NGR-33-008-053

GRAVITATIONAL RADIATION FROM NEUTRON STARS

CHAU, WAI-YIN
Department of Physics and Astronomy
Columbia University, N.Y.

ABSTRACT N67-12934

Graviatational radiation from pulsating and rotating objects is calculated using the formula obtained with the weak field limit of general relativity. The cases of rotation and oscillation are first considered separately. Then the effects of roatation on radial oscillations are investigated. Numerical estimates are made with data relevant to a neutron star. It is concluded that most of the energy a neutron star may acquire during its formation is dissipated rapidly, unless the rotation is quite slow.

Author

INTRODUCTION

The energy loss by gravitational radiation from a system of bodies moving with velocities small compared to that of light is given by the weak field limit to general relativity as (Landau and Lifshitz, 1962)

$$-\frac{d\mathcal{E}}{dt} = \frac{G}{45c^5}\,\dddot{\mathcal{D}}_{\alpha\beta}^{\,2} \quad * \tag{1}$$

where $\mathcal{D}_{\alpha\beta}$ is the quadrupole moment tensor of the mass distribution, defined as

$$\mathcal{D}_{\alpha\beta} = \int \rho(\underset{\sim}{x})\left(3x_\alpha x_\beta - \delta_{\alpha\beta}\, x_\gamma^2\right) d\underset{\sim}{x} \tag{2}$$

Angular momentum may also be lost through gravitational radiation. the rate is given by (Peters, 1964)

$$-\frac{dL_i}{dt} = \frac{2G}{45c^5}\,\epsilon_{ijk}\,\ddot{\mathcal{D}}_{jm}\,\dddot{\mathcal{D}}_{km} \tag{3}$$

---

* Greek letters range from 1 to 3, and summation over repeated indices is understood, unless stated otherwise.

where L_i is the i-th component of the angular momentum vector $\underset{\sim}{L}$ and ϵ_{ijk} is a completely antisymmetric unit pseudo-tensor.

Makung use of these equations, we calculate the loss of energy and angular momentum from various oscillationg and rotating systems. From this we estimate the damping time of the motion.

ROTATING ELLIPSOID

Gravitational radiation from a rotating ellipsoid of mass m, uniform density ρ and semi-axes $(a_1 a_2 a_3)$ has been calculated. (C. W. Chin, 1965) We give here a simpler derivation, which will be useful for us later.

Let the angular velocity $\underset{\sim}{\Omega}$ be in the z-direction. In the body set of axes $(x'y'z')$, all off-diagonal elements of the quadrupole moment about the origin vanish because of the reflection symmetry. The diagonal elements are given by

$$D'_{ii} = \frac{m}{5}\left(3a_i^2 - a_\alpha a_\alpha\right) \tag{4}$$

where i=1, 2, 3 and is <u>not</u> summed over.

In terms of the space set (xyz), we have

$$\begin{aligned} x' &= x\cos\Omega t + y\sin\Omega t \\ y' &= -x\sin\Omega t + y\cos\Omega t \\ z' &= z \end{aligned} \tag{5}$$

while for D_{ij} in the space set, we have

$$D_{ij} = D'_{\alpha\beta}\frac{\partial x'_\alpha}{\partial x_i}\frac{\partial x'_\beta}{\partial x_j} \tag{6}$$

Further reductions lead to

$$-\frac{d\mathcal{E}}{dt} = \frac{32}{125}\frac{G}{c^5} m^2\Omega^6 (a_2^2 - a_1^2)^2 \tag{7}$$

If the configuration does not differ much from that of a sphere of radius R, we can put

$$a_2 - a_1 = \eta R \tag{8}$$

where η is a small quantity. Equation (7) can then be written as

$$-\frac{d\mathcal{E}}{dt} \approx \frac{128}{125}\frac{G}{c^5} m^2\Omega^6 R^4 \eta^2 \tag{9}$$

The rotational energy of the ellipsoid may be calculated classically and is given by

$$\mathcal{E} = \frac{1}{10} m(a_1^2 + a_2^2)\Omega^2 \approx \frac{m}{5} R^2\Omega^2 \tag{10}$$

Substituting this into equation (7), we readily find

$$\Omega^4 = \frac{\Omega_0^4}{1 + Kt} \tag{11}$$

where

$$K = \Omega_0^4 \frac{128}{25}\frac{Gm}{c^5}\frac{(a_1^2 - a_2^2)^2}{(a_1^2 + a_2^2)} \tag{12}$$

$$\approx \Omega_0^4 \frac{256}{25}\frac{Gm}{c^5}\eta^2 R^2$$

with η small. Ω_0 is the angular velocity at t=0.

In the case of a Jacobi ellipsoid, η^2 also depends on Ω^2, and hence eqn. (11) is no longer completely correct. We also remark that it is not clear whether a rotating neutron star can be a Jacobi ellipsoid.

NON-RADIAL OSCILLATIONS OF A SPHERE

Consider an axisymmetric oscillation of a spherical mass of incompressible fluid of constant density ρ and radius R. The boundary surface can be described by

$$r(\theta) = R\left\{1 + \alpha_2 P_2(\theta) + \ldots + \alpha_n P_n(\theta)\right\} \tag{13}$$

where the α's are functions of time. We assume $\alpha_n \ll 1$. The P_n's are Legendre functions. Because the fluid is of constant density, equation is sufficient for the calculation of the D_{ij}. Only one diagonal element need be evaluated sinee the off-diagonal elements vanish because of the axial symmetry and since $D_{11} = D_{22} = -\frac{1}{2} D_{33}$ because the trace of $D_{\alpha\beta}$ vanishes. We find

$$D_{33} = \frac{8\pi\rho}{5} R^5 \left\{\alpha_2 + \sum_{n=2\ldots} \frac{10\alpha_n^2\, n(n+1)}{(2n-1)(2n+1)(2n+3)} + \sum_{n=2\ldots} \frac{30\alpha_n \alpha_{n+2}(n+1)(n+2)}{(2n+1)(2n+3)(2n+5)} + O(\alpha_n^3)\right\} \tag{14}$$

To find the energy loss, we assume:

$$\alpha_n = \alpha_{n0} \sin \sigma_n t \tag{15}$$

where α_{n0} is the amplitude, assumed to vary only slowly with time so that in calculating $\dddot{D}_{ij}$ we can treat it as constant.

Up to terms linear in α_n, P_2 is the only contributing mode. We then find for the energy loss from the P_2-mode, averaged over a full cycle in this linearised approximation

$$-\frac{d\mathcal{E}_2}{dt} = \frac{3}{125} \frac{G}{c^5} m^2 R^4 \sigma_2^6 \alpha_{20}^2 \tag{16}$$

Now the energy $\mathcal{E}_n$ of the P_n oscillation is given by (Rayleigh 1945)

$$\mathcal{E}_n = \pi \rho R^5 \frac{1}{n(2n+1)} \alpha_{n0}^2 \sigma_n^2 \tag{17}$$

From equations (16) and (17), we have

$$\alpha_{20}(t) = \alpha_{20}(0) e^{-K_2 t} \tag{18}$$

where

$$K_2 = \frac{4}{25} \frac{GmR^2}{c^5} \sigma_2^4 \tag{19}$$

We thus see that the P_2 mode oscillation is damped exponentially. Thus after a short time, only the higher modes remain. From equation (14), we see that the coefficient of the coupled term $\alpha_n \alpha_{n+2}$ is of the same order of magnitude as the square term α_n^2. Thus, for the mode with the largest amplitude, we can neglect the cross term and take the square term to be the only contribution to $\mathcal{D}_{33}$, in which case the energy loss expression from this n-th mode becoms

$$-\frac{d\mathcal{E}_n}{dt} = \frac{G}{c^5}\ \frac{1024}{15}\left[\frac{\pi\rho n(n+1)}{(2n-1)(2n+1)(2n+3)}\right]^2 \sigma_n^6 \alpha_{n0}^4 R^{10} \qquad (20)$$

Equations (17) and (20) then would give

$$\alpha_{n0}^2(t) = \frac{\alpha_{n0}^2(0)}{1+k_n \alpha_{n0}^2 t} \qquad (21)$$

where

$$k_n = 256\ \frac{GmR^2\sigma_n^4}{c^5}\ \frac{n^3(n+1)^2}{(2n-1)^2(2n+1)(2n+3)^2} \qquad (22)$$

Thus, the higher modes would be damped at a much slower rate than the P_2 -mode. However, in the non-linear domain, the dynamic coupling between the various modes should be considered. This coupling could give a stronger damping because of energy transfer from the higher modes into the P_2 mode. Furthermore, the oscillations would not be purely harmonic as given by equation (15). Thus our analysis should be interpreted as giving an essentially qualitative description of the actual picture.

Suppose now the above pulsating object is also rotating slowly. To the first order in Ω/σ, the energy loss remains unchanged because it should be even in Ω. Angular momentum, however, will be lost at a rate given by equation (3) if the rotation is about an axis other than that of symmetry, say the x-axis. The loss rate can then be readily calculated using the same technique that led to equation (7). The rèsult when we consider only the P_2 -mode is

$$-\frac{dL_1}{dt} = \frac{9}{25}\frac{Gm^2}{c^5}R^4\alpha_{20}^2\sigma_2^4\Omega \qquad (23)$$

For higher modes, the numerical coefficient would be different and α_{n0}^4 would replace α_{20}^2. From equation (23), it is seen that for small (Ω/σ), the rotation is damped much more slowly than the pulsation. By the time the pulsation becomes insignificant, mush of the rotation would still persist.

THE EFFECT OF ROTATION ON RADIAL OSCILLATIONS

In the foregoing we have been considering only the directly radiating modes. A purely radial mode does not radiate; hence energy might be stored indefinitely in the mode, at least if neutrino processes and the like are disregarded. However, rotation would destroy the spherical symmetry of the system, again leading to gravitational radiation.

In this section we consider these effects of rotation taking all terms of order $(\Omega/\sigma)^2$ into account. We consider a sphere of radius R and of uniform density pulsating in the lowest radial mode when the Lagrangian displacement is given by $\xi_r = \alpha r e^{i\sigma_0 t}$ with α a constant. If this sphere is given a small uniform rotation, the oscillations are altered both because the equilibrium shape is changed and because the pulsation equations include centrifugal and coriolis force terms.

When axisymmetric perturbations are considered, the linearized equation of motion for $\underset{\sim}{\xi}$ in a rotating frame can be written as in spherical polar coordinates (Ledoux and Walraven, 1958)

$$-\sigma^2 \xi_r - 2i\sigma\Omega \sin\theta\, \xi_\phi = -\frac{\partial \Phi'}{\partial r} + \frac{\rho'}{\rho^2}\frac{\partial p}{\partial r} - \frac{1}{\rho}\frac{\partial p'}{\partial r} \tag{24}$$

$$-\sigma^2 \xi_\theta - 2i\sigma\Omega \cos\theta\, \xi_\phi = -\frac{1}{r}\frac{\partial \Phi'}{\partial \theta} - \frac{1}{\rho r}\frac{\partial p'}{\partial \theta} + \frac{\rho'}{\rho^2 r}\frac{\partial p}{\partial \theta} \tag{25}$$

$$-\sigma^2 \xi_\phi + 2i\sigma\Omega\left(\xi_r \sin\theta + \xi_\theta \cos\theta\right) = 0 \tag{26}$$

Here unprimed quantities indicate the equilibrium values of a uniformly rotating spheroid of constant density, and the primed ones are the Eulerian perturbed values. $\underset{\sim}{\xi}$ is a Lagrangian displacement with an assumed time dependence $e^{i\sigma t}$.

To solve these equations, we expand $\underset{\sim}{\xi}$ and σ^2 in powers of Ω retaining terms up to those of order Ω^2 :

$$\begin{aligned} \sigma^2 &= \sigma_0^2 + \Omega\sigma_1^2 + \Omega^2\sigma_2^2 \\ \xi_r &= \xi_{r0} + \Omega\xi_{r1} + \Omega^2\xi_{r2} \\ \xi_\theta &= \qquad\quad \Omega\xi_{\theta 1} + \Omega^2\xi_{\theta 2} \\ \xi_\phi &= \qquad\quad \Omega\xi_{\phi 1} + \Omega^2\xi_{\phi 2} \end{aligned} \tag{27}$$

For oscillations that are originally axisymmetric, $\sigma_1 = 0$. (Clement, 1965)

Also the right-hand side of (25) is of order Ω^2, because the pressure and the density perturbations should be independent of the sign of Ω. Therefore by substituting (27) into (25) and (26), and comparing terms of different orders in Ω, we obtain

$$\begin{aligned} \xi_{\theta 1} &= \xi_{r1} = 0 \\ \Omega\, \xi_{\phi 1} &= 2i \frac{\Omega}{\sigma} \sin\theta\, \xi_{r0} \\ \xi_{\phi 2} &= 0 \\ \sin\theta\, \xi_{r2} + \cos\theta\, \xi_{\theta 2} &= 0 \end{aligned} \tag{28}$$

To solve for ξ_{r2} or $\xi_{\theta 2}$, we now turn to equation (25). The right-hand side of the equation involves the perturbed quantities ρ', p', and Φ'. These can be expressed in terms of the equilibrium values by making use of the continuity equation, Poisson's equation and the adiabatic relation between pressure and density variations respectively, as follows:

$$\begin{aligned} \rho' &= -\rho \operatorname{div} \boldsymbol{\xi} \\ \nabla^2 \Phi' &= -4\pi G \rho \operatorname{div} \boldsymbol{\xi} \\ p' &= -\gamma p \operatorname{div} \boldsymbol{\xi} - \boldsymbol{\xi} \cdot \boldsymbol{\nabla} p \end{aligned} \tag{29}$$

In equation (29), the equilibrium quantities are again those of a uniformly rotating mass of fluid of constant density. Limiting ourselves to small rotation, the pressure is given by (Lamb, 1932; Chandrasekhar 1962)

$$\frac{p}{\rho} = \frac{1}{2}\Omega^2 r^2 \sin^2\theta - \pi G\rho \Big\{ \frac{2}{3}r^2 - \frac{2}{3}R^2\big(1 - \frac{2}{3}e^2\big) - \frac{4}{15}e^2 R^2 + \frac{2}{15}e^2 r^2 (2 - 3\sin^2\theta) \Big\} \tag{30}$$

where we have made p vanish on the boundary surface

$$r(\theta) = R\Big[1 + e^2\big(\frac{1}{2}\sin^2\theta - \frac{1}{3}\big)\Big] \tag{31}$$

The semi-axis (a_1 , $a_2 = a_1$, a_3) of the rotating ellipsoid is related to R by

$$a_1 = R\big(1 + \frac{1}{6}e^2\big)$$

$$a_3 = R\big(1 - \frac{1}{3}e^2\big) \tag{32}$$

where e is the eccentricity given by

$$e^2 = \frac{15\,\Omega^2}{8\pi G\rho} \tag{33}$$

By making use of (28) , (29) , and (30) , we can rewrite equation

(25) to the second order of Ω :

$$-\sigma_0^2 \xi_{\theta 2} + 2\sin 2\theta \, \alpha r = -\frac{1}{r}\frac{\partial \Phi_2'}{\partial \theta} - \frac{15}{4}\alpha r \sin 2\theta$$
$$+ \frac{1}{r}\left\{ \frac{15}{4}\alpha r^2 \sin 2\theta \left(\gamma + \frac{2}{3}\right) - \frac{4}{3}\pi G \rho r \frac{\partial \xi_{r2}}{\partial \theta} \right.$$
$$\left. - \gamma \pi G \rho \left(\frac{2}{3} r^2 - \frac{2}{3} R^2\right) \frac{\partial}{\partial \theta} \operatorname{div}(\underset{\sim}{\xi}) \right\} \tag{34}$$

An examination of (34) indicates that in the desired solution, the θ - dependence of $\xi_{\theta 2}$ may be taken as $\sin 2\theta$. Thus by also making use of equation (28) , we can write

$$\xi_{\theta 2} = f(r) \sin 2\theta$$
$$\xi_{r2} = -2 f(r) \cos^2 \theta \tag{35}$$

and for the internal gravitational potential:

$$\Phi_2(r, \theta) = g_0(r) + g_2(r) P_2(\theta) \tag{36}$$

with $g_0(r)$, $g_2(r)$ and $f(r)$ still to be determined.

By substituting (35) and (36) into (34) , we would then have

$$-\sigma_0^2 f(r) + 2\alpha r = \frac{3}{2}\frac{g_2(r)}{r} - \frac{15}{4}\alpha r + \frac{15}{4}\alpha r\left(\gamma + \frac{2}{3}\right)$$

$$-\frac{8}{3}\pi G\rho f(r) - \frac{1}{r}\gamma\pi G\rho\left(\frac{2}{3}r^2 - \frac{2}{3}R^2\right)\frac{\partial}{\partial\theta}\operatorname{div}\underset{\sim}{\xi}\ \frac{1}{\sin 2\theta}$$

$$(37)$$

By inspection, we see that $f(r) = \lambda r$ gives a solution provided that λ satisfies

$$\lambda\left[\frac{8}{3}\pi G\rho - \sigma_0^2\right] = \frac{3}{2}k_2 - \frac{13}{4}\alpha + \frac{15}{4}\alpha\gamma \qquad (38)$$

where we have made use of the fact that in this case $\frac{\partial}{\partial\theta}\operatorname{div}\underset{\sim}{\xi} = 0$ while from Poisson's equation, we have $g_2(r) = k_2 r^2$

The constant k_2 can be determined in a straightforward manner by demanding that the gravitational potential and its derivatives are continuous on the disturbed boundary. The result is

$$k_2 = \left(\frac{16}{9}\pi G\rho\lambda - \frac{299}{30}\alpha\right) \qquad (39)$$

which therefore gives a λ from equation (38)

$$\lambda = \frac{\alpha\left(\frac{1092}{60} - \frac{15}{4}\gamma\right)}{\sigma_0^2} \qquad (40)$$

Summing up, our solution for the Lagrangian displacement is:

$$\xi_r = \alpha r - 2\Omega^2 \lambda r \cos^2\theta$$
$$\xi_\theta = \Omega^2 \lambda r \sin 2\theta$$
$$\xi_\phi = 2i \frac{\Omega}{\sigma} \alpha r \sin\theta \tag{41}$$

and the new boundary becomes:

$$r'(\theta) = r(\theta)\left\{1 + \alpha \cos\sigma t - 2\Omega^2 \lambda \cos^2\theta \cos\sigma t\right\} \tag{42}$$

with $r(\theta)$ given by equation (31).

We remark here that although our solutions are obtained by inspection from the θ-equation (25), they satisfy all the boundary conditions. Furthermore, equation (24) gives an expression for σ_2^2 :

$$-\alpha \sigma_2^2 + 2\sigma_0^2 \lambda = -2k_2 + (1-3\gamma)\alpha + \frac{8}{3}\gamma \pi G \rho \lambda \tag{43}$$

On substituting for k_2 and λ the values as given by equations (39) and (40), we have

$$\sigma_2^2 = \frac{2}{3}(5 - 3\gamma) \tag{44}$$

which agrees with the corresponding expression of Ledoux. (Ledoux, 1945)

With (41) and (42) , the quadrupole moment tensor can be readily evaluated, the time-dependent part of which is :

$$\mathcal{D}_{33} = -\frac{8}{15}\pi\rho\alpha R^5 \cos\sigma t \left(\frac{45}{4}\gamma - \frac{9}{5}\right)\frac{\Omega^2}{\sigma_0^2} \qquad (45)$$

The time-averaged energy loss is then given by equation (1) as:

$$-\frac{d\mathcal{E}}{dt} = \frac{1}{375}\frac{G}{c^5} m^2 R^4 \alpha^2 \sigma_0^6 \left(\frac{45}{4}\gamma - \frac{9}{5}\right)^2 \left(\frac{\Omega^2}{\sigma_0^2}\right)^2 \qquad (46)$$

Since no angular momentum is lost in this case, the angular velocity Ω will remain constant. Thus, all the energy loss is at the expense of oscillation. With ξ given by equation (41) , the total pulsation energy is:

$$\mathcal{E} = \frac{3}{20} m R^2 \alpha^2 \sigma_0^2 \qquad (47)$$

A comparison of (46) and (47) shows that the energy decreases exponentially given by

$$\alpha(t) = \alpha(0)\, e^{-K_0 t} \qquad (48)$$

where

$$K_0 = \frac{2}{225}\frac{G m R^2}{c^5}\Omega^4 \left(\frac{45}{4}\gamma - \frac{9}{5}\right)^2 \qquad (49)$$

NUMERICAL ESTIMATES AND CONCLUSION

We now assemble the various expressions for the damping of oscillation, using data relevant to a neutron-star,* mass $= 1\, M_\odot$, $R = 10^6$ cm. For the oscillation frequency, we use $\sigma = 3 \times 10^3$ cps for the radial mode (Tsuruta, Wright and Cameron, 1965) and the expression for an incompressible fluid for the non-radial ones (Lamb, 1932)

$$\sigma_n^2 = \frac{2n(n-1)}{(2n+1)} \frac{4}{3} \pi G \rho \tag{50}$$

For the P_2 -oscillation, equations (18) and (19) gives:

$$\alpha_{20}(t) = \alpha_{20}(0)\, e^{-K_2 t}$$

where

$$\begin{aligned} K_2 &= \frac{4}{25}\left(\frac{G M_\odot}{c^5}\right) \frac{m}{M_\odot} R^2 \sigma_2^4 \\ &= (8.8 \times 10^{-28}) \frac{m}{M_\odot} R^2 \sigma_2^4 \\ &= 1.2 \times 10^1 \ \mathrm{sec}^{-1} \end{aligned} \tag{51}$$

For P_4 -oscillation, equations (21) and (22) give:

$$\alpha_{40}^2(t) = \frac{\alpha_{40}^2(0)}{1 + K_4\, \alpha_{40}^2\, t}$$

where

$$\begin{aligned} K_4 &= (4.1 \times 10^{-26}) \frac{m}{M_\odot} R^2 \sigma_4^4 \\ &= 4.3 \times 10^3 \ \mathrm{sec}^{-1} \end{aligned} \tag{52}$$

---

* The problem of gravitational radiation from neutron stars has been considered by a number of investigators, the first of whom we wish to acknowledge is Prof. J.A. Wheeler.

For rotation and nearly radial oscillations, equations (48) and (49) give:

$$\alpha(t) = \alpha(o)\, e^{-k_o t}$$

where

$$K_o = (1.1 \times 10^{-26}) \frac{m}{m_\odot} R^2 \Omega^4$$
$$= 1.1 \times 10^{-14} \Omega^4 \qquad (53)$$

In equation(53), we have used a $\gamma \sim 1.5$, as a crude compromise between $\gamma = 4/3$ for an extremely relativistic, completely degenerate fermi gas and $\gamma = 5/3$ for a perfect, monoatomic gas. We also remark that K_o is independent of the oscillation frequency in contrast to K_2 and K_4.

From these expressions, we can easily obtain an upper limit to the total oscillation energy after any lenghth of time by taking the oriiginal amplitude to be unity. We remember that we have assumed uniform density for the equilibrium configuration in our work, which is of course not true for a neutron-star. However, our calculations would still give a very good idea under what conditions the oscillation energy can still be large enough to be of interest in phenomena in supernova remanants. This can be compared with the result of Finzi(Finzi, 1965), who showed that radial oscillatons can be effectively damped by the ν -processes at a rate given by:

$$-\frac{d\mathcal{E}}{dt} = 8.38 \times 10^{18} \times \alpha^8 \text{ ergs gm}^{-1} \text{ sec}^{-1} \qquad (54)$$

~~The results of damping in the various cases are tabulated as follows:~~

The results of damping in the various cases are tabulated as follows:

Mode	Energy in ergs at time			
	Initial ($\alpha = 1$)	1 day	1 year	10^3 years
P_0 ($\Omega = 10^2\ sec^{-1}$)	2.7×10^{51}	2.3×10^{51}	6.0×10^{22}	0
P_0 ($\Omega = 10\ sec^{-1}$)	2.7×10^{51}	2.7×10^{51}	2.7×10^{51}	4.1×10^{48}
P_0 ($\Omega = 1\ sec^{-1}$)	2.7×10^{51}	2.7×10^{51}	2.7×10^{51}	2.7×10^{51}
P_0 ($\Omega = 0$, ν-process)	2.7×10^{51}	2.1×10^{49}	3.0×10^{48}	3.0×10^{47}
P_2	1.8×10^{52}	0	0	0
P_4	1.4×10^{52}	3.8×10^{43}	1.1×10^{41}	1.1×10^{38}

From the table, we thus see that the only significant surviving mode after a time of 10^3 years would be radial modes if $\Omega \lesssim 10\ sec^{-1}$. By comparison, the angular velocity of bifurcation for an incompressible object would be about $3.5 \times 10^3\ sec^{-1}$, while the centrifugal acceleration at the equator would become comparable to the gravitational acceleration for Ω about equal to $1.4 \times 10^4\ sec^{-1}$

The small angular velocity required for effective energy storage poses a serious problem with regard to the angular momentum of the neutron star. If we assume that the neutron star is formed by contraction from an original star of mass $= 1 M_\odot$ and radius $\approx 10^{11} cm$, then if angular momentum is conserved, the angular velcoity of the original star would have to be $10^{-9}sec^{-1}$, much slower than that for normal stars. It thus appears that a considerable amount of angular momentum has to be lost for an effective energy storage in the radial modes. Of course, angular momentum can be lost through gravitational radiation. The rate, however, is extremely insignificant unless the neutron star forms a Jacobi ellipsoid of large asymmetry.

The research reported in this paper has in part been supported by the National Aeronautics and Space Administration under Contract NGR 33-008-053, by the Air Force Office of Scientific Research under contract AF 49 (638) 1358, and by a John Tyndall Fellowship from Columbia University during the academic year 1965-66.

I wish to thank Professor L. Woltjer, who suggested the problem and very patiently and constantly gave guidance till its completion.

BIBLIOGRAPHY

Chandrasekhar, S. Ap. J. 136, 1037, (1962)

Chin, C.W. P.R. 139, B761, (1965)

Clement, M.J. Ap. J. 141, 210, (1965)

Finzi, A.. P.R.L. 15, 599, (1965)

Lamb, H. Hydrodynamics, Cambridge University Press, 6th Edition, 1932

Landau and Lifshitz, The Classical Theory of Fields, Pergaman Press, 1962, section 104

Ledoux, P. Ap. J. 102, 149, (1945)

Ledoux and Walraven, Hdb. d. Phys., ed. S. Flugge (Berlin Springer-Verlag) 51, 527, (1958)

Peters, P.C. P.R. 136, 1229, (1964)

Rayleigh, J. The Theory of Sound, Dover Ed., section 364, (1945)

Tsuruta, S., Wright, J., and Cameron, A.G.W. Nature, 206, 1137, (1965)