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THE DYNAMIC BEHAVIOR OF A FOIL IN THE
PRESENCE OF A LUBRICATING FILM
by
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ABSTRACT

Equations for the oscillations of a foil over a lubricating fluid film are derived and are simplified by a small parameter expansion. A few particular cases are discussed, and a linearized solution is obtained for the case of a massless, perfectly flexible foil moving at a speed U over an incompressible film. The solution reveals the phenomenon that small disturbances in the film thickness, as well as symmetrical large disturbances, propagate at a speed U/2.

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NOMENCLATURE

a   Half square wavelet width

$\bar{a}$ Normalized half square wavelet width $\equiv a/\tau^{1/4}$

a_n Foil acceleration component normal to local foil direction

a_s Foil acceleration component along local foil direction

A   Height of square wavelet

C   Compressibility parameter $\equiv \dfrac{p_a \, h_o}{T_o} \, \epsilon^{-2/3}$

d   Local foil thickness

d_o Unstretched foil thickness

$\bar{d}$ Dimensionless foil thickness d/d_o

D   Flexural rigidity of tape per unit width

E   Modulus of elasticity

f   Small perturbation $\equiv H - 1$

f_o Initial small disturbance

F   Fourier transform of f

F_o Transform of initial disturbance

h   Clearance

h_o Asymptotic clearance

H	Dimensionless clearance $\equiv h/h_o$
j,k,ℓ,m,n	Exponential measures
M	Bending moment per unit width
$\bar{M}$	Normalized moment
M_p	Mass parameter $\equiv \dfrac{\sigma_o U_o}{12\mu}\epsilon$
p	Pressure under foil
p_a	Ambient pressure
Q	Shear force per unit width
R	Local radius of curvature of foil
s	Distance along tape; Fourier transform parameter
$\bar{s}$	Normalized distance along tape $\equiv \dfrac{s}{h_o}\epsilon^{1/3}$
S_p	Stiffness parameter $\equiv \dfrac{D\,\epsilon^{2/3}}{T_o\,h_o{}^2}$
T	Tension per unit width of foil
T_o	Tension per unit width of foil at $s = \infty$
$\bar{T}$	Normalized tension $\equiv T/T_o$
T_p	Tension parameter $\equiv \dfrac{T_o}{E\,d_o}$
t	Time
U	Foil velocity component normal to local foil direction
U_o	Foil velocity at $s = \infty$
$\bar{U}$	Dimensionless velocity component in foil direction $\equiv \dfrac{U}{U_o}$
W	Foil velocity component normal to local foil direction

$\overline{W}$ Normalized velocity component normal to foil

direction $\equiv \dfrac{W}{U_o} \epsilon^{-1/3}$

x   Longitudinal coordinate

ϵ Dimensionless parameter $\dfrac{6\mu\, U_o}{T_o}$

χ Dimensionless curvature

λ $\equiv 2\pi\,/$(wavelength of Fourier component)

$\overline{\lambda}$ $\lambda\,\tau^{1/4}$

μ Viscosity

ν Poisson ratio

π Dimensionless pressure $\dfrac{p - p_a}{T_o/h_o} \epsilon^{-2/3}$

σ Foil mass per unit area

σ_o Unstretched foil mass per unit area

$\overline{\sigma}$ Dimensionless foil mass per unit area

τ Fluid shear traction on foil; dimensionless time

$\dfrac{t\, U_o}{2\, h_o} \epsilon^{-1/3}$

θ Slope

ξ Normalized longitudinal coordinate $\equiv \dfrac{x}{h_o} \epsilon^{1/3}$

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1.0 INTRODUCTION

The dynamics of a flexible foil serving as a boundary for the flow of a thin fluid film is a problem of considerable interest and application. The use and manufacture of magnetic tapes, paper, and plastic foils are some of the fields in which such applications arise.

Previous studies of this elasto-hydrodynamic problem were confined to steady state. It is the purpose of this paper to formulate and obtain a solution for a simple model involving transient phenomena in order to gain understanding and make it possible to interpret and devise techniques for the solution of more complex configurations which arise in practice.

2.0 DESCRIPTION OF THE MODEL

Consider an infinitely wide foil stretched between two far apart guides (Fig. 1) and moving at a very small distance, h_o, from a solid surface. Neglecting gravity, the equilibrium configuration of the foil may be found to be a straight line connecting the supports. At time $t = 0$, a disturbance in the shape of the foil is introduced, and the problem is to find its subsequent history.

The coordinates used in the mathematical description of the problem are t (time) and x (stationary position coordinate measured along the solid surface). Vectorial resolutions into components are made <u>along</u> the instantaneous positions of the foil (subscript s) and <u>normal</u> to it (subscript n). Some of the equations are more conveniently derived using the foil length s as a coordinate, rather than the stationary coordinate x.
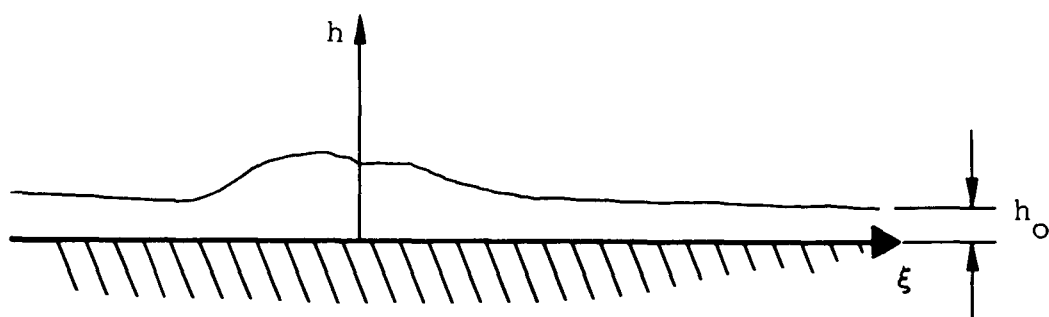
Fig. 1   Schematic View of Problem Under Consideration

The conversion between the two coordinates is

$$\left(\frac{\partial s}{\partial x}\right)_t = \left[1 + \left(\frac{\partial h}{\partial x}\right)_t^2\right]^{1/2} \tag{1}$$

where h is the clearance.

3.0 ASSUMPTIONS

(1) The flow is assumed to be planar.

(2) The fluid velocity profile is parabolic, and the fluid inertia is neglected.

(3) For consistency with assumption (1), the approximation is made that the lateral strain in the foil is zero.

(4) The stresses normal to the foil and the effect of the transverse shears on the deformation are neglected in analogy with the theory of plates.

(5) The rotational inertia of the foil is neglected.

4.0 BASIC EQUATIONS

Using assumptions (1) and (2), the fluid continuity is expressed by the Reynolds equation,

$$\frac{\partial}{\partial x}\left(h^3 p \frac{\partial p}{\partial x}\right) = 6\mu \frac{\partial(u\,ph)}{\partial x} + 12\mu \frac{\partial(ph)}{\partial t} \tag{2a}$$

where p, u are the pressure and the tangential component of the tape speed, respectively, which are functions of position and time. Based on

the same approximation is the expression for the fluid shear tractions on
the foil

$$\mathcal{T} = -\frac{h}{2}\frac{\partial p}{\partial x} - \mu\frac{U}{h}$$

(2b)

The three dynamic equations on an element of foil are:

$$p - p_a - \frac{T}{R} + \frac{dQ}{ds} = \sigma a_n$$

(2c)

$$\frac{dT}{ds} + \frac{Q}{R} + \mathcal{T} = \sigma a_s$$

(2d)

$$\frac{dM}{ds} - Q = 0$$

(2e)

The right hand side of the last equation vanishes due to
assumption (5). In the above equations $T, \sigma, Q, M, R, a_n, a_s$ denote
tension per unit width, foil mass per unit area, normal shear force,
bending moment, radius of curvature, normal and tangential acceleration
components, respectively. The aforementioned variables are functions of
position and time. With assumptions (3) and (4), the stress-strain
relations result in the equations,

$$d = d_o - \frac{\nu(1+\nu^2)}{E} T$$

(2f)

$$\frac{1-\nu^2}{E}\frac{\partial\left(\frac{T}{d}\right)}{\partial t} = \frac{\partial U}{\partial s}$$

(2g)

$$M = \frac{D}{R}$$

(2h)

where d is the local tape thickness which is a function of position and time, and D is the flexural rigidity per unit width:

$$D \equiv \frac{E d^3}{12(1 - \nu^2)}$$

The continuity equation for the tape is

$$\frac{\partial}{\partial x}\left(\sigma \cdot d\left[U \cos \theta - W \sin \theta\right]\right) = -\frac{\partial}{\partial t}\left(d \cdot \sigma\right) \tag{2j}$$

where $\theta = \theta(x, t)$ is the slope of the foil and $W(x, t)$ is the normal component of the tape speed. The acceleration components of an element of foil are*:

$$a_t = \frac{DU}{Dt} - \frac{D\theta}{Dt} W \tag{2ℓ}$$

$$a_n = \frac{DW}{Dt} + \frac{D\theta}{Dt} U \tag{2m}$$

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* The operator Df/Dt denotes the convective derivative, i.e., the rate of change of the property f following a material point of the tape. In (x, t) and (s, t) expressions, this becomes, respectively:

$$\frac{Df}{Dt} \equiv \left(\frac{\partial f}{\partial x}\right)_t \left(U \cos \theta - W \sin \theta\right) + \left(\frac{\partial f}{\partial t}\right)_x$$

$$\equiv \left(\frac{\partial f}{\partial s}\right)_t \left(\frac{Ds}{Dt}\right) + \left(\frac{\partial f}{\partial t}\right)_s$$

Finally, the following kinematical relations are needed to complete the formulation:

$$\frac{1}{R} = - \frac{\frac{\partial^2 h}{\partial x^2}}{\left[1 + \left(\frac{\partial h}{\partial x}\right)^2\right]^{3/2}} \tag{2n}$$

$$\frac{Dh}{Dt} = U \sin\theta + W \cos\theta \tag{2o}$$

$$\left(\frac{\partial h}{\partial x}\right)_t = tg\,\theta \tag{2p}$$

5.0 ANALYSIS

In the following, the equations will be nondimensionalized and then simplified by means of an asymptotic analysis. The analysis revolves around the dimensionless group $\epsilon = \dfrac{6\mu\, U_o}{T_o}$ which in view of past experience[1], is chosen as a perturbation parameter. The dependent variables will be normalized with respect to their characteristic magnitude so that the new dimensionless quantity will be of order unity. The independent variables will be normalized with respect to their characteristic interval of change so that differentiation with respect to the resulting dimensionless variable will not change the order of magnitude of the differentiated dimensionless variable. Those characteristic dimensions which are unknown a priori are formed with the aid of unknown exponents which are to be determined later.

Thus,

$$\bar{U} \equiv \frac{U}{U_o} \tag{3a}$$

$$\bar{d} \equiv \frac{d}{d_o} \tag{3b}$$

$$\bar{G} \equiv \frac{G}{G_o} \tag{3c}$$

$$H \equiv \frac{h}{h_o} \tag{3d}$$

$$\bar{T} \equiv \frac{T}{T_o} \tag{3e}$$

$$\Pi \equiv \frac{p - p_a}{\frac{T_o}{h_o} \epsilon^k} \tag{3f}$$

$$\chi \equiv \frac{h_o / \epsilon^n}{R} \tag{3g}$$

$$\bar{W} \equiv \frac{W}{U_o \epsilon^\ell} \tag{3h}$$

$$\tau \equiv \frac{t}{\frac{2h_o}{U_o}/\epsilon^j} \tag{3i}$$

$$\bar{s} \equiv \frac{s}{h_o/\epsilon^m} \tag{3j}$$

$$\xi \equiv \frac{x}{h_o/\epsilon^m} \tag{3k}$$

The new variables Eq. (3) are substituted in Eqs. (2). In addition, Eq. (2h) is substituted in Eq. (2e) and Eqs. (2b), (2e), (2 l), (2m) into Eqs. (2c), (2d). Finally, Eq. (2p) is expanded into a trigonometric series as follows:

$$\theta = arc\, tg\left(\epsilon^m \frac{\partial H}{\partial \xi}\right) = \epsilon^m \frac{\partial H}{\partial \xi} - \frac{\epsilon^{3m}\left(\frac{\partial H}{\partial \xi}\right)^3}{3} + \cdots \tag{4}$$

and the transformation

$$\left(\frac{\partial}{\partial \bar{s}}\right)_t \equiv \left[1 - \frac{1}{2}\frac{\partial H}{\partial \xi}\epsilon^{2m}\cdots\right]\frac{\partial}{\partial \xi} \tag{5}$$

is applied. With these changes, Eqs. (2) become:

$$\epsilon^{2m+2k}\frac{\partial}{\partial\xi}\left(H^3\left[\pi+\frac{p_a h_o}{T_o \epsilon^k}\right]\frac{\partial\pi}{\partial\xi}\right) = \epsilon^{1+m+k}\frac{\partial\left(\bar{U}\left[\pi+\frac{p_a h_o}{T_o \epsilon^k}\right]H\right)}{\partial\xi} + \epsilon^{1+k+j}\frac{\partial\left(\pi+\frac{p_a h_o}{T_o \epsilon^k}\right]H)}{\partial\tau} \tag{6a}$$

$$\frac{\partial\bar{T}}{\partial\xi}\left[1-\tfrac{1}{2}\left(\frac{\partial H}{\partial\xi}\right)^2\epsilon^{2m}\cdots\right]+\tfrac{1}{2}\frac{D\epsilon^{2n}}{T_o h_o^2}\frac{\partial^2\chi}{\partial\xi}\left[1-\tfrac{1}{2}\left(\frac{\partial H}{\partial\xi}\right)^2\epsilon^{2m}\cdots\right]-\frac{\epsilon^k}{2}H\frac{\partial\pi}{\partial\xi}-\frac{\epsilon^{l-m}}{6}\frac{\bar{U}}{H} = \frac{\sigma_o U_o}{12\mu}\epsilon^{1+j-m}\bar{a}\left[\frac{D\bar{U}}{Dt}\frac{D}{Dt}\left(\frac{\partial H}{\partial\xi}\right)\bar{w}\epsilon^{l+m}\right] \tag{6b}$$

$$\pi-\epsilon^{n-k}\bar{T}\chi + \frac{D\epsilon^{n+2m-k}}{T_o h_o^2}\frac{\partial^2\chi}{\partial\xi^2}\left[1+\cdots\right] = \frac{\sigma_o U_o}{12\mu}\epsilon^{1+j-k}\bar{a}\left(\frac{D\bar{w}}{D\tau}\epsilon^l + \frac{D}{D\tau}\left(\frac{\partial H}{\partial\xi}\right)\bar{U}\epsilon^m+\cdots\right) \tag{6c}$$

$$\bar{d} = 1 - \nu(1+\nu^2)\frac{T_o}{Ed_o}\bar{T} \tag{6d}$$

$$\frac{1-\nu^2}{2}\frac{T_o}{Ed_o}\frac{\partial\left(\frac{\bar{T}}{\bar{d}}\right)}{\partial\tau} = \epsilon^{m-j}\frac{\partial\bar{U}}{\partial\xi}\left[1-\tfrac{1}{2}\frac{\partial H}{\partial\xi}\epsilon^{2m}\right] \tag{6e}$$

$$2\epsilon^{m-j}\frac{\partial}{\partial\xi}\left(\bar{a}\bar{d}\left[\bar{U}\left(1-\left(\frac{\partial H}{\partial\xi}\right)^2\epsilon^{2m}\cdots\right]-\bar{w}\epsilon^{l+m}\frac{\partial H}{\partial\xi}\cdots\right) = -\frac{\partial\left(\bar{a}\bar{d}\right)}{\partial\tau} \tag{6f}$$

$$\chi = -\epsilon^{2m-n}\frac{\partial^2 H}{\partial\xi^2}\left[1-\tfrac{3}{2}\left(\frac{\partial H}{\partial\xi}\right)^2\epsilon^{2m}\cdots\right] \tag{6g}$$

$$\epsilon^j\frac{DH}{D\tau} = 2\left(\bar{U}\frac{\partial H}{\partial\xi}\epsilon^m + \bar{w}\epsilon^l\cdots\right) \tag{6h}$$

The already mentioned perturbation process with $\epsilon \to 0$ will be considered now. In the physical problem which is of interest here, namely, the time dependent problem with interaction between the fluid-mechanical effects and the elastic effects in the foil, all the terms of Eq. (6a) are to be of the same order, hence:

$$2m + 2k = 1 + m + k \tag{7a}$$

$$2m + 2k = 1 + k + j \tag{7b}$$

Since it is desired that χ and $\dfrac{\partial^2 H}{\partial \xi^2}$ will be of order unity it follows from Eq. (6j) that it must be true that

$$2m - n = 0 \tag{7c}$$

By similar reasons it is concluded from Eq. (6c) that

$$n - k = 0 \tag{7d}$$

As a result of Eq. (6h):

$$\ell = j$$
$$m = j \tag{7e}$$

(Note that the last condition is redundant). The solution of Equations (7) is:

$$m = 1/3 \tag{8a}$$
$$n = 2/3 \tag{8b}$$
$$k = 2/3 \tag{8c}$$
$$j = 1/3 \tag{8d}$$
$$1 = 1/3 \tag{8e}$$

It is further concluded from Eq. (6c) that foil stiffness is important when the stiffness parameter

$$S_p \equiv \frac{D \epsilon^{2/3}}{T_o h_o^2} \sim O(1) \tag{9}$$

and foil mass is important when the mass parameter

$$M_p \equiv \frac{\sigma_o U_o}{12 \mu} \epsilon \sim O(1) \tag{10}$$

Variations in tension, speed and thickness are significant when the extensibility parameter

$$T_p \equiv \frac{T_o}{E d_o} \sim O(1) \tag{11}$$

and the effect of fluid compressibility is appreciable when the compressibility parameter

$$\frac{1}{C_p} \equiv \frac{T_o \epsilon^{2/3}}{p_a h_o} \sim O(1) \tag{12}$$

The functions H, π, $\overline{T}$, $\overline{d}$, $\overline{U}$, $\overline{W}$, x, $\overline{\delta}$ are now expanded in a series of the form:

$$P = P_O + P_1 \epsilon^{2/3} + \ldots . \tag{13}$$

where P stands for any one of the above property functions. Substitution of the expansion Eq. (13), into the equations (6) collecting terms of equal powers of $\epsilon^{2/3}$, the zeroth approximation with the omission of the subscript zero becomes:*

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*If consistent higher order approximation equations are desired it is necessary to use more accurate fluid mechanical equations and at a certain stage, also, more accurate elastic equations.

$$\frac{\partial}{\partial \xi}\left(H^3[\pi + C_p]\frac{\partial \pi}{\partial \xi}\right) = \frac{\partial(\bar{U}[\pi + C_p]H)}{\partial \xi} + \frac{\partial([\pi + C_p]H)}{\partial \tau} \tag{14a}$$

$$\frac{\partial \bar{T}}{\partial \xi} = M_p \bar{\sigma}\frac{D\bar{U}}{D\tau} \tag{14b}$$

$$\pi - T\chi + S_p \frac{\partial^2 \chi}{\partial \xi^2} = M_p \bar{\sigma}\left[\frac{D\bar{W}}{D\tau} + \frac{D}{D\tau}\left(\frac{\partial H}{\partial \xi}\right)\right] \tag{14c}$$

$$\bar{d} = 1 - \nu'(1 + \nu^2)T_p \bar{T} \tag{14d}$$

$$\frac{1 - \nu^2}{2}T_p \frac{\partial(\frac{\bar{T}}{\bar{d}})}{\partial \tau} = \frac{\partial \bar{U}}{\partial \xi} \tag{14e}$$

$$\frac{\partial}{\partial \xi}\left(\bar{\sigma}\bar{d}\bar{U}\right) = -\frac{1}{2}\frac{\partial(\bar{\sigma}\bar{d})}{\partial \tau} \tag{14f}$$

$$\chi = -\frac{\partial^2 H}{\partial \xi^2} \tag{14g}$$

$$\frac{DH}{D\tau} = 2\left(\bar{U}\frac{\partial H}{\partial \xi} + \bar{W}\right) \tag{14h}$$

where the operator $\frac{D}{D\tau}$ to the zeroth approximation is:

$$\frac{D}{D\tau} = 2\bar{U}\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \tau}$$

6.0 SPECIAL CASES

6.1 Inexstensible Foil

In this case $T_p \sim 0(\epsilon^{2/3})$. If at $\xi = -\infty$,

$\bar{T} = 1$, $\bar{U} = 1$ and if initially $\bar{\sigma} (\xi) \equiv 1$ then $\bar{d} = 1$. $\bar{\sigma} = 1$ at all times and

$$\frac{\partial}{\partial \xi} \left(H^3 [C_p + \pi] \frac{\partial \pi}{\partial \xi} \right) = \frac{\partial}{\partial \xi} \left([C_p + \pi] H \right) + \frac{\partial}{\partial \tau} \left([C_p + \pi] H \right) \qquad (15a)$$

$$\pi + \frac{\partial^2 H}{\partial \xi^2} - S_p \frac{\partial^4 H}{\partial \xi^4} = 2 M_p \left(\frac{\partial^2 H}{\partial \xi^2} + \frac{\partial^2 H}{\partial \tau \partial \xi} + \frac{1}{4} \frac{\partial^2 H}{\partial \tau^2} \right) \qquad (15b)$$

It is seen that in this case the effect of the foil mass appears only transversely. If extensionality is not negligible then the foil mass is of equal significance or insignificance transversely and longitudinally.

6.2 Massless Foil

In this case $M_p \sim 0(\epsilon^{2/3})$. If at $\xi = -\infty$, $\bar{T} = 1$, then $\bar{U} = 1$, $\bar{\sigma} = 1$, $\bar{d} = 1 - \nu (1 + \nu^2) T_p$ at all times and the equations reduce to

$$\frac{\partial}{\partial \xi} \left(H^3 [C_p + \pi] \frac{\partial \pi}{\partial \xi} \right) = \frac{\partial}{\partial \xi} \left([C_p + \pi] H \right) + \frac{\partial}{\partial \tau} \left([C_p + \pi] H \right) \qquad (16a)$$

$$\pi + \frac{\partial^2 H}{\partial \xi^2} - S_p \frac{\partial^4 H}{\partial \xi^4} = 0 \qquad (16b)$$

6.3 Solid Moving at U_o; No Axial Bulk Motion of Foil

Two cases must be distinguished here:

Case a: Large disturbances causing longitudinal foil speeds of order U_o are considered. In this case the speed in the Quette terms in the Reynolds equation and in the fluid shear expression has to be modified to include the sum of U_o and U_{foil}. In dimensionless form, to the zeroth approximation, the only modification in Eqs. (14) will be the replacement of $\bar{U}$ by $1 + \bar{U}$ in Eq. (14a). (The effect on the fluid shear is, anyway, of order $\epsilon^{2/3}$.)

Case b: Only longitudinal disturbances which do not affect the fluid mechanics will be considered. Thus, foil speeds of order $U_o \, \epsilon^{2/3}$ or less are considered. To the zeroth order approximation:

$$\frac{D}{D\tau} \equiv \frac{\partial}{\partial\tau} + O\left(\epsilon^{2/3}\right)$$

$$\frac{\partial H}{\partial\tau} = 2\bar{W}$$

$$\frac{\partial \bar{T}}{\partial \xi} = O\left(\epsilon^{2/3}\right)$$

The basic equations become to the zeroth order approximation

$$\frac{\partial}{\partial\xi}\left(H^3[\pi + c_p]\frac{\partial\pi}{\partial\xi}\right) = \frac{\partial([\pi + c_p]H)}{\partial\xi} + \frac{\partial([\pi + c_p]H)}{\partial\tau} \qquad (17a)$$

$$\pi + \frac{\partial^2 H}{\partial\xi^2} - S_p\frac{\partial^4 H}{\partial\xi^4} = 2M_p\bar{\sigma}\frac{\partial^2 H}{\partial\tau^2} \qquad (17b)$$

6.4   Massless, Perfectly Flexible Foil, Incompressible

Flow

The equations reduce to

$$\frac{\partial}{\partial \xi}\left(H^3 \frac{\partial^3 H}{\partial \xi^3}\right) + \left(\frac{\partial H}{\partial \xi}\right) = -\frac{\partial H}{\partial \tau} \tag{18}$$

Writing the equation in terms of a coordinate $\xi' = \xi - \tau$ moving at a speed $\frac{d\xi}{d\tau} = 1$, it becomes:

$$H^3 H^{IV} + 3 H^2 H' H''' = -\frac{\partial H}{\partial \tau}$$

It is clear, therefore, that symmetrical disturbances will make $\frac{\partial H}{\partial \tau}$ symmetrical and will stay symmetrical as time goes on. Thus, the axis of symmetry of such disturbances travels at a dimensionless velocity of $\frac{d\xi}{d\tau} = 1$ which corresponds to an actual velocity of U/2. It will be shown below that small disturbances of arbitrary shape also propagate at a velocity of U/2. This is not necessarily true, however, for large non-symmetrical disturbances. For example, it can be shown that the "nodal" point, H = 1, of an antisymmetrical large disturbance moves at a speed different from U/2.

If the disturbance is small relative to the magnitude of the clearance, the equation may be linearized as follows:

$$H = 1 + f(\xi, \tau) \tag{19}$$

$$\frac{\partial^4 f}{\partial \xi^4} + \frac{\partial f}{\partial \xi} = -\frac{\partial f}{\partial \tau} \tag{20}$$

with $f = f_o(\xi)$ at $\tau = 0$.

The solution is obtained by taking the Fourier transform:

$$F(s,\tau) \equiv \int_{-\infty}^{\infty} f(\xi,\tau) e^{-2\pi \xi s i} d\xi \tag{21}$$

The transformed differential equation is

$$\frac{dF}{d\tau} + \left(2\pi s i + 16 \pi^4 s^4\right) F = 0 \tag{22}$$

and its solution

$$F(s,\tau) = F_o(s) e^{-\left(2\pi s i + 16 \pi^4 s^4\right)\tau} \tag{23}$$

where $F_o(s)$ is the transformed initial condition. Inversion is made by two successive uses of the convolution theorem and the use of a new variable $\lambda = 2\pi s$. It is found that

$$H(\xi,\tau) = 1 + \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{0}^{\infty} f_o(\xi') e^{-\lambda^4 \tau} \cos \lambda(\xi - \xi' - \tau) \, d\lambda \, d\xi' \tag{24}$$

Eq.(24) describes wave motion in a dissipative nondispersive medium. Each Fourier component of the disturbance decays at a rate which is proportional to $e^{-\lambda^4 \tau}$ and travels at a constant speed of $U_o/2$. A particular example is given in the appendix.

The above result has been obtained for a case with an infinite distance in terms of ξ between the disturbance and the boundary. Since ξ is a stretched coordinate, this result applies in practice, to cases with finite length between the supports, as long as the propagation of the disturbance has not yet reached within an order of $h_o/\epsilon^{1/3}$ from the boundary. Furthermore, this result applies to the central region of self-acting foil bearing as long as the disturbance is far enough from the inlet and exit regions.

APPENDIX

In this section a specific example is considered in order to enhance the physical understanding of the phenomenon. Imagine an initial disturbance in the form of a square wavelet

$$f_o(\xi) = A \qquad\qquad -a < \xi < a$$

$$f_o(\xi) = 0 \qquad\qquad \xi < -a \; , \; a < \xi$$

Let the point $\xi = \tau$ be followed. Putting $\lambda \tau^{1/4} = \bar{\lambda}$ and $\dfrac{a}{\tau^{1/4}} = \bar{a}$

Eq. (24) becomes

$$H(\tau,\tau) = 1 + \frac{2A}{\pi} \int_0^\infty \frac{e^{-\bar{\lambda}^4} \sin \bar{a}\bar{\lambda}}{\bar{\lambda}} d\bar{\lambda}$$

This integral is numerically evaluated[*] and the result is depicted in Fig. A1. When $\bar{a}$ is small (large τ) the integral may be approximated by:

$$\frac{H(\tau,\tau)-1}{2A/\pi} \cong \bar{a} \int_0^\infty e^{-\bar{\lambda}^4} d\bar{\lambda} - \frac{\bar{a}^3}{3!} \int_0^\infty \bar{\lambda}^2 e^{-\bar{\lambda}^4} d\bar{\lambda} + \ldots$$

$$= 0.906402 \left(\frac{a}{\tau^{1/4}}\right) - 0.051059 \left(\frac{a}{\tau^{1/4}}\right)^3 + \ldots$$

The asymptotic approximation is represented by a broken line in Fig. A1.

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[*]The apparent singularity in the integral is overcome by using the relation

$$\frac{H(\tau,\tau)-1}{2A/\pi} = \bar{a}\cdot\delta - \frac{\bar{a}^3\delta^3}{3\cdot3!} + \int_\delta^\infty \frac{e^{-\bar{\lambda}^4} \sin \bar{a}\bar{\lambda}}{\bar{\lambda}} d\bar{\lambda} + O\left([\delta\cdot\bar{a}]^5\right)$$

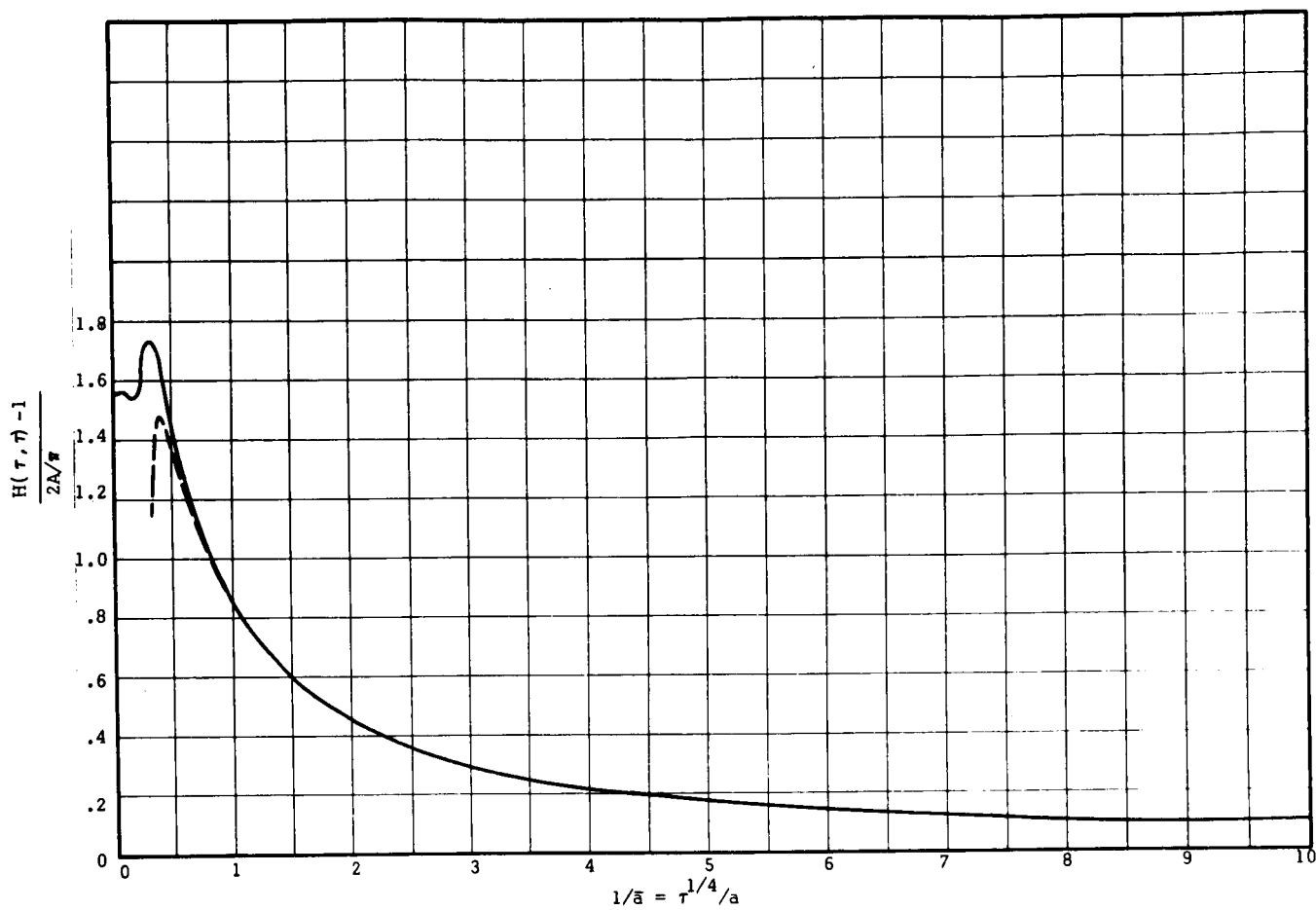where δ is a properly chosen small number.

Fig. A-1 Time History of the Symmetry Point of a Disturbance in the form of a Square Wavelet

Since the original profile of the disturbance is flat, the symmetry point is initially in a region of uniform clearance and therefore does not change its elevation. The excess fluid spreads out by forming exit regions with their corresponding undulations at the two ends of the wavelet. Thus for a period of time, the symmetry point may be in a region of curvature convex towards the h_o line and is, thus, being pushed away from it until it flattens again. This explains the initial undulations shown in Fig. A1.

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Equations for the oscillations of a foil over a lubricating fluid film are derived and are simplified by a small parameter expansion. A few particular cases are discussed, and a linearized solution is obtained for the case of a massless, perfectly flexible foil moving at a speed U over an incompressible film. The solution reveals the phenomenon that small disturbances in the film thickness, as well as symmetrical large disturbances, propagate at a speed $U/2$.

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