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*Separation of Two Bodies in Space—
A Machine Programmed Analysis Using the
Lagrange Equations and Eulerian Angles*

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CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

May 15, 1966

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ABSTRACT

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This report presents a development of the Lagrangian equations of motion and a FORTRAN computer program for the motion of two rigid bodies in space, separating as a result of any one, or a combination of, the following force mechanisms: springs with ball ends, springs with one end guided, pyrotechnics, rockets, cold-gas jets, air pistons, and coulomb drag. Two constraints, treated by the method of Lagrange multipliers, are included as options. These constraints are the *pin-puller delay* and the *hard-mounted spring constraint*. The pin-puller delay represents the situation when one discrete separation device (e.g., a pin puller) actuates later than the others, providing a ball and socket type of joint prior to final separation-device firing. The hard-mounted spring constraint takes into account the fact that sliding, at the tip of a guided spring, is usually prohibited either by design or by inherent friction. The forces arising from the various mechanisms are represented by the customary mathematical models. No approximations are used in the derivation of the classical equations of motion; they are numerically integrated by the Adams-Moulton method.

Author

I. INTRODUCTION

The separation of two bodies in space is distinguished from the many other current applications of dynamics to space technology because it is associated with every space mission. Thus, the problem is of unique, practical

importance, and the need for an analytical tool to solve this dynamic problem is clear. The purpose of the work presented in this report has been to develop such a tool that is appropriate to a variety of separation schemes.

The basic approach to the problem is a straightforward application of the Lagrange equations augmented by undetermined multipliers. The mathematical complexity of the resultant equations is primarily due to the large number of degrees of freedom (12) and the coordinate transformations involved. Many of the more complex and tedious derivations have been relegated to the Appendixes.

The computer program, which consists of three chain links, is written in the FORTRAN/FAP system, and is compatible with the JPL/IBM 7094 installation.

The *input link* reads the input data and performs initial conversions. The locations and directions of all force-

producing devices are converted to the appropriate body-fixed coordinate system (CS), and initial conditions for the coordinates are generated.

The *integration link* applies the JPL integration subroutine FMARK to the differential equations. Separate subroutines for each force type compute the contributions to the generalized forces. Small matrix operation subroutines are used throughout this link.

The *plotting link* accepts the data to be plotted, generates scale factors, and prepares the other necessary plotting instructions.

II. DESCRIPTION OF THE PHYSICAL PROBLEM AND THE CORRESPONDING MATHEMATICAL MODEL

A. Physical Problem

All space exploration missions require, at some point in their lifetime, a separation of two bodies. The separation of an aerodynamic shroud from a launch vehicle, the separation of a spacecraft from a launch vehicle, and the separation of an exploratory module from its parent spacecraft are a few of the more common requirements.

In connection with the common types of separations that will be encountered, the following force-producing devices are considered to be significant: rockets, pyrotechnics, cold-gas jets, pneumatic pistons, coulomb drag, and springs.

Two constraint conditions are considered to be important aspects of the physical problem: (1) there is a dispersion among latching device release times, and (2) guided spring tips will not slip on their bearing surfaces.

B. Mathematical Model

The mathematical model assumes two rigid bodies subjected to the action of forces from the following types of idealized devices:

1. Linear springs hard mounted on one body and bearing on the other (either with frictionless slippage or no slippage)
2. Linear springs between the two bodies with universal joints at both ends
3. Pyrotechnic devices supplying impulsive forces
4. Cold-gas jets—adiabatic processes of an ideal gas
5. Pneumatic pistons—adiabatic processes of an ideal gas
6. Coulomb drag between the bodies
7. Rockets (constant force), including linear changes in inertia properties

The primary assumptions concerning the forces are implicitly contained in the descriptions of these idealized devices. It has been assumed that the bodies do not have separation distances that are significant relative to the distance from the common center of mass (CM) to the center of any external force-field. The equations of motion are written in a reference frame that has fixed directions in inertial space, but that moves with the pre-separation trajectory of the two bodies; this frame is taken to be an inertial one. The results are then correct (when used in combination with a trajectory solution for the common CM) if there are no forces acting other than the ones considered in the analysis, or if there is an external force-field exerting approximately the same vector force per unit mass on each element of the system.

III. SYMBOLS AND CONVENTIONS

A. Symbols

To make the analysis more useful for reference when writing input for the computer program, a descriptive listing is made at the beginning of each section detailing the notation for that particular section.

For clarity, the symbols that are used repeatedly throughout the text are listed below.

1. Scalars

- a_{ij} element of the A_I matrix (i^{th} row, j^{th} column)
- b_{ij} element of the A_{II} matrix
- c_{ij} generalized element of either A_I or A_{II}
- k spring rate
- m mass
- \mathcal{Q} generalized force
- q generalized coordinate
- T kinetic energy
- \mathbf{T} matrix product $A_I A_{II}^{-1}$
- t time
- Δt length of integration interval
- x, y, z inertial coordinates of the CM
- x', y', z' body-fixed coordinates
- x'', y'', z'' drawing board coordinates
- θ, ϕ, ψ Euler angles (see Appendix A)
- $\lambda_x, \lambda_y, \lambda_z$ direction cosines with body I-fixed coordinate axes
- μ_x, μ_y, μ_z direction cosines with drawing board axes when referring to body I
- μ'_x, μ'_y, μ'_z direction cosines with drawing board axes when referring to body II
- ρ_x, ρ_y, ρ_z direction cosines with body II-fixed coordinate axes
- $\omega_{x''}, \omega_{y''}, \omega_{z''}$ components of the initial angular velocity of body II in the drawing board CS
- $\omega_{Ix'}, \omega_{Iy'}, \omega_{Iz'}$ components of the initial angular velocity of body I about the body I-fixed CS

$\omega_{IIx'}, \omega_{IIy'}, \omega_{IIz'}$ components of the initial angular velocity of body II about the body II-fixed CS

2. Vectors and Matrixes

- A_I (3×3) orthogonal transformation matrix that takes vectors in the inertial CS into vectors in the body I-fixed CS (see Appendix A)
- A_{II} (3×3) orthogonal transformation matrix that takes vectors in the inertial CS into vectors in the body II-fixed CS (see Appendix A)
- e unit vector or column matrix in direction indicated by subscript
- \mathbf{I} inertia matrix
- r $r_{II} - r_I$
- r'_{Ii} location vector of i^{th} device in the body I-fixed CS
- r'_{IIi} location vector of i^{th} device in the body II-fixed CS
- r_I $\begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix}$
- r_{II} $\begin{bmatrix} x_{II} \\ y_{II} \\ z_{II} \end{bmatrix}$
- v velocity vector or column matrix
- ρ $\begin{bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{bmatrix}$
- λ $\begin{bmatrix} \lambda_x \\ \lambda_y \\ \lambda_z \end{bmatrix}$
- ω angular velocity vector or column matrix

3. Subscripts

- i, j, r, s nonspecific indexes
- k, l specific indexes
- I refers to body I
- II refers to body II
- 0 initial value

B. Notation Conventions

1. Vector-Matrix

In the interests of economy and clarity, the same notation is used for vectors and their associated (3×1) column matrixes. The symbolism \mathbf{v} is used where it is to be considered a vector, as in $(\mathbf{v} \times \boldsymbol{\omega})$ or $(\mathbf{f} \cdot \mathbf{r})$, and where it is to be considered a column matrix, as in $\mathbf{v}^T \mathbf{I} \mathbf{v}$.

2. Trigonometric Function

Since the sine and cosine functions of the Euler angles occur quite frequently throughout the text, $\cos \theta$ is replaced by $(c\theta)$ and $\sin \theta$ is replaced by $(s\theta)$. The other Euler angles are treated similarly. Such expressions, or groups of expressions, are always enclosed in parentheses [e.g., $(-c\theta s\phi s\psi + c\phi c\psi)$].

3. Miscellaneous

The miscellaneous notation conventions used in this report are as follows:

Bold face lower case letters vector, (3×1) matrix

| | |
|--|--|
| Bold face capital letters | square matrix |
| $\mathbf{a}_{(i)}$ | (3×1) matrix formed from i^{th} row of corresponding (3×3) matrix |
| $\mathbf{a}^{(i)}$ | (3×1) matrix formed from i^{th} column of corresponding (3×3) matrix |
| Dot (\cdot) between vectors | scalar product (e.g., $\mathbf{f} \cdot \mathbf{r}$) |
| Multiplication sign (\times) between vectors | cross product (e.g., $\mathbf{f} \times \mathbf{r}$) |
| Dot ($\dot{\cdot}$) over symbol | derivative with respect to time |
| Two dots ($\ddot{\cdot}$) over symbol | second derivative with respect to time |
| $(\)^T$ | transpose |
| $(\)^{-1}$ | inverse |
| $ \mathbf{v} $ | magnitude of \mathbf{v} |
| δ | variational operator |
| δ_{ij} | Kronecker delta |

IV. COORDINATE SYSTEMS

The four coordinate systems used in this analysis are: (1) drawing board CS, (2) inertial CS, (3) body I-fixed CS, and (4) body II-fixed CS.

A. Symbols

- c (subscript) refers to common CM of bodies I and II
- \mathbf{r} location vector
- $\boldsymbol{\epsilon}$ location vector of CM indicated by subscript
- $\lambda_{ix}, \lambda_{iy}, \lambda_{iz}$ cosines of angles between rocket directions, spring directions, etc., and drawing-board axes when applicable to body I
- $\mu_{ix}, \mu_{iy}, \mu_{iz}$ cosines of angles between rocket directions, spring directions, etc., and drawing-board axes when applicable to body II

- $\mu'_{ix}, \mu'_{iy}, \mu'_{iz}$ cosines of angles between rocket directions, spring directions, etc., and body I-fixed axes
- $\rho_{ix}, \rho_{iy}, \rho_{iz}$ cosines of angles between rocket directions, spring directions, etc., and body II-fixed axes
- ω angular velocity about axis indicated by subscript

B. Generalized Coordinates

To apply the Lagrange equations, it is necessary to choose a set of generalized coordinates. By definition, the generalized coordinates must be independent and must fully describe the position and orientation of the system. This analysis employs the three inertial coordinates of each CM and the three Euler angles for each

body (coordinates are defined in Subsection D). These coordinates form an independent and fully descriptive set unless certain special values of either θ -coordinate are reached. The methods used to circumvent this difficulty are described in Subsections F and G.

C. Drawing Board Coordinates

The 3-dimensional orthogonal drawing board coordinate system may be selected by the user, subject to the following restrictions:

1. If the hard-mounted spring constraint is to be present, the z'' axis must be perpendicular to the separation plane (this situation is assumed in Appendix B, Eq. B-1).
2. Since the solutions for θ , ϕ , ψ , pitch and yaw rates, etc., depend on the definition of the body-fixed CS, which is in turn controlled by the definition of the drawing board CS, it will usually be convenient to make the drawing board CS parallel to any symmetry or geometric axes that the bodies may have.

D. Inertial Coordinates

The point at which the common CM would be located if there were no separation is used as the origin of the inertial CS. The directions of these axes are taken to be fixed in the inertial CS attached to the fixed stars. The inertial CS is initially parallel to the drawing board CS. Because of its motion in the gravitational fields of the Sun, the planets, the galaxy, etc, this CS is not a true inertial system; however, as long as the bodies are sufficiently close to the origin to be in essentially the same gravitational field, they will move as though this CS is exactly an inertial system. The results of this analysis are then deviations from the pre-separation trajectory. To provide initial conditions for new trajectory calculations, it is recommended that the translation results be considered in conjunction with the initial trajectory.

Since the orientation and location of the two rigid bodies relative to the stars and planets is not considered, certain force effects must necessarily be neglected. For example, solar pressure effects are not included in the analysis. The pre-separation trajectory, however, includes the translational effects of solar pressure, based on the orientation relative to the Sun before separation (solar pressure is a function of paint patterns, etc., so that it is usually a function of orientation). Then, in translation, it is only the deviation from pre-separation solar pressure effects that is neglected. Solar pressure torques are neglected completely.

E. CM Body-Fixed Coordinates and Eulerian Angles

A body-fixed CS is associated with each body. The respective CM's are the origins of these systems, and they are the references used for locating force-producing devices.

The orientation of each system is specified by a set of Eulerian angles that are generated in the following manner (Fig. 1):

1. Roll through angle ϕ
2. Pitch through an angle θ
3. Roll through an angle ψ

A more complete description of the Eulerian angles is given in Appendix A.

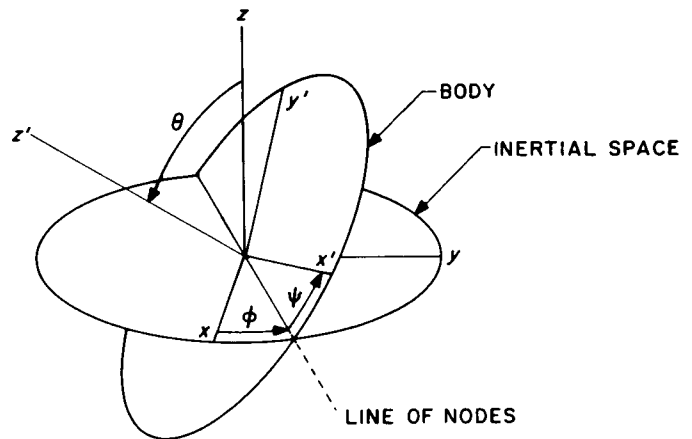


Fig. 1. Eulerian angles

F. Coordinate Conversions From Drawing Board CS to Body-Fixed CS

Although it would be convenient to have the body-fixed CS initially parallel to the drawing board or inertial CS, it is not feasible. In both Section VI and Appendix C, $\sin \theta$ appears in some denominators; this corresponds to dependence among the coordinates when either $\sin \theta_I$ or $\sin \theta_{II}$ vanishes. To avoid this, the body-fixed CS are defined so that ordinary initial conditions do not have either θ equal to zero. Also, provision must be made to rotate (redefine) the body-fixed CS relative to the body if $\sin \theta$ approaches zero.

The CM body-fixed CS (Fig. 2) is defined so that the initial value of θ is 90 deg. The conversions between

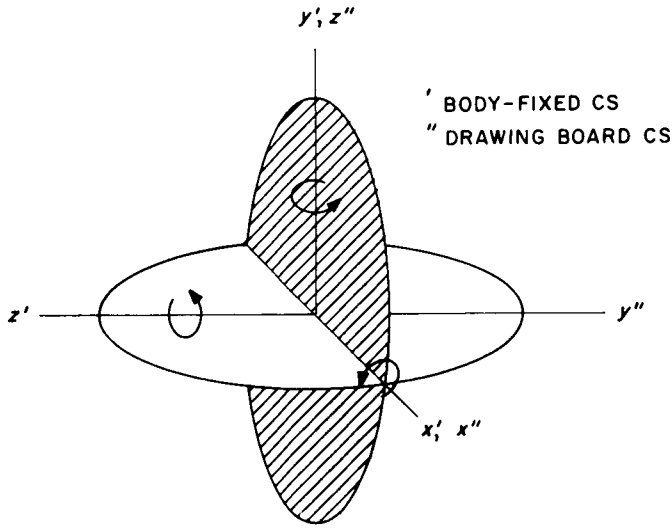


Fig. 2. Relation between drawing board coordinates and body-fixed coordinates

drawing board coordinates and body-fixed coordinates are given in Eqs. (1) through (3).

$$\begin{aligned} x'_{1i} &= x''_i - \epsilon''_{1x} \\ x'_{2i} &= x''_i - \epsilon''_{11x} \end{aligned} \quad (1)$$

$$\begin{aligned} y'_{1i} &= z''_i - \epsilon''_{1z} \\ y'_{2i} &= z''_i - \epsilon''_{11z} \end{aligned} \quad (2)$$

$$\begin{aligned} z'_{1i} &= -(y''_i - \epsilon''_{1y}) \\ z'_{2i} &= -(y''_i - \epsilon''_{11y}) \end{aligned} \quad (3)$$

In addition to coordinate conversions, there are also direction cosine and inertia matrix conversions that must be made (Eqs. 4 through 12).

$$\begin{aligned} \lambda_{ix} &= \mu_{ix} \\ \rho_{ix} &= \mu'_{ix} \end{aligned} \quad (4)$$

$$\begin{aligned} \lambda_{iy} &= \mu_{iy} \\ \rho_{iy} &= \mu'_{iy} \end{aligned} \quad (5)$$

$$\begin{aligned} \lambda_{iz} &= -\mu_{iy} \\ \rho_{iz} &= -\mu'_{iy} \end{aligned} \quad (6)$$

$$I_{x'x'} = I_{x''x''} \quad (7)$$

$$I_{x'y'} = I_{x''z''} \quad (8)$$

$$I_{x'z'} = I_{x''y''} \quad (9)$$

$$I_{y'y'} = I_{z''z''} \quad (10)$$

$$I_{y'z'} = -I_{y''z''} \quad (11)$$

$$I_{z'z'} = I_{y''y''} \quad (12)$$

For simplicity in notation, the primes on the subscripts of the I's are dropped for the remainder of this analysis.

G. Body-Fixed CS Redefinition

If $\theta \leq 0.1$ and $\dot{\theta} < 0$, or if $\theta \geq 3.04$ and $\dot{\theta} > 0$, all body-fixed coordinates, direction cosines, and inertia matrix elements relating to the body concerned are converted according to Eqs. (13) through (24); this is sufficient to assure that $\sin \theta$ will not approach zero.

$$x'_{i \text{ new}} = x'_{i \text{ old}} \quad (13)$$

$$y'_{i \text{ new}} = z'_{i \text{ old}} \quad (14)$$

$$z'_{i \text{ new}} = -y'_{i \text{ old}} \quad (15)$$

$$\lambda_{ix \text{ new}} = \lambda_{ix \text{ old}} \quad (16)$$

$$\rho_{ix \text{ new}} = \rho_{ix \text{ old}}$$

$$\lambda_{iy \text{ new}} = \lambda_{iz \text{ old}} \quad (17)$$

$$\rho_{iy \text{ new}} = \rho_{iz \text{ old}}$$

$$\lambda_{iz \text{ new}} = -\lambda_{iy \text{ old}} \quad (18)$$

$$\rho_{iz \text{ new}} = -\rho_{iy \text{ old}}$$

$$I_{xx \text{ new}} = I_{xx \text{ old}} \quad (19)$$

$$I_{xy \text{ new}} = I_{xz \text{ old}} \quad (20)$$

$$I_{xz \text{ new}} = -I_{xy \text{ old}} \quad (21)$$

$$I_{yy \text{ new}} = I_{zz \text{ old}} \quad (22)$$

$$I_{yz \text{ new}} = -I_{yz \text{ old}} \quad (23)$$

$$I_{zz \text{ new}} = I_{yy \text{ old}} \quad (24)$$

The foregoing redefinition is accomplished by rotating the body-fixed CS about the x' axis until y'_{new} coincides with z'_{old} , and z'_{new} coincides with $-y'_{\text{old}}$. Since the Eulerian angles are defined relative to the inertial CS, they now assume new values (see Fig. 3).

Straightforward application of the spherical trigonometric laws of cosines and sines to the spherical triangle containing the old and new Eulerian angles (Fig. 4) gives Eqs. (25) through (27).

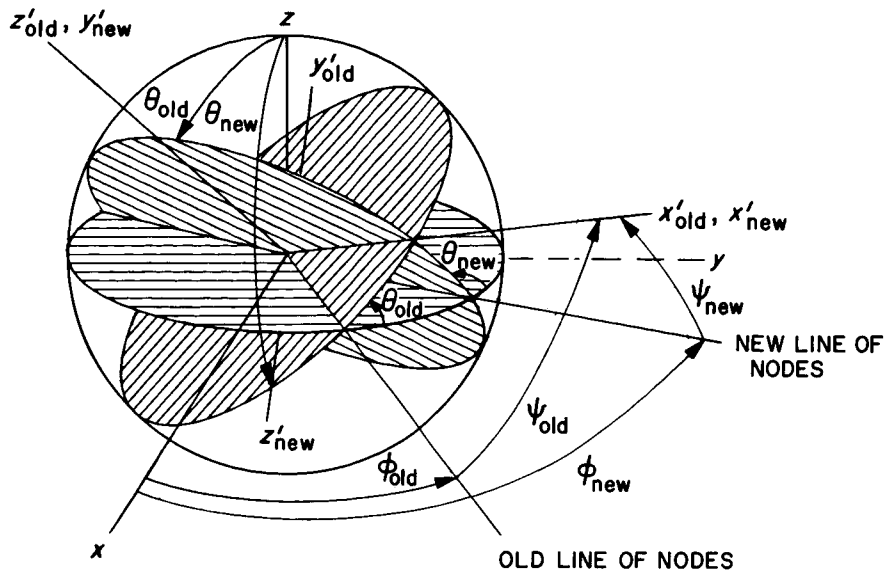


Fig. 3. Eulerian angle change due to CS redefinition

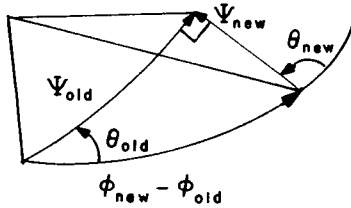


Fig. 4. Spherical triangles containing new and old Eulerian angles

The quadrant of θ_{new} may be obtained from Eq. (25). For the three ranges of $\cos \psi_{old}$, θ_{new} has a value within the indicated range:

$$\cos \psi_{old} < 0, \quad 0 < \theta_{new} < \frac{\pi}{2}$$

$$\cos \psi_{old} = 0, \quad \theta_{new} = \frac{\pi}{2}$$

$$\cos \psi_{old} > 0, \quad \frac{\pi}{2} < \theta_{new} < \pi$$

$$\cos \theta_{new} = -\sin \theta_{old} \cos \psi_{old} \quad (25)$$

$$\sin(\phi_{new} - \phi_{old}) = \frac{\sin \psi_{old}}{(1 - \sin^2 \theta_{old} \cos^2 \psi_{old})^{1/2}} \quad (26)$$

$$\sin \psi_{new} = \frac{\sin \psi_{old} \sin \theta_{old}}{(1 - \sin^2 \theta_{old} \cos^2 \psi_{old})^{1/2}} \quad (27)$$

The quadrant of $\phi_{new} - \phi_{old}$ may be obtained from Eq. (29) and Fig. 3. For the two ranges of $\cos \theta_{old}$, the quadrant of $\phi_{new} - \phi_{old}$ is as follows:

$\cos \theta_{old} > 0$, $\phi_{new} - \phi_{old}$ is in the same quadrant as ψ_{old}

$\cos \theta_{old} < 0$, $\phi_{new} - \phi_{old}$ is in the same quadrant as $2\pi - \psi_{old}$

The quadrant of ψ_{new} may be obtained from Eq. (27) and Fig. 3. For the two ranges of $\sin \psi_{old}$, the quadrant of ψ_{new} is as follows:

$\sin \psi_{old} \geq 0$, ψ_{new} is in the same quadrant as θ_{old}

$\sin \psi_{old} < 0$, ψ_{new} is in the same quadrant as $2\pi - \theta_{old}$

These redefinitions are carried out such that the angles have the ranges of $0 < \theta < \pi$, $0 \leq \phi < 2\pi$, and $0 \leq \psi < 2\pi$.

Since the computer does not have subroutines for the arc sine or arc cosine, it is necessary to restate the new Eulerian angles in terms of arc tangents (Eqs. 28 through 30).

$$\theta_{new} = \tan^{-1} \frac{(1 - \sin^2 \theta_{old} \cos^2 \psi_{old})^{1/2}}{\sin \theta_{old} \cos \psi_{old}} \quad (28)$$

$$\phi_{new} = \phi_{old} + \tan^{-1} \frac{\tan \psi_{old}}{\cos \theta_{old}} \quad (29)$$

$$\psi_{new} = \tan^{-1} (\tan \theta_{old} \sin \psi_{old}) \quad (30)$$

H. Initial Values

If the locations of the CM's of bodies I and II are ϵ_I'' and ϵ_{II}'' in the drawing board CS, then

$$\epsilon_{cx}'' = \frac{m_I \epsilon_{Iz}'' + m_{II} \epsilon_{IIz}''}{m_I + m_{II}} \quad (31)$$

$$\epsilon_{cy}'' = \frac{m_I \epsilon_{Iy}'' + m_{II} \epsilon_{IIy}''}{m_I + m_{II}} \quad (32)$$

$$\epsilon_{cz}'' = \frac{m_I \epsilon_{Iz}'' + m_{II} \epsilon_{IIz}''}{m_I + m_{II}} \quad (33)$$

and the initial values of the generalized coordinates are as given in Eqs. (34) through (37).

$$\begin{aligned} x_{I0} &= \epsilon_{Ix}'' - \epsilon_{cx}'' & (34) \\ x_{II0} &= \epsilon_{IIx}'' - \epsilon_{cx}'' \end{aligned}$$

$$\begin{aligned} y_{I0} &= \epsilon_{Iy}'' - \epsilon_{cy}'' & (35) \\ y_{II0} &= \epsilon_{IIy}'' - \epsilon_{cy}'' \end{aligned}$$

$$\begin{aligned} z_{I0} &= \epsilon_{Iz}'' - \epsilon_{cz}'' & (36) \\ z_{II0} &= \epsilon_{IIz}'' - \epsilon_{cz}'' \end{aligned}$$

$$\begin{aligned} \theta_{I0} = \theta_{II0} &= \frac{\pi}{2}, & \phi_{I0} = \phi_{II0} &= 0 & (37) \\ \psi_{I0} &= \psi_{II0} = 0 \end{aligned}$$

The initial values of the components of the translational velocities are

$$\dot{x}_{I0} = \omega_{Iy}' z_{I0} - \omega_{Iz}' y_{I0}$$

$$\dot{y}_{I0} = \omega_{Iz}' x_{I0} - \omega_{Ix}' z_{I0}$$

$$\dot{z}_{I0} = \omega_{Ix}' y_{I0} - \omega_{Iy}' x_{I0}$$

and

$$\dot{x}_{II0} = \omega_{IIy}' z_{II0} - \omega_{IIz}' y_{II0}$$

$$\dot{y}_{II0} = \omega_{IIz}' x_{II0} - \omega_{IIx}' z_{II0}$$

$$\dot{z}_{II0} = \omega_{IIx}' y_{II0} - \omega_{IIy}' x_{II0} \quad (38)$$

The components of the initial angular velocities are determined in Appendix C, Eqs. (C-10) through (C-12). Note that initially

$$\omega_{Ix}' = \omega_{IIx}' = \omega_x'' \quad (39)$$

$$\omega_{Iy}' = \omega_{Iy}'_0 + \omega_z'' \quad (40)$$

$$\omega_{IIy}' = \omega_z''$$

$$\omega_{Iz}' = \omega_{IIz}' = -\omega_y'' \quad (41)$$

Where ω_x'' , ω_y'' , ω_z'' are the initial angular velocities of body II in the drawing board CS; the bodies are assumed to be connected at $t = 0$, but a relative roll rate about z'' , namely $\omega_{Iy}'_0$, is permitted.

V. SOLUTION TECHNIQUE

Because of the ease with which the constraint conditions can be included in the Lagrangian formulation, this method was chosen for the derivation of the equations of motion.

A. Symbols

f_i force due to i^{th} device

Q generalized force

q generalized coordinate

T kinetic energy

t time

λ_l Lagrange multipliers

B. Lagrange Equations

The Lagrange equations, in the form most useful for this problem, are given by

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_s} \right) - \frac{\partial T}{\partial q_s} = Q_s \quad (42)$$

The generalized forces for the chosen coordinates are given by Eqs. (43) through (48) (see Appendix D).

$$Q_x = \Sigma f_i \cdot e_{ix} \quad (43)$$

$$Q_y = \Sigma f_i \cdot e_{iy} \quad (44)$$

$$Q_z = \Sigma f_i \cdot e_{iz} \quad (45)$$

$$Q_\theta = \Sigma (\text{all torques about } \theta \text{ axis}) \quad (46)$$

$$Q_\phi = \Sigma (\text{all torques about } \phi \text{ axis}) \quad (47)$$

$$Q_\psi = \Sigma (\text{all torques about } \psi \text{ axis}) \quad (48)$$

The θ axis is defined as a line passing through the CM perpendicular to the plane in which θ is measured; the ϕ axis and the ψ axis are defined similarly.

C. Constraints

Since the coordinates are no longer independent when constraints are present, Eq. (42) cannot be directly applied.

There are several methods of incorporating constraints; the method used here is the Lagrange multiplier technique. This technique uses nearly all of the mathematical machinery available in the unconstrained case, and is readily adaptable to numerical solution.

It can be shown¹ that if the constraints can be put in the form

$$\sum_s e_{ls} dq_s + e_{lt} dt = 0 \quad (49)$$

where l is used to indicate the different constraint equations, the equations of motion may be written in the form

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_s} \right) - \frac{\partial T}{\partial q_s} = Q_s + \sum_l \lambda_l e_{ls} \quad (50)$$

$s = 1, 2, \dots, 12$

together with

$$\sum_s e_{ls} \dot{q}_s + e_{lt} = 0$$

$$l = 1, 2, \dots, l_m, \quad l_m = \text{number of constraints} \quad (51)$$

where λ_l are the undetermined Lagrange multipliers. These make $12 + l_m$ equations in the $12 + l_m$ unknowns

$$[q_s (s = 1, \dots, 12), \quad \text{and} \quad \lambda_l (l = 1, \dots, l_m)]$$

that must be solved.

Although more equations must now be solved, most of the terms are calculated in the same manner as when no constraints are acting.

Since the constraints to be dealt with are both scleronomous and holonomic, they are of the form

$$f_l(q_1, \dots, q_{12}) = 0 \quad (52)$$

from which we obtain

$$\sum_s \frac{\partial f_l}{\partial q_s} dq_s = 0, \quad \sum_s \frac{\partial f_l}{\partial q_s} \dot{q}_s = 0 \quad (53)$$

and

$$e_{ls} = \frac{\partial f_l}{\partial q_s}, \quad e_{lt} = 0 \quad (54)$$

To make the constraint Eqs. (51) compatible (for numerical solution) with the Lagrange equations, they must be differentiated once more, with respect to time, giving

$$\sum_s (\dot{e}_{ls} \dot{q}_s + e_{ls} \ddot{q}_s) = 0 \quad (55)$$

or

$$\sum_s e_{ls} \ddot{q}_s = r_l \quad (56)$$

where

$$r_l = - \sum_s \dot{e}_{ls} \dot{q}_s \quad (57)$$

Equations (50) and (56) are then the complete set for the $12 + l_m$ unknowns.

D. Units

Since rocket thrusts, spring forces, etc., will normally be reported in pounds (force), and masses will normally be reported as earth-surface weights (pounds mass), and dimensional locations will normally be reported in inches, it will be convenient to employ the following system:

| Quantity | Unit |
|----------|------|
| Force | lbf |
| Mass | lbm |
| Time | sec |
| Length | in. |

This is not a consistent system of units; that is, the equations of motion will not be dimensionally correct if the values reported in this system are entered directly into the equations. Dimensional consistency is obtained by multiplying the generalized forces by the constant $k = 385.7 \text{ in./sec}^2$.

¹Goldstein, H., *Classical Mechanics*, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1959, pp. 40-43.

VI. FORCES

The forces must be expressed explicitly so that they may be entered into the equations of motion. The forces are divided into groups (depending on how they are produced) and are then added appropriately. For any particular problem, however, most of the forces will be absent. The program (Sections IX through XIII) has been written so that the sets of force calculations may be avoided to save computation time.

A. Components of \mathcal{Q}

\mathcal{Q} generalized force

\mathcal{D} component of \mathcal{Q} due to coulomb drag forces

\mathcal{E} component of \mathcal{Q} due to explosive forces (pyrotechnics)

\mathcal{J} component of \mathcal{Q} due to cold-gas jet forces

\mathcal{N} component of \mathcal{Q} due to pneumatic forces

\mathcal{R} component of \mathcal{Q} due to hot rocket forces

\mathcal{S} component of \mathcal{Q} due to body II hard-mounted springs

\mathcal{M} component of \mathcal{Q} due to universally-jointed springs

The summation equation for the components of \mathcal{Q} is

$$\mathcal{Q}_s = \mathcal{D}_s + \mathcal{E}_s + \mathcal{J}_s + \mathcal{N}_s + \mathcal{R}_s + \mathcal{S}_s + \mathcal{M}_s \quad (58)$$

where

$$s = \text{I}_x, \dots, \text{II}_\psi$$

B. Coulomb Drag

A coulomb drag force is assumed to act between two points, one on each body, which are initially coincident.

The drag force ceases when the separation distance exceeds some selected value. This model represents the usual idealization of electrical disconnects and similar devices. The forces are constant during their action time and take the direction of \mathbf{v}_i (defined below).

1. Symbols

D_i estimated potential drag force at point i

D_i magnitude of i^{th} drag force

\mathbf{d}_i separation distance vector at point i

d_i separation distance at point i (see Eqs. 59 and 60)

d_{if} separation distance beyond which i^{th} drag force ceases

\mathbf{v}_i separation velocity at point i (see Appendix E)

$\mathbf{e}_\theta, \mathbf{e}_\phi, \mathbf{e}_\psi$ unit vectors in the $\dot{\theta}, \dot{\phi},$ and $\dot{\psi}$ directions

2. Distance d_i , Between a Point on Body I and a Point on Body II

Clearly

$$\mathbf{d}_i = \mathbf{r}_{\text{I}i} - \mathbf{r}_{\text{II}i} = \mathbf{r} + \mathbf{A}_\text{I}^\text{T} \mathbf{r}'_{\text{I}i} - \mathbf{A}_\text{II}^\text{T} \mathbf{r}'_{\text{II}i} \quad (59)$$

where

$$\mathbf{d}_i = \begin{bmatrix} d_{ix} \\ d_{iy} \\ d_{iz} \end{bmatrix}, \quad \mathbf{r}'_{\text{I}i} = \begin{bmatrix} x'_{\text{I}i} \\ y'_{\text{I}i} \\ z'_{\text{I}i} \end{bmatrix} \quad (60)$$

$$\mathbf{r}'_{\text{II}i} = \begin{bmatrix} x'_{\text{II}i} \\ y'_{\text{II}i} \\ z'_{\text{II}i} \end{bmatrix}, \quad \mathbf{r} = \mathbf{r}_\text{I} - \mathbf{r}_\text{II}$$

3. Generalized Forces

The generalized forces are given in Eqs. (61) through (66).

$$\mathcal{Q}_{\text{I}x} = \sum_i \left(-D_i \frac{v_{ix}}{|\mathbf{v}_i|} \right) = -\mathcal{Q}_{\text{II}x} \quad (61)$$

$$\mathcal{Q}_{\text{I}y} = \sum_i \left(-D_i \frac{v_{iy}}{|\mathbf{v}_i|} \right) = -\mathcal{Q}_{\text{II}y} \quad (62)$$

$$\mathcal{D}_{Iz} = \sum_i \left(-D_i \frac{v_{iz}}{|v_i|} \right) = -\mathcal{D}_{IIz} \quad (63)$$

$$\mathcal{D}_{Io} = \left\{ \sum_i \left(-\frac{D_i}{|v_i|} \right) [(A_I^T r_{Ii}) \times v_i] \right\} \cdot e_{Io} \quad (64)$$

$$\mathcal{D}_{I\phi} = \left\{ \sum_i \left(-\frac{D_i}{|v_i|} \right) [(A_I^T r_{Ii}) \times v_i] \right\} \cdot e_{I\phi} \quad (65)$$

$$\mathcal{D}_{I\psi} = \left\{ \sum_i \left(-\frac{D_i}{|v_i|} \right) [(A_I^T r_{Ii}) \times v_i] \right\} \cdot e_{I\psi} \quad (66)$$

$$\begin{aligned} D_i &= D_i & \text{if } d_i \leq d_{if} \\ D_i &= 0 & \text{if } d_i > d_{if} \end{aligned} \quad (67)$$

\mathcal{D}_{IIo} , $\mathcal{D}_{II\phi}$, and $\mathcal{D}_{II\psi}$ are found by replacing I by II and reversing sign in Eqs. (64), (65), and (66). The unit vectors are

$$\begin{aligned} e_{Io} &= \begin{bmatrix} (c\phi_I) \\ (s\phi_I) \\ 0 \end{bmatrix}, & e_{I\phi} &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, & e_{I\psi} &= \begin{bmatrix} (s\theta_I s\phi_I) \\ (-s\theta_I c\phi_I) \\ (c\theta_I) \end{bmatrix} \\ e_{IIo} &= \begin{bmatrix} (c\phi_{II}) \\ (s\phi_{II}) \\ 0 \end{bmatrix}, & e_{II\phi} &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, & e_{II\psi} &= \begin{bmatrix} (s\theta_{II} s\phi_{II}) \\ (-s\theta_{II} c\phi_{II}) \\ (c\theta_{II}) \end{bmatrix} \end{aligned} \quad (68)$$

C. Pyrotechnics

Forces due to pyrotechnic devices are considered to be impulsive in nature, and are taken to act between the two bodies, as in the case of bolt cutters, explosive bolts, and linear separation charges. The devices are assumed to have fixed locations and directions relative to body II.

1. Symbols

- E_i force due to i^{th} pyrotechnic device
- I_i total impulse of i^{th} device
- t_i firing time of i^{th} device
- Δt integration interval
- ρ_i direction cosine vector of the i^{th} device in the body II CS.

2. Forces

The impulses are idealized as constant forces acting over one integration interval. Thus

$$\begin{aligned} E_i &= 0 & \text{for } t < t_i - \frac{\Delta t}{2} & \text{ or } t > t_i + \frac{\Delta t}{2} \\ E_i &= \frac{I_i}{\Delta t} & \text{for } t_i - \frac{\Delta t}{2} \leq t \leq t_i + \frac{\Delta t}{2} \end{aligned} \quad (69)$$

The generalized forces are given in Eqs. (70) through (78).

$$\mathcal{E}_{I_x} = \sum_i E_i \mathbf{b}^{(1)T} \cdot \boldsymbol{\rho}_i \frac{(x_I - x_{II})}{|x_I - x_{II}|} = -\mathcal{E}_{II_x} \quad (70)$$

$$\mathcal{E}_{I_y} = \sum_i E_i \mathbf{b}^{(2)T} \cdot \boldsymbol{\rho}_i \frac{(x_I - x_{II})}{|x_I - x_{II}|} = -\mathcal{E}_{II_y} \quad (71)$$

$$\mathcal{E}_{I_z} = \sum_i E_i \mathbf{b}^{(3)T} \cdot \boldsymbol{\rho}_i \frac{(x_I - x_{II})}{|x_I - x_{II}|} = -\mathcal{E}_{II_z} \quad (72)$$

$$\mathcal{E}_{I_\theta} = \left\{ \sum_i E_i [(\mathbf{A}_{II}^T \mathbf{r}'_{IIi} + \mathbf{r}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I_\theta} \quad (73)$$

$$\mathcal{E}_{I_\phi} = \left\{ \sum_i E_i [(\mathbf{A}_{II}^T \mathbf{r}'_{IIi} + \mathbf{r}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I_\phi} \quad (74)$$

$$\mathcal{E}_{I_\psi} = \left\{ \sum_i E_i [(\mathbf{A}_{II}^T \mathbf{r}'_{IIi} + \mathbf{r}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I_\psi} \quad (75)$$

$$\mathcal{E}_{II_\theta} = \left\{ \mathbf{A}_{II}^T \left[\sum_i E_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \right\} \cdot \mathbf{e}_{II_\theta} \quad (76)$$

$$\mathcal{E}_{II_\phi} = \left\{ \mathbf{A}_{II}^T \left[\sum_i E_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \right\} \cdot \mathbf{e}_{II_\phi} \quad (77)$$

$$\mathcal{E}_{II_\psi} = \left\{ \mathbf{A}_{II}^T \left[\sum_i E_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \right\} \cdot \mathbf{e}_{II_\psi} \quad (78)$$

D. Cold-Gas Jets

Cold-gas jet reaction systems are assumed to be of the constant volume, unregulated type. The *adiabatic process of an ideal gas approximation* is made in this section.

1. Symbols

| | |
|--------------------------|---|
| A_{ie} | area at exit of nozzle jet i |
| A_{ig} | area at throat of nozzle jet i |
| C_{i1}, C_{i2}, C_{i3} | constants for jet i depending on nozzle geometry and properties of working fluid |
| f_{i1}, f_{i2}, f_{i3} | constants for jet i depending on initial conditions, nozzle geometry, and properties of working fluid |
| \mathbf{J}_i | vector force of jet i |
| J_i | magnitude of the vector force of jet i |
| J_{st} | value of jet force below which J_i may be considered zero |
| M_{ie} | mach number at exit of jet i |
| p | pressure |
| p_{i0} | initial pressure in bottle for jet i |
| R | gas constant for working fluid as in $pV = RT$ |
| T | absolute temperature |
| T_{i0} | initial temperature in bottle for jet i |
| t | time |

- t_{ie} time at which i^{th} jet fires
- t_{if} time at which i^{th} jet cuts off
- t_i $t - t_{ie}$
- V_{i0} volume of bottle for jet i
- v specific volume
- α_i i^{th} nozzle divergence half angle
- γ ratio of specific heats, c_p/c_v , for working fluid
- λ_i nozzle divergence factor for jet i
- $\lambda_{ix}, \lambda_{iy}, \lambda_{iz}$ cosines of angles between thrust vector for jet i and body I axes
- $\rho_{ix}, \rho_{iy}, \rho_{iz}$ cosines of angles between thrust vector for jet i and body II axes

It is assumed (in Appendix F) that the total mass expelled by the cold-gas jets is negligible compared to the total mass of the body.

2. Forces

The vector force is given in Eq. (79), where either

$$\mathbf{J}_i = J_i (\lambda_{ix} \mathbf{e}_{Ix'} + \lambda_{iy} \mathbf{e}_{Iy'} + \lambda_{iz} \mathbf{e}_{Iz'})$$

or

$$\mathbf{J}_i = J_i (\rho_{ix} \mathbf{e}_{IIx'} + \rho_{iy} \mathbf{e}_{IIy'} + \rho_{iz} \mathbf{e}_{IIz'}) \tag{79}$$

This subsection contains all of the definitions necessary to use the force equations. Appendix F details the derivation of the following equations.

$$t_{ie} \leq t \leq t_{if}, \quad J_i = \frac{f_{i1}}{(1 + f_{i2} t_i)^{f_3}} \tag{80}$$

$$t < t_{ie} \quad \text{or} \quad t > t_{if} \quad \text{or} \quad J_i < J_{st}, \quad J_i = 0 \tag{81}$$

Where

$$f_{i1} = (-\lambda_i C_{i1} C_{i2} + A_{ie} C_{i3}) p_{ito} \tag{82}$$

$$f_{i2} = \left(\frac{\gamma - 1}{2}\right) \left(\frac{2}{\gamma + 1}\right)^{\gamma + 1/[2(\gamma - 1)]} (\gamma R T_{it0})^{1/2} \frac{A_{ig}}{V_{i0}} \tag{83}$$

$$f_3 = \frac{2\gamma}{\gamma - 1} \tag{84}$$

And where

$$\lambda_i = \frac{1}{2} (1 + \cos \alpha_i) \tag{85}$$

$$C_{i1} = -A_{ig} \left(\frac{2}{\gamma + 1}\right)^{\gamma + 1/[2(\gamma - 1)]} \left(\frac{\gamma}{R}\right)^{1/2} \tag{86}$$

$$C_{i2} = \frac{M_{ie} (\gamma R)^{1/2}}{\left(1 + \frac{\gamma - 1}{2} M_{ie}^2\right)^{1/2}} \tag{87}$$

$$C_{i3} = \left(1 + \frac{\gamma - 1}{2} M_{ie}^2\right)^{-\gamma/(\gamma-1)} \quad (88)$$

M_{ie} or the nozzle area ratio is to be determined from Eq. (89).

$$\frac{A_{ig}}{A_{ie}} = \left(\frac{\gamma + 1}{2}\right)^{\gamma+1/[2(\gamma-1)]} M_{ie} \left(1 + \frac{\gamma - 1}{2} M_{ie}^2\right)^{-\gamma+1/[2(\gamma-1)]} \quad (89)$$

The generalized forces are given in Eqs. (90) through (102).

$$\mathcal{J}_{Ix} = \mathbf{a}^{(1)T} \left[\sum_i J_i \boldsymbol{\lambda}_i \right] \quad (90)$$

$$\mathcal{J}_{Iy} = \mathbf{a}^{(2)T} \left[\sum_i J_i \boldsymbol{\lambda}_i \right] \quad (91)$$

$$\mathcal{J}_{Iz} = \mathbf{a}^{(3)T} \left[\sum_i J_i \boldsymbol{\lambda}_i \right] \quad (92)$$

$$\mathcal{J}_{I\theta} = \left[\sum_i J_i (\mathbf{r}'_{i1} \times \boldsymbol{\lambda}_i) \right] \cdot \mathbf{e}'_{i\theta} \quad (93)$$

$$\mathcal{J}_{I\phi} = \left[\sum_i J_i (\mathbf{r}'_{i1} \times \boldsymbol{\lambda}_i) \right] \cdot \mathbf{e}'_{i\phi} \quad (94)$$

$$\mathcal{J}_{I\psi} = \left[\sum_i J_i (\mathbf{r}'_{i1} \times \boldsymbol{\lambda}_i) \right] \cdot \mathbf{e}'_{i\psi} \quad (95)$$

$$\mathcal{J}_{IIx} = \mathbf{b}^{(1)T} \left[\sum_i J_i \boldsymbol{\rho}_i \right] \quad (96)$$

$$\mathcal{J}_{IIy} = \mathbf{b}^{(2)T} \left[\sum_i J_i \boldsymbol{\rho}_i \right] \quad (97)$$

$$\mathcal{J}_{IIz} = \mathbf{b}^{(3)T} \left[\sum_i J_i \boldsymbol{\rho}_i \right] \quad (98)$$

$$\mathcal{J}_{II\theta} = \left[\sum_i J_i (\mathbf{r}'_{i2} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{i\theta} \quad (99)$$

$$\mathcal{J}_{II\phi} = \left[\sum_i J_i (\mathbf{r}'_{i2} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{i\phi} \quad (100)$$

$$\mathcal{J}_{II\psi} = \left[\sum_i J_i (\mathbf{r}'_{i2} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{i\psi} \quad (101)$$

where

$$\mathbf{e}'_{i\theta} = \begin{bmatrix} (c\psi) \\ -(s\psi) \\ 0 \end{bmatrix}$$

$$\mathbf{e}'_{i\phi} = \mathbf{c}^{(3)}$$

$$\mathbf{e}'_{i\psi} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (102)$$

E. Pneumatics

Pneumatic ejection systems are modeled as a number of piston-cylinder assemblies fixed in body II and connected to a central reservoir (with or without regulation). It is assumed that the gas process is adiabatic, and that the gas is ideal.

1. Symbols

- A area of piston face
- J regulator existence index: if 0, there is no regulator; if 1, there is a regulator that remains closed after $p_b = p_p$; if 2, there is a regulator that remains open after $p_b = p_p$
- j_i displacement of i^{th} piston from closed position (see Appendix B)
- $j_{eqp, i}$ value of j_i such that $p_b = p_p$
- $j_{max, i}$ distance at which i^{th} piston separates
- N_i magnitude of the i^{th} pneumatic force
- \mathbf{N}_i pneumatic force vector of the i^{th} piston
- p pressure
- V_b volume of gas bottle, including all lines upstream of the faces of the closed pistons
- b (subscript) gas bottle
- i (subscript) refers to the i^{th} piston assembly or a part of the assembly
- p (subscript) piston chamber
- 0 (subscript) initial conditions
- γ ratio of specific heats for working fluids
- $\rho_{ix}, \rho_{iy}, \rho_{iz}$ cosines of angles between piston rod i and the $x'_{II}, y'_{II}, z'_{II}$ axes

2. Forces

Using the results of Appendix H, the forces are calculated as shown in Eq. (103).

$$\mathbf{N}_i = N_i (\rho_{ix} \mathbf{e}_{IIx'} + \rho_{iy} \mathbf{e}_{IIy'} + \rho_{iz} \mathbf{e}_{IIz'}) \quad (103)$$

The factors that must be taken into consideration when calculating the value of N_i are: whether or not there is a pressure regulator and, if there is, whether or not it stays open after $p_b = p_p$.

$$j_{eqp, i} = \frac{V_b}{\gamma A_i} \left(\frac{p_{b0}}{p_{pi}} - 1 \right) \quad (104)$$

Where $J = 0$,

if

$$j_i < j_{max, i}$$

then

$$N_i = p_{b0} A_i \left(1 + \frac{\sum_i A_i j_i}{V_b} \right)^{-\gamma} \quad (105)$$

if

$$j_i \geq j_{max, i}$$

then

$$N_i = 0 \quad (106)$$

Where $J = 1$,

if

$$j_i \leq j_{eqp, i}$$

then

$$N_i = p_{pi} A_i \tag{107}$$

if

$$j_{eqp, i} < j_i < j_{max, i}$$

then

$$N_i = p_{pi} A_i \left(\frac{j_{eqp, i}}{j_i} \right)^\gamma \tag{108}$$

if

$$j_{max, i} \leq j_i$$

then

$$N_i = 0 \tag{109}$$

Where $J = 2$,

if

$$j_i \leq j_{eqp, i}$$

then

$$N_i = p_{pi} A_i \tag{110}$$

if

$$j_{eqp, i} < j_i < j_{max, i}$$

then

$$N_i = p_{pi} A_i \left(1 + \frac{\sum_k A_k b_k}{V_b \sum_k A_k C_k} \right)^{-\gamma} \tag{111}$$

if

$$j_{max, i} \leq j_i$$

then

$$N_i = 0 \tag{112}$$

Where

$$b_k = j_k \quad \text{if} \quad j_k < j_{eqp, k} \tag{113}$$

$$b_k = j_k - j_{eqp, k} \quad \text{if} \quad j_{eqp, k} \leq j_k \tag{114}$$

and

$$C_k = j_k \quad \text{if} \quad j_k < j_{eqp, k} \tag{115}$$

$$C_k = j_{eqp, k} \quad \text{if} \quad j_{eqp, k} \leq j_k \tag{116}$$

The generalized forces are given in Eqs. (117) through (126).

$$\mathcal{N}_{Ix} = \mathbf{b}^{(1)T} \cdot \left[\sum_i N_i \boldsymbol{\rho}_i \right] \frac{x_I - x_{II}}{|x_I - x_{II}|} = -\mathcal{N}_{IIx} \quad (117)$$

$$\mathcal{N}_{Iy} = \mathbf{b}^{(2)T} \cdot \left[\sum_i N_i \boldsymbol{\rho}_i \right] \frac{y_I - y_{II}}{|y_I - y_{II}|} = -\mathcal{N}_{IIy} \quad (118)$$

$$\mathcal{N}_{Iz} = \mathbf{b}^{(3)T} \cdot \left[\sum_i N_i \boldsymbol{\rho}_i \right] \frac{z_I - z_{II}}{|z_I - z_{II}|} = -\mathcal{N}_{IIz} \quad (119)$$

$$\mathcal{N}_{I\theta} = \left[\sum_i N_i (\mathbf{A}_I^T \mathbf{r}'_{ii}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}_{I\theta} \quad (120)$$

$$\mathcal{N}_{I\phi} = \left[\sum_i N_i (\mathbf{A}_I^T \mathbf{r}'_{ii}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}_{I\phi} \quad (121)$$

$$\mathcal{N}_{I\psi} = \left[\sum_i N_i (\mathbf{A}_I^T \mathbf{r}'_{ii}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}_{I\psi} \quad (122)$$

$$\mathcal{N}_{II\theta} = - \left[\sum_i N_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{II\theta} \quad (123)$$

$$\mathcal{N}_{II\phi} = - \left[\sum_i N_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{II\phi} \quad (124)$$

$$\mathcal{N}_{II\psi} = - \left[\sum_i N_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{II\psi} \quad (125)$$

Where

$$\mathbf{e}_{I\theta} = \begin{bmatrix} (c\phi) \\ (s\phi) \\ 0 \end{bmatrix}, \quad \mathbf{e}_{I\phi} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{e}_{I\psi} = \mathbf{c}^{(3)} \quad (126)$$

The unit vectors with regard to the body II-fixed CS ($\mathbf{e}'_{II\theta}$, $\mathbf{e}'_{II\phi}$, $\mathbf{e}'_{II\psi}$), are given in Eqs. (102).

F. Rockets

Provision is made for attaching body-fixed, constant-force rockets to either body I or body II.

1. Symbols

$\dot{I}_{xxIIi}, \dots, \dot{I}_{zzIIi}$ moment of inertia degradation rates

$\dot{m}_{Ii}, \dot{m}_{IIi}$ mass degradation rates

\mathbf{R}_i vector thrust developed by i^{th} rocket

R_i magnitude of thrust developed by i^{th} rocket

R_i rated thrust level of i^{th} rocket

t_{ie} time at which i^{th} rocket starts firing

t_{if} time at which i^{th} rocket ceases firing

$\lambda_{ix}, \lambda_{iy}, \lambda_{iz}$ cosines of angles between thrust vector for rocket i and the x'_i, y'_i, z'_i axes

2. Forces

The forces described in this subsection are of the same form as those described in Section VI D, except that the rocket forces are constant with time.

If t is not between t_{ie} and t_{if} , rocket i develops no thrust; however, if t is between t_{ie} and t_{if} , it is assumed to develop full thrust. Therefore, the thrust magnitude is as described by Eq. (127)

$$\begin{aligned} \text{If } t < t_{ie} & \quad \text{then } R_i = 0 \\ \text{If } t_{ie} \leq t \leq t_{if} & \quad \text{then } R_i = R_i \\ \text{If } t_{if} < t & \quad \text{then } R_i = 0 \end{aligned} \tag{127}$$

Consequently, the components of the rocket contribution to the generalized forces are as given in Eqs. (128) through (139).

$$R_{Ix} = \mathbf{a}^{(1)T} \cdot \left[\sum_i R_i \lambda_i \right] \tag{128}$$

$$R_{Iy} = \mathbf{a}^{(2)T} \cdot \left[\sum_i R_i \lambda_i \right] \tag{129}$$

$$R_{Iz} = \mathbf{a}^{(3)T} \cdot \left[\sum_i R_i \lambda_i \right] \tag{130}$$

$$R_{Io} = \left[\sum_i R_i (\mathbf{r}'_{Ii} \times \lambda_i) \right] \cdot \mathbf{e}'_{Io} \tag{131}$$

$$R_{I\phi} = \left[\sum_i R_i (\mathbf{r}'_{Ii} \times \lambda_i) \right] \cdot \mathbf{e}'_{I\phi} \tag{132}$$

$$R_{I\psi} = \left[\sum_i R_i (\mathbf{r}'_{Ii} \times \lambda_i) \right] \cdot \mathbf{e}'_{I\psi} \tag{133}$$

$$R_{IIx} = \mathbf{b}^{(1)T} \cdot \left[\sum_i R_i \rho_i \right] \tag{134}$$

$$R_{IIy} = \mathbf{b}^{(2)T} \cdot \left[\sum_i R_i \rho_i \right] \tag{135}$$

$$R_{IIz} = \mathbf{b}^{(3)T} \cdot \left[\sum_i R_i \rho_i \right] \tag{136}$$

$$R_{IIo} = \left[\sum_i R_i (\mathbf{r}'_{IIi} \times \rho_i) \right] \cdot \mathbf{e}'_{IIo} \tag{137}$$

$$R_{II\phi} = \left[\sum_i R_i (\mathbf{r}'_{IIi} \times \rho_i) \right] \cdot \mathbf{e}'_{II\phi} \tag{138}$$

$$R_{II\psi} = \left[\sum_i R_i (\mathbf{r}'_{IIi} \times \rho_i) \right] \cdot \mathbf{e}'_{II\psi} \tag{139}$$

3. Mass and Moment of Inertia Decreases

During the firing of the rockets, the mass and the moments of inertia (of the body on which the rocket is mounted) will decrease. The amount of mass remaining is shown in Eqs. (140) through (145).

$$\text{If } t \leq t_{ie} \quad \text{then } m_I = m_{I0} \tag{140}$$

$$t_{ie} \leq t < t_{if} \quad \text{then } m_I = m_{I0} - (t - t_{ie}) \sum_i \dot{m}_{Ii} \tag{141}$$

$$t_{if} \leq t \quad \text{then } m_I = m_{I0} - (t_{if} - t_{ie}) \sum_i \dot{m}_{Ii} \tag{142}$$

$$t \leq t_{ie} \quad \text{then } m_{II} = m_{II0} \tag{143}$$

$$t_{ie} < t < t_{if} \quad \text{then} \quad m_{II} = m_{II0} - (t - t_{ie}) \sum_i m_{IIi} \quad (144)$$

$$t_{if} \leq t \quad \text{then} \quad m_{II} = m_{II0} - (t_{if} - t_{ie}) \sum_i m_{IIi} \quad (145)$$

The principal moments of inertia will decrease in a similar fashion. It is assumed that the inertia decrease can be adequately approximated by a linearization. It is also assumed that the mass loss has no effect on the products of inertia.

G. Hard-Mounted Springs on Body II

The usual linear-spring assumption has been made in this section; however, there is provision for the use of an experimental spring efficiency factor. No provision has been made for the spring-tip sliding friction. In practice, the static coefficient of friction is usually several times higher than the necessary value to prevent slippage so that the constraint mode will almost always be present (see Appendix H).

1. Symbols

- j_i length of extension of i^{th} spring
- N restriction index (if $N = 0$, there is no constraint; if $N > 1$, the constraint is operating)
- r_{IIi} location of tip of spring i in the inertial CS (see Appendix B)
- S_i force in i^{th} spring
- S_{if} residual spring force
- S_{i0} initial force in i^{th} spring
- η spring efficiency factor
- $\rho_{ix}, \rho_{iy}, \rho_{iz}$ cosines of angles between spring i and body II axes

2. Longitudinal Forces in Springs

The vector representation of the i^{th} spring force is given by Eqs. (146) and (147).

If

$$0 \leq k_i j_i < S_{i0} - S_{if}$$

then

$$S_i = \eta^2 (S_{i0} - k_i j_i) (\rho_{ix} \mathbf{e}_{IIx'} + \rho_{iy} \mathbf{e}_{IIy'} + \rho_{iz} \mathbf{e}_{IIz'}) \quad (146)$$

If

$$k_i j_i \geq S_{i0} - S_{if}$$

then

$$S_i = 0 \quad (147)$$

3. Generalized Forces

Because the Lagrange multipliers automatically supply the forces necessary to prevent slippage, only the longitudinal forces in the springs need be considered.

Then

$$\mathcal{J}_{IIx} = -\mathbf{a}^{(1)T} \sum_i S_i \boldsymbol{\rho}_i = -\mathcal{J}_{Ix} \quad (148)$$

$$\mathcal{E}_{IIy} = -\mathbf{a}^{(2)T} \sum_i S_i \boldsymbol{\rho}_i = -\mathcal{E}_{Iy} \quad (149)$$

$$\mathcal{E}_{IIz} = -\mathbf{a}^{(3)T} \sum_i S_i \boldsymbol{\rho}_i = -\mathcal{E}_{Iz} \quad (150)$$

$$\mathcal{E}_{II\theta} = - \left[\sum_i S_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{I\theta} \quad (151)$$

$$\mathcal{E}_{II\phi} = - \left[\sum_i S_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{I\phi} \quad (152)$$

$$\mathcal{E}_{II\psi} = - \left[\sum_i S_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \cdot \mathbf{e}'_{I\psi} \quad (153)$$

$$\mathcal{E}_{I\theta} = \left\{ \sum_i S_i [(\mathbf{r}'_{IIi} - \mathbf{r}_I) \times (\mathbf{A}_{II}^{-1} \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I\theta} \quad (154)$$

$$\mathcal{E}_{I\phi} = \left\{ \sum_i S_i [(\mathbf{r}'_{IIi} - \mathbf{r}_I) \times (\mathbf{A}_{II}^{-1} \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I\phi} \quad (155)$$

$$\mathcal{E}_{I\psi} = \left\{ \sum_i S_i [(\mathbf{r}'_{IIi} - \mathbf{r}_I) \times (\mathbf{A}_{II}^{-1} \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I\psi} \quad (156)$$

where the unit vectors have been previously defined.

The equation for \mathbf{r}_{IIi} is derived in Appendix B:

$$\mathbf{r}_{IIi} = \mathbf{r}_{II} + \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{A}_{II}^T \boldsymbol{\rho}_i$$

Clearly, $\mathbf{r}_{IIi} - \mathbf{r}_I$ is the vector connecting body I's CM and the tip of spring i , but expressed in inertial coordinates.

H. Universally-Jointed Springs

In actual practice, universally-jointed springs are rarely used. They do, however, closely approximate hard-mounted springs when the guiding is poor, and the relative motion is small (see Appendix H).

1. Symbols

- \mathbf{d}_i vector connecting spring attachment points
- k_i spring rate of i^{th} spring
- \mathbf{W}_i force in i^{th} spring
- \mathbf{W}_{if} residual spring force
- \mathbf{W}_{i0} initial force in i^{th} spring
- w_i a temporary notation convention (see Eq. 159)
- η spring efficiency factor

2. Forces

With universal joints at both ends, the spring force lines up with the vector connecting the two attachment points.

If

$$0 \leq k_i (|d_i| - |d_{i0}|) < W_{i0} - W_{if}$$

then

$$W_i = \eta^2 [W_{i0} - k_i (|d_i| - |d_{i0}|)] \frac{d_i}{|d_i|} \quad (157)$$

If

$$k_i d_i \geq W_{i0} - W_{if}$$

then

$$W_i = 0 \quad (158)$$

For simplicity in notation, let

$$w_i = \frac{\eta^2 [W_{i0} - k_i (|d_i| - |d_{i0}|)]}{|d_i|} \quad (159)$$

Then

$$W_i = w_i d_i \quad (160)$$

The generalized forces are given in Eqs. (161) through (166).

$$\mathcal{M}_{Ix} = \sum_i w_i d_{ix} = -\mathcal{M}_{IIx} \quad (161)$$

$$\mathcal{M}_{Iy} = \sum_i w_i d_{iy} = -\mathcal{M}_{IIy} \quad (162)$$

$$\mathcal{M}_{Iz} = \sum_i w_i d_{iz} = -\mathcal{M}_{IIz} \quad (163)$$

$$\mathcal{M}_{I\theta} = \left\{ \sum_i w_i [(A_I^T r_i) \times d_i] \right\} \cdot e_{I\theta} \quad (164)$$

$$\mathcal{M}_{I\phi} = \left\{ \sum_i w_i [(A_I^T r_i) \times d_i] \right\} \cdot e_{I\phi} \quad (165)$$

$$\mathcal{M}_{I\psi} = \left\{ \sum_i w_i [(A_I^T r_i) \times d_i] \right\} \cdot e_{I\psi} \quad (166)$$

The values of $\mathcal{M}_{II\theta, \phi, \psi}$ are obtained by replacing I by II in Eqs. (164) through (166).

VII. EQUATIONS OF MOTION

A. Unconstrained Case

The unconstrained equations of motion are determined from the Lagrange formulation as outlined in Section VB. The details of the calculation of the terms may be found in Appendix I. The equations of motion are given in Eqs. (167) through (172).

$$m\ddot{x} = \mathcal{Q}'_x \equiv p'_x \quad (\text{from } q = x) \quad (167)$$

$$m\ddot{y} = \mathcal{Q}'_y \equiv p'_y \quad (\text{from } q = y) \quad (168)$$

$$m\ddot{z} = \mathcal{Q}'_z \equiv p'_z \quad (\text{from } q = z) \quad (169)$$

$$m_{11}\ddot{\theta} + m_{12}\ddot{\phi} + m_{13}\ddot{\psi} = p_1 + \mathcal{Q}'_\theta \equiv p'_1 \quad (\text{from } q = \theta) \quad (170)$$

$$m_{21}\ddot{\theta} + m_{22}\ddot{\phi} + m_{23}\ddot{\psi} = p_2 + \mathcal{Q}'_\phi \equiv p'_2 \quad (\text{from } q = \phi) \quad (171)$$

$$m_{31}\ddot{\theta} + m_{32}\ddot{\phi} + m_{33}\ddot{\psi} = p_3 + \mathcal{Q}'_\psi \equiv p'_3 \quad (\text{from } q = \psi) \quad (172)$$

where

$$m_{11} = I_{xx}(c^2 \psi) - 2I_{xy}(s\psi c\psi) + I_{yy}(s^2 \psi) \quad (173)$$

$$m_{12} = [(I_{xx} - I_{yy})(s\psi c\psi) + I_{xy}(c^2 \psi - s^2 \psi)](s\theta) + [I_{xz}(c\psi) - I_{yz}(s\psi)](c\theta) \quad (174)$$

$$m_{13} = I_{xz}(c\psi) - I_{yz}(s\psi) \quad (175)$$

$$m_{21} = m_{12} \quad (176)$$

$$m_{22} = (s^2 \theta) [I_{xx}(s^2 \psi) + 2I_{xy}(s\psi c\psi) + I_{yy}(c^2 \psi)] + 2(s\theta c\theta) [I_{xz}(s\psi) + I_{yz}(c\psi)] + (c^2 \theta) I_{zz} \quad (177)$$

$$m_{23} = (s\theta) [I_{xz}(s\psi) + I_{yz}(c\psi)] + (c\theta) I_{zz} \quad (178)$$

$$m_{31} = m_{13} \quad (179)$$

$$m_{32} = m_{23} \quad (180)$$

$$m_{33} = I_{zz} \quad (181)$$

$$\begin{aligned} p_1 = & (s\theta) \dot{\phi} \dot{\psi} [4I_{xy}(s\psi c\psi) - (I_{xx} - I_{yy})(c^2 \psi - s^2 \psi) - I_{zz}] + 2\dot{\theta} \dot{\psi} [(I_{xx} - I_{yy})(s\psi c\psi) + I_{xy}(c^2 \psi - s^2 \psi)] \\ & + \dot{\psi}^2 [I_{xz}(s\psi) + I_{yz}(c\psi)] + \dot{\phi}^2 (s\theta c\theta) [I_{xx}(s^2 \psi) + 2I_{xy}(s\psi c\psi) + I_{yy}(c^2 \psi) - I_{zz}] \\ & + \dot{\phi}^2 (c^2 \theta - s^2 \theta) [I_{xz}(s\psi) + I_{yz}(c\psi)] + 2(c\theta) [I_{xz}(s\psi) + I_{yz}(c\psi)] \end{aligned} \quad (182)$$

$$\begin{aligned} p_2 = & -2\dot{\phi} \{ \dot{\theta} (s\theta c\theta) [I_{xx}(s^2 \psi) + 2I_{xy}(s\psi c\psi) + I_{yy}(c^2 \psi) - I_{zz}] + \dot{\psi} (s^2 \theta) [(I_{xx} - I_{yy})(s\psi c\psi) \\ & + I_{xy}(c^2 \psi - s^2 \psi)] + \dot{\theta} (c^2 \theta - s^2 \theta) [I_{xz}(s\psi) + I_{yz}(c\psi)] + \dot{\psi} (s\theta c\theta) [I_{xz}(c\psi) - I_{yz}(s\psi)] \} \\ & - \dot{\theta} \{ \dot{\theta} (c\theta) [(I_{xx} - I_{yy})(s\psi c\psi) + I_{xy}(c^2 \psi - s^2 \psi)] + \dot{\psi} (s\theta) [(I_{xx} - I_{yy})(c^2 \psi - s^2 \psi) - 4I_{xy}(s\psi c\psi)] \\ & - \dot{\theta} (s\theta) [I_{xz}(c\psi) - I_{yz}(s\psi)] \} - \dot{\psi} \{ \dot{\psi} (s\theta) [I_{xz}(c\psi) - I_{yz}(s\psi)] - \dot{\theta} (s\theta) I_{zz} \} \end{aligned} \quad (183)$$

$$\begin{aligned} p_3 = & -2\dot{\phi} \{ \dot{\theta} (c\theta) [I_{xz}(s\psi) + I_{yz}(c\psi)] \} + \dot{\phi} (s\theta) \{ (c^2 \psi - s^2 \psi) [I_{xx} - I_{yy}] \dot{\theta} + I_{xy} \dot{\phi} (s\theta) \} + \dot{\theta} I_{zz} \\ & + (I_{xx} - I_{yy})(s\psi c\psi) [\dot{\phi}^2 (s^2 \theta) - \dot{\theta}^2] - 4I_{xy} \dot{\theta} \dot{\phi} (s\theta s\psi c\psi) - [I_{xz}(c\psi) - I_{yz}(s\psi)] \dot{\phi}^2 (s\theta c\theta) \\ & - I_{xy} \dot{\theta}^2 (c^2 \psi - s^2 \psi) \end{aligned} \quad (184)$$

It is understood that Eqs. (167) through (184) are to be applied to both body I and body II.

$$\mathcal{Q}'_s = k\mathcal{Q}_s, \quad \text{where } k = 385.7 \text{ in./sec}^2 \quad (185)$$

The unconstrained equations of motion can then be written as

$$M\ddot{q} = f \tag{186}$$

where

$$M = \begin{bmatrix} m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{11I} & m_{12I} & m_{13I} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{21I} & m_{22I} & m_{23I} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{31I} & m_{32I} & m_{33I} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{I1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{II} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{II} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{11II} & m_{12II} & m_{13II} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{21II} & m_{22II} & m_{23II} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{31II} & m_{32II} & m_{33II} \end{bmatrix} \tag{187}$$

$$\ddot{q} = \begin{bmatrix} \ddot{x}_I \\ \ddot{y}_I \\ \ddot{z}_I \\ \ddot{\theta}_I \\ \ddot{\phi}_I \\ \ddot{\psi}_I \\ \ddot{x}_{II} \\ \ddot{y}_{II} \\ \ddot{z}_{II} \\ \ddot{\theta}_{II} \\ \ddot{\phi}_{II} \\ \ddot{\psi}_{II} \end{bmatrix} \quad f = \begin{bmatrix} p'_{xI} \\ p'_{yI} \\ p'_{zI} \\ p'_{\theta I} \\ p'_{\phi I} \\ p'_{\psi I} \\ p'_{xII} \\ p'_{yII} \\ p'_{zII} \\ p'_{\theta II} \\ p'_{\phi II} \\ p'_{\psi II} \end{bmatrix} \tag{188}$$

$$\tag{189}$$

B. Constrained Case

Since it is assumed that both constraint conditions will not be required at the same time, a set of constrained equations of motion has been derived for each constraint condition. These equations, with their attendant definitions, are given in Appendix J (see Appendixes K and L). Each set consists of 15 second-order differential equations.

C. Numerical Solution

In either the constrained or unconstrained case, the equations of motion are solved for the accelerations by Cramer's rule. The accelerations are then doubly integrated by the Adams-Moulton technique.

VIII. LIMITATIONS ON THE ANALYSIS AND SOLUTION ACCURACY

The limitations on the use of this program are established by the mathematical model described in this report. Considerable flexibility within the model can be obtained by ingenious use of its facets; the force equations and constraint modes need not be used for their nominal purposes. For example, the pin-puller delay can be used in conjunction with a very large m_1 to obtain a fixed-point constraint on body II. A large number of other potentialities exist.

In view of the complexity of the program and the wide range of possible problem situations, computational ac-

curacy is difficult to define. With the generalized coordinates given, the forces are obtained by straightforward noniterative procedures, so that the only errors introduced in computing these forces would be because of truncation. The relatively unknown area is then the integration of the equations of motion. As detailed in the computer program portion of this report (Sections IX through XIII), the equations are integrated with a routine that changes step size in an effort to maintain a constant integration error that is a function of the initial step size selected. It is recommended that different step sizes be tried on a new problem; a comparison of the results will indicate approximately the accuracy of integration.

IX. DETAILED DESCRIPTION OF THE COMPUTER PROGRAM: INPUT LINK, INTEGRATION LINK, AND PLOTTING LINK

The program is written in the FORTRAN/FAP system, and is divided into three chain links (input, integration, and plotting). The first link reads and prints the input data; the second link integrates the differential equations; the third link prepares output data for plotting (see Figs. 5 and 6). The detailed description of the integration technique (FMARK) is relegated to Appendix M. Rather than

the conventional flow-chart approach, the discussion of the links is referenced to the actual FORTRAN listing (see Appendix N).

A. Input Link

All appropriate input data are read and printed at the beginning of this link. Force input and initialization begins after FORMAT statement 845, and is controlled by the values of the indicators. These indicators are: ICD, coulomb drag; IPR, pyrotechnic; ICG, cold gas; IPN, pneumatic; IRK, rockets; ISP, hard-mounted springs; ISU, universally jointed springs; and IPN, pin-puller constraint. Input for a force is bypassed if its indicator is zero. If the indicator is non-zero, input/output is performed along with the initial calculations on the forces and conversion from drawing board CS to body-fixed CS. All parameters needed for the force calculations are transmitted to the integration link through COMMON storage. No special subroutines are used in the input link.

Of special note is the sentinel ICLAG, which, at the beginning of the link = 1. If the hard-mounted spring constraint is used, ICLAG = -1 (statement 6552); however, if the pin-puller constraint is used, ICLAG = 0 (above statement 43). Both constraints cannot be working at the same time, but the spring constraint can be used after the pin-puller constraint terminates.

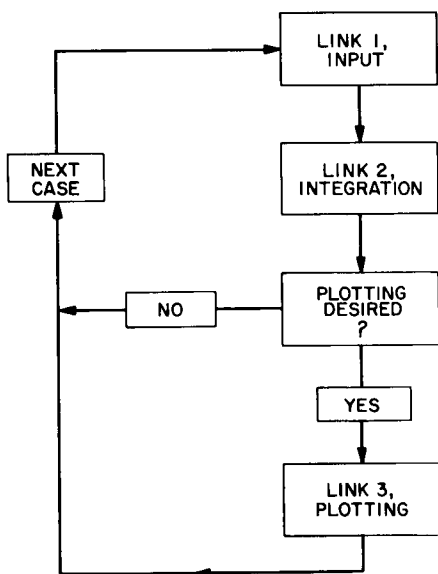


Fig. 5. Program main flow

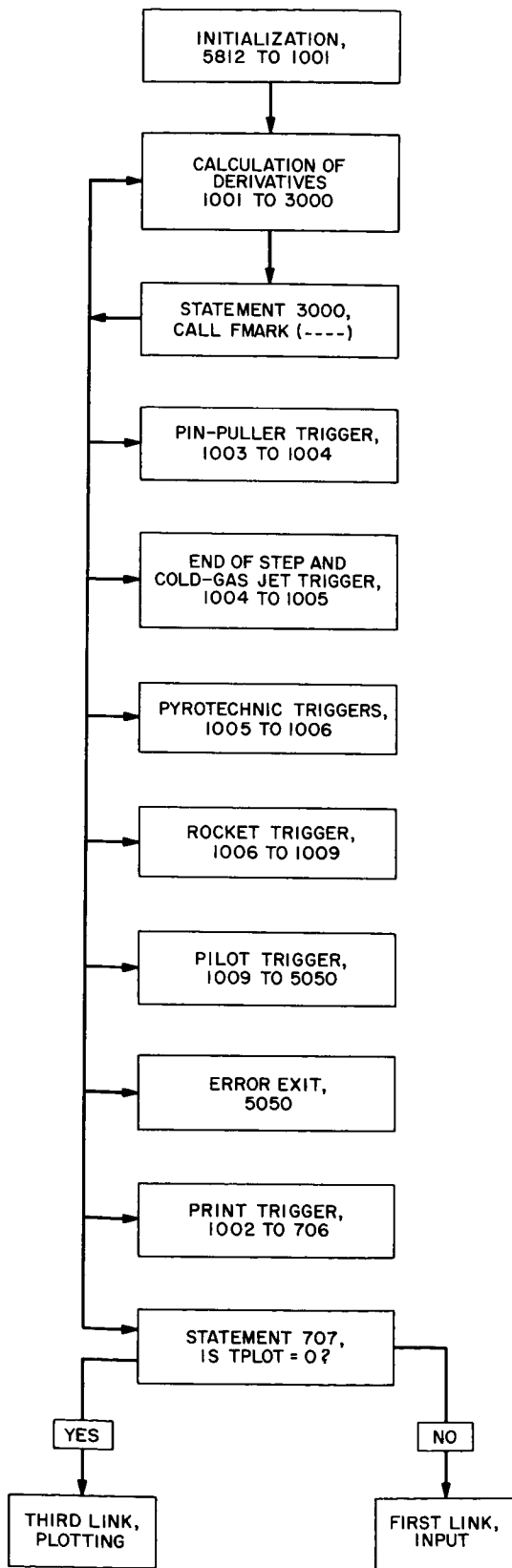


Fig. 6. Flow of integration link

B. Integration Link

1. Integration Subroutine, FMARK

The FMARK integration subroutine (statement 3000) applies the Adams-Moulton method with automatic error control. The computed GO TO statement, following the call to FMARK, is an integral part of the subroutine. The variables in the calling sequence are defined as follows:

| | |
|---------------------|--|
| KIK | fixed-point variable that must correspond to the variable of the computed GO TO statement |
| HB | an array of dimension 6 [i.e., HB (1), . . . , HB (6)] that contains parameters that monitor the Adams-Moulton automatic error control |
| HB (1) = Δt | step size at the start of integration |
| HB (2) = 1 | used to prevent unnecessary reduction in the step size |
| HB (3) = 0.00025 | minimum allowable step size |
| HB (4) = 0.01 | maximum allowable step size |
| HB (5) = 10^{-8} | relative lower bound on truncation error |
| HB (6) = 10^{-6} | relative upper bound on truncation error |
| NH | an array of dimension 6 [i.e., NH (1), . . . , NH (6)] that supplies FMARK with needed parameters |
| NH (1) = 24 | total number of first order differential equations |
| NH (2) = NH (1) | |
| NH (3) | not used |
| NH (4) | not used |
| NH (5) = 4 | number of backward differences used |
| NH (6) | not used |
| IVP | set to zero, denoting that the independent variable is carried in single precision |
| IPHI | set to 4 denoting Adams-Moulton automatic error control mode |
| 1,1 | first value of 1, in the calling sequence, used to supply the location |

(to FMARK) of the portion of the program where the derivatives are calculated (FMARK sets KIK = 1, then executes the computed GO TO statement); second value of 1 needed to suppress option in FMARK

- 2,2 first value of 2, in the calling sequence, used to supply the location (to FMARK) of the *end of step area* (FMARK sets KIK = 2 to transfer to the *end of step area*); second value of 2 is used to initialize triggers
- (I, TRG) 8 sets of variables containing a fixed point and a floating point number that define a *trigger* (when T = TRG, KIK set = to I; computed GO TO statement is executed)
- 0 denotes end of calling sequence

When FMARK is entered (i.e., when statement 3000 is executed) it expects the *initial* value for the independent variable T stored in the *highest* location in COMMON. Since T could be carried in double precision, a dummy variable (ZERO) is adjacent to T. FMARK expects the 24 initial values of the dependent variables to be stored in COMMON just below T and ZERO. FMARK will transfer control to the area of the program where the derivatives are to be calculated; it expects these derivatives supplied in COMMON just below the values of the independent variables. Thus, the first COMMON of this link is

COMMON T, ZERO, V1, V2, YDOT1, YDOT2

DIMENSION V1 (18), V2 (6), YDOT1 (18), YDOT2 (6)

where YDOT1 (I) is the derivative of V1 (I) and the derivative area of the program computes YDOT1, YDOT2 using V1, V2. After the derivatives have been calculated and placed in COMMON, control is returned to FMARK by

CALL ROUT(0)

Triggers can be either *on* or *off*. Turning a trigger on can be accomplished by

CALL TRMOD(I,1)

where I is the number of the trigger in the calling sequence (the first trigger is number 0). A trigger is turned off by

CALL TRMOD(I,0)

Control is returned to FMARK from a trigger by

CALL ROUT(0)

The following is a list of the triggers:

| Trigger No. | Calling sequence designation | Program location | Function |
|-------------|------------------------------|------------------|---------------------------|
| 0 | 2,TRIG3 | 1004 | Cold gas on |
| 1 | 3,TRIG2 | 1003 | Pin-puller constraint off |
| 2 | 4,TRIG1 | 1002 | Print output |
| 3 | 5,TRIG4 | 1005 | First pyrotechnic shot |
| 4 | 6,TROC | 1006 | Rockets on and off |
| 5 | 7,PYRO2 | 1007 | Second pyrotechnic shot |
| 6 | 8,PYRO3 | 1008 | Third pyrotechnic shot |
| 7 | 9,PLOT | 1009 | Plot output |

In certain cases, control is not returned to FMARK by calling ROUT(0). If a discontinuity has occurred in a derivative (e.g., pyrotechnic), the program again calls FMARK. This is done to recompute the backward differences.

The main function of the end of step area is to verify that θ is not near zero. If θ is near zero, the transformation is made and the integration continues.

2. Main Program

The DO loop that ends at statement 5812 puts the hard-mounted spring locations and directions in a more convenient form for later computation. From statement 5812 to statement 6677, the maximum integration time (WIL1) is tested to see if the integration will be terminated when the forces become zero (yes IFORCE = -1, no IFORCE = 0). Triggers are set up and turned on from 55 to 704. The program, up to the call to subroutine GET(1001), sets up initial conditions for the dependent variables and the arrays of HB and NH along with miscellaneous sentinels.

The area of the link beginning with statement 1001 and ending at statement 6007 calculates the derivatives for FMARK. Just before 3000, there is logic to either CALL ROUT(0), the normal return to FMARK from the derivative area, or CALL FMARK(. . .). The latter branch is taken the first time through the derivative area and when a discontinuity has occurred in the derivatives.

The function of GET(1001) (which communicates to the link through COMMON) is to compute the transfor-

mation matrixes A_I, A_{II} , and \dot{A}_I, \dot{A}_{II} , coefficients m_{ij} and terms p_i for the unconstrained equations of motion and, if necessary, the matrix F and terms r_i for the constrained equations of motion. After GET, the link checks the force indicators (3032 to 67) and computes the necessary forces. The twelve force components are computed from the particular force components in the DO loop starting at 69. If $ICLAG \leq 0$, the constrained case must be solved (414). This is done from statements 74 up to 75. The subroutine SOLVE finds the solution for the 15 linear equations of the constrained case. The derivatives for the unconstrained case are computed from 75 to 6007.

The remainder of the link (the area below statement 3000) contains the triggers and the end of step area. Control is transferred to 1003 when the pin-puller constraint terminates. In some cases, the spring constraint will be used after the pin-puller constraint. Thus, a test is made for the spring constraint ($N \geq 2$ yes). $ICLAG$ is set to the proper value and a printout is made.

Statement 1004 serves as the cold-gas jet trigger and the end of step area. The array TCO contains the time cold-gas jets are turned on—in ascending order. The fixed-point variable (ICOLD) denotes the next jet to be turned on; thus $ICOLD = 1, ICG$. If the trigger has just been activated [i.e., $T = TC(ICOLD)$], control is transferred to 8010. Statement 8007 is executed after the last jet has been turned on, otherwise 8006 is executed. In either case, $LDER = 1$ for a restart. If $LDER < 0(200)$, a force has just ceased and a restart has to be made. If $ISS(1) + ISS(2) \neq 0(6519)$, the spring constraint is acting and a spring must be switched. The area of the program from statement 6522 up to 1005 checks the size of θ_I and θ_{II} . If either value of θ is close to zero, a rotation of the body axes is made. Exit to FMARK is made when statement 426 transfers to 403.

The first pyrotechnic trigger starts at 1005, the second at 1007, and the third at 1008. The logic assumes that the triggers are fixed in chronological order. The firing of the triggers is completed in Δt sec (Runge-Kutta integration is used over this interval). The rocket trigger is placed at statement 1006, and the plot trigger is at 1009. For the plot trigger, all possible variables are output on tape B1; they will be read in for plotting in the next link. Statement 5050 is executed if an error occurs in FMARK. A typical error would be an activated trigger that has an execution time less than that of the current value of the independent variable.

The output trigger is placed at 1002. The subroutine STEP enters FMARK and finds the current step size.

Body axis rates are computed and placed in the array CB (301). Other variables computed are speed (V), rate about instantaneous axis (W1), and magnitude of body axis rate vector (WP1). Separation distance and velocity are computed below 350. If a constraint is working, the constraint values FQQ1, FQQ2, and FQQ3 are printed (6663). The constraint values correspond to d_{ix}, d_{iy} , and d_{iz} for the pin-puller and $g_x, g_z, e_{ax}e_{bx} - e_{az}e_{bz}$ for the spring constraint. If $IHY \neq 0$, the transformation matrixes (A_I, A_{II}) are output (6670). The logic from 8001 to 6501 controls the termination of the program. If $IFORCE = 0$, the job terminates only when the independent variable, T, is greater than WIL1 or when all forces cease. A force must be on ($SUN \neq 0$) before termination for null force ($SUN = 0$). If $TPLOT = 0(706)$, plotting is not done, and control returns to the input link. Statement 5000 prints input errors. Control goes to 5002 if a 3×3 matrix used to find the rotational acceleration is singular.

3. Other Subroutines

a. Constraint subroutine. The subroutine GET calculates those parameters needed to find the acceleration components, the rotational matrixes ($A_I, A_{II}, \dot{A}_I, \dot{A}_{II}$) and, if necessary, the 15×15 constraint matrix F.

Initially, the subroutine sets $IM = 0$ to denote calculations for body I; $IM = 1$ denotes calculations for body II. After computations for body I trigonometric functions (after 5004), A and AD are computed—note that $IM2 = IM + 1$ stores the A_I, \dot{A}_I and A_{II}, \dot{A}_{II} in different areas. A test is made just before 48, and m_{ij} and p_i for either body I or II are computed. The control then goes to statement 41. Between 101 and 3, matrixes needed for the constrained case ($ICLAG \leq 0$) are computed (see Table 1). At 104, IM is tested to see if body II computations have been made. If the computations have been made, control goes to 43 to test whether the constrained or unconstrained case is running. Calculations needed for both pin-puller and spring constraints are made after statement 106, then a test is made to determine which constraint is acting. The subroutine from 111 up to 110 calculates elements of the F matrix and r_1, r_2 , and r_3 —stored in CA (1), CA (2), CA (3)—for the pin-puller. All code below 110 calculates the same values for the spring constraint.

b. Force subroutines. Almost all the routines use the variable LDER, which controls integration restart. At each entry, a test is made in these routines to see if a force terminates. If a force does terminate, a discontinuity can occur in the force function, and an integration restart

Table 1. Identification of matrixes

| Matrix | Statement |
|--|-----------------|
| A_i | A (1,1,1) |
| A_{II} | A (1,1,2) |
| \dot{A}_i | AD (1,1,1) |
| \dot{A}_{II} | AD (1,1,2) |
| $\frac{\partial A_i}{\partial q_k}$ $k = 4,5,6$ | WAP (1,1, K-3) |
| $\frac{\partial A_{II}}{\partial q_k}$ $k = 10,11,12$ | WAP (1,1, K-6) |
| $\frac{d}{dt} \frac{\partial A_i}{\partial q_k}$ $k = 4,5,6$ | WAPP (1,1, K-3) |
| $\frac{d}{dt} \frac{\partial A_{II}}{\partial q_k}$ $k = 10,11,12$ | WAPP (1,1, K-6) |
| r_{ii}' | WRP1 (1,1) |
| r_{im}' | WRP1 (1,2) |
| r_{iII}' | WRP2 (1,1) |
| r_{iIm}' | WRP2 (1,2) |
| ρ_i | WRH (1,1) |
| ρ_m | WRH (1,2) |
| T | TEE (1,1) |
| \dot{T} | TD (1,1) |

must be made. In this case, LDER is set to -1; restart is made when the end of step area is entered.

The input and output calling sequence for the force subroutines is presented for a clearer understanding of the program.

Coulomb drag, input

- D(I) magnitude of i^{th} drag force (D_i)
- DF(I) separation distance beyond which i^{th} drag force ceases (d_{if})
- XC1(I), YC1(I), ZC1(I) location of i^{th} drag force on body I in body I CS (x'_{ii} , y'_{ii} , z'_{ii})
- XC2(I), YC2(I), ZC2(I) location of i^{th} drag force on body II in body II CS (x'_{IIi} , y'_{IIi} , z'_{IIi})
- ICD number of drag forces

Coulomb drag, output

- DD(J) component of Q_j due to coulomb drag forces (Q_j)
- LDER = 0, no restart
< 0, restart to be made after end of step

ICD1 number of drag forces that have been turned off

Pyrotechnics, input

- PI(I) total impulse of i^{th} pyrotechnic device (I_i)
- TP(I) firing time of i^{th} device (t_i)
- XP2(I), YP2(I), ZP2(I) location of i^{th} device on body II in body II CS (x'_{IIi} , y'_{IIi} , z'_{IIi})
- UPX(I), UPY(I), UPZ(I) direction cosine vector of the i^{th} device in body II CS (ρ_{xi} , ρ_{yi} , ρ_{zi})
- IPR number of devices

Pyrotechnics, output

EP(J) component of Q_j due to pyrotechnics (\mathcal{E}_j)

Cold-gas jets, input

- FG1(I), FG2(I) characteristic force constants of jet i (f_{i1} , f_{i2})
- FG3 gas-system characteristic (f_3)
- UGX(I), UGY(I), UGZ(I) cosines of angles between thrust vector for jet i and body I axis (λ_{ix} , λ_{iy} , λ_{iz})
- VGX(I), VGY(I), VGZ(I) cosines of angles between thrust vector for jet i and body II axis (ρ_{ix} , ρ_{iy} , ρ_{iz})
- XG1(I), YG1(I), ZG1(I) location of i^{th} jet on body I in body I CS (x'_{ii} , y'_{ii} , z'_{ii})
- XG2(I), YG2(I), ZG2(I) location of i^{th} jet on body II in body II CS (x'_{IIi} , y'_{IIi} , z'_{IIi})
- GST value of jet force below which J_i may be considered zero (J_{st})
- TCG(I) time which i^{th} jet cuts off (t_{if})
- TCO(I) time which i^{th} jet begins (t_{ie})
- ICG number of jets

Cold-gas jets, output

GJ (J) component of Q_j due to cold-gas forces (G_j)
 LDER =0, no restart
 <0, restart to be made after end of step
 ICGI number of jets that have been turned off

Pneumatic forces, input

JN regulator existence index: if 0, there is no regulator; if 1, there is a regulator that remains closed after $p_b = p_p$; if 2, there is a regulator that remains open after $p_b = p_p$ (J)
 PBO initial bottle pressure (p_{b0})
 VBN volume of gas bottle, including all lines upstream of the faces of the closed pistons (V_b)
 GAMN ratio of specific heats for working fluid (γ)
 AN (I) area of i^{th} piston face (A_i)
 RAX (I) distance at which i^{th} piston separates ($j_{max, i}$)
 VNX (I), VNY (I), VNZ (I) cosines of angles between piston rod i and the x'_{II} , y'_{II} , z'_{II} axes (ρ_{ix} , ρ_{iy} , ρ_{iz})
 XN1 (I), YN1 (I), ZN1 (I) location of i^{th} piston on body I in body I CS (x'_{Ii} , y'_{Ii} , z'_{Ii})
 XN2 (I), YN2 (I), ZN2 (I) location of i^{th} piston on body II in body II CS (x'_{IIi} , y'_{IIi} , z'_{IIi})
 PPN (I) $(p_{pi}) A_i$
 EQP (I) $\dot{j}_{eqp, i}$
 PNEM (I) $(\dot{j}_{eqp, i}) A_i$
 IPN number of pneumatic forces

Pneumatic forces, output

XN (J) component of Q_j due to pneumatic forces (N_j)
 LDER =0, no restart
 <0, restart to be made after end of step
 IPN1 number of pneumatics that have been turned off

Rockets, input

IRK number of rockets
 RR (I) magnitude of thrust developed by i^{th} rocket (R_i)
 TE (I) time at which i^{th} rocket starts firing (t_{ie})
 TF (I) time at which i^{th} rocket ceases firing (t_{if})
 RUM the sum $\Sigma \dot{m}_{Ii}$
 PUM the sum $\Sigma \dot{m}_{IIi}$
 URX (I), URY (I), URZ (I) cosines of angles between thrust vector for rocket i and the x'_I , y'_I , z'_I axes (λ_{ix} , λ_{iy} , λ_{iz})
 VRX (I), VRY (I), VRZ (I) cosines of angles between thrust vector for rocket i and the x'_{II} , y'_{II} , z'_{II} axes (ρ_{ix} , ρ_{iy} , ρ_{iz})
 FM10, FM20 initial values of mass
 XR1 (I), YR1 (I), ZR1 (I) location of i^{th} rocket on body I in body I CS (x'_{Ii} , y'_{Ii} , z'_{Ii})
 XR2 (I), YR2 (I), ZR2 (I) location of i^{th} rocket on body II in body II CS (x'_{IIi} , y'_{IIi} , z'_{IIi})
 SUN1 the sum $\Sigma \dot{I}_{zzIi}$
 SUN2 the sum $\Sigma \dot{I}_{yyIi}$
 SUN3 the sum $\Sigma \dot{I}_{zzIIi}$
 SAN1 the sum $\Sigma \dot{I}_{xxIIi}$

SAN2 the sum $\Sigma \dot{I}_{yyIIi}$
 SAN3 the sum $\Sigma \dot{I}_{zzIIi}$
 XX10, YY10, ZZ10 initial values of moments of inertia, body I
 XX20, YY20, ZZ20 initial values of moments of inertia, body II

Rockets, output

XX1, YY1, ZZ1 computed values of moments of inertia, body I
 XX2, YY2, ZZ2 computed values of moments of inertia, body II
 FM1, FM2 computed values of mass (m_i), (m_{II})
 R (J) component of \mathcal{Q}_j due to rocket forces (\mathcal{R}_j)

Triggers, rather than the variable LDER, are used to initiate restarts for the rocket forces.

Hard-mounted springs, input

WRP1 (1,I), WRP1 (2,I), WRP1 (3,I) location of i^{th} spring on body I in body I CS (x'_{ii}), (y'_{ii}), (z'_{ii})
 WRP2 (1,I), WRP2 (2,I), WRP2 (3,I) location of i^{th} spring on body II in body II CS (x'_{IIi}), (y'_{IIi}), (z'_{IIi})
 WRH (1,I), WRH (2,I), WRH (3,I) direction cosines between spring i and body II axes (ρ_{iix}), (ρ_{iIy}), (ρ_{iIz})
 SO (I), SK (I), SIOQ initial force, spring constant, and residual force for i^{th} spring (corrected for spring efficiency— $\eta^2 S_{i0}$, $\eta^2 k_i$, $\eta^2 S_{if}$)
 ISP number of springs
 N restriction index

Hard-mounted springs, output

ICLAG set to 1 when constraint goes off
 S (J) component of \mathcal{Q}_j due to body II hard-mounted springs (\mathcal{F}_j)

LDER =0, no restart
 <0, restart to be made after end of step
 ISPI number of springs that have come off
 ISS (1), ISS (2) values controlling interchange of springs in constraint condition, in the case that the springs being used for constraint are not the last two to cease acting; if ISS (1) = ISS (2) = 0, no transfer required—if ISS (1) or ISS (2) is non-zero, transfer made by subroutine SWITCH

Universally-jointed springs, input

ISU number of springs
 WO (I), UK (I), WF (I) initial force, spring constant, and final force of spring i (corrected for spring efficiency— $\eta^2 W_{i0}$, $\eta^2 k_i$, $\eta^2 W_{if}$)
 XU1 (I), YU1 (I), ZU1 (I) spring location on body I in body I CS (x'_{iI}), (y'_{iI}), (z'_{iI})
 XU2 (I), YU2 (I), ZU2 (I) spring location on body II in body II CS (x'_{IIi}), (y'_{IIi}), (z'_{IIi})

Universally-jointed springs, output

W (J) component of \mathcal{Q}_j due to universally-jointed springs (\mathcal{W}_j)
 LDER =0, no restart
 <0, restart to be made after end of step
 ISU1 number of springs that have come off

c. Matrix algebra subroutines. To aid in these calculations, the following subroutines are used:

MULT1 (A,B,C,M1,M2,M3) calculates $A^T B = C$ where A is $M1 \times M2$, B is $M2 \times M3$
 MULT2 (A,B,C,M1,M2,M3) calculates $A B^T = C$ where A is $M1 \times M2$, B is $M2 \times M3$
 MULT3 (A,B,C,M1,M2,M3) calculates $A B = C$ where A is $M1 \times M2$, B is $M2 \times M3$

| | | | |
|--------------------|---|---------------|---|
| MCR (A,B,C,J) | forms the dot product of the 3×1 vector B with the j^{th} row of the 3×3 matrix A and stores the result in C | MCROS (A,B,C) | forms $A \times B = C$ where A,B,C are dimensioned 3×1 (15×15 linear set for the constraint is solved by subroutine SOLVE ²) |
| MCS (A,B,C,J) | forms the dot product of the 3×1 vector B with the j^{th} column of the 3×3 matrix B and stores the result in C | | |
| MADD (A,B,C,M1,M2) | forms $A + B = C$ with dimension $M1 \times M2$ | | |
| MSUB (A,B,C,M1,M2) | forms $A - B = C$ with dimension $M1 \times M2$ | | |
| MSCAL (A,B,C) | forms $AB = C$ where A is 1×1 , B is 3×1 | | |

C. Plotting Link

All of the values that can be plotted are written on tape B1. At the beginning of the plotting link, plot indicators are written into the array LOT. If variable I is to be plotted, LOT (I) is nonzero.

²Moler, C. B., *Numerical Matrix Inversion With Iterative Improvement*, Technical Report No. 32-394, Jet Propulsion Laboratory, Pasadena, California, April 15, 1963.

X. INPUT DESCRIPTION AND FORMAT

The input format is given in the form of a series of tables, some of which may be omitted, depending on whether or not the corresponding type of force is to act. A complete list of input symbols and dimensions is collected here for reference.

A. Input List With Dimensions

1. General Inertial and Geometrical Characteristics

| | |
|--|---|
| $I_{xxI}, I_{yyI}, \dots, I_{zzII}$ | elements of inertia matrixes in the drawing board CS: lbm-in. ² |
| i_A | transformation matrix indicator: =1, A_I, A_{II} printed; =0, A_I, A_{II} not printed |
| m_I, m_{II} | masses: lbm |
| Δt | initial integration interval: sec |
| t_{PR} | printout interval: sec |
| t_0 | time at which program begins: sec |
| $\epsilon''_{Ix}, \dots, \epsilon''_{IIz}$ | coordinates in the drawing board CS of the CM of bodies I and II: in. |
| $\omega''_x, \omega''_y, \omega''_z$ | rates about drawing board axes just before separation: deg/sec |

$\omega_{Iy'0}$ spinup roll of body I relative to body II: deg/sec

2. Forces

| | |
|----------|---|
| i_{CD} | coulomb drag force indicator, number of drag forces present: up to 8 |
| i_{PY} | pyrotechnic force indicator, number of pyrotechnic devices present: up to 3 |
| i_{CG} | cold-gas jet indicator, number of cold-gas jets: up to 8 |
| i_{PN} | pneumatics indicator, number of pneumatic pistons: up to 8 |
| i_{RK} | rockets indicator, number of rockets: up to 8 |
| i_{SP} | spring indicator, number of hard-mounted springs: up to 8 |
| i_{SU} | universally-jointed spring indicator, number of springs: up to 8 |
| i_{CN} | constraint index: if =0, no constraint; if =1, pin-puller constraint is present |

a. Coulomb drag

D_i magnitude of i^{th} drag force: lbf
 d_{if} separation distance beyond which i^{th} drag force ceases: in.
 x'_i, y'_i, z'_i drawing board locations of i^{th} drag force: in.

b. Pyrotechnics

I_i total impulse of i^{th} device: lbf-sec
 t_i firing time of i^{th} device: sec (program requires that $t_1 \leq t_2 \leq t_3$)
 x'_i, y'_i, z'_i pyrotechnic locations: in.
 $\mu'_{ix}, \mu'_{iy}, \mu'_{iz}$ cosines of the acute angles between the drawing board axes and the i^{th} device, except that μ'_{iy} is negative

c. Cold-gas jets

f_{i1}, f_{i2}, f_{i3} constants for jet i depending on initial conditions, nozzle geometry, and properties of working fluid: lbf, 1/sec, 1
 J_{ST} value of jet force below which J_i may be considered zero: lbf
 t_{i1} time at which i^{th} jet turns on: sec
 t_{i2} time at which i^{th} jet cuts off: sec
 x'_i, y'_i, z'_i cold-gas jet locations: in.
 $\mu_{ix}, \mu_{iy}, \mu_{iz}$ cosines of angles between thrust vector for jet i and body I axes
 $\mu'_{ix}, \mu'_{iy}, \mu'_{iz}$ cosines of angles between thrust vector for jet i and body II axes

d. Pneumatics

A_i area of i^{th} piston face: in.²
 J regulator existence index: if 0, there is no regulator; if 1, there is a regulator that remains closed after $p_b = p_p$; if 2, there is a regulator that remains open after $p_b = p_p$
 $j_{max, i}$ distance at which i^{th} piston separates: in.
 p_{b0} initial pressure in gas bottle: lbf (absolute)/in.²

p_{bi} pressure beyond regulator (if any) for i^{th} piston: lbf (absolute)/in.²
 V_b volume of gas bottle, including all lines upstream of the faces of the closed pistons: in.³
 x'_i, y'_i, z'_i pneumatics locations: in.
 γ ratio of specific heats for working fluid
 $\mu_{ix}, \mu_{iy}, \mu_{iz}$ cosines of angles between piston rod i and the $x'_{II}, y'_{II}, z'_{II}$ axes

e. Rockets

$\dot{I}_{xxIIi}, \dots, \dot{I}_{zzIIi}$ rate of decrease of principal moments of inertia due to rocket i : lbm in.²/sec
 $\dot{m}_{Ii}, \dot{m}_{IIi}$ mass decrease rate due to i^{th} rocket: lbf/sec
 R_i magnitude of thrust developed by i^{th} rocket: lbf
 t_{ie} time at which i^{th} rocket starts firing: sec
 t_{if} time at which i^{th} rocket ceases firing: sec
 x'_i, y'_i, z'_i rocket locations: in.
 $\mu_{ix}, \mu_{iy}, \mu_{iz}$ cosines of angles between thrust vector for rocket i and the drawing board axes (μ if on body I, μ' if on body II; if on body I, all $\mu'_i = 0$, if on body II, all $\mu_i = 0$)

f. Hard-mounted springs

k_i i^{th} spring constant: lbf/in.
 N restriction index (if $N = 0$, there is no constraint; if $N > 1$, the constraint is operating)
 S_{i0} initial force in i^{th} spring: lbf
 x'_i, y'_i, z'_i spring location: in.
 η spring efficiency factor
 $\mu'_{ix}, \mu'_{iy}, \mu'_{iz}$ cosines of angles between spring i and body II axes

g. Universally-jointed springs

k_i spring rate of i^{th} spring: lbf/in.

- W_{i0} initial force in i^{th} spring: lbf
- $x'_{i1}, y'_{i1}, z'_{i1}$ location of end of spring i in body I: in.
- $x'_{i2}, y'_{i2}, z'_{i2}$ location of end of spring i in body II: in.
- η spring efficiency factor

h. Constraints

- t_G time of firing of delayed pin-puller: sec
- x'_P, y'_P, z'_P location of delayed pin-puller: in.

i. Plots

- TPLOT time increment for plotting: sec
- LOT _{i} plot indicators: a nonzero value calls for plot i
- NCAMERA camera 1 requires photographic processing; camera 2 is a fast-copy device

given in its corresponding table, and the groups are to be stacked according to the order of the tables. Table 2 gives the mass, geometry, and indicator input, and Tables 3 through 9 give the force inputs. If a particular force is not to act(i.e., its indicator number is zero) its table and corresponding cards are to be omitted. Table 10 gives the pin-puller constraint input. Table 11 gives the plotting directions and completes the input, and Table 12 defines the plot indicators.

Table 2. Mass, geometry, indicators

| Format type | Input | | | | | |
|---------------|-------------------------------|------------------|------------------|-------------------|-------------------|-------------------|
| (2X, A70) | Comment (up to 70 characters) | | | | | |
| (E12.6, 2I12) | t_{CUT} | Case No. | i_A | | | |
| 2 | I_{xxI} | I_{xyI} | I_{xzI} | I_{yyI} | I_{yzI} | I_{zzI} |
| 2 | I_{xxII} | I_{xyII} | I_{xzII} | I_{yyII} | I_{yzII} | I_{zzII} |
| 2 | m_I | m_{II} | Δt | t_{PR} | t_0 | |
| 2 | $\epsilon_{x'I}$ | $\epsilon_{y'I}$ | $\epsilon_{z'I}$ | $\epsilon_{x'II}$ | $\epsilon_{y'II}$ | $\epsilon_{z'II}$ |
| 2 | $\omega_{x''}$ | $\omega_{y''}$ | $\omega_{z''}$ | $\omega_{Iy'0}$ | | |
| 1 | i_{CD} | i_{PY} | i_{CG} | i_{PN} | i_{RK} | i_{SP} |
| 1 | i_{SU} | i_{CN} | | | | |

Table 3. Coulomb drag

| Format type | Input* | | | | | |
|-------------|----------|-------|---------|---------|---------|--|
| 2 | d_{1f} | D_1 | x_1'' | y_1'' | z_1'' | |
| 2 | d_{2f} | D_2 | x_2'' | y_2'' | z_2'' | |
| . | | | | | | |
| . | | | | | | |
| 2 | d_{kf} | D_k | x_k'' | y_k'' | z_k'' | |

*k = i_{CD}

Table 4. Pyrotechnics

| Format type | Input* | | | | | |
|-------------|-------------|-------------|-------------|---------|---------|---------|
| 2 | I_1 | t_1 | | | | |
| 2 | $\mu_{1x'}$ | $\mu_{1y'}$ | $\mu_{1z'}$ | x_1'' | y_1'' | z_1'' |
| . | | | | | | |
| . | | | | | | |
| 2 | I_k | t_k | | | | |
| 2 | $\mu_{kx'}$ | $\mu_{ky'}$ | $\mu_{kz'}$ | x_k'' | y_k'' | z_k'' |

*k = i_{PY}

B. Input Format

Input data takes the form of a data card deck. To simplify input procedure, only three types of card format are used (with the exception of the first two cards).

For format type 1, the FORTRAN code is 6I12. These cards will contain up to six numbers per card, and each number is allotted 12 columns. The numbers must be integers (without a decimal point) and right adjusted.

For format type 2, the FORTRAN code is 6E12.8. These cards also will contain up to six numbers per card, with each number allotted 12 columns. These numbers are not integers (must have a decimal point), need not be right adjusted, and may have an exponent.

For format type 3, the FORTRAN code is 14I5. These cards are similar to those of format type 1, except that each number is allotted only 5 columns.

The input arrays for each group of input data are given in Tables 2 through 12. Each group is to be ordered as

Table 5. Cold-gas jets

| Format type | Input ^a (body I) | | | | | |
|-------------|-----------------------------|----------------|----------------|---------------|-------------|-------------|
| 1 | 1 | k | | | | |
| 2 | f_{11} | f_{12} | t_{11} | t_{12} | | |
| 2 | μ_{1x} | μ_{1y} | μ_{1z} | x_1'' | y_1'' | z_1'' |
| 2 | f_{21} | f_{22} | t_{21} | t_{22} | | |
| 2 | μ_{2x} | μ_{2y} | μ_{2z} | x_2'' | y_2'' | z_2'' |
| . | . | . | . | . | . | . |
| 2 | f_{k1} | f_{k2} | t_{k1} | t_{k2} | | |
| 2 | μ_{kx} | μ_{ky} | μ_{kz} | x_k'' | y_k'' | z_k'' |
| (body II) | | | | | | |
| 1 | 2 | l | | | | |
| 2 | $f^{(k+1)1}$ | $f^{(k+1)2}$ | $t^{(k+1),1}$ | $t^{(k+1),2}$ | | |
| 2 | $\mu^{(k+1)x}$ | $\mu^{(k+1)y}$ | $\mu^{(k+1)z}$ | x_{k+1}'' | y_{k+1}'' | z_{k+1}'' |
| . | . | . | . | . | . | . |
| 2 | $f^{(k+l)1}$ | $f^{(k+l)2}$ | $t^{(k+l),1}$ | $t^{(k+l),2}$ | | |
| 2 | $\mu^{(k+l)x}$ | $\mu^{(k+l)y}$ | $\mu^{(k+l)z}$ | x_{k+l}'' | y_{k+l}'' | z_{k+l}'' |

^ak = number of jets on body I; l = number of jets on body II

Table 7. Rockets

| Format type | Input ^a | | | | | |
|-------------|--------------------|------------------|------------------|------------------|------------------|------------------|
| 2 | μ_{1x} | μ_{1y} | μ_{1z} | μ_{1x}' | μ_{1y}' | μ_{1z}' |
| 2 | μ_{2x} | μ_{2y} | μ_{2z} | μ_{2x}' | μ_{2y}' | μ_{2z}' |
| . | . | . | . | . | . | . |
| 2 | μ_{kx} | μ_{ky} | μ_{kz} | μ_{kx}' | μ_{ky}' | μ_{kz}' |
| 2 | t_{10} | t_{1f} | | | | |
| 2 | t_{20} | t_{2f} | | | | |
| . | . | . | . | . | . | . |
| 2 | \dot{t}_{k0} | \dot{t}_{kf} | | | | |
| 2 | \dot{I}_{xx11} | \dot{I}_{yy11} | \dot{I}_{zz11} | \dot{I}_{xx11} | \dot{I}_{yy11} | \dot{I}_{zz11} |
| 2 | \dot{I}_{xx12} | \dot{I}_{yy12} | \dot{I}_{zz12} | \dot{I}_{xx12} | \dot{I}_{yy12} | \dot{I}_{zz12} |
| . | . | . | . | . | . | . |
| 2 | \dot{I}_{xx1k} | \dot{I}_{yy1k} | \dot{I}_{zz1k} | \dot{I}_{xx1k} | \dot{I}_{yy1k} | \dot{I}_{zz1k} |
| 2 | \dot{m}_{11} | \dot{m}_{111} | R_1 | x_1'' | y_1'' | z_1'' |
| 2 | \dot{m}_{21} | \dot{m}_{211} | R_2 | x_2'' | y_2'' | z_2'' |
| . | . | . | . | . | . | . |
| 2 | \dot{m}_{k1} | \dot{m}_{k11} | R_k | x_k'' | y_k'' | z_k'' |

^ak = i_{RK}

Table 6. Pneumatics

| Format type | Input ^a | | | | | |
|-------------|--------------------|-------------|-------------|---------|---------|---------|
| 1 | J | | | | | |
| 2 | p_{b0} | V_b | γ | | | |
| 2 | A_1 | $j_{max,1}$ | p_{p1} | | | |
| 2 | μ_{1x}' | μ_{1y}' | μ_{1z}' | x_1'' | y_1'' | z_1'' |
| 2 | A_2 | $j_{max,2}$ | p_{p2} | | | |
| 2 | μ_{2x}' | μ_{2y}' | μ_{2z}' | x_2'' | y_2'' | z_2'' |
| . | . | . | . | . | . | . |
| 2 | A_k | $j_{max,k}$ | p_{pk} | | | |
| 2 | μ_{kx}' | μ_{ky}' | μ_{kz}' | x_k'' | y_k'' | z_k'' |

^ak = i_{PN}

Table 8. Hard-mounted springs on body II

| Format type | Input ^a | | | | | |
|-------------|--------------------|-------------|-------------|---------|---------|---------|
| 2 | η | | | | | |
| 1 | N | | | | | |
| 2 | μ_{1x}' | μ_{1y}' | μ_{1z}' | x_1'' | y_1'' | z_1'' |
| 2 | S_{10} | k_1 | S_{1f} | | | |
| 2 | μ_{2x}' | μ_{2y}' | μ_{2z}' | x_2'' | y_2'' | z_2'' |
| 2 | S_{20} | k_2 | S_{2f} | | | |
| . | . | . | . | . | . | . |
| 2 | μ_{ix}' | μ_{iy}' | μ_{iz}' | x_i'' | y_i'' | z_i'' |
| 2 | S_{i0} | k_i | S_{if} | | | |

^ai = i_{SP}

Table 9. Universally-jointed springs

| Format type | Input ^a | | | | | |
|-------------|--------------------|------------|------------|------------|------------|------------|
| 1 | η | | | | | |
| 2 | x_{11}'' | y_{11}'' | z_{11}'' | x_{11}'' | y_{11}'' | z_{11}'' |
| 2 | W_{10} | k_1 | W_{1f} | | | |
| . | . | . | . | . | . | . |
| 2 | x_{i1}'' | y_{i1}'' | z_{i1}'' | x_{i1}'' | y_{i1}'' | z_{i1}'' |
| 2 | W_{i0} | k_i | W_{if} | | | |

^ai = i_{SU}

Table 10. Pin-puller constraint

| Format type | Input | | | |
|-------------|-------|---------|---------|---------|
| 2 | t_G | x_p'' | y_p'' | z_p'' |

Table 11. Plot

| Format type | Input ^a |
|--|---|
| 2 | T PLOT |
| 3 | LOT ₁ LOT ₂ , . . . , LOT ₁₄ |
| ^a Enter T PLOT = 0. if no plots are desired | |

Table 12. Plots—indicators and arguments

| Plot No. i | Type of plot ^a |
|--|--|
| 1 | x_I, y_I, z_I VS t |
| 2 | x_{II}, y_{II}, z_{II} VS t |
| 3 | $\dot{x}_I, \dot{y}_I, \dot{z}_I$ VS t |
| 4 | $\dot{x}_{II}, \dot{y}_{II}, \dot{z}_{II}$ VS t |
| 5 | $\ddot{x}_I, \ddot{y}_I, \ddot{z}_I$ VS t |
| 6 | $\ddot{x}_{II}, \ddot{y}_{II}, \ddot{z}_{II}$ VS t |
| 7 | θ_I, ϕ_I, ψ_I VS t |
| 8 | $\theta_{II}, \phi_{II}, \psi_{II}$ VS t |
| 9 | $\dot{\theta}_I, \dot{\phi}_I, \dot{\psi}_I$ VS t |
| 10 | $\dot{\theta}_{II}, \dot{\phi}_{II}, \dot{\psi}_{II}$ VS t |
| 11 | $\ddot{\theta}_I, \ddot{\phi}_I, \ddot{\psi}_I$ VS t |
| 12 | $\ddot{\theta}_{II}, \ddot{\phi}_{II}, \ddot{\psi}_{II}$ VS t |
| 13 | $\dot{\theta}_I$ VS θ_I $\dot{\phi}_I$ VS ϕ_I $\dot{\psi}_I$ VS ψ_I |
| 14 | $\dot{\theta}_{II}$ VS θ_{II} $\dot{\phi}_{II}$ VS ϕ_{II} $\dot{\psi}_{II}$ VS ψ_{II} |
| ^a Each type of plot is identified by an indicator LOT _i . If LOT _i is not zero, plot type i will be included in the output. | |

XI. OUTPUT DESCRIPTION AND FORMAT

Program output consists of the generalized coordinates, velocities and useful auxiliary quantities. A listing of all output quantities and dimensions is given below.

A. Output List with Dimensions

- d_s separation distance
= $[(x_I - x_{II})^2 + (y_I - y_{II})^2 + (z_I - z_{II})^2]^{1/2}$: in.
- t time: sec
- v_s separation velocity
= $[(\dot{x}_I - \dot{x}_{II})^2 + (\dot{y}_I - \dot{y}_{II})^2 + (\dot{z}_I - \dot{z}_{II})^2]^{1/2}$: in./sec
- v_I speed of CM: in./sec
- x_I, y_I, z_I inertial coordinates: in.
- $\dot{x}_I, \dot{y}_I, \dot{z}_I$ inertial components of velocity of CM: in./sec

- θ_I, ϕ_I, ψ_I Eulerian angles: deg
- $\dot{\theta}_I, \dot{\phi}_I, \dot{\psi}_I$ rates of change of Eulerian angles: deg/sec
- $\omega_x''^I, \omega_y''^I, \omega_z''^I$ rotation rates about drawing board axes: deg/sec
- ω_I rotation rate about instantaneous axis: deg/sec
- ω_{PYI} magnitude of vector sum of pitch and yaw rates: deg/sec

All of the output listed for body I also applies for body II.

B. Output Format

All quantities in the output are self-explanatory; the format is illustrated in the sample problem of Section XIII.

XII. ERROR DIAGNOSIS AND CHECKOUT

Because of the flexibility of the program, there are few error diagnoses that can be made internally. Should an error occur within the integration routine FMARK, the statement "ERROR OCCURS IN FMARK" is printed. Should the (12×12) or (15×15) matrix currently being inverted to solve for the generalized accelerations become singular, an appropriate statement is printed.

XIII. SAMPLE PROBLEM

The purpose of the sample problem is to illustrate the operational use of the computer program. For this reason, the system has been presented in idealized form.

A. Problem Statement

A capsule is to be separated from a spacecraft by means of four springs hard mounted on the spacecraft. The spring tips are constrained not to slip on their bearing surfaces. After separation, the capsule is spun up by means of 3 cold-gas jets. Subsequent to spinup, a retro-rocket is fired. The properties of the bodies and the force mechanisms are as follows (see Fig. 7):

Spacecraft, body II

$$\begin{aligned} m &= 1000 \text{ lbm} \\ I_{xx} &= 200 \text{ slug-ft}^2 \\ I_{yy} &= 205 \text{ slug-ft}^2 \\ I_{zz} &= 100 \text{ slug-ft}^2 \\ I_{xy} &= I_{xz} = I_{yz} = 0 \end{aligned}$$

Capsule, body I

$$\begin{aligned} m &= 1800 \text{ lbm} \\ I_{xx} &= I_{yy} = 250 \text{ slug-ft}^2 \\ I_{zz} &= 100 \text{ slug-ft}^2 \\ I_{xy} &= I_{xz} = I_{yz} = 0 \end{aligned}$$

Rocket motor

$$\begin{aligned} \text{Thrust} &= 100 \text{ lbf (constant)} \\ \text{Starting time} &= \text{separation} + 10 \text{ sec} \\ \text{Duration} &= 5 \text{ sec} \end{aligned}$$

Springs

$$\begin{aligned} k_1 &= k_2 = k_3 = 100 \text{ lbf/in.} \\ k_4 &= 80 \text{ lbf/in.} \\ \text{Initial compression} &= 4 \text{ in.} \\ \text{Final force} &= 0 \end{aligned}$$

The springs are parallel to the z'' axis; they have 100% efficiency. Initial angular velocities are zero.

Gas jets

Each nozzle has its own reservoir and is well insulated. From Eqs. (82) through (84), the properties of the nozzles and the working fluid yield the force law

$$J_i = \frac{50}{(1+t)^{1.5}}, \quad i = 1, 2, 3$$

The jets are to fire at separation +2 sec and continue for 5 sec.

B. Choice of the Drawing Board CS

Since the hard mounted spring constraint is present, the z'' axis must be chosen perpendicular to the separation plane (see Fig. 8).

C. Output

The computer printout for the sample problem is presented in Appendix N, pp. N-79-N-84. Because of the length of the complete printout, only a few typical output data sets are shown.

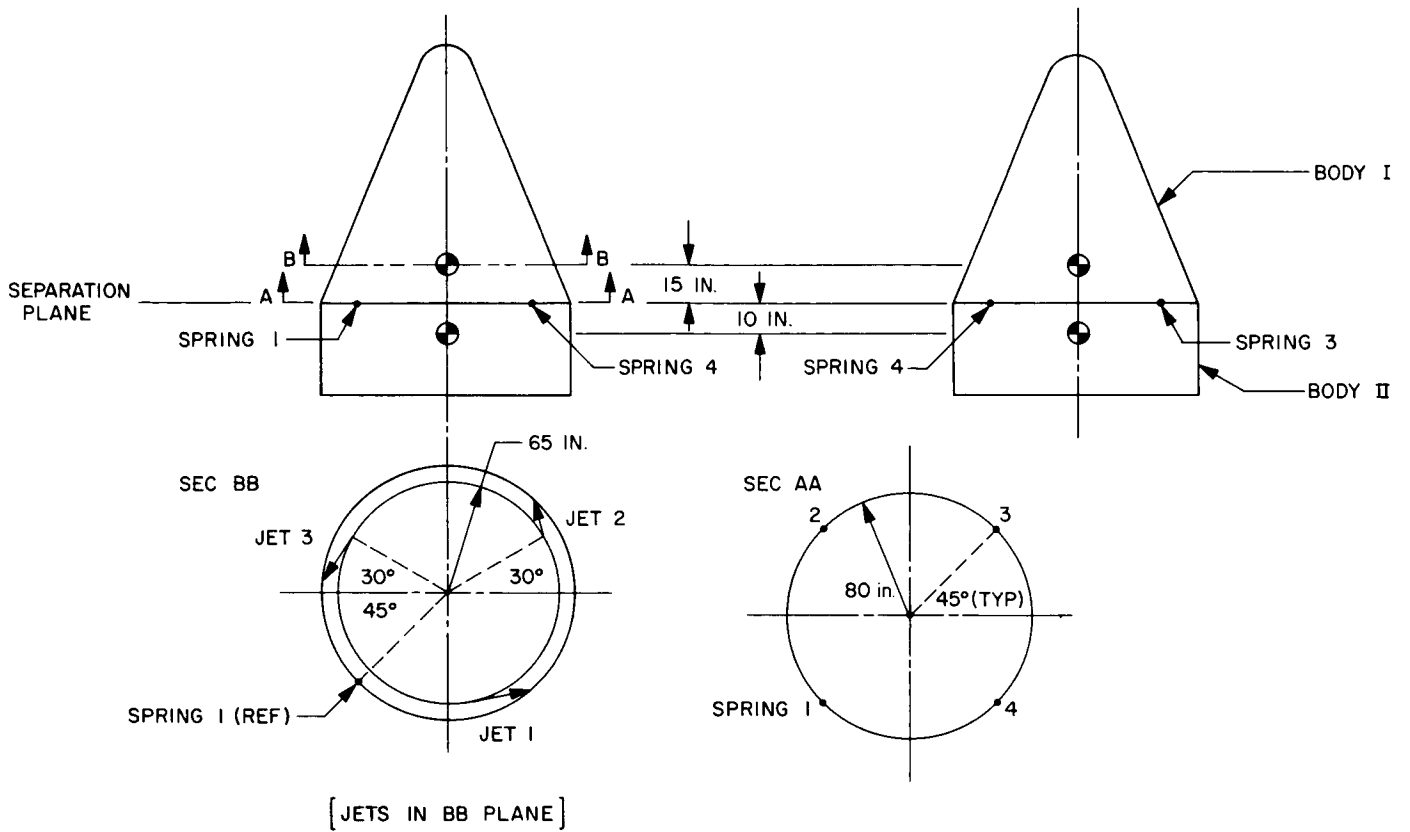


Fig. 7. Problem geometry

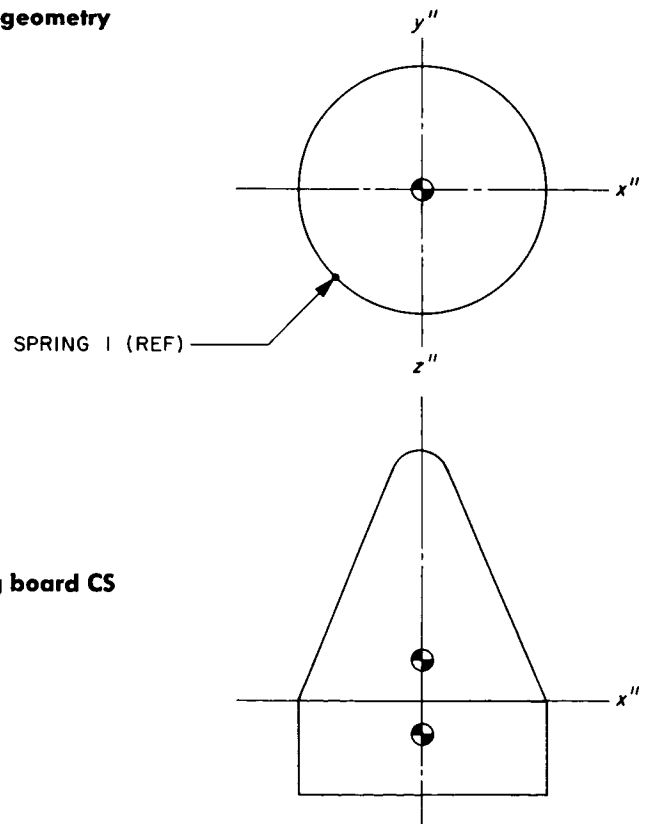


Fig. 8. Selection of drawing board CS

APPENDIX A

Eulerian Angles and Vector Component Transformations

To use the Lagrange equations it is necessary to define three independent parameters that specify the orientation of each body. A common choice for these parameters is the set of Eulerian angles. There is not complete uniformity in the definitions of the Eulerian angles; the definitions used in this report are those of Goldstein.¹

A. Symbols

| | |
|-----------------------------------|---|
| A, B, C, D | rotation matrixes used in describing Eulerian angle generation |
| $A^{-1}, B^{-1}, C^{-1}, D^{-1}$ | inverse of the A, B, C, D matrixes |
| a_{ij} | elements of the A_I matrix (i^{th} row, j^{th} column) |
| b_{ij} | elements of the A_{II} matrix (i^{th} row, j^{th} column) |
| c_{ij} | elements of the A_I or A_{II} matrix (i^{th} row, j^{th} column) |
| r_r | location vector of the point whose index is i |
| v_a | a vector in the xyz CS |
| v_x, v_y, v_z | components of the v_a in the xyz CS |
| $v_{a'}$ | a vector in the $x'y'z'$ CS |
| $v_{x'}, v_{y'}, v_{z'}$ | components of $v_{a'}$ in the $x'y'z'$ CS |
| v_α | a vector in the $\xi\eta\zeta$ CS |
| v_ξ, v_η, v_ζ | components of v_α in the $\xi\eta\zeta$ CS |
| $v_{\alpha'}$ | a vector in the $\xi'\eta'\zeta'$ CS |
| $v_{\xi'}, v_{\eta'}, v_{\zeta'}$ | components of $v_{\alpha'}$ in the $\xi'\eta'\zeta'$ CS |
| x, y, z | the inertial CS |
| x', y', z' | the body-fixed CS |
| ξ, η, ζ | an intermediate CS used in the definition of the Eulerian angles |
| ξ', η', ζ' | another intermediate CS used in the definition of the Eulerian angles |

B. Generation of Eulerian Angles

First, the x, y, z system (in which a vector is denoted v_a with components v_x, v_y, v_z) is rotated by an angle ϕ ,

counterclockwise, about the z axis (Fig. A-1a). The resultant CS is the $\xi\eta\zeta$ system; a vector in these coordinates is denoted v_α with components v_ξ, v_η, v_ζ .

Second, the intermediate axes, $\xi\eta\zeta$, are rotated about the ξ axis, counterclockwise, by the angle θ to produce the $\xi'\eta'\zeta'$ axes (see Fig. A-1b). A vector in this CS, denoted $v_{\alpha'}$, has components $v_{\xi'}, v_{\eta'}, v_{\zeta'}$. The ξ' axis also is known as the line of nodes.

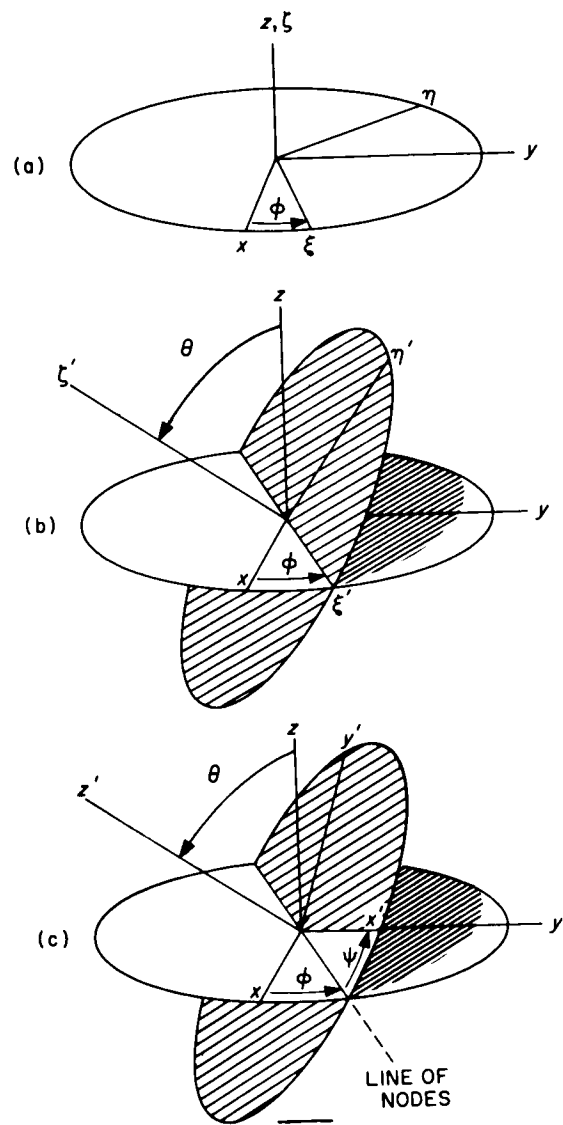


Fig. A-1. Generation of Eulerian angles

¹Goldstein, H., *Classical Mechanics*, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1959, pp. 107-109.

Third, the $\xi'\eta'\zeta'$ axes are rotated counterclockwise by an angle ψ about the ζ' axis to produce the $x'y'z'$ system of axes (see Fig. A-1c). The $x'y'z'$ system is the body-

fixed system, and the xyz system is the space-fixed (or inertial) CS. A vector in the $x'y'z'$ system is denoted $v_{a'}$ with components $v_{x'}, v_{y'}, v_{z'}$.

C. Vector Component Transformations

The elements of the complete rotation matrix **A** can be obtained by writing the matrix as the product of the separate rotation matrixes.

Thus

$$v_a = D v_{a'} \quad \text{where} \quad D = \begin{bmatrix} c\phi & s\phi & 0 \\ -s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A-1)$$

$$v_{a'} = C v_a \quad \text{where} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \quad (A-2)$$

$$v_{a''} = B v_{a'} \quad \text{where} \quad B = \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A-3)$$

$$v_{a''} = BCD v_a = A v_a \quad \text{where} \quad A = BCD \quad (A-4)$$

$$A = \begin{bmatrix} c\phi c\psi - c\theta s\phi s\psi & s\phi c\psi + c\theta c\phi s\psi & s\theta s\psi \\ -c\phi s\psi - c\theta s\phi c\psi & -s\phi s\psi + c\theta c\phi c\psi & s\theta c\psi \\ s\theta s\phi & -s\theta c\phi & c\theta \end{bmatrix} \quad (A-5)$$

For simplicity of notation, the elements of the A_I and A_{II} matrixes are denoted a_{ij} and b_{ij} , respectively. The elements of a nonspecific **A** matrix are denoted c_{ij} .

The inverse transformation is

$$v_a = A^{-1} v_{a'} \quad (A-6)$$

Since the **BCD** matrixes are orthogonal, their product, **A**, is orthogonal, and the inverse of **A** is its transpose.

$$A^{-1} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \quad (A-7)$$

$$A_{II}^{-1} = \begin{bmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} \quad (A-8)$$

The complete transformation for location vectors is

$$r_i = r + A^{-1} r'_i \quad (A-9)$$

$$r'_i = A (r_i - r) \quad (A-10)$$

APPENDIX B

Length of Extension of a Spring Hard-Mounted on Body II

To determine the length of extension of a hard-mounted spring on body II, it is necessary to define the bearing plane in the body I CS, express $\bar{\mathbf{r}}_i$ in the body I CS, and then solve for j_i .

A. Symbols

$\bar{\mathbf{r}}_{IIi}$ location of the tip of spring i in body II CS

\mathbf{T} matrix product $\mathbf{A}_I \mathbf{A}_{II}^T$

Because of the redefinition of the initial CS, the bearing plane is perpendicular to the y'_i axis, and \mathbf{r}'_{IIi} is in the plane. The equation for the plane is

$$y'_i = y'_i \quad (\text{B-1})$$

Clearly

$$\bar{\mathbf{r}}_{IIi} = \mathbf{r}'_{IIi} + j_i \boldsymbol{\rho}_i$$

In the inertial CS

$$\bar{\mathbf{r}}_{IIi} = \mathbf{r}_{II} + \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{A}_{II}^T \boldsymbol{\rho}_i \quad (\text{B-3})$$

In the body I CS

$$\bar{\mathbf{r}}_{Ii} = \mathbf{A}_I (\mathbf{r}_{II} - \mathbf{r}_I) + \mathbf{A}_I \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{A}_I \mathbf{A}_{II}^T \boldsymbol{\rho}_i \quad (\text{B-4})$$

$$\bar{\mathbf{r}}_{Ii} = -\mathbf{A}_I \mathbf{r} + \mathbf{A}_I \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{T}^T \boldsymbol{\rho}_i \quad (\text{B-5})$$

where a substitution of \mathbf{T} has been used selectively for later simplicity.

Now the constraint condition is that $\bar{\mathbf{r}}'_{Ii}$ must have its tip in the bearing plane. Thus

$$y'_{Ii} = -\mathbf{a}_{(2)}^T \mathbf{r} + \mathbf{a}_{(2)}^T \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{t}^{(2)T} \boldsymbol{\rho}_i \quad (\text{B-6})$$

and

$$j_i \mathbf{t}^{(2)T} \boldsymbol{\rho}_i = +y'_{Ii} + \mathbf{a}_{(2)}^T \mathbf{r} - \mathbf{a}_{(2)}^T \mathbf{A}_{II}^T \mathbf{r}'_{IIi} \quad (\text{B-7})$$

The left-hand side of the above equation is simply $\mathbf{a}_2^T \cdot \mathbf{d}_i$ (see Coulomb Drag Section, Eq. 59). Thus

$$j_i = \frac{\mathbf{a}_{(2)}^T \cdot \mathbf{d}_i}{\mathbf{t}^{(2)T} \cdot \boldsymbol{\rho}_i} \quad (\text{B-8})$$

APPENDIX C

Conversions Between Rates About Eulerian Axes and Rates About Body Axes

Since initial angular rates will normally be reported as body-axis angular rates, and since body-axis rates are often needed as the final result, conversions between Euler angle rates and body rates must be established.

A. Symbols

With the exception of those listed below, the symbols used in this appendix are the same as in Appendix A.

- $\boldsymbol{\omega}$ angular velocity
- $\boldsymbol{\omega}_\theta$ Eulerian θ component of $\boldsymbol{\omega}$
- $(\boldsymbol{\omega}_\theta)_{z'}$ component along the body x' axis due to $\boldsymbol{\omega}_\theta$
- $\boldsymbol{\omega}_{z'}$ body x' component of $\boldsymbol{\omega}$
- \mathbf{E} identity matrix

From the definition of $(\boldsymbol{\omega}_\theta)_{x'}$, etc., there is

$$\boldsymbol{\omega}_{x'} = (\boldsymbol{\omega}_\theta)_{x'} + (\boldsymbol{\omega}_\phi)_{x'} + (\boldsymbol{\omega}_\psi)_{x'} \quad (\text{C-1})$$

$$\boldsymbol{\omega}_{y'} = (\boldsymbol{\omega}_\theta)_{y'} + (\boldsymbol{\omega}_\phi)_{y'} + (\boldsymbol{\omega}_\psi)_{y'} \quad (\text{C-2})$$

$$\boldsymbol{\omega}_{z'} = (\boldsymbol{\omega}_\theta)_{z'} + (\boldsymbol{\omega}_\phi)_{z'} + (\boldsymbol{\omega}_\psi)_{z'} \quad (\text{C-3})$$

From geometry, it is evident that

$$\begin{bmatrix} (\boldsymbol{\omega}_\phi)_{x'} \\ (\boldsymbol{\omega}_\phi)_{y'} \\ (\boldsymbol{\omega}_\phi)_{z'} \end{bmatrix} = \mathbf{A} \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix} \quad (\text{C-4})$$

$$\begin{bmatrix} (\boldsymbol{\omega}_\theta)_{x'} \\ (\boldsymbol{\omega}_\theta)_{y'} \\ (\boldsymbol{\omega}_\theta)_{z'} \end{bmatrix} = \mathbf{B} \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \end{bmatrix} \quad (\text{C-5})$$

$$\begin{bmatrix} (\omega\psi)_{x'} \\ (\omega\psi)_{y'} \\ (\omega\psi)_{z'} \end{bmatrix} = \mathbf{E} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \quad (\text{C-6})$$

The inverse transformations may be found by a linear inversion (such as the use of Cramer's rule) of Eqs. (C-7) through (C-9). The result of this inversion is

Putting Eqs. (C-4), (C-5), (C-6) into Eqs. (C-1), (C-2), and (C-3) gives the components of ω with respect to the body axes in terms of the rates about the Eulerian axes.

$$\dot{\theta} = \omega_{x'} \cos \psi - \omega_{y'} \sin \psi \quad (\text{C-10})$$

$$\omega_{x'} = \dot{\phi} \sin \theta \sin \psi + \dot{\theta} \cos \psi \quad (\text{C-7})$$

$$\dot{\phi} = \frac{1}{\sin \theta} [\omega_{x'} \sin \psi + \omega_{y'} \cos \psi] \quad (\text{C-11})$$

$$\omega_{y'} = \dot{\phi} \sin \theta \cos \psi - \dot{\theta} \sin \psi \quad (\text{C-8})$$

$$\omega_{z'} = \dot{\phi} \cos \theta + \dot{\psi} \quad (\text{C-9})$$

$$\dot{\psi} = \omega_{z'} - \frac{\cos \theta}{\sin \theta} [\omega_{x'} \sin \psi + \omega_{y'} \cos \psi] \quad (\text{C-12})$$

APPENDIX D

Determination of Generalized Forces

The generalized forces are defined by means of a virtual work principle: the virtual work done by a force, \mathbf{f} , acting through a virtual displacement, $\delta \mathbf{r}$, is equal to the virtual work done by the generalized forces, \mathcal{Q}_k , acting through the corresponding virtual generalized displacements, δq_k . That is

$$\mathbf{f}^T (\delta \mathbf{r}) = \sum_k \mathcal{Q}_k \delta q_k \quad (\text{D-1})$$

Consider some point i on body I and let this be the point of action for a force \mathbf{f}_i . In inertial coordinates

$$\mathbf{r}_i = \mathbf{r}_I + \mathbf{A}_I^T \mathbf{r}'_i$$

and

$$\delta \mathbf{r}_i = \delta \mathbf{r}_I + \delta [\mathbf{A}_I^T \mathbf{r}'_i] \quad (\text{D-2})$$

$$\delta \mathbf{r}_i = \delta \mathbf{r}_I + \left[\frac{\partial \mathbf{A}_I^T}{\partial \theta_I} \delta \theta_I + \frac{\partial \mathbf{A}_I^T}{\partial \phi_I} \delta \phi_I + \frac{\partial \mathbf{A}_I^T}{\partial \psi_I} \delta \psi_I \right] \mathbf{r}'_i \quad (\text{D-3})$$

$$\delta \mathbf{r}_i = \delta \mathbf{r}_I + \left[\frac{\partial \mathbf{A}_I^T}{\partial \theta_I} \mathbf{A}_I \delta \theta_I + \frac{\partial \mathbf{A}_I^T}{\partial \phi_I} \mathbf{A}_I \delta \phi_I + \frac{\partial \mathbf{A}_I^T}{\partial \psi_I} \mathbf{A}_I \delta \psi_I \right] \mathbf{r}_{Ii} \quad (\text{D-4})$$

where \mathbf{r}_{Ii} is \mathbf{r}'_i expressed in the inertial CS.

Applying the virtual work principle

$$\sum_k \mathcal{Q}_k \delta q_k = \mathbf{f}_i^T \left\{ \left[\frac{\partial \mathbf{A}_I^T}{\partial \theta_I} \mathbf{A}_I \delta \theta_I + \frac{\partial \mathbf{A}_I^T}{\partial \phi_I} \mathbf{A}_I \delta \phi_I + \frac{\partial \mathbf{A}_I^T}{\partial \psi_I} \mathbf{A}_I \delta \psi_I \right] \mathbf{r}_{Ii} + \delta \mathbf{r}_I \right\} \quad (\text{D-5})$$

Since the δq_k are independent, the generalized forces are

$$\mathcal{Q}_{zI} = f_{iz}, \mathcal{Q}_{yI} = f_{iy}, \mathcal{Q}_{xI} = f_{ix} \quad (\text{D-6})$$

$$Q_{\theta I} = \mathbf{f}_i^T \frac{\partial \mathbf{A}_I^T}{\partial \theta_I} \mathbf{A}_I \mathbf{r}_{Ii} \quad (\text{D-7})$$

$$Q_{\phi I} = \mathbf{f}_i^T \frac{\partial \mathbf{A}_I^T}{\partial \phi_I} \mathbf{A}_I \mathbf{r}_{Ii} \quad (\text{D-8})$$

$$Q_{\psi I} = \mathbf{f}_i^T \frac{\partial \mathbf{A}_I^T}{\partial \psi_I} \mathbf{A}_I \mathbf{r}_{Ii} \quad (\text{D-9})$$

Equation (D-6) establishes the desired result for the linear coordinates. The remainder of this Appendix establishes the results for the angular coordinates.

It will now be shown that the matrix product $\mathbf{A}^T \dot{\mathbf{A}}$ is skew symmetric. Since \mathbf{A}_I is orthogonal

$$\mathbf{a}^{(i)T} \mathbf{a}^{(j)} = \delta_{ij} \quad (\text{D-10})$$

Let $\mathbf{H} = \mathbf{A}_I^T \dot{\mathbf{A}}_I$, with elements h_{ij}

Then

$$h_{ij} = \mathbf{a}^{(i)T} \dot{\mathbf{a}}^{(j)} \quad (\text{D-11})$$

$$h_{ij} + h_{ji} = \mathbf{a}^{(i)T} \dot{\mathbf{a}}^{(j)} + \mathbf{a}^{(j)T} \dot{\mathbf{a}}^{(i)}$$

$$h_{ij} + h_{ji} = \mathbf{a}^{(i)T} \dot{\mathbf{a}}^{(j)} + \dot{\mathbf{a}}^{(i)T} \mathbf{a}^{(j)} = \frac{d}{dt} [\mathbf{a}^{(i)T} \mathbf{a}^{(j)}] = \frac{d}{dt} [\delta_{ij}] = 0$$

and

$$h_{ij} = -h_{ji} \quad (\text{D-12})$$

It may be easily verified that the product of a skew-symmetric matrix and a column matrix is equivalent to the vector cross product of two corresponding vectors

$$\boldsymbol{\omega} \times \mathbf{r} = \mathbf{Lr}$$

where

$$\mathbf{L} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}, \quad \boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \quad (\text{D-13})$$

Now consider two coordinate systems, $\Sigma_1(x_1, x_2, x_3)$ and $\Sigma_2(y_1, y_2, y_3)$. \mathbf{D} is the transformation matrix from Σ_2 to Σ_1 . Then, for a vector \mathbf{r}

$$\mathbf{r}_1 = \mathbf{D} \mathbf{r}_2 \quad (\text{D-14})$$

$$\dot{\mathbf{r}}_1 = \mathbf{D} \dot{\mathbf{r}}_2 + \dot{\mathbf{D}} \mathbf{r}_2 = \mathbf{D} [\dot{\mathbf{r}}_2 + \mathbf{D}^T \dot{\mathbf{D}} \mathbf{r}_2] \quad (\text{D-15})$$

where the subscript on \mathbf{r} indicates the coordinate system in which it is expressed. By analogy to elementary kinematics, we have

$$\mathbf{D}^T \dot{\mathbf{D}} \mathbf{r}_2 = \boldsymbol{\omega} \times \mathbf{r}_2$$

where

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_{\beta 1} \\ \omega_{\beta 2} \\ \omega_{\beta 3} \end{bmatrix} \quad (\text{D-16})$$

is the angular velocity of Σ_2 relative to Σ_1 expressed in Σ_2 .

Since \mathbf{A}_I is the transformation from the inertial CS to the body I-fixed CS, it is clear, from the above discussion, that

$$\mathbf{A}_I^T \dot{\mathbf{A}}_I = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

where

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{D-17}$$

is the angular velocity of the inertial CS relative to the body I CS, expressed in the inertial CS.

Then

$$\dot{\mathbf{A}}_I^T \mathbf{A}_I = \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix} \tag{D-18}$$

By the total derivative rule

$$\dot{\mathbf{A}}_I^T \mathbf{A}_I = \left[\frac{\partial \mathbf{A}_I^T}{\partial \theta} \dot{\theta} + \frac{\partial \mathbf{A}_I^T}{\partial \phi} \dot{\phi} + \frac{\partial \mathbf{A}_I^T}{\partial \psi} \dot{\psi} \right] \mathbf{A}_I \tag{D-19}$$

Let

$$\mathbf{B}_\theta \equiv [b_{\theta ij}] = \frac{\partial \mathbf{A}_I^T}{\partial \theta} \mathbf{A}_I, \mathbf{B}_\phi \equiv [b_{\phi ij}] = \frac{\partial \mathbf{A}_I^T}{\partial \phi} \mathbf{A}_I, \mathbf{B}_\psi \equiv [b_{\psi ij}] = \frac{\partial \mathbf{A}_I^T}{\partial \psi} \mathbf{A}_I$$

By considering equality for the (2,3) elements in (D-19), we obtain

$$\omega_x = b_{\theta 23} \dot{\theta} + b_{\phi 23} \dot{\phi} + b_{\psi 23} \dot{\psi} \tag{D-20}$$

but from vector addition

$$\omega_x = (-e_{\theta x}) \dot{\theta} + (-e_{\phi x}) \dot{\phi} + (-e_{\psi x}) \dot{\psi} \tag{D-21}$$

where

$$\mathbf{e}_\theta = \begin{bmatrix} e_{\theta x} \\ e_{\theta y} \\ e_{\theta z} \end{bmatrix}, \quad \mathbf{e}_\phi = \begin{bmatrix} e_{\phi x} \\ e_{\phi y} \\ e_{\phi z} \end{bmatrix}, \quad \mathbf{e}_\psi = \begin{bmatrix} e_{\psi x} \\ e_{\psi y} \\ e_{\psi z} \end{bmatrix}$$

are the unit vectors along the positive rotation axes for θ, ϕ, ψ when the inertial CS is considered fixed.

Now, since $\dot{\theta}, \dot{\phi},$ and $\dot{\psi}$ are independent, Eqs. (D-20) and (D-21) yield

$$b_{\theta 23} = -e_{\theta x} \quad b_{\phi 23} = -e_{\phi x} \quad b_{\psi 23} = -e_{\psi x} \tag{D-22}$$

By writing equations similar to (D-20) and (D-21) for ω_y and ω_z , it can be shown that

$$\mathbf{B}_\theta = \begin{bmatrix} 0 & -e_{\theta z} & e_{\theta y} \\ e_{\theta z} & 0 & -e_{\theta x} \\ -e_{\theta y} & e_{\theta x} & 0 \end{bmatrix}, \quad \mathbf{B}_\phi = \begin{bmatrix} 0 & -e_{\phi z} & e_{\phi y} \\ e_{\phi z} & 0 & -e_{\phi x} \\ -e_{\phi y} & e_{\phi x} & 0 \end{bmatrix}, \quad \mathbf{B}_\psi = \begin{bmatrix} 0 & -e_{\psi z} & e_{\psi y} \\ e_{\psi z} & 0 & -e_{\psi x} \\ -e_{\psi y} & e_{\psi x} & 0 \end{bmatrix} \tag{D-23}$$

Equation (D-23) into (D-7) gives

$$\mathcal{Q}_{\theta I} = \mathbf{f}_I^T \mathbf{B}_\theta \mathbf{r}_{Ii} \tag{D-24}$$

Application of Eqs. (D-13) through (D-24) gives

$$\begin{aligned}
 Q_{\theta I} &= \mathbf{f}_i \cdot (\mathbf{e}_\theta \times \mathbf{r}_{Ii}) \\
 Q_{\theta I} &= -\mathbf{f}_i \cdot (\mathbf{r}_{Ii} \times \mathbf{e}_\theta) = -(\mathbf{r}_{Ii} \times \mathbf{e}_\theta) \cdot \mathbf{f}_i \\
 Q_{\theta I} &= (\mathbf{r}_{Ii} \times \mathbf{f}_i) \cdot \mathbf{e}_{\theta I} \\
 Q_{\theta I} &= (\text{torque vector}) \cdot \mathbf{e}_\theta \\
 Q_{\theta I} &= \text{component of torque along } \mathbf{e}_\theta
 \end{aligned} \tag{D-25}$$

Similarly

$$Q_{\phi I} = \text{component of torque along } \mathbf{e}_\phi \tag{D-26}$$

$$Q_{\psi I} = \text{component of torque along } \mathbf{e}_\psi \tag{D-27}$$

It is valid to sum over i in Eqs. (D-25) and (D-6) to obtain the generalized forces for many \mathbf{f}_k . This analysis also applies to body II.

APPENDIX E

Velocity of Separation Between a Point on Body I and a Point on Body II

The velocity of separation between initially coincident points on body I and body II is required in order to compute the directions of coulomb drag forces. By definition

$$\mathbf{v}_i = \frac{d(\mathbf{d}_i)}{dt} \tag{E-1}$$

$$\mathbf{v}_i = \dot{\mathbf{r}} + \dot{\mathbf{A}}_I^T \mathbf{r}'_{Ii} - \dot{\mathbf{A}}_{II}^T \mathbf{r}'_{IIi} \tag{E-2}$$

The time derivatives of the elements of the \mathbf{A} matrixes are

$$c_{11} = (-c\theta s\phi s\psi + c\phi c\psi) \tag{E-3}$$

$$\dot{c}_{11} = (s\theta s\phi s\psi) \dot{\theta} - \dot{c}_{12} \dot{\phi} + c_{21} \dot{\psi} \tag{E-4}$$

$$c_{12} = (c\theta c\phi s\psi + s\phi c\psi) \tag{E-5}$$

$$\dot{c}_{12} = -(s\theta c\phi s\psi) \dot{\theta} + \dot{c}_{11} \dot{\phi} + c_{22} \dot{\psi} \tag{E-6}$$

$$c_{13} = (s\theta c\psi) \tag{E-7}$$

$$\dot{c}_{13} = (c\theta s\psi) \dot{\theta} + c_{23} \dot{\psi} \tag{E-8}$$

$$c_{21} = (-c\theta s\phi c\psi - c\phi s\psi) \tag{E-9}$$

$$\dot{c}_{21} = (s\theta s\phi c\psi) \dot{\theta} - c_{22} \dot{\phi} - c_{11} \dot{\psi} \tag{E-10}$$

$$c_{22} = (c\theta c\phi c\psi - s\phi s\psi) \tag{E-11}$$

$$\dot{c}_{22} = -(s\theta c\phi c\psi) \dot{\theta} + c_{21} \dot{\phi} - c_{12} \dot{\psi} \tag{E-12}$$

$$c_{23} = (s\theta c\psi) \tag{E-13}$$

$$\dot{c}_{23} = (c\theta c\psi) \dot{\theta} - c_{13} \dot{\psi} \quad (\text{E-14})$$

$$c_{31} = (s\theta s\phi) \quad (\text{E-15})$$

$$\dot{c}_{31} = (c\theta s\phi) \dot{\theta} - c_{32} \dot{\phi} \quad (\text{E-16})$$

$$c_{32} = -(s\theta c\phi) \quad (\text{E-17})$$

$$\dot{c}_{32} = -(c\theta c\phi) \dot{\theta} + c_{31} \dot{\phi} \quad (\text{E-18})$$

$$c_{33} = c\theta \quad (\text{E-19})$$

$$\dot{c}_{33} = -(s\theta) \dot{\theta} \quad (\text{E-20})$$

APPENDIX F

Derivation of Force vs Time Relation for an Adiabatic Compressed Gas Jet

The classical ideal gas relations are utilized to determine the force-time relation for the cold gas-jet model (Fig. F-1).

A. Symbols

| | |
|-----------------|--|
| A | area |
| a | speed of sound |
| C_1, C_2, C_3 | constants depending on working fluid properties and/or nozzle geometry |
| F | thrust |
| M | mach number |
| m | mass of gas in tank |
| \dot{m} | mass flow rate |
| p | static pressure |
| p' | external static pressure |
| R | gas constant |
| T | absolute temperature |
| u | velocity |
| V | volume upstream of throat |
| α | nozzle divergence half-angle |
| γ | ratio of specific heats for working fluid |
| λ | nozzle divergence correction factor, $\frac{1}{2}(1 + \cos \alpha)$ |
| ρ | density |
| τ | time |
| 0 (subscript) | initial condition |

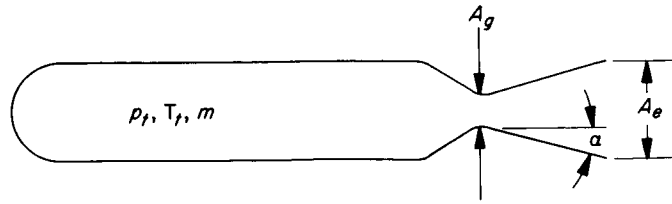


Fig. F-1. Compressed gas jet

B. Derivation

$$F = -\lambda u_e \dot{m} + (p_e - p') A_e \tag{F-1}$$

Assuming a perfect gas and an adiabatic, isentropic process

$$\dot{m} = \rho_g u_g A_g \tag{F-2}$$

$$a = (\gamma RT)^{1/2} \tag{F-3}$$

$$\rho = \rho_t \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/(\gamma - 1)} \tag{F-4}$$

$$\frac{u}{a_t} = M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/2} \tag{F-5}$$

$$p = \rho RT \tag{F-6}$$

$$\rho_g = \rho_t \left(\frac{2}{\gamma + 1}\right)^{1/\gamma - 1} = \frac{p_t}{RT_t} \left(\frac{2}{\gamma + 1}\right)^{-1/(\gamma - 1)} \tag{F-7}$$

$$u_g = a_t \left(\frac{2}{\gamma + 1}\right)^{1/2} = \left(\frac{2}{\gamma + 1}\right)^{1/2} (\gamma RT_t)^{1/2} \tag{F-8}$$

$$\dot{m} = -\left(\frac{2}{\gamma + 1}\right)^{(\gamma + 1)/2(\gamma - 1)} \left(\frac{\gamma}{RT_t}\right)^{1/2} p_t A_g \equiv C_1 \frac{p_t}{(T_t)^{1/2}} \tag{F-9}$$

$$u_e = M_e \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1/2} (\gamma RT_t)^{1/2} \equiv C_2 (T_t)^{1/2} \tag{F-10}$$

M_e is implicitly determined by

$$\frac{A_g}{A_e} = \left(\frac{\gamma + 1}{2}\right)^{(\gamma + 1)/[2(\gamma - 1)]} M_e \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-[(\gamma + 1)/2(\gamma - 1)]} \tag{F-11}$$

$$p_e = C_3 p_t$$

where

$$C_3 \equiv \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-[\gamma/(\gamma - 1)]} \tag{F-12}$$

$$F = -\lambda C_1 C_2 p_t + A_e C_3 p_t - A_e p' = (\lambda C_1 C_2 + A_e C_3) p_t - A_e p' \tag{F-13}$$

$$m = \rho_t V = \frac{V p_t}{RT_t} \tag{F-14}$$

Now

$$T_t = \left(\frac{p_t}{p_{t_0}} \right)^{(\gamma-1)/\gamma} T_{t_0} \quad (F-15)$$

so

$$m = \frac{V p_t}{R T_{t_0} \left(\frac{p_t}{p_{t_0}} \right)^{(\gamma-1)/\gamma}} = \frac{V (p_{t_0})^{(\gamma-1)/\gamma}}{R T_{t_0}} p_t^{1/\gamma} \quad (F-16)$$

$$\dot{m} = C_1 p_t \left(\frac{p_t}{p_{t_0}} \right)^{-[(\gamma-1)/2\gamma]} T_{t_0}^{-1/2} = C_1 \left(\frac{p_{t_0}^{(\gamma-1)/\gamma}}{T_{t_0}} \right)^{1/2} p_t^{(\gamma+1)/2\gamma} \quad (F-17)$$

Using Eqs. (F-15) and (F-16) to eliminate p_t from Eq. (F-17) gives

$$\dot{m} = C_1 \frac{\left(\frac{p_{t_0}^{(\gamma-1)/\gamma}}{T_{t_0}} \right)^{1/2} p_t^{(\gamma+1)/2\gamma}}{\left[\frac{V (p_{t_0}^{(\gamma-1)/\gamma})}{R \left(\frac{p_{t_0}}{T_{t_0}} \right)} p_t^{1/\gamma} \right]^{(\gamma+1)/2}} m^{(\gamma+1)/2} \quad (F-18)$$

$$\dot{m} = C_1 \left(\frac{R}{V} \right)^{(\gamma+1)/2} \left(\frac{p_{t_0}^{(\gamma-1)/\gamma}}{T_{t_0}} \right)^{-(\gamma/2)} m^{(\gamma+1)/2} \quad (F-19)$$

$$m = \left[m_0^{-(\gamma-1)/2} - C_1 \left(\frac{\gamma-1}{2} \right) \left(\frac{R}{V} \right)^{(\gamma+1)/2} \left(\frac{T_{t_0}}{p_{t_0}^{\gamma-1/\gamma}} \right)^{-(\gamma/2)} \tau \right]^{-[2/(\gamma-1)]} \quad (F-20)$$

$$m = \left[\left(\frac{R T_{t_0}}{V p_{t_0}} \right)^{(\gamma-1)/2} - C_1 \left(\frac{\gamma-1}{2} \right) \left(\frac{R}{V} \right)^{(\gamma+1)/2} \left(\frac{T_{t_0}}{p_{t_0}^{\gamma-1/\gamma}} \right)^{\gamma/2} \tau \right]^{-[2/(\gamma-1)]} \quad (F-21)$$

$$p_t = \left(\frac{R T_{t_0}}{V p_{t_0}} \right)^\gamma m^\gamma \quad (F-22)$$

Therefore, after combining terms

$$p_t = p_{t_0} \left[1 + \left(\frac{\gamma-1}{2} \right) \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/2(\gamma-1)} (\gamma R T_{t_0})^{1/2} \frac{A_g}{V} \tau \right]^{-[2\gamma/(\gamma-1)]} \quad (F-23)$$

Since $p' = 0$, the thrust is

$$F = p_{t_0} \left[\frac{1}{2} (1 + \cos \alpha) \gamma \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/2(\gamma-1)} \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{-1/2} M_e A_g + \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{-[\gamma/(\gamma-1)]} A_e \right] \left[1 + \left(\frac{\gamma-1}{2} \right) \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/2(\gamma-1)} (\gamma R T_{t_0})^{1/2} \left(\frac{A_g}{V} \right) \tau \right]^{-[2\gamma/(\gamma-1)]} \quad (F-24)$$

APPENDIX G

Derivation of Force vs Distance Relation for an Adiabatic Pneumatic Ejection System

The ideal gas laws are used to derive the force-distance relation for a pneumatic separation system. The existence of a regulator that either stays open or stays closed after the regulation pressure has been reached is considered.

A. Symbols

| | |
|-----------------|---|
| A | area of piston face |
| b (subscript) | gas bottle |
| c_p | specific heat at constant pressure |
| c_v | specific heat at constant volume |
| E | internal energy |
| j | displacement of piston from closed position |
| j (subscript) | as a function of j |
| j_{eq} | The j at which $p_b = p_p$, and the regulator ceases to work |
| N_i | force due to the i^{th} pneumatic piston |
| p | pressure |
| p (subscript) | piston chamber |
| R | universal gas constant |
| T | absolute temperature |
| V_b | volume of gas bottle, including all lines upstream of the face of the closed piston |
| W | work |
| γ | ratio of specific heats, c_p/c_v |
| ρ | density |
| 0 | initial condition |
| '(prime) | equivalent initial conditions for regulatorless motion when $j \geq j_{eq}$ |

As may be seen from Figs. G-1 and G-2, the force on the piston is

$$N_i = p_i A_i \quad (G-1)$$

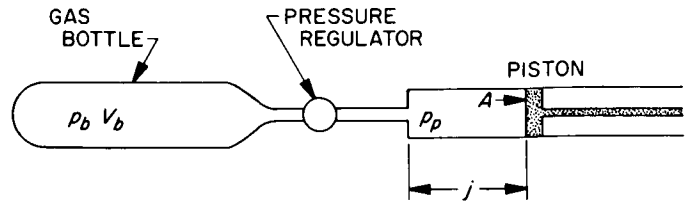


Fig. G-1. Pneumatic separation device with pressure regulator

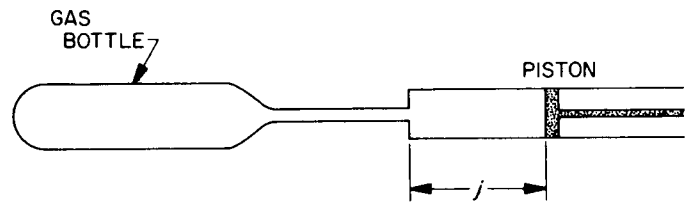


Fig. G-2. Pneumatic separation device without pressure regulator

The two cases to be considered are: (1) with a pressure regulator in the circuit, and (2) without a pressure regulator in the circuit. With a regulator, Eq. (G-2) is valid until $p_b = p_p$; without a regulator, Eq. (G-3) is valid.

$$p_p = \text{constant} \quad (G-2)$$

$$p_p = p_b = p \quad (G-3)$$

B. With Pressure Regulator

Using the law of conservation of energy on the gas contained within the system

$$E_0 = \rho_{b0} V_b c_v T_{b0} \quad (G-4)$$

$$E_j = \rho_b V_b c_v T_b + \rho_p c_v T_p A j \quad (G-5)$$

$$W_j = p_p A j \quad (G-6)$$

Assuming adiabatic conditions

$$E_0 = E_j + W_j \quad (G-7)$$

$$\rho_{b0} V_b c_v T_{b0} = \rho_b V_b c_v T_b + \rho_p c_v T_p A j + p_p A j \quad (G-8)$$

Assuming that the working fluid is a perfect gas, Eqs. (G-9), (G-10), and (G-11) are valid. Equations (G-9)

through (G-11) will then yield Eq. (G-12) which, in turn, can be reduced to Eq. (G-13).

$$p = \rho RT \quad (G-9)$$

$$R = c_p - c_v \quad (G-10)$$

$$\frac{c_v}{R} = \frac{c_v}{c_p - c_v} = \frac{1}{\frac{c_p}{c_v} - 1} = \frac{1}{\gamma - 1} \quad (G-11)$$

$$\frac{c_v}{R} V_b p_{b0} = \frac{c_v}{R} V_b p_b + \frac{c_v}{R} p_p A_j + p_p A_j \quad (G-12)$$

$$V_b p_{b0} = V_b p_b + \gamma p_p A_j \quad (G-13)$$

Therefore

$$p_b = p_{b0} - \gamma p_p \frac{A_j}{V_b} \quad (G-14)$$

The force, N , is $p_p A$ as long as $p_b \geq p_p$, when $p_b = p_p$, which occurs at

$$j = j_{eqp} = \frac{V_b}{\gamma A} \left(\frac{p_{b0}}{p_p} - 1 \right) \quad (G-15)$$

then this is no longer true.

When $j > j_{eqp}$, the system acts as if there were no regulator. The new initial conditions are

$$j' = j - j_{eqp} \quad (G-16)$$

$$p'_{b0} = p_p \quad (G-17)$$

and, if the pressure regulator stays closed

$$V'_b = A j_{eqp} \quad (G-18)$$

If the regulator stays open

$$V'_b = V_b + A j_{eqp} \quad (G-19)$$

C. Without Pressure Regulator

Using the law of conservation of energy gives

$$E_0 = \rho_0 V_b c_v T_0 \quad (G-20)$$

$$E_j = \rho c_v T (V_b + A_j) \quad (G-21)$$

$$W_j = \int_0^j p A dj \quad (G-22)$$

Assuming adiabatic conditions

$$E_0 = E_j + W_j \quad (G-23)$$

$$\rho_0 V_b c_v T_0 = \rho c_v T (V_b + A_j) + \int_0^j p A dj \quad (G-24)$$

Assuming a perfect gas

$$\frac{c_v}{R} V_b p_{b0} = \frac{c_v}{R} (V_b + A_j) p + \int_0^j p A dj \quad (G-25)$$

$$p_{b0} = \left(1 + \frac{A_j}{V_b} \right) p + (\gamma - 1) \frac{A}{V_b} \int_0^j p dj \quad (G-26)$$

Taking the derivative of Eq. (G-27) with respect to j results in

$$0 = \left(1 + \frac{A_j}{V_b} \right) \frac{dp}{dj} + \frac{A}{V_b} p + (\gamma - 1) \frac{A}{V_b} p \quad (G-27)$$

which reduces to

$$\frac{V_b}{\gamma A} \frac{dp}{p} = - \frac{dj}{1 + \frac{A}{V_b} j} \quad (G-28)$$

Integrating Eq. (G-28) gives

$$p = C \left(1 + \frac{A_j}{V_b} \right)^{-\gamma} \quad (G-29)$$

C is determined by setting $p = p_{b0}$ when $j = 0$.

$$p_{b0} = C, \text{ so} \quad (G-30)$$

$$p = p_{b0} \left(1 + \frac{A_j}{V_b} \right)^{-\gamma} \quad (G-31)$$

APPENDIX H

Derivation of Spring Efficiency Factor Relation

Experience has shown that separation velocities calculated by equating initial spring potential energy to eventual body kinetic energy do not agree with experimental results. Typical differences are large enough to warrant inclusion of this effect in the analysis.

A. Symbols

- d_0 initial spring compression distance = S_0/k
- d_f final spring compression distance = S_f/k
- k true spring rate
- k' artificial spring rate that gives correct velocity
- S_0 initial spring force
- S_f final spring force
- S'_0 modified initial spring force
- S'_f modified final spring force
- v_a actual separation velocity
- v_p separation velocity predicted using k
- η spring efficiency factor

B. Definition and Determination of η

Experimental determination of η does not lend itself to calculation of a different value for each spring; therefore, one value of η is used for all the springs. η is defined as

$$\eta = \frac{\text{actual separation velocity of bodies}}{\text{velocity predicted assuming perfectly efficient springs}} \quad (\text{H-1})$$

In the analysis, η is included by calculating an artificial spring rate that yields the correct separation velocity.

It is assumed that the actual separation velocity is determined from a test in which all angular rates are zero (i.e., only rectilinear motion is involved), so that the separation process can be modeled as a simple 2-deg of freedom system. Since only relative motions between the two bodies need be considered, this 2-deg of freedom

system degenerates to a 1-deg of freedom spring-mass system with effective mass m and composite spring constant k .

In the determination of k' , it will be required that the initial spring compression distance, d_0 , and the final spring compression distance, d_f , be preserved, rather than the initial and final forces.

C. Effect of η on Spring Rate

The differential equation representing the 1-deg of freedom model is

$$m\ddot{x} = k(d_0 - x) \quad (\text{H-2})$$

where

$$x = (\text{separation distance between CM's}) - (\text{initial separation distance})$$

The initial values are

$$x(0) = \dot{x}(0) = 0$$

The solution to Eq. (H-2) is

$$\left. \begin{aligned} x(t) &= d_0 \left[1 - \cos \left(\frac{k}{m} \right)^{1/2} t \right], \\ \dot{x}(t) &= d_0 \left(\frac{k}{m} \right)^{1/2} \sin \left(\frac{k}{m} \right)^{1/2} t \end{aligned} \right\} 0 \leq x \leq (d_0 - d_f) \quad (\text{H-3})$$

Let t_f be the time of complete separation ($x = d_0 - d_f$) for the perfectly efficient spring case, and let t'_f be the corresponding value for the actual case. Then, from Eqs. (H-1) and (H-3)

$$\eta = \frac{v_a}{v_p} = \left(\frac{k'}{k} \right)^{1/2} \frac{\sin \left(\frac{k'}{m} \right)^{1/2} t'_f}{\sin \left(\frac{k}{m} \right)^{1/2} t_f} \quad (\text{H-4})$$

Now when $x = d_0 - d_f$, from Eq. (H-3)

$$d_0 - d_f = d_0 \left[1 - \cos \left(\frac{k}{m} \right)^{1/2} t_f \right]$$

and

$$d_0 - d_f = d_0 \left[1 - \cos \left(\frac{k'}{m} \right)^{1/2} t_f' \right]$$

Thus

$$\cos \left(\frac{k}{m} \right)^{1/2} t_f = \cos \left(\frac{k'}{m} \right)^{1/2} t_f'$$

which implies that

$$\sin \left(\frac{k}{m} \right)^{1/2} t_f = \pm \sin \left(\frac{k'}{m} \right)^{1/2} t_f' \quad (\text{H-5})$$

since

$$v_a > 0 \quad \text{and} \quad v_p > 0$$

$$\sin \left(\frac{k}{m} \right)^{1/2} t_f = + \sin \left(\frac{k'}{m} \right)^{1/2} t_f' \quad (\text{H-6})$$

and Eq. (H-4) becomes

$$\eta = \left(\frac{k'}{k} \right)^{1/2}$$

Then

$$k' = \eta^2 k \quad (\text{H-7})$$

To preserve d_0 and d_f with this new spring constant, new values of S_0 and S_f must be used. These values are

$$S'_0 = \eta^2 S_0 \quad S'_f = \eta^2 S_f \quad (\text{H-8})$$

APPENDIX I

Calculation of Kinetic Energy, $\partial T / \partial \mathbf{q}_k$, and $d/dt (\partial T / \partial \dot{\mathbf{q}}_k)$ as Functions of the Generalized Coordinates

The quantities T , $\partial T / \partial \mathbf{q}_k$, and $d/dt (\partial T / \partial \dot{\mathbf{q}}_k)$ are required as explicit functions of the generalized coordinates in the equations of motion.

A. Symbols

\mathbf{I} inertia matrix

I components of the inertia matrix

$\dot{\mathbf{I}}$ derivative of inertia matrix [$\dot{I}_{jk} = \sum_i (\dot{I}_{jk})_i$]

$\dot{m} = \sum_i \dot{m}_i$

T kinetic energy of the system

\mathbf{v} velocity vector of CM, $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$, in inertial coordinates

$\boldsymbol{\omega}$ angular velocity about body-fixed axes, $\begin{bmatrix} \omega_{x'} \\ \omega_{y'} \\ \omega_{z'} \end{bmatrix}$

B. Calculation of $T(\mathbf{q}_{11}, \dots, \mathbf{q}_{12})$

$$2T = m_I \mathbf{v}_I^T \mathbf{v}_I + m_{II} \mathbf{v}_{II}^T \mathbf{v}_{II} + \boldsymbol{\omega}_I^T \mathbf{I}_I \boldsymbol{\omega}_I + \boldsymbol{\omega}_{II}^T \mathbf{I}_{II} \boldsymbol{\omega}_{II} \quad (\text{I-1})$$

Using the results of Appendix D

$$\omega = \begin{bmatrix} \dot{\phi}(s\theta s\psi) + \dot{\theta}(c\psi) \\ \dot{\phi}(s\theta c\psi) - \dot{\theta}(s\psi) \\ \dot{\phi}(c\theta) + \dot{\psi} \end{bmatrix}$$

Thus we have

$$T(q_1, \dots, q_{12}) \tag{I-2}$$

C. Calculation of $\partial T/\partial q_s$

Clearly

$$\frac{\partial T}{\partial q_s} = 0 \quad \text{for } q_s = x_I, x_{II}, y_I, y_{II}, z_I, z_{II}, \phi_I, \phi_{II} \tag{I-3}$$

The derivatives with respect to the other coordinates are

$$\frac{\partial T}{\partial q_s} = \omega_I^T \mathbf{I}_I \frac{\partial \omega_I}{\partial q_s} + \omega_{II}^T \mathbf{I}_{II} \frac{\partial \omega_{II}}{\partial q_s} \tag{I-4}$$

where use has been made of the fact that the inertia matrixes are symmetric. Since the first term of Eq. (I-4) is only a function of θ_I and ψ_I , and the second term is only a function of θ_{II} and ψ_{II}

$$\frac{\partial T}{\partial \theta} = \omega^T \mathbf{I} \frac{\partial \omega}{\partial \theta}, \quad \frac{\partial T}{\partial \psi} = \omega^T \mathbf{I} \frac{\partial \omega}{\partial \psi} \tag{I-5}$$

where

$$\frac{\partial \omega}{\partial \theta} = \begin{bmatrix} \dot{\phi}(c\theta s\psi) \\ \dot{\phi}(c\theta c\psi) \\ -\dot{\phi}(s\theta) \end{bmatrix} \tag{I-6}$$

$$\frac{\partial \omega}{\partial \psi} = \begin{bmatrix} \dot{\phi}(s\theta c\psi) + \dot{\theta}(s\psi) \\ -\dot{\phi}(s\theta s\psi) - \dot{\theta}(c\psi) \\ 0 \end{bmatrix} \tag{I-7}$$

The explicit forms are

$$\begin{aligned} \frac{\partial T}{\partial \theta} = & [\mathbf{I}_{xx}(s^2\psi) + 2\mathbf{I}_{xy}(s\psi c\psi) + \mathbf{I}_{yy}(c^2\psi) - \mathbf{I}_{zz}] \dot{\phi}^2(s\theta c\theta) \\ & + (\mathbf{I}_{xx} - \mathbf{I}_{yy}) \dot{\theta} \dot{\phi}(c\theta s\psi c\psi) + \mathbf{I}_{xy} \dot{\theta} \dot{\phi}(c\theta)(c^2\psi - s^2\psi) \\ & + \mathbf{I}_{xz} \dot{\phi}^2(s\psi)(c^2\theta - s^2\theta) + [\mathbf{I}_{xz}(s\psi) + \mathbf{I}_{yz}(c\psi)] \dot{\phi} \dot{\psi}(c\theta) \\ & + [\mathbf{I}_{yz}(s\psi) - \mathbf{I}_{xz}(c\psi)] \dot{\theta} \dot{\phi}(s\theta) + \mathbf{I}_{yz} \dot{\phi}^2(c\psi)(c^2\theta - s^2\theta) - \mathbf{I}_{zz} \dot{\phi} \dot{\psi}(s\theta) \end{aligned} \tag{I-8}$$

$$\begin{aligned} \frac{\partial T}{\partial \psi} = & (\mathbf{I}_{xx} - \mathbf{I}_{yy})(s\psi c\psi) [\dot{\phi}^2(s^2\theta) - \dot{\theta}^2] + [\mathbf{I}_{xx} - \mathbf{I}_{yy}] \dot{\theta} \dot{\phi}(s\theta)(c^2\psi - s^2\psi) \\ & + \mathbf{I}_{xy} \dot{\phi}^2(s^2\theta)(c^2\psi - s^2\psi) - 4\mathbf{I}_{xy} \dot{\theta} \dot{\phi}(s\theta s\psi c\psi) \\ & + [\mathbf{I}_{xz}(c\psi) - \mathbf{I}_{yz}(s\psi)] [\dot{\phi}(c\theta) + \dot{\psi}] \dot{\phi}(s\theta) - \mathbf{I}_{xy} \dot{\theta}^2(c^2\psi - s^2\psi) \\ & - \dot{\theta} [\dot{\phi}(c\theta) + \dot{\psi}] [\mathbf{I}_{xz}(s\psi) + \mathbf{I}_{yz}(c\psi)] \end{aligned} \tag{I-9}$$

D. Calculation of $\partial T/\partial \dot{q}_s$

From Eq. (I-1) and the definition of v

$$\frac{\partial T}{\partial \dot{x}} = m\dot{x} \tag{I-10}$$

$$\frac{\partial T}{\partial \dot{y}} = m\dot{y} \tag{I-11}$$

$$\frac{\partial T}{\partial \dot{z}} = m\dot{z} \tag{I-12}$$

For the other coordinates

$$\frac{\partial T}{\partial \dot{q}_s} = \omega_I^T \mathbf{I}_I \frac{\partial \omega_I}{\partial \dot{q}_s} + \omega_{II}^T \mathbf{I}_{II} \frac{\partial \omega_{II}}{\partial \dot{q}_s} \tag{I-13}$$

Because of the independence of the first and second terms

$$\frac{\partial T}{\partial \dot{\theta}} = \omega^T \mathbf{I} \frac{\partial \omega}{\partial \dot{\theta}}, \quad \frac{\partial T}{\partial \dot{\phi}} = \omega^T \mathbf{I} \frac{\partial \omega}{\partial \dot{\phi}}, \quad \frac{\partial T}{\partial \dot{\psi}} = \omega^T \mathbf{I} \frac{\partial \omega}{\partial \dot{\psi}} \tag{I-14}$$

where

$$\frac{\partial \omega}{\partial \dot{\theta}} = \begin{bmatrix} (c\psi) \\ -(s\psi) \\ 0 \end{bmatrix}, \quad \frac{\partial \omega}{\partial \dot{\phi}} = \begin{bmatrix} (s\theta \ s\psi) \\ (s\theta \ c\psi) \\ (c\theta) \end{bmatrix}, \quad \frac{\partial \omega}{\partial \dot{\psi}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{I-15}$$

E. Calculation of $d/dt (\partial T/\partial \dot{q}_s)$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}} \right) = m\ddot{x} \tag{I-16}$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}} \right) = m\ddot{y} \tag{I-17}$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{z}} \right) = m\ddot{z} \tag{I-18}$$

For the other coordinates

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_s} \right) = \dot{\omega}_I^T \mathbf{I}_I \frac{\partial \omega_I}{\partial \dot{q}_s} + \omega_I^T \mathbf{I}_I \frac{d}{dt} \left[\frac{\partial \omega_I}{\partial \dot{q}_s} \right] + \dot{\omega}_{II}^T \mathbf{I}_{II} \frac{\partial \omega_{II}}{\partial \dot{q}_s} + \omega_{II}^T \mathbf{I}_{II} \frac{\partial \omega_{II}}{\partial \dot{q}_s} \tag{I-19}$$

Because of the independence of the first terms in Eq. (I-19) from the last three

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_s} \right) = \dot{\omega}^T \mathbf{I} \frac{\partial \omega}{\partial \dot{q}_s} + \omega^T \mathbf{I} \frac{d}{dt} \left[\frac{\partial \omega}{\partial \dot{q}_s} \right] \tag{I-20}$$

Equation (I-20) applies subscripted with a I when $q_s = \theta_I, \phi_I, \psi_I$, and subscripted with a II when $q_s = \theta_{II}, \phi_{II}, \psi_{II}$.

$$\dot{\omega}_I = \begin{bmatrix} \ddot{\phi} (s\theta \ s\psi) + \dot{\phi} \dot{\theta} (c\theta \ s\psi) + \dot{\phi} \dot{\psi} (s\theta \ c\psi) + \ddot{\theta} (c\psi) - \dot{\theta} \dot{\psi} (s\psi) \\ \ddot{\phi} (s\theta \ c\psi) + \dot{\phi} \dot{\theta} (c\theta \ c\psi) - \dot{\phi} \dot{\psi} (c\theta \ s\psi) - \ddot{\theta} (s\psi) - \dot{\theta} \dot{\psi} (c\psi) \\ \ddot{\phi} (c\theta) - \dot{\phi} \dot{\theta} (s\theta) + \ddot{\psi} \end{bmatrix}$$

$$\frac{d}{dt} \left[\frac{\partial \omega}{\partial \dot{\theta}} \right] = \begin{bmatrix} (-s\psi) \dot{\psi} \\ (-c\psi) \dot{\psi} \\ 0 \end{bmatrix} \tag{I-21}$$

$$\frac{d}{dt} \left[\frac{\partial \omega}{\partial \dot{\phi}} \right] = \begin{bmatrix} (c\theta s\psi) \dot{\theta} + (s\theta c\psi) \dot{\psi} \\ (c\theta c\psi) \dot{\theta} - (s\theta s\psi) \dot{\psi} \\ (-s\theta) \dot{\theta} \end{bmatrix} \quad (I-22)$$

$$\frac{d}{dt} \left[\frac{\partial \omega}{\partial \dot{\psi}} \right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (I-23)$$

Therefore, the explicit forms of $d/dt [\partial T/\partial q_s]$ for $q_s = \theta, \phi, \psi$ are

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \theta} \right) &= \ddot{\phi} \{ (s\theta) [(I_{xx} - I_{yy}) (s\psi c\psi) + I_{xy} (c^2 \psi - s^2 \psi)] + (c\theta) [I_{xz} (c\psi) - I_{yz} (s\psi)] \} \\ &+ \ddot{\psi} \{ I_{xz} (c\psi) - I_{yz} (s\psi) \} + \dot{\phi} \{ (c\theta) \theta [(I_{xx} - I_{yy}) (s\psi c\psi) + I_{xy} (c^2 \psi - s^2 \psi)] \\ &+ (s\theta) \dot{\psi} [(I_{xx} - I_{yy}) (c^2 \psi - s^2 \psi) - 4I_{xy} (s\psi c\psi)] - (s\theta) \dot{\theta} [I_{xz} (c\psi) - I_{yz} (s\psi)] \\ &- (c\theta) \dot{\psi} [I_{xz} (s\psi) + I_{yz} (c\psi)] \} - 2\dot{\theta} \dot{\psi} \{ [I_{xx} - I_{yy}] (s\psi c\psi) + I_{xy} (c^2 \psi - s^2 \psi) \} \\ &- \dot{\psi}^2 \{ I_{xz} (s\psi) + I_{yz} (c\psi) \} \end{aligned} \quad (I-24)$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\phi}} \right) &= \ddot{\theta} \{ (s\theta) [(I_{xx} - I_{yy}) (s\psi c\psi) + I_{xy} (c^2 \psi - s^2 \psi)] + (c\theta) [I_{xz} (c\psi) - I_{yz} (s\psi)] \} \\ &+ \ddot{\phi} \{ (s^2 \theta) [I_{xx} (s^2 \psi) + 2I_{xy} (s\psi c\psi) + I_{yy} (c^2 \psi)] + 2(s\theta c\theta) [I_{xz} (s\psi) + I_{yz} (c\psi)] + (c^2 \theta) I_{zz} \} \\ &+ \ddot{\psi} \{ (s\theta) [I_{xz} (s\psi) + I_{yz} (c\psi)] + (c\theta) I_{zz} \} + 2\dot{\phi} \{ \dot{\theta} (s\theta c\theta) [I_{xx} (s^2 \psi) + 2I_{xy} (s\psi c\psi) + I_{yy} (c^2 \psi) - I_{zz}] \\ &+ \dot{\psi} (s^2 \theta) [(I_{xx} - I_{yy}) (s\psi c\psi) + I_{xy} (c^2 \psi - s^2 \psi)] + \dot{\theta} (c^2 \theta - s^2 \theta) [I_{xz} (s\psi) + I_{yz} (c\psi)] \\ &+ \dot{\psi} (s\theta c\theta) [I_{xz} (c\psi) - I_{yz} (s\psi)] \} + \dot{\theta} \{ \dot{\theta} (c\theta) [(I_{xx} - I_{yy}) (s\psi c\psi) + I_{xy} (c^2 \psi - s^2 \psi)] \\ &+ \dot{\psi} (s\theta) [(I_{xx} - I_{yy}) (c^2 \psi - s^2 \psi) - 4I_{xy} (s\psi c\psi)] - \dot{\theta} (s\theta) [I_{xz} (c\psi) - I_{yz} (s\psi)] \} \\ &+ \dot{\psi} \{ \dot{\psi} (s\theta) [I_{xz} (s\psi) - I_{yz} (c\psi)] - \dot{\theta} (s\theta) I_{zz} \} \end{aligned} \quad (I-25)$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\psi}} \right) &= \ddot{\theta} \{ I_{xz} (c\psi) - I_{yz} (s\psi) \} + \ddot{\phi} \{ (s\theta) [I_{xz} (s\psi) + I_{yz} (c\psi)] + (c\theta) I_{zz} \} \\ &+ \ddot{\psi} I_{zz} + \dot{\phi} \{ \dot{\theta} (c\theta) [I_{xz} (s\psi) + I_{yz} (c\psi)] + \dot{\psi} (s\theta) [I_{xz} (c\psi) - I_{yz} (s\psi)] - \dot{\theta} (s\theta) I_{zz} \} \\ &- \dot{\theta} \dot{\psi} \{ I_{xz} (s\psi) + I_{yz} (c\psi) \} \end{aligned} \quad (I-26)$$

APPENDIX J

Constraints

As discussed in Section VII, the constraint conditions are incorporated by means of the Lagrange multiplier technique. It is assumed that the spring constraint and the pin-puller constraint do not act simultaneously.

The equations of motion for the constrained case are

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_s} - \frac{\partial T}{\partial q_s} = \mathcal{Q}'_s + \sum_l \lambda_l e_{ls} \quad s = x_1, y_1, \dots, \phi_{II}, \psi_{II} \quad (J-1)$$

$$\sum_s e_{ls} \ddot{q}_s = r_l \quad l = 1, 2, \dots, l_m \quad (J-2)$$

where the constraint equations are

$$\begin{aligned} f_1 &= 0 \\ f_2 &= 0 \\ &\dots\dots\dots \\ f_{l_m} &= 0 \end{aligned} \quad (J-3)$$

and where

$$e_{ls} = \frac{\partial f_l}{\partial q_s} \quad (J-4)$$

$$e_{lt} = \frac{\partial f_l}{\partial t} = 0 \quad (J-5)$$

$$r_l = -\sum_s \dot{e}_{ls} \dot{q}_s \quad (J-6)$$

The pin-puller delay constraint is the condition that, until $t = t_i$, a point i on body I remains coincident with a point on body II. Then

$$d_i = 0 \Rightarrow \mathbf{d}_i = 0 \Rightarrow d_{ix} = 0, \quad d_{iy} = 0, \quad d_{iz} = 0$$

Let

$$f_1 = d_{ix}, \quad f_2 = d_{iy}, \quad f_3 = d_{iz}$$

and let

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \quad \mathbf{r} = \begin{bmatrix} x_I - x_{II} \\ y_I - y_{II} \\ z_I - z_{II} \end{bmatrix} \quad \mathbf{r}'_I = \begin{bmatrix} x'_{Ii} \\ y'_{Ii} \\ z'_{Ii} \end{bmatrix} \quad \mathbf{r}'_{IIi} = \begin{bmatrix} x'_{IIi} \\ y'_{IIi} \\ z'_{IIi} \end{bmatrix} \quad \mathbf{e}_s = \begin{bmatrix} e_{1s} \\ e_{2s} \\ e_{3s} \end{bmatrix} \quad (J-7)$$

Then

$$\mathbf{f} = \mathbf{A}_{II}^T \mathbf{r}'_{IIi} - \mathbf{A}_I^T \mathbf{r}'_{Ii} - \mathbf{r} \quad (J-8)$$

and

$$\mathbf{e}_s = \frac{\partial \mathbf{f}}{\partial q_s} = \frac{\partial \mathbf{A}_{II}^T}{\partial q_s} \mathbf{r}'_{IIi} = \frac{\partial \mathbf{A}_I^T}{\partial q_s} \mathbf{r}'_{Ii} - \frac{\partial \mathbf{r}}{\partial q_s} \quad (J-9)$$

where

$$\frac{\partial a_{ij}}{\partial \theta_{II}} = \frac{\partial a_{ij}}{\partial \phi_{II}} = \frac{\partial a_{ij}}{\partial \psi_{II}} = \frac{\partial b_{ij}}{\partial \theta_I} = \frac{\partial b_{ij}}{\partial \phi_I} = \frac{\partial b_{ij}}{\partial \psi_I} = \frac{\partial c_{ij}}{\partial x_I} = \frac{\partial c_{ij}}{\partial y_I} = \frac{\partial c_{ij}}{\partial z_I} = \frac{\partial c_{ij}}{\partial x_{II}} = \frac{\partial c_{ij}}{\partial y_{II}} = \frac{\partial c_{ij}}{\partial z_{II}} = 0 \quad (J-10)$$

for $i, j = 1, 2, 3$.

$$\frac{\partial \mathbf{A}}{\partial \theta} = \begin{bmatrix} (s\theta \ s\phi \ s\psi) & (-s\theta \ c\phi \ s\psi) & (c\theta \ s\psi) \\ (s\theta \ s\phi \ c\psi) & (-s\theta \ c\phi \ c\psi) & (c\theta \ c\psi) \\ (c\theta \ s\phi) & (-c\theta \ c\phi) & (-s\theta) \end{bmatrix} \quad (J-11)$$

$$\frac{\partial \mathbf{A}}{\partial \phi} = \begin{bmatrix} -c_{12} & c_{11} & 0 \\ -c_{22} & c_{21} & 0 \\ -c_{32} & c_{31} & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{A}}{\partial \psi} = \begin{bmatrix} c_{21} & c_{22} & c_{23} \\ -c_{11} & -c_{12} & -c_{13} \\ 0 & 0 & 0 \end{bmatrix} \tag{J-12}$$

and

$$\frac{\partial \mathbf{r}}{\partial x_I} = \frac{\partial \mathbf{r}}{\partial \theta_{II}} = \frac{\partial \mathbf{r}}{\partial \phi_I} = \frac{\partial \mathbf{r}}{\partial \phi_{II}} = \frac{\partial \mathbf{r}}{\partial \psi_I} = \frac{\partial \mathbf{r}}{\partial \psi_{II}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{J-13}$$

$$\frac{\partial \mathbf{r}}{\partial x_I} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial y_I} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial z_I} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial x_{II}} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial y_{II}} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial z_{II}} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \tag{J-14}$$

Now

$$\dot{\mathbf{e}}_s = \frac{d}{dt} \left[\frac{\partial \mathbf{A}_{II}^T}{\partial q_s} \right] \mathbf{r}'_{IIi} - \frac{d}{dt} \left[\frac{\partial \mathbf{A}_I^T}{\partial q_s} \right] \mathbf{r}'_{Ii} - \frac{d}{dt} \left[\frac{\partial \mathbf{r}}{\partial q_s} \right] \tag{J-15}$$

where

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}}{\partial \theta} \right] = \begin{bmatrix} (c\theta s\phi s\psi) \dot{\theta} + (s\theta c\phi s\psi) \dot{\phi} + (s\theta s\phi c\psi) \dot{\psi}, & (-c\theta c\phi s\psi) \dot{\theta} + (s\theta s\phi s\psi) \dot{\phi} - (s\theta c\phi c\psi) \dot{\psi}, & -(s\theta s\psi) \dot{\theta} + (c\theta c\psi) \dot{\psi} \\ (c\theta s\phi c\psi) \dot{\theta} + (s\theta c\phi c\psi) \dot{\psi} - (s\theta s\phi s\psi) \dot{\psi}, & -(c\theta c\phi c\psi) \dot{\theta} + (s\theta s\phi c\psi) \dot{\phi} + (s\theta c\phi s\psi) \dot{\phi}, & -(s\theta c\psi) \dot{\theta} - (c\theta s\psi) \dot{\psi} \\ -(s\theta s\phi) \dot{\theta} + (c\theta c\phi) \dot{\phi}, & (s\theta c\phi) \dot{\theta} + (c\theta s\phi) \dot{\phi}, & -(c\theta) \dot{\theta} \end{bmatrix} \tag{J-16}$$

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}}{\partial \phi} \right] = \begin{bmatrix} -\dot{c}_{12} & \dot{c}_{11} & 0 \\ -\dot{c}_{22} & \dot{c}_{21} & 0 \\ -\dot{c}_{32} & \dot{c}_{31} & 0 \end{bmatrix} \tag{J-17}$$

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}}{\partial \psi} \right] = \begin{bmatrix} \dot{c}_{21} & \dot{c}_{22} & \dot{c}_{23} \\ -\dot{c}_{11} & -\dot{c}_{12} & -\dot{c}_{13} \\ 0 & 0 & 0 \end{bmatrix} \tag{J-18}$$

and the last term of Eq. (J-15) vanishes.

Then, with r_1, r_2, r_3 given by Eq. (J-6), the constrained equation of motion is

$$[\mathbf{F}] \begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathcal{Q}' \\ \mathbf{r} \end{bmatrix} \tag{J-19}$$

$$\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} \ddot{x}_I \\ \ddot{y}_I \\ \ddot{z}_I \\ \ddot{\theta}_I \\ \ddot{\phi}_I \\ \ddot{\psi}_I \\ \ddot{x}_{II} \\ \ddot{y}_{II} \\ \ddot{z}_{II} \\ \ddot{\theta}_{II} \\ \ddot{\phi}_{II} \\ \ddot{\psi}_{II} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \quad \begin{bmatrix} Q' \\ r \end{bmatrix} = \begin{bmatrix} Q'_{xI} \\ Q'_{yI} \\ Q'_{zI} \\ p'_{1I} \\ p'_{2I} \\ p'_{3I} \\ Q'_{xII} \\ Q'_{yII} \\ Q'_{zII} \\ p'_{1II} \\ p'_{2II} \\ p'_{3II} \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} \tag{J-20}$$

$$\tag{J-21}$$

$$[F] = \begin{bmatrix} m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & m_{11I} & m_{12I} & m_{13I} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\theta I} & e_{2\theta I} & e_{3\theta I} \\ 0 & 0 & 0 & m_{21I} & m_{22I} & m_{23I} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\phi I} & e_{2\phi I} & 0 \\ 0 & 0 & 0 & m_{31I} & m_{32I} & m_{33I} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\psi I} & e_{2\psi I} & e_{3\psi I} \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{II} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{II} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{II} & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{11II} & m_{12II} & m_{13II} & e_{1\theta II} & e_{2\theta II} & e_{3\theta II} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{21II} & m_{22II} & m_{23II} & e_{1\phi II} & e_{2\phi II} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{31II} & m_{32II} & m_{33II} & e_{1\psi II} & e_{2\psi II} & e_{3\psi II} \\ -1 & 0 & 0 & e_{1\theta I} & e_{1\phi I} & e_{1\psi I} & 1 & 0 & 0 & e_{1\theta II} & e_{1\phi II} & e_{1\psi II} & 0 & 0 & 0 \\ 0 & -1 & 0 & e_{2\theta I} & e_{2\phi I} & e_{2\psi I} & 0 & 1 & 0 & e_{2\theta II} & e_{2\phi II} & e_{2\psi II} & 0 & 0 & 0 \\ 0 & 0 & -1 & e_{3\theta I} & 0 & e_{3\psi I} & 0 & 0 & 1 & e_{3\theta II} & 0 & e_{3\psi II} & 0 & 0 & 0 \end{bmatrix} \tag{J-22}$$

At each step of the integration during which the constraint is acting, Eq. (J-19) is solved for the accelerations \ddot{q}_s , which are then integrated.

The hard-mounted spring constraint is the condition that, at the tips of the hard-mounted springs, neither translation nor rotation in a plane perpendicular to the springs is permitted. Thus, $g_x = g_z = 0$ (see Appendix K) and $h = 0$ (see Appendix L).

Let

$$\mathbf{r} = \begin{bmatrix} x_I - x_{II} \\ y_I - y_{II} \\ z_I - z_{II} \end{bmatrix}, \quad \mathbf{r}_{Il} = \begin{bmatrix} x'_{Il} \\ y'_{Il} \\ z'_{Il} \end{bmatrix}, \quad \mathbf{r}_{Im} = \begin{bmatrix} x'_{Im} \\ y'_{Im} \\ z'_{Im} \end{bmatrix}$$

$$\mathbf{v}_k = \begin{bmatrix} v_{kx} \\ v_{ky} \\ v_{kz} \end{bmatrix}, \quad \mathbf{d}_k = \begin{bmatrix} d_{kx} \\ d_{ky} \\ d_{kz} \end{bmatrix}, \quad \boldsymbol{\rho}_k = \begin{bmatrix} \rho_{kx} \\ \rho_{ky} \\ \rho_{kz} \end{bmatrix} \tag{J-23}$$

The symbols l, m refer to the first and second springs in the input, and k refers to either.

The constraint equations are then

$$f_1 = g_x = \mathbf{b}_{(1)}^T \mathbf{r} + \mathbf{t}_{(1)}^T \mathbf{r}'_l - \mathcal{X}'_{II} - j_l \rho_{lx} = 0 \quad (\text{J-24})$$

$$f_2 = g_z = \mathbf{b}_{(3)}^T \mathbf{r} + \mathbf{t}_{(3)}^T \mathbf{r}'_l - \mathcal{Z}'_{II} - j_l \rho_{lz} = 0 \quad (\text{J-25})$$

Since $h = 0$ only when the numerator (Eq. L-13, Appendix L) vanishes

$$f_3 = \mathbf{t}_{(3)}^T \mathbf{r} (\mathcal{X}'_{II} - j_l \rho_{lx} - \mathcal{X}'_{Im} - j_m \rho_{mx}) - \mathbf{t}_{(1)}^T \mathbf{r} (\mathcal{Z}'_{II} + j_l \rho_{lz} - \mathcal{Z}'_{Im} - j_m \rho_{mx}) = 0 \quad (\text{J-26})$$

We will have need for

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}_{II}}{\partial q_s} \right]$$

and

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}_I}{\partial q_s} \right], \quad s = 1, \dots, 12$$

For convenience, only

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}_{II}}{\partial q_s} \right]$$

is computed; the rest are found by replacing II by I, and b by a .

$$\frac{d}{dt} \left(\frac{\partial b_{11}}{\partial \theta_{II}} \right) = (c\theta_{II} s\phi_{II} s\psi_{II}) \dot{\theta}_{II} + (s\theta_{II} c\phi_{II} s\psi_{II}) \dot{\phi}_{II} + (s\theta_{II} s\phi_{II} c\psi_{II}) \dot{\psi}_{II} \quad (\text{J-27})$$

$$\frac{d}{dt} \left(\frac{\partial b_{12}}{\partial \theta_{II}} \right) = - (c\theta_{II} c\phi_{II} s\phi_{II}) \dot{\theta}_{II} + (s\theta_{II} s\phi_{II} s\psi_{II}) \dot{\phi}_{II} - (s\theta_{II} c\phi_{II} c\psi_{II}) \dot{\psi}_{II} \quad (\text{J-28})$$

$$\frac{d}{dt} \left(\frac{\partial b_{13}}{\partial \theta_{II}} \right) = - (s\theta_{II} s\psi_{II}) \dot{\theta}_{II} + c\theta_{II} c\psi_{II} \dot{\psi}_{II} \quad (\text{J-29})$$

$$\frac{d}{dt} \left(\frac{\partial b_{21}}{\partial \theta_{II}} \right) = (c\theta_{II} s\phi_{II} c\psi_{II}) \dot{\theta}_{II} + (s\theta_{II} c\phi_{II} c\psi_{II}) \dot{\phi}_{II} - (s\theta_{II} s\phi_{II} s\psi_{II}) \dot{\psi}_{II} \quad (\text{J-30})$$

$$\frac{d}{dt} \left(\frac{\partial b_{22}}{\partial \theta_{II}} \right) = - (c\theta_{II} c\phi_{II} c\psi_{II}) \dot{\theta}_{II} + (s\theta_{II} s\phi_{II} c\psi_{II}) \dot{\phi}_{II} + (s\theta_{II} c\phi_{II} s\psi_{II}) \dot{\psi}_{II} \quad (\text{J-31})$$

$$\frac{d}{dt} \left(\frac{\partial b_{23}}{\partial \theta_{II}} \right) = - (s\theta_{II} c\psi_{II}) \dot{\theta}_{II} - (c\theta_{II} s\psi_{II}) \dot{\psi}_{II} \quad (\text{J-32})$$

$$\frac{d}{dt} \left(\frac{\partial b_{31}}{\partial \theta_{II}} \right) = - (s\theta_{II} s\phi_{II}) \dot{\theta}_{II} + (c\theta_{II} c\phi_{II}) \dot{\phi}_{II} \quad (\text{J-33})$$

$$\frac{d}{dt} \left(\frac{\partial b_{32}}{\partial \theta_{II}} \right) = (s\theta_{II} c\phi_{II}) \dot{\theta}_{II} + (c\theta_{II} s\phi_{II}) \dot{\phi}_{II} \quad (\text{J-34})$$

$$\frac{d}{dt} \left(\frac{\partial b_{33}}{\partial \theta_{II}} \right) = - (c\theta_{II}) \dot{\theta}_{II} \quad (\text{J-35})$$

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}_{II}}{\partial \phi_{II}} \right] = \begin{bmatrix} -\dot{b}_{12} & \dot{b}_{11} & 0 \\ -\dot{b}_{22} & \dot{b}_{21} & 0 \\ -\dot{b}_{32} & \dot{b}_{31} & 0 \end{bmatrix} \quad (\text{J-36})$$

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}_{II}}{\partial \psi_{II}} \right] = \begin{bmatrix} \dot{b}_{21} & \dot{b}_{22} & \dot{b}_{23} \\ -\dot{b}_{11} & -\dot{b}_{12} & -\dot{b}_{13} \\ 0 & 0 & 0 \end{bmatrix} \quad (\text{J-37})$$

Clearly

$$\frac{d}{dt} \left[\frac{\partial \mathbf{A}_{II}}{\partial q_s} \right] = 0 \quad \text{for } s = I_x, \dots, II_z \quad (\text{J-38})$$

Now

$$\frac{\partial \mathbf{r}}{\partial x_I} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (\text{J-39})$$

$$\frac{\partial \mathbf{r}}{\partial y_I} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (\text{J-40})$$

$$\frac{\partial \mathbf{r}}{\partial z_I} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (\text{J-41})$$

$$\frac{\partial \mathbf{r}}{\partial x_{II}} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad (\text{J-42})$$

$$\frac{\partial \mathbf{r}}{\partial y_{II}} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad (\text{J-43})$$

$$\frac{\partial \mathbf{r}}{\partial z_{II}} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad (\text{J-44})$$

$$\frac{\partial \mathbf{r}}{\partial q_s} = 0 \quad \text{for all other } q_s \quad (\text{J-45})$$

The following expressions also will be needed

$$\dot{\mathbf{T}} = \dot{\mathbf{A}}_{II} \mathbf{A}_I^T + \mathbf{A}_{II} \dot{\mathbf{A}}_I^T \quad (\text{J-46})$$

$$\frac{\partial T}{\partial q_s} = \frac{\partial \mathbf{A}_{II}}{\partial q_s} \mathbf{A}_I^T + \mathbf{A}_{II} \frac{\partial \mathbf{A}_I^T}{\partial q_s} \quad (\text{J-47})$$

$$\frac{d}{dt} \left[\frac{\partial T}{\partial q_s} \right] = \frac{d}{dt} \left[\frac{\partial \mathbf{A}_{II}}{\partial q_s} \right] \mathbf{A}_I^T + \frac{\partial \mathbf{A}_{II}}{\partial q_s} \dot{\mathbf{A}}_I^T + \dot{\mathbf{A}}_{II} \frac{\partial \mathbf{A}_I^T}{\partial q_s} + \mathbf{A}_{II} \frac{d}{dt} \left[\frac{\partial \mathbf{A}_I^T}{\partial q_s} \right] \quad (\text{J-48})$$

Now let

$$f_k = \mathbf{a}^{(2)T} \mathbf{d}_k \quad (J-49)$$

$$g_k = \mathbf{a}^{(2)T} \mathbf{A}_{II}^T \boldsymbol{\rho}_k \quad (J-50)$$

Then, from Appendix B

$$j_k = \frac{f_k}{g_k} \quad (J-51)$$

$$\dot{f}_k = \dot{\mathbf{a}}^{(2)T} \mathbf{d}_k + \mathbf{a}^{(2)T} \mathbf{v}_k \quad (J-52)$$

$$\dot{g}_k = \dot{\mathbf{a}}^{(2)T} \mathbf{A}_{II}^T \boldsymbol{\rho}_k + \mathbf{a}^{(2)T} \dot{\mathbf{A}}_{II}^T \boldsymbol{\rho}_k \quad (J-53)$$

and

$$\dot{j}_k = \frac{1}{g_k} (\dot{f}_k - j_k \dot{g}_k) \quad (J-54)$$

$$\frac{\partial f_k}{\partial q_s} = \frac{\partial \mathbf{a}^{(2)T}}{\partial q_s} \mathbf{d}_k + \mathbf{a}^{(2)T} \left(\frac{\partial \mathbf{r}}{\partial q_s} + \frac{\partial \mathbf{A}_I^T}{\partial q_s} \mathbf{r}'_{Ik} - \frac{\partial \mathbf{A}_{II}^T}{\partial q_s} \mathbf{r}'_{IIk} \right) \quad (J-55)$$

$$\frac{\partial g_k}{\partial q_s} = \frac{\partial \mathbf{a}^{(2)T}}{\partial q_s} \mathbf{A}_{II}^T \boldsymbol{\rho}_k + \mathbf{a}^{(2)T} \frac{\partial \mathbf{A}_{II}^T}{\partial q_s} \boldsymbol{\rho}_k \quad (J-56)$$

$$\frac{\partial j_k}{\partial q_s} = \frac{1}{g_k} \left[\frac{\partial f_k}{\partial q_s} - j_k \frac{\partial g_k}{\partial q_s} \right] \quad (J-57)$$

Now let

$$f_k = \frac{\partial f_k}{\partial q_s} - j_k \frac{\partial g_k}{\partial q_s} \quad (J-58)$$

Then

$$\begin{aligned} \frac{d}{dt} \left[\frac{\partial f_k}{\partial q_s} \right] &= \frac{d}{dt} \left[\frac{\partial \mathbf{a}^{(2)T}}{\partial q_s} \right] \mathbf{d}_k + \frac{\partial \mathbf{a}^{(2)T}}{\partial q_s} \mathbf{v}_k + \dot{\mathbf{a}}^{(2)T} \left[\frac{\partial \mathbf{r}}{\partial q_s} + \frac{\partial \mathbf{A}_I^T}{\partial q_s} \mathbf{r}'_{Ik} - \frac{\partial \mathbf{A}_{II}^T}{\partial q_s} \mathbf{r}'_{IIk} \right] \\ &+ \mathbf{a}^{(2)T} \left[\frac{d}{dt} \left(\frac{\partial \mathbf{A}_I^T}{\partial q_s} \right) \mathbf{r}'_{Ik} - \frac{d}{dt} \left(\frac{\partial \mathbf{A}_{II}^T}{\partial q_s} \right) \mathbf{r}'_{IIk} \right] \end{aligned} \quad (J-59)$$

$$\begin{aligned} \frac{d}{dt} \left[\frac{\partial g_k}{\partial q_s} \right] &= \frac{d}{dt} \left[\frac{\partial \mathbf{a}^{(2)T}}{\partial q_s} \right] \mathbf{A}_{II}^T \boldsymbol{\rho}_k + \frac{\partial \mathbf{a}^{(2)T}}{\partial q_s} \dot{\mathbf{A}}_{II}^T \boldsymbol{\rho}_k \\ &+ \dot{\mathbf{a}}^{(2)T} \frac{\partial \mathbf{A}_{II}^T}{\partial q_s} \boldsymbol{\rho}_k + \mathbf{a}^{(2)T} \frac{d}{dt} \left[\frac{\partial \mathbf{A}_{II}^T}{\partial q_s} \right] \boldsymbol{\rho}_k \end{aligned} \quad (J-60)$$

Then

$$\dot{j}_k = \frac{d}{dt} \left(\frac{\partial f_k}{\partial q_s} \right) - j_k \left(\frac{\partial g_k}{\partial q_s} \right) - j_k \frac{d}{dt} \left(\frac{\partial g_k}{\partial q_s} \right) \quad (J-61)$$

and

$$\frac{d}{dt} \left(\frac{\partial j_k}{\partial q_s} \right) = \frac{1}{g_k} \left[\dot{j}_k - \left(\frac{\partial j_k}{\partial q_s} \right) \frac{d}{dt} \left(\frac{\partial g_k}{\partial q_s} \right) \right] \quad (J-62)$$

Thus, for the coefficients e_{ls} we have

$$e_{15} = \frac{\partial \mathbf{b}_{(1)}^T}{\partial q_s} \mathbf{r} + \mathbf{b}_{(1)}^T \frac{\partial \mathbf{r}}{\partial q_s} + \frac{\partial \mathbf{t}_{(1)}^T}{\partial q_s} \mathbf{r}'_{1l} - \frac{\partial j_l}{\partial q_s} \rho_{1z} \quad (\text{J-63})$$

$$e_{25} = \frac{\partial \mathbf{b}_{(3)}^T}{\partial q_s} \mathbf{r} + \mathbf{b}_{(3)}^T \frac{\partial \mathbf{r}}{\partial q_s} + \frac{\partial \mathbf{t}_{(3)}^T}{\partial q_s} \mathbf{r}'_{1l} - \frac{\partial j_l}{\partial q_s} \rho_{1z} \quad (\text{J-64})$$

$$e_{35} = \frac{\partial \mathbf{t}_{(3)}^T}{\partial q_s} (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) (x'_{11l} + j_l \rho_{1z} - x'_{11m} - j_m \rho_{mz}) + \mathbf{t}_{(3)}^T (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) \left(\frac{\partial j_l}{\partial q_s} \rho_{1z} - \frac{\partial j_m}{\partial q_s} \rho_{mz} \right) \\ - \frac{\partial \mathbf{t}_{(1)}^T}{\partial q_s} (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) (z'_{11l} + j_l \rho_{1z} - z'_{11m} - j_m \rho_{mz}) - \mathbf{t}_{(1)}^T (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) \left(\frac{\partial j_l}{\partial q_s} \rho_{1z} - \frac{\partial j_m}{\partial q_s} \rho_{mz} \right) \quad (\text{J-65})$$

and

$$\dot{e}_{15} = \frac{d}{dt} \left(\frac{\partial \mathbf{b}_{(1)}^T}{\partial q_s} \right) \mathbf{r} + \frac{\partial \mathbf{b}_{(1)}^T}{\partial q_s} \dot{\mathbf{r}} + \dot{\mathbf{b}}_{(1)}^T \frac{\partial \mathbf{r}}{\partial q_s} + \frac{d}{dt} \left(\frac{\partial \mathbf{t}_{(1)}^T}{\partial q_s} \right) \mathbf{r}'_{1l} - \frac{d}{dt} \left(\frac{\partial j_l}{\partial q_s} \right) \rho_{1z} \quad (\text{J-66})$$

$$\dot{e}_{25} = \frac{d}{dt} \left(\frac{\partial \mathbf{b}_{(3)}^T}{\partial q_s} \right) \mathbf{r} + \frac{\partial \mathbf{b}_{(3)}^T}{\partial q_s} \dot{\mathbf{r}} + \dot{\mathbf{b}}_{(3)}^T \frac{\partial \mathbf{r}}{\partial q_s} + \frac{d}{dt} \left(\frac{\partial \mathbf{t}_{(3)}^T}{\partial q_s} \right) \mathbf{r}'_{1l} - \frac{d}{dt} \left(\frac{\partial j_l}{\partial q_s} \right) \rho_{1z} \quad (\text{J-67})$$

$$\dot{e}_{35} = \frac{d}{dt} \left[\frac{\partial \mathbf{t}_{(3)}^T}{\partial q_s} \right] (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) (x'_{11l} + j_l \rho_{1z} - x'_{11m} - j_m \rho_{mz}) + \frac{\partial \mathbf{t}_{(3)}^T}{\partial q_s} (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) (\dot{j}_l \rho_{1z} - \dot{j}_m \rho_{mz}) \\ + \dot{\mathbf{t}}_{(3)}^T (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) \left(\frac{\partial j_l}{\partial q_s} \rho_{1z} - \frac{\partial j_m}{\partial q_s} \rho_{mz} \right) + \mathbf{t}_{(3)}^T (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) \left[\frac{d}{dt} \left(\frac{\partial j_l}{\partial q_s} \right) \rho_{1z} - \frac{d}{dt} \left(\frac{\partial j_m}{\partial q_s} \right) \rho_{mz} \right] \\ - \frac{d}{dt} \left(\frac{\partial \mathbf{t}_{(1)}^T}{\partial q_s} \right) (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) (z'_{11l} + j_l \rho_{1z} - z'_{11m} - j_m \rho_{mz}) - \frac{\partial \mathbf{t}_{(1)}^T}{\partial q_s} (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) (\dot{j}_l \rho_{1z} - \dot{j}_m \rho_{mz}) \\ - \dot{\mathbf{t}}_{(1)}^T (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) \left(\frac{\partial j_l}{\partial q_s} \rho_{1z} - \frac{\partial j_m}{\partial q_s} \rho_{mz} \right) - \mathbf{t}_{(1)}^T (\mathbf{r}'_{1l} - \mathbf{r}'_{1m}) \left[\frac{d}{dt} \left(\frac{\partial j_l}{\partial q_s} \right) \rho_{1z} - \frac{d}{dt} \left(\frac{\partial j_m}{\partial q_s} \right) \rho_{mz} \right] \quad (\text{J-68})$$

The constrained equations of motion are

$$[\mathbf{F}] \begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathcal{Q}' \\ \mathbf{r} \end{bmatrix} \quad (\text{J-69})$$

where

$$\begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{z}_1 \\ \ddot{\theta}_1 \\ \ddot{\phi}_1 \\ \ddot{\psi}_1 \\ \ddot{x}_{II} \\ \ddot{y}_{II} \\ \ddot{z}_{II} \\ \ddot{\theta}_{II} \\ \ddot{\phi}_{II} \\ \ddot{\psi}_{II} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \quad \begin{bmatrix} \mathcal{Q}' \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}'_{zI} \\ \mathcal{Q}'_{yI} \\ \mathcal{Q}'_{zI} \\ \rho'_{1I} \\ \rho'_{2I} \\ \rho'_{3I} \\ \mathcal{Q}'_{zII} \\ \mathcal{Q}'_{yII} \\ \mathcal{Q}'_{zII} \\ \rho'_{1II} \\ \rho'_{2II} \\ \rho'_{3II} \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} \quad (\text{J-70})$$

$$[F] = \begin{bmatrix}
 m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{1xI} & e_{2xI} & e_{3xI} \\
 0 & m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{1yI} & e_{2yI} & e_{3yI} \\
 0 & 0 & m_I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{1zI} & e_{2zI} & e_{3zI} \\
 0 & 0 & 0 & m_{11I} & m_{12I} & m_{13I} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\theta I} & e_{2\theta I} & e_{3\theta I} \\
 0 & 0 & 0 & m_{21I} & m_{22I} & m_{23I} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\phi I} & e_{2\phi I} & e_{3\phi I} \\
 0 & 0 & 0 & m_{31I} & m_{32I} & m_{33I} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\psi I} & e_{2\psi I} & e_{3\psi I} \\
 0 & 0 & 0 & 0 & 0 & 0 & m_{1I} & 0 & 0 & 0 & 0 & 0 & e_{1xII} & e_{2xII} & e_{3xII} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{1I} & 0 & 0 & 0 & 0 & e_{1yII} & e_{2yII} & e_{3yII} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{1I} & 0 & 0 & 0 & e_{1zII} & e_{2zII} & e_{3zII} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{11II} & m_{12II} & m_{13II} & e_{1\theta II} & e_{2\theta II} & e_{3\theta II} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{21II} & m_{22II} & m_{23II} & e_{1\phi II} & e_{2\phi II} & e_{3\phi II} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{31II} & m_{32II} & m_{33II} & e_{1\psi II} & e_{2\psi II} & e_{3\psi II} \\
 e_{1xI} & e_{1yI} & e_{1zI} & e_{1\theta I} & e_{1\phi I} & e_{1\psi I} & e_{1xII} & e_{1yII} & e_{1zII} & e_{1\theta II} & e_{1\phi II} & e_{1\psi II} & 0 & 0 & 0 \\
 e_{2xI} & e_{2yI} & e_{2zI} & e_{2\theta I} & e_{2\phi I} & e_{2\psi I} & e_{2xII} & e_{2yII} & e_{2zII} & e_{2\theta II} & e_{2\phi II} & e_{2\psi II} & 0 & 0 & 0 \\
 e_{3xI} & e_{3yI} & e_{3zI} & e_{3\theta I} & e_{3\phi I} & e_{3\psi I} & e_{3xII} & e_{3yII} & e_{3zII} & e_{3\theta II} & e_{3\phi II} & e_{3\psi II} & 0 & 0 & 0
 \end{bmatrix} \tag{J-71}$$

APPENDIX K

Distance in the $x_{II}' z_{II}'$ Plane Between the Initial Point of Contact on Body I of a Spring Hard-Mounted on Body II and the Tip of the Spring

The distance between the initial and current contact point for a hard-mounted spring tip is required to determine the no-slippage constraint condition.

A. Symbols

- d_i vector distance between point i on body I and point i on body II
- g vector distance (as defined in the title of this Appendix) $g = g_x e_{IIx}' + g_z e_{IIz}'$
- j_i length of extension of spring i (see Appendix B)
- l subscript identifying first hard-mounted spring
- \bar{r}_{IIi} location in inertial space of tip of spring i
- \bar{r}'_{IIi} location in body II CS of the tip of spring i
- T matrix product $A_{II} A_I^{-1}$
- $t_{i,j}$ elements of T matrix (i^{th} row, j^{th} column)

B. Determination of g_i

The vector g consists of just the x and z components of g_i in the body II CS. Now

$$A_{II} d_i = A_{II} r + A_{II} A_I^T r'_{II} - A_{II} A_{II}^T \bar{r}'_{IIi} = A_{II} r + T r'_{II} - r'_{IIi} \tag{K-1}$$

Therefore

$$g_x = b_{(1)}^T r + t_{(1)}^T r'_{II} - \bar{x}'_{IIi}, \quad g_z = b_{(3)}^T r + t_{(3)}^T r'_{II} - \bar{z}'_{IIi} \tag{K-2}$$

where the first spring has been specified.

Clearly

$$\bar{x}_{II} = x'_{II} + j_l \rho_{lx} \quad (\text{K-3})$$

$$\bar{z}_{II} = z'_{II} + j_l \rho_{lz} \quad (\text{K-4})$$

APPENDIX L

Angle of Rotation in the $x'_{II} z'_{II}$ Plane Between Body I Pads and Body II Spring Tips

The angle of rotation in the $x'_{II} z'_{II}$ plane is required to specify the constraint condition that no such rotation shall occur.

A. Symbols

- a** a vector on body I from the initial point of contact of spring tip l to the initial point of contact of spring tip m
- b** a vector joining the tip of spring l to the tip of spring m
- e_a** unit vector along the projection of **a** on the $x'_{II} z'_{II}$ plane
- e_b** unit vector along the projection of **b** on the $x'_{II} z'_{II}$ plane
- h** angle of rotation in the $x'_{II} z'_{II}$ plane between body I pads and body II spring tips
- l, m** subscripts identifying the first two hard-mounted springs
- p_i** location vector of tip of spring i (see Appendix B)
- T** matrix product $A_{II} A_I^{-1}$ (see Appendix K)
- t_{i,j}** elements of the **T** matrix (i^{th} row, j^{th} column)

B. Calculation of h

During any one integration interval, if the rotation is constrained, the rotation will be small, so it is not necessary to consider angles other than those in the first and fourth quadrants.

Expression of the definitions of **a** and **b** yields Eqs. (L-1) and (L-2).

$$\mathbf{a} = \mathbf{r}'_{I_l} - \mathbf{r}'_{I_m} \quad (\text{L-1})$$

$$\mathbf{b} = \mathbf{p}_l - \mathbf{p}_m \quad (\text{L-2})$$

Expressed in body II coordinates, Eqs. (L-1) and (L-2) become

$$\begin{aligned} \mathbf{a} &= \mathbf{A}_{II} \mathbf{A}_I^{-1} [(x'_{I_l} - x'_{I_m}) \mathbf{e}_{Ix'} + (y'_{I_l} - y'_{I_m}) \mathbf{e}_{Iy'} + (z'_{I_l} - z'_{I_m}) \mathbf{e}_{Iz'}] \\ &= [t_{11}(x'_{I_l} - x'_{I_m}) + t_{12}(y'_{I_l} - y'_{I_m}) + t_{13}(z'_{I_l} - z'_{I_m})] \mathbf{e}_{IIx'} \\ &\quad + [t_{21}(x'_{I_l} - x'_{I_m}) + t_{22}(y'_{I_l} - y'_{I_m}) + t_{23}(z'_{I_l} - z'_{I_m})] \mathbf{e}_{IIy'} \\ &\quad + [t_{31}(x'_{I_l} - x'_{I_m}) + t_{32}(y'_{I_l} - y'_{I_m}) + t_{33}(z'_{I_l} - z'_{I_m})] \mathbf{e}_{IIz'} \end{aligned} \quad (\text{L-3})$$

$$\begin{aligned} \mathbf{b} &= (x'_{IIl} - x'_{IIm} + j_l \rho_{lx} - j_m \rho_{mx}) \mathbf{e}_{IIx'} \\ &\quad + (y'_{IIl} - y'_{IIm} + j_l \rho_{ly} - j_m \rho_{my}) \mathbf{e}_{IIy'} \\ &\quad + (z'_{IIl} - z'_{IIm} + j_l \rho_{lz} - j_m \rho_{mz}) \mathbf{e}_{IIz'} \end{aligned} \tag{L-4}$$

Let

$$\begin{aligned} \Delta_a &= (a_{IIx'}^2 + a_{IIz'}^2)^{1/2} \\ &= \{ [t_{11}(x'_{IIl} - x'_{IIm}) + t_{12}(y'_{IIl} - y'_{IIm}) + t_{13}(z'_{IIl} - z'_{IIm})]^2 \\ &\quad + [t_{31}(x'_{IIl} - x'_{IIm}) + t_{32}(y'_{IIl} - y'_{IIm}) + t_{33}(z'_{IIl} - z'_{IIm})]^2 \}^{1/2} \end{aligned} \tag{L-5}$$

$$\begin{aligned} \Delta_b &= (b_{IIx'}^2 + b_{IIz'}^2)^{1/2} \\ &= [(x'_{IIl} - x'_{IIm} + j_l \rho_{lx} - j_m \rho_{mx})^2 + (z'_{IIl} - z'_{IIm} + j_l \rho_{lz} - j_m \rho_{mz})^2]^{1/2} \end{aligned} \tag{L-6}$$

Then

$$\begin{aligned} \mathbf{e}_a &= \left(\frac{a_{IIx'}}{\Delta_a} \right) \mathbf{e}_{IIx'} + \left(\frac{a_{IIz'}}{\Delta_a} \right) \mathbf{e}_{IIz'} = e_{ax} \mathbf{e}_{IIx'} + e_{az} \mathbf{e}_{IIz'} \\ \mathbf{e}_b &= \left(\frac{b_{IIx'}}{\Delta_b} \right) \mathbf{e}_{IIx'} + \left(\frac{b_{IIz'}}{\Delta_b} \right) \mathbf{e}_{IIz'} = e_{bx} \mathbf{e}_{IIx'} + e_{bz} \mathbf{e}_{IIz'} \end{aligned} \tag{L-7}$$

and

$$\begin{aligned} e_{ax} &= \frac{[t_{11}(x'_{IIl} - x'_{IIm}) + t_{12}(y'_{IIl} - y'_{IIm}) + t_{13}(z'_{IIl} - z'_{IIm})]}{\Delta_a} \\ e_{az} &= \frac{[t_{31}(x'_{IIl} - x'_{IIm}) + t_{32}(y'_{IIl} - y'_{IIm}) + t_{33}(z'_{IIl} - z'_{IIm})]}{\Delta_a} \end{aligned} \tag{L-8}$$

$$e_{bx} = \frac{x'_{IIl} - x'_{IIm} + j_l \rho_{lx} - j_m \rho_{mx}}{\Delta_b} \tag{L-9}$$

$$e_{bz} = \frac{z'_{IIl} - z'_{IIm} + j_l \rho_{lz} - j_m \rho_{mz}}{\Delta_b} \tag{L-10}$$

Since \mathbf{e}_a and \mathbf{e}_b are unit vectors

$$\sin h = |\mathbf{e}_a \times \mathbf{e}_b| = e_{ax} e_{bz} - e_{az} e_{bx} \tag{L-11}$$

$$\cos h = \mathbf{e}_a \cdot \mathbf{e}_b = e_{ax} e_{bx} + e_{az} e_{bz} \tag{L-12}$$

Thus

$$h = \tan^{-1} \left(\frac{e_{ax} e_{bz} - e_{az} e_{bx}}{e_{ax} e_{bx} + e_{az} e_{bz}} \right) \tag{L-13}$$

APPENDIX M

FMARK Integration Method

The integration method used by FMARK is the fourth order Adams-Moulton predictor formula with one correction. Automatic error control is exercised by comparing the value of the predicted with the corrected result. Fourth order Runge-Kutta integration is used to build backward differences, initially or for a restart.

The classical fourth order Runge-Kutta equations follow. Let the differential equations to be solved be of the form

$$\dot{y}_j = f_j(t, \bar{y}) \quad j = 1, n \quad (\text{M-1})$$

where symbolically

$$\bar{y} = (y_1, \dots, y_n) \quad (\text{M-2})$$

Let

$$\begin{aligned} y_{j,\eta} \quad \text{and} \quad f_{j,\eta} \quad \text{denote} \\ y_j \quad \text{and} \quad f_j \quad \text{at} \quad t = t_\eta \end{aligned} \quad (\text{M-3})$$

Thus

$$\bar{y}_\eta = (y_{1,\eta}, \dots, y_{n,\eta}) \quad (\text{M-4})$$

Let

$$\bar{K}_i = (K_{1i}, \dots, K_{ni}) \quad (\text{M-5})$$

and

$$K_{j1} = h f_j(t, \bar{y}_\eta) \quad (\text{M-6})$$

$$K_{j2} = h f_j\left(t + \frac{h}{2}, \quad \bar{y}_\eta + \frac{\bar{K}_2}{2}\right) \quad (\text{M-7})$$

$$K_{j3} = h f_j\left(t + \frac{h}{2}, \quad \bar{y}_\eta + \frac{\bar{K}_2}{2}\right) \quad (\text{M-8})$$

$$K_{j4} = h f_j(t + h, \quad \bar{y}_\eta + \bar{K}_3) \quad (\text{M-9})$$

The numerical integration equation is then

$$\bar{y}_{\eta+1} = \bar{y}_\eta + \frac{1}{6}(\bar{K}_1 + 2\bar{K}_2 + 2\bar{K}_3 + \bar{K}_4) \quad (\text{M-10})$$

where $y_{j,\eta+1}$ denotes y_j at $t+h$.

The Adams-Moulton fourth order predictor-corrector equations follow, where $\bar{y}_{\eta+1}^p$ = the predicted values, and $\bar{y}_{\eta+1}^c$ = the corrected value

$$\bar{y}_{\eta+1}^p = \bar{y}_\eta + h \left(\sum_{i=0}^4 a_i \nabla^i \right) \dot{\bar{y}}_\eta \quad (\text{M-11})$$

where

$$\begin{aligned}\nabla^0 \dot{\bar{y}}_\eta &= \dot{\bar{y}}_\eta \\ \nabla^1 \dot{\bar{y}}_\eta &= \dot{\bar{y}}_\eta - \dot{\bar{y}}_{\eta-1} \\ \nabla^2 \dot{\bar{y}}_\eta &= \nabla^1 \dot{\bar{y}}_\eta - \nabla^1 \dot{\bar{y}}_{\eta-1} \\ &\text{etc.}\end{aligned}\tag{M-12}$$

and

$$a_0 = 1, \quad a_1 = \frac{1}{2}, \quad a_2 = \frac{5}{12}, \quad a_3 = \frac{3}{8}, \quad a_4 = \frac{251}{720}\tag{M-13}$$

Also

$$\dot{\bar{y}}_{j,\eta+1}^P = f_j(t_{\eta+1}, \bar{y}_{\eta+1})\tag{M-14}$$

and the correction becomes

$$\dot{\bar{y}}_{j,\eta+1}^C = \dot{\bar{y}}_{j,\eta} + h \left(\sum_{i=0}^4 b_i \nabla^i \right) \dot{\bar{y}}_{\eta+1}^P\tag{M-15}$$

where

$$b_0 = a_0, \quad b_{m+1} = a_{m+1} - a_m, \quad m = 1, 2, 3\tag{M-16}$$

Let

$$E_{\eta+1} = \max_j \frac{|y_{j,\eta+1}^P - y_{j,\eta+1}^C|}{D_j}\tag{M-17}$$

where

$$D_j = \max |y_{j,\eta+1}^C, \hat{y}|\tag{M-18}$$

$E_{\eta+1}$ represents the maximum error in any of the dependent variables due to truncation error in the step t_η to $t_{\eta+1}$. The user, through the array HB, provides the

- \bar{E} upper bound on $E_{\eta+1}$
- \hat{E} lower bound on $E_{\eta+1}$
- h_{\max} maximum allowable step size
- h_{\min} minimum allowable step size
- \hat{y} constant used to prevent unnecessary reduction in h whenever $|y_{j,\eta+1}|$ is small

If $E_{\eta+1} \leq \hat{E}$ for four successive steps, h is doubled. If $\hat{E} < E_{\eta+1} \leq \bar{E}$, h is unchanged. If $E_{\eta+1} > \bar{E}$, h is halved.

If a discontinuity occurs in some \dot{y}_j it is obvious that the Adams-Moulton technique with the backward differences can not be continued. In this case, a restart must be made (i.e., Runge-Kutta integration is used for four steps). With these points, Adams-Moulton integration is continued.

APPENDIX N
FORTRAN Listing and Sample Problem Printout

GENERAL INPUT

C

```

1 FORMAT(6E12.8)
2 FORMAT(6I12)
EQUIVALENCE (T,T),(ZERO,ZERO),
1 (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
11)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
COMMON A,B,AD,BD
DIMENSION YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
DIMENSION DM(3,3),DN(3,3)
COMMON W11,TCO
DIMENSION TCG(6),TCO(6)
DIMENSION FM(3,4),FEL(3),AONST(3),CM(3,3),DUD(3)
DIMENSION DO(12),DF(8),D(8),XC1(8),YC1(8),ZC1(8),XC2(8),YC2(8),
1ZC2(8)
DIMENSION EP(12),PI(3),TP(6),XP1(3),YP1(3),ZP1(3),XP2(3),YP2(3),
1ZP2(3),UPX(3),UPY(3),UPZ(3)
DIMENSION G(12),GA(6),PA(6),VA(6),PAD(6),VAD(6),SGX(6),SGY(6),
1SGZ(6),UAX(3),UAY(3),UAZ(3),UPAX(3),UPAY(3),UPAZ(3),VAX(3),VAY(3)
1,VAZ(3),VPAX(3),VPAY(3),VPAZ(3)
1,GG(6)
DIMENSION BINV(15)
DIMENSION H(12)
DIMENSION GJ(12),FG1(8),FG2(8),XG1(8),YG1(8),ZG1(8),XG2(8),
1YG2(8),ZG2(8),UGX(8),UGY(8),UGZ(8),VGX(8),VGY(8),VGZ(8)
DIMENSION XN(12),AN(8),RAX(8),PPN(8),VNX(8),VNY(8),VNZ(8),XN1(8),
1YN1(8),ZN1(8),XN2(8),YN2(8),ZN2(8),PNEH(8),EQP(8)
DIMENSION R(12),RR(16),XR1(16),YR1(16),ZR1(16),XR2(16),YN2(16),
1ZR2(16),URX(16),URY(16),URZ(16),VRX(16),VRY(16),VRZ(16),TE(16),
1TF(16),TRK(32)
DIMENSION SIOB(12)
DIMENSION S(12),XS1(8),YS1(8),ZS1(8),XS2(8),YS2(8),ZS2(8),USX(8),
1USY(8),USZ(8),SD(8),SK(8)
DIMENSION W(12),WD(8),UK(8),XU1(8),YU1(8),ZU1(8),XU2(8),YU2(8),
1ZU2(8)
DIMENSION Q(12)
COMMON XX1,XY1,XZ1,YY1,YZ1,ZZ1,XX2,XY2,XZ2,YY2,YZ2,ZZ2,
1FM10,FM20,DEL,TPR,TD,EX1,EY1,EZ1,EX2,EY2,EZ2,WX,WY,WZ,
2W1Y0,ICD,IPR,IAC,IHY,ICG,IPN,IRK,ISP,ISU,ICN
COMMON DF,D,XC,YC,ZC,XC1,XC2,YC1,YC2,ZC1,ZC2
COMMON PI,TP,UX,UY,UZ,XP,YP,ZP,UPX,UPY,UPZ,XP1,XP2,YP1,YP2,
1ZP1,ZP2
COMMON TA,TB,TC,TD,IAC1,IAC2,MAC,NAC,IGLAG,GA,PA,VA,PAD,
1VAD,SGX,SGY,SGZ,UX,UY,UZ,PUX,PUY,PUZ,VAX,VPAX,VAY,
2VPAY,VAZ,VPAZ,UAX,UPAX,UAY,UPAY,UAZ,UPAZ,GG
COMMON HD,HK,HE,TA,PPH,DHB,AH,PHBO,VHB,GAMH,JH,DEQP,HYDA,
1HYDB,HYDC,HE,TA,DA,HYDD,HYDE,DB
COMMON GJ,GST,FG3,IJLAG,MAXES,MAXES,FG1,FG2,TCO,TCG,
1UX,UY,UZ,XG,YG,ZG,XG1,XG2,YG1,YG2,ZG1,ZG2,VGX,VGY,
2VGZ,UGX,UGY,UGZ
COMMON XN,JN,PBO,VBN,GAMN,AN,RAX,PPN,XXN,YN,ZN,VNX,VNY,

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GENERAL INPUT

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1VNZ, XN1, XN2, YN1, YN2, ZN1, ZN2, EGP, PPM, PNEM
COMMON R, VEX, VRY, VRZ, URX, URY, URZ, TE, TF, TROC, SUN1,
1SUN2, SUN3, SAN1, SAN2, SAN3, XX10, YY10, ZZ10, XX20, YY20, ZZ20,
2FKD1, FMD2, RR, XR, YR, ZR, XR1, XR2, YR1, YR2, ZR1, ZR2, RUM, PUM,
3TRK
COMMON SIOG
COMMON S, SETA, XMU, EPSLN, DELTS, XS1, XS2, YS1, YS2, ZS1, ZS2,
1SD, SK
COMMON W, ETA, WD, UK, XU, YU, ZU, XU1, XU2, YU1, YU2, ZU1, ZU2, TG
DIMENSION WIOG(8)
COMMON WIOG
COMMON DM, DN, R1, R2, P3, P21, P22, P23, ZTEST, F, CA, XI1, YI1, ZI1, XI2,
XYI2, ZI2, USX, USY, USZ
COMMON XP, YP, ZP, XI1, XI2, ZI1, ZI2, ICLAG, TRIG2, ELM, LIRK, N, XS, YS, ZS
COMMON WL
DIMENSION WL(8)
DIMENSION F(15,15), CA(3)
ICLAG=1
449 FORMAT (1HD,28HBODY 1-FIXED AXES X Y Z)
450 FORMAT (1HD,28HBODY 2-FIXED AXES X Y Z)
451 FORMAT (1H ,28HDRAWING BOARD AXES X Z -Y)
READ INPUT TAPE 5,1011,(G(I),I=1,12)
1011 FORMAT(12A6)
READ INPUT TAPE 5,1012,WI1,IL1,IHY
1012 FORMAT(E12.8,2I12)
READ INPUT TAPE 5,1,XX1,XY1,XZ1,YY1,YZ1,ZZ1
READ INPUT TAPE 5,1,XX2,XY2,XZ2,YY2,YZ2,ZZ2
READ INPUT TAPE 5,1,FM10,FM20,DEL,TPR,TD
READ INPUT TAPE 5,1,EX1,EY1,EZ1,EX2,EY2,EZ2
READ INPUT TAPE 5,1,WX,WY,WZ,WIYD
READ INPUT TAPE 5,2,ICD,IFR,ICG,IPN,IRK,ISP
READ INPUT TAPE 5,2,ISU,IQN,IAC
WRITE OUTPUT TAPE 6,308,(G(I),I=1,12),IL1
308 FORMAT(1H1,12A6/5HD0CASE,15/11HDINPUT DATA/)
WRITE OUTPUT TAPE 6,309,DEL,TPR,TD,WIYD
309 FORMAT(1HD,13HDELTA TIME = E10.3, 4H SEC,3X,13HDELTA PRINT=
1E10.3,4H SEC,3X,13HTIME (INITIAL)E10.3,4H SEC,3X,13HSPIN-UP RATE
1E10.3, 8H DEG/SEC )
J=1
WRITE OUTPUT TAPE 6,310,J,FM10
310 FORMAT(1HD,14HINPUT FOR BODY 11,5X,7HMASS = E15.7,4H LBM)
WRITE OUTPUT TAPE 6,311
311 FORMAT(1HD,10X,25HINERTIA MATRIX LBM IN SQ ,15X,19HRATES ABOUT DR
1AWING, 6X,17HCOORDS IN DRAWING / 51X,18HBOARD AXIS DEG/SEC,7X,
121HBOARD CS OF CM-INCHES )
WRITE OUTPUT TAPE 6,312,XX1,YY1,WX,EX1,XY1,YZ1,WY,EY1,XZ1,ZZ1,WZ,
1 EZ1
312 FORMAT(1H ,5HIXX E15.7,5X,5HIYY E15.7,5X,5HX E15.7,5X,5HX
1 E15.7/6H IXY E15.7,5X,5HIYZ E15.7,5X,5HY E15.7,5X,5HY
1 E15.7/6H IXZ E15.7,5X,5HIZZ E15.7,5X,5HZ E15.7,5X,5HZ
1 E15.7 //)
J=2
WRITE OUTPUT TAPE 6,310,J,FM20
WRITE OUTPUT TAPE 6,311
WRITE OUTPUT TAPE 6,312,XX2,YY2,WX,EX2,XY2,YZ2,WY,EY2,XZ2,ZZ2,WZ,
1 EZ2

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GENERAL INPUT

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WRITE OUTPUT TAPE 6,313,ICD,IPR      ,ICG,IPN,IRK,ISP,ISU,ION
313 FORMAT(1H ,6HICD = 11,4X,6HIPR = 11,4X
16HICG = 11,4X,6HIPN = 11,4X,6HIRK = 11,4X,6HISP = 11,4X,6HISU = 11
1,4X,6HION = 11 //)
WRITE OUTPUT TAPE 6,449
WRITE OUTPUT TAPE 6,451
WRITE OUTPUT TAPE 6,450
WRITE OUTPUT TAPE 6,451
WRITE OUTPUT TAPE 6,845
845 FORMAT(1H1)
DO 3 K=1,12
DD(K)=0.0
3 CONTINUE
IF (ICD) 5000,4,5
C
C COULOMB DRAG INPUT
5 WRITE OUTPUT TAPE 6,314
314 FORMAT(1H0,10HCOULOMB DRAG INPUT /5X,8HD(I)-LBF,13X,8HDF(I)-IN,10X
1 ,40HDRAWING BOARD LOCATIONS OF DRAG FORCE-IN /49X,1HX,20X,1HY,
1 20X,1HZ / )
DO 6 I=1,ICD
READ INPUT TAPE 5,1,DF(I),D(I),XC,YC,ZC
WRITE OUTPUT TAPE 6,315,D(I),DF(I),XC,YC,ZC
315 FORMAT(1H ,E15.7,6X,E15.7,6X,E15.7,6X,E15.7,6X,E15.7 )
XC1(I)=XC-EX1
XC2(I)=XC-EX2
YC1(I)=YC-EZ1
YC2(I)=YC-EZ2
ZC1(I)=EZ1-YC
ZC2(I)=EZ2-YC
6 CONTINUE
C
C PYROTECHNIC INPUT
4 DO 7 K=1,12
EP(K)=0.0
7 CONTINUE
IF (IPR) 5000,18,9
9 WRITE OUTPUT TAPE 6,354
TP(2)=0.0
TP(3)=0.0
354 FORMAT(1H0,17HPYROTECHNIC INPUT / 11X,37HCOSINE DRAW BOARD AXIS AN
10 ITH DEVICE,30X,24HPYROTECHNIC LOCATIONS-IN / 6X,2HUX,19X,2HUY,
119X,2HUZ,20X,1HX,20X,1HY,20X,1HZ )
DO 10 I=1,IPR
READ INPUT TAPE 5,1,PI(I),TP(I)
READ INPUT TAPE 5,1,UX,UY,UZ,XP,YP,ZP
WRITE OUTPUT TAPE 6,355,UX,UY,UZ,XP,YP,ZP
355 FORMAT(1H ,E15.7,6X,E15.7,6X,E15.7,6X,E15.7,6X,E15.7,6X,E15.7 )
UPX(I)=UX
UPY(I)=UY
UPZ(I)=-UY
XP1(I)=XP-EX1
XP2(I)=XP-EX2
YP1(I)=YP-EZ1
YP2(I)=YP-EZ2
ZP1(I)=EZ1-YP

```

GENERAL INPUT

```

ZP2(I)=EY2-YP
10 CONTINUE
WRITE OUTPUT TAPE 6,356
356 FORMAT(1H ,1X,12HTOTAL IMPULSE,9X,11MFIRING TIME / 1X,
11EHITH DEVICE-LBF SEC,5X,12HITM PYRO-SEC )
DO 357 I=1,IPR
WRITE OUTPUT TAPE 6,355,P1(I),TP(I)
P1(I)=P1(I)/DEL
357 CONTINUE
C
C COLD GAS JET INPUT
18 DO 20 K=1,12
GJ(K)=0.0
20 CONTINUE
IF (ICG) 5000,21,22
22 READ INPUT TAPE 5,1,68T,763
723 READ INPUT TAPE 5,2,IJLAG,K
WRITE OUTPUT TAPE 6,369,F63,68T
WRITE OUTPUT TAPE 6,370,IJLAG
IF (IJLAG-1) 720,721,720
721 MAXES=1
MAXES=K
GO TO 722
369 FORMAT(2H0,16HCOLD GAS JET INPUT /4H F3 E15.7,5X,20XVALUE JET FORC
1E BECOMES ZERO E15.7,4H LBF )
370 FORMAT(1H ,8X,40NCOSINE ANGLE THRUST AND DRAIN BOARD-CODY 11,20H,
12SHCOLD GAS JET LOCATIONS-IN /
16X,20X,19X,20UY,19X,20UZ,20X,19X,20V,20X,19V,20X,19W,20X,19W )
720 MAXES=ICG-K+1
MAXES=ICG
722 DO 23 I=MAXES,MAXES
READ INPUT TAPE 5,3,F61(I),F62(I),TCO(I),TOS(I)
READ INPUT TAPE 5,4,UX,UY,UZ,X5,Y6,Z6
WRITE OUTPUT TAPE 6,355,UX,UY,UZ,X5,Y6,Z6
XG1(I)=XG-EX1
XG2(I)=XG-EX2
YG1(I)=ZG-EZ1
YG2(I)=ZG-EZ2
ZG1(I)=EY1-Y6
ZG2(I)=EY2-Y6
IF (IJLAG-1) 24,25,24
24 VGX(I)=UX
VGY(I)=UZ
VGZ(I)=-UY
UGX(I)=0.0
UGY(I)=0.0
UGZ(I)=0.0
GO TO 23
25 VGX(I)=0.0
VGY(I)=0.0
VGZ(I)=0.0
UGX(I)=UX
UGY(I)=UZ
UGZ(I)=-UY
23 CONTINUE
IF (MAXES-ICG) 723,8001,8001

```

```

GENERAL INPUT

SUN2=0.0
SUN3=0.0
SAN1=0.0
SAN2=0.0
SAN3=0.0
DO 3024 I=1,IRK
READ INPUT TAPE 5,1,UX,UY,UZ,VX,VY,VZ
WRITE OUTPUT TAPE 6,355,UX,UY,UZ,VX,VY,VZ
SUN1=SUN1+UX
SUN2=SUN2+UY
SUN3=SUN3+UZ
SAN1=SAN1+VX
SAN2=SAN2+VY
SAN3=SAN3+VZ
3024 CONTINUE
XX10=XX1
YY10=YY1
ZZ10=ZZ1
XX20=XX2
YY20=YY2
ZZ20=ZZ2
WRITE OUTPUT TAPE 6,379
379 FORMAT(1H ,6X,24HMASS FLOW RATE LBM/SEC,10X,16HTHRUST MAGNITUDE,
1 18X,16HROCKET LOCATIONS / 1H ,
1 4X,6HBODY 1 ,15X,6HBODY 2 ,11X,3HLCBF,19X,1HX,20X,1HY,20X,1HZ )
RUN=0.0
PUN=0.0
DO 380 I=1,IRK
READ INPUT TAPE 5,1,FMD1,FMD2,RR(I),XR,YR,ZR
WRITE OUTPUT TAPE 6,355,FMD1,FMD2,RR(I),XR,YR,ZR
XR1(I)=XR-EX1
XR2(I)=XR-EX2
YR1(I)=ZR-EZ1
YR2(I)=ZR-EZ2
ZR1(I)=EY1-YR
ZR2(I)=EY2-YR
RUN=RUN+FMD1
PUN=PUN+FMD2
380 CONTINUE
DO 3025 I=1,IRK
J=2*I-1
TRK(J)=TE(I)
TRK(J+1)=TF(I)
3025 CONTINUE
J=IRK+IRK
DO 3026 I=1,J
K=I
3027 K=K+1
IF (K-J ) 3029,3029,3026
3029 IF (TRK(I)-TRK(K)) 3027,3027,3026
3026 TEMP=TRK(K)
TRK(K)=TRK(I)
TRK(I)=TEMP
GO TO 3029
3028 CONTINUE
LIRK=1

```

GENERAL INPUT

```

C
C   SPRING INPUT
31 DO 36 K=1,12
   S(K)=0.0
36 CONTINUE
   IF (ISP) 5000,37,38
38 READ INPUT TAPE 5,1,SETA,XMU,EPSLN,DELTA
   IQ=1
   READ INPUT TAPE 5,2,N
   IF(N-2) 6551,6552,6552
6552 ICLAG=-1
6551 CONTINUE
   WRITE OUTPUT TAPE 6,381,SETA,N
381 FORMAT(1HD,12HSPRING INPUT,/,2X,10HEFFICIENCY,5X,
114HRESTRICT INDEX, /
1E15.7,110)
   SETA=SETA*SETA
   WRITE OUTPUT TAPE 6,382
382 FORMAT(1H ,15X,28HCOSINE DRAW BOARD AND SPRING,39X,19HSPRING LOCAT
11CN5-IN /6X,2HUZ,19X,2HUY,19X,2HUX,20X,1HX,20X,1HY,20X,1HZ )
   DO 39 I=1,ISP
   READ INPUT TAPE 5,1,UX,UY,UZ,XS,YS,ZS
   WRITE OUTPUT TAPE 6,355,UX,UY,UZ,XS,YS,ZS
   XS1(I)=XS-EX1
   XS2(I)=XS-EX2
   YS1(I)=YS-EZ1
   YS2(I)=YS-EZ2
   ZS1(I)=EZ1-YS
   ZS2(I)=EZ2-YS
   USX(I)=UX
   USY(I)=UY
   USZ(I)=-UY
   READ INPUT TAPE 5,1,SO(I),SK(I),SIOQ(I)
39 CONTINUE
   WRITE OUTPUT TAPE 6,383
383 FORMAT(1H ,1X,13HINITIAL FORCE,7X,15HSPRING CONSTANT,
17X,14HRESIDUAL FORCE/,
26X,3HLBF,17X,6HLBF/IN,14X,3HLBF)
   DO 384 I=1,ISP
   WRITE OUTPUT TAPE 6,355,SO(I),SK(I),SIOQ(I)
   SK(I)=SK(I)*SETA
   SO(I)=SO(I)*SETA
   SIOQ(I)=SIOQ(I)*SETA
384 CONTINUE
C
C   UNIVERSALLY JOINTED SPRING INPUT
37 DO 40 K=1,12
   W(K)=0.0
40 CONTINUE
   IF (ISU) 5000,41,42
42 READ INPUT TAPE 5,1,ETA
   WRITE OUTPUT TAPE 6,385,ETA
385 FORMAT(1HD,32HUNIVERSALLY JOINTED SPRING INPUT / 1X,
118HEFFICIENCY FACTOR E15.7 /
117X,23HLOCATIONS ITH SPRING-IN/,
17X,2HX1,19X,2HY1,19X,2HZ1,19X,2HX2,19X,2HY2,19X,2HZ2)

```

GENERAL INPUT

```

ETA=ETA*ETA
DO 43 I=1,18U
READ INPUT TAPE 5,1,XU,YU,ZU,XUD,YUD,ZUD
WRITE OUTPUT TAPE 6,355,XU,YU,ZU,XUD,YUD,ZUD
READ INPUT TAPE 5,1,WO(I),UK(I),WIOQ(I)
WIOQ(I)=WIOQ(I)*ETA
UK(I)=UK(I)*ETA
WO(I)=WO(I)*ETA
XU1(I)=XU-EX1
XU2(I)=XUD-EX2
YU1(I)=YU-EZ1
YU2(I)=YUD-EZ2
ZU1(I)=YU1-YU
ZU2(I)=YU2-YUD
43 CONTINUE
TG=0
WRITE OUTPUT TAPE 6,7312
7312 FORMAT(16H0 INITIAL FORCE,9X,11HSPRING RATE,7X,14HRESIDUAL FORCE/
1,6X,3HLBF,16X,6HLBF/IN,16X,3HLBF)
DO 7311 I=1,18U
7311 WRITE OUTPUT TAPE 6,355 ,WO(I),UK(I),WIOQ(I)
C
41 IF(ION)5000,44,45
45 READ INPUT TAPE 5,1,TG, XP,YP,ZP
IQ=-1
WRITE OUTPUT TAPE 6,386,XP,YP,ZP,TG
386 FORMAT(1HD,1GHPIN-PULLER INPUT / 13X,30HLOCATION DELAYED PIN PULLE
1R-IN,23X,11HFIRING TIME ,10X /7X,1HX,20X,1HY,20X,
11HZ,19X,3HSEC,18X/1X,
1 E15.7,6X,E15.7,6X,E15.7,6X,
1 E15.7,6X,E15.7)
XI1=XP-EX1
XI2=XP-EX2
YI1=ZP-EZ1
YI2=ZP-EZ2
ZI1=EI1-YP
ZI2=EI2-YP
ICLAG=0
TRIG2=TG
44 CONTINUE
46 CALL CHAIN(2,3)
C
C ERROR
5000 WRITE OUTPUT TAPE 6,5001
5001 FORMAT(1HD,16HERROR INPUT DATA )
4000 CALL EXIT
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)

```



```
SUBROUTINE SW(I,J,A)
SUBROUTINE SW(I,J,A)
DIMENSION A(1)
B=A(I)
A(I)=A(J)
A(J)=B
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```

DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)
DIMENSION FOIE(8)
EQUIVALENCE (T,T) ,(ZERO,ZERO) ,
1 (X1,V1(1)) ,(Y1,V1(2)) ,(Z1,V1(3)) ,(T1,V1(4)) ,(T2,V1(5))
1 ,(P1,V1(6)) ,(P2,V1(7)) ,(S1,V1(8)) ,(S2,V1(9)) ,(X1X,V1(10)) ,(Y1Y,V1(
11)) ,(Z1Z,V1(12)) ,(T1T,V1(13)) ,(T2T,V1(14)) ,(P1P,V1(15)) ,(P2P,V1(1
16)) ,(S1S,V1(17)) ,(S2S,V1(18)) ,(X2,V2(1)) ,(Y2,V2(2)) ,(Z2,V2(3)) ,
1 (X2X,V2(4)) ,(Y2Y,V2(5)) ,(Z2Z,V2(6))
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
COMMON A,B,AD,BD
DIMENSION YDOT2(6) ,YDOT1(18) ,V2(6) ,V1(18) ,NH(6) ,HB(6)
DIMENSION A(3,3,1) ,B(3,3) ,AD(3,3) ,BD(3,3)
DIMENSION DN(3,3) ,DN(3,3)
COMMON WIL1,TCO
DIMENSION TCG(8) ,TCO(8)
DIMENSION FM(3,4) ,FEL(3) ,ACNST(3) ,CH(3,3) ,DUD(3)
DIMENSION DD(12) ,DF(8) ,D(8) ,XC1(8) ,YC1(8) ,ZC1(8) ,XC2(8) ,YC2(8) ,
1ZC2(8)
DIMENSION EP(12) ,PI(3) ,TP(6) ,XP1(3) ,YP1(3) ,ZP1(3) ,XP2(3) ,YP2(3) ,
1ZP2(3) ,UPX(3) ,UPY(3) ,UPZ(3)
DIMENSION G(12) ,GA(6) ,PA(6) ,VA(6) ,PAD(6) ,VAD(6) ,SGX(6) ,SGY(6) ,
1SGZ(6) ,UAX(3) ,UAY(3) ,UAZ(3) ,UFAX(3) ,UPAY(3) ,UPAZ(3) ,VAX(3) ,VAY(3)
1 ,VAZ(3) ,VPAX(3) ,VPAY(3) ,VPAZ(3)
1 ,GG(6)
DIMENSION BINV(15)
DIMENSION M(12)
DIMENSION GJ(12) ,FG1(8) ,FG2(8) ,XG1(8) ,YG1(8) ,ZG1(8) ,XG2(8) ,
1YG2(8) ,ZG2(8) ,UGX(8) ,UGY(8) ,UGZ(8) ,VGX(8) ,VGY(8) ,VGZ(8)
DIMENSION XN(12) ,AN(8) ,RAX(8) ,PFN(8) ,VNX(8) ,VNY(8) ,VNZ(8) ,XN1(8) ,
1YN1(8) ,ZN1(8) ,XN2(8) ,YN2(8) ,ZN2(8) ,PNEM(8) ,EQP(8)
DIMENSION R(12) ,RR(16) ,XR1(16) ,YR1(16) ,ZR1(16) ,XR2(16) ,YR2(16) ,
1ZR2(16) ,URX(16) ,URY(16) ,URZ(16) ,VRX(16) ,VRY(16) ,VRZ(16) ,TE(16) ,
1TF(16) ,TRK(32)
DIMENSION SIOQ(12)
DIMENSION S(12) ,XS1(8) ,YS1(8) ,ZS1(8) ,XS2(8) ,YS2(8) ,ZS2(8) ,USX(8) ,
1USY(8) ,USZ(8) ,SO(8) ,SK(8)
DIMENSION W(12) ,WO(8) ,UK(8) ,XU1(8) ,YU1(8) ,ZU1(8) ,XU2(8) ,YU2(8) ,
1ZU2(8)
DIMENSION Q(12)
COMMON XX1 ,XY1 ,XZ1 ,YY1 ,YZ1 ,ZZ1 ,XX2 ,XY2 ,XZ2 ,YY2 ,YZ2 ,ZZ2 ,
1FM10 ,FM20 ,DEL ,TPR ,TD ,EX1 ,EY1 ,EZ1 ,EX2 ,EY2 ,EZ2 ,WX ,WY ,WZ ,
2W1YD ,ICD ,IPR ,IAC ,IHY ,ICG ,IPN ,IRK ,ISP ,ISU ,ICN
COMMON DF ,D ,XC ,YC ,ZC ,XC1 ,XC2 ,YC1 ,YC2 ,ZC1 ,ZC2
COMMON PI ,TP ,UX ,UY ,UZ ,XP ,YP ,ZP ,UPX ,UPY ,UPZ ,XP1 ,XP2 ,YP1 ,YP2 ,
1ZP1 ,ZP2
COMMON TA ,TB ,TC ,TD ,IAC1 ,IAC2 ,MAC ,NAC ,IGLAG ,GA ,PA ,VA ,PAD ,
1VAD ,SGX ,SGY ,SGZ ,UX ,UY ,UZ ,PUX ,PUY ,PUZ ,VAX ,VPAX ,VAY ,
2VPAY ,VAZ ,VPAZ ,UAX ,UPAX ,UAY ,UPAY ,UAZ ,UPAZ ,GG
COMMON HD ,HK ,HETA ,PPH ,DHB ,AH ,PHBO ,VHB ,GAMH ,JH ,DEQP ,HYDA ,
1HYDB ,HYDC ,HETA ,DA ,HYDD ,HYDE ,DB
COMMON GJ ,GST ,FG3 ,IJLAG ,MAXES ,NAXES ,FG1 ,FG2 ,TCO ,TCG ,
1UX ,UY ,UZ ,XG ,YG ,ZG ,XG1 ,XG2 ,YG1 ,YG2 ,ZG1 ,ZG2 ,VGX ,VGY ,
2VGZ ,UGX ,UGY ,UGZ
COMMON XN ,JN ,PBO ,VBN ,GAMN ,AN ,RAX ,PPN ,XXN ,YN ,ZN ,VNX ,VNY ,
1VNZ ,XN1 ,XN2 ,YN1 ,YN2 ,ZN1 ,ZN2 ,EQP ,PPN ,PNEM
COMMON R ,VRX ,VRY ,VRZ ,URX ,URY ,URZ ,TE ,TF ,TROC ,SUM1 ,

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DIMENSION BA(3,3) ,WR(3),WT1(3),WT2(3) ,WT3(3)

1SUN2,SUN3,SAN1,SAN2,SAN3,XX10,YY10,ZZ10,XX20,YY20,ZZ20,
2FMD1,FMD2,RR,XR,YR,ZR,XR1,XR2 ,YR1,YR2,ZR1,ZR2,RUM,PUM,
3TRK
COMMON SIOQ
COMMON S,SETA,XMU,EPSLN,DELTS,XS1,XS2,YS1,YS2,ZS1,ZS2,
1SD,SK
COMMON W,ETA,W0,UK,XU,YU,ZU,XU1,XU2,YU1,YU2,ZU1,ZU2,TG
DIMENSION WIOQ(8)
COMMON WIOQ
COMMON DN,DN,R1,R2,P3,P21,P22,P23,ZTEST,F,CA,XI1,YI1,ZI1,XI2,
XYI2,ZI2,USX,USY,USZ
COMMON XI1,XI2,ZI1,ZI2,ICLA6,TRIG2,ELM,LIRK,M,XS,Y8,Z8
COMMON ML
COMMON SAVE,BINV,E,NH,MB,H,Q
COMMON FQQ1,FQQ2,FQQ3
DIMENSION WRP1(3,8),WRP2(3,8),WRH(3,8)
COMMON WRP1,WRP2,WRH
DIMENSION ISP1(8)
DIMENSION ISS(2),FJ(8),WPP1(3)
2 FORMAT(6I12)
1 FORMAT(6E12.8)
DO 5812 I=1,ISP
WRP1(1,I)=XS1(I)
WRP1(2,I)=YS1(I)
WRP1(3,I)=ZS1(I)
WRP2(1,I)=XS2(I)
WRP2(2,I)=YS2(I)
WRP2(3,I)=ZS2(I)
WRH(1,I)=USX(I)
WRH(2,I)=USY(I)
WRH(3,I)=USZ(I)
5812 CONTINUE
IF(WIL1)6675,6675,6676
6675 WIL1=-WIL1
IFORCE=-1
GO TO 6677
6676 IFORCE=0
6677 CONTINUE
FSHT=0.
IF(IAC)54,55,54
54 FSHT=-FM10
55 CONTINUE
IQ=0
TRIG1=TD
TRIG2=TC
CALL TRMOD(7,0)
CALL TRMOD(0,0)
CALL TRMOD(1,0)
CALL TRMOD(2,1)
CALL TRMOD(3,0)
CALL TRMOD(4,0)
CALL TRMOD(5,0)
CALL TRMOD(6,0)
IF(ICG)8005,8004,8005
8005 TRIG3=TCO(1)
CALL TRMOD(0,1)

```

```

        DIMENSION BA(3,3) ,WR(3),WT1(3),WT2(3) ,WT3(3)

        ICOLD=1
8004 CONTINUE
        IF (IRK) 5000,31,32
        32 CALL TRMOD(4,1)
        LIRK=1
        31 CONTINUE
        IF (ION) 5000,44,45
        45 CALL TRMOD(1,1)
        44 CONTINUE
        IF (IPR) 46,46,47
        47 IF (IPR-2) 469,470,471
        471 CALL TRMOD(6,1)
        PYR3=TP(3)-DEL/2.0
        TP(3)=PYR3
        TP(6)=TP(3)+DEL
        470 CALL TRMOD(5,1)
        PYR2=TP(2)-DEL/2.0
        TP(2)=PYR2
        TP(5)=TP(2)+DEL
        469 CALL TRMOD(3,1)
        TRIG4=TP(1)-DEL/2.0
        TP(1)=TRIG4
        TP(4)=TP(1)+DEL

C
        46 TEMP=XZ1
        READ INPUT TAPE 5,1,TPLOT
        IF (TPLOT) 705,704,705
        705 NTZ=8
        REWIND NTZ
        PLOT=TD
        CALL TRMOD(7,1)
        XPTS=0.0
        CALL CAMERA(18,1)
        CALL SET(500)
        WRITE OUTPUT TAPE 38,6263,(WL(I),I=1,8)
        6263 FORMAT(40X,8A6)
        704 XZ1=-XY1
        WRITE OUTPUT TAPE 6,899
        899 FORMAT(1H1)
        XY1=TEMP
        TEMP=ZZ1
        ZZ1=YY1
        YY1=TEMP
        TEMP=XZ2
        XZ2=-XY2
        XY2=TEMP
        TEMP=ZZ2
        ZZ2=YY2
        YY2=TEMP
        YZ1=-YZ1
        YZ2=-YZ2
        DIMENSION CBQ(3),GAQ1(3,3,3)
C
        INERTIA MATRIX TRANSFORMED
        MAXE 8=1
        MAXE 8=1
        ELK=57.2957795131
    
```

```

DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

TEMP=WZ/ELK
WZ=-WY/ELK
WY=TEMP
WX=WX/ELK
C INITIAL VALUES
DOL=DEL
IMOD=0
LAM=0
L1=0
L2=0
L3=0
LX=25
SUM=FM10+FM20
FM1=FM10
FM2=FM20
ECX=(FM1*EX1+FM2*EX2)/SUM
ECY=(FM1*EY1+FM2*EY2)/SUM
ECZ=(FM1*EZ1+FM2*EZ2)/SUM
YMU=0.99*XMU
IFINI=0
JKUT=0
LKUT=0
HB(1)=DEL
HB(2)=1.0
HB(4)=MINIF(.01,TPR*.2)
HB(3)=HB(4)*.05
HB(5)=1.0E-8
HB(6)=1.0E-6
DO 511 K=1,6
NH(K)=0.0
511 CONTINUE
NH(5)=4
DIMENSION CA(3) ,E(3,3) ,X(3)
1F(15,15) ,ML(30)
NH(1)=24
78 NH(2)=24
80 V1(4)=1.5707963
V1(5)=V1(4)
V1(6)=0.0
V1(7)=0.0
V1(8)=0.0
V1(9)=0.0
W1Y0=W1Y0/ELK
V1(13)=WX
V1(14)=WX
V1(15)=WY+W1Y0
V1(16)=WY
V1(17)=WZ
V1(18)=WZ
IPHI=4
CONST=385.7
JIK=1
ITER=1
LOW=0
IILA6=0
X10=EX1-ECX

```

```

DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

X20=EX2-ECX
Y10=EY1-ECY
Y20=EY2-ECY
Z10=EZ1-ECZ
Z20=EZ2-ECZ
V1(1)=X10
V2(1)=X20
V1(2)=Y10
V2(2)=Y20
V1(3)=Z10
V2(3)=Z20
V1(10)=-WZ*V1(3)-(WY+W1Y0)*V1(2)
V1(11)=(WY+W1Y0)*V1(1)-WX*V1(3)
V1(12)=WX*V1(2)+WZ*V1(1)
V2(4)=-WZ*V2(3)-WY*V2(2)
V2(5)=WY*V2(1)-WX*V2(3)
V2(6)=WX*V2(2)+WZ*V2(1)
DO 512 I=1,9
YDOT1(I)=V1(I+9)
YDOT1(I+9)=0.0
512 CONTINUE
DO 513 I=1,3
YDOT2(I)=V2(I+3)
YDOT2(I+3)=0.0
513 CONTINUE
IVP=0
T=TD
ZERO=0.0
IKUT=0
SMALL=0.0001
DIMENSION CX(3) ,CZ(3) ,SAVE(68) ,CB(3) ,CC(3) ,GAMMA(3)
HLL=180.0/3.14159265
SN=0.0
279 DO 277 J=1,18
SAVE(J)=V1(J)
SAVE(J+24)=YDOT1(J)
277 CONTINUE
DO 278 J=1,6
SAVE(J+61)=GA(J)
SAVE(J+54)=NH(J)
SAVE(J+48)=HB(J)
SAVE(J+18)=V2(J)
SAVE(J+42)=YDOT2(J)
278 CONTINUE
SAVE(61)=T
ZTEST=0.64E-07
LDER=1
IWLL=1
IXFT=0
MAP=DEL*.5
FMA51=FM1/CONST
FMA52=FM2/CONST
XX10=XX1
YY10=YY1
ZZ10=ZZ1
XX20=XX2

```

```

DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

YY20=YY2
ZZ20=ZZ2
ICD1=0
ICG1=0
IPN1=0
ISP1=0
ISU1=0
ISS(1)=0
ISS(2)=0

C
C DER1
C
C DER2
C
1001 CALL GET
3032 IF(ICD) 50,51,50
50 CALL DRAG(D,DF,DD,XC1,YC1,ZC1,XC2,YC2,ZC2,ICD,LDER,ICD1)
51 IF(IPR) 52,57,52
52 CALL PYRO(PI,TP,XP1,YP1,ZP1,XP2,YP2,ZP2,UPX,UPY,UPZ,EP,IPR)
57 IF(ICG) 58,59,58
58 CALL COLGAS(ICG,GJ,FG1,FG2,FG3,UGX,UGY,UGZ,VGX,VGY,VGZ,XG1,YG1,
12G1,XG2,YG2,ZG2,GST,TCG,LDER,ICG1)
59 IF(IPN) 60,61,60
60 CALL PNEUM(IPN,JN,PBO,VBN,GAMN,AN,RAX,VNX,VNY,VNZ,XN1,YN1,ZN1,XN2,
1YN2,ZN2,XN,PPN,EQP,FNEM,LDER,IPN1)
61 IF(IRK) 62,63,62
62 CALL ROCKET(IRK,R,RR,TE,TF,RUN,FUN,FM1,FM2,URX,URY,URZ,FM10,FM20,X
1R1,YR1,ZR1,XR2,YR2,ZR2,VRX,VRY,VRZ,SUN1,SUN2,SUN3,SAN1,SAN2,SAN3,
2XX10,YY10,ZZ10,XX20,YY20,ZZ20,XX1,YY1,ZZ1,XX2,YY2,ZZ2)
63 IF(ISP) 66,67,66
66 CALL SPRING(WRP1,WRP2,WRH,SD,SK,SIOQ,ISP,N,ICLAG,S,FJ,LDER,ISP1,
1ISS)
67 IF(ISU) 68,69,68
68 CALL UNIVSL(ISU,W0,UK,WIOF,W,XU1,YU1,ZU1,XU2,YU2,ZU2,LDER,ISU1)
69 DO 70 K=1,12
Q(K)=DD(K)+EP(K)+GJ(K)+XN(K)+R(K)+S(K)+W(K)
70 CONTINUE
Q(3)=Q(3)+FSMT
R1=R1+Q(7)*CONST
R2=R2+Q(9)*CONST
P3=P3+Q(11)*CONST
P21=P21+Q(8)*CONST
P22=P22+Q(10)*CONST
P23=P23+Q(12)*CONST
414 IF(ICLAG) 74,74,75
74 CONTINUE
F(1,1)=FM1
F(2,2)=FM1
F(3,3)=FM1
F(7,7)=FM2
F(8,8)=FM2
F(9,9)=FM2
3034 J=15
6010 FORMAT(8E15.5)
IN=1
DQ=(2.)**(-26)

```

```

        DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

700 DO 701 I=1,3
      YDOT2(I+3)=D.0
      K=2*I
      WL(I)=Q(K-1)*CONST
      WL(I+6)=Q(K)*CONST
      WL(I+12)=CA(I)
701 CONTINUE
      WL(4)=R1
      WL(5)=R2
      WL(6)=P3
      WL(10)=P21
      WL(11)=P22
      WL(12)=P23
      CALL SOLVE(15,F,WL,IN,DQ,10,BINV,IN3)
      IF(IN3)6005,6006,6002
C      F MATRIX IS SINGULAR
6005 WRITE OUTPUT TAPE 6,6003
6003 FORMAT(22H1 F MATRIX IS SINGULAR)
      CALL DUMP
C      OVERFLOW IN F MATRIX INVERSION
6006 WRITE OUTPUT TAPE 6,6004
6004 FORMAT(37H1 F MATRIX INVERSION DID NOT CONVERGE)
      CALL DUMP
6002 CONTINUE
3037 DO 702 I=1,9
      YDOT1(I+9)=D.0
702 CONTINUE
      DO 703 J=1,3
      K=J*2
      YDOT1(K+12)=BINV(J+9)
      YDOT1(J+9)=BINV(J)
      YDOT1(K+11)=BINV(J+3)
      YDOT2(J+3)=BINV(J+6)
703 CONTINUE
      YDOT2(1)=V2(4)
      YDOT2(2)=V2(5)
      YDOT2(3)=V2(6)
      YDOT1(1)=V1(10)
      YDOT1(2)=V1(11)
      YDOT1(3)=V1(12)
      YDOT1(4)=V1(13)
      YDOT1(5)=V1(14)
      YDOT1(6)=V1(15)
      YDOT1(7)=V1(16)
      YDOT1(8)=V1(17)
      YDOT1(9)=V1(18)
      GO TO 6007
75 CONST2=CONST/FM2
      YDOT2(4)=Q(2)*CONST2
      YDOT2(5)=Q(4)*CONST2
      YDOT2(6)=Q(6)*CONST2
      YDOT2(1)=V2(4)
      YDOT2(2)=V2(5)
      YDOT2(3)=V2(6)
      D2233=DM(2,2)*DM(3,3)
      D23=DM(2,3)*DM(2,3)

```



```

DIMENSION BA(3,3) ,WR(3),WT1(3),WT2(3) ,WT3(3)

D1323=DM(1,3)*DM(2,3)
D1223=DM(1,2)*DM(2,3)
D1233=DM(1,2)*DM(3,3)
D1322=DM(1,3)*DM(2,2)
CONS1=CONST/FM1
DET=DM(1,1)*(D2233-D23)+DM(1,2)*(D1323-D1233)+DM(1,3)*(D1223-D1322)
1)
IF (DET) 214,5002,214
214 YDOT1(10)=Q(1)*CONS1
YDOT1(11)=Q(3)*CONS1
YDOT1(12)=Q(5)*CONS1
YDOT1(2)=V1(11)
YDOT1(3)=V1(12)
YDOT1(1)=V1(10)
YDOT1(13)=R1*(D2233-D23)+R2*(D1323-D1233)+P3*(D1223-D1322)
YDOT1(13)=YDOT1(13)/DET
YDOT1(15)=R1*(DM(2,3)*DM(3,1)-DM(2,1)*DM(3,3))
1 +R2*(DM(1,1)*DM(3,3)-DM(1,3)*DM(1,3))
1 +P3*(DM(1,3)*DM(2,1)-DM(1,1)*DM(2,3))
YDOT1(15)=YDOT1(15)/DET
YDOT1(17)=R1*(DM(2,1)*DM(3,2)-DM(2,2)*DM(3,1))
1 +R2*(DM(1,2)*DM(3,1)-DM(1,1)*DM(3,2))
1 +P3*(DM(1,1)*DM(2,2)-DM(1,2)*DM(1,2))
YDOT1(17)=YDOT1(17)/DET
D2233=DN(2,2)*DN(3,3)
D23=DN(2,3)*DN(2,3)
D1323=DN(1,3)*DN(2,3)
D1233=DN(1,2)*DN(3,3)
D1223=DN(1,2)*DN(2,3)
D1322=DN(1,3)*DN(2,2)
DET=DN(1,1)*(D2233-D23)+DN(1,2)*(D1323-D1233)+DN(1,3)*(D1223-D1322)
1)
IF (DET) 213,5002,213
213 CONTINUE
YDOT1(14)=P21*(D2233-D23)+P22*(D1323-D1233)+P23*(D1223-D1322)
YDOT1(14)=YDOT1(14)/DET
YDOT1(16)=P21*(DN(2,3)*DN(3,1)-DN(2,1)*DN(3,3))
1 +P22*(DN(1,1)*DN(3,3)-DN(1,3)*DN(1,3))
1 +P23*(DN(1,3)*DN(2,1)-DN(1,1)*DN(2,3))
YDOT1(16)=YDOT1(16)/DET
YDOT1(18)=P21*(DN(2,1)*DN(3,2)-DN(2,2)*DN(3,1))
1 +P22*(DN(1,2)*DN(3,1)-DN(1,1)*DN(3,2))
1 +P23*(DN(1,1)*DN(2,2)-DN(1,2)*DN(1,2))
YDOT1(18)=YDOT1(18)/DET
YDOT1(4)=V1(13)
YDOT1(5)=V1(14)
YDOT1(6)=V1(15)
YDOT1(7)=V1(16)
YDOT1(8)=V1(17)
YDOT1(9)=V1(18)
6007 CONTINUE
IF (LDER) 636,636,631
636 CALL ROUT (0)
631 LDER=0
C
3000 CALL FMARK (KIK,HB,NH,IVP,IPH1,1,1,2,2,TR163,3,TR162,4,TR161,

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DIMENSION BA(3,3) ,WR(3),WT1(3),WT2(3) ,WT3(3)
15,TRIG4,6,TROC,7,PYRO2,8,PYRO3,9,PLOT,0)
GO TO (1001,1004,1003,1002,1005,1006,1007,1008,1009,5050),KIK
C
C DER2 STARTS AT 1001
C PRINT-OUT STARTS AT 1002
C PIN-PULLER CONSTRAINT OFF AT 1003
C EOS AND COLD GAS JET AT 1004
C PYROTECHNIC TRIGGER STARTS AT 1005
C SECOND PYROTECHNIC AT 1007
C THIRD PYROTECHNIC AT 1008
C ROCKET TRIGGER AT 1006
C PLOT TRIGGER AT 1009
C
1003 CALL TRMOD (1,0)
IF(N-2)7337,7334,7334
7337 ICLAG=1
7338 FORMAT(30HOPIN-PULLER CONSTRAINT OFF T=,E15.7)
WRITE OUTPUT TAPE 6,7338,T
GO TO 1001
7334 ICLAG=-1
WRITE OUTPUT TAPE 6,7333,T
7333 FORMAT(52HOPIN-PULLER CONSTRAINT OFF SPRING CONSTRAINT ON T=
1E15.7)
GO TO 1001
1004 IF(T-TCO(ICOLD))200,8010,200
8010 IF(ICOLD-ICG)8006,8007,8006
8006 ICOLD=ICOLD+1
TRIG3=TCO(ICOLD)
LDER=1
GO TO 1001
8007 CALL TRMOD(0,0)
TCO(ICG)=-3.
LDER=1
GO TO 1001
C
C THETA TEST
C
200 IF(LDER)6518,6519,6519
6518 LDER=1
6519 IF(ISS(1)+ISS(2))6521,6522,6521
6521 DO 3232 I=1,2
CT=COSF(V1(I+3))
ST=SINF(V1(I+3))
CP=COSF(V1(I+5))
SP=SINF(V1(I+5))
CS=COSF(V1(I+7))
SS=SINF(V1(I+7))
A(1,1,I)=-CT*SP*SS+CP*CS
A(1,2,I)=CT*CP*SS+SP*CS
A(1,3,I)=ST*SS
A(2,1,I)=-CT*SP*CS-CP*SS
A(2,2,I)=CT*CP*CS-SP*SS
A(2,3,I)=ST*CS
A(3,1,I)=ST*SP
A(3,2,I)=-ST*CP
3232 A(3,3,I)=CT
DO 6523 J=1,2

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DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

I=ISS(J)
IF(1) 6524 ,6523,6524
6524 CONTINUE
CALL MULT2 (B,A,BA,3,3,3)
WR(1)=V1(1)-V2(1)
WR(2)=V1(2)-V2(2)
WR(3)=V1(3)-V2(3)
CALL MULT1 (A,WRP1(1,1),WT1,3,3,1)
CALL MULT1 (B,WRP2(1,1),WT2,3,3,1)
CALL MSUB (WT1,WT2,WT3,3,1)
CALL MADD (WT3,WR,WT1,3,1)
CALL MSR (A,WT1,WT2(1),2)
CALL MSC (BA,WRH(1,1),WT2(2),2)
FJ(I)=WT2(1)/WT2(2)
CALL MSCAL(FJ(I),WRH(1,1),WT1)
CALL MADD(WT1,WRP2(1,1),WT1,3,1)
CALL MULT1(BA,WT1,WT2,3,3,1)
CALL MULT3(A,WR,WT1,3,3,1)
CALL MSUB(WT2,WT1,WPP1,3,1)
CALL SWITCH(WRP1,WRP2,WRH,SD,SK,SIOQ,J,I,T)
ISS(J)=0
DO 6525 K=1,3
6525 WRP1(K,J)=WPP1(K)
CALL MSR(B,WR,WT1,1)
CALL MSR(BA,WRP1(1,J),WT1(2),1)
6523 CONTINUE
6522 CONTINUE
ITHETA=0
TH=T1
THDOT=T1T
PH=P1
PDOT=P1P
SI=S1
SDOT=S1S
234 LPHI=0
PHSI=PH
DOT=PDOT
235 IF(DOT) 236,260,260
264 PHSI=PHSI+6.2831853
LDER=1
236 IF(PHSI) 264,256,256
258 PHSI=PHSI-6.2831853
LDER=1
260 IF(PHSI-6.2831853) 256,256,258
256 IF(LPHI) 261,257,261
257 LPHI=1
PH=PHSI
PHSI=SI
DOT=SDOT
GO TO 235
261 LPHI=0
SI=PHSI
IF(THDOT) 259,263,263
259 IF(TH-0.1) 237,413,413
263 IF(TH-3.04) 413,413,237
237 IF(ITHETA) 240,239,240

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      DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

239 DO 241 I=1,ICD
      TEMP=YC1(I)
      YC1(I)=-ZC1(I)
      ZC1(I)=TEMP
241 CONTINUE
      TEMP=XY1
      XY1=XZ1
      XZ1=-TEMP
      TEMP=YY1
      YY1=ZZ1
      ZZ1=TEMP
      YZ1=-YZ1
      DO 242 I=1,IPR
      TEMP=YP1(I)
      YP1(I)=-ZP1(I)
      ZP1(I)=TEMP
242 CONTINUE
      TEMP=YY1D
      YY1D=ZZ1D
      ZZ1D=TEMP
      DO 243 I=1,ICG
      TEMP=UGZ(I)
      UGZ(I)=-UGY(I)
      UGY(I)=TEMP
      TEMP=YG1(I)
      YG1(I)=-ZG1(I)
      ZG1(I)=TEMP
243 CONTINUE
      DO 244 I=1,IPN
      TEMP=YN1(I)
      YN1(I)=-ZN1(I)
      ZN1(I)=TEMP
244 CONTINUE
      DO 245 I=1,IRK
      TEMP=URZ(I)
      URZ(I)=-URY(I)
      URY(I)=TEMP
      TEMP=YR1(I)
      YR1(I)=-ZR1(I)
      ZR1(I)=TEMP
245 CONTINUE
      DO 514 I=1,IAC1
      TEMP=UAZ(I)
      UAZ(I)=-UAY(I)
      UAY(I)=TEMP
      TEMP=UPAZ(I)
      UPAZ(I)=-UPAY(I)
      UPAY(I)=TEMP
514 CONTINUE
      DO 246 I=1,ISP
      TEMP=Y81(I)
      Y81(I)=-Z81(I)
      Z81(I)=TEMP
246 CONTINUE
      DO 247 I=1,ISU
      TEMP=YU1(I)
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DIMENSION BA(3,3) ,WR(3),WT1(3),WT2(3) ,WT3(3)

YU1(I)=-ZU1(I)
ZU1(I)=TEMP
247 CONTINUE
TEMP=Y11
Y11=-Z11
Z11=TEMP
WRITE OUTPUT TAPE 6,449
MAXES=MAXES+1
IF (MAXES-4) 461,461,460
460 MAXES=1
461 GO TO (455,456,457,458),MAXES
455 WRITE OUTPUT TAPE 6,451
GO TO 248
456 WRITE OUTPUT TAPE 6,452
GO TO 248
457 WRITE OUTPUT TAPE 6,453
GO TO 248
458 WRITE OUTPUT TAPE 6,454
GO TO 248
248 DO 249 I=1,ICD
TEMP=YC2(I)
YC2(I)=-ZC2(I)
ZC2(I)=TEMP
249 CONTINUE
TEMP=XY2
XY2=XZ2
XZ2=-TEMP
TEMP=YY2
YY2=ZZ2
ZZ2=TEMP
YZ2=-YZ2
TEMP=YY2D
YY2D=ZZ2D
ZZ2D=TEMP
DO 250 I=1,IPR
TEMP=UPZ(I)
UPZ(I)=-UPY(I)
UPY(I)=TEMP
TEMP=YP2(I)
YP2(I)=-ZP2(I)
ZP2(I)=TEMP
250 CONTINUE
DO 251 I=1,ICG
TEMP=VGZ(I)
VGZ(I)=-VGY(I)
VGY(I)=TEMP
TEMP=YG2(I)
YG2(I)=-ZG2(I)
ZG2(I)=TEMP
251 CONTINUE
DO 252 I=1,IPN
TEMP=VNZ(I)
VNZ(I)=-VNY(I)
VNY(I)=TEMP
TEMP=YN2(I)
YN2(I)=-ZN2(I)

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      DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

      ZN2(I)=TEMP
252 CONTINUE
      DO 253 I=1,IRK
        TEMP=VRZ(I)
        VRZ(I)=-VRY(I)
        VRY(I)=TEMP
        TEMP=YR2(I)
        YR2(I)=-ZR2(I)
        ZR2(I)=TEMP
253 CONTINUE
      DO 254 I=1,ISP
        TEMP=USZ(I)
        USZ(I)=-USY(I)
        USY(I)=TEMP
        TEMP=YS2(I)
        YS2(I)=-ZS2(I)
        ZS2(I)=TEMP
254 CONTINUE
      J=IAC1+1
      DO 515 I=J,IAC2
        TEMP=VAZ(I)
        VAZ(I)=-VAY(I)
        VAY(I)=TEMP
        TEMP=VPAZ(I)
        VPAZ(I)=-VPAY(I)
        VPAY(I)=TEMP
515 CONTINUE
      DO 255 I=1,ISU
        TEMP=YU2(I)
        YU2(I)=-ZU2(I)
        ZU2(I)=TEMP
255 CONTINUE
      TEMP=YI2
      YI2=-ZI2
      ZI2=TEMP
      NAXES=NAXES+1
      IF (NAXES-4) 462,462,463
463 NAXES=1
462 WRITE OUTPUT TAPE 6,450
      GO TO (455,456,457,458),NAXES
248 TEMP=SINF(TH)*COSF(SI)
      IF (TEMP) 850,851,850
851 TNEW=1.57079633
      GO TO 852
850 CONTINUE
      TNEW=ATANF( ABSF((SQRTF(1.0-TEMP**2))/TEMP) )
852 PNEW=ATANF( ABSF(TANF(SI)/COSF(TH)) )
      SNEW=ATANF( ABSF(TANF(TH)*SINF(SI)) )
      IF (COSF(SI)) 265,267,266
267 TNEW=1.57079633
      GO TO 265
268 TNEW=3.14159265-TNEW
265 IF (COSF(TH)) 269,266,266
266 L=1
      ANGLE=81
      GO TO 270

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      DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

269 L=1
      ANGLE=6.2831853-SI
270 IF (ANGLE-3.14159265) 271,262,262
271 IF (ANGLE-1.57079633) 430,430,431
430 K=1
      GO TO 432
431 K=2
432 GO TO (433,434) ,L
262 IF (ANGLE-4.712388981) 437,437,438
437 K=3
      GO TO 432
438 K=4
      GO TO 432
433 GO TO (439,440,441,442) ,K
440 PNEW=3.14159265-PNEW
      GO TO 439
441 PNEW=3.14159265+PNEW
      GO TO 439
442 PNEW=6.2831853-PNEW
439 PH=PNEW
      IF (SINF(SI)) 443,444,444
443 L=2
      ANGLE=TH
      GO TO 270
444 L=2
      ANGLE=6.2831853-TH
      GO TO 270
434 GO TO (445,446,447,448) ,K
446 SNEW=3.14159265-SNEW
      GO TO 445
447 SNEW=3.14159265+SNEW
      GO TO 445
448 SNEW=6.2831853-SNEW
445 SI=SNEW
      TH=TNEW
      WRITE OUTPUT TAPE 6,8000,TH,ITHETA
8000 FORMAT(1H ,E15.7,16)
      LDER=1
413 IF (ITHETA) 272,273,272
273 T1=TH
      TH=T2
      S1=SI
      P1=PH
      PDOT=P2P
      SDOT=S2S
      PH=P2
      THDOT=T2T
      SI=S2
      ITHETA=1
      GO TO 234
C      THETA MODIFIED RETURN TO FMARK
272 TRIG3=TRIG3+DEL
      T2=TH
      S2=SI
      P2=PH
449 FORMAT(1H0,20HBODY 1-FIXED AXES X Y Z )

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DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

450 FORMAT(1HD,28HBODY 2-FIXED AXES X Y Z )
451 FORMAT(1H ,28HDRAWING BOARD AXES X Z -Y )
452 FORMAT(1H ,28HDRAWING BOARD AXES X -Y -Z )
453 FORMAT(1H ,28HDRAWING BOARD AXES X -Z Y )
454 FORMAT(1H ,28HDRAWING BOARD AXES X Y Z )
DO 317 J=1,6
SAVE(J+18)=V2(J)
SAVE(J+42)=YDOT2(J)
317 CONTINUE
DO 318 J=1,18
SAVE(J)=V1(J)
SAVE(J+24)=YDOT1(J)
318 CONTINUE
SAVE(81)=T
ZERO=D.0
IF(IMOD) 425,426,428
426 IF(LDER) 5425,403,5425
425 DO 427 J=1,6
NH(J)=SAVE(J+54)
HB(J)=SAVE(J+48)
427 CONTINUE
5425 CONTINUE
IMOD=0
LDER=1
GO TO 1001
C
C PYROTECHNIC TRIGGER
C
1005 IF(IKUT) 423,424,425
424 TRIG4=TRIG4+DEL
TP(1)=T
TP(4)=TRIG4
IKUT=1
3001 IPHI=2
HB(1)=DEL*.2
632 LDER=1
GO TO 1001
423 CALL TRMOD(3,0)
415 IKUT=0
JPHI=4
IF(TP(2)) 633,600,633
633 TP(4)=0.0
DEP=TP(2)
IF (TRIG4-DEP) 602,634,634
634 JPHI=2
GO TO 602
C FORCE CEASES TO ACT - - - SET INDEX = ZERO
600 IPR=0
601 HB(1)=DEL
DO 420 J=1,12
EP(J)=D.0
420 CONTINUE
602 IPHI=JPHI
GO TO 632
C
C 2ND PYROTECHNIC TRIGGER

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      DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)
C
1007 IF (JKUT) 465,466,465
466 PYRO2=PYRO2+DEL
      TP(2)=T
      TP(5)=PYRO2
      JKUT=1
      GO TO 3001
465 CALL TRMOD (5,0)
      JPHI=4
      IF (TP(3)) 635,600,635
635 TP(5)=D.D
      DEP=TP(3)
      IF (FYRO2-DEP) 602,634,634
C
C      3RD PYROTECHNIC TRIGGER
C
1008 IF (LKUT) 467,468,467
468 PYRO3=PYRO3+DEL
      TP(3)=T
      TP(6)=PYRO3
      LKUT=1
      GO TO 3001
467 CALL TRMOD (6,0)
      JPHI=4
      GO TO 600
C
C      ROCKET TRIGGER
C
1006 I=IRK+IRK
      IF (TROC-TRK(I)) 428,429,429
428 LIRK=LIRK+1
      TROC=TRK(LIRK)
      LDER=1
      GO TO 1001
429 CALL TRMOD (4,0)
      IRK=0
      LDER=1
      GO TO 1001
C
C      PLOT TRIGGER
C
1009 DO 4050 K=1,3
      J=2*K
      WL(K)=V1(J+2)*HLL
      WL(K+3)=V1(J+3)*HLL
      WL(K+6)=V1(J+11)*HLL
      WL(K+9)=V1(J+12)*HLL
      WL(K+12)=YDOT1(J+11)*HLL
      WL(K+15)=YDOT1(J+12)*HLL
4050 CONTINUE
      WRITE TAPE NT2,V1(1) ,V1(2) ,V1(3) ,V2(1) ,V2(2) ,V2(3) ,
1 V1(10) ,V1(11) ,V1(12) ,V2(4) ,V2(5) ,V2(6) ,YDOT1(10) ,
1 YDOT1(11) ,YDOT1(12) ,YDOT2(4) ,YDOT2(5) ,YDOT2(6) ,
1 (WL(K) ,K=1,18)
      PLOT=PLOT+TPLOT
      XPTS=XPTS+1.D

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      DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

      TEND=T
      CALL ROUT(D)
505D CALL ERR (DEL,TPR,TP,TG,TA,TB,TC,TD,TRIG1,TRIG2,TRIG3,
1 TRIG4,TROC,PYRO2,PYRO3,PLOT )
C
C PRINT ROUTINE
C
1002 TRIG1=TRIG1+TPR
      CALL STEP(DELP)
      WRITE OUTPUT TAPE 6,300,T,DELP
300  FORMAT(1HD, //BH TIME = E15.7,1X,7HSECONDS,5X,10HSTEP SIZE=E15.7/)
      P=X1
      Y=Y1
      Z=Z1
      XDOT=X1X
      YDOT=Y1Y
      ZDOT=Z1Z
      TH=T1
      PH=P1
      SI=S1
      TDOT=T1T
      SDOT=S1S
      PDOT=P1P
      J=1
301  CA(1)=PDOT*SINF(TH)
      CA(2)=SINF(SI)
      CA(3)=COSF(SI)
      CB(1)=(CA(1)*CA(2)+TDOT*CA(3))*ELK
      CB(2)=- (PDOT*COSF(TH)+SDOT) *ELK
      CB(3)=(CA(1)*CA(3)-TDOT*CA(2))*ELK
      WPY=CB(1)*CB(1)+CB(2)*CB(2)
      W1=SQRTF(WPY+CB(3)*CB(3))
      WPY=SQRTF(WPY)
C CONVERT RADIAN'S TO DEGREE'S
      TH=TH*ELK
      PH=PH*ELK
      SI=SI*ELK
      TDOT=TDOT*ELK
      SDOT=SDOT*ELK
      PDOT=PDOT*ELK
      V=SQRTF(XDOT*XDOT+YDOT*YDOT+ZDOT*ZDOT)
      CA(1)=CA(1)*ELK
      CA(2)=CA(2)*ELK
      CA(3)=CA(3)*ELK
      WRITE OUTPUT TAPE 6,302,J
302  FORMAT(1H , 5HBODY I1)
      WRITE OUTPUT TAPE 6,352,V
352  FORMAT(1H ,14HSPEED OF CM = E15.7, 7H IN/SEC )
      WRITE OUTPUT TAPE 6,303,W1,WPY
303  FORMAT(1H ,31HRATE ABOUT INSTANTANEOUS AXIS= E15.7,1X,7HDEG/SEC,
1 7X,41HMAGNITUDE VECTOR SUM OF PITCH-YAW RATES= E15.7 )
      WRITE OUTPUT TAPE 6,304
304  FORMAT(1HD,22HINERTIAL COORDS-INCHES,3X,19H1ST DERIVATIVE /SEC,
16X,20HEULER ANGLES-DEGREE'S,5X,19H1ST DERIVATIVE /SEC ,8X,
14X,10HRATE ABOUT/102X,17HBODY AXES DEG/SEC)
      WRITE OUTPUT TAPE 6,305,P,XDOT,TH,TDOT,CB(1),Y,YDOT,PH,PDOT,CB(2),

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DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

1 Z,ZDOT,SI ,SDOT,CB(3)
305 FORMAT(1H ,1HX,4X,E15.7,5X,1HX,4X,E15.7,5X,5HTheta,E15.7,5X,5HTheta
1A,E15.7,5X,1HX,4X,E15.7/
1 1X, 1HY,4X,E15.7,5X,1HY,4X,E15.7,5X,5HPhi ,E15.7,5X,5HPhi
1 ,E15.7,5X,1HY,4X,E15.7/
1 1X, 1HZ,4X,E15.7,5X,1HZ,4X,E15.7,5X,5HPsi ,E15.7,5X,5HPsi
1 ,E15.7,5X,1HZ,4X,E15.7 )
IF (J-1) 306,307,306
307 TH=T2
PH=P2
SI=S2
TDOT=T2T
PDOT=P2P
SDOT=S2S
P=X2
Y=Y2
Z=Z2
XDOT=X2X
YDOT=Y2Y
ZDOT=Z2Z
J=2
GO TO 301
306 DO 350 J=1,6
NH(J)=SAVE(J+54)
HB(J)=SAVE(J+48)
350 CONTINUE
CA(1)=SQRTF((X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2)+(Z1-Z2)*(Z1-Z2))
CA(2)=SQRTF((X1X-X2X)*(X1X-X2X)+(Y1Y-Y2Y)*(Y1Y-Y2Y)+(Z1Z-Z2Z)*
1 (Z1Z-Z2Z))
WRITE OUTPUT TAPE 6,353,CA(1),CA(2)
353 FORMAT(1HD,2ZHSEPARATION DISTANCE = E15.7, 3H IN ,20X,
1 2ZHSEPARATION VELOCITY = E15.7, 7H IN/SEC )
IF (ICLAG) 6663,6663,6662
6663 WRITE OUTPUT TAPE 6,6661,FQQ1,FQQ2,FQQ3
6661 FORMAT(1HD,17HCONSTRAINT VALUES,3E16.7)
10171 FORMAT(1X,1P13E10.3/)
6662 CONTINUE
IF (IHY) 6670,8001,6670
6670 WRITE OUTPUT TAPE 6,6671,((A(I,J,1),J=1,3),(B(I,J),J=1,3),I=1,3)
6671 FORMAT(1HD,22X,2HA1,48X,2HA2/(3E15.6,5X,3E15.6))
8001 CONTINUE
SUN=0.0
DO 232 K=1,12
SUN=SUN+ABSF(Q(K))
232 CONTINUE
IF (IFORCE) 6678,405,6678
6678 IF (SUN) 404,404,6680
6680 IFORCE=1
GO TO 403
404 IF (IFORCE) 405,405,706
405 IF (WIL1-T) 706,6501,6501
6501 CONTINUE
403 CALL ROUT(D)
706 IF (TPLOT) 707,706,707
707 YDOT1(12)=TEND
YDOT1(11)=XPT8

```

```

        DIMENSION BA(3,3) ,WR(3) ,WT1(3) ,WT2(3) ,WT3(3)

        YDOT1(13)=TPLOT
        CALL CHAIN(3,3)
    708 CALL CHAIN(1,3)
C
C   ERROR
    5000 WRITE OUTPUT TAPE 6,5001
    5001 FORMAT(1H0,16HERROR INPUT DATA )
    4000 CALL EXIT
C
    5002 WRITE OUTPUT TAPE 6,5003
    5003 FORMAT(1H ,9HDELTA = 0,5X,48HSEE EQUATION 241,SECTION VII-EQUATION
        1S OF MOTION )
        CALL DUMP
        END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0,0)

```

SUBROUTINE GET

```

SUBROUTINE GET
EQUIVALENCE (T,T), (ZERO,ZERO),
1      (X1,V1(1)), (Y1,V1(2)), (Z1,V1(3)), (T1,V1(4)), (T2,V1(5))
1, (P1,V1(6)), (P2,V1(7)), (S1,V1(8)), (S2,V1(9)), (X1X,V1(10)), (Y1Y,V1(
11)), (Z1Z,V1(12)), (T1T,V1(13)), (T2T,V1(14)), (P1P,V1(15)), (P2P,V1(1
16)), (S1S,V1(17)), (S2S,V1(18)), (X2,V2(1)), (Y2,V2(2)), (Z2,V2(3)),
1(X2X,V2(4)), (Y2Y,V2(5)), (Z2Z,V2(6))
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
COMMON A,AD
DIMENSION      YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
DIMENSION DN(3,3),DN(3,3)
COMMON W11,TCO
DIMENSION TCG(6),TCO(6)
DIMENSION FM(3,4),FEL(3),ACNST(3),CM(3,3),DUD(3)
DIMENSION DD(12),DF(8),D(8),XC1(8),YC1(8),ZC1(8),XC2(8),YC2(8),
1ZC2(8)
DIMENSION EP(12),PI(3),TP(6),XP1(3),YP1(3),ZP1(3),XP2(3),YP2(3),
1ZP2(3),UPX(3),UPY(3),UPZ(3)
DIMENSION G(12),GA(6),PA(6),VA(6),PAD(6),VAD(6),SGX(6),SGY(6),
1SGZ(6),UAX(3),UAY(3),UAZ(3),UPAX(3),UPAY(3),UPAZ(3),VAX(3),VAY(3)
1,VAZ(3),VPAX(3),VPAY(3),VPAZ(3)
1,GG(6)
DIMENSION BINV(15)
DIMENSION H(12)
DIMENSION GJ(12),FG1(8),FG2(8),XG1(8),YG1(8),ZG1(8),XG2(8),
1YG2(8),ZG2(8),UGX(8),UGY(8),UGZ(8),VGX(8),VGY(8),VGZ(8)
DIMENSION XN(12),AN(8),RAX(8),PPN(8),VNX(8),VNY(8),VNZ(8),XN1(8),
1YN1(8),ZN1(8),XN2(8),YN2(8),ZN2(8),PNEM(8),EQP(8)
DIMENSION R(12),RR(16),XR1(16),YR1(16),ZR1(16),XR2(16),YR2(16),
1ZR2(16),URX(16),URY(16),URZ(16),VRX(16),VRY(16),VRZ(16),TE(16),
1TF(16),TRK(32)
DIMENSION SIOQ(12)
DIMENSION S(12),XS1(8),YS1(8),ZS1(8),XS2(8),YS2(8),ZS2(8),USX(8),
1USY(8),USZ(8),SD(8),SK(8)
DIMENSION W(12),WO(8),UK(8),XU1(8),YU1(8),ZU1(8),XU2(8),YU2(8),
1ZU2(8)
DIMENSION Q(12)
COMMON XX1,XY1,XZ1,YY1,YZ1,ZZ1,XX2,XY2,XZ2,YY2,YZ2,ZZ2,
1FM10,FM20,DEL,TPR,TD,EX1,EY1,EZ1,EX2,EY2,EZ2,WX,WY,WZ,
2W1YD,ICD,IPR,IAC,IHY,ICG,IPN,IRK,ISP,ISU,ION
COMMON DF,D,XC,YC,ZC,XC1,XC2,YC1,YC2,ZC1,ZC2
COMMON P1,TP,UX,UY,UZ,XP,YP,ZP,UPX,UPY,UPZ,XP1,XP2,YP1,YP2,
1ZP1,ZP2
COMMON TA,TB,TC,TD,IAC1,IAC2,MAC,NAC,IJLAG,GA,PA,VA,PAD,
1VAD,SGX,SGY,SGZ,UX,UY,UZ,PUX,PUY,PUZ,VAX,VPAX,VAY,
2VPAY,VAZ,VPZ,UAX,UPAX,UAY,UPAY,UAZ,UPAZ,GG
COMMON HD,HK,HETA,DA,PHBO,VHB,GAMH,JH,DEQP,HYDA,
1HYDB,HYDC,HETA,DA,HYDD,HYDE,DB
COMMON GJ,GST,FG3,IJLAG,MAXES,NAXES,FG1,FG2,TCO,TCG,
1UX,UY,UZ,XG,YG,ZG,XG1,XG2,YG1,YG2,ZG1,ZG2,VGX,VGY,
2VGZ,UGX,UGY,UGZ
COMMON XN,JN,PBO,VBN,GAMN,AN,RAX,PPN,XXN,YN,ZN,VNX,VNY,
1VNZ,XN1,XN2,YN1,YN2,ZN1,ZN2,EQP,PPN,PNEM
COMMON R,VRX,VRY,VRZ,URX,URY,URZ,TE,TF,TROC,SUN1,
1SUN2,SUN3,SAN1,SAN2,SAN3,XX1D,YY1D,ZZ1D,XX2D,YY2D,ZZ2D,
2FMD1,FMD2,RR,XR,YR,ZR,XR1,XR2,YR1,YR2,ZR1,ZR2,RUM,PUM

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SUBROUTINE GET

3TRK
COMMON SIOQ
COMMON S,SETA,XMU,EP,SLN,DELTS,XS1,XS2,YS1,YS2,ZS1,ZS2,
1SD,SK
COMMON W,ETA,W0,UK,XU,YU,ZU,XU1,XU2,YU1,YU2,ZU1,ZU2,TG
DIMENSION WIOQ(8)
COMMON WIOQ
COMMON DN,DN,R1,R2,P3,P21,P22,P23,ZTEST,F,CA,XI1,YI1,ZI1,XI2,
XY12,Z12,USX,USY,USZ
COMMON XI1,XI2,ZI1,ZI2,ICLAG,TRI62,ELM,LIRK,N,X8,Y8,Z8
COMMON WL
COMMON SAVE,BINV,E,NH,HB,H,0
COMMON FQQ1,FQQ2,FQQ3
DIMENSION SAVE(68),E(3,3),CA(3),F(15,15),WL(30)
DIMENSION WR(3),WRD(3),WAP(3,3,6),
1WAPP(3,3,6),WTP(3,3,6),WTPP(3,3,6),WD(3,2),W(3,2),WJ(2),WJP(12,2)
2,WJD(2),WJPP(12,2),A(3,3,2),AD(3,3,2),WRP(3,6)
DIMENSION WT1(3),WT2(3),WT3(3,3),WT4(3,3),WT5(6),WT6(12),WT7(12),
1WT8(12)
DIMENSION TEE(3,3),RIL(3),RIK(3),WTD(3,3)
DIMENSION SHT(12)
DIMENSION GED(2)
DIMENSION WRP1(3,8),WRP2(3,8),WRH(3,8)
COMMON WRP1,WRP2,WRH
DIMENSION WRPQ(3,2)
DO 63 I=1,15
DO 63 J=1,15
F(I,J)=0.0
63 CONTINUE
DO 36 I=1,3
SHT(I)=V1(I+9)
SHT(I+6)=V2(I+3)
J=2 *I
SHT(I+3)=V1(J+11)
SHT(I+9)=V1(J+12)
36 CONTINUE
5004 IM=0
ST=SINF(T1)
CT=COSE(T1)
CP=COSE(P1)
SP=SINF(P1)
CS=COSE(S1)
SS=SINF(S1)
49 IF (ABSF(CT)-ZTEST) 50,50,51
50 CT=0.0
51 IF (ABSF(ST)-ZTEST) 52,52,53
52 ST=0.0
53 IF (ABSF(CP)-ZTEST) 54,54,55
54 CP=0.0
55 IF (ABSF(SP)-ZTEST) 56,56,57
56 SP=0.0
57 IF (ABSF(CS)-ZTEST) 58,58,59
58 CS=0.0
59 IF (ABSF(SS)-ZTEST) 60,60,61
60 SS=0.0
61 CONTINUE

```

SUBROUTINE GET

CTSP=CT*SP
 CTCF=CT*CF
 CTSS=CT*SS
 CTCB=CT*CB
 CPCS=CP*CS
 SPCS=SF*CS
 STSS=ST*SS
 CPSS=CP*SS
 SPSS=SP*SS
 STCS=ST*CS
 CSPSS=CT*SPSS
 CCPSS=CT*CPSS
 CSPCS=CT*SPCS
 CCPCS=CT*CPCS
 STSP=ST*SP
 STCF=ST*CF

C

CT2=CT*CT
 ST2=ST*ST
 CB2=CB*CB
 SS2=SS*SS
 CS2=CS*CS
 STCT=ST*CT
 CT2=CT*CT
 CO1=CB2-SS2
 THD=V1 (IM+13)
 PHD=V1 (IM+15)
 PSD=V1 (IM+17)
 IM2=IM+1
 A(1,1,IM2)=-CFSS+CPCS
 A(1,2,IM2)=CCPSS+SPCS
 A(1,3,IM2)=STSS
 A(2,1,IM2)=-CSPCS-CPSS
 A(2,2,IM2)=CCPCS-SPSS
 A(2,3,IM2)=STCS
 A(3,1,IM2)=STSP
 A(3,2,IM2)=-STCF
 A(3,3,IM2)=CT
 AD(1,1,IM2)=ST*SS*THD-(CCPSS+SPCS)*PHD-(CSPCS+CPSS)*PSD
 AD(1,2,IM2)=-ST*CF*SS*THD-(CSPSS-CPCS)*PHD+(CCPCS-SPSS)*PSD
 AD(1,3,IM2)=CT*SS*THD+STCS*PSD
 AD(2,1,IM2)=ST*SP*CS*THD-(CCPCS-SPSS)*PHD+(CSPSS-CPCS)*PSD
 AD(2,2,IM2)=-ST*CPCS*THD-(CSPCS+CPSS)*PHD-(CCPSS+SPCS)*PSD
 AD(2,3,IM2)=CTCS*THD-STSS*PSD
 AD(3,1,IM2)=CT*SP*THD+STCF*PHD
 AD(3,2,IM2)=-CT*CF*THD+STSP*PHD
 AD(3,3,IM2)=-ST*THD
 IF (IM) 47,48,47

C

C

FORM DH MATRIX AND P1,P2,P3 FOR BODY 1
 48 DIFI=(XX1-YY1)*CSS
 DH(1,1)=XX1*CS2-2.0*XY1*CSS+YY1*SS2
 TER=XX1*SS2+2.0*XY1*CSS+YY1*CS2
 DIFM=XZ1*CS-YZ1*SS
 DIFP=XZ1*SS+YZ1*CS
 DIFT=DIFP*ST

```

SUBROUTINE GET

CO2=CO1*XY1
DH(1,2)=(DIFI+CO2)*ST+DIFM*CT
DH(1,3)=DIFM
DH(3,1)=DH(1,3)
DH(2,1)=DH(1,2)
DH(2,2)=ST2*TER+2.0*DIFT*CT+ZZ1*CT2
DH(2,3)=ST*DIFP+ZZ1*CT
DH(3,2)=DH(2,3)
DH(3,3)=ZZ1
R1=ST*P1P*S1S*(4.0+CSS*XY1-(XX1-YY1)*CO1-ZZ1)+2.0*T1T*S1S*(DIFI+
1 CO2)
1 +S1S*S1S*DIFP+P1P*P1P*STCT*(TER-ZZ1)+P1P*P1P*(CT2-ST2)*DIFP
1+2.*P1P*S1S*CT*(XZ1+SS+YZ1*CS)
R2=-2.0*P1P*(T1T*STCT*(TER-ZZ1)+S1S*ST2*(DIFI+CO2)+T1T*(CT2-ST2)*
1 DIFP+S1S*STCT*DIFM)-T1T*(T1T*CT*(DIFI+CO2)+S1S*ST*(XX1-YY1)*
1 CO1-4.0*XY1*CSS)-T1T*ST*DIFM)-S1S*(S1S*ST*DIFM-T1T*ST*ZZ1)
P3=-2.0*P1P*(T1T*CT*DIFP) +P1P*ST*(CO1*((XX1-YY1)*T1T
1 +XY1*P1P*ST)+T1T*ZZ1)+DIFI*(P1P*P1P*ST2-T1T*T1T)
1 -4.0*XY1*T1T*P1P*ST*CSS+DIFM*P1P*P1P*STCT-CO2*T1T*T1T
GO TO 41

C
C FORM DN MATRIX AND P21,P22,P23 FOR BODY2
47 DIFI=(XX2-YY2)*CSS
TER=XX2*CS2+2.0*XY2*CSS+YY2*CS2
DIFP=XZ2*SS+YZ2*CS
DIFM=XZ2*CS-YZ2*SS
DIFT=DIFP*ST
CO2=CO1*XY2
DN(1,1)=XX2*CS2-2.0*XY2*CSS+YY2*SS2
DN(1,2)=(DIFI+CO2)*ST+DIFM*CT
DN(1,3)=DIFM
DN(2,1)=DN(1,2)
DN(2,2)=ST2*TER+2.0*DIFT*CT+ZZ2*CT2
DN(2,3)=ST*DIFP+ZZ2*CT
DN(3,1)=DN(1,3)
DN(3,2)=DN(2,3)
DN(3,3)=ZZ2
P21=ST*P2P*S2S*(4.0+CSS*XY2-(XX2-YY2)*CO1-ZZ2)+2.0*T2T*S2S*(DIFI+
1 CO2)
1 +S2S*S2S*DIFP+P2P*P2P*STCT*(TER-ZZ2)+P2P*P2P*(CT2-ST2)*DIFP
1+2.*P2P*S2S*CT*(XZ2+SS+YZ2*CS)
P22=-2.0*P2P*(T2T*STCT*(TER-ZZ2)+S2S*ST2*(DIFI+CO2)+T2T*(CT2-ST2)*
1 DIFP+S2S*STCT*DIFM)-T2T*(T2T*CT*(DIFI+CO2)+S2S*ST*(XX2-YY2)*
1 CO1-4.0*XY2*CSS)-T2T*ST*DIFM)-S2S*(S2S*ST*DIFM-T2T*ST*ZZ2)
P23=-2.0*P2P*(T2T*CT*DIFP) +P2P*ST*(CO1*((XX2-YY2)*T2T
1 +XY2*P2P*ST)+T2T*ZZ2)+DIFI*(P2P*P2P*ST2-T2T*T2T)
1 -4.0*XY2*T2T*P2P*ST*CSS+DIFM*P2P*P2P*STCT-CO2*T2T*T2T
41 IF (ICLAG) 101,101,104

C
C CONSTRUCT R PARTIAL MATRIX
C
101 DO 1 I=1,3
DO 1 J=1,6
WRP(I,J)=0.
1 CONTINUE
DO12 I=1,3

```



```

SUBROUTINE GET
  WAP(1,1)=1.
  WAP(1,1+3)=-1.
12 CONTINUE
  IM1=IM*3+1
  STSPSS=STSS*SP
  WAP(1,1,IM1)=STSPSS
  WAP(1,2,IM1)=-CPSS*ST
  WAP(1,3,IM1)=CTSS
  WAP(2,1,IM1)=STSP*CS
  WAP(2,2,IM1)=-STCP*CS
  WAP(2,3,IM1)=CTCS
  WAP(3,1,IM1)=CTSP
  WAP(3,2,IM1)=-CTCP
  WAP(3,3,IM1)=-ST
  DO 2 J=1,3
    WAP(J,1,IM1+1)=-A(J,2,IM2)
    WAP(J,2,IM1+1)=A(J,1,IM2)
    WAP(1,J,IM1+2)=A(2,J,IM2)
    WAP(J,3,IM1+1)=0.
    WAP(2,J,IM1+2)=-A(1,J,IM2)
    WAP(3,J,IM1+2)=0.
  2 CONTINUE
  WAPP(1,1,IM1)=CSPSS*THD+STSS*CP*PHD+SPCS*ST*PSD
  WAPP(1,2,IM1)=-CCPSS*THD+ST*SPSS*PHD-STCS*CP*PSD
  WAPP(1,3,IM1)=-STSS*THD+CTCS*PSD
  WAPP(2,1,IM1)=CSPCS*THD+CPCS*ST*PHD-STSS*SP*PSD
  WAPP(2,2,IM1)=-CCPCS*THD+STSP*CS*PHD+STCP*SS*PSD
  WAPP(2,3,IM1)=-STCS*THD-CTSS*PSD
  WAPP(3,1,IM1)=-STSP*THD+CTCP*PHD
  WAPP(3,2,IM1)=STCP*THD+CTSP*PHD
  WAPP(3,3,IM1)=-CT*THD
  DO 3 I=1,3
    WAPP(I,1,IM1+1)=-AD(I,2,IM2)
    WAPP(I,2,IM1+1)=AD(I,1,IM2)
    WAPP(I,3,IM1+1)=0.
    WAPP(1,I,IM1+2)=AD(2,I,IM2)
    WAPP(2,I,IM1+2)=-AD(1,I,IM2)
    WAPP(3,I,IM1+2)=0.
  3 CONTINUE
104 IF(IM)43,44,43
C
C   MATRICES B AND BD
44 ST=SINF(T2)
   CT=COSE(T2)
   CP=COSE(P2)
   SP=SINF(P2)
   CS=COSE(S2)
   SS=SINF(S2)
   IM=1
   GO TO 49
43 IF(ICLAG)106,106,105
105 RETURN
106 DO 102 K=1,3
   DO 102 J=1,3
     F(K+9,J+9)=DN(K,J)
102 CONTINUE

```

```

SUBROUTINE GET

DO 100 K=1,3
DO 100 J=1,3
F(K+3,J+3)=DM(K,J)
100 CONTINUE
DO 13 I=1,3
WR(I)=V1(I)-V2(I)
WRD(I)=V1(I+9)-V2(I+3)
13 CONTINUE
IF (ICLAG) 110,111,111
111 WRPQ(1,1)=XI1
WRPQ(2,1)=YI1
WRPQ(3,1)=ZI1
WRPQ(1,2)=XI2
WRPQ(2,2)=YI2
WRPQ(3,2)=ZI2
DO 115 I=1,3
CA(I)=0.
F(I+12,I)=-WRP(I,1)
F(I+12,I+6)=1.
J=I+3
CALL MULTI(WAP(1,1,J),WRPQ(1,2),F(13,I+9),3,3,1)
CALL MULTI(WAP(1,1,I),WRPQ(1,1),F(13,I+3),3,3,1)
DO 117 K=1,3
117 F(K+12,I+3)=-F(K+12,I+3)
115 CONTINUE
DO 116 I=1,3
J=I+3
CALL MULTI(WAPP(1,1,J),WRPQ(1,2),WT1(1),3,3,1)
CALL MULTI(WAPP(1,1,I),WRPQ(1,1),WT2(1),3,3,1)
F(I,I+12)=F(I+12,I)
F(I+6,I+12)=F(I+12,I+6)
DO 116 K=1,3
F(I+3,K+12)=F(K+12,I+3)
F(I+9,K+12)=F(K+12,I+9)
116 CA(K)=CA(K)-SHT(I+9)*WT1(K)+SHT(I+3)*WT2(K)
CALL MULTI(A(1,1,1),WRPQ(1,1),WT1(1),3,3,1)
CALL MULTI(A(1,1,2),WRPQ(1,2),WT2(1),3,3,1)
FRQ1=WR(1)+WT1(1)-WT2(1)
FRQ2=WR(2)+WT1(2)-WT2(2)
FRQ3=WR(3)+WT1(3)-WT2(3)
RETURN
110 CONTINUE
C
C COMPUTE T
C
CALL MULT2(A(1,1,2),A,TEE,3,3,3)
C
C THE FOLLOWING COMPUTES THE PARTIAL OF T WITH RESPECT TO Q
C AND ITS TIME DERIVATIVE
C
DO 10 I=1,3
CALL MULT2(A(1,1,2),WAP(1,1,I),WTP(1,1,I),3,3,3)
CALL MULT2(AD(1,1,2),WAP(1,1,I),WT3,3,3,3)
CALL MULT2(A(1,1,2),WAPP(1,1,I),WT4,3,3,3)
CALL MADD(WT3,WT4,WTPP(1,1,I),3,3)
J=I+3

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SUBROUTINE GET

CALL MULT2(WAP(1,1,J),A,WTP(1,1,J),3,3,3)
CALL MULT2(WAFP(1,1,J),A,WT3,3,3,3)
CALL MULT2(WAP(1,1,J),AD,WT4,3,3,3)
CALL MADD(WT3,WT4,WTFP(1,1,J),3,3)
10 CONTINUE

C
C COMPUTE T DOT
C
CALL MULT2(AD(1,1,2),A,WT3,3,3,3)
CALL MULT2(A(1,1,2),AD,WT4,3,3,3)
CALL MADD(WT3,WT4,WTD,3,3)
DO 25 I=1,2

C
C COMPUTE D
C
CALL MULT1(A,WRP1(1,I),WT1,3,3,1)
CALL MULT1(A(1,1,2),WRP2(1,I),WT2,3,3,1)
CALL MADD(WT1,WR,WT1,3,1)
CALL MSUB(WT1,WT2,W0(1,I),3,1)

C
C COMPUTE V
C
CALL MULT1(AD,WRP1(1,I),WT1,3,3,1)
CALL MULT1(AD(1,1,2),WRP2(1,I),WT2,3,3,1)
CALL MADD(WT1,WRD,WT1,3,1)
CALL MSUB(WT1,WT2,WV(1,I),3,1)

C
C FORM JI
C
CALL MSR(A,W0(1,I),WF,2)
CALL MULT1(A(1,1,2),WRH(1,I),WT1,3,3,1)
CALL MSR(A,WT1,WG,2)
WJ(I)=WF/WG

C
C FORM THE PARTIALS OF JI WITH RESPECT TO QS
C
DO 15 J=1,3
WT8(J)=0.
CALL MSR(A,WRP(1,J),WT1,2)
CALL MSR(A,WRP(1,J+3),WT1(2),2)
WJP(J,I)=WT1/WG
WJP(J+6,I)=WT1(2)/WG
WT8(J+6)=0.
CALL MSR(WAP(1,1,J),W0(1,I),WT1,2)
CALL MULT1(WAP(1,1,J),WRP1(1,I),WT2,3,3,1)
CALL MSR(A,WT2,WT1(2),2)
CALL MULT1(A(1,1,2),WRH(1,I),WT2,3,3,1)
CALL MSR(WAP(1,1,J),WT2,WT1(3),2)
WT8(J+3)=WT1(3)
WJP(J+3,I)=(WT1(1)+WT1(2)-WT1(3)*WJ(1))/WG
CALL MULT1(WAP(1,1,J+3),WRP2(1,I),WT1,3,3,1)
CALL MSR(A,WT1,WT2,2)
CALL MULT1(WAP(1,1,J+3),WRH(1,I),WT1,3,3,1)
CALL MSR(A,WT1,WT2(2),2)
WT8(J+9)=WT2
WJP(J+9,I)=- (WT2(1)+WT2(2)*WJ(1))/WG

```

```

SUBROUTINE GET
15 CONTINUE
C
C   COMPUTE J1 DOT
C
CALL MSR (A,WV(1,1),WTS(2),2)
CALL MSR (AD,W0(1,1),WTS,2)
CALL MULT1(A(1,1,2),WRH(1,1),WT1,3,3,1)
CALL MSR (AD,WT1,WTS(3),2)
CALL MULT1(AD(1,1,2),WRH(1,1),WT1,3,3,1)
CALL MSR (A,WT1,WTS(4),2)
WJD(I)=(WTS(1)+WTS(2)-WJ(I)*(WTS(3)+WTS(4)))/WG
GED(I)=WTS(3)+WTS(4)
C
C   FORM THE FIRST TIME DERIVATIVE OF J1
C   WITH RESPECT TO Q8
C
DO 17 J=1,3
CALL MSR (AD,WRP(1,J),WT6(J),2)
WT7(J)=0.
CALL MSR (AD,WRP(1,J+3),WT6(J+6),2)
WT7(J+6)=0.
CALL MSR (WAPP(1,1,J),W0(1,1),WTS,2)
CALL MSR (WAP(1,1,J),WV(1,1),WTS(2),2)
CALL MULT1(WAP(1,1,J),WRP1(1,1),WT1,3,3,1)
CALL MSR (AD,WT1,WTS(3),2)
CALL MULT1(WAPP(1,1,J),WRP1(1,1),WT1,3,3,1)
CALL MSR (A,WT1,WTS(4),2)
WT6(J+3)=WTS(1)+WTS(2)+WTS(3)+WTS(4)
CALL MULT1(A(1,1,2),WRH(1,1),WT1,3,3,1)
CALL MSR (WAPP(1,1,J),WT1,WTS,2)
CALL MULT1(AD(1,1,2),WRH(1,1),WT1,3,3,1)
CALL MSR (WAP(1,1,J),WT1,WTS(2),2)
WT7(J+3)=WTS(1)+WTS(2)
CALL MULT1(WAP(1,1,J+3),WRP2(1,1),WT1,3,3,1)
CALL MSR (AD,WT1,WTS(1),2)
CALL MULT1(WAPP(1,1,J+3),WRP2(1,1),WT1,3,3,1)
CALL MSR (A,WT1,WTS(2),2)
WT6(J+9)=-WTS(1)-WTS(2)
CALL MULT1(WAP(1,1,J+3),WRH(1,1),WT1,3,3,1)
CALL MSR (AD,WT1,WTS(1),2)
CALL MULT1(WAPP(1,1,J+3),WRH(1,1),WT1,3,3,1)
CALL MSR (A,WT1,WTS(2),2)
WT7(J+9)=WTS(1)+WTS(2)
17 CONTINUE
DO 18 J=1,12
FDK=WTS(J)-WJD(I)*WTS(J)-WJ(I)*WT7(J)
WJPP(J,I)=(FDK-WJP(J,I)*GED(I))/WG
18 CONTINUE
25 CONTINUE
DO 24 I=1,3
24 RIL(I)=0.
CALL MSUB(WRP1(1,1),WRP1(1,2),WT1,3,1)
DO 27 I=1,12
F(13,I)=-WJP(I,1)*WRH(1,1)
F(14,I)=-WJP(I,1)*WRH(3,1)
CALL MSR(TEE,WT1,WTS(4),3)

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SUBROUTINE GET

WT5(1)=WJP(I,1)*WRH(1,1)-WJP(I,2)*WRH(1,2)
WT2(1)=WT5(4)*WT5(1)
CALL MSR(TEE,WT1,WT5(5),1)
WT5(2)=WJP(I,1)*WRH(3,1)-WJP(I,2)*WRH(3,2)
WT2(2)=WT5(5)*WT5(2)
F(15,I)=WT2(1)-WT2(2)
RIK(1)=-WJPP(I,1)*WRH(1,1)
RIK(2)=-WJPP(I,1)*WRH(3,1)
CALL MSR(WTD(1,1),WT1,WT2(1),3)
CALL MSR(WTD(1,1),WT1,WT2(2),1)
RIK(3)=WT2(1)*WT5(1)+WT5(4)*(WJPP(I,1)*WRH(1,1)-WJPP(I,2)*WRH(1,2)
1)-WT2(2)*WT5(2)-WT5(5)*(WJPP(I,1)*WRH(3,1)-WJPP(I,2)*WRH(3,2))
DO 27 J=1,3
RIL(J)=RIL(J)+RIK(J)*SHT(I)
27 CONTINUE
DO 30 I=1,3
DO 31 J=1,3,2
J1=12+(J+1)/2
CALL MSR(A(1,1,2),WRP(1,I),WT2,J)
F(J1,I)=F(J1,I)+WT2(1)
CALL MSR(A(1,1,2),WRP(1,I+3),WT2,J)
F(J1,I+6)=F(J1,I+6)+WT2(1)
CALL MSR(WAP(1,1,I+3),WR,WT2(2),J)
F(J1,I+9)=F(J1,I+9)+WT2(2)
CALL MSR(WTP(1,1,I),WRP1,WT2(3),J)
F(J1,I+3)=F(J1,I+3)+WT2(3)
CALL MSR(WTP(1,1,I+3),WRP1,WT2,J)
F(J1,I+9)=F(J1,I+9)+WT2(1)
31 CONTINUE
CALL MSUB(WRP1(1,1),WRP1(1,2),WT1,3,1)
DO 32 J=1,3,2
WT5(J)=WRP2(J,1)+WJ(1)*WRH(J,1)-WRP2(J,2)-WJ(2)*WRH(J,2)
WT6(J)=WT5(J)
CALL MSCAL(WT5(J),WT1,WT3(1,J))
J1=4-J
CALL MSR(WTP(1,1,I),WT3(1,J),WT2,J1)
CALL MSR(WTP(1,1,I+3),WT3(1,J),WT2(2),J1)
GRQ=FLOATF(2-J)
F(15,I+3)=F(15,I+3)+GRQ*WT2(1)
F(15,I+9)=F(15,I+9)+GRQ*WT2(2)
32 CONTINUE
DO 33 J=1,3,2
CALL MSR(WAPP(1,1,I+3),WR,WT5,J)
CALL MSR(WAP(1,1,I+3),WRD,WT5(2),J)
CALL MSR(WTPP(1,1,I+3),WRP1,WT5(3),J)
J1=(J+1)/2
RIL(J1)=RIL(J1)+(WT5(1)+WT5(2)+WT5(3))*SHT(I+9)
CALL MSR(WTPP(1,1,I),WRP1,WT5(4),J)
RIL(J1)=RIL(J1)+SHT(I+3)*WT5(4)
CALL MSR(AD(1,1,2),WRP(1,1),WT5(5),J)
RIL(J1)=RIL(J1)+WT5(5)*SHT(I)
CALL MSR(AD(1,1,2),WRP(1,I+3),WT5(6),J)
RIL(J1)=RIL(J1)+WT5(6)*SHT(I+6)
33 CONTINUE
DO 34 J=1,3,2
J1=4-J

```

```

SUBROUTINE GET

      QGQ=WJD(1)*WRH(J,1)-WJD(2)*WRH(J,2)
      CALL MSCAL(QGQ,WT1,WT2)
      CALL MSR(WTP(1,1,I),WT2,WT5,J1)
      GRQ=FLOATF(2-J)
      CALL MSR(WTPP(1,1,I),WT3(1,J),WT5(2),J1)
      RIL(3)=RIL(3)+SHT(I+3)*GRQ*(WT5(1)+WT5(2))
      CALL MSR(WTP(1,1,I+3),WT2,WT5,J1)
      CALL MSR(WTPP(1,1,I+3),WT3(1,J),WT5(2),J1)
      RIL(3)=RIL(3)+SHT(I+9)*GRQ*(WT5(1)+WT5(2))
34  CONTINUE
30  CONTINUE
      CA(1)=-RIL(1)
      CA(2)=-RIL(2)
      CA(3)=-RIL(3)
      DO 37 I=1,3
      DO 37 J=1,12
37  F(J,I+12)=F(I+12,J)
      DO 65 I=1,3,2
      CALL MSR(A(1,1,2),WR,WT5(1),I)
      CALL MSR(TEE,WRP1,WT5(2),I)
      WT5(3)=-WRP2(I,1)
      WT5(4)=-WJ(1)*WRH(I,1)
      FQ=WT5(1)+WT5(2)+WT5(3)+WT5(4)
      IF(I-2)66,65,67
66  FQQ1=FQ
1234 FORMAT(6E20.7)
      GO TO 63
67  FQQ2=FQ
65  CONTINUE
1007 FORMAT(6E20.7)
      FQQ3=0.
      DO 68 J=1,3,2
      J1=4-J
      CALL MSCAL(WT6(J),WT1,WT5(1))
      CALL MSR(TEE,WT5(1),CUF,J1)
      FQQ3=FQQ3+FLOATF(2-J)*CUF
68  CONTINUE
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)

```

```

SUBROUTINE SFRING(WRP1,WRF2,WRH,SD,SK,SIOQ,ISP,N,ICLAG,S,
SUBROUTINE SFRING(WRP1,WRF2,WRH,SD,SK,SIOQ,ISP,N,ICLAG,S,
1FJ,LDER,LSP1,ISS)
DIMENSION ISS(1),FJ(1)
  DIMENSION WRP1(3,8)
  DIMENSION ISP1(8)
  DIMENSION ICRP(12)
  COMMON T,ZERO,V1,V2,YDOT1,YDOT2
  DIMENSION          YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
  EQUIVALENCE (T,T),(ZERO,ZERO),
1  (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
  COMMON A,B,AD,BD
  DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
  DIMENSION EEK(3,4)
  DIMENSION WT1(3),WT2(3),WT3(3),BA(3,3),WT4(3),WT5(3),WT6(3)
  DIMENSION WRP1(3,1),WRF2(3,1),WRH(3,1),SD(1),SK(1),SIOQ(1)
  XS(1),FOIE(1),WR(3)
  DO 1 I=1,12
    ICRP(I)=0
    S(I)=0.0
1 CONTINUE
  TUF=0.
  CP=COSF(P1)
  SP=SINF(P1)
  EEK(1,1)=CP
  EEK(2,1)=SP
  EEK(3,1)=0.
  ST=SINF(T1)
  CT=COSF(T1)
  EEK(1,2)=ST*SP
  EEK(2,2)=-ST*CP
  EEK(3,2)=CT
  EEK(1,3)=COSF(S2)
  EEK(2,3)=-SINF(S2)
  EEK(3,3)=0.
  LSP=0
  DO 200 I=1,3
    WR(I)=V1(I)-V2(I)
    WT4(I)=0.
    WT5(I)=0.
    WT6(I)=0.
200 CONTINUE
  CALL MULT2 (B,A,BA,3,3,3)
  DO 100 I=1,ISP
    CALL MULT1 (A,WRP1(1,I),WT1,3,3,1)
    CALL MULT1 (B,WRF2(1,I),WT2,3,3,1)
    CALL MSUB (WT1,WT2,WT3,3,1)
    CALL MADD (WT3,WR,WT1,3,1)
    CALL MSR (A,WT1,WT2(1),2)
    CALL MSC (BA,WRH(1,I),WT2(2),2)
    FJ(I)=WT2(1)/WT2(2)
    PUT=SK(I)*FJ(I)
    PUT1 =PUT+SIOQ(1)

```

```

SUBROUTINE SPRING(WRP1,WRP2,WRH,SD,SK,SIGG,ISP,N,ICLAG,S,
FOIE =FUT1-SD(I)
IF(FOIE )3,3,6
6 LSP=LSP+1
ICRP(I)=1
GO TO 100
3 XS=(SD(I)-FUT)*SQRT(WRH(1,I)**2+WRH(2,I)**2+WRH(3,I)**2)
C
CALL MSCAL(XS,WRH(1,I),WT1)
CALL MADD(WT4,WT1,WT4,3,1)
C
CALL MCROS (WRP2(1,I),WT1(1),WT2)
CALL MSUB (WT5,WT2,WT5,3,1)
C
CALL MULT1 (B,WRP2(1,I),WT2,3,3,1)
CALL MULT1 (B,WRH(1,I),WT3,3,3,1)
CALL MSCAL(FJ(I),WT3,WT3)
CALL MADD (WT2,WT3,WT2,3,1)
CALL MSUB (WT2,WR,WT2,3,1)
CALL MULT1(B,WT1,WT3,3,3,1)
CALL MCROS (WT2,WT3,WT1)
CALL MADD (WT1,WT6,WT6,3,1)
100 CONTINUE
DO 201 J=1,3
J1=2*J-1
CALL MSC(A,WT4,S(J1),J)
S(J1+1)=-S(J1)
201 CONTINUE
S(12)=WT5(3)
CALL MSC(WT6,E EK(1,2),S(11),1)
CALL MSC(WT6,E EK(1,1),S(7),1)
S(9)=WT6(3)
CALL MSC(WT5,E EK(1,3),S(8),1)
CALL MSC(B,WT5,S(10),3)
IF(N-2)11,20,20
20 IF(ISP-LSP-2)11,12,12
12 KL=3
IF(ICRP(1))13,14,13
13 DO 15 L=3,ISP
IF(ICRP(L))15,16,15
16 ISS(1)=L
KL=L+1
GO TO 14
15 CONTINUE
14 IF(ICRP(2))17, 8,17
17 IF(KL-ISP)33,33,7
33 DO 19 L1=KL,ISP
IF(ICRP(L1))19,21,19
21 ISS(2)=L1
GO TO 8
19 CONTINUE
11 ICLAG=1
IF(LSP-ISP)8,7,8
C
FOR CE CEASES TO ACT - - - SET INDEX = ZERO
7 ISP=0
ICLAG=1
8 IF(LSP-LSP1)18,18,25

```



```
SUBROUTINE SPRING(WRP1,WRP2,WRH,SD,SK,SIOQ,ISP,N,ICLAG,S,  
25 LDER=-1  
   LSF1=LSP  
18 RETURN  
   END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```

SUBROUTINE PYRO (FI,TF,XP1,YF1,ZP1,XP2,YF2,ZP2,UPX,UPY,UPZ,EP,IPR)

SUBROUTINE PYRO (FI,TF,XP1,YF1,ZP1,XP2,YF2,ZP2,UPX,UPY,UPZ,EP,IPR)
EQUIVALENCE (T,T),(ZERO,ZERO),
1      (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(F1,V1(6)),(F2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
11)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
COMMON A,AD,BD
DIMENSION A(3,3,2),AD(3,3),BD(3,3)
DIMENSION YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
DIMENSION PI(3),TP(6),EP(12),XP1(3),YP1(3),ZP1(3),XP2(3),YP2(3),
1 ZP2(3),UPX(3),UPY(3),UPZ(3),CA(3),CB(3)
DIMENSION RHO(3),WT1(3),RP1(3),RP2(3),WT2(3,2),WT3(3,2),WT4(
*3)
E=0.
DO 4 J=1,12
EP(J)=0.0
4 CONTINUE
CONX=1.0
CONY=1.0
CONZ=1.0
IF (X1-X2) 8,9,9
8 CONX=-1.0
9 IF (Y1-Y2) 10,11,11
10 CONY=-1.0
11 IF (Z1-Z2) 12,13,13
12 CONZ=-1
13 DO 100 I=1,IPR
DIMENSION IT(3)
IF (T-TP(I+3)) 1,20,22
20 IT(I)=0
GO TO 25
22 IF (IT(I)) 25,100,25
1 IF (T-TP(I)) 100,24,23
23 IF (IT(I)) 25,100,25
24 IT(I)=1
25 E=PI(I)
RP1(1) =XP1(I)
RP1(2) =YP1(I)
RP1(3) =ZP1(I)
RP2(1) =XP2(I)
RP2(2) =YP2(I)
RP2(3) =ZP2(I)
RHO(1) =UPX(I)
RHO(2) =UPY(I)
RHO(3) =UPZ(I)
CALL MULT1(A(1,1,2),RHO,WT1,3,3,1)
DO 30 J=1,3
WT2(J,1)=0.
WT2(J,2)=0.
30 EP(2*J-1)=EP(2*J-1)+E*WT1(J)
CALL MCROS(RP2,RHO,WT3)
CALL MCROS(RP1,RHO,WT3(1,2))
DO 31 J=1,3
WT2(J,1)=WT2(J,1)+E*WT3(J,1)

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```

SUBROUTINE FYRO (FI,TF,XP1,YF1,ZP1,XP2,YF2,ZP2,UPX,UPY,UFZ,EP,IFR)

31 WT2(J,2)=WT2(J,2)+E*WT3(J,2)
100 CONTINUE
   IF(E) 34,6,34
34 DO 32 J=1,2
   CP=COSF(V1(J+5))
   SP=SINF(V1(J+5))
   CT=COSF(V1(J+3))
   ST=SINF(V1(J+3))
   SNG=(-1)**J
   J1=3-J
   CALL MULT1(A(1,1,J1),WT2(1,J),WT1,3,3,1)
   EP(J+6)=EP(J+6)+SNG*(WT1(1)*CP+WT1(2)*SP)
   EP(J+8)=EP(J+8)+SNG*WT1(3)
   EP(J+10)=EP(J+10)+SNG*(ST*SF+WT1(1)-ST*CP*WT1(2)
1+CT*WT1(3))
32 CONTINUE
   CONX=-CONX
   CONZ=-CONZ
   CONY=-CONY
   EP(1)=-EP(1)*CONX
   EP(3)=-EP(3)*CONY
   EP(5)=-EP(5)*CONZ
7 EP(2)=-EP(1)
EP(4)=-EP(3)
EP(6)=-EP(5)
6 RETURN
END (1,0,0,0,0,0,1,0,0,1,0,0,0,0)

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```

SUBROUTINE PNEUM(IPN, JN, FBO, VBN, GAMN, AN, RAX, UNX, UNY, UNZ,
SUBROUTINE PNEUM(IPN, JN, FBO, VBN, GAMN, AN, RAX, UNX, UNY, UNZ,
1 XN1, YN1, ZN1, XN2, YN2, ZN2, XN, FPN, EQF, PNEM, LDER, IPN1)
COMMON T, ZERO, V1, V2, YDOT1, YDOT2
DIMENSION YDOT2(6), YDOT1(18), V2(6), V1(18), NH(6), HB(6)
EQUIVALENCE (T, T), (ZERO, ZERO),
1 (X1, V1(1)), (Y1, V1(2)), (Z1, V1(3)), (T1, V1(4)), (T2, V1(5))
1, (F1, V1(6)), (F2, V1(7)), (S1, V1(8)), (S2, V1(9)), (X1X, V1(10)), (Y1Y, V1(
11)), (Z1Z, V1(12)), (T1T, V1(13)), (T2T, V1(14)), (F1F, V1(15)), (F2F, V1(1
16)), (S1S, V1(17)), (S2S, V1(18)), (X2, V2(1)), (Y2, V2(2)), (Z2, V2(3)),
1 (X2X, V2(4)), (Y2Y, V2(5)), (Z2Z, V2(6))
COMMON A, B, AD, BD
DIMENSION A(3,3), B(3,3), AD(3,3), BD(3,3)
DIMENSION AN(8), RAX(8), UNX(8), UNY(8), UNZ(8), XN1(8), XN2(8), YN1(8),
1 YN2(8), ZN1(8), ZN2(8), FPN(8), XN(12), EQF(8), CA(3), CB(3), CC(3), DEX(8
1), PNEM(8)
DO 17 K=1, 12
XN(K)=0.0
17 CONTINUE
CONX=1.0
CONY=1.0
CONZ=1.0
IF (X1-X2) 19, 20, 20
19 CONX=-1.0
20 IF (Y1-Y2) 21, 22, 22
21 CONY=-1.0
22 IF (Z1-Z2) 23, 24, 24
23 CONZ=-1.0
24 LPM=0
SUM=0.0
DO 1 I=1, IPN
DO 2 K=1, 3
CA(K)=A(1, K)*XN1(I)+A(2, K)*YN1(I)+A(3, K)*ZN1(I)
CB(K)=B(1, K)*UNX(I)+B(2, K)*UNY(I)+B(3, K)*UNZ(I)
CC(K)=B(1, K)*XN2(I)+B(2, K)*YN2(I)+B(3, K)*ZN2(I)
2 CONTINUE
DEX(I)=- (A(1, 2)*(X2+CC(1))-X1-CA(1))+A(2, 2)*(Y2+CC(2))-Y1-CA(2)
1 +A(3, 2)*(Z2+CC(3))-Z1-CA(3)) / (A(1, 2)*CB(1)+A(2, 2)*CB(2)
1+A(3, 2)*CB(3))
DEX(I)=ABS(DEX(I))
SUM=SUM+AN(I)*DEX(I)
1 CONTINUE
DO 100 I=1, IPN
IF (JN-1) 3, 5, 9
3 IF (DEX(I)-RAX(I)) 4, 15, 15
15 LPM=LPM+1
GO TO 100
4 YN=FBO*AN(I)*((1.0+SUM/VBN)**GAMN)
GO TO 50
5 IF (DEX(I)-EQF(I)) 6, 6, 7
6 YN=FPN(I)
GO TO 50
7 IF (DEX(I)-RAX(I)) 8, 15, 15
8 YN=FPN(I)*(EQF(I)/DEX(I)**GAMN)
GO TO 50
9 IF (DEX(I)-EQF(I)) 6, 6, 10
10 IF (DEX(I)-RAX(I)) 11, 15, 15

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SUBROUTINE FNEUM(IPN, JN, FBO, VBN, GAMN, AN, RAX, UNX, UNY, UNZ,
11 SUMN=0.0
SUMD=0.0
DO 12 J=1, IPN
IF (DEX(J)-EQF(J)) 13,14,14
13 SUMN=SUMN+AN(J)*DEX(J)
SUMD=SUMD+AN(J)*DEX(J)
GO TO 12
14 SUMN=SUMN+AN(J)*DEX(J)-FNEM(J)
SUMD=SUMD+FNEM(J)
12 CONTINUE
YN=FPN(1)/(1.0+SUMN/(VBN+SUMD))*GAMN
50 DO 51 K=1,3
CA(K)=A(1,K)*XN1(I)+A(2,K)*YN1(I)+A(3,K)*ZN1(I)
CB(K)=B(1,K)*UNX(I)+B(2,K)*UNY(I)+B(3,K)*UNZ(I)
51 CONTINUE
XN(1)=XN(1)+YN*CB(1)
XN(3)=XN(3)+YN*CB(2)
XN(5)=XN(5)+YN*CB(3)
XN(7)=XN(7)+YN*((CB(1)*CA(3)-CB(3)*CA(1))*SINF(P1)
+ (CB(3)*CA(2)-CB(2)*CA(3))*COSF(P1))
XN(9)=XN(9)+YN*(CB(2)*CA(1)-CB(1)*CA(2))
XN(11)=XN(11)+YN*((CB(3)*CA(2)-CB(2)*CA(3))*A(3,1)+(CB(1)*CA(3)-
1 CB(3)*CA(1))*A(3,2)+(CB(2)*CA(1)-CB(1)*CA(2))*A(3,3))
CB(1)=UNX(I)*ZN2(I)-UNZ(I)*XN2(I)
CA(1)=UNZ(I)*YN2(I)-UNY(I)*ZN2(I)
CA(2)=UNZ(I)*XN2(I)-UNX(I)*ZN2(I)
CA(3)=UNY(I)*XN2(I)-UNX(I)*YN2(I)
XN(8)=XN(8)-YN*(CA(1)*COSF(S2)+CA(2)*SINF(S2))
XN(10)=XN(10)-YN*(CA(1)*B(1,3)+CB(1)*B(2,3)+CA(3)*B(3,3))
XN(12)=XN(12)-YN*CA(3)
100 CONTINUE
CONX=-CONX
CONY=-CONY
CONZ=-CONZ
XN(1)=XN(1)*CONX
XN(3)=XN(3)*CONY
XN(5)=XN(5)*CONZ
53 XN(2)=-XN(1)
XN(4)=-XN(3)
XN(6)=-XN(5)
54 IF (LPM-IPN) 18,16,18
C FORCE CEASES TO ACT - - - SET INDEX = ZERO
16 IPN=0
18 IF (LPM-IPN1) 31,31,30
30 LDER=-1
IPN1=LPM
31 RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)

```

```

SUBROUTINE DRAG(D,DF,DD,XC1,YC1,ZC1,XC2,YC2,ZC2,ICD,LDER,ICD1)

SUBROUTINE DRAG(D,DF,DD,XC1,YC1,ZC1,XC2,YC2,ZC2,ICD,LDER,ICD1)
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
DIMENSION YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
EQUIVALENCE (T,T),(ZERO,ZERO),
1 (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(F1,V1(6)),(F2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
11)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1F,V1(15)),(P2P,V1(1
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
COMMON A,B,AD,BD
DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
DIMENSION D(8),DD(12),DF(8),XC1(8),YC1(8),ZC1(8),XC2(8),YC2(8),
1ZC2(8),CA(3),CB(3),CC(3)
LDRAG=0
DO 4 J=1,12
DD(J)=0.0
4 CONTINUE
DO 100 I=1,ICD
DO 1 J=1,3
CA(J)=A(1,J)*XC1(I)+A(2,J)*YC1(I)+A(3,J)*ZC1(I)
CB(J)=B(1,J)*XC2(I)+B(2,J)*YC2(I)+B(3,J)*ZC2(I)
1 CONTINUE
DIX=X1-X2+CA(1)-CB(1)
DIY=Y1-Y2+CA(2)-CB(2)
DIZ=Z1-Z2+CA(3)-CB(3)
XD=SQRTF(DIX**2+DIY**2+DIZ**2)
IF(XD-DF(I)) 2,7,7
7 LDRAG=LDRAG+1
WRITE OUTPUT TAPE 6,IG1,T,X1,Y1,Z1,X2,Y2,Z2,XD,DF(I)
101 FORMAT(SF14.6)
GO TO 100
2 DO 3 J=1,3
CC(J)=AD(1,J)*XC1(I)+AD(2,J)*YC1(I)+AD(3,J)*ZC1(I)
1 -(BC(1,J)*XC2(I)+BD(2,J)*YC2(I)+BD(3,J)*ZC2(I))
3 CONTINUE
VIX=X1X-X2X+CC(1)
VIY=Y1Y-Y2Y+CC(2)
VIZ=Z1Z-Z2Z+CC(3)
V=SQRTF(VIX**2+VIY**2+VIZ**2)
V=D(1)/V
DD(1)=DD(1)-VIX*V
DD(3)=DD(3)-VIY*V
DD(5)=DD(5)-VIZ*V
DD(7)=DD(7)-V*((VIX*CA(3)-VIZ*CA(1))*SINF(P1)
1 +(VIZ*CA(2)-VIX*CA(3))*COSF(P1))
DD(8)=DD(8)+V*((VIX*CB(3)-VIZ*CB(1))*SINF(P2)
1 +(VIZ*CB(2)-VIX*CB(3))*COSF(P2))
DD(9)=DD(9)-V*(VIY*CA(1)-VIX*CA(2))
DD(10)=DD(10)+V*(VIY*CB(1)-VIX*CB(2))
DD(11)=DD(11)-V*((VIZ*CA(2)-VIY*CA(3))*A(1,3)+(VIX*CA(3)-VIZ*CA(1)
1)*A(2,3)+(VIY*CA(1)-VIX*CA(2))*A(3,3))
DD(12)=DD(12)+V*((VIZ*CB(2)-VIY*CB(3))*B(1,3)+(VIX*CB(3)-VIZ*CB(1)
1)*B(2,3)+(VIY*CB(1)-VIX*CB(2))*B(3,3))
100 CONTINUE
6 DD(2)=-DD(1)
DD(4)=-DD(3)

```

```
      SUBROUTINE DRAG(D,DF,DD,XC1,YC1,ZC1,XC2,YC2,ZC2,ICD,LDER,ICD1)

      DD(6)=-DD(5)
      5 IF(LDRAG-ICD) 9,8,9
C     FORCE CEASES TO ACT - - - SET INDEX = ZERO
      8 ICD=0
      9 IF(LDRAG-ICD1)11,11,10
     10 LDER=-1
         ICD1=LDRAG
     11 RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0)
```

```

SUBROUTINE COLGAS(ICG,GJ,FG1,FG2,FG3,VGX,VGY,VGZ,UGX,UGY,UGZ,
SUBROUTINE COLGAS(ICG,GJ,FG1,FG2,FG3,VGX,VGY,VGZ,UGX,UGY,UGZ,
1 XG1,YG1,ZG1,XG2,YG2,ZG2,GST,TCG,LDER,ICG1)
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
DIMENSION YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
EQUIVALENCE (T,T),(ZERO,ZERO),
1 (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(F1,V1(6)),(F2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(F1F,V1(15)),(F2F,V1(
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
COMMON A,B,AD,BD
DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
DIMENSION GJ(12),FG1(8),FG2(8),VGX(8),VGY(8),VGZ(8),UGX(8),UGY(8),
1 UGZ(8),XG1(8),YG1(8),ZG1(8),XG2(8),YG2(8),ZG2(8),CA(3),CB(3)
1,TCG(8)
COMMON WIL1,TCO
DIMENSION TCO(8)
LGAS=0
DO 1 K=1,12
GJ(K)=0.D
1 CONTINUE
DO 100 I=1,ICG
IF (T-TCO(I)) 100,15,15
15 CONTINUE
IF (T-TCG(I)) 5,5,9
9 LGAS=LGAS+1
GO TO 100
5 XJ=FG1(I)/(1.+FG2(I)*(T-TCO(I)))*FG3
IF (ABS(XJ)-GST) 9,6,6
6 DO 2 J=1,3
CA(J)=A(1,J)*VGX(I)+A(2,J)*VGY(I)+A(3,J)*VGZ(I)
2 CONTINUE
3 DO 8 J=1,3
4 CB(J)=B(1,J)*UGX(I)+B(2,J)*UGY(I)+B(3,J)*UGZ(I)
8 CONTINUE
GJ(2)=GJ(2)+XJ*CB(1)
GJ(4)=GJ(4)+XJ*CB(2)
GJ(6)=GJ(6)+XJ*CB(3)
7 GJ(1)=GJ(1)+XJ*CA(1)
GJ(3)=GJ(3)+XJ*CA(2)
GJ(5)=GJ(5)+XJ*CA(3)
CA(1)=VGZ(I)*YG1(I)-VGY(I)*ZG1(I)
CA(2)=VGZ(I)*XG1(I)-VGX(I)*ZG1(I)
CA(3)=VGY(I)*XG1(I)-VGX(I)*YG1(I)
CB(1)=UGZ(I)*YG2(I)-UGY(I)*ZG2(I)
CB(2)=UGZ(I)*XG2(I)-UGX(I)*ZG2(I)
CB(3)=UGY(I)*XG2(I)-UGX(I)*YG2(I)
GJ(7)=GJ(7)+XJ*(CA(1)*COSF(S1)+CA(2)*SINF(S1))
GJ(8)=GJ(8)+XJ*(CB(1)*COSF(S2)+CB(2)*SINF(S2))
GJ(9)=GJ(9)+XJ*(CA(1)*A(1,3)-CA(2)*A(2,3)+CA(3)*A(3,3))
GJ(10)=GJ(10)+XJ*(CB(1)*B(1,3)-CB(2)*B(2,3)+CB(3)*B(3,3))
GJ(11)=GJ(11)+XJ*CA(3)
GJ(12)=GJ(12)+XJ*CB(3)
100 CONTINUE
IF (LGAS-ICG) 11,10,11
C FORCE CEASES TO ACT - - - SET INDEX = ZERO

```



```
      SUBROUTINE COLGAS(ICG,GJ,FG1,FG2,FG3,VGX,VGY,VGZ,UGX,UGY,UGZ,  
10 ICG=0  
11 IF(LGAS-ICG1)13,13,12  
12 LDER=-1  
      ICG1=LGAS  
13 RETURN  
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0)
```

```

SUBROUTINE ROCKET(IRK,R,RR,TE,TF,RUM,FUM,FM1,FM2,URX,URY,URZ,
SUBROUTINE ROCKET(IRK,R,RR,TE,TF,RUM,FUM,FM1,FM2,URX,URY,URZ,
1FM1D,FM2D,XR1,YR1,ZR1,XR2,YR2,ZR2,VRX,VRY,VRZ,SUN1,SUN2,SUN3,
2SAN1,SAN2,SAN3,XX1D,YY1D,ZZ1D,XX2D,YY2D,ZZ2D,XX1,YY1,ZZ1,XX2,YY2,
3ZZ2)
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
DIMENSION YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
EQUIVALENCE (T,T),(ZERO,ZERO),
1 (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(F1,V1(6)),(F2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
11)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
COMMON A,B,AD,BD
DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
DIMENSION R(12),RR(16),URX(16),URY(16),URZ(16),VRX(16),VRY(16),
1VRZ(16),XR1(16),YR1(16),ZR1(16),XR2(16),YR2(16),ZR2(16)
1,TE(16),TF(16),CA(3)
LROC=0
DO 1 I=1,12
R(I)=0.D
1 CONTINUE
DO 30 I=1,IRK
IF (T-TE(I)) 2,3,3
3 IF (T-TF(I)) 8,8,5
5 S=TF(I)-TE(I)
FM1=FM1D-RUM*S
FM2=FM2D-FUM*S
XX1=XX1D-SUN1*S
YY1=YY1D-SUN2*S
ZZ1=ZZ1D-SUN3*S
XX2=XX2D-SAN1*S
YY2=YY2D-SAN2*S
ZZ2=ZZ2D-SAN3*S
LROC=LROC+1
IF (LROC-IRK) 2,40,40
C FORCE CEASES TO ACT - - - SET INDEX = ZERO
40 IRK=0
2 GO TO 30
8 S=T-TE(I)
FM1=FM1D-RUM*S
FM2=FM2D-FUM*S
XX1=XX1D-SUN1*S
YY1=YY1D-SUN2*S
ZZ1=ZZ1D-SUN3*S
XX2=XX2D-SAN1*S
YY2=YY2D-SAN2*S
ZZ2=ZZ2D-SAN3*S
DO 21 K=1,3
CA(K)=A(1,K)*URX(I)+A(2,K)*URY(I)+A(3,K)*URZ(I)
21 CONTINUE
R(1)=R(1)+RR(I)*CA(1)
R(3)=R(3)+RR(I)*CA(2)
R(5)=R(5)+RR(I)*CA(3)
CA(1)=URZ(I)*YR1(I)-URY(I)*ZR1(I)
CA(2)=URZ(I)*XR1(I)-URX(I)*ZR1(I)
CA(3)=URY(I)*XR1(I)-URX(I)*YR1(I)

```

```
SUBROUTINE ROCKET(IRK,R,RR,TE,TF,RUM,FUM,FM1,FM2,URX,URY,URZ,
R(7)=R(7)+RR(I)*(CA(1)*COSF(S1)+CA(2)*SINF(S1))
R(9)=R(9)+RR(I)*(CA(1)*A(1,3)-CA(2)*A(2,3)+CA(3)*A(3,3))
R(11)=R(11)+RR(I)*CA(3)
52 DO 51 K=1,3
CA(K)=B(1,K)*VRX(I)+B(2,K)*VRY(I)+B(3,K)*VRZ(I)
51 CONTINUE
R(2)=R(2)+RR(I)*CA(1)
R(4)=R(4)+RR(I)*CA(2)
R(6)=R(6)+RR(I)*CA(3)
53 CA(1)=VRZ(I)*YR2(I)-VRY(I)*ZR2(I)
CA(2)=VRZ(I)*XR2(I)-VRX(I)*ZR2(I)
CA(3)=VRY(I)*XR2(I)-VRX(I)*YR2(I)
R(8)=R(8)+RR(I)*(CA(1)*COSF(S2)+CA(2)*SINF(S2))
R(10)=R(10)+RR(I)*(CA(1)*B(1,3)-CA(2)*B(2,3)+CA(3)*B(3,3))
R(12)=R(12)+RR(I)*CA(3)
30 CONTINUE
32 RETURN
END(1,0,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```

SUBROUTINE UNIVSL (ISU,WG,UK,WIOQ,W,XU1,YU1,ZU1,XU2,YU2,ZU2,LDER,
SUBROUTINE UNIVSL (ISU,WG,UK,WIOQ,W,XU1,YU1,ZU1,XU2,YU2,ZU2,LDER,
1 ISU1)
DIMENSION WIOQ (1)
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
DIMENSION YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
EQUIVALENCE (T,T),(ZERO,ZERO),
1 (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
11)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1F,V1(15)),(P2F,V1(
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
COMMON A,B,AD,BD
DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
DIMENSION WG(8),W(12),XU1(8),YU1(8),ZU1(8),XU2(8),YU2(8),ZU2(8)
1,UK(8),CA(3),CB(3)
20 LUN=0
DO 1 I=1,12
W(I)=D.0
1 CONTINUE
DO 100 I=1,ISU
DO 2 K=1,3
CA(K)=A(1,K)*XU1(I)+A(2,K)*YU1(I)+A(3,K)*ZU1(I)
CB(K)=B(1,K)*XU2(I)+B(2,K)*YU2(I)+B(3,K)*ZU2(I)
2 CONTINUE
DIX=X1+CA(1)-X2-CB(1)
DIY=Y1+CA(2)-Y2-CB(2)
DIZ=Z1+CA(3)-Z2-CB(3)
DD=SQRTF(DIX**2+DIY**2+DIZ**2)
PUT=UK(I)*DD
IF (ABSF (DUT) -ABSF (WG(I)+WIOQ(I))) 3,3,9
9 LUN=LUN+1
GO TO 100
3 IF(DD) 6,7,6
7 DIZ=1.0
XW=WG(I)
GO TO 8
6 XW=WG(I)/DD-UK(I)
8 W(1)=W(1)+XW*DIX
W(3)=W(3)+XW*DIY
W(5)=W(5)+XW*DIZ
W(9)=W(9)+XW*(DIY*CA(1)-DIX*CA(2))
W(7)=W(7)+XW*((DIX*CA(3)-DIZ*CA(1))*SINF(P1)+(DIZ*CA(2)-DIY*CA(3))
1 *COSF(P1))
W(11)=W(11)+XW*((DIZ*CA(2)-DIY*CA(3))*A(1,3)+(DIX*CA(3)-DIZ*CA(1))
1 *A(2,3)+(DIY*CA(1)-DIX*CA(2))*A(3,3))
W(8)=W(8)-XW*((DIX*CB(3)-DIZ*CB(1))*SINF(P2)+(DIZ*CB(2)-DIY*CB(3))
1 *COSF(P2))
W(10)=W(10)-XW*(DIY*CB(1)-DIX*CB(2))
W(12)=W(12)-XW*((DIZ*CB(2)-DIY*CB(3))*B(1,3)+(DIX*CB(3)-DIZ*CB(1))
1 *B(2,3)+(DIY*CB(1)-DIX*CB(2))*B(3,3))
100 CONTINUE
4 W(2)=-W(1)
W(4)=-W(3)
W(6)=-W(5)
5 IF(LUN-ISU) 11,10,11
C FORCE CEASES TO ACT - - - SET INDEX = ZERO

```

```
SUBROUTINE UNIVSL (ISU, WG, UK, WIOQ, W, XU1, YU1, ZU1, XU2, YU2, ZU2, LDER,  
10 ISU=0  
11 IF (LUN-ISU1) 13, 13, 12  
12 LDER=-1  
   ISU1=LUN  
13 RE TURN  
   END (1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0)
```

```
SUBROUTINE MCROS (A,B,C)
SUBROUTINE MCROS (A,B,C)
DIMENSION A(1),B(1),C(1)
C(1)=A(2)*B(3)-A(3)*B(2)
C(2)=-A(1)*B(3)+A(3)*B(1)
C(3)=A(1)*B(2)-A(2)*B(1)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE MSR(A,B,C,J)
SUBROUTINE MSR(A,B,C,J)
DIMENSION A(3,3),B(3)
C=0.
DO 1 I=1,3
1 C=C+B(I)*A(J,I)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0)
```

```
SUBROUTINE MSC (A,B,C,J)
SUBROUTINE MSC (A,B,C,J)
DIMENSION A(3,3),B(3)
C=D.
DO 1 I=1,3
1 C=C+B(I)*A(I,J)
RETURN
END(1,D,D,D,D,D,1,D,D,1,D,D,D,D)
```



```
SUBROUTINE MSCAL(A,B,C)
SUBROUTINE MSCAL(A,B,C)
DIMENSION B(3),C(3)
DO 1 I=1,3
1 C(I)=A*B(I)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE MADD(A,B,C,M1,M2)
SUBROUTINE MADD(A,B,C,M1,M2)
DIMENSION A(3,3),B(3,3),C(3,3)
DO 1 I=1,M1
DO 1 J=1,M2
1 C(I,J)=A(I,J)+B(I,J)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE MSUB(A,B,C,M1,M2)
SUBROUTINE MSUB(A,B,C,M1,M2)
DIMENSION A(3,3),B(3,3),C(3,3)
DO 1 I=1,M1
DO 1 J=1,M2
1 C(I,J)=A(I,J)-B(I,J)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0)
```

```
SUBROUTINE MULT3(A,B,C,M1,M2,M3)

SUBROUTINE MULT3(A,B,C,M1,M2,M3)
DIMENSION A(3,3),B(3,3),C(3,3)
DO 2 I=1,M1
DO 2 K=1,M3
C(I,K)=0.
DO 2 J=1,M2
2 C(I,K)=C(I,K)+A(I,J)*B(J,K)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE MULT2(A,B,C,M1,M2,M3)
SUBROUTINE MULT2(A,B,C,M1,M2,M3)
DIMENSION A(3,3),B(3,3),C(3,3)
DO 2 I=1,M1
DO 2 K=1,M3
C(I,K)=0.
DO 2 J=1,M2
2 C(I,K)=A(I,J)*B(K,J)+C(I,K)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE MULT1(A,B,C,M1,M2,M3)

SUBROUTINE MULT1(A,B,C,M1,M2,M3)
DIMENSION A(3,3),B(3,3),C(3,3)
DO 2 I=1,M1
DO 2 K=1,M3
C(I,K)=0.
DO 2 J=1,M2
2 C(I,K)=A(J,I)*B(J,K)+C(I,K)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE SWITCH(A1,A2,A3,A4,A5,A6,I,J,T)
SUBROUTINE SWITCH(A1,A2,A3,A4,A5,A6,I,J,T)
DIMENSION A1(3,1),A2(3,1),A3(3,1)
DO 1 K=1,3
CALL SW1(I,J,A1(K,1))
CALL SW1(I,J,A2(K,1))
1 CALL SW1(I,J,A3(K,1))
CALL SW(I,J,A4)
CALL SW(I,J,A5)
CALL SW(I,J,A6)
WRITE OUTPUT TAPE 6,100,I,J,T
100 FORMAT (7H SPRING,I3,19H INTERCHANGED WITH,I3,7H AT T=E11.4)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE SW(I,J,A)
SUBROUTINE SW(I,J,A)
DIMENSION A(1)
B=A(I)
A(I)=A(J)
A(J)=B
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```



```
SUBROUTINE SW1(I,J,A)
SUBROUTINE SW1(I,J,A)
  DIMENSION A(3,1)
  Q=A(1,J)
  A(1,J)=A(1,I)
  A(1,I)=Q
  RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```

SUBROUTINE ERR (DEL,TPR,TP,TC,TA,TB,TC,TD,TRIG1,TRIG2,TRIG3,
SUBROUTINE ERR (DEL,TPR,TP,TC,TA,TB,TC,TD,TRIG1,TRIG2,TRIG3,
1 TRIG4,TROC,PYRO2,PYRO3,PLOT )
EQUIVALENCE (T,T),(ZERO,ZERO),
1 (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(F1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
11)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
COMMON T,ZERO,V1,V2,YDOT1,YDOT2
DIMENSION YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
COMMON A,B,AD,BD
DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
WRITE OUTPUT TAPE 6,1
1 FORMAT(1H1,21HERROR OCCURS IN FMARK )
WRITE OUTPUT TAPE 6,2,T
2 FORMAT(1H0,7HTIME = E15.7/)
WRITE OUTPUT TAPE 6,7,DEL,TPR,TP
7 FORMAT (1H0,9HSTEP SIZE,E16.5 /
111H PRINT STEP,E16.5/
118H PYRO FIRING TIMES,E16.5)
WRITE OUTPUT TAPE 6,8,TC,TA,TB,TC,TD
8 FORMAT (1H ,15HFIN PULLER TIME,E16.5/
119H ROCKET TIMES--- TA,E16.5,2HTB,E16.5,2HTC,E16.5,2HTD,E16.5)
IF (T-TRIG1)10,9,9
10 WRITE OUTPUT TAPE 6,11,TRIG1
11 FORMAT (1H ,13HPRINT TRIGGERE16.5)
9 IF (T-TRIG2)12,13,13
12 WRITE OUTPUT TAPE 6,14,TRIG2
14 FORMAT (1H ,18HFIN PULLER TRIGGER E16.5)
13 IF (T-TRIG3) 15,16,16
15 WRITE OUTPUT TAPE 6,17,TRIG3
17 FORMAT (1H ,19HEND OF STEP TRIGGER E16.5)
16 IF (T-TRIG4) 18,19,19
18 WRITE OUTPUT TAPE 6,20,TRIG4
20 FORMAT (1H ,16H1ST PYRO TRIGGER E16.5)
19 IF (T-PYRO2) 21,22,22
21 WRITE OUTPUT TAPE 6,23,PYRO2
23 FORMAT (1H ,16H2ND PYRO TRIGGER E16.5)
22 IF (T-PYRO3) 24,25,25
24 WRITE OUTPUT TAPE 6,26,PYRO3
26 FORMAT (1H ,16H3RD PYRO TRIGGER E16.5)
25 IF (T-TROC) 27,28,28
27 WRITE OUTPUT TAPE 6,29,TROC
29 FORMAT (1H ,14HROCKET TRIGGER E16.5)
28 IF (T-PLOT) 30,31,31
30 WRITE OUTPUT TAPE 6,32,PLOT
32 FORMAT (1H ,12HPLOT TRIGGER E16.5)
31 CONTINUE
DO 3 I=1,18
WRITE OUTPUT TAPE 6,4,I,V1(I),I,YDOT1(I)
3 CONTINUE
4 FORMAT(1H ,3HV1( 12,1H) E15.7,5X,6HYDOT1( 12,1H)E15.7)
DO 5 I=1,6
WRITE OUTPUT TAPE 6,6,I,V2(I),I,YDOT2(I)
5 CONTINUE
6 FORMAT(1H ,3HV2( 12,1H) E15.7,5X,6HYDOT2( 12,1H)E15.7)

```

SUBROUTINE ERR (DEL, TFR, TF, TG, TA, TB, TC, TD, TRIG1, TRIG2, TRIG3,

CALL DUMP

RETURN

END (1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)

```

SOLVE   LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT

      SUBROUTINE SOLVE(NN,AA,BB,IN,EPS,ITMAX,X,ITT)
C      SOLVES AX=B WHERE A IS NXN MATRIX AND B IS NX1 VECTOR
C      IN=
C          1 FOR FIRST ENTRY
C          2 FOR SUBSEQUENT ENTRIES WITH NEW B
C          3 TO RESTORE AA AND BB
C      EPS AND ITMAX ARE PARAMETERS IN THE ITERATION
C      ITT=
C          -1 IF AA IS SINGULAR
C           0 IF NOT CONVERGENT
C          NUMBER OF ITERATIONS IF CONVERGENT
C      CALLS FAP SUBROUTINES EQUIL,DOT AND DAD
C
      DIMENSION AA(15,15),A(15,15),KA(15,15),B(15),BB(15),X(15),Z(15),
1CM(15),RM(15),JCP(15),IRP(15)
      EQUIVALENCE (A,KA)
C
C      MA MUST = DECLARED DIMENSION OF SYSTEM
C
      MA=15
C
C
C      MA1=MA+1
      GO TO (1,2,3),IN
1  N=NN
      IP=1
C
C      EQUILIBRATION
C
      CALL EQUIL1(AA,KA,N,MA)
      DO 510 I=1,N
          KT=KA(I,1)
          DO 503 J=2,N
              IF(KT-KA(I,J)) 502,503,503
502          KT=KA(I,J)
503          CONTINUE
              RM(I)=2.0**KT
              DO 509 J=1,N
509          KA(I,J)=KA(I,J)-KT
510          CONTINUE
              DO 520 J=1,N
                  KT=KA(1,J)
                  DO 513 I=2,N
                      IF(KT-KA(I,J))512,513,513
512          KT=KA(I,J)
513          CONTINUE
                      CM(J)=2.0**KT
                      DO 519 I=1,N
519          KA(I,J)=KA(I,J)-KT
520          CONTINUE
                  CALL EQUIL2
                  DO 530 I=1,N
530          BB(I)=BB(I)/RM(I)
C

```

```

SOLVE   LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT

C       SAVE EQUILIBRATED DATA
C
DO 548 I=1,N
      DO 548 J=1,N
548     A(I,J)=AA(I,J)
      IF=2

C
C       GAUSSIAN ELIMINATION WITH COMPLETE PIVOTING
C
      NM1=N-1
      DO 99 M=1,NM1
        TOP=ABSF(A(M,M))
        IMAX=M
        JMAX=M
        DO 12 I=M,N
          DO 12 J=M,N
            IF(TOP-ABSF(A(I,J)))10,12,12
10          TOP=ABSF(A(I,J))
            IMAX=I
            JMAX=J
12        CONTINUE
        IF(TOP)14,13,14
13        IT=-1
        IP=10
        ITT=IT
        RETURN
14        IRP(M)=IMAX
        JCP(M)=JMAX
        IF(JMAX-M)29,23,21
21        DO 22 I=1,N
          TEMP=A(I,M)
          A(I,M)=A(I,JMAX)
          A(I,JMAX)=TEMP
22        IF(IMAX-M)29,29,24
23        DO 25 J=1,N
          TEMP=A(M,J)
          A(M,J)=A(IMAX,J)
          A(IMAX,J)=TEMP
25        MF1=M+1
29        DO 33 I=MF1,N
          EM=A(I,M)/A(M,M)
          A(I,M)=EM
          DO 32 J=MF1,N
32          A(I,J)=A(I,J)-A(M,J)*EM
33        CONTINUE
99      CONTINUE
C       STORAGE FOR A NOW CONTAINS TRIANGULAR L AND U SO THAT (L+I)*U=A
      IF(A(N,N))101,93,101
93      IT=-1
      IP=10
      ITT=IT
      RETURN
101     IRP(N)=N
        JCP(N)=N
C
C       DUPLICATE INTERCHANGES IN DATA

```

```

SOLVE   LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT
C
      DO 219 J=1,N
        IF(J-JF) 211,219,211
211      DO 212 I=1,N
          TEMP=AA(I,J)
          AA(I,J)=AA(I,JP)
212      AA(I,JP)=TEMP
219 CONTINUE
      DO 229 I=1,N
        IF=IRP(I)
        IF(I-IF) 221,229,221
221      DO 222 J=1,N
          TEMP=AA(I,J)
          AA(I,J)=AA(IP,J)
222      AA(IP,J)=TEMP
          TEMP=BB(I)
          BB(I)=BB(IP)
          BB(IP)=TEMP
229 CONTINUE
      IP=3
      GO TO 199
C
C   PROCESS NEW RIGHT HAND SIDE
C
      2 N=N
      IP=4
      DO 601 I=1,N
601      BB(I)=BB(I)/RM(I)
      DO 609 M=1,NM1
        IF=IRP(M)
          TEMP=BB(M)
          BB(M)=BB(IP)
          BB(IP)=TEMP
609 CONTINUE
      GO TO 199
C
C   SOLVE FOR FIRST APPROXIMATION TO X
C
199 DO 200 I=1,N
      IM1=I-1
      BEE=BB(I)
200      B(I)=-DOT(IM1,A,I,MA,B,1,1,BEE,1.0)
      DO 201 K=1,N
        L=K-1
        I=N-L
        IA=I*MA1
        IX=I+1
        BE=B(I)
        D=A(I,I)
201      X(I)=-DOT(L,A,IA,MA,X,IX,1,BE,D)
      IP=5
C
C   ITERATIVE IMPROVEMENT
C
      IF(ITMAX) 370,370,300

```

```

SOLVE   LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT

300 TOP=0.0
  DO 303 I=1,N
    IF (TOP-ABSF(X(I))) 302,303,303
302   TOP=ABSF(X(I))
303 CONTINUE
  EPSX=EPS*TOP
  DO 369 IT=1,ITMAX
C   FIND RESIDUALS
    DO 319 I=1,N
      BEE=BB(I)
319   Z(I)=-DOT(N,AA,I,MA,X,1,1,BEE,1.0)
      IF=11
C   FIND INCREMENT
    DO 329 I=1,N
      IM1=I-1
      ZEE=Z(I)
329   B(I)=-DOT(IM1,A,I,MA,B,1,1,ZEE,1.0)
    DO 339 K=1,N
      L=K-1
      I=N-L
      IA=I*MA1
      IZ=I+1
      BE=B(I)
      D=A(I,I)
339   Z(I)=-DOT(L,A,IA,MA,Z,IZ,1,BE,D)
      IF=6
C   INCREMENT AND TEST
    TOP=0.0
    DO 342 I=1,N
      TEMP=X(I)
      ZEE=Z(I)
      X(I)=DAD(TEMP,ZEE)
      DELX=ABSF(X(I)-TEMP)
      IF (TOP-DELX) 341,342,342
341   TOP=DELX
342   CONTINUE
      IF=7
      IF (TOP-EPSX) 381,381,369
369 CONTINUE
370 IT=0
381 DO 383 K=1,N
      I=N-K+1
      IP=JCF(I)
      TEMP=X(I)
      X(I)=X(IP)
383   X(IP)=TEMP
    DO 385 I=1,N
385   X(I)=X(I)/CM(I)
      IF=9
      ITT=IT
      RETURN
C
C   RESTORE AA AND BB
C
3   NF1=N+1
  DO 709 K=1,N

```

SOLVE LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT

```
      I=NP1-K
      IP=IRP(I)
      IF(1-IP)701,709,701
701   DO 702 J=1,N
        TEMP=AA(I,J)
        AA(I,J)=AA(IP,J)
702   AA(IF,J)=TEMP
        TEMP=BB(I)
        BB(I)=BB(IP)
        BB(IF)=TEMP
709  CONTINUE
      DO 719 K=1,N
        J=NP1-K
        JF=JCP(J)
        IF(JF-J)711,719,711
711   DO 712 I=1,N
        TEMP=AA(I,J)
        AA(I,J)=AA(I,JP)
712   AA(I,JP)=TEMP
719  CONTINUE
      DO 721 I=1,N
        R=RM(I)
        BB(I)=BB(I)*R
        DO 721 J=1,N
721   AA(I,J)=AA(I,J)*R
        DO 722 J=1,N
        C=CM(J)
        DO 722 I=1,N
722   AA(I,J)=AA(I,J)*C
      IP=12
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```


SEPARATION OF TWO BODIES IN SPACE

```

C
COMMON Y,TFLOT
DIMENSION Y(38),W(15 000),LOT(14),A(36),X(4),S(4),BUF(100)
DO 5040 I=1,14
5040 LOT(I)=0
READ INPUT TAPE 5,1,(LOT(I),I=1,14)
1 FORMAT (14I5)
NT3=38
NT2=8
NT1=18
2 FORMAT(E12.8,112)
NO=0
PI2=360.0
S(1)=119.0
S(2)=903.0
S(3)=1023.0
S(4)=0.0
X(1)=0.0
NPTS=Y(37)
MPTS=NPTS+NPTS
ZERO=0.0
REWIND NT2
L=0
3 L=L+1
IF (L-15) 100,101,101
100 IF (LOT(L)) 5,3,5
5 CALL ADV(18)
IF (LOT(L-12)) 7,7,8
7 I=3*L-3
READ TAPE NT2,(A(K),K=1,36)
IREAD=1
DO 6 K=1,36
Y(K)=ABSF(A(K))
6 CONTINUE
W(1)=A(I+1)
W(2)=A(I+2)
W(3)=A(I+3)
J=0
DO 9 II=2,NPTS
READ TAPE NT2,(A(K),K=1,36)
J=J+3
DO 9 JJ=1,3
K=I+JJ
IF (ABSF(A(K))-Y(K)) 106,106,107
107 Y(K)=ABSF(A(K))
106 KK=J+JJ
W(KK)=A(K)
9 CONTINUE
REWIND NT2
10 NO=NO+1
I=3*L-3+IREAD
FINAL=Y(I)
IF (FINAL-1.0) 606,605,605
606 WRITE OUTPUT TAPE NT3,750,FINAL
750 FORMAT(1H1,F13.5)

```

SEPARATION OF TWO BODIES IN SPACE

```
GO TO 751
605 WRITE OUTPUT TAPE NT3,500,FINAL
500 FORMAT(1H1,F13.2)
751 CALL GRID (NT1,20,20,119,903)
    IF(L-6) 26,26,11
    26 WRITE OUTPUT TAPE 6,660
660 FORMAT(1HG)
    CALL SET(444)
    WRITE OUTPUT TAPE NT3,501
501 FORMAT (1H ,6HINCHES,4X,3HG.0)
    IF (L-2) 90,19,20
    90 JJ=1
    GO TO 91
    19 JJ=2
    GO TO 91
    20 WRITE OUTPUT TAPE NT3,508
508 FORMAT (1H ,5H/ SEC)
    IF (L-4) 90,19,93
    93 WRITE OUTPUT TAPE NT3,508
    IF(L-6) 90,19,11
    91 FINAL=-FINAL
    CALL SET(895)
    IF(FINAL+1.0) 780,780,781
781 WRITE OUTPUT TAPE NT3,754,FINAL
    GO TO 782
780 WRITE OUTPUT TAPE NT3,502,FINAL
502 FORMAT(1H ,F13.2)
782 CALL SET(911)
    WRITE OUTPUT TAPE NT3,503,Y(38)
503 FORMAT (1H ,14X,3HG.0,4RX,7HSECONDS,46X,F7.2)
    IF (L-2) 22,22,23
    23 IF (L-4) 24,24,25
    24 WRITE OUTPUT TAPE NT3,509,NO
509 FORMAT (1HD,16HVELOCITY VS TIME,100X,7HPLOT NO,12)
    GO TO 12
    25 WRITE OUTPUT TAPE NT3,510,NO
510 FORMAT (1HD,20HACCELERATION VS TIME,96X,7HPLOT NO,12)
    GO TO 12
    22 WRITE OUTPUT TAPE NT3,504,NO
504 FORMAT (1HD,16HDISTANCE VS TIME,100X,7HPLOT NO,12)
    12 IF (IREAD-2) 14,15,16
    14 WRITE OUTPUT TAPE NT3,505,JJ
505 FORMAT (1HG,1HX,12)
    GO TO 17
    15 WRITE OUTPUT TAPE NT3,506,JJ
506 FORMAT (1HD,1HY,12)
    GO TO 17
    16 WRITE OUTPUT TAPE NT3,507,JJ
507 FORMAT (1HG,1HZ,12)
    17 I=3*L-3+IREAD
        X(2)=-Y(I)
        X(3)=Y(38)
        X(4)=Y(I)
        IF(Y(I)) 600,601,600
601 X(2)=-0.01
    X(4)=0.01
```

SEPARATION OF TWO BODIES IN SPACE

```

600 T=0.0
    CALL SDINIT (BUF,100,X,S,NT1)
    II=IREAD-3
    DO 94 J=1,NPTS
        II=II+3
        CALL SDNFUT (T,W(II),BUF,0)
        T=T+TFLOT
94 CONTINUE
    CALL STERM (NT1,BUF)
    IREAD=IREAD+1
    IF (IREAD-3) 10,10,3
55 J=1
150 NO=NO+1
    IF (FINAL-1.0) 757,758,758
757 WRITE OUTPUT TAPE NT3,750,FINAL
    GO TO 759
758 WRITE OUTPUT TAPE NT3,500,FINAL
759 CALL GRID (NT1,20,20,119,903)
    11 WRITE OUTPUT TAPE 6,660
        CALL SET(444)
        WRITE OUTPUT TAPE NT3,511
511 FORMAT (1H ,7HDEGREES,3X,3HD.0)
    IF (L-9) 27,28,28
    28 WRITE OUTPUT TAPE NT3,508
        IF (L-11) 27,29,30
    30 IF (L-12) 27,29,27
    29 WRITE OUTPUT TAPE NT3,508
    27 CALL SET(895)
        FINAL=-FINAL
        IF (FINAL+1.0) 752,752,753
753 WRITE OUTPUT TAPE NT3,754,FINAL
754 FORMAT(1H ,F13.5)
    GO TO 755
752 WRITE OUTPUT TAPE NT3,502,FINAL
755 CALL SET(911)
    IF (L-12) 31,31,32
    31 WRITE OUTPUT TAPE NT3,503,Y(38)
    GO TO 33
    32 FINAL=-FINAL
609 WRITE OUTPUT TAPE NT3,512,FINAL
512 FORMAT (1H ,14X,3HD.0,47X,7HDEGREES,48X,F6.1)
33 IF (L-8) 36,34,35
36 JJ=1
    GO TO 39
34 JJ=2
39 WRITE OUTPUT TAPE NT3,513,NO
513 FORMAT (1HD,13HANGLE VS TIME,103X,7HPLOT NO,12)
    GO TO 40
35 IF (L-10) 41,37,38
41 JJ=1
    GO TO 700
37 JJ=2
700 WRITE OUTPUT TAPE NT3,509,NO
    GO TO 40
38 IF(L-12) 701,44,45
701 JJ=1

```

SEPARATION OF TWO BODIES IN SPACE

```
GO TO 702
44 JJ=2
702 WRITE OUTPUT TAPE NT3,510,NO
40 IF (IREAD-2) 46,47,48
46 WRITE OUTPUT TAPE NT3,514,JJ
514 FORMAT (1HD,5HTHETA,I2)
GO TO 49
47 WRITE OUTPUT TAPE NT3,515,JJ
515 FORMAT (1HD,3HPHI,I2)
GO TO 49
48 WRITE OUTPUT TAPE NT3,516,JJ
516 FORMAT (1HD,3HPSI,I2)
GO TO 49
45 WRITE OUTPUT TAPE NT3,517,NO
517 FORMAT (1HD,17HVELOCITY VS ANGLE,99 X,7HPLOT NO,I2)
IF(L-13) 770,771,770
770 JJ=2
GO TO 40
771 JJ=1
GO TO 40
49 IF(L-12) 17,17,53
53 X(3)=PI2
CALL SDINIT (BUF,100,X,S,NT1)
I=J
IF( J-MPTS) 65,151,152
65 CALL SDNPUT (W(J),W(J+1),BUF,D)
I=J+2
151 CALL SDNPUT (W(I),W(I+1),BUF,D)
66 IF (W(I)-W(J)) 69,68,67
68 J=I
I=I+2
IF(I-MPTS) 65,152,152
69 K=I
IGO=1
70 IF (W(K+2)-W(K)) 71,71,72
72 J=K+2
CALL STERM (NT1,BUF)
GO TO 150
71 K=K+2
CALL SDNPUT (W(K),W(K+1),BUF,D)
IF(K-MPTS) 73,73,74
73 GO TO (70,75),IGO
152 CALL STERM(NT1,BUF)
74 IREAD=IREAD+1
IF (IREAD-3) 78,78,3
67 K=I
IGO=2
75 IF (W(K+2)-W(K)) 72,71,71
8 IREAD=1
78 Y(19)=180.D
Y(22)=180.D
DO 105 I=1,2
Y(I+19)=360.D
Y(I+22)=360.D
105 CONTINUE
I=0
```

SEPARATION OF TWO BODIES IN SPACE

```
READ TAPE NT2, (A(K), K=1, 36)
IF (L-13) 102, 103, 102
102 I=3
103 K=18+I*READ+1
X(2)=-Y(K)
X(4)=Y(K)
IF (Y(K)) 602, 603, 602
603 X(2)=-0.01
X(4)=0.01
602 FJNAL=Y(K)
W(1)=A(K)
W(2)=A(K+6)
Y(K+6)=ABS(F(A(K+6)))
J=1
DO 104 II=2, NPTS
J=J+2
READ TAPE NT2, (A(K), K=1, 36)
W(J)=A(K)
W(J+1)=A(K+6)
IF (ABS(F(W(J+1)))-Y(K+6)) 104, 104, 110
110 Y(K+6)=ABS(F(W(J+1)))
104 CONTINUE
REWIND NT2
GO TO 55
101 CALL CHAIN(1, 3)
END(1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0)
```

Sample Problem

CASE 23904

INPUT DATA

DELTA TIME = 1.000E-02 SEC DELTA PRINT= 0.100E-00 SEC TIME (INITIAL) 0. SEC SPIN-UP RATE 0. DEG/SEC

INPUT FOR BODY1 MASS = 0.1000000E 04 LBM

| INERTIA MATRIX LBM IN SQ | | | | RATES ABOUT DRAWING BOARD AXIS DEG/SEC | | COORDS IN DRAWING BOARD CS OF CM-INCHES | |
|--------------------------|---------------|-----|---------------|--|----|---|---------------|
| IXX | 0.9272000E 06 | IYY | 0.9504000E 06 | X | 0. | X | 0. |
| IXY | 0. | IYZ | 0. | Y | 0. | Y | 0. |
| IXZ | 0. | IZZ | 0.4636000E 06 | Z | 0. | Z | 0.1500000E 02 |

INPUT FOR BODY2 MASS = 0.1800000E 04 LBM

| INERTIA MATRIX LBM IN SQ | | | | RATES ABOUT DRAWING BOARD AXIS DEG/SEC | | COORDS IN DRAWING BOARD CS OF CM-INCHES | |
|--------------------------|---------------|-----|---------------|--|----|---|----------------|
| IXX | 0.1159000E 07 | IYY | 0.1159000E 07 | X | 0. | X | 0. |
| IXY | 0. | IYZ | 0. | Y | 0. | Y | 0. |
| IXZ | 0. | IZZ | 0.4636000E 06 | Z | 0. | Z | -0.1000000E 02 |

ICD = 0 IPR = 0 ICG = 3 IPN = 0 IRK = 1 ISP = 4 ISU = 0 ION = 0

BODY 1-FIXED AXES X Y Z
DRAWING BOARD AXES X Z -Y

BODY 2-FIXED AXES X Y Z
DRAWING BOARD AXES X Z -Y

COLD GAS JET INPUT

| | | | | | | |
|-----------------|----------------|---|---------------|-----------------|---------------------------|---------------|
| F3 | 0.1500000E 01 | VALUE JET FORCE BECOMES ZERO | D. | LBF | | |
| | | COSINE ANGLE THRUST AND DRAW BOARD-BODY 1 | | | COLD GAS JET LOCATIONS-IN | |
| | UX | UY | UZ | X | Y | Z |
| | -0.1000000E 01 | 0. | 0. | 0. | -0.6500000E 02 | 0.1500000E 02 |
| | 0.5000000E 00 | -0.8660000E 00 | 0. | 0.5640000E 02 | 0.3250000E 02 | 0.1500000E 02 |
| | 0.5000000E 00 | 0.8660000E 00 | 0. | -0.5640000E 02 | 0.3250000E 02 | 0.1500000E 02 |
| CONSTANTS JET I | | | | TIME JET I GOES | | |
| F1 LBF | F2 1/SEC | ON SEC | OFF SEC | | | |
| 0.5000000E 02 | 0.1000000E 01 | 0.2000000E 01 | 0.7000000E 01 | | | |
| 0.5000000E 02 | 0.1000000E 01 | 0.2000000E 01 | 0.7000000E 01 | | | |
| 0.5000000E 02 | 0.1000000E 01 | 0.2000000E 01 | 0.7000000E 01 | | | |

ROCKETS INPUT

| | | | | | |
|--|----------------|---|------------------|------------------|----|
| | | COSINE ANGLE THRUST AND DRAW BOARD | | | |
| | UX | BODY 1 UY | UZ | BODY 2 UY | UZ |
| | 0. | 0. | 0.1000000E 01 | 0. | 0. |
| | TIME FIRING | TIME FIRING | | | |
| | STARTS SEC | CEASES SEC | | | |
| | 0.1000000E 02 | 0.1500000E 02 | | | |
| | | DECREASES IN PRINCIPAL MOMENTS OF INERTIA LBM IN SQ/SEC | | | |
| | XX | BODY 1 YY | ZZ | BODY 2 YY | ZZ |
| | 0. | 0. | 0. | 0. | 0. |
| | MASS FLOW RATE | LBM/SEC | THRUST MAGNITUDE | ROCKET LOCATIONS | |
| | BODY 1 | BODY 2 | LBF | Y | Z |
| | 0. | 0. | 0.1000000E 03 | 0. | 0. |

SPRING INPUT

| | | | | | |
|---------------|------------------------------|----------------|---------------|---------------------|----------------|
| EFFICIENCY | RESTRICT INDEX | | | | |
| 0.1000000E 01 | 2 | | | | |
| | COSINE DRAW BOARD AND SPRING | | | SPRING LOCATIONS-IN | |
| | UX | UY | UZ | X | Y |
| | 0. | 0. | 0.1000000E 01 | -0.5660000E 02 | -0.5660000E 02 |
| | 0. | 0. | 0.1000000E 01 | -0.5660000E 02 | 0.5660000E 02 |
| | 0. | 0. | 0.1000000E 01 | 0.5660000E 02 | 0.5660000E 02 |
| | 0. | 0. | 0.1000000E 01 | 0.5660000E 02 | -0.5660000E 02 |
| INITIAL FORCE | SPRING CONSTANT | RESIDUAL FORCE | | | |
| LBF | LBF/IN | LBF | | | |
| 0.4000000E 03 | 0.1000000E 03 | -0. | | | |
| 0.4000000E 03 | 0.1000000E 03 | -0. | | | |
| 0.4000000E 03 | 0.1000000E 03 | -0. | | | |
| 0.3200000E 03 | 0.8000000E 02 | -0. | | | |

TIME = 0. SECONDS STEP SIZE= 1.000000E-02

BODY 1
 SPEED OF CM = 0. IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0. DEG/SEC MAGNITUDE VECTOR SUM OF PITCH-YAW RATES= 0.

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|---------------|---------------------|----|----------------------|--------------|---------------------|-----|------------------------------|-----|
| X | 0. | X | 0. | THETA | 0.900000E 02 | THETA | 0. | X | 0. |
| Y | 0. | Y | 0. | PHI | 0. | PHI | 0. | Y | -0. |
| Z | 0.1607143E 02 | Z | 0. | PSI | 0. | PSI | -0. | Z | 0. |

BODY 2
 SPEED OF CM = 0. IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0. DEG/SEC MAGNITUDE VECTOR SUM OF PITCH-YAW RATES= 0.

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|-----|----------------------|--------------|---------------------|-----|------------------------------|-----|
| X | 0. | X | -0. | THETA | 0.900000E 02 | THETA | 0. | X | 0. |
| Y | 0. | Y | 0. | PHI | 0. | PHI | 0. | Y | -0. |
| Z | -0.8928571E 01 | Z | 0. | PSI | 0. | PSI | -0. | Z | 0. |

SEPARATION DISTANCE = 0.250000E 02 IN SEPARATION VELOCITY = 0. IN/SEC

CONSTRAINT VALUES 0.1907349E-05 -0.1907349E-05 -0.

| A1 | | | A2 | | |
|--------------|---------------|--------------|--------------|---------------|--------------|
| 1.000000E 00 | 0. | 0. | 1.000000E 00 | 0. | 0. |
| -0. | 0. | 1.000000E 00 | -0. | 0. | 1.000000E 00 |
| 0. | -1.000000E 00 | 0. | 0. | -1.000000E 00 | 0. |

SPRING 2 INTERCHANGED WITH 3 AT T= 0.9000E-01

TIME = 0.100000E-00 SECONDS STEP SIZE= 0.1979060E-08

BODY 1
 SPEED OF CM = 0.3856862E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.1558812E 01 DEG/SEC MAGNITUDE VECTOR SUM OF PITCH-YAW RATES= 0.1558812E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | 0.6388861E-02 | X | -0.1005825E-00 | THETA | 0.9017740E 02 | THETA | -0.1139844E 01 | X | -0.1139847E 01 |
| Y | -0.6918635E-02 | Y | 0.1048707E-00 | PHI | 0.2023377E-03 | PHI | 0.2991826E-02 | Y | -0.1063316E 01 |
| Z | 0.1847882E 02 | Z | 0.3856835E 02 | PSI | 0.3598242E 03 | PSI | 0.1063326E 01 | Z | -0.5050041E-03 |

BODY 2
 SPEED OF CM = 0.2142701E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.1657113E 01 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.1657113E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | -0.3549367E-02 | X | 0.5587918E-01 | THETA | 0.8985526E 02 | THETA | 0.1195752E 01 | X | 0.1195756E 01 |
| Y | 0.3843686E-02 | Y | -0.5826150E-01 | PHI | 0.3599993E 03 | PHI | 0.2933975E-02 | Y | 0.1147254E 01 |
| Z | -0.1026601E 02 | Z | -0.2142686E 02 | PSI | 0.1463798E-00 | PSI | -0.1147261E 01 | Z | -0.1209621E-03 |

SEPARATION DISTANCE = 0.2874483E 02 IN SEPARATION VELOCITY = 0.5999563E 02 IN/SEC

CONSTRAINT VALUES 0.1382828E-04 0.1192093E-04 -0.3784180E-02

| A1 | | | A2 | | |
|--------------|---------------|---------------|---------------|---------------|--------------|
| 0.999995E 00 | 0.130300E-04 | -0.306778E-02 | 0.999997E 00 | -0.576893E-05 | 0.255480E-02 |
| 0.306780E-02 | -0.309619E-02 | 0.999990E 00 | -0.255478E-02 | 0.252612E-02 | 0.999994E 00 |
| 0.353144E-05 | -0.999995E 00 | -0.309621E-02 | -0.122226E-04 | -0.999997E 00 | 0.252610E-02 |

TIME = 0.2000000E-00 SECONDS STEP SIZE= 0.3841706E-08
 BODY 1
 SPEED OF CM = 0.3878719E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.2460264E 01 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.2460264E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | -0.3952325E-02 | X | -0.1033879E-00 | THETA | 0.9000176E 02 | THETA | -0.1785013E 01 | X | -0.1785013E 01 |
| Y | 0.3893239E-02 | Y | 0.1080947E-00 | PHI | 0.4212135E-03 | PHI | -0.4926649E-03 | Y | -0.1693112E 01 |
| Z | 0.2235669E 02 | Z | 0.3878690E 02 | PSI | 0.3599907E 03 | PSI | 0.1693112E 01 | Z | -0.7811086E-03 |

BODY 2
 SPEED OF CM = 0.2154844E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.2395754E 01 DEG/SEC MAGNITUDE VECTOR SUM OF PITCH-YAW RATES= 0.2395754E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | 0.2195736E-02 | X | 0.5743775E-01 | THETA | 0.9002480E 02 | THETA | 0.1718265E 01 | X | 0.1718265E 01 |
| Y | -0.2162911E-02 | Y | -0.6005260E-01 | PHI | 0.3599995E 03 | PHI | -0.6665016E-03 | Y | 0.1669491E 01 |
| Z | -0.1242038E 02 | Z | -0.2154828E 02 | PSI | 0.3599817E 03 | PSI | -0.1669491E 01 | Z | -0.1181567E-03 |

SEPARATION DISTANCE = 0.3477707E 02 IN SEPARATION VELOCITY = 0.6033563E 02 IN/SEC

| A1 | | | A2 | | |
|--------------|---------------|---------------|---------------|---------------|---------------|
| 1.000000E 00 | 0.735653E-05 | -0.161592E-03 | 1.000000E 00 | -0.904473E-05 | -0.319127E-03 |
| 0.161592E-03 | -0.307213E-04 | 1.000000E 00 | 0.319123E-03 | -0.432788E-03 | 1.000000E 00 |
| 0.735156E-05 | -1.000000E 00 | -0.307225E-04 | -0.918284E-05 | -1.000000E 00 | -0.432786E-03 |

TIME = 0.3000000E-00 SECONDS STEP SIZE= 1.0000000E-02
 BODY 1
 SPEED OF CM = 0.3878719E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.2460264E 01 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.2460264E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | -0.1429112E-01 | X | -0.1033879E-00 | THETA | 0.8982326E 02 | THETA | -0.1785002E 01 | X | -0.1785012E 01 |
| Y | 0.1470271E-01 | Y | 0.1080947E-00 | PHI | 0.9500863E-04 | PHI | -0.6031450E-02 | Y | -0.1693114E 01 |
| Z | 0.2623538E 02 | Z | 0.3878690E 02 | PSI | 0.1600579E-00 | PSI | 0.1693132E 01 | Z | -0.1044927E-02 |

BODY 2
 SPEED OF CM = 0.2154844E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.2395754E 01 DEG/SEC MAGNITUDE VECTOR SUM OF PITCH-YAW RATES= 0.2395754E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | 0.7939511E-02 | X | 0.5743775E-01 | THETA | 0.9019662E 02 | THETA | 0.1718256E 01 | X | 0.1718265E 01 |
| Y | -0.8168170E-02 | Y | -0.6005260E-01 | PHI | 0.3599992E 03 | PHI | -0.5673247E-02 | Y | 0.1669491E 01 |
| Z | -0.1457521E 02 | Z | -0.2154828E 02 | PSI | 0.3598148E 03 | PSI | -0.1669511E 01 | Z | -0.1180400E-03 |

SEPARATION DISTANCE = 0.4081060E 02 IN SEPARATION VELOCITY = 0.6033563E 02 IN/SEC

| A1 | | | A2 | | |
|---------------|---------------|--------------|---------------|---------------|---------------|
| 0.999996E 00 | 0.102754E-04 | 0.279352E-02 | 0.999995E 00 | -0.363127E-05 | -0.323299E-02 |
| -0.279354E-02 | 0.308467E-02 | 0.999991E 00 | 0.323296E-02 | -0.343173E-02 | 0.999989E 00 |
| 0.165821E-05 | -0.999995E 00 | 0.308468E-02 | -0.147260E-04 | -0.999994E 00 | -0.343170E-02 |

TIME = 0.4000000E-00 SECONDS STEP SIZE= 1.0000000E-02
 BODY 1
 SPEED OF CM = 0.3878719E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.2460264E 01 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.2460264E 01

| | | | | | | | | | |
|---|----------------|---|----------------|-------|---------------|-------|----------------|-------------------|----------------|
| X | -0.7368218E 02 | X | -0.9488960E 01 | THETA | 0.9558828E 02 | THETA | -0.8919962E 01 | BODY AXES DEG/SEC | |
| Y | 0.1297472E 03 | Y | 0.1690635E 02 | PHI | 0.1971992E 03 | PHI | -0.5528139E 03 | X | -0.2503493E 01 |
| Z | 0.2252492E 04 | Z | 0.2306667E 03 | PSI | 0.3593318E 03 | PSI | -0.5375655E 02 | Y | -0.5693560E-01 |
| | | | | | | | | Z | -0.5502550E 03 |

BODY 2
 SPEED OF CM = 0.2154844E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.2395753E 01 DEG/SEC MAGNITUDE VECTOR SUM OF PITCH-YAW RATES= 0.2395753E 01

| | | | | | | | | | |
|------------------------|----------------|---------------------|----------------------|-------|---------------------|------------------------------|----------------|---|----------------|
| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | RATE ABOUT BODY AXES DEG/SEC | | | |
| X | 0.1145206E 01 | X | 0.5743775E-01 | THETA | 0.1220430E 03 | THETA | 0.1363998E 01 | X | 0.1718224E 01 |
| Y | -0.1197209E 01 | Y | -0.6005260E-01 | PHI | 0.3488800E 03 | PHI | -0.1232695E 01 | Y | 0.1669532E 01 |
| Z | -0.4412309E 03 | Z | -0.2154828E 02 | PSI | 0.3225498E 03 | PSI | -0.2323547E 01 | Z | -0.1147725E-03 |

SEPARATION DISTANCE = 0.2697942E 04 IN SEPARATION VELOCITY = 0.2529652E 03 IN/SEC

| | | | | | |
|---------------|---------------|---------------|---------------|---------------|---------------|
| A1 | | | A2 | | |
| -0.954882E 00 | -0.296759E-00 | -0.116058E-01 | 0.841199E 00 | 0.163447E-00 | -0.515431E 00 |
| -0.399221E-01 | 0.895372E-01 | 0.995183E 00 | 0.515420E 00 | -0.530567E 00 | 0.672934E 00 |
| -0.294290E-00 | 0.950746E 00 | -0.973447E-01 | -0.163482E-00 | -0.831735E 00 | -0.530556E 00 |

TIME = 0.1989999E 02 SECONDS STEP SIZE= 1.0000000E-02
 BODY 1
 SPEED OF CM = 0.2314800E 03 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.5502608E 03 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.2444964E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | -0.7178438E 02 | X | -0.9488960E 01 | THETA | 0.8878715E 02 | THETA | 0.5557124E 02 | X | -0.1459889E 01 |
| Y | 0.1263659E 03 | Y | 0.1690635E 02 | PHI | 0.3072290E 03 | PHI | -0.5475667E 03 | Y | 0.1961268E 01 |
| Z | 0.2206359E 04 | Z | 0.2306667E 03 | PSI | 0.5948272E 01 | PSI | 0.9628836E 01 | Z | -0.5502554E 03 |

BODY 2
 SPEED OF CM = 0.2154844E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.2395753E 01 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.2395753E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | 0.1133719E 01 | X | 0.5743775E-01 | THETA | 0.1217694E 03 | THETA | 0.1372404E 01 | X | 0.1718225E 01 |
| Y | -0.1185198E 01 | Y | -0.6005260E-01 | PHI | 0.3491248E 03 | PHI | -0.1216021E 01 | Y | 0.1669532E 01 |
| Z | -0.4369213E 03 | Z | -0.2154828E 02 | PSI | 0.3230132E 03 | PSI | -0.2309770E 01 | Z | -0.1147592E-03 |

SEPARATION DISTANCE = 0.2647360E 04 IN SEPARATION VELOCITY = 0.2529652E 03 IN/SEC

| A1 | | | A2 | | |
|---------------|---------------|--------------|---------------|---------------|---------------|
| 0.603492E 00 | -0.790609E 00 | 0.103607E-00 | 0.844191E 00 | 0.160367E-00 | -0.511491E 00 |
| -0.459342E-01 | 0.952499E-01 | 0.994393E 00 | 0.511480E 00 | -0.526513E 00 | 0.679097E 00 |
| -0.796045E 00 | -0.604867E 00 | 0.211666E-01 | -0.160402E-00 | -0.834905E 00 | -0.526502E 00 |

TIME = 0.1999999E 02 SECONDS STEP SIZE= 1.0000000E-02
 BODY 1
 SPEED OF CM = 0.2314800E 03 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.5502607E 03 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.2486550E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | -0.7273328E 02 | X | -0.9488960E 01 | THETA | 0.9393443E 02 | THETA | 0.3960344E 02 | X | -0.2241270E 01 |
| Y | 0.1280565E 03 | Y | 0.1690635E 02 | PHI | 0.2524090E 03 | PHI | -0.5001292E 03 | Y | 0.1076865E 01 |
| Z | 0.2229426E 04 | Z | 0.2306667E 03 | PSI | 0.4360648E 01 | PSI | -0.3882392E 02 | Z | -0.5502551E 03 |

BODY 2
 SPEED OF CM = 0.2154844E 02 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.2395753E 01 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.2395753E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT BODY AXES DEG/SEC | |
|------------------------|----------------|---------------------|----------------|----------------------|---------------|---------------------|----------------|------------------------------|----------------|
| X | 0.1139463E 01 | X | 0.5743775E-01 | THETA | 0.1219064E 03 | THETA | 0.1368219E 01 | X | 0.1718224E 01 |
| Y | -0.1191204E 01 | Y | -0.6005260E-01 | PHI | 0.3490028E 03 | PHI | -0.1224345E 01 | Y | 0.1669532E 01 |
| Z | -0.4390761E 03 | Z | -0.2154828E 02 | PSI | 0.3227818E 03 | PSI | -0.2316640E 01 | Z | -0.1147325E-03 |

SEPARATION DISTANCE = 0.2672651E 04 IN SEPARATION VELOCITY = 0.2529652E 03 IN/SEC

| A1 | | | A2 | | |
|---------------|---------------|---------------|---------------|---------------|---------------|
| -0.306318E-00 | -0.948902E 00 | 0.758550E-01 | 0.842698E 00 | 0.161904E-00 | -0.513466E 00 |
| -0.422379E-01 | 0.931555E-01 | 0.994755E 00 | 0.513455E 00 | -0.528544E 00 | 0.676021E 00 |
| -0.950992E 00 | 0.301508E-00 | -0.686149E-01 | -0.161939E-00 | -0.833323E 00 | -0.528534E 00 |

TIME = 0.2009999E 02 SECONDS STEP SIZE= 1.0000000E-02
 BODY 1
 SPEED OF CM = 0.2314800E 03 IN/SEC
 RATE ABOUT INSTANTANEOUS AXIS= 0.5502607E 03 DEG/SEC MAGNITUDE VECTOR SUM OF FITCH-YAW RATES= 0.2504141E 01

| INERTIAL COORDS-INCHES | | 1ST DERIVATIVE /SEC | | EULER ANGLES-DEGREES | | 1ST DERIVATIVE /SEC | | RATE ABOUT | |
|------------------------|--|---------------------|--|----------------------|--|---------------------|--|------------|--|
|------------------------|--|---------------------|--|----------------------|--|---------------------|--|------------|--|

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