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Ionospheric Effects on 400 Mc Radar Signals

by

H. UNZ

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ABSTRACT

The effects of the ionosphere on 400 Mc radar signals sent vertically downward from a satellite at the height of 300 km have been investigated. The time delay and the virtual height, the attenuation factor and the Faraday rotation have been calculated.

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INTRODUCTION

One of the main methods of investigating the surface of the moon and the planets will be by analyzing the reflections of radar signals. By sending radar signals from satellites to the surface of the earth, correlations between radar reflections and the characteristics of the surface may be found and used later on for unknown surfaces. In this case the ionosphere will affect the radar signals to a certain degree and the aim of the present paper is to discuss those effects and calculate their order of magnitude.

In the present analysis of the effects of the ionosphere on radar signals we will assume the radar frequency to be 400 Mc, sent in pulses of 1 μs duration. It will be assumed that the satellite will send the radar signals at a height of 300 km (the maximum electron density of the F2-layer) directly vertically downward and we will find the total effects of the ionosphere from this height. The radar signals are to be sent at Lawrence, Kansas where the total geomagnetic field B_o = 0.7 Gauss at the surface and the angle of inclination from the horizonal is about ϕ = 65° — 70°. [Peoples, 1965; U.S.A. Airforce, 1963] .

The different layers of the ionosphere are given as follows:

D layer	70-100 km (only daytime)
E layer	100-150 km
F_1 layer	150-250 km
F_2 layer	250-400 km

The average electron density of each layer may be found in Figure 1.

The Appleton-Hartree equation for propagation of radio waves in the ionosphere is given in the form [Ratcliffe, 1959; Budden, 1961] :

$$n^2 = 1 - \frac{X}{1 - iZ - \dfrac{\frac{1}{2} Y_T^2}{1-X-iZ} \pm \sqrt{\left(\dfrac{\frac{1}{2} Y_T^2}{1-X-iZ}\right)^2 + Y_L^2}} \qquad (1a)$$

where we define:

$n = \mu - ix$ = comple refractive index

$\mu = \frac{c}{v}$ = wave refractive index

x = absorption coefficient

$\omega_N = \sqrt{\frac{N e^2}{\varepsilon_o m}}$ = plasma frequency

$\omega_H = \frac{|e| B_o}{m}$ = gyro-magnetic (cyclotron) frequency

$\omega_L = \omega_H \sin \phi$

$\omega_T = \omega_H \cos \phi$

ν = collision frequency of electrons with heavy particles (neutrals)

$X = \frac{\omega_N^2}{\omega^2}$

$Y = \frac{\omega_H}{\omega}$

$Y_L = \frac{\omega_L}{\omega}$

$Y_T = \frac{\omega_T}{\omega}$

$Z = \frac{\nu}{\omega}$

Rationalized MKSA system of units will be used in our calculations.

The polarization R of the wave will be given by:

$$R = \frac{-i}{Y_L} \left[\frac{\frac{1}{2} Y_T^2}{1-X-iZ} \mp \sqrt{\left(\frac{\frac{1}{2} Y_T^2}{1-X-iZ}\right)^2 + Y_L^2} \right] \qquad (1b)$$

For the case of quasi-longitudinal (QL) approximation, one has the condition [Ratcliffe, 1959; Budden, 1961] :

$$\frac{\frac{1}{2} Y_T^2}{Y_L} << |1 - X - i Z| = \sqrt{(1-X)^2 + Z^2} \qquad (2a)$$

under condition (2a) one will obtain from (1):

$$n^2_{QL} = 1 - \frac{X}{1 - iZ \pm | Y_L |} \qquad (2b)$$

$$R_{QL} - \mp \; i \qquad (2c)$$

The polarization R being that which corresponds to negative values of Y_L as for a wave passing downwards through the ionosphere in the Northern hemisphere [Ratcliffe, 1959]. Equation (2c) shows that the QL wave in the ionosphere is circularly polarized. In the future we will refer to the case of the upper sign as the ordinary wave and to the case of the lower sign as the extraordinary wave, though some confusion exists for the QL case [Ratcliffe, 1959].

IONOSPHERIC PARAMETERS

In the present paper we discuss radar signals of the frequency:

$$f = 400 \text{ Mc} \qquad \omega = 8\pi \times 10^8 \ 1/s \qquad (3)$$

which are sent from satellite at a height of 300 km through the ionosphere to the ground. Using the definitions in Section 1 and the electron density model given in Figure 1, one may find:

TABLE A			
height (km)	electron density $N\left(\frac{1}{cm^3}\right)$	plasma frequency $f_N\left(\frac{1}{sec}\right)$	$X = \frac{\omega_N^2}{\omega^2}$
300	4×10^6	18 Mc	0.002
50	0	0	0

Using the electron collision frequency given in Figure 2, one may find:

TABLE B		
height (km)	collision frequency $\nu\left(\frac{1}{sec}\right)$	$Z = \frac{\nu}{\omega}$
300 II	70	$2.8 \times 10^{-8} \approx 0$
300 III	10^3	$4 \times 10^{-7} \approx 0$
50 I	3×10^7	0.012

Using the geomagnetic field for Lawrence, Kansas given in Figure 3, one may find:

TABLE C						
height (km)	B_{oL} (Gauss)	B_{oT} (Gauss)	f_{oL} (1/sec)	f_{oT} (1/sec)	$Y_L = \frac{\omega_L}{\omega}$	$Y_T = \frac{\omega_T}{\omega}$
300	0.56	0.18	1 Mc	0.32Mc	0.0025	0.0008
50	0.63	0.25	1.1 Mc	0.45Mc	0.0027	0.0011

From Tables A,B,C, one may find:

TABLE D		
height km	$\frac{1}{2} \frac{Y_T^2}{Y_L}$	$\sqrt{(1-X)^2 + Z^2}$
300	0.00013	0.998
50	0.00022	1.00072

From table D one can see that the condition (2a) for QL approximation exists in our case and the wave propagation is given by (2b) and (2c). Since one has $Z \ll 1$ and $|Y_L| \ll 1$ one may develop (2b) to obtain:

$$n^2_{QL} = 1 - X\,(1 + iZ \mp Y_L) \qquad (4a)$$

Since $X \ll 1$ one may develop (4a) to obtain:

$$n_{QL} = \mu - i\varkappa = 1 - \frac{1}{2} X\,(1 + iZ \mp Y_L) \qquad (4b)$$

From (4b) one obtains for our case:

$$\mu = 1 - \frac{1}{2} X\,(1 \mp Y_L) \qquad (5a)$$

$$\varkappa = \frac{1}{2} XZ \qquad (5b)$$

where $\mu = \frac{c}{v}$ is the real refractive index and $\varkappa$ is the absorption coefficient.

THE VIRTUAL HEIGHT

When the satellite is sending radar signals vertically downward towards the earth from an assumed height of h_o = 300 km, the signals are delayed by the existence of the ionosphere and this time delay t is of interest to us. Let us define the virtual height of the satellite h' in the form:

$$h' = ct \tag{6a}$$

where t is the delay time of the signal through the ionosphere. The virtual height h' is defined as the height at which the satellite should be in free space with no ionosphere, in order that the time delay of the radar signal from the satellite to the receiving station on earth will be delayed at the same delay time t of the signal through the ionosphere when the satellite is at actual height h_o = 300 km. The delay time t is given by

$$t = \int_0^{h_o} \frac{dz}{v_g} \tag{6b}$$

where v_g is the group velocity of the signal.

Assuming a wave propagation in the form $e^{i(\omega t-kx)}$ one may define:

$$v_p = \frac{\omega}{k} \qquad \mu = \frac{c}{v_p} = \frac{ck}{\omega} \tag{7a}$$

where v_p is the phase velocity and μ is the wave refractive index.

Similarly one may define:

$$v_g = \frac{d\omega}{dk} \qquad \mu' = \frac{c}{v_g} = c\,\frac{dk}{d\omega} \tag{7b}$$

where v_g is the group velocity and μ' is the group refractive index.

From the definitions (7a) and (7b) one obtains the simple relationship:

$$\mu' = \frac{d}{d\omega}(\omega\mu) \tag{7c}$$

Substituting (6b) in (6a) and using (7b) one obtains:

$$h' = \int_0^{h_o} \mu'\, dz \tag{8}$$

From (8) one may obtain:

$$\Delta h = h' - h_o = \int_o^{h_o} (\mu' - 1)\, dz \qquad (9)$$

where Δh represents the difference between the virtual height and the actual height of the satellite. From (9) one has

$$\Delta h = c\ \Delta t \qquad (10)$$

where Δt represents the additional time delay of the radar signal because of the ionosphere.

From Table C one can see that $|Y_L| << 1$ and therefore (5a) becomes:

$$\mu = 1 - \frac{1}{2} \quad X = 1 - \frac{1}{2} \quad \frac{\omega_N{}^2}{\omega^2} \qquad (11)$$

Using (11) in (7c) one obtains:

$$\mu' = \frac{d}{d\omega} (\omega\mu) = \frac{d}{d\omega} \left(\omega - \frac{1}{2} \quad \frac{\omega_N{}^2}{\omega} \right)$$

$$= 1 + \frac{1}{2} \quad \frac{\omega_N{}^2}{\omega^2} = 1 + \frac{1}{2} X \qquad (12)$$

Substituting (12) in (9) one obtains:

$$\Delta h = \frac{1}{2} \int_o^{h_o} X\, dz \qquad (13)$$

Using in (13) the definitions of Section 1, one obtains:

$$\Delta h = \frac{1}{2\omega^2} \int_o^{h_o} \omega_N{}^2\, dz = \frac{e^2}{2\omega^2 \varepsilon_o m} \int_o^{300km} N(z)\, dz \qquad (14)$$

The integral $\int_o^{h_o} N(z)\, dz$ is called the total electron content of the ionosphere. (Arendt, Papayoanou and Soicher, 1965).

In order to evaluate Δh from (14) we have to evaluate first the total electron content of the ionosphere below h_o = 300 km the actual position of the satellite. Since Figure 1 represents the average electron density model (U. S. Airforce, 1963) we can evaluate the electron content by linear approximation from the average day curve:

TABLE E

height (km)	electron density $N\left(\frac{1}{cm^3}\right)$	average electron density $N_{av}\left(\frac{1}{cm^3}\right)$
0	0	
50	0	0
100	4×10^4	0.2×10^5
150	1×10^5	0.7×10^5
200	2×10^5	1.5×10^5
250	8×10^5	5.0×10^5
300	4×10^6	24.0×10^5
	Total	$31.4 \times 10^5 \frac{1}{cm^3}$

The total electron content will be given from Table E by:

$$\int_0^{300km} N\,(z)\,dz = 31.4 \times 10^5 \frac{1}{cm^3} \times 50 \times 10^5 \text{ cm} = 1.57 \times 10^{13} \frac{1}{cm^2} \quad (15)$$

The total electron content found in (15) is of the same order of magnitude as given by Arendt, et al (1965) . It should be pointed out that the difference in magnitudes of the results comes because in his paper he has the F2 layer maximum at 250 km height with maximum electron density of $1.5 \times 10^5 \div 6 \times 10^5\ 1/cm^3$ while in our Figure 1 we have an average curve of electron density with an F2 layer maximum at 300 km and maximum electron density of $40 \times 10^5\ 1/cm^3$.

Substituting (15) in (14) one obtains, using (3):

$$\Delta h = \frac{e^2}{(2\omega^2 \epsilon_o m)^2} \left[1.57 \times 10^{17} \frac{1}{m^2} \right] = 40 \text{ m.} \qquad (16)$$

Equation (16) shows that the delay time of the signal because of the ionosphere is equivalent to rasing the height of the satellite in free space by 40 meters or by $\frac{40 \times 100}{300 \times 1000}\ \% = 0.0133\%$. Using (16) in (10) one may find the extra-delay time of the radar signal because of the ionosphere to be:

$$\Delta t = \frac{\Delta h}{c} = 0.133 \quad \mu\text{sec.} \qquad (17a)$$

The number of cycles of the radar signal which enter in the time delay because of the ionosphere is given by:

$$\frac{\Delta t}{T} = f \quad \Delta t = \frac{f}{c} \Delta h = \frac{\Delta h}{\lambda} = \frac{40\text{m}}{0.75\text{m}} = 53.33 \qquad (17b)$$

The additional height is equal to $53 \frac{1}{3}$ wavelengths of the radar signal. Of course the above results are approximate only, but they give an idea about the order of magnitudes involved.

Assuming that we use radar pulses of $t_o = 1\ \mu$ sec one will have:

$$\frac{\Delta t}{t_o} = 0.133 = 13.3\% \qquad (17c)$$

The additional time delay because of the ionosphere will be only 13.3% of the time of each of the radar pulses.

ATTENUATION FACTOR

It is found that the changes of the value of the collision frequency ν affect the propagation of radio waves far less than changes of the electron number density N (Budden, 1961). For many purposes it is therefore permissible to treat ν as a constant over a small range of the height z. This is especially true at high frequencies (ν >> 1 Mc) where the wavelength is small compared with the scale height, which is about 10 km.

The curves of electron collision frequency γ versus height are given in Figure 2. There is some variance between the sources regarding the value of the collision frequency, particularly above 100 km, where the collision frequency is not as large. Curve I in Figure 2 has been given by Benson (1965), curve II has been given by Budden (1961), and curve III has been drawn based on data given by Ginzburg (1963). In order to find the higher attenuation, we will base our calculations in the present section on curves I and III, which give the higher values for collision frequencies. It should be pointed out that additional data for collision frequency has been given by Mitra (1952) and by others, and it is distributed between and around the curves in Figure 2.

Using WKB approximation (Budden, 1961) one could write the propagating wave in a downward positive z direction for a medium with varying refractive index $n(z) = \mu(z) - i\,\chi(z)$ for the field components, in the form:

$$Ce^{i\left(\omega t - k\int_{h_o}^{z} n(z)\,dz\right)} = Ce^{-k\int_{h_o}^{z} (z)\,dz}\; e^{i\left(\omega t - k\int_{h_o}^{z} \mu(z)\,dz\right)} \quad (18a)$$

where $k = \omega\sqrt{\mu_o \varepsilon_o} = \frac{\omega}{c}$. The attenuation factor A will be defined as the ratio of the received field $E_R(z)$ to the transmitted field $E_T(h_o)$ and using (18a) will give:

$$A = \left| \frac{E_R(z)}{E_T(h_o)} \right| = e^{-\frac{\omega}{c}\int_{h_o}^{z} x(z)\,dz} \tag{18b}$$

The attenuation factor in decibel is given by:

$$A^{db} = -20 \log_{10} A \tag{18c}$$

Substituting (18b) in (18c) one obtains:

$$A^{db} = +20\,(\log_{10} e)\,\frac{\omega}{c}\int_{h_o}^{z} \chi(z)\,dz \tag{18d}$$

where $\log_{10} e = 0.4343$.

Substituting (5b) in (18d) one obtains for our case:

$$A^{db} = 4.343\,\frac{\omega}{c}\int_{h_o}^{z} X\,Z\,dz \tag{18e}$$

Using the definitions in Section 1, (18e) becomes:

$$A^{db} = 4.343\,\frac{\omega}{c}\,\frac{1}{\omega^3}\int_{h_o}^{z} \omega_N^2\,\nu\,dz \tag{18f}$$

and from (18f) one obtains:

$$A^{db} = 4.343\,\frac{e^2}{\omega^2 c\,\varepsilon_o m}\int_{0}^{300km} N(z)\,\nu(z)\,dz \tag{19}$$

Equation (19) gives us the total attenuation in the ionosphere in db. It should be noted that at $z < 50$ km $N(z) = 0$ so the attenuation will be the same all the way to the ground. Let us first evaluate the integral in (19) using Figure 1 and curves I, III in Figure 2.

TABLE F

height (km)	electron density $N\left(\frac{1}{cm^3}\right)$	collision frequency $\nu\left(\frac{1}{sec}\right)$	$N\gamma$	average of $N\gamma$
0	0	9.5×10^{11}	0	
50	0	2×10^{7}	0	0
100	4×10^{4}	4×10^{5}	1.6×10^{10}	8×10^{9}
150	1×10^{5}	4×10^{4}	4×10^{9}	10×10^{9}
200	2×10^{5}	10^{4}	2×10^{9}	3×10^{9}
250	8×10^{5}	2.5×10^{3}	2×10^{9}	2×10^{9}
300	4×10^{6}	1×10^{3}	4×10^{9}	3×10^{9}
			Total	$26 \times 10^{9} \frac{1}{cm^3 sec}$

Using Table F one may find the valueof the integral in (19) to give:

$$\int_0^{300km} N(z)\,\nu(z)\,dz = 26 \times 10^{9} \frac{1}{cm^3 sec} \times 50 \times 10^{5} cm =$$

$$1.3 \times 10^{17} \frac{1}{cm^2 sec} \qquad (20)$$

Substituting (20) in (19) one obtains:

$$A^{db} = 4.343 \frac{e^2}{\omega_c^2 \varepsilon_o m} \left[1.3 \times 10^{21} \frac{1}{m^2 sec}\right] \simeq 0.01 \text{ db} \qquad (21a)$$

Using (18b) and (21a) one has:

$$0.01 = 10 \log_{10} \frac{P_T}{P_R} \qquad (22a)$$

where P_T is the transmitted power from the satellite and P_R is the received power at the ground. From (22a) one obtains:

$$\frac{P_T}{P_R} = 10^{0.001} = 1 + 0.001 \times 2.3026 = 1.0023 \qquad (22b)$$

The power loss of the radar signal through the ionosphere is 0.23%. From (22) we see that for all practical purposes at this radar frequency of 400 Mc the attenuation of the signal in the ionosphere could be neglected.

FARADAY ROTATION

In Lawrence, Kansas the geomagnetic field is given by: (Peoples, 1965; USA Airforce, 1963)

$$B_o = 0.7 \text{ Gauss } \phi = 68^o \qquad (23)$$

where B_o is the magnitude of the field and ϕ is the inclination angle from the horizontal. Since the goemagnetic field is to a very good approximation a dipole field with the dipole at the center of the earth and since the dipole field vary as $\frac{1}{r^3}$ we could take:

$$B = B_o \frac{R^3}{(R + z)^3} = \frac{B_o}{(1 + \frac{z}{R})^3} \qquad (24a)$$

where the radius of the earth given by:

$$R = \frac{40,000}{2\pi} \text{ km} = 6,370 \text{ km} \qquad (24b)$$

It will be assumed that the inclination angle ϕ remains the same for all heights.

From (24) one obtains:

$$B = B_o \left(1 + \frac{z^{km}}{6,370}\right)^{-3} \simeq B_o \left[1 - \frac{3\, z^{km}}{6,370}\right] \qquad (24c)$$

and one also obtains:

$$B_L = B \sin\phi \qquad B_T = B \cos\phi \qquad (24d)$$

Figure 3 has been drawn by using (24c) and (24d) taking B_o = 0.7 Gauss and $\phi = 68^o$.

In Appendix A we will derive the total Faraday rotation angle Ω for two circularly polarized waves in the ionosphere (Van Allen, 1956), to give:

$$\Omega = \frac{\omega}{2c} \int (\mu_o - \mu_x) \, dz \tag{25}$$

Using the result in (5a) one has:

$$\mu_o = 1 - \frac{1}{2} X + \frac{1}{2} X Y_L \tag{26a}$$

$$\mu_x = 1 - \frac{1}{2} X - \frac{1}{2} X Y_L \tag{26b}$$

Substituting (26) in (25) one obtains:

$$\Omega = \frac{\omega}{2c} \int_0^{300km} X Y_L \, dz \tag{27}$$

Using the definitions in Section 1 in (27) one obtains:

$$\Omega = \frac{|e|^3}{2c \, \varepsilon_o m^2 \omega^2} \int_0^{300km} N(z) \, B_L (z) \, dz \tag{28}$$

Using Figure 1 and Figure 2 one may evaluate the integral in (28):

TABLE G				
height (km)	electron density $N\left(\frac{1}{cm^3}\right)$	geomagnetic field B_L (Gauss)	$N\ B_L$	average $N\ B_L$
0	0	0.64	0	
				0
50	0	0.63	0	
				0.124×10^5
100	4×10^4	0.62	2.48×10^4	
				0.424×10^5
150	1×10^5	0.60	0.60×10^5	
				0.890×10^5
200	2×10^5	0.59	1.18×10^5	
				2.910×10^5
250	8×10^5	0.58	4.64×10^5	
				13.570×10^5
300	4×10^6	0.56	2.24×10^6	
			Total	17.918×10^5 $\frac{\text{Gauss}}{\text{cm}^3}$

Using Table G one may find the integral in (28) to give:

$$\int_0^{300km} N\ (z)\ B_L\ (z)\ dz = 17.918 \times 10^5 \ \frac{\text{Gauss}}{\text{cm}^3} \times 50 \times 10^5 \text{ cm} =$$

$$9 \times 10^{12} \ \frac{\text{Gauss}}{\text{cm}^2} = 9 \times 10^{12} \ \frac{\text{V sec}}{\text{m}^4} \qquad (29)$$

Substituting (29) in (28) one obtains:

$$\Omega = \frac{|e|^3}{2c\ \epsilon_o m^2 \omega^2} \left[9 \times 10^{12} \right] = 1.32 \text{ Rad.} \cong 75^\circ \qquad (30)$$

The Faraday rotation in the ionosphere will be 75°. The electric field vector while passing through the ionosphere will rotate its plane of polarization by 75°. The derivation of the equation for total Faraday rotation (25) will be given in Appendix A.

SUMMARY

In the present paper the effects of the ionosphere on a 400 Mc radar signals, sent from a satellite at a height of 300 km vertically downward to the ground, have been investigated. Taking the geomagnetic field in Lawrence, Kansas it has been found that in the propagation of the waves through the ionosphere the quasi-longitudinal (QL) approximation will be in order and that one will have two circularly polarized waves, the ordinary and the extraordinary modes of propagation.

It has been found that the time delay of the signal because of the ionosphere will be equivalent to increasing the satellite height from the ground (300km) by 40 meters in free space.

It has been found that the attenuation of the radar signal through the ionosphere is approximately $\frac{1}{100}$ db which is equivalent to a 0.23% decrease in the power while propagating through the ionosphere.

It has been found that the Faraday rotation of the total electric vector plane of polarization while passing through the ionosphere will be 75^{o}.

In all the above calculations an average day time electron density profile has been assumed [U.S.A. Airforce, 1963], and the numerical values mentioned above give the order of magnitude of the ionospheric effects on 400 Mc radar signals, rather than exact results.

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APPENDIX A

FARADAY ROTATION

In the present section we will derive the formula for Faraday rotation Van Allen, 1956 given in (25). Assuming a QL circularly polarized wave $e^{i(\omega t - k\mu z)}$ propagates in the ionosphere, where $k = \frac{\omega}{c}$ and μ is the refractive index. From the above one may find that:

$$d\,(\omega t - k\,\mu\,z) = \omega dt - k\,\mu\,dz = d\ C = 0 \tag{A-1}$$

where (A-1) may be used to find the phase velocity:

$$v_p = \frac{dz}{dt} = \frac{\omega}{k\,\mu} = \frac{c}{\mu} \tag{A-2}$$

If one takes a plane z = Const the circularly polarized wave will have the electric field vector rotate by angle α where

$$d\,\alpha = \omega\,dt \tag{A-3a}$$

By using (A-1) in (A-3a) one obtains also

$$d\,\alpha = k\,\mu\,dz = \frac{\omega}{c}\,\mu\,dz \tag{A-3b}$$

There will be rotation of the circularly polarized wave for a given time t = Const as we move along the z axis. Since the ordinary and the extra-ordinary QL circularly polarized waves electric field vectors rotate in two opposite directions, as may be seen from (2c), one will denote the corresponding angle of rotation α_o and α_x as in Figure 4. The total electric field E is the vector sum of E_o and E_x and it rotates from its original position by an angle Ω. From the geometry of the parallelogram in Figure 4 one obtains:

$$\alpha_o - \Omega = \alpha_x + \Omega \tag{A-4a}$$

From which we obtain:

$$2\,\Omega = \alpha_o - \alpha_x \tag{A-4b}$$

Which could be rewritten in the form:

$$d\,\Omega = \frac{1}{2} \left[d\,\alpha_o - d\,\alpha_x \right] \tag{A-4c}$$

Substituting (A-3b) in (A-4c) for the ordinary and the extraordinary wave, one obtains:

$$d\,\Omega = \frac{1}{2} \frac{\omega}{c} \left[\mu_o\,dz - \mu_x\,dz \right] = \frac{\omega}{2c} \left(\mu_o - \mu_x \right) dz \tag{A-5a}$$

and from (A-5a) one has:

$$\Omega = \frac{\omega}{2c} \int \left(\mu_o - \mu_x \right) dz \tag{A-5b}$$

Equation (A-5b) is identical with (25) and has been given previously [Van Allen, 1956; Ginzburg, 1963].

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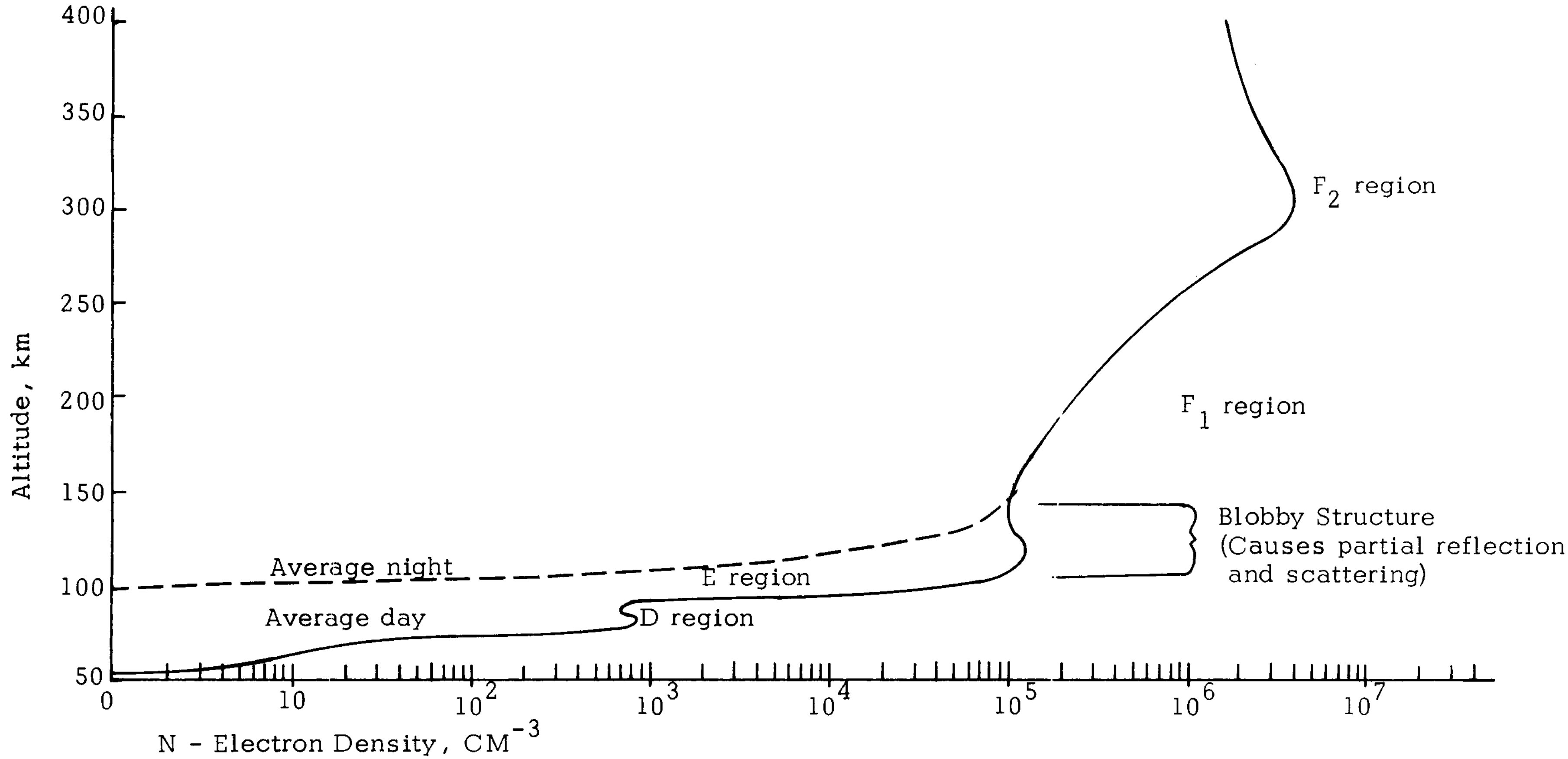

Figure 1. An Electron Density Model in the Ionosphere

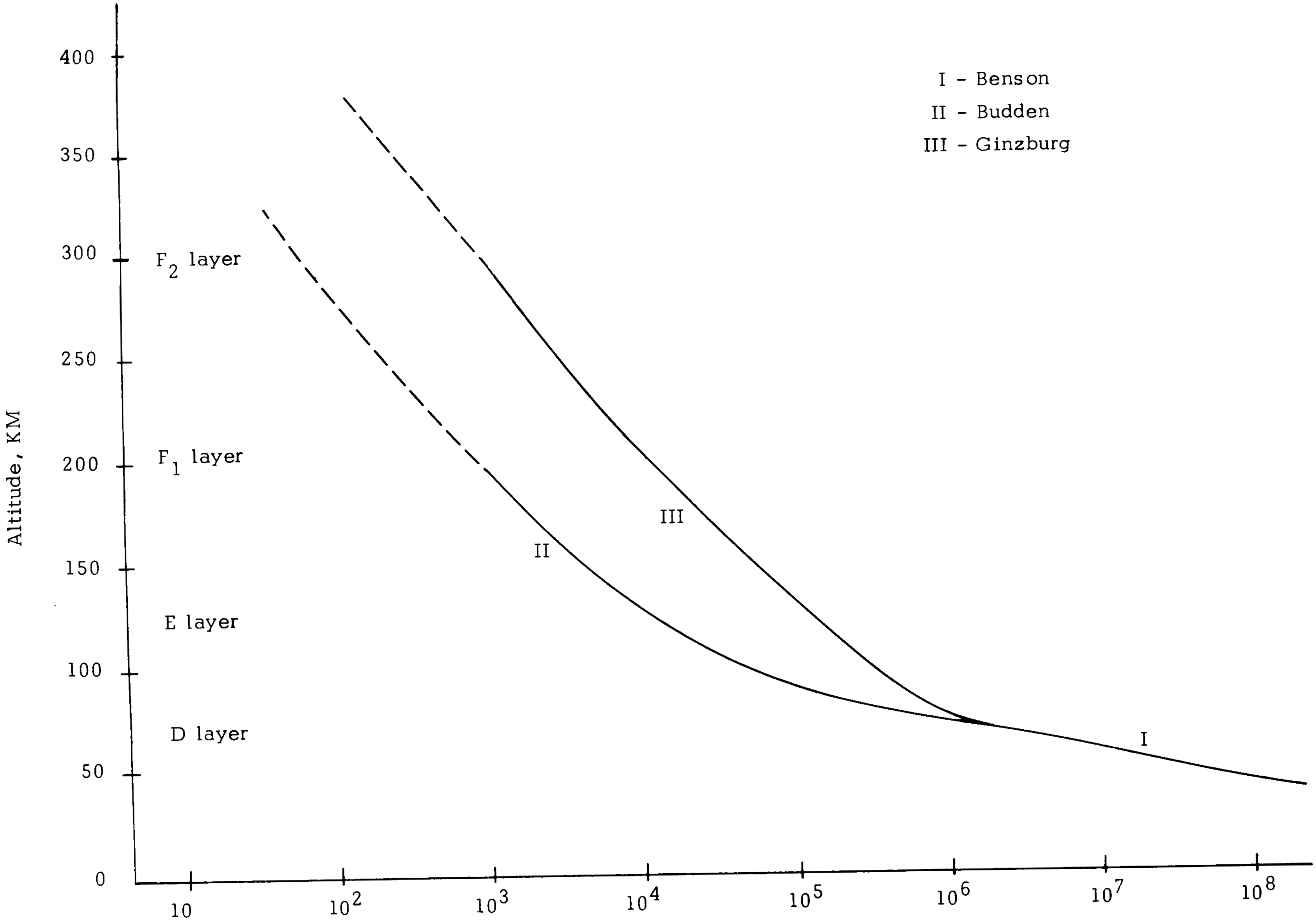

Figure 2. Electron Collision Frequency ν 1/sec

Lawrence, Kansas

$B_o = 0.7$ Gauss, Inclination (from horizontal) = 68^o

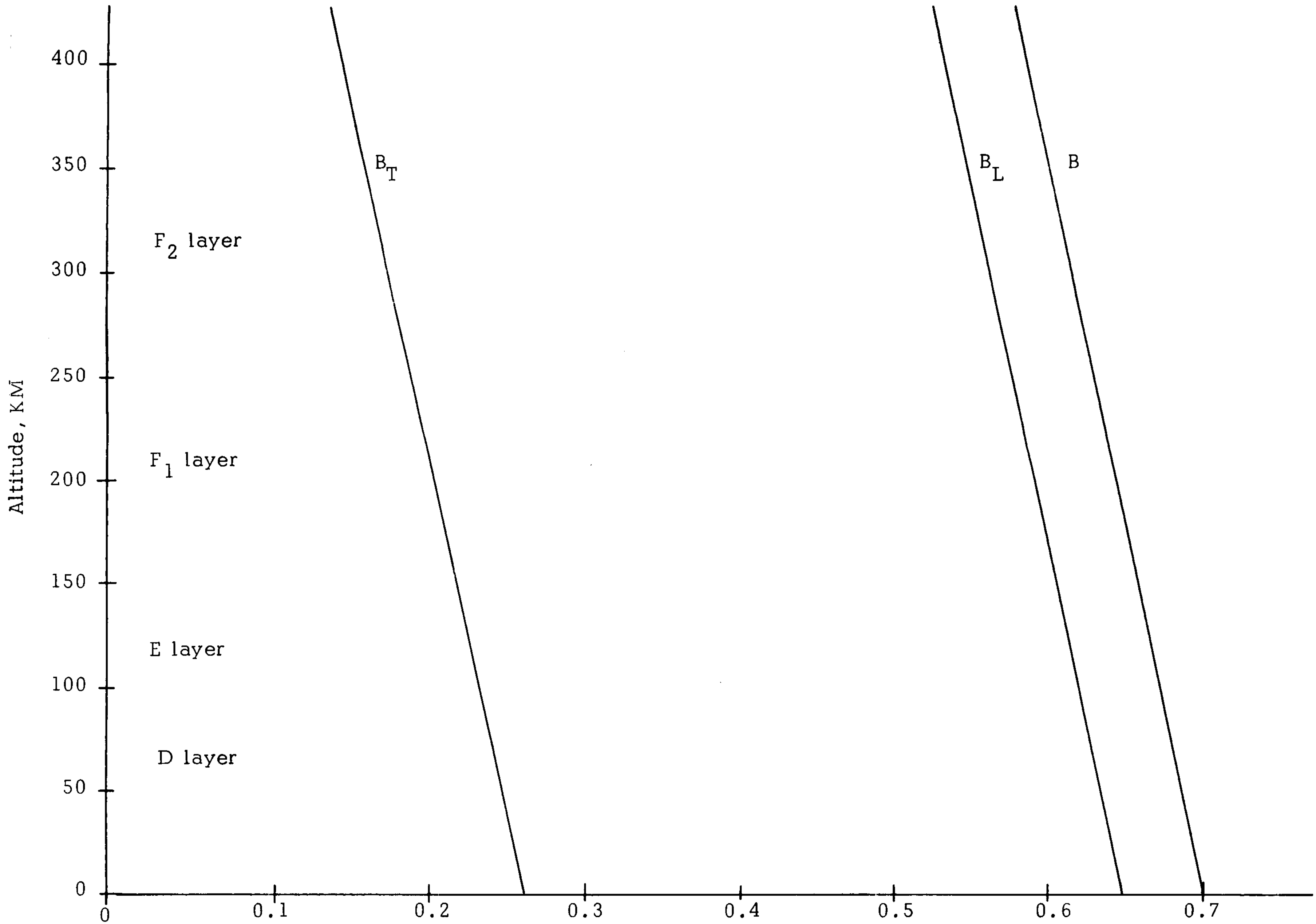

Figure 3. Geomagnetic Field, Gauss

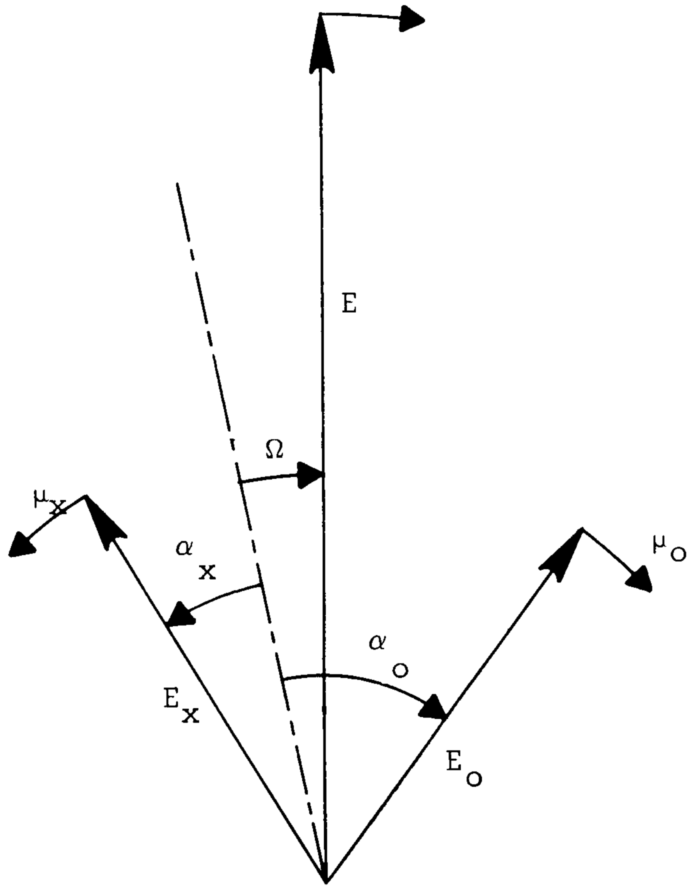

Figure 4. Faraday Rotation

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