SM. GIHRF

National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS-5-9299

ST - PF - CR - SP
-10511-

COMPTON EFFECT ON RELATIVISTIC ELECTRONS IN THE ATMOSPHERE OF THE SUN

by

A. A. Korchak
Yu. B. Ponomarenko

(USSR)

29 AUGUST 1966

ST-PF-CR-SP-10511

COMPTON-EFFECT ON RELATIVISTIC ELECTRONS IN THE ATMOSPHERE OF THE SUN *

Geomagnetizm i Aeronomiya
Tom 4, vyp. 3, 417-423
Izdatel'stvo "NAUKA", 1966

by A. A. Korchak &
Yu. B. Ponomarenko

SUMMARY

12241

The Compton-effect is considered of thermal of photons on relativistic electrons in the atmosphere of the Sun. The calculation of the cross section and power of emission for hard photons is performed for two particular cases: of isotropic distribution of thermal photons and at their radial propagation. The form is investigated of the frequency spectrum of Compton photons for the monoenergetic and the power-law spectrum of relativistic electrons. It is shown that for a power-law energetic spectrum of electrons with the exponent $\varkappa$ the emission power increases with the rise of the heliographic longitude of the flare θ according to the law $(\sin\theta/2)^{\varkappa+1}$.

Author

*
* *

The Compton-effect on relativistic electrons ("inverse Compton-effect") in the Galaxy and near the Sun was considered in the works [1, 2] in connection with the question as to whether the galactic cosmic rays may have an electron component. However, upon the recent detection of the electron component of galactic and solar cosmic rays [3, 4] the interest toward the possible role of the Compton-effect in cosmic conditions has risen considerably. Thus, for example, it is shown in [5, 6] that the Compton-effect of thermal photons on the electron component of galactic and metagalactic cosmic rays may contribute to the observed intensity of the isotropic background of γ- rays. The possible role of the "inverse" Compton-effect in the generation of X-ray and γ-ray emission during solar flares is discussed in the works [7—11]. Finally, the works [12, 13] point to the possible role of the Compton-effect in "radiostars".

In connection with the above it is of interest to find practical formulas for the spectrum and the intensity of X-ray and γ-ray emissions occurring at Compton scattering of thermal photons on relativistic electrons in cosmic conditions, and in particular in the Sun's atmosphere.

* O KOMPTON EFFEKTE NA RELATIVISTSKIKH ELEKTRONAKH V ATMOSFERE SOLNTSA

As in [5] we shall make use for calculations the following expression for the Compton cross section in the general invariant form

$$d\sigma_k(\varepsilon, E_\gamma, E, \nu_1, \nu_2, \nu) = 2r_e^2\left(\frac{E}{mc^2}\right)^2\frac{1}{\varkappa_1^2}\left\{4\left(\frac{1}{\varkappa_1}+\frac{1}{\varkappa_2}\right)^2 - 4\left(\frac{1}{\varkappa_1}+\frac{1}{\varkappa_2}\right)+ \right. \tag{1}$$

$$\left. + \left(\frac{\varkappa_1}{\varkappa_2}+\frac{\varkappa_2}{\varkappa_1}\right)\right\} d\Omega_2 = \frac{1}{2}r_e^2\gamma^2\left(\frac{E_\gamma}{\varepsilon}\right)^2\frac{1}{\nu_1^2}A\, d\Omega_2,$$

where

$$A = \gamma^2\frac{\nu}{\nu_1\nu_2}\left[\gamma^2\frac{\nu}{\nu_1\nu_2} - 2\right] + \frac{\nu_1 - \alpha\nu}{\nu_1} + \frac{\nu_1}{\nu_1 - \alpha\nu}, \quad \gamma = \frac{mc^2}{E}, \quad \alpha = \frac{E_\gamma}{E}; \; \varkappa_1 = -\frac{2\varepsilon E\nu_1}{(mc^2)^2}, \quad \varkappa_2 = \frac{2EE_\gamma\nu_2}{(mc^2)^2}.$$

At the same time the energy of the scattered photon is

$$E_\gamma = \varepsilon\nu_1 \Big/ \left(\nu_2 + \frac{\varepsilon}{E}\nu\right) = \Phi(\varepsilon, E, \nu_1, \nu_2, \nu). \tag{2}$$

The following denotations are used un the expressions (1) and (2):

$$\nu_{1,2} = 1 - \beta\cos\theta_{1,2}, \qquad \nu = 1 - \cos\theta;$$

ε and E are respectively the energies of a photon and electron prior to scattering; $r_e = e^2/mc^2$; $\underline{e}$ and $\underline{m}$ are the charge and the mass of the electron; θ is the angle between the photon pulses k_1 and k_2 prior and after scattering; θ_1 and θ_2 are the angles between these pulses and the initial electron pulse p_1 ; Ω_1 and Ω_2 are the corresponding solid angles.

We shall consider that the relativistic electrons and the thermal photons are distributed isotropically and that the distribution of photons by energies is Planck. The effective cross section $\sigma(E_\gamma, E)$ for the formation of photons with energy E_γ at scattering of thermal photons with Planck distribution on electrons with energy E will then have the form

$$\sigma(E_\gamma, E) = \int \nu_1(\sigma_k\, d\Omega_2)\left(\frac{n}{n_0}d\varepsilon\frac{d\Omega_1}{4\pi}\right)\frac{d\Omega_3}{4\pi}\delta(E_\gamma - \Phi), \tag{3}$$

where $\sigma_k d\Omega_2$ is the Compton cross section (1); $d\Omega_3$ is the solid angle characterizing the direction of the initial electron pulse;

$$n = \frac{n_0}{aT^3}\frac{\varepsilon^2}{e^{\varepsilon/T} - 1} \qquad (a = 2.404)$$

is the Planck distribution function of thermal photons with temperature T (in energetic units).

In the integral (3) we shall count the angles $\Omega_{1,2}$ from the direction of photon pulse k_1 and the angle Ω_3 — from a certain arbitrary direction. At the same time

$$\cos\theta = \cos\theta_2\cos\theta_1 + \cos\varphi\sin\theta_2\sin\theta_1,$$
$$d\Omega_2 = \sin\theta_2 d\theta_2 d\varphi, \quad d\Omega_1 = \sin\theta_1 d\theta_1 d\varphi_0, \tag{4}$$

where φ is the angle between the planes $(\mathbf{k}_1\mathbf{p}_1)$ and $(\mathbf{k}_2\mathbf{p}_1)$, while the origin of the count φ_0 is arbitrary. Since in (3) the integrand does not depend on Ω_3 or φ_0, when integrating over these angles and then utilizing the equality

$$(e^{\varepsilon/T}-1)^{-1}=\sum_{k=1}^{\infty} e^{-k\varepsilon/T}$$

and the property of the δ-function, we shall obtain

$$\sigma(E_\gamma, E)=\frac{r_e^2}{4a}\,\frac{\gamma^2}{T}\sum_{k=1}^{\infty} k^{-2} I(T/k), \tag{5}$$

$$I(T)=\int_0^{2\pi} d\varphi \int \sin\theta_2\, d\theta_2 \sin\theta_1\, d\theta_1 \frac{A}{\nu_2}\left(\frac{\varepsilon}{T}\right)^2 e^{-\varepsilon/T}, \tag{6}$$

where $\varepsilon = E_\gamma \nu_2 / (\nu_1 - a\nu)$, and the quantity A is determined by expression (1). In the integral (6) the region of integration over θ_1 and θ_2 is determined by the conditions

$$0 \leqslant \theta_{1,2} \leqslant \pi, \qquad \varepsilon > 0, \tag{7}$$

the second condition being equivalent to the condition $E_\gamma < (\nu_1/\nu)E$, obtained at integration of the δ-function in (3).

The Compton emission in the Sun's atmosphere may notably exceed the thermal emission of the corona only in the region $E_\gamma \gg 1$ kev. Since the temperature of Sun's photosphere $T \approx 1$ ev, we shall consider in the following that $E_\gamma / T \gg 1$. Moreover, for the problem under consideration the greatest interest is offered by the case of relativistic electrons ($\beta \sim 1$) and of not too great energies E_γ, when the following inequalities are fulfilled:

$$\frac{E_\gamma}{T}\beta \gg 1, \quad a=\frac{E_\gamma}{E} \ll 1+\beta. \tag{8}$$

At fulfillment of one of these inequalities the principal contribution to the integral (6) is made by the small angles θ_2, for because of the presence of the exponential factor the integrand decreases rapidly with the rise of θ_2 from zero. The second inequality allows to neglect the quantity θ_2 by comparison with ν_1 everywhere in (6). As a result of integration (see Appendix) we obtain the following expression for I:

$$I(\beta, x)=\frac{4\pi(1+\beta)}{\beta^2} g'(\beta, x) e^{-x}, \tag{9}$$

where, $x = (E_\gamma / T)(1-\beta)/(1+\beta)$ and the function

$$g'(\beta x)=\frac{1+\beta^2}{2\beta^2}+\frac{(1+\beta)^2}{2\beta^2}x+\frac{1+\beta}{\beta^2}x\left[1+\frac{1}{2}(1+\beta)x\right]e^x E_i(-x), * \tag{10}$$

varies slowly with the variation of x. Considered in the following will be the relativistic electrons for which $E \gg mc^2$. According to (5), (9), the cross section for such electrons has the following form:

---

* (E_i is an integral exponential function

$$\sigma(E_\gamma, E) = \frac{3}{4a}\sigma_T\left(\frac{mc^2}{E}\right)^2 \frac{1}{T}\sum_{k=1}^{\infty} k^{-2} e^{-kx} g\ (kx), \tag{11}$$

where

$$\sigma_T = \frac{8}{3}\pi r_e^2 \approx 6{,}65\cdot 10^{-25}\ \text{см}^2,$$

$$g(x) = g'(\beta = 1,\ x) = 1 + 2x + 2x(1+x)e^x E_i(-x), \tag{11}$$

$$x = \frac{E_\gamma}{4T}\left(\frac{mc^2}{E}\right)^2.$$

It should be noted that the expression (11) may be obtained from (5) by utilizing at integration the approximate equality

$$\cos\theta \approx \cos\theta_1, \tag{12}$$

following from (4) at $\theta_2 = 0$. This result is easy to understand, for at fulfillment of the c onditions (8) the greatest contribution to the integral (6) is made by the small angles θ_2.

The function g (x) in (11) depends little on x in the interval $(0, \infty)$; at the ends of this interval it reaches the maximum value $g_{\max} = 1$, and for $x \approx 0{,}7$ it has a unique minimum $g_{\min} \approx 0{,}6$ (see the Figure). This is why at finding the spectral emission power

$$p(E_\gamma, E) = cn_0\sigma(E_\gamma, E)E_\gamma \tag{13}$$

it is sufficient to retain in the sum (11) the first term (see Figure, where $f(x) = p(x) / 8\pi r_e^2 a^{-1} cn_0$, and the dashed line is the first term of the sum). At the same time we shall find that the power is maximum at $x \approx 1$ and is equal to

$$p_{\max} = \frac{8\pi r_e^2 h}{ae} cn_0 g(1) \approx 0{,}3\sigma_T h\, cn_0 \text{ erg/sec cps} \tag{14}$$

It is characteristic that the value of the maximum is determined only by the concentration of thermal photons.

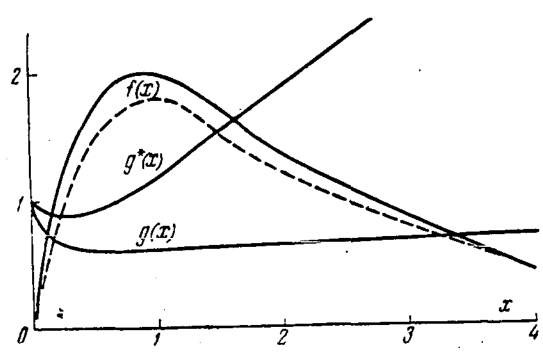

Fig. 1

Let us compute the spectral emission power for a power-law spectrum of electrons

$$N(E) = KE^{-\varkappa}, \quad \text{where} \quad K = (\varkappa - 1)N_0E_0^{\varkappa-1}. \tag{15}$$

Here $\varkappa > 1$ is a constant; N_0 is the total concentration of relativistic electrons. E_0 is the boundary of the power-law spectrum of electrons from the side of small energies. From (11), (13), (15) we obtain

$$P(E_\gamma)=\int_{E_0}^{\infty} p(E_\gamma E)\,KE^{-\varkappa}\,dE=\frac{3}{2a}\sigma_T(\varkappa-1)\,cN_0n_0x_0^{(1-\varkappa)/2}I(x_0), \quad (16)$$

where

$$I(x_0)=\sum_{k=1}^{\infty}k^{-2}\int_0^{x_0}e^{-kx}\,x^{(\varkappa-1)/2}g(kx)\,dx,$$

but

$$x_0=\frac{E_\gamma}{4T}\left(\frac{mc^2}{E_0}\right)^2. \quad (17)$$

In the extreme case of small E_γ, when $x_0\ll 1$, the integral in (16) is

$$I=\sum_{k=1}^{\infty}k^{-2}\int_0^{x_0}x^{(\varkappa-1)/2}dx=\frac{\pi^2}{3(\varkappa+1)}x_0^{(\varkappa+1)/2}$$

and for the power we obtain the following expression:

$$P(E_\gamma)=\frac{\pi^2}{8a}\sigma_T c n_0 N_0\frac{\varkappa-1}{\varkappa+1}\left(\frac{mc^2}{E_0}\right)^2\frac{E_\gamma}{T}. \quad (18)$$

In the opposite extreme case $x_0\gg 1$ in the integral (17) may be postulated $x_0=\infty$, reducing the integral to tabular [15] and the final expression for I and P will have the following form

$$I=\frac{\varkappa^2+4\varkappa+11}{(\varkappa+3)(\varkappa+5)}\Gamma\left(\frac{\varkappa+1}{2}\right)\zeta\left(\frac{\varkappa+5}{2}\right), \quad (19)$$

$$P(E_\gamma)=f(\varkappa)\sigma_T c n_0 N_0\left(\frac{E_0}{mc^2}\right)^{\varkappa-1}\left(\frac{E_\gamma}{4T}\right)^{(1-\varkappa)/2},$$

$$f(\varkappa)=\frac{3}{2a}\frac{(\varkappa^2+4\varkappa+11)(\varkappa-1)}{(\varkappa+3)(\varkappa+5)}\Gamma\left(\frac{\varkappa+1}{2}\right)\zeta\left(\frac{\varkappa+5}{2}\right);$$

ζ is the zeta-function of Riemann. For $\varkappa=1{,}5, 2, 3, 4, 5$ the function $f(\varkappa)$ is respecttively equal to 0.216, 0.410, 0.9, 1.79, 3.62.

Therefore, according to (18), in the presence of a boundary in the exponential energy spectrum of electrons $E_0\gg mc^2$ the spectral emission power $P(E_\gamma)$ rises linearly with E_γ, for small E_γ, and according to (19), for great values of we have

$$P(E_\gamma)\sim E_\gamma^{\frac{1-\varkappa}{2}}$$

(just as for the synchrotron radiation). The maximum of power is reached for a certain intermediate value E_γ^m, which may be estimated by equating the expressions (18) and (19). After rather simple transformations we shall obtain

$$E_\gamma^m=4T\varphi(\varkappa)\,(E_0/mc^2)^2, \quad (20)$$

where $\varphi(\varkappa)=$ 0.6, 0.71, 0.94, 1.16 and 1.38 at $\varkappa=1.5, 2, 3, 4, 5$ respectively.

It should be noted that while E_γ^m is proportional to TE_0^2, the maximum power

$P(E_\gamma^m)$ does not depend on T and E_0 and is only a function of $\varkappa$ it is easy to verify this with the aid of expressions (18) — (20).

It was assumed above that the distribution of thermal photons is isotropic. In connection with the X-ray emission of solar flares this assumption is to some extent corroborated only for flares occurring at small heights in the chromosphere. If the emitting region is situated high in the corona, the other extreme case will prove to be more natural, when all photons move from the Sun along the radius, while the distribution of relativistic electrons is isotropic as previously. In this case the angle θ between primary photons moving in radial direction and the scattered photons moving along the visual ray is fixed, and the cross section $d\sigma$ for the formation of photons with energy E_γ and pulse direction in the solid angle $d\Omega_2$ has the form (cf. with (3))

$$d\sigma = d\Omega_2 \int \nu_1 \delta(E_\gamma - \Phi)\, \sigma_k \frac{n}{n_0}\, d\varepsilon \frac{d\Omega_3}{4\pi}, \tag{21}$$

where all the denotations have the same sense as in (1) — (4). We shall count from the direction of the scattered photon k_2 ; then, upon integration of the δ — function over ε we shall obtain for $d\sigma$ the expression (5) with the factor having here at the same time (cf. (6))

$$I(T) = \int d\varphi \int d(\cos\theta_2) \frac{A}{\nu_2}\left(\frac{\varepsilon}{T}\right)^2 e^{-\varepsilon/T} \tag{22}$$

$$\cos\theta_1 = \cos\theta\cos\theta_2 + \cos\varphi\sin\theta\sin\theta_2,$$

but the denotations are the same as in (5) and (6).

At fulfillment of conditions (8) the basic contribution to integral (22) is made by the small angles θ_2 , for which the second equality (22) passes to (12); utilizing the latter and neglecting in the integral (22) the quatities proportional to a we shall obtain upon integration over θ_2 the expression for $d\sigma$, having at $\beta \approx 1$ the following form (cf. with (11)):

$$d\sigma = d\Omega_2 \frac{3}{16\pi a}\sigma_T \left(\frac{mc^2}{E}\right)^2 \frac{1}{T}\sum_{k=1}^{\infty} k^{-2} e^{-kz} g^*(kz),$$

$$g^*(z) = 1 - z - 2z^2 e^z E_i(-z), \quad z = \frac{E_\gamma}{2T(1-\cos\theta)}\left(\frac{mc^2}{E}\right)^2. \tag{23}$$

The graph of the function $g^*(z)$ is plotted in Fig. 1.

Let us consider the cross section $d\sigma$, defining (23) as a function of the angle θ. At $\theta = 0$ (the flare is observed at the center of the disk) the cross section becomes zero. (At $\theta = 0$ the cross section (23) is not difficult to find exactly; it is an exponentially small quantity $\theta = 0$). as θ increases so does the cross section, reaching its maximum at $\theta \approx \pi/2$ (the flare takes place near the limb). The effect of cross section increase with θ is easy to understand directly with the aid of (1) — (3). At $\theta = 0$ no scattering takes place, that is, the energy of primary photons does not vary; this is why the observer perceives only hard photons of Planck distribution, of which the number is exponentially small. The principal part of Planck photons has an enery $\sim T$ and it may acquire an energy $E_\gamma \gg T$ only at scattering by sufficiently large angles θ.

For the exponential spectrum (15), taking into account (23), we obtain (cf. (16))

$$dP = d\Omega_2 \frac{3}{16a\pi} \sigma_T c n_0 N_0 (\varkappa - 1)(1 - \cos\theta)^{(\varkappa+1)/2} \left(\frac{E_0}{mc^2}\right)^{\varkappa-1} \left(\frac{E_\gamma}{2T}\right)^{(1-\varkappa)/2} I(z_0), \tag{24}$$

where $I(z_0)$ is obtained from (17) by substitution of $\underline{g}$, $\underline{x}$, x_0 by $\underline{g}^*$, $\underline{z}$, z_0 (see (23)), with, at the same time

$$z_0 = \frac{E_\gamma}{2T(1-\cos\theta)} \left(\frac{mc^2}{E_0}\right)^2. \tag{25}$$

Let us consider (24) in extreme cases of small and great z_0. At $z_0 \ll 1$ we obtain analogously to (18) that the power $dP \sim E_\gamma / T$, and at $z_0 \gg 1$ (for small angles θ only one such case is possible) we shall find analogously to (19) that $dP \sim (1-\cos\theta)^{(\varkappa+1)/2} (E_\gamma / T)^{(1-\varkappa)/2}$. At the same time, the energy E_γ^m for which the power dP is maximum, is proportional to $(1-\cos\theta) T (E_0 / mc^2)^2$ (cf. (20)).

Therefore, if the X-ray emission at flares sets in on account of Compton scattering of thermal photons on relativistic electrons with an exponential energetic distribution (15), in the region of hard photons, for which

$$(E_\gamma / 4T)(mc^2 / E_0) \gg 1,$$

the dependence of the emitted power on the angle 0 is as follows:

$$dP(\theta) \sim (\sin\theta / 2)^{\varkappa+1}, \tag{26}$$

where θ is the angular distance of the emitting region from the central meridian. This dependence means that, other conditions being equal, the greatest flux of hard X- ray radiation must be observed for flares occurring near the limb and at a great height behind the Sun's limb. Such a peculiarity of the Compton emission allows for a simple verification of this hypothesis by experimental means.

The authors are sincerely indebted to S. I. Syrovatskiy for his discussion of the results of this work.

***** THE END *****

APPENDIX

Assume that the inequality

$$1 - \beta \gg 2\alpha. \tag{I}$$

is fulfilled.

Then it is possible to neglect in (6) the quantity $\alpha\nu$ by comparison with ν_1 for any values of the angles, and the integral (5) will be written in the following form:

$$I = \beta^{-2} \int\int d\nu_1\, d\nu_2 \frac{A_0}{\nu_2} \left(\frac{\varepsilon_0}{T}\right)^2 e^{-\varepsilon_0/T}, \tag{II}$$

where

$$A_0 = \int_0^{2\pi} d\varphi \left[\gamma^2 \frac{\nu}{\nu_1\nu_2}\left(\gamma^2 \frac{\nu}{\nu_1\nu_2} - 2\right) + 2\right],$$

$$\varepsilon_0 = \frac{E_\gamma \nu_2}{\nu_1} \qquad (1 - \beta \leqslant \nu_{1,2} \leqslant 1 + \beta). \tag{III}$$

Substituting (III) into (II) and performing the integration over φ, we shall find that $A_0 = A_0(\nu_1\nu_2)$ is a quadratic trinomial relative to $1/\nu_1$. Upon integration over ν_1 in (II), we shall obtain

$$I = \beta^{-2} \int \frac{E_\nu}{T} d\nu_2 \left\{ e^{-\varepsilon_0/T} \left[A_0 + \frac{T^2}{E_\gamma^2 \nu_2^2} \frac{\partial^2 A_0}{d(1/\nu_1)^2} \right] \right\}_{\nu_1 = 1-\beta}^{\nu_1 = 1+\beta} \quad (1-\beta \leqslant \nu_2 \leqslant 1+\beta). \tag{IV}$$

At fulfillment of the first inequality (8) the substitution of the lower limit in (IV) provides exponentially small terms that may be dropped. In the remaining integral we may neglect the terms proportional to derivatives from A_0, for the estimates of the corresponding integrals show that they are proportional to the first and second powers of the parameter $T/E_\gamma\beta \ll 1$. The integral (IV), in whuch the derivatives from A_0 were dropped, is expressed by a function of E_i ; this expression is reduced to the expression (9) for $\nu_2 = 1 - \beta$ (small angles θ_2) (the value of the integral for $\nu_2 = 1 + \beta$ is exponentially small). If $(E_\gamma / T)\beta \gg 1$, it is sufficient to require that the inequality $\nu_1 \gg \alpha\nu$ be fulfilled only for small angles θ_2, inasmuch as they make the principal contribution to the value of the integral (II). As is shown by calculation, this leads instead of (I) to the second inequality (8).

*** END OF APPENDIX ***

Contract No. NAS-5-9299
Consultants & Designers, Inc.
Arlington, Virginia

Translated by ANDRE L. BRICHANT
on 30 - 31 August 1966

N. B. This is the last paper under the current contract terminating today.

****

REFERENCES

1. E. FEENBERG, H. PRIMAKOFF. Phys. Rev., 73, 449, 1948.
2. T. M. DONAHUE, - Ibid., 84, 972, 1951.
3. I. A. EARL. Ibid. 6, 125, 1961.
4. P. MAYER, R. VOGT. - Phys. Rev. Letters, 6, 193, 1961; 8, 387, 1962.
5. V. L. GINZBURG, S. I. SYROVATSKIY. ZhETF, 46, 5, 1964.
6. I. E. FELTON, A. P. MORRISON, Phys. Rev. Letters, 10, 453, 1963.
7. I. M. GORDON. Astronom. Zh, 37, 934, 1960.
8. P. E. GUSEYNOV, Ibid., 38, 5, 1961; 40, 584, 1963.
9. I. S. SHKLOVSKIY. Ibid., 41, 676, 1965.
10. V. V. ZHELEZNYAKOV. Ibid., 42, 1, 96, 1964.
11. A. A. KORCHAK. Geom. i Aeronom. 5, 1, 32, 1965; Kosm. Issl., 3, 5, 1965.
12. I. S. SHKLOVSKIY. Astronom. Zh., 4;, 801, 1964.
13. V. L. GINZBURG. Dokl. AN SSSR, 152, 557, 1962; Astronom. Ts. 267, 1962.
14. A. I. AKHIZER, V. B. BERESTETSKIY. Kvantovaya Elektrodinamika Fizmatgiz, 1950.
15. I. S. GRADSHTEYN, I. M. RYZHIK. Tablitsy summ integralov i proizvedeniy (Tables for sums and products of integrals). Fizmatgiz, 1963.