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REMARKS ON PRACTICAL SPECTROSCOPIC TEMPERATURE MEASUREMENTS IN PLASMAS

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SUMMARY

Spectroscopic techniques for the determination of plasma parameters are being applied to many laboratory plasmas. An attempt is made to show that the lack of spatial resolution can lead to drastic errors in spectroscopically determined temperatures even in uniform plasmas with thin boundary layers. Although the need for such resolution is emphasized, certain semiquantitative conclusions can be drawn from spatially unresolved spectra.

INTRODUCTION

Perhaps the most widely used spectroscopic method (refs. 1 to 3) for measuring the temperature of an optically thin plasma is the spectral line intensity ratio method, in which the ratio of the intensities of spectral lines emitted by the same element but originating in different molecular, atomic, or ionic states (sometimes different stages of ionization) yields an excitation temperature.

Serious errors in the excitation temperature obtained by this spectroscopic method can result if the plasma is not entirely uniform but has temperature gradients in the line of sight. Even thin boundary layers with temperatures just slightly different from the bulk of the plasma can give rise to large errors in the temperature determination if spatial resolution (for methods see, for example, ref. 1, ch. 7) is not achieved with sufficient precision. The same situation arises whenever there is an exponential dependence of the measured quantities upon temperature. (A number of techniques were reviewed by Greenshields (ref. 4).) Usually the worst errors will arise, however, if intensity ratios of lines originating in different stages of ionization (or, possibly, dissociation) are used.

Error estimates can be derived from previously developed theory (refs. 1 to 5). However, it appears that in practical plasma spectroscopy such sources of error are frequently not taken into account properly. The purpose of this paper is to point out, for the laboratory investigator, the magnitude of possible errors and to indicate methods for corrections. In particular it is shown in this report that the average temperature obtained, if spatial temperature variations are neglected, can be arbitrarily erroneous in the general case. A simple criterion is given for estimating the effect of the presence of boundary layers in otherwise uniform plasmas. An alternate method is discussed which is not subject to the same difficulty.

SYMBOLS

A	transition probability, 1/second
E_{21}	energy difference between states 1 and 2, joules
I	intensity, joules/meter^3-second
L	depth of plasma, meters
n	number density, 1/meter^3
T	temperature, degrees Kelvin
h	Planck's constant, joule-seconds
k	Boltzmann's constant, joules/degree Kelvin
$r_{21} = \frac{\alpha_{i,2}}{\alpha_{i,1}}$	
x	coordinate, meters
$\alpha_i = \frac{n_i}{n}$	
ϵ	parameter (see eq. (6))
ν	frequency, 1/second
ρ	constant (see eq. (2))
τ	volume, meters^3
$< >$	average

Subscripts:

av	average
i	initial state
1,2	running index
I	region I
II	region II

APPLICATION OF SPECTRAL LINE INTENSITY RATIO METHOD TO SPATIALLY NONUNIFORM PLASMAS

An optically thin plasma is never in complete local thermodynamic equilibrium (LTE); for example, it is not in equilibrium with the radiation field so that LTE relations, such as the Boltzmann factor and the Saha equation, are not necessarily applicable. Griem (ref. 1, ch. 6) has derived validity criteria for LTE relations for optically thin plasmas excited by thermal electron collision processes.

For simplicity, it is assumed herein that the LTE relations hold; the conclusions, however, are valid also in the more general case of corona equilibrium with only small (but mathematically cumbersome) modifications, because the general form of the relations remains unchanged to a fair approximation (refs. 2 and 3). The assumption implies that the plasma constituent (for example, the electrons) which is responsible for excitation and ionization has a Maxwellian energy distribution characterized by the temperature T. The intensity I of a spectral line is usually expressed as

$$I = h\nu A n_i$$

in terms of the photon energy $h\nu$, the transition probability A, and the number density of radiating systems in its initial state n_i. It is convenient to introduce the fraction $\alpha_i = \frac{n_i}{n}$ of the total number density n of radiating systems. The intensity ratio of two lines

$$\frac{I_2}{I_1} = \frac{\nu_2 A_2 \alpha_{i,2}}{\nu_1 A_1 \alpha_{i,1}} \tag{1}$$

is independent of n and is easily measurable, as only a relative calibration of the detectors measuring I_1 and I_2 is required. The ratio $r_{21} = \frac{\alpha_{i,2}}{\alpha_{i,1}}$ is obtained from thermodynamics and is usually an exponential function

$$r_{21} = \rho_{21} \exp\left(-\frac{E_{21}}{kT}\right) \tag{2}$$

of the temperature T where ρ_{21} and E_{21} are constants or possibly weak functions of T, obtainable from atomic physics and thermodynamics (ref. 1, ch. 6).

The spectral line intensity ratio method consists of determining T from

$$T = \frac{-E_{21}}{k \log\left(\frac{I_2 \nu_1 A_1}{I_1 \nu_2 A_2 \rho_{21}}\right)} \tag{3}$$

after measuring the ratio $\frac{I_2}{I_1}$.

If the plasma is nonuniform, the argument is usually made that "some sort of average temperature" will result. The average temperature which then results from equation (3)

$$T_{av} = \frac{-E_{21}}{k \log\left(\frac{\nu_1 A_1}{\nu_2 A_2 \rho_{21}} \frac{\int I_2 \, d\tau}{\int I_1 \, d\tau}\right)} \tag{4}$$

(the integration is carried out over the region of observation) is, in general, different by an arbitrarily large factor from a properly defined spatial average temperature

$$<T> = \frac{\int T \, d\tau}{\int d\tau} \equiv \frac{1}{\int d\tau} \int \frac{-E_{21} \, d\tau}{k \log\left(\frac{\nu_1 A_1 I_2}{\nu_2 A_2 \rho_{21} I_1}\right)} \tag{5}$$

or any other well-defined average temperature.

ESTIMATE OF ERRORS

Errors incurred by using the intensity ratio method without regard to nonuniformities may be estimated in practical cases by assuming the existence of a low-temperature boundary layer in an otherwise uniform plasma.

Let L be the depth of the plasma along the line of sight x and let intensities

$$I = \begin{cases} I_I & \text{for } 0 \leqq x \leqq \epsilon L \\ I_{II} & \text{for } \epsilon L < x \leqq L \end{cases}$$

be emitted in two uniform regions. The parameter $\epsilon << 1$ is a measure of the depth of the thin boundary layer.

The "average" temperature obtained without regard to nonuniformities T_{av} results from

$$\frac{\int I_2 \, d\tau}{\int I_1 \, d\tau} = \frac{\epsilon I_{2,I} + (1 - \epsilon) I_{2,II}}{\epsilon I_{1,I} + (1 - \epsilon) I_{1,II}} \tag{6}$$

rather than from $I_{2,II}/I_{1,II}$ as it should for a sufficiently thin boundary layer.

The error will, in general, be small only if the conditions

$$I_{1,I} << \frac{(1 - \epsilon)}{\epsilon} I_{1,II} \tag{7a}$$

and

$$I_{2,I} << \frac{(1 - \epsilon)}{\epsilon} I_{2,II} \tag{7b}$$

are both satisfied. (There may be instances in which the error is accidentally small despite a failure to meet these requirements if, for example, both conditions are violated and the effects happen to cancel partially.)

EXAMPLES

It is assumed that a laboratory plasma is under examination in which there are regions I and II with different temperatures and, consequently, with different levels of emission from each of the spectral lines considered. If one decided to measure the temperature of this plasma from the intensity ratio of the spectral lines of carbon III and carbon IV at 2.297×10^{-7} and 2.53×10^{-7} meter, respectively, figure 1 (taken from ref. 2) would be used to find a temperature corresponding to the measured intensity ratio. Without regard to spatial resolution, a temperature would be determined on the basis of equation (4), where the measured ratio of the integrated intensities (over the region of observation) is given in equation (6).

In order to find this ratio, it is necessary to know the intensities of the two spectral lines as functions of the temperature. The intensities can be calculated from the same relations which are the basis for figure 1 and which are not repeated herein because they can be found in the literature (see, for example, refs. 1, 2, 5, and 6); they have been calculated from equilibrium relations which were corrected for semicorona type of deviations from LTE by the method described in reference 2, rather than from straight LTE relations. (If more sophisticated semicorona corrections (ref. 1, ch. 6) had been used, the result would have been essentially the same as far as differences between T_{av} and $\langle T \rangle$ are concerned.) Figure 2 shows the resulting intensities on arbitrary but equal scales for the spectral lines of carbon III and carbon IV.

Table I presents the temperatures in regions I and II in the first two columns and the average temperature computed from equation (5) in the third column. In the fourth column is given the temperature arrived at if the intensity ratio, obtained without space resolution, was used to compute T_{av} from equation (4). The parameter ϵ was assumed to be equal to one-tenth in all cases.

It is seen from table I that a boundary layer of 10 percent at about half of the temperature of the bulk of the plasma would lead to a "measured" temperature which is more than 20 percent below a well-defined average temperature for the plasma. At temperatures in excess of 90 000° K the discrepancy can be significantly larger (for example, a factor of three in the case considered in refs. 2 and 3). In general, errors will be most serious if the temperature in the bulk of the plasma is much higher than the temperature at which one of the spectral lines used in the ratio method has maximum emission. Thus, in figure 2 the emission in the carbon III line peaks at a temperature of about 40 000° K and drops to less than 1 percent of the peak intensity at 90 000° K. The emission from the boundary layer at 40 000° K in this carbon III line exceeds therefore the emission from the bulk of the plasma which is at 90 000° K. The two spectral lines are essentially emitted in separate regions of the plasma, and then it is evidently not meaningful to form an intensity ratio and to use it in a formula (eq. (3)) which was derived under the assumption of a uniform plasma.

On the other hand, if both spectral lines are near or well below the temperature of peak emission, it is seen that the error does not exceed 10 percent and is actually smaller in some of the more extreme cases due to partial cancellation in the ratio in equation (6). (See parenthetical remark at the end of the previous section.)

Note, however, that the same value of T_{av} which resulted from a plasma at 90 000° K with a 10 percent boundary layer at 40 000° K would also result from the same measurement on a plasma at 180 000° K (not shown in table I) and a 10 percent boundary layer at either about 30 000° or 50 000° K. The temperature resulting from a single line intensity ratio is therefore not unique in the absence of spatial resolution.

The fact that intensity ratios involving different spectral lines may give approximately the same value for T_{av} does not necessarily imply that this temperature is representative for the plasma. Spectral lines of approximately the same excitation energy and the same stage of ionization tend to have very similar intensity-temperature characteristics. The argument becomes tenable only if spectral lines of vastly differing excitation energies and different stages of ionization, if possible of different chemical elements, are used.

A METHOD INDEPENDENT OF SPATIAL RESOLUTION

If spatial resolution cannot be readily obtained, it is still possible to draw certain conclusions by inspection of the observed spectrum (refs. 5 and 6) if the plasma conditions are such that LTE holds or that ionization and excitation equations of the same functional form as the LTE relations apply. The appearance of a certain spectral line implies that somewhere in the plasma the excitation conditions are appropriate; conversely, the absence of a spectral line indicates that nowhere in the plasma are the excitation conditions right, supposing, of course, the corresponding element is present.

Although these statements may appear to be quite weak, they do actually supply a considerable amount of information in many situations, because the minimum temperature at which a given spectral line is observable is usually very well defined. This point is illustrated by figures 3 and 4 which show calculated intensities of carbon and argon lines, respectively, as functions of temperature. (See table II for relevant parameters for argon.) These intensities are based upon the same relations as figures 2.2, 2.3, and 2.4 in reference 2 with the corresponding atomic quantities (ref. 7). For easier comparison all curves were normalized to unity at maximum intensity. The appearance of, say, the line of argon III in a plasma would imply that somewhere the temperature is at least about 20 000° K, whereas its absence would mean that everywhere throughout the plasma the temperature is lower (or possibly much higher) than this limit.

By combining the limits and the information supplied by the presence and absence of spectral lines originating in excited states of widely different excitation potential, it is possible to narrow the limits of the overall temperature considerably in many situations. This method has been successfully applied to astrophysical and solar problems (ref. 5). It has also been used in the laboratory. In a hypothetical argon plasma of approximately

10^{16} atoms/cm^3, the absence of argon I lines, the predominance of argon II lines, the presence of weak argon III lines, and the absence of argon lines from still higher stages of ionization would indicate a lower limit of about 20 000° K for the temperature in the coolest part of the device and an upper limit of about 30 000° K for the excitation temperature in the hottest part, with a likely value of about 25 000° K for an average excitation temperature.

CONCLUDING REMARKS

It has been demonstrated that the spectral line intensity ratio method for spectroscopic measurements of temperature in nonuniform plasmas is usually plagued by large errors if adequate space resolution is not achieved, although the nonuniformity may be confined to an extremely thin boundary layer. The importance of spatially resolved spectroscopic temperature measurements must therefore be emphasized. General conclusions of a semiquantitative nature can be drawn from spatially unresolved spectra even for strongly nonuniform plasmas; information about temperature ranges and limits for unspecified regions of the plasma and the plasma as a whole can be deduced. If the precision requirements are not too high this method would appear to be adequate in view of its ease of use, especially in many engineering applications.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., July 20, 1966,
129-01-05-09-23.

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TABLE I.- A PROPER AVERAGE VALUE AND THE HYPOTHETICAL RESULT OF A MEASUREMENT MADE WITHOUT SPATIAL RESOLUTION

T_I, °K	T_{II}, °K	$<T>$, °K (computed from eq. (5))	T_{av}, °K (computed from eq. (4))
80 000	90 000	89 000	88 000
70 000	90 000	88 000	82 000
60 000	90 000	87 000	76 000
50 000	90 000	86 000	72 000
40 000	90 000	85 000	67 000
50 000	60 000	59 000	59 000
40 000	60 000	58 000	57 000
30 000	60 000	57 000	59 000
60 000	80 000	78 000	74 000
50 000	80 000	77 000	72 000
40 000	80 000	76 000	66 500
30 000	80 000	75 000	73 000

TABLE II.- PARAMETERS OF ARGON SPECTRAL LINES

Ion	Wavelength, m	Ionization potential, V	Excitation potential, V
Ar I	4.2007×10^{-7}	15.69	14.44
Ar II	4.6096×10^{-7}	27.5	21.05
Ar III	2.7248×10^{-7}	40.8	27.83
Ar IV	2.8094×10^{-7}	59.6	35.50
Ar V	4.639×10^{-8}	75	26.86

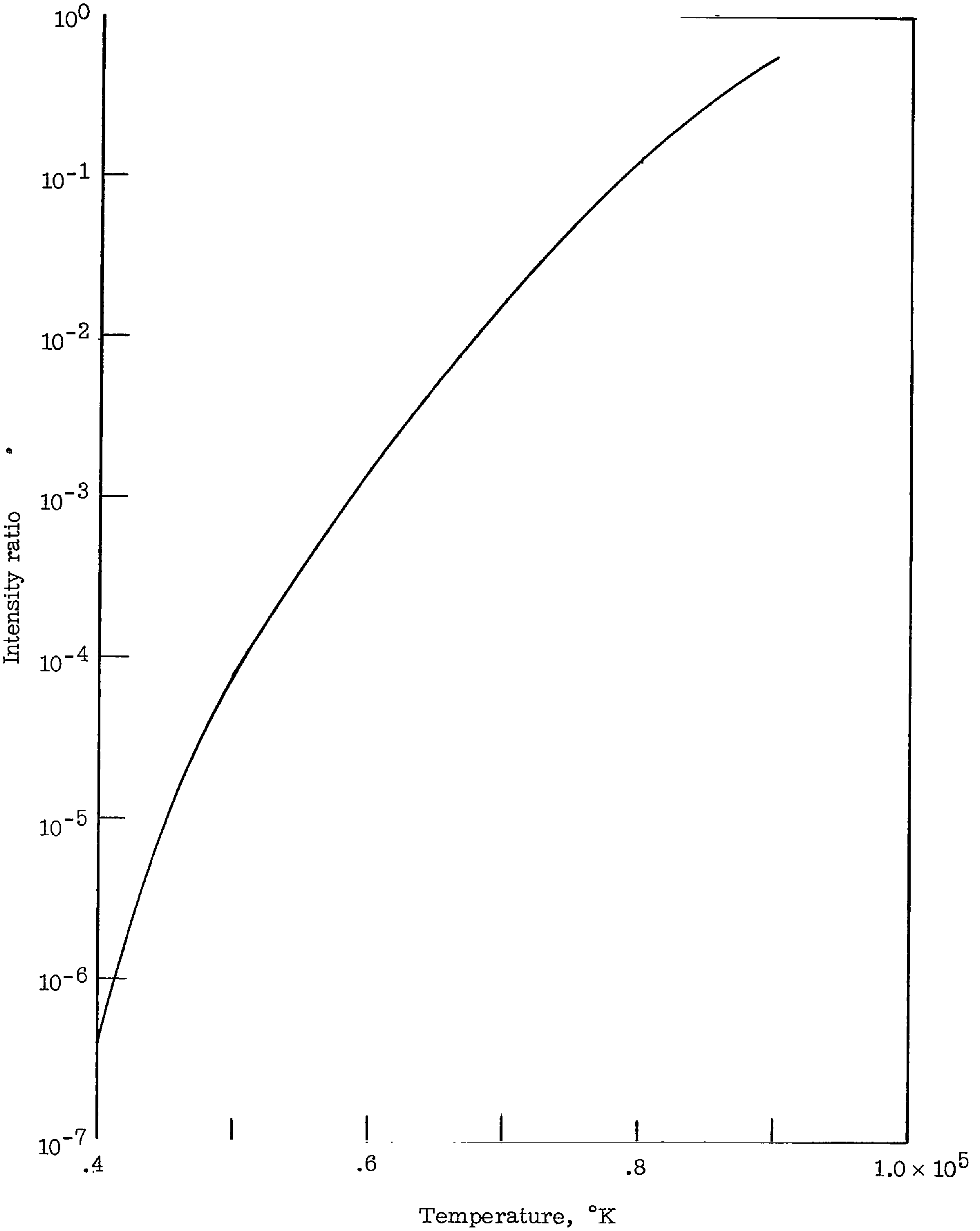

Figure 1.- Intensity ratio of carbon IV and carbon III lines at 2.530×10^{-7} and 2.297×10^{-7} meter, respectively.

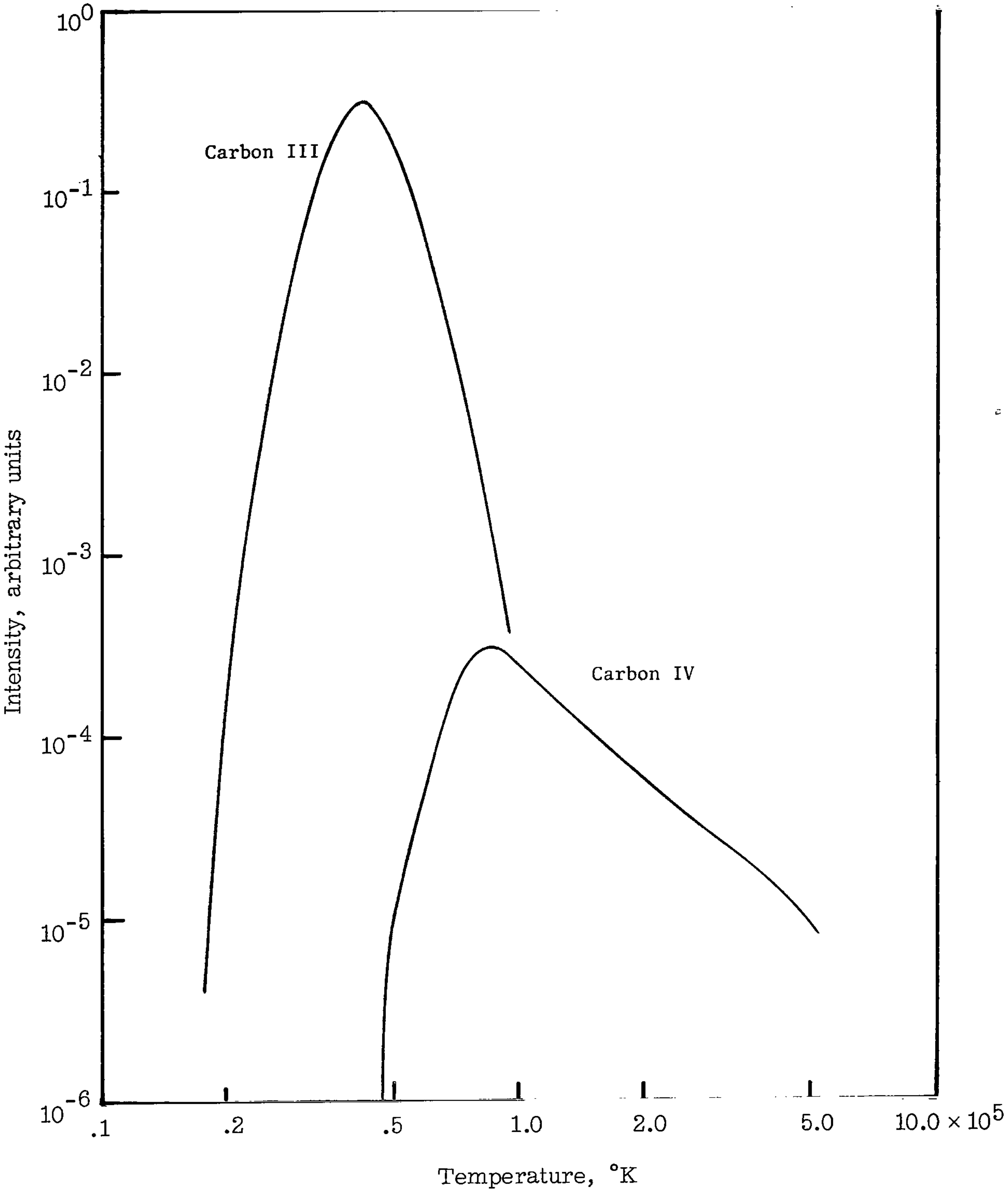

Figure 2.- Intensity, in arbitrary units, of carbon III and carbon IV spectral lines at 2.297×10^{-7} and 2.530×10^{-7} meter, respectively, as a function of temperature.

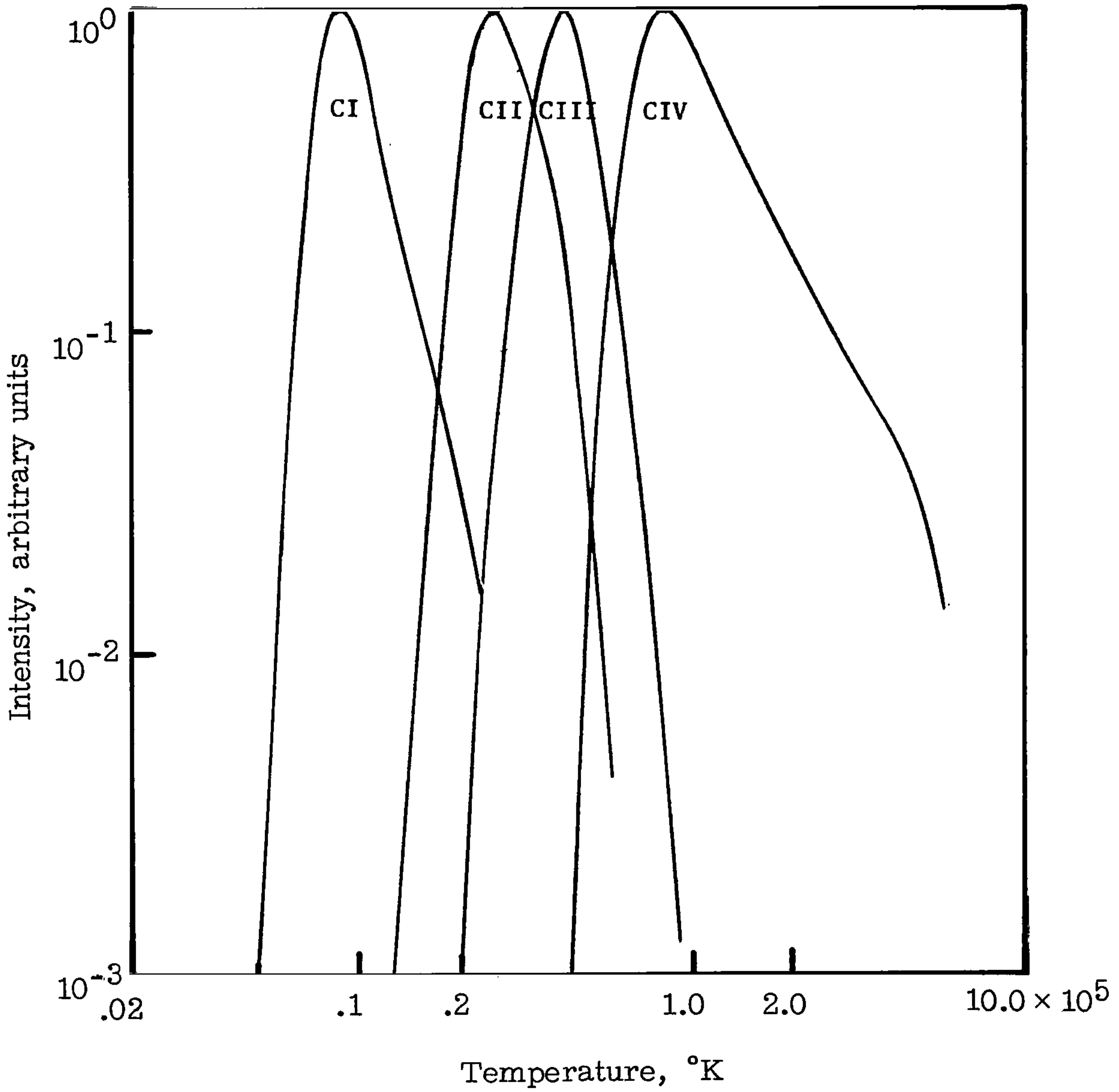

Figure 3.- Intensities of carbon I through IV lines as functions of temperature on a scale normalized separately for each line to be unity at peak intensity.

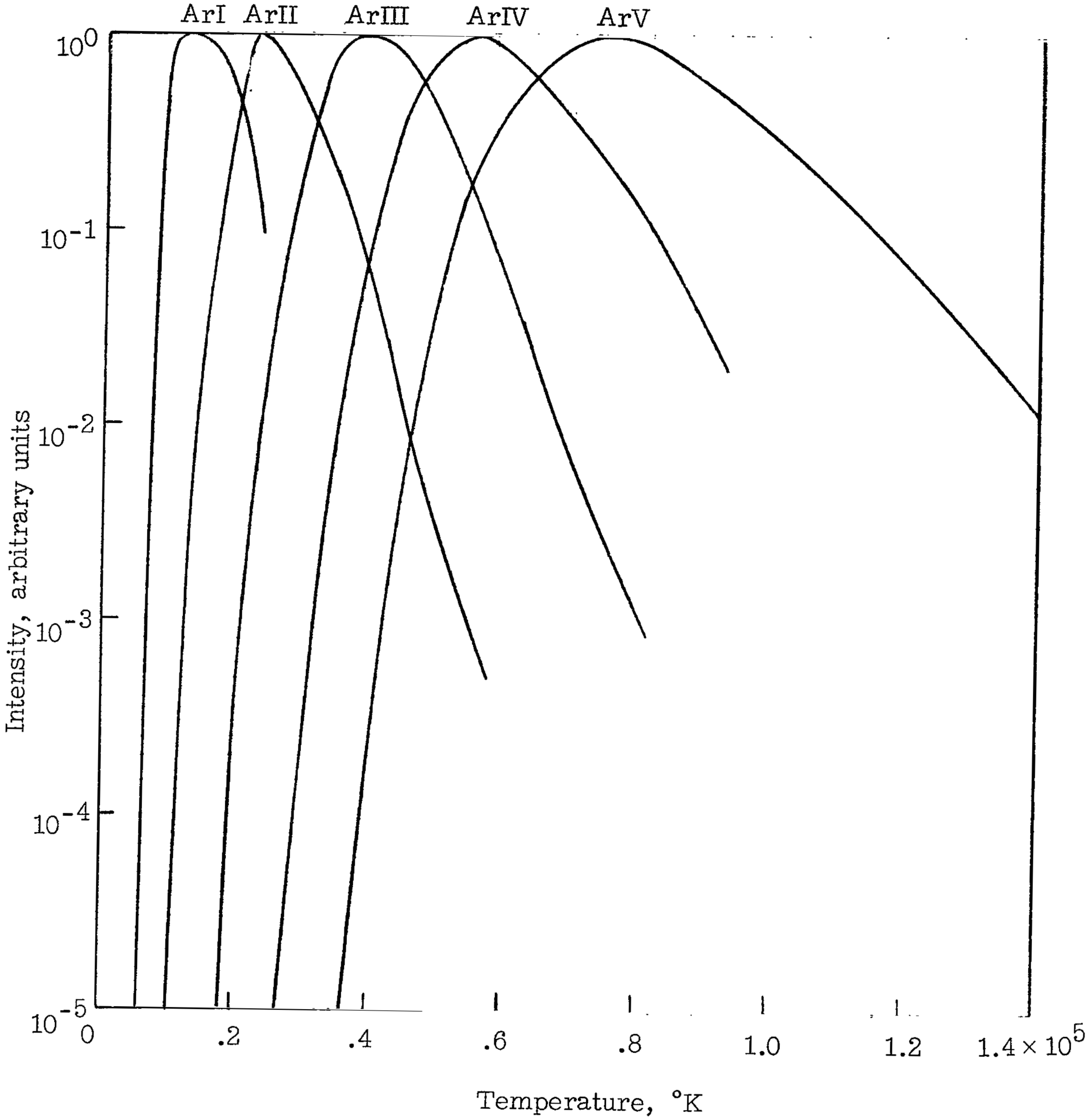

Figure 4.- Intensities of argon I through V lines as functions of temperature on a scale normalized separately for each line to be unity at peak intensity.

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