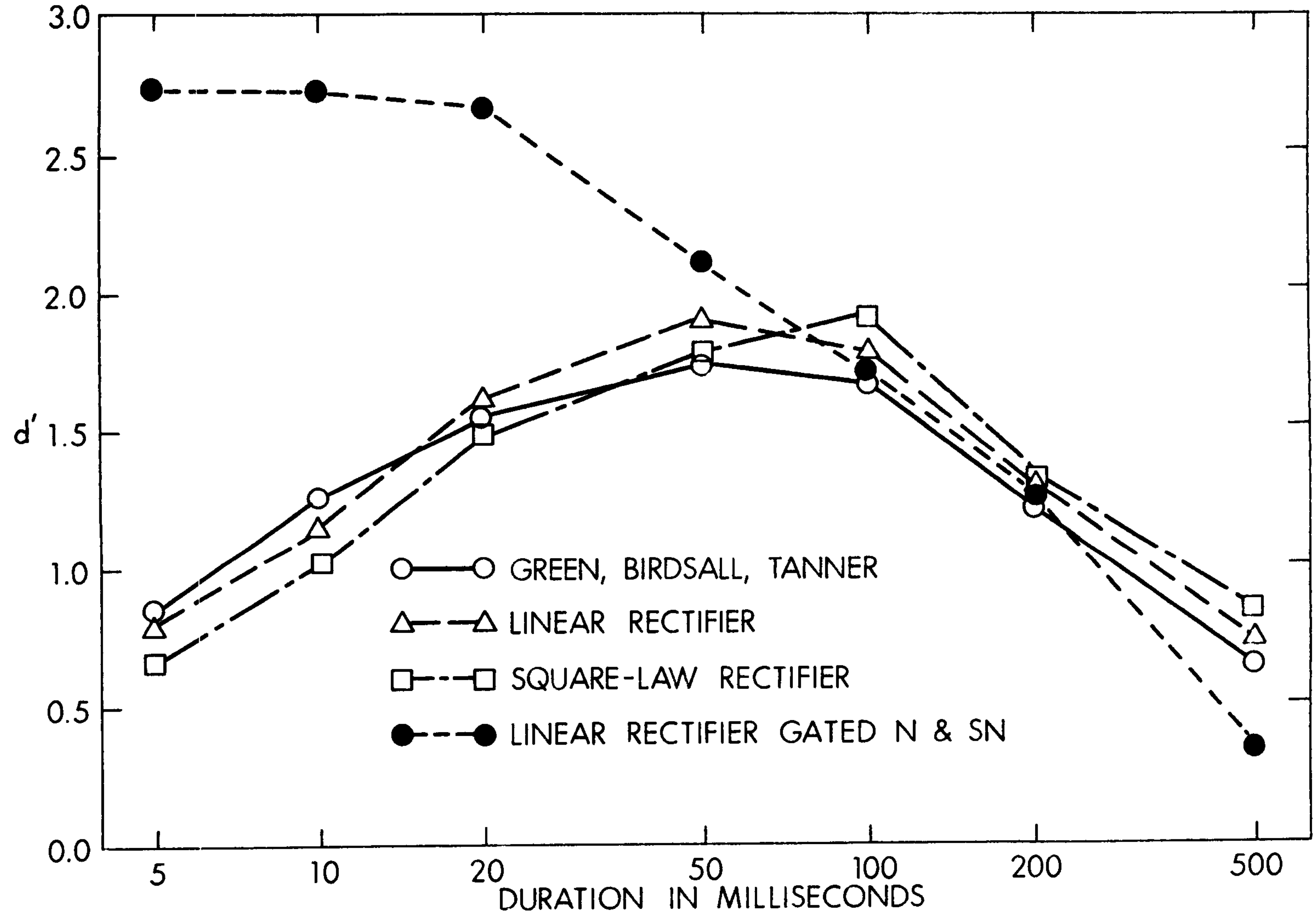

Fig. 7. Detection vs duration for electrical model and human observers. (See text for discussion.)

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do show a substantial improvement. For the human observers, detection falls off sharply between 20 msec and 10 msec. This fact suggests that people either employ a narrower filter (or a differently shaped one) than that used in the model or that they find the short durations difficult for some reason not accounted for in our models.

XI. DISTRIBUTION FUNCTIONS

The probability distributions of voltage obtained using the combination filter (525 Hz minus 500 Hz) with the running averager proved to be considerably more nearly normal than the Rayleigh-Rice distributions, or than the distributions obtained with the 50 Hz filter when employed as an envelope detector (short decay time and random samples of N and of SN). Several parameters of these distributions are presented in Table I for continuous noise and a 100 msec signal.

One of the surprising facts apparent in Table I is that σ_{sn} increases steadily with signal level. This means that the slope of the corresponding ROC lines on normal-normal coordinate paper would decrease monotonically with an increase of signal. This is contrary to the results published in Table II of the earlier paper which showed the slope decreasing with signal level to an A/σ of about 2.5 and then increasing back toward unity. A re-examination of the earlier table reveals the source of the disagreement. The table was based on graphical plotting of ROC lines on normal-normal paper. For high signal levels there is not much paper left, and measurements of slope are imprecise. If we base our reasoning on computation instead of graphical methods we arrive at a different finding.

Rice has shown that as the value of A/σ is increased, the standard deviation of the SN distribution approaches σ, the rms noise voltage out

TABLE I

Data for 100 msec Signal and Continuous Noise

525 Minus 500 Hz Filter

$(2E/N_o)^{\frac{1}{2}}$	Mean	σ	M/σ	σ_n/σ_{sn}	Skewness	Kurtosis	z	d_z
				Linear Detector				
Noise	10.00	0.82	12.25	--	0.22	0.03	--	--
3.00	11.16	1.04	10.78	0.79	0.14	-0.11	0.87	1.23
3.72	11.73	1.10	10.62	0.75	0.20	0.03	1.27	1.80
4.47	12.48	1.17	10.65	0.70	0.12	-0.23	1.74	2.46
5.50	13.57	1.27	10.72	0.65	0.12	-0.01	2.38	3.36
7.00	15.42	1.44	10.72	0.57	0.08	0.00	3.27	4.62
				Square-law Detector				
Noise	10.00	1.39	7.21	--	0.48	0.35	--	--
3.72	13.58	2.26	6.00	0.62	0.46	0.52	1.35	1.91

of the filter. If we derive the mode, mean, and standard deviation of the Rayleigh distribution we find that the mode equals σ (rms noise voltage), the mean of the distribution equals $\sigma\sqrt{\pi/2}$ and the standard deviation equals $\sigma\sqrt{2 - \pi/2}$. Since the standard deviation of the SN distribution approaches σ; the ratio, σ_n/σ_{sn} for the Rayleigh-Rice distributions approaches, not unity, as suggested in the earlier article, but $(\sqrt{2 - \pi/2})/1 = 0.65$. An experimental check, using the 50 Hz filter and sampling the N and SN envelopes randomly, yielded $\sigma_n/\sigma_{sn} = 0.64$, for a value of $A/\sigma = 4.5$. The table in the earlier article incorrectly reported a slope of 0.92 for this signal level.

Green and Swets (1966) in their discussion of an energy detector model have shown that for the ideal rectangular filter, the M/σ ratio for a gaussian noise input to the filter is $(WT)^{\frac{1}{2}}$. [3] Mathews and

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[3] Green and Swets (1966) have shown that with a gaussian noise input and an ideal rectangular filter, a square-law (energy) detector will yield a Chi-square distribution. If 2WT samples of noise are taken, the Chi-square distribution will have 2WT degrees of freedom. Since the mean of the Chi-square distribution is the number of degrees of freedom and the variance is twice the mean, the mean to sigma ratio will be $(df/2)^{\frac{1}{2}}$, and for the ideal filter $(WT)^{\frac{1}{2}}$.

Pfafflin (1965) in a different derivation arrived at the same value for a rectangular filter but find that for a differently shaped filter the ratio is $\sqrt{2}$ larger.

For the running averager, the value of T is somewhat uncertain since the decay is exponential, and for the filter of Table I the value of W is also difficult to specify. We therefore employed the 50 Hz steep-sided filter in a similar pair of measurements using both a square-law and a linear detector. The M/σ ratios were 7.70 for the linear detector and 5.45 for the square law.

Assuming with Mathews and Pfafflin that our M/σ ratio for the square-law detector, is $\sqrt{2}$ times that for the ideal filter, and knowing the equivalent rectangular bandwidth of the filter, 55 Hz, we can solve for T. This proves to be 0.27. We can now use this value of T to determine W for the filter of Table I. It is 96 Hz, a value intermediate between the 3 dB bandwidth, 78 Hz and the equivalent rectangular width, 110 Hz.

XII. BANDWIDTH, AVERAGING TIME, AND DISTRIBUTION FUNCTIONS

The near-normal distributions of Table I and the fact that the 50 Hz filter with a short averaging time (envelope detector) yields functions closely resembling those of Rayleigh and Rice, raise the question whether it is the bandwidth or the averaging time that is responsible for the differences. Accordingly we sampled noise distributions from several filters, employing three averaging times 50, 100, and 200 msec. The half-power bandwidths of the filters were 12.5 Hz (single-tuned), 25 Hz, 50 Hz (the filter employed in several of the previous experiments), 100 Hz, 128 Hz (a very steep-sided filter), and 150 Hz. The results are presented in Table II. The standard deviations are adjusted to a mean of 10.00.

Examination of the table reveals a consistent trend toward the normal as the bandwidth and the averaging time are increased. The narrowest filter with the shortest averaging time has a skewness about

TABLE II

	Skewness			Kurtosis			S. D.			M/σ Ratio		
	T = 50	100	200	50	100	200	50	100	200	50	100	200
Bandwidth												
12.5	0.76	0.70	0.58	0.76	0.44	0.44	2.46	1.98	1.44	4.06	5.04	6.96
25	0.47	0.50	0.39	0.43	0.50	0.16	2.49	1.95	1.53	4.01	5.12	7.02
50	0.46	0.38	0.34	0.06	0.30	0.03	1.94	1.38	0.98	5.15	7.24	10.24
100	0.32	0.27	0.13	-.05	-.18	-.16	1.31	0.98	0.73	7.63	10.24	13.74
128	0.24	0.25	0.18	-.08	-.00	-.09	1.36	0.93	0.72	7.35	10.78	13.88
150	0.29	0.22	0.07	-.06	0.14	0.06	1.12	0.81	0.57	8.96	12.37	17.47

that of the Rayleigh distribution (0.62 for the Rayleigh), but is considerable more leptokurtic. The kurtosis for the Rayleigh distribution is 0.10. The 50 Hz filter is intermediate between the Rayleigh and the normal, and the wider filters approach more closely to the normal.

The standard deviations also diminish, as we would expect them to, as we increase either the bandwidth or the averaging time. This means that the M/σ ratio grows larger as either bandwidth or averaging time is increased. This too is to be expected, since the M/σ ratio is some function of $(WT)^{\frac{1}{2}}$ and this will, or course, increase with an increase in either W or T.

XIII. DISCUSSION

There is always the danger in employing models, whether physical or mathematical, of identifying the model with the thing modeled. I hope we can avoid doing so in the present context. Obviously our model is not the ear. The ear does its filtering mechanically and probably neurally, using both excitation and inhibition in the process.[4]

[4] I am using "ear" to stand for the auditory system involved in the detection process, not just for the peripheral organ.

Furthermore, the ear is a multitude of filters which can apparently be combined in a variety of ways, as the data on frequency uncertainty suggest. [See Tanner, Swets, and Green (1956), Green (1958), Creelman

(1960), and Swets (1963).] The present model was devised to detect a single frequency, primarily to answer the question: can a single fixed-frequency filter yield data like those of the experiment by Green et al., or is it necessary to postulate (as I did in the earlier paper) a bandwidth which is adjusted to the duration of the signal? The answer is clearly that for detecting various durations of a single frequency, a single filter is adequate when followed by a running-average device. This does not mean, however, that ten such filters could detect ten different frequencies in a frequency uncertainty experiment as well as the human observer can. Combining the outputs of ten filters in an OR gate, for example, would have the effect of multiplying the false-alarm rate by ten for a given level of detection [$P(Y|SN)$]. The ear must use a different method of combining filters, since for it ten frequencies are not appreciably worse than three (Bourbon 1966).

The particular filter used in the electrical model is probably not wholly appropriate. The data of the Tucker and Evans experiment suggest a narrower bandwidth or steeper skirts for the ear's filter than those employed in the electrical model. They found poorer detection for 10 msec than for 20 msec and 50 msec for gated N and SN, whereas the model showed about uniform detection over this range.

The half-wave rectifier does seem appropriate, since in the ear stimulation appears to occur during the rarefaction half-cycle. The fact that the fibers with which we are probably concerned fire at a rate which is less than the frequency of the stimulus suggests that some integration is necessary before the fiber can fire--for a weak stimulus several cycles of stimulus are required to achieve firing, for stronger stimuli, fewer (Moushegian, 1964).

The remainder of the integration and averaging is probably performed by centers higher than the cochlea. This appears to follow from the fact that while integration continues for durations up to several hundred msec, there are no latencies of this order to be found in the auditory nerve.

I believe that the model has served to point out some weaknesses in our mathematical theories as they apply to audition. The principal one is evident when we employ continuous noise with a gated signal. A basic condition of TSD is that N and SN be sampled in the same way. With continuous noise, we are asking the subject to gate himself in the same way as the experimenter gates the signal. Even when we signal the onset and termination of the stimulus interval (by a light), he is unable to do so. The interval was so signaled in the Tucker-Evans experiment, but detection for continuous noise was (for short durations) considerably poorer than for gated noise, though contrary to this for longer durations. The slightly poorer detection for gated N and SN at long durations appears to be a property of both the model and people. For gated noise the conditions of TSD are more nearly satisfied, and the expected peak at the reciprocal of the filter bandwidth appears to occur.

The second weakness of our mathematical models lies in the nature of the N and SN distribution functions they assume. Neither the normal distributions commonly employed, nor the Rayleigh and Rice distributions (for the phase-unknown ideal detector) are completely appropriate. If we assume that the ear employs some sort of running average of its input, and all of the psychophysical data we have examined support this hypothesis, we are forced to the conclusion that the distribution functions lie somewhere between the Rayleigh-Rice and the normal. We need no _ad hoc_ explanation of the difference of variance between the N and SN distributions, and can also account for the fact that the ROC curves from rating-scale data do not quite fit the normal, unequal variance model. The fact that they more nearly fit the Rayleigh-Rice model than the normal again suggests that the ear's filter is narrower than that employed in the model. The narrower the filter, the more skewed the N distribution will be. Unfortunately the ROC curve is not a sufficiently sensitive indicator to permit distinguishing definitely between the subtly different distribution shapes with which we are concerned.

The mathematical approach taken by McGill (1966) appears to hold promise. He proposes a Poisson counting process where the events counted are nerve impulses and for any particular neuron follow a Poisson distribution. In its present stage of development, his model appears to have the same weakness as the TSD model, in being unable to cope with continuous noise and gated signal, but the addition of some kind of leak appears possible. A bin that would drop counts at a rate which is a function of the number of counts in the bin would be functionally similar to our leaky integrator, and more nearly like what we can imagine the nervous system doing. If the leak also followed a Poisson process, probably the mathematics might not be so difficult as in the TSD model for the kind of distribution functions with which we appear to be concerned.

Probably the most important inference to be drawn from the experiments reported here (including those of Green, Birdsall, and Tanner) is that the nervous system appears to employ a running average of the filtered stimulus in its processing of auditory information. The conclusion is certainly in agreement with Zwislocki's findings about temporal integration in hearing, and is not outrageous physiologically. Licklider (1951 and 1955) implies a similar model for the binaural system when he speaks of a device which takes a "running correlation" of its inputs. At present no theory of binaural release from masking employs such a model, and none can, at the present time, account for the effect of signal duration on the detection process.

XIV. SUMMARY AND CONCLUSIONS

The present paper describes several versions of an electrical model of part of the auditory system. The model consists of a narrow filter followed by a half-wave rectifier (in some cases a square-law), and it in turn, by an integrator having a long decay time. The integrator acts as a running averager, and with a constant voltage input reaches a constant voltage at its output. The electrical model was able to generate data very similar to those of an experiment by Green, Birdsall, and Tanner where a continuous noise and a gated signal of constant energy was employed. The two sets of data agree over a range of signal durations from 10 msec to 500 msec. The model further predicted that gated noise and signal should show better detection at short durations than continuous noise and gated signal. A psychophysical experiment to test this prediction found it to be supported.

From the psychophysical experiments described, and from the experiments with the model, several conclusions can be drawn:

1. In the detection of a single frequency (gated) in a continuous noise background, the auditory system performs as if it were taking a running average of the output of a rectifier following a narrow filter.

2. While the rectifier employed by the ear is probably a half-wave device, the use of a square-law (energy) detector in mathematical developments does not introduce any serious error.

3. The prediction based on the electrical model, that detection should be better at short durations for gated N and SN than for continuous noise, is supported by psychophysical findings.

4. The probability density functions yielded by the model (and by inference those employed by human observers) lie between the normal and

those of Rayleigh and Rice. The SN distribution has more variance than the N distribution, and the variance increases with an increase in the signal level. Whether this variance approaches an asymptote, as is the case for the Rice distributions, is not known at the present time.

5. The running averager is similar in its time constant to the integration time of the ear proposed by Zwislocki. In the monaural system it plays a role like that of the "running correlator" proposed by Licklider for the binaural system.

ACKNOWLEDGMENTS

This work was supported by the U. S. Navy Bureau of Ships and under a grant from the National Aeronautics and Space Administration. I wish to express my gratitude to Audley D. Gaston, Jr., who helped greatly with the electronics of the model, and to both Gaston and Thomas L. Nichols for writing computer programs to do all my hard work for me. I also wish to thank Charles S. Watson and Wilson P. Tanner, Jr., for many helpful suggestions during the preparation of the manuscript.

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