THEORY OF THE LUNAR PHOTOELECTRON ATMOSPHERE

A Dissertation

Presented to

the Faculty of the Graduate School

University of Missouri at Rolla

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

by

Hal Emerson McCloud Jr.

August 1966

ABSTRACT

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By considering the mechanics of an electron projected into an arbitrary potential field together with Poisson's equation it is possible to set up a differential equation for the electrostatic potential. The equation may be integrated and the photoelectron density found as a function of height for four approximate models. A simple model based upon monoenergetic photons and monoenergetic electrons ejected vertically upward yields rough estimates of the parameters of interest. Another model takes into account the fact that the photoelectrons are ejected at all angles. A general model takes into account the illumination of the lunar surface by solar (black body) radiation and also the distribution in energy of the electrons ejected for each photon energy. An adiabatic gas model of the photoelectron atmosphere provides an independent check of the results.

Assuming a metalic surface the electron density is of the order of 10^4 electrons/cm^3 at a half-height of the order of 1.5 cm above the lunar surface. The charge distribution produces an electrostatic force field capable of levitating particles of the order of 10^{-14} gm.

Author

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J.P. Wesley, Dissertation Supervisor

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CHAPTER I

INTRODUCTION

With the advent of the space age the problem of determining the

Moon's environment has become increasingly important. As the time for

the landing of the first man on the lunar surface draws near it is be-

coming of the upmost importance to know what conditions exist near to

and on the surface. This knowledge is necessary in order to design the

spacecraft and the various supporting equipment which will be neces-

sary once the landing is accomplished.

Many theories have been put forth concerning the various charac-

teristics of the lunar surface (Markov, 1962). There has been much

controversy concerning the correctness of these theories. While each

of these theories may have its good points, generally each one fails

to be consistent with all of the observed phenomenon.

Prior to the close up pictures obtained by Luna 9 (Winston, 1966)

and Surveyor spacecrafts (Jaffe, L. D. et al, 1966), it was assumed by

many (eg., Gold, 1955; Goodwin, 1958) that the lunar surface was covered

with a layer of dust. There was much disagreement as to the thickness

of the proposed dust layer and as to whether the surface was entirely

covered or only partially covered with the dust. Radlova (1939) sug-

gested that the variations in the surface brightness which are observed

might be accounted for by assuming the lunar surface to be only par-

tially covered by a continuous or dense layer of dust, the true surface

being exposed in places. The question arose as to where the dust came from and how it became distributed over the surface. There was speculation that the Moon's gravitational field swept the dust up from space over the ages, or that the dust resulted from the erosion of the lunar surface by a variety of physical forces. Platt (1958) postulated that the lunar surface might be covered by dust particles consisting of aggregates of unsaturated and free-radical molecular species rich in carbon, nitrogen, and oxygen.

The best evidence from observed reflectivity and close up photographs taken by Luna 9 and Surveyor space craft indicates that the surface of the Moon is extremely granular, having the appearance of sand. Wesselink (1948) estimates from temperature measurements that the grain size of the dust is something less than 0.03 centimeters. Microwave measurements indicate that the lunar surface is smooth on the order of 10 centimeters (Evans and Pettengill, 1963). In photographs taken by the Ranger 9 space craft on the order of a few miles above the lunar surface (Schurmeier, Heacock, and Wolte, 1966), the surface appears to have smooth rolling features.

To account for the lunar mares and the large scale smoothing, Urey (1956) and DuFresne (1956) assumed that the Moon was formed in a cold state from meteoritic material and postulated that the impact of meteors, both large and small, with the surface produced large amounts of dust. At the same time that the dust was formed, gases and water were released which had been held in the pulverized material. These gases and water formed a temporary atmosphere. The water fell as

precipitation which caused the dust to gather in surface depressions.
However, the Moon's retention of an atmosphere for an adequate length
of time and at a high enough density to allow precipitation seems rather
unlikely.

Considering the conditions which exist on the lunar surface and
comparing them with those which exist in dry caves here on Earth,
Goodwin (1958) proposed that they are similar. In both cases there
would be no disturbance of the surface by the flow of water or the move-
ment of an atmosphere. To account for the lunar mares and the large
scale smoothing, Gold (1955) theorized that the dust behaved similar to
a liquid with no internal viscosity and that it tended to flow to
points of lower gravitational potential energy. Under the assumption
that seismic activity exists on the Moon, Gilvarry (1957) advanced the
hypothesis that frequent and strong tremors caused dust particles to
jump and roll into depressions.

Whipple (1951) believes that corpuscular radiation from the Sun
acts to sinter dust particles together and prevents any flow over the
surface. An investigation of this possible sealing of the dust layer
by solar wind sputtering has been made by Wehner and his associates
(1965). Measurements made in the laboratory indicate that sputtering
could definitely produce sealing of the surface. Photographs taken only
a few feet above the surface by Luna 9 and by Surveyor (Winston, 1966)
indicate that it consists of a rather firm porous material. The gas
jets on Surveyor failed to stir up any dust whatsoever. In view of
this evidence, Whipple's hypothesis seems to be essentially correct.

While the surface may be sintered at the present time, there must have been a period when mass transport of dust or magma took place in order to provide for the large scale smoothing or apparent erosion on the 10 centimeter scale. The physical forces which could cause such an erosion of the lunar surface must be considerably different from those which are encountered on the Earth's surface. The processes which erode the Earth's surface are primarily associated with the existence of the atmosphere. It is possible that the physical forces which produce erosion of the lunar surface are of such small magnitude that they would go completely unnoticed here on the Earth's surface. The erosion may be produced by some process or processes which cannot occur when any appreciable atmosphere is present, such as the Earth possesses.

The Moon's atmosphere is very rarefied if it exists at all. Using kinetic theory, Sytinskaya (1963) arrives at the conclusion that only the heaviest gases such as krypton could be present in any appreciable amount near the lunar surface and that for all practical purposes the atmosphere should be regarded as nonexistent. The formation of an atmosphere of xenon and krypton on the Moon is considered by Edwards and Borst (1958). They list four mechanisms by which an atmosphere of the heavier gases might be formed. Attempts to detect a lunar atmosphere by spectroscopic methods (Teyfel, 1959) have given either negative or completely inconclusive results. The French astronomer Dollfus (1956) by measuring the brightness of light scattered in the supposed atmosphere arrived at a density not exceeding 10^{-9} times the density of air at the Earth's surface. This corresponds to a density on the order of

10^{10} molecules per cubic centimeter. This would be a very thin atmosphere and therefore could play little part in the actual eroding of the lunar surface.

It has been suggested by Fielder (1961) that in the absence of a conventional atmosphere there could be an electrical 'space charge' close to the lunar surface during the lunar day due to radiation from the Sun and the solar wind. Brandt (1960) speculates that the electron density close to the surface of the Moon may be on the order of 10^3 or 10^4 electron per cubic centimeter. It is possible that this atmosphere of photoelectrons may provide a mechanism by which positively ionized dust particles could be transported over the surface of the Moon. Gold (1955, 1956) gives the following mechanisms which he believes are capable of producing transport of dust particles over the lunar surface: (1) The night time condensation and the day time evaporation of volatile substances on the surfaces of the dust particles, (2) Thermal motion of the fine dust particles, making a layer of dust behave as a gas of very high molecular weight, (3) Photoemission of electrons from the surface of dust particles under the effect of the Sun's radiation, (4) Explosions of micrometeorites, (5) Motion of the rarefied residual atmosphere. Gold's suggestion of the transport of dust over the surface by photoemission of electrons is similar to Fielders (1961) suggestion of the possibility of an atmosphere of photoelectrons close to the lunar surface.

At least two attempts have been made to determine experimentally the electron density near the lunar surface. By observing the

occultations of a radio star by the Moon's disc, Elsmore and Whitfield (1955) arrived at an upper limit for the Moon's atmosphere of 10^{-12} of that of the Earth's atmosphere at sea level. More accurate measurements by Costain, Elsmore and Whitfield (1956) set an upper limit of 10^{-13} for the surface density as compared to that of the Earth's at sea level. This corresponds to an electron density at the lunar surface of approximately 10^3 electrons per cubic centimeter.

Due to the Moon lacking any appreciable atmosphere, the surface is exposed to the total spectrum of the Sun's radiation whereas the Earth's surface is shielded from the radiation in the shorter wavelengths of the spectrum. In addition, the presence of the Earth's atmosphere would cause any accumulation of charge above the surface to be dissipated by conduction. The Moon may possess an atmosphere consisting of photoelectrons close to the surface due to the lack of a more conventional atmosphere.

Although the idea of photoemission of electrons and an accompanying electron atmosphere close to the lunar surface has been postulated, no apparent effort has been made to determine theoretically the precise magnitude of the effect. It would be very interesting to know the density of such a photoelectron atmosphere. Once the photoelectron density is known it would be possible to estimate the maximum mass of a positively charged dust particle that could be levitated by the electrostatic field. Once the photoelectron density is known other interesting effects produced by the electron atmosphere close to the lunar surface can be investigated. The electron atmosphere might

possibly provide a means of communications over the lunar surface.

An electron atmosphere produced by the photoelectric effect is not limited to the moon, it will exist near the surface of any object in space illuminated by sunlight. Space stations, space craft, the planet Mercury and the asteroids will all have such a photoelectron atmosphere.

It may be possible to obtain a direct conversion of solar energy (radiation of short wavelength) to electrical energy by collecting the photoelectrons. The kinetic energy of the electrons can, in principle, be converted to a direct current at a low voltage. This might be a practical means for converting solar energy for use in space. While the efficiency of such a device would be quite low, the simplicity and reliability might make it practical.

Quite independent of possible applications, the determination of the charge distribution produced by the ejection of photoelectrons from a surface illuminated by black-body radiation is an interesting theoretical problem well worth solving in its own right.

CHAPTER II

GENERAL THEORY

Electromagnetic radiation in the ultraviolet region and at
shorter wavelengths will eject photoelectrons from a surface (Hughes
and DuBridge, 1932) and (Simon and Suhramann, 1958). It will be pos-
tulated here that the lunar surface consists of metals in their reduced
or uncombined state. This is a reasonable assumption due to the reduc-
tion of metal compounds by the action of the solar wind, soft X-rays,
and cosmic rays (Wehner and associates, 1965). The particles com-
prising the lunar surface overlap one another and only their upper sur-
faces will be exposed to the reducing effect of the solar wind and
radiation. For this reason the surface (while not forming a continuous
conductor) may appear to be a metallic surface to incident ultraviolet
radiation. Since the free atoms do not form a continuous conductor,
the surface will have a low electrical conductivity for microwaves
(Hey and Hughes, 1959). Since the reduced metals are only a few atoms
thick on the surface the thermal conductivity of the surface will
remain low (Jaeger and Harper, 1950). The ejection of photoelectrons
from the particles on the lunar surface will produce a charge distri-
bution above the surface and an accompanying electrostatic field.

The electrostatic field will support positively charged parti-
cles, the size of the particles being determined by the magnitude of
the field and the magnitude of the positive charge on the particle. It

may be assumed that minute particles will be present due to meteoritic impacts and inflow of gravitationally trapped cosmic dust particles. Such particles will become positively charged by the photoelectric process and bombardment by the solar wind and cosmic rays. Once the small dust particles are positively charged, the electrostatic field produced by the photoelectrons will be able to levitate them above the surface. The supported dust particles can then be transported laterally by nonuniformities in the electrostatic field caused by the irregularities of the lunar surface. The transport of the dust particles may also come about through collisions of the dust particles with photoelectrons, solar protons, or cosmic rays. It may be assumed that the positively charged dust particles will tend to return to the lunar surface at places where the surface is least positively charged. This would mean that the dust particles would tend to collect in depressions where shadows would yield no photoelectrons and no positive charge. This then provides a possible mechanism for the apparent erosion of the lunar surface.

In this investigation of a possible photoelectron atmosphere close to the lunar surface four models are considered. They are:

1) _Simplest Model_ in which it is assumed that all photoelectrons are ejected monoenergetically perpendicular to the surface by monochromatic radiation,

2) _Modified Simple Model_ in which it is assumed that the photoelectrons are ejected monoenergetically with a specified angular distribution by monochromatic radiation,

3) <u>General</u> <u>Model</u> in which it is assumed that the surface is irradiated by black body radiation,

4) <u>Adiabatic</u> <u>Gas</u> <u>Model</u> in which the photoelectron atmosphere is assumed to form an adiabatic gaseous atmosphere.

These four models are then compared in order to establish confidence in the results.

From the dynamics of the electrons moving in any potential field it is possible to find a relationship between the particle density (i.e., probability of finding a particle in a given element of volume in a given time interval) and the potential. Poisson's equation gives another independent relationship between the electron density and the potential. By eliminating the particle density a nonlinear differential equation is obtained for the potential as a function of the height above the surface. Once a solution has been obtained the electron density may be readily found and the various applications investigated.

CHAPTER III

SIMPLEST MODEL

A very simple model will be considered first in order to lay the ground work for more realistic models that will be considered subsequently. The present model serves as an introduction to some of the concepts and techniques useful in solving the more general models. The present model may also give some idea as to the order of magnitude of the quantities involved.

For the present model it is assumed that monoenergetic photons eject photoelectrons normal to an infinite plane surface. The further simplifying assumption is made that all electrons ejected have identical kinetic energies as they leave the surface. A steady state situation exists in that electrons return to the surface at the same rate at which they are being ejected from the surface by the incident radiation. Because of this equilibrium condition there is no net current flowing through any plane lying above the surface.

I. Poisson's Equation

The electric potential at any point above the surface is specified by Poisson's equation, which is

$$d^2\phi/dz^2 = 4\pi e\rho \quad , \qquad\qquad (3.1)$$

where ρ is the density of the photoelectrons, ϕ is the electric potential at a distance z above the surface from which the electrons are ejected, and e is the magnitude of the electronic charge. Gaussian units are used throughout unless otherwise indicated. The potential energy U, of an electron at an electric potential ϕ is

$$U = -e\phi \quad . \qquad (3.2)$$

Replacing ϕ in Poisson's equation (3.1) by the potential energy U, gives

$$d^2U/dz^2 = -4\pi e^2\rho \quad . \qquad (3.3)$$

Another expression for ρ in terms of U, z, or both must now be found in order to obtain a solution of equation (3.3).

II. Density ρ as a Function of Velocity v_z

The upward directed current per unit area, j, of the photoelectrons may be written

$$j = -e\rho v_z/2 \quad , \qquad (3.4)$$

where v_z is the velocity of any one of the electrons as it passes upward through the point z. The factor of 2 occurs in the denominator since ρ is the total density of electrons both rising and falling. The

current j, may also be expressed in terms of electron flux n_o, or

$$j = -en_o \quad , \quad (3.5)$$

where n_o (a constant for all $z \leq z_m$, where z_m is the maximum height) is the number of electrons passing upward through a unit area per second. Combining equations (3.4) and (3.5) gives

$$\rho = 2n_o/v_z \quad . \quad (3.6)$$

Using equation (3.6), Poisson's equation may now be written

$$d^2U/dz^2 = -8\pi e^2 n_o/v_z \quad . \quad (3.7)$$

III. Velocity v_z as Function of the Potential Energy U

Each electron at a point z above the surface possesses a potential energy U. Conservation of energy then requires

$$mv_z^2/2 + U = E_t \quad , \quad (3.8)$$

where E_t is the total energy of one electron at any point above the surface. The total energy may be found in terms of the boundary conditions at the z = 0 plane. At the plane where z = 0, equation (3.8) becomes

$$mv_{zo}^2/2 + U_o = E_t \qquad , \qquad (3.9)$$

where the subscript zero is used to designate the values that the quantities possess at the z = 0 plane.

According to the Einstein photoelectric equation, the maximum kinetic energy that a photoelectron can have as it leaves the surface is

$$(mv^2/2)_{max} = E - W \qquad , \qquad (3.10)$$

where v is the velocity of ejection, E is the energy of the incoming photon and W is the work function for the surface. The kinetic energies of the photoelectrons ejected from a surface by monoenergetic photons of energy E are distributed over a range of values from zero to this maximum energy (E - W), equation (3.10). It is found experimentally that for most metals and for photons of wavelength in the ultraviolet region of the spectrum, the average kinetic energy of the electrons ejected may be expressed as some fraction λ of the maximum kinetic energy of ejection (Hughes and DuBridge, 1932). This average value will be taken as the single value for the kinetic energy of all the ejected electrons. The initial kinetic energy E_o of the ejected photoelectron thus becomes

$$E_o = mv_{zo}^2/2 = \lambda(E - W) \qquad , \qquad (3.11)$$

where according to Hughes and DuBridge (1932), for metals, $\lambda \approx 0.4$.

Combining equations (3.9) and (3.11) gives

$$E_t = \lambda(E - W) + U_o = E_o + U_o \quad . \tag{3.12}$$

The energy equation (3.8) may be solved for the velocity v_z, giving

$$v_z = \pm \ [2(E_t - U)/m]^{1/2} \quad . \tag{3.13}$$

The positive sign in equation (3.11) holds for electrons traveling upward from the surface and the negative sign is for the case where electrons are returning to the surface.

At some distance z_m above the surface the electron reaches a turning point. At the turning point the velocity of an electron becomes zero. It is convenient to choose the potential energy U to be zero at the turning point. From equation (3.13), it may be seen that for the velocity at the turning point to be zero and the potential energy U set equal to zero, the total energy E_t must be taken equal to zero. This means, from equation (3.12), that U_o must be chosen such that

$$E_t = 0 = E_o + U_o \quad . \tag{3.14}$$

The expression (3.13) for velocity then simplifies to

$$v_z = \pm(-2U/m)^{1/2} \quad . \tag{3.15}$$

Substituting equation (3.15) into (3.7) yields a differential equation in U alone which is

$$d^2U/dz^2 = -b/(-U)^{1/2} \quad , \tag{3.16}$$

where the constant b is defined to be

$$b = 8\pi e^2 (m/2)^{1/2} n_o \quad . \tag{3.17}$$

Care must be taken to apply equation (3.16) only within its range of validity. It will not be a solution above the $z = z_m$ plane where the source vanishes. To account for the source being zero for $z \geq z_m$, a unit step function may be inserted so that the entire solution is zero above the $z = z_m$ plane. The unit step function is defined by

$$S(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases} \quad . \tag{3.18}$$

With the insertion of the step function S(-U), U being always negative, into equation (3.16) then yields the desired differential equation,

$$d^2U/dz^2 = -b \ S(-U)/(-U)^{1/2} \quad . \tag{3.19}$$

IV. Integration of the Differential Equation

The assumption has been made that all electrons are ejected with the same initial kinetic energy. It may also be assumed that they are ejected at a uniform rate. With these two assumptions the upward flux n_o is a constant for all values of z and depends only on the flux of incoming energy. Multiplying both sides of equation (3.19) by dU/dz and integrating with respect to z yields

$$(dU/dz)^2 = 4b(-U)^{1/2} \quad . \tag{3.20}$$

The constant of integration has been chosen as zero since the electric field, varying as dU/dz, and U vanish together at $z = z_m$.

Taking the square root of equation (3.20), choosing the plus sign, and integrating yields

$$-U = (9b/4)^{2/3}(z_m - z)^{4/3} \quad , \tag{3.21}$$

the constant of integration having been chosen so that U = 0 for $z = z_m$. This result, equation (3.21), gives the desired relation of U as a function of the height above the surface z.

The density of electrons may now be found by taking the second derivative of equation (3.21) and substituting into equation (3.3) or from equations (3.6) and (3.15); which yields

$$\rho = \rho_0/(1 - z/z_m)^{2/3} \quad , \qquad (3.22)$$

where the constant ρ_0 , the electron density at the surface, using equation (3.17), becomes

$$\rho_0 = (9b/4)^{2/3}\Big/9\pi e^2 z_m{}^{2/3} = (2mn_0{}^2\big/9\pi e^2 z_m{}^2)^{1/3} \quad , \qquad (3.23)$$

where the constants n_0 and z_m are yet to be determined. This result, equation (3.22), is plotted in Figure 3.1 . Although ρ/ρ_0 approaches infinity as z/z_m approaches unity, the total area under the curve remains finite and is proportional to the positive charge produced on the surface by the ejection of the photoelectrons.

In order to obtain z_m, equation (3.21) at $z = 0$ yields

$$-U_0 = (9b/4)^{2/3} z_m{}^{4/3} \quad , \qquad (3.24)$$

where U_0 is the potential energy of an electron at the surface. Solving this equation for z_m and using equation (3.14) for the value of U_0, it is found that

$$z_m = (4/9b)^{1/2}E_0^{3/4} = (E_0^3/162\pi^2 e^4 mn_0^2)^{1/4} \quad , \qquad (3.25)$$

where b was obtained from equation (3.17) and where E_0 is defined by equation (3.11). Only the constants n_0 and E_0 remain to be determined.
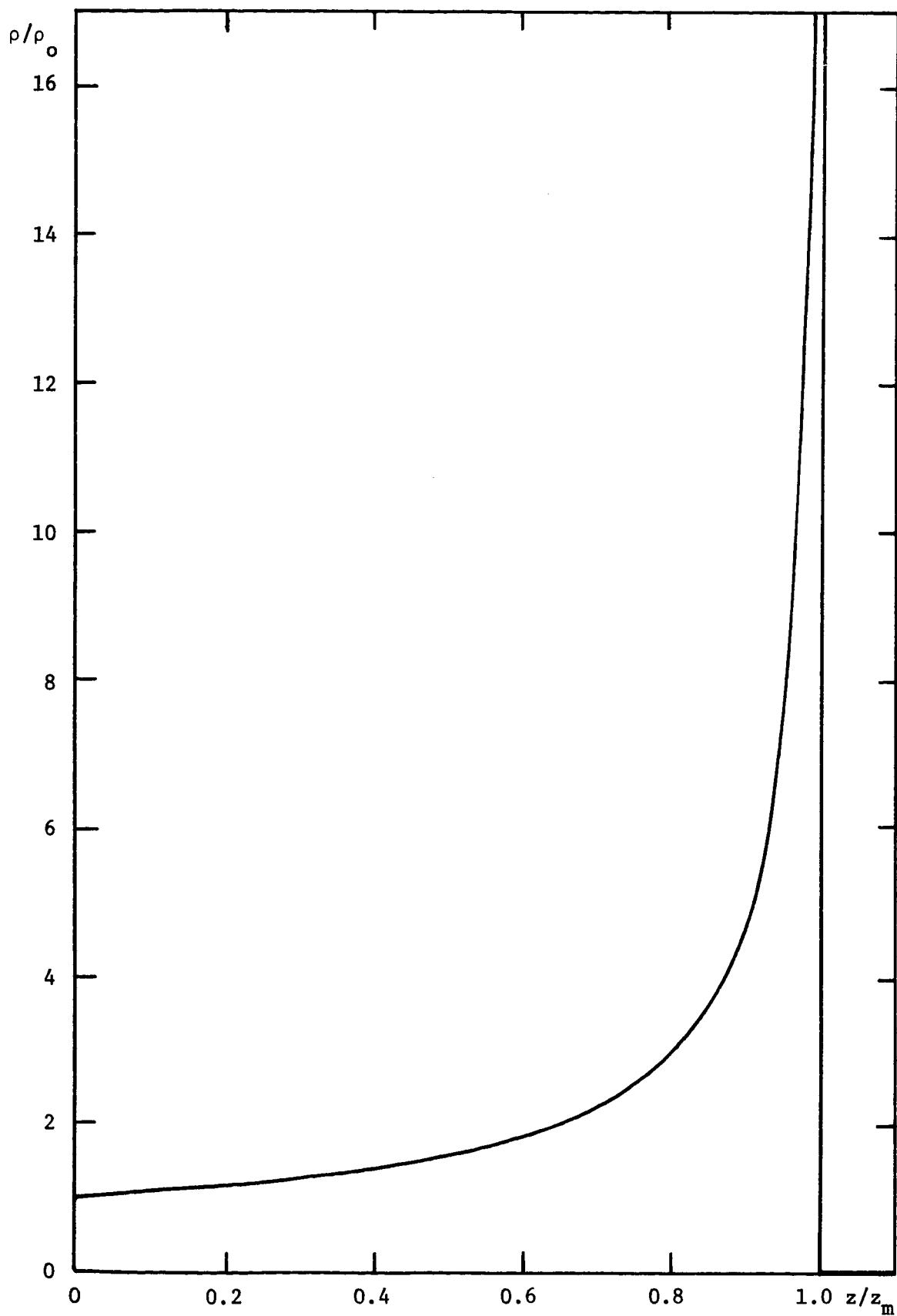
Figure 3.1 Plot of ρ/ρ_o as a function of z/z_m for the
Simplest Model, equation (3.22)

V. Evaluation of the Photoelectron Flux, n_o

It may be assumed that n_o, the flux of electrons leaving the surface, will be proportional to the flux of the incident photons (the photoelectron current being much less than the saturation current). The source of the electromagnetic radiation for the present problem is the Sun. Assuming that the Sun radiates as a perfect black body and using the Planck black body radiation formula (French, 1958), the number of photons radiated by the Sun per second per square centimeter of area with the energy E, is

$$dn_s/dE = (E^2/6c^2\pi^2\hbar^3)/(\epsilon^{E/kT} -1) \quad , \qquad (3.26)$$

where T is the temperature of the photosphere and c is the velocity of light in vacuum. Since the work function W for the lunar surface corresponds to a photon energy in the ultraviolet region of the spectrum (for most metals) and the solar energy is a maximum in the infrared region, only those values of E/kT which are much greater than unity need be considered, W/kT >> 1. Consequently, equation (3.26) reduces to the Wien formula (French, 1958)

$$dn_s/dE = b_s E^2 \epsilon^{-E/kT} \quad , \qquad (3.27)$$

where b_s is defined to be

$$b_s = (6c^2\pi^2\hbar^3)^{-1} \qquad . \tag{3.28}$$

To obtain the photon flux at the lunar surface it is necessary to decrease the flux from the Sun's surface by the factor $(r_s/r)^2$, where r_s is the solar radius and r is the distance from the Sun to the Moon. The photon flux at the lunar surface becomes

$$dn_p/dE = (r_s/r)^2 (dn_s/dE) \qquad , \tag{3.29}$$

where n_p is the number of photons striking per unit area per second at the distance r from the Sun.

The relationship between the number of photons which strike a surface to the number of photoelectrons ejected is a rather complicated function of the photon energy. The yield of photoelectrons ejected by photons of energy E is given by

$$dn_o/dE = fB \ F(E) \ (dn_p/dE) \qquad , \tag{3.30}$$

where f is a factor which indicates the fraction of the photons which eject electrons (i.e., the quantum efficiency of the process), B is a constant for a given material and $F(E)$ is the spectral distribution function (Hughes and DuBridge, 1932). Experimentally (Plenard and Becker, 1928) the spectral distribution may be fitted sufficiently well for the present purposes by the simple function

$$F(E) = (1 - W/E)^2 \quad .$$ (3.31)

Using equation (3.31), equation (3.30) becomes

$$dn_o/dE = fB(1 - W/E)^2(dn_p/dE) \quad ,$$ (3.32)

where the constant B and the efficiency f must be determined. From equation (3.32), using equations (3.27) and (3.29), the total flux of electrons from the lunar surface is

$$n_o = \int_W^\infty fBb_s(r_s/r)^2(E - W)^2\varepsilon^{-E/kT} \, dE \quad .$$ (3.33)

where the constant b_s is defined by equation (3.28). Upon performing the integration, equation (3.33) yields

$$n_o = 2fBb_s(r_s/r)^2(kT)^3\varepsilon^{-W/kT} \quad ,$$ (3.34)

where the constants f and B remain to be evaluated and suitable values must be chosen for the work function for the surface, and the temperature of the Sun's photosphere.

In order to evaluate the constant B the relationship between the energy flux of the incident photons and the energy flux of the ejected electrons may be considered. Equating the energy flux of the ejected photoelectrons to the energy flux of the incident photons reduced by

the energy lost in the work function and the efficiency factor for the surface, yields

$$\int_W^\infty \lambda(E - W)(dn_o/dE) \, dE = \int_W^\infty f(E - W)(dn_p/dE) \, dE \quad , \quad (3.35)$$

where the integral on the left is the energy flux of the ejected electrons and the integral on the right is the effective energy flux of the incident photons. Using equations (3.32), (3.34), and (3.27), equation (3.35) may be solved for B, yielding

$$B = \int_W^\infty \lambda(E - W)E^2 \varepsilon^{-E/kT} \, dE \bigg/ \int_W^\infty (E - W)^3 \varepsilon^{-E/kT} \, dE \quad . \quad (3.36)$$

The integrals in equation (3.36) may be evaluated, yielding

$$\int_W^\infty \lambda(E - W)E^2 \varepsilon^{-E/kT} \, dE$$

$$= 6\lambda(kT)^4 [1 + (2/3)(W/kT) + (1/6)(W/kT)^2] \, \varepsilon^{-W/kT} \quad , \quad (3.37)$$

and

$$\int_W^\infty (E - W)^3 \varepsilon^{-E/kT} \, dE = 6(kT)^4 \varepsilon^{-W/kT} \quad . \quad (3.38)$$

Substituting these results, equations (3.37) and (3.38), into equation (3.36) yields

$$B = (\lambda/6)(6 + 4x + x^2) \quad , \tag{3.39}$$

where x is defined to be

$$x \equiv W/kT \quad . \tag{3.40}$$

The efficiency factor f, may be determined from the experimental data relating the photon flux at a given surface to the photoelectron flux at the surface. Once the photon flux and the photoelectron flux for a particular photon energy is known, equation (3.32) may be used to determine the efficiency factor f, for the particular surface. Measurements of the quantum efficiency have been made (Kenty, 1931; Plenard and Becker, 1928). These measurements give a value for f on the order of 10^{-3} .

The constant $r_s^2 b_s/r^2$ may be evaluated from the known value of the solar constant R_o (i.e., the total energy flux over all wavelengths of sunlight at the Earth's mean radius from the Sun). The total energy flux at the distance r may be found by using equation (3.26) and integrating equation (3.29) over the entire spectrum, which yields

$$R_o = \pi^4(kT)^4 r_s^2 b_s/15r^2 \quad , \tag{3.41}$$

where the constant b_s is defined by equation (3.28); or

$$r_s^2 b_s/r^2 = 15R_o/\pi^4(kT)^4 \quad . \tag{3.42}$$

The flux n_o, may now be written, using equation (3.42) and (3.39), as

$$n_o = 30 f B R_o \epsilon^{-W/kT}/\pi^4 kT$$

$$= (5 f R_o \lambda/\pi^4 kT)(6 + 4x + x^2)\epsilon^{-x} \quad , \tag{3.43}$$

where x is defined by equation (3.40).

VI. Evaluation of the Effective Mean Photon Energy, E

The Sun does not, in fact, radiate monoenergetic photons, therefore some reasonable average estimate must be obtained for the parameter E in equation (3.11). The value of E in equation (3.11) may be taken as the average energy of those photons which are effective in ejecting photoelectrons; thus, from equation (3.27) through (3.32), the average value for E is

$$E = \langle E \rangle =$$

$$\int_W^\infty E(1 - W/E)^2 (dn_s/dE)\ dE \Bigg/ \int_W^\infty (1 - W/E)^2 (dn_s/dE)\ dE \quad , \tag{3.44}$$

where the factor $(1 - W/E)^2$ is the spectral distribution function, equation (3.31). The integrals in equation (3.44) may be evaluated to yield

$$\int_W^\infty E(1 - W/E)^2 (dn_s/dE)\ dE = 2(kT)^3 Wb_s (1 + 3kT/W)\epsilon^{-W/kT} \quad , \tag{3.45}$$

and

$$\int_{W}^{\infty} (1 - W/E)^2 (dn_s/dE) \ dE = 2(kT)^3 b_s \varepsilon^{-W/kT} \quad , \qquad (3.46)$$

where the constant b_s is defined by equation (3.27). Substituting the results of equations (3.45) and (3.46) into equation (3.44) yields

$$E = W + 3kT \quad , \qquad (3.47)$$

the value of the energy E of the incident photons which will be used in equation (3.11) for the simplest model. The average kinetic energy of a photoelectron leaving the surface, E_o , from equations (3.11) and (3.47) then becomes

$$E_o = 3\lambda kT \quad . \qquad (3.48)$$

VII. Surface Charge Density and Half Height

The surface charge density is positive and may be obtained by summing the charge density above the surface in order to find the total negative charge above the surface; thus,

$$\sigma = e \int_{0}^{z_m} \rho \ dz \quad . \qquad (3.49)$$

Substituting equation (3.22) into (3.49) and integrating yields

$$\sigma = 3e\rho_o z_m \quad , \tag{3.50}$$

where ρ_o is given by equation (3.23) and z_m is given by equation (3.25).

The half height h, defined as the point at which half the charge lies below and half above, of the charge distribution may be found from

$$e \int_o^h \rho dz = \sigma/2 \quad . \tag{3.51}$$

Upon substitution of equation (3.22) and integrating, equation (3.51) yields

$$h = 7z_m/8 \quad . \tag{3.52}$$

VIII. Numerical Estimates of the Parameters

The measured value for R_o is 1.33×10^6 ergs per square centimeter per second (Handbook of Physics and Chemistry, 1963). The cutoff wavelength ($\lambda_c = hc/W$) for most metals falls in the ultraviolet region of the spectrum (Hughes and DuBridge, 1932). A rough estimate of the photoelectron density may be obtained by considering a cutoff wavelength of 3000 angstroms. This value falls within the ultraviolet region of the spectrum. The value for W is then found to be 6.63×10^{-12} ergs. The temperature of the Sun's photosphere is approximately 6000° Kelvin.

From equation (3.40), $x = W/kT = 8.01$; assuming the value of $\lambda = 0.4$ ($\lambda = 0.40 \pm 0.05$, Hughes and DuBridge, 1932), equation (3.43)

yields the estimate

$$n_o = 1.1 \times 10^{15} f \ cm^{-2}sec^{-1} \quad . \tag{3.53}$$

The efficiency factor f may be estimated by assuming that the lunar surface is covered with uncombined metallic atoms. For wavelengths in the ultraviolet region and shorter the surface will then appear to be a continuous metallic surface. The value of $f = 10^{-3}$ fits experimental data for several metals, where $10^{-4} \leq f \leq 10^{-2}$ (Plenard and Becker, 1928; Kenty, 1931), the flux density becomes

$$n_o = 1.1 \times 10^{12} \ cm^{-2}sec^{-1} \quad . \tag{3.54}$$

From equations (3.25), (3.48) and (3.54), z_m is found to be

$$z_m = (\lambda^3 k^3 T^3 / 6\pi^2 e^4 m n_o^2)^{1/4} = 1.8 \ cm \quad . \tag{3.55}$$

The electron density at the surface is then found from equations (3.6), (3.11) and (3.47), to be

$$\rho_o = n_o (2m/3\lambda kT)^{1/2}$$

$$= 5fR_o (2m\lambda/3\pi^8 k^3 T^3)^{1/2}(6 + 4x + x^2)e^{-x}$$

$$= 4.8 \times 10^4 \ cm^{-3} \quad . \tag{3.56}$$

From equations (3.50), (3.56) and (3.55) the surface charge density becomes

$$\sigma = 1.3 \times 10^{-4} \text{ statcoul cm}^{-2} \quad . \tag{3.57}$$

The half height of the charge above the surface may be found from equation (3.51) and is

$$h = 1.6 \text{ cm} \quad . \tag{3.58}$$

These results are tabulated in Table 3.1.

Table 3.1   Parameters for the Simplest Model

Quantity	Symbol	Value
Solar constant	R_0	1.33×10^6 ergs cm^{-2} sec^{-1}
Solar temperature	T	$6000°$ K
Approximate work function	W	6.6×10^{-12} ergs
Average energy	$\langle E \rangle$	9.1×10^{-12} ergs
Most probable fraction of maximum kinetic energy	λ	0.4
Efficiency factor	f	10^{-3}
Electron flux	n_0	1.1×10^{12} cm^{-2} sec^{-1}
Electron density at z = 0	ρ_0	4.8×10^4 cm^{-3}
Maximum height	z_m	1.8 cm
Half height	h	1.6 cm
Surface charge density	σ	1.3×10^{-4} statcoul cm^{-2}

CHAPTER IV

MODIFIED SIMPLE MODEL

In the previous chapter the assumption was made that all electrons were ejected normal to the surface with identical velocities. Actually photoelectrons will be ejected at all angles. To take this into consideration, the present model assumes that the initial total kinetic energy E_o of each electron is the same, but that electrons are emitted as some distribution in the direction with respect to the normal to the surface. The vertical motion will be independent of the lateral motion.

I. Vertical Component of Velocity, v_z, as Function of Potential, U

The part of the kinetic energy associated with the vertical motion $mv_z^2/2$, varies from zero to the maximum E_o. The value of E_o is assumed to be the average energy of the emitted electrons which is related to the energy of the incident photons by

$$E_o = \lambda(E - W) \qquad , \qquad (4.1)$$

where the quantities λ, E, and W are defined in equations (3.10) and (3.11). The total kinetic energy of the ejected electrons in terms of the velocities becomes

$$E_o = m(v_{xo}^2 + v_{zo}^2)/2 \qquad , \qquad (4.2)$$

where v_{xo} represents the component of the velocity in the lateral direction at $z = 0$ and v_{zo} is the vertical component of the velocity at $z = 0$. Solving equation (4.2) for v_{zo} yields

$$v_{zo} = [2 \cos^2\theta \; \lambda(E - W)/m]^{1/2} \tag{4.3}$$

where θ is the angle the electron makes with respect to the normal upon ejection.

The motion of the electron in the direction transverse to the z axis is of no interest in this problem. From energy considerations, the total energy E_z associated with the motion in the z direction is

$$E_z = (mv_z^2/2) + U \quad , \tag{4.4}$$

where $U = -e\phi$ is the potential energy of the charge at the electrostatic potential ϕ. At the $z = 0$ plane, equation (4.4) becomes

$$E_z = (mv_{zo}^2/2) + U_o \quad . \tag{4.5}$$

Combining equation (4.3) and (4.5) gives

$$E_z = E_o \cos^2\theta + U_o \quad , \tag{4.6}$$

and the energy equation, equation (4.4), gives

$$(mv_z^2/2) + U = E_o \cos^2\theta + U_o \quad . \tag{4.7}$$

Solving equation (4.7) for the velocity v_z as a function of the initial angle θ and the potential energy U, gives

$$v_z = (2/m)^{1/2}(E_o \cos^2\theta + U_o - U)^{1/2} \quad . \tag{4.8}$$

A velocity v_z is defined only as long as

$$E_o \cos^2\theta + U_o - U \geq 0 \quad . \tag{4.9}$$

II. Initial Angular Distribution of Photoelectrons

If the number of photoelectrons ejected per unit area per unit time is n_o, the flux per unit solid angle $dn_o/d\Omega$, may be represented by

$$dn_o/d\Omega = n_o g(\theta) \quad . \tag{4.10}$$

The function $g(\theta)$ must then satisfy the normalizing condition

$$\int_0^{\pi/2} 2\pi g(\theta) \sin \theta \, d\theta = 1 \quad . \tag{4.11}$$

Upon integration over the azmithal coordinate, equation (4.10) becomes

$$dn_o/d\theta = 2\pi n_o \sin \theta \ g(\theta) \quad . \quad\quad\quad (4.12)$$

The density of electrons $d\rho/d\theta$ created above the surface by electrons which are ejected from the surface at an angle θ to the normal may be written as

$$d\rho/d\theta = 2(dn_o/d\theta) \ S(v_z)/v_z \quad , \quad\quad\quad (4.13)$$

where the factor of 2 allows for the inclusion of the returning electrons and $S(v_z)$ is a unit step function as defined by equation (3.16). Using equations (4.8), (4.9), and (4.12), equation (4.13) may be written as

$$d\rho/d\theta = \frac{2\pi(2m)^{1/2}n_o \sin \theta \ g(\theta) \ S(v_z)}{(E_o \cos^2\theta + U_o - U)^{1/2}} \quad . \quad\quad\quad (4.14)$$

In order to find the total density due to electrons ejected at all angles an integration over the variable θ must be performed.

The angular distribution is found experimentally (Ives, Olpin and Johnsrud, 1928) to favor the upward directed electrons, the transverse flux going to zero. The distribution is dependent upon the value of the electrostatic field at the surface. While this function $g(\theta)$ is derivable from a detailed consideration of the scattering of the photoelectrons in the material before they escape, here it will be sufficient to approximate the distribution by the cosine of the angle θ, assuming a

low value for the electrostatic field at the surface; thus,

$$g(\theta) \approx (1/\pi) \cos \theta \quad , \qquad (4.15)$$

where the factor of $1/\pi$ is a normalization factor, equation (4.11). This choice, equation (4.15), fits the experimental data quite well (Ives and Fry, 1922).

III. Derivation of Differential Equation

Making the substitutions

$$y = \cos \theta \quad , \quad V = U - U_o \quad , \qquad (4.16)$$

equation (4.14) and (4.15) yield

$$\rho = -2n_o (2m)^{1/2} \int_1^0 [y \, S(E_o y^2 - V)/(E_o y^2 - V)^{1/2}] \, dy \quad . \qquad (4.17)$$

The definition of y, equation (4.16), and the unit step function, equation (3.16), yield the condition

$$1 \geq y \geq (V/E_o)^{1/2} \quad ; \qquad (4.18)$$

therefore the upper limit on the integral becomes $(V/E_o)^{1/2}$ rather than zero. Equation (4.17) then becomes

$$\rho = 2n_0(2m)^{1/2} \ S(E_0 - V) \int_{\sqrt{V/E_0}}^{1} y(E_0 y^2 - V)^{-1/2} \ dy \qquad ; \qquad (4.19)$$

the unit step function remains due to condition (4.18) that $V/E_0 \leq 1$. Performing the integration of equation (4.19) yields

$$\rho = 2(2m)^{1/2}(n_0/E_0) \ S(E_0 - V)^{1/2} \qquad . \qquad (4.20)$$

Using the definition of V, equation (4.16), Poisson's equation, equation (3.1), may be written as

$$d^2V/dz^2 = -4\pi e^2 \rho \qquad . \qquad (4.21)$$

Substituting equation (4.20) into (4.21) yields the desired differential equation

$$d^2V/dz^2 = -b_1 \ S(E_0 - V) \ (E_0 - V)^{1/2} \qquad , \qquad (4.22)$$

where the constant b_1 is defined to be

$$b_1 = 8\pi e^2 (2m)^{1/2} n_0/E_0 \qquad . \qquad (4.23)$$

IV. Integration of the Differential Equation

This equation, equation (4.22), may be integrated by multiplying both sides by dV/dz; which yields

$$(dV/dz)^2 = (4b_1/3) \ S(E_o - V) \ (E_o - V)^{3/2} \quad , \quad (4.24)$$

the constant of integration having been chosen as zero so that the field which varies as dV/dz vanishes when $V = E_o$. The final integration may be performed upon taking the square root and dropping the superfluous step function; thus

$$(E_o - V) = (b_1/12)^2 (z_m - z)^4 \quad , \quad (4.25)$$

where the constant of integration z_m has been chosen so that $U = 0$ or $V = E_o$ at the turning point $z = z_m$. The constant z_m may be obtained from equation (4.25) by setting $z = 0$ and noting that $V = 0$ at $z = 0$, as it must according to the definition (4.16); thus,

$$z_m = (144E_o/b_1^2)^{1/4} = (9E_o^3/8\pi^2 e^4 m n_o^2)^{1/4} \quad , \quad (4.26)$$

where b_1 was obtained from equation (4.23). Using the same average value for E_o, equation (3.48), as in the simple model, equation (4.26) may be written

$$z_m = (243\lambda^3 k^3 T^3 / 8\pi^2 e^4 m n_o^2)^{1/4} \quad , \tag{4.27}$$

which may be compared with equation (3.55).

The density of electrons may now be found by taking the second derivative of equation (4.25) and using equation (4.21); which yields

$$\rho = \rho_o (1 - z/z_m)^2 \quad , \tag{4.28}$$

where ρ_o is given by

$$\rho_o = b_1^2 z_m^2 / 48\pi e^2 \quad , \tag{4.29}$$

where equation (4.28) may be compared with equation (3.22) for the Simplest Model. Combining equations (4.29), (4.26) and (4.23), yields

$$\rho_o = n_o (2m/E_o)^{1/2} = n_o (2m/3\lambda kT)^{1/2} \quad , \tag{4.30}$$

where the value of E_o has been chosen as the appropriate average value from equation (3.48). This result is identical, as it should be, to equation (3.56). A plot of equation (4.28) is shown in Figure 4.1.

V. Surface Charge Density and Half Height

From equations (4.28) and (3.49) the surface charge density becomes
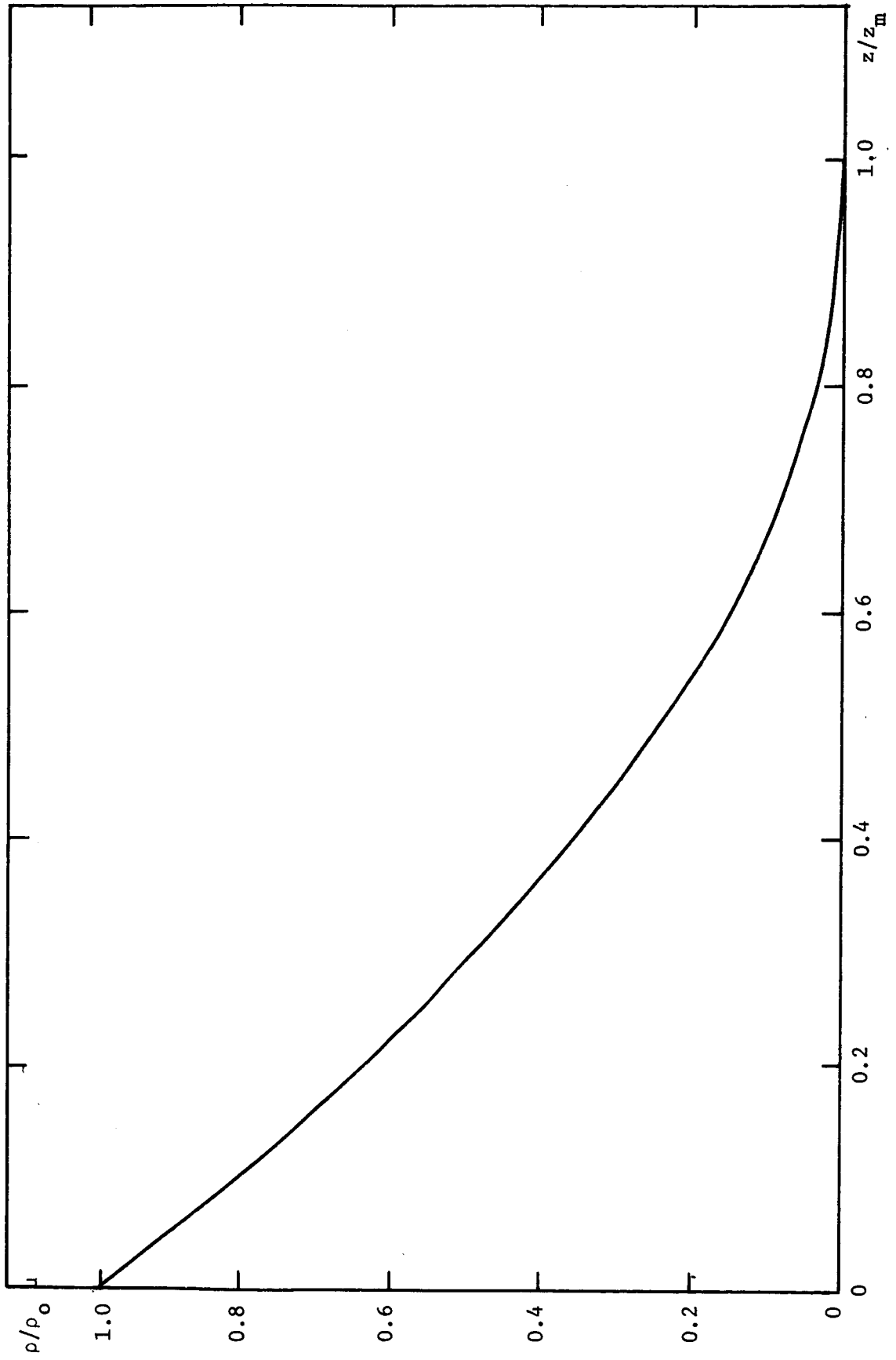
Figure 4.1 Plot of ρ/ρ_o as a function of z/z_m for the Modified Simple Model, equation (4.28)

$$\sigma = e\rho_o \int_0^{z_m} (1 - z/z_m)^2 \, dz \quad . \tag{4.31}$$

Upon integration, equation (4.31) yields

$$\sigma = e\rho_o z_m/3 \quad , \tag{4.32}$$

where ρ_o and z_m may be found from equations (4.30) and (4.27).

The half height of the charge distribution may be found from equation (3.51) upon substitution of equation (4.28) and integrating; thus,

$$h = [1 - (1/2)^{1/3}] z_m = 0.206 \, z_m \quad . \tag{4.33}$$

VI. Numerical Estimates of the Parameters

Using the values for k, T, λ, E_o, and n_o from Table 3.1 , the value for z_m, equation (4.27), is found to be

$$z_m = [243(\lambda kT)^3/8\pi^2 e^4 m n_o^2]^{1/4} = 6.6 \text{ cm} \quad . \tag{4.34}$$

The electron density at the surface from equation (4.30) or (3.56) is found to be

$$\rho_o = 4.8 \times 10^4 \text{ cm}^{-3} \quad . \tag{4.35}$$

The numerical value of σ is then found from equation (4.32), (4.34), and (4.35) to be

$$\sigma = 5.0 \times 10^{-5} \text{ statcoul cm}^{-2} \quad . \tag{4.36}$$

The half height is found from equations (4.33) and (4.34) to be

$$h = 1.4 \text{ cm} \quad . \tag{4.37}$$

These results are tabulated in Table 4.1.

Table 4.1 Parameters for Modified Simple Model (The parameters remaining the same as in the Simple Model are listed in Table 3.1)

Quantity	Symbol	Value
Electron density	ρ_o	$4.8 \times 10^4 \text{ cm}^{-3}$
Maximum height	z_m	6.6 cm
Half height	h	1.4 cm
Surface charge density	σ	$5.0 \times 10^{-5} \text{ statcoul cm}^{-2}$

CHAPTER V

A GENERAL MODEL

A more general and realistic model may now be considered by using

the techniques developed in the preceding two chapters. Two aspects of

the problem which were neglected in the previous chapters and which will

now be considered are:

1) The photoelectrons emitted from a surface by monochromatic

radiation display a distribution ᵒᶠ ᵏᶦⁿᵉᵗᶦᶜ ᵉⁿᵉʳᵍᶦᵉˢ ᵃⁿᵈ ⁿᵒᵗ ˢᶦᵐᵖˡʸ ᵃ

single energy as ᵃˢ ˢᵘᵐ̲ed in the previous two chapters (the average value

indicated by equation (3.11)).

2) The lunar surface is illuminated by black body radiation from

the Sun and not by monochromatic radiation as assumed in the previous

two chapters (the average frequency indicated by equation (3.44)).

I. Kinetic Energy Distribution of Photoelectrons

Ejected by Monochromatic Radiation

The kinetic energy distribution of photoelectrons ejected by

monochromatic radiation may be represented by the function $G(E')$, the

fraction of photoelectrons emitted per unit energy interval at the

energy E'. This function has been obtained experimentally (Hughes and

Dubridge, 1932; and Simon and Suhramann, 1958).

In principle, one might try to account for this distribution by

first assuming the electrons in the metal are distributed in energy

according to the usual Fermi energy distribution for the metal. The kinetic energy of the ejected electron would then be distributed according to the energy difference of the photon energy and the electrons' energy in the metal (Dubridge, 1933). This theory is not sufficient, however, since it fails to take into account the depth of the electron when it receives the photon's energy. The number of photoelectrons generated at a particular depth will depend upon the attenuation of photons with the depth of penetration (a few atomic distances). The photoelectron proceeding from a particular depth to the surface will lose energy by multiple coulomb scattering and as a result straggling of the photoelectrons will occur (Leighton, 1959). The situation is very complicated and no theory yielding satisfactory agreement with observation has as yet been proposed.

In view of the lack of an adequate theory, it will be necessary to choose a function $G(E')$ which gives some reasonable fit to the empirical data. Replacing the kinetic energy of the photoelectrons E' by the dimensionless parameter ξ, where

$$\xi = E'/(E - W) \quad , \tag{5.1}$$

the distribution function $G(E')$ becomes a distribution in ξ, where, according to Einstein's photoelectric equation, ξ varies from zero to unity. Noting the fact that $G(\xi)$ must be zero at both $\xi = 0$, and 1, a function which fits the data reasonably well may be obtained by adjusting two arbitrary parameters λ and d in the following expression

$$G(\xi) = N \, [\varepsilon^{-\xi(\xi-2\lambda)/d^2} - (\varepsilon^{-(1-2\lambda)/d^2} - 1)\xi - 1] \qquad , \qquad (5.2)$$

where N is an appropriate normalizing constant defined by

$$\int_0^1 G(\xi) \, d\xi = 1 \qquad , \qquad (5.3)$$

The constant λ is approximately at the maximum of the curve and corresponds to the average value of $\langle E'/(E - W) \rangle = \lambda \approx 0.4$, equation (3.11). The constant d determines the half width of the expression. The value of $1/d$ is of the order of 3 for a reasonable fit to the experimental data (e.g., Hughes and DuBridge, 1932).

II. Electron Density

If the distribution of photon energies according to the black body formula, equation (3.26), is now included, the flux of electrons from the surface of fractional kinetic energy $\xi = E'/(E - W)$ due to photons of energy E per unit fractional kinetic energy $d\xi$ and per unit photon energy dE is given by

$$d^2 n_o/dE \, d\xi = b_2 G(\xi) \, F(E) \, E^2 \varepsilon^{-E/kT} \qquad , \qquad (5.4)$$

from equations (3.27), (3.28), (3.30), and (3.42), where the b_2 is defined to be

$$b_2 = [15fBR_0/(\pi kT)^4] \qquad , \qquad (5.5)$$

where B is given by equation (3.39).

Including the distribution over all angles, equations (4.11) and (4.12), the flux per unit angle θ per unit photon energy and per unit fractional energy ξ becomes

$$d^3n_0/dE \, d\xi \, d\theta = 2\pi b_2 \sin \theta \, g(\theta) \, G(\xi) \, F(E) \, E^2 \epsilon^{-E/kT} \qquad . \qquad (5.6)$$

From equations (3.6), (4.8), (4.16), and (5.1) the electron density above the surface becomes

$$\rho = 2\pi b_2 \int_W^\infty F(E) E^2 \epsilon^{-E/kT} \, dE \int_0^1 G(\xi) \, d\xi$$

$$\times \int_0^{\pi/2} \frac{g(\theta) \sin \theta \, S[\xi(E - W) \cos^2\theta - V]}{[\xi(E - W) \cos^2\theta - V]^{1/2}} \, d\theta \qquad (5.7)$$

The angular distribution $g(\theta)$, of the emitted electrons will be assumed to follow the empirical curve specified by equation (4.15). Making the substitution

$$y = \sqrt{\xi} \cos \theta \qquad , \qquad (5.8)$$

in equation (5.7) yields

$$\rho = 2b_2 \int_W^\infty F(E)E^2 \varepsilon^{-E/kT} \, dE \int_0^1 \left\{ G(\xi) \quad S[\xi(E-W)-V] \Big/ \xi \right\} d\xi$$

$$\times \int_0^{\sqrt{\xi}} \left\{ y \Big/ [y^2(E-W)-V]^{1/2} \right\} dy \quad . \tag{5.9}$$

This result, equation (5.9), is the expression for the electron density in its most general form.

Upon integration over the variable y, equation (5.9) yields

$$\rho = 2b_2 \int_W^\infty [F(E)E^2 \varepsilon^{-E/kT} \Big/ (E-W)] \quad dE$$

$$\times \int_0^1 \left\{ G(\xi) \quad [\xi(E-W)-V]^{1/2} \, S[\xi(E-W)-V] \Big/ \xi \right\} d\xi \quad . \tag{5.10}$$

III. Approximation 1) and Solution of Differential Equation

If the function $G(\xi)$, equation (5.3), is substituted into equation (5.10), the resultant integral-differential equation which is obtained by substituting equation (5.10) into equation (4.21), becomes completely intractable. Some idea as to the effect of the $G(\xi)$ distribution on the final result may be estimated, however, by considering the approximation

$$G(\xi) \approx N\xi(1-\xi) \quad . \tag{5.11}$$

This approximation is a parabola with its maximum value at 1/2 rather

than the value of λ (≈ 0.4) as used in equation (5.2). By using the normalization condition, equation (5.3), the value of N in equation (5.11) is found to be

$$N = 6 \quad . \tag{5.12}$$

Substituting the approximation for $G(\xi)$, equation (5.11) into the equation for ρ, equation (5.10), yields

$$\rho = 12b_2 \int_W^\infty [F(E)E^2 \varepsilon^{-E/kT} /(E - W)] \quad dE$$

$$\times \int_0^1 (1 - \xi) [\xi(E - W) - V]^{1/2} S[\xi(E - W) - V] d\xi \quad . \tag{5.13}$$

Upon integration over ξ, equation (5.13) yields

$$\rho = (16b_2/5) \int_W^\infty [F(E)E^2 \varepsilon^{-E/kT}(E - W - V)^{5/2} S(E - W - V) /(E - W)^3] dE \quad . \tag{5.14}$$

Substituting in F(E) from equation (3.31) and making the substitution

$$y^2 = E - W - V \quad , \tag{5.15}$$

in equation (5.14) yields

$$\rho = (32b_2/5)\epsilon^{-(W + V)/kT}\int_0^\infty y^6(y^2 + V)^{-1}\epsilon^{-y^2/kT}\, dy \qquad . \qquad (5.16)$$

If it is assumed that $V/kT \ll 1$, the factor $(y^2 + V)^{-1}$ in equation (5.16) may be expanded in a power series and yields to the first order in V

$$\rho = (32b_2/5)\epsilon^{-(W + V)/kT}\int_0^\infty (y^4 - y^2 V)\epsilon^{-y^2/kT}\, dy \qquad , \qquad (5.17)$$

a result valid for small values of V (near the surface $z = 0$). Upon integrating and using equation (5.5), equation (5.17) yields

$$\rho = 96fBR_o(2m/\pi^7k^5T^5)^{1/2}\epsilon^{-W/kT}(3kT/2 - V)\epsilon^{-V/kT} \qquad . \qquad (5.18)$$

The desired nonlinear differential equation may now be found by substituting equation (5.18) into equation (4.21); this yields

$$d^2V/dz^2 = -b_3(3kT/2 - V)^{1/2}\epsilon^{-V/kT} \qquad , \qquad (5.19)$$

where the constant b_3 is defined to be

$$b_3 = 384fBR_o e^2(2m/\pi^5k^5T^5)^{1/2}\epsilon^{-W/kT}$$

$$= 64fR_o e^2(2m\lambda^2/\pi^5k^5T^5)^{1/2}(6 + 4x + x^2)\epsilon^{-x} \qquad , \qquad (5.20)$$

where $x = W/kT$.

Equation (5.19) may be integrated by multiplying both sides by dV/dz and integrating; thus,

$$(dV/dz)^2 = 4b_3(kT)^2(1/4 - V/kT)\epsilon^{-VkT} \quad , \tag{5.21}$$

where the boundary condition is used that $dV/dz \rightarrow 0$ as V becomes large. Making the substitution

$$u^2 = 1/4 - V/2kT \quad , \tag{5.22}$$

taking the square root and rearranging terms, equation (5.21) yields

$$-\int_{1/2}^{\sqrt{1/4 - V/2kT}} \epsilon^{(1/4 - u^2)} du = (b_3^{1/2}/2) z \quad . \tag{5.23}$$

The integral in equation (5.23) is an error function and the equation becomes

$$erf(1/2) - erf(\sqrt{1/4 - V/2kT}) = (b_3/\pi)^{1/2}\epsilon^{-1/4} z \quad . \tag{5.24}$$

The assumption has been made that $V/kT << 1$ and therefore the square root term within the error function in equation (5.24) may be expanded and equation (5.24) becomes

$$erf(1/2) - erf[(1 - V/kT)/2] = (b_3/\pi)^{1/2}\epsilon^{-1/4}z \quad . \tag{5.25}$$

For small values of V/kT, the error function may be expanded in a Taylor series about the point 1/2 and equation (5.25) yields, to the first power in V,

$$V \approx b_3^{1/2} kT \, z \quad , \tag{5.26}$$

for small values of V/kT.

IV. Approximation 2) and Solution of Differential Equation

Equation (5.26) is valid only for small values of V/kT. It will now be shown that it is possible to derive an approximate solution for V which is valid for all values of z. Returning to equation (5.10), and using the mean value theorem for the ξ integral, there is a $\bar{\xi}$ such that

$$\rho = 2b_2 \int_W^\infty \left\{ F(E)E^2 \varepsilon^{-E/kT} [\bar{\xi}(E - W) - V]^{1/2} S[\bar{\xi}(E - W) - V] \middle/ \bar{\xi}(E - W) \right\} dE$$

$$\times \int_0^1 G(\xi) \, d\xi \quad , \tag{5.27}$$

where b_2 is defined by equation (5.5). Using the normalization condition for $G(\xi)$, equation (5.3), equation (5.27) yields

$$\rho = 2b_2 \int_W^\infty \left(\frac{F(E)E^2 \varepsilon^{-E/kT} [\bar{\xi}(E - W) - V]^{1/2} S[\bar{\xi}(E - W) - V]}{\bar{\xi}(E - W)} \right) dE \quad . \tag{5.28}$$

Using equation (3.31) for the function F(E), assuming that $\lambda \equiv \bar{\xi}$, and letting

$$E_o = \lambda(E - W) \qquad , \qquad (5.29)$$

equation (5.28) yields

$$\rho = (2b_2/\lambda^3)\epsilon^{-(W+V/\lambda kT)} \int_0^\infty E_o(E_o - V)^{1/2}\epsilon^{-E_o/\lambda kT}S(E_o - V)\, dE \qquad . \qquad (5.30)$$

Making the substitution

$$y^2 = E_o - V \qquad , \qquad (5.31)$$

in equation (5.30), the density of electrons becomes

$$\rho = (4b_2/\lambda^3)\epsilon^{-(W+V)/kT} \int_0^\infty y^2(y^2 + V)\epsilon^{-y^2/\lambda kT}\, dy \qquad . \qquad (5.32)$$

Where the step function in equation (5.30) makes the lower limit on the integral in equation (5.32) zero. Upon integration, equation (5.32) yields

$$\rho = b_4(3\lambda kT/2 + V)\epsilon^{-V/\lambda kT} \qquad , \qquad (5.33)$$

where the constant b_4 is

$$b_4 = 15BR_0 f(2m/\pi^7\lambda^3k^5T^5)^{1/2}\epsilon^{-W/kT}$$

$$= (5R_0 f/2)(2m/\pi^7\lambda k^5T^5)^{1/2}(6 + 4x + x^2)\epsilon^{-x} \quad , \quad (5.34)$$

where $x = W/kT$. The electron density, equation (5.33), may also be written from equation (3.39) in the form

$$\rho = \rho_0(1 + 2V/3\lambda kT)\epsilon^{-V/\lambda kT} \quad , \quad\quad (5.35)$$

where the constant ρ_0 is given by

$$\rho_0 = 15R_0 f(m\lambda/8\pi^7k^3T^3)^{1/2}(6 + 4x + x^2)\epsilon^{-x} \quad , \quad\quad (5.36)$$

where the quantity x is defined by equation (3.40). This result may be compared with equation (3.57).

The desired differential equation may now be obtained by substituting equation (5.33) into equation (4.21); which yields

$$d^2V/dz^2 = -4\pi e^2 b_4 f(3\lambda kT/2 + V)\epsilon^{-V/\lambda kT} \quad . \quad\quad (5.37)$$

Multiplying both sides of equation (5.37) by dV/dz and integrating with respect to z yields

$$(dV/dz)^2 = b_5(5/4 + V/2\lambda kT)\epsilon^{-V/\lambda kT} \quad , \quad\quad (5.38)$$

where the electric field which is proportional to dV/dz vanishes as V approaches infinity and the constant b_5 is defined by using equation (3.39)

$$b_5 = 40f\lambda R_o e^2 (6 + 4x + x^2)(2m\lambda/\pi^5 kT)^{1/2} \epsilon^{-x} \quad . \qquad (5.39)$$

Taking the square root of equation (5.38) and separating the variables yields

$$\int_0^V (5/4 + V/2\lambda kT)^{-1/2} \epsilon^{V/2\lambda kT} \, dV = b_5^{1/2} z \quad , \qquad (5.40)$$

where the potential energy V is chosen to be zero on the lunar surface. By making the substitution

$$u^2 = 5/4 + V/2\lambda kT \quad , \qquad (5.41)$$

equation (5.40) yields

$$\epsilon^{-5/4} \int_{\sqrt{5/4}}^{\sqrt{5/4 + V/2\lambda kT}} \epsilon^{u^2} \, du = b_6 z \quad , \qquad (5.42)$$

where the constant b_6 is defined by

$$b_6 = [25e^4 R_o^2 f^2 (6 + 4x + x^2)^2 m/2\pi^5 \lambda k^5 T^5]^{1/4} \epsilon^{-x/2} \quad , \qquad (5.43)$$

where $x = W/kT$. The integral in equation (5.42) is of the form of Dawson's Integral, which is defined by

$$D(y) = \epsilon^{-y^2} \int_0^y \epsilon^{t^2} \, dt \quad . \tag{5.44}$$

The value of the Dawson Integral may be obtained from published tables such as in the "Handbook of Mathematical Functions" (Abramowitz and Stegun, 1964). The solution of equation (5.42) may then be expressed in terms of Dawson's Integrals, equation (5.44), and is

$$D(\sqrt{5/4 + V/2\lambda kT}\,)\epsilon^{V/2\lambda kT} - D(\sqrt{5/4}\,) = b_6 \, z \quad . \tag{5.45}$$

A plot of $V/2\lambda kT$ as a function of z is shown in Figure 5.1.

Equation (5.44) may be obtained in a power series by expanding the exponential in the integrand; thus,

$$D(y) = \epsilon^{-y^2} \int_0^y \sum_{n=0}^{\infty} (1/n!)t^{2n} \, dt \quad . \tag{5.46}$$

Upon integration of equation (5.46) the desired power series becomes

$$D(y) = \epsilon^{-y^2} \sum_{n=0}^{\infty} [1/n!(2n + 1)] \, y^{2n+1} \quad . \tag{5.47}$$

The solution, equation (5.45), may then be expressed as a power series,

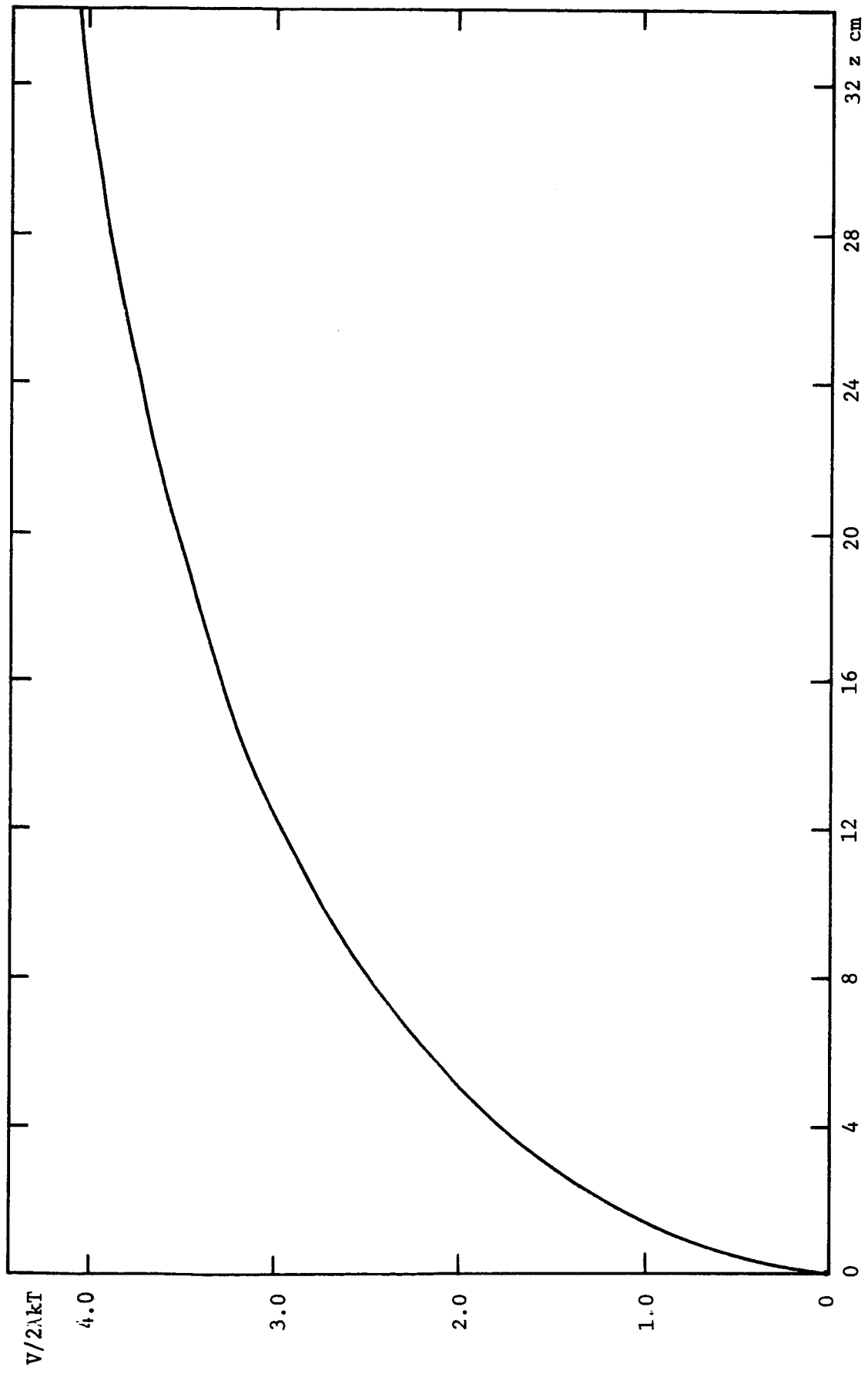$$\sum_{n=0}^{\infty} [(1/n!(2n + 1))(5/4 + V/2\lambda kT)^{(2n+1)/2}] - 1.82 = b_6 \, z \quad . \tag{5.48}$$

Figure 5.1 Plot of $V/2\lambda kT$ as a function of z, equation (5.45)

The series in equation (5.48) converges for all values of V, but does not converge very rapidly for values of $V \gg 2\lambda kT$.

For small values of $V/2\lambda kT$ the integral in equation (5.42) may be evaluated by the mean value theorem for integrals; thus, evaluating the integral at the midpoint yields

$$[(5/4 + V/2\lambda kT)^{1/2} - (5/4)^{1/2}] \, \epsilon^{(\sqrt{5/4 + V/2\lambda kT} + \sqrt{5/4})^2/4}$$

$$\approx \epsilon^{5/4} b_6 \, z \quad . \tag{5.49}$$

For small values of $V/2\lambda kT$ the terms in equation (5.49) or (5.48) may be again expanded in a power series yielding to the first power in V

$$V \approx (2\sqrt{5} \, \lambda kT b_6) z \quad . \tag{5.50}$$

The validity of the Approximation 2) may now be estimated by comparing it with the Approximation 1) (more accurate for small V) in the previous section which was obtained only for small values V (or z). The slope of V as a function of z for z = 0 according to the more accurate Approximation 1), equation (5.26), has the value of $\sqrt{b_3} \, kT$ ergs cm^{-1}; while the present Approximation 2), equation (5.50) yields the comparable value of $2 \sqrt{5} \, \lambda kT b_6$ erg cm^{-1}. The proper behavior of the Approximation 2) for large z, thus, establishes the over-all utility of the second approximation. The previous approximation, Approximation 1), need concern us no further.

For large values of the argument the difference between two

For large values of the argument the difference between two Dawson integrals times the exponential ϵ^{y^2} may be obtained as an asymptotic series by integrating equation (5.44) by parts; thus,

$$\epsilon^{t_2^2} D(t_2) - \epsilon^{t_1^2} D(t_1) = \epsilon^{y^2}(2y)^{-1}$$

$$+ \epsilon^{y^2} \sum_{n=1}^{\infty} [2(2n-1)!/(n-1)! \ (2y)^{2n+1}] \ \left.\begin{matrix} y = t_2 \\ y = t_1 \end{matrix}\right| \qquad . \qquad (5.51)$$

As applied to equation (5.42) the solution may be expressed in the asymptotic series

$$(5 + 2V/\lambda kT)^{-1/2} [1 + \sum_{n=1}^{\infty} 2(2n-1)!/(n-1)!(5 + 2V/\lambda kT)^n] \ \epsilon^{V/2\lambda kT}$$

$$- 1.82 = b_6 \ z \qquad . \qquad (5.52)$$

As $V \to \infty$ equation (5.52) yields

$$V \approx 2\lambda kT \ \ln(b_6 \ z) \qquad . \qquad (5.53)$$

Once V is known as a function of z, the electron density may be obtained from equation (5.33) and the known relation between V and z. The plot of ρ/ρ_o as a function of z is shown in Figure 5.2. An expression for ρ as a function of z for small values of z may be found analytically by substituting equation (5.50) into equation (5.35) and expanding the exponential under the assumption that z is small; thus,
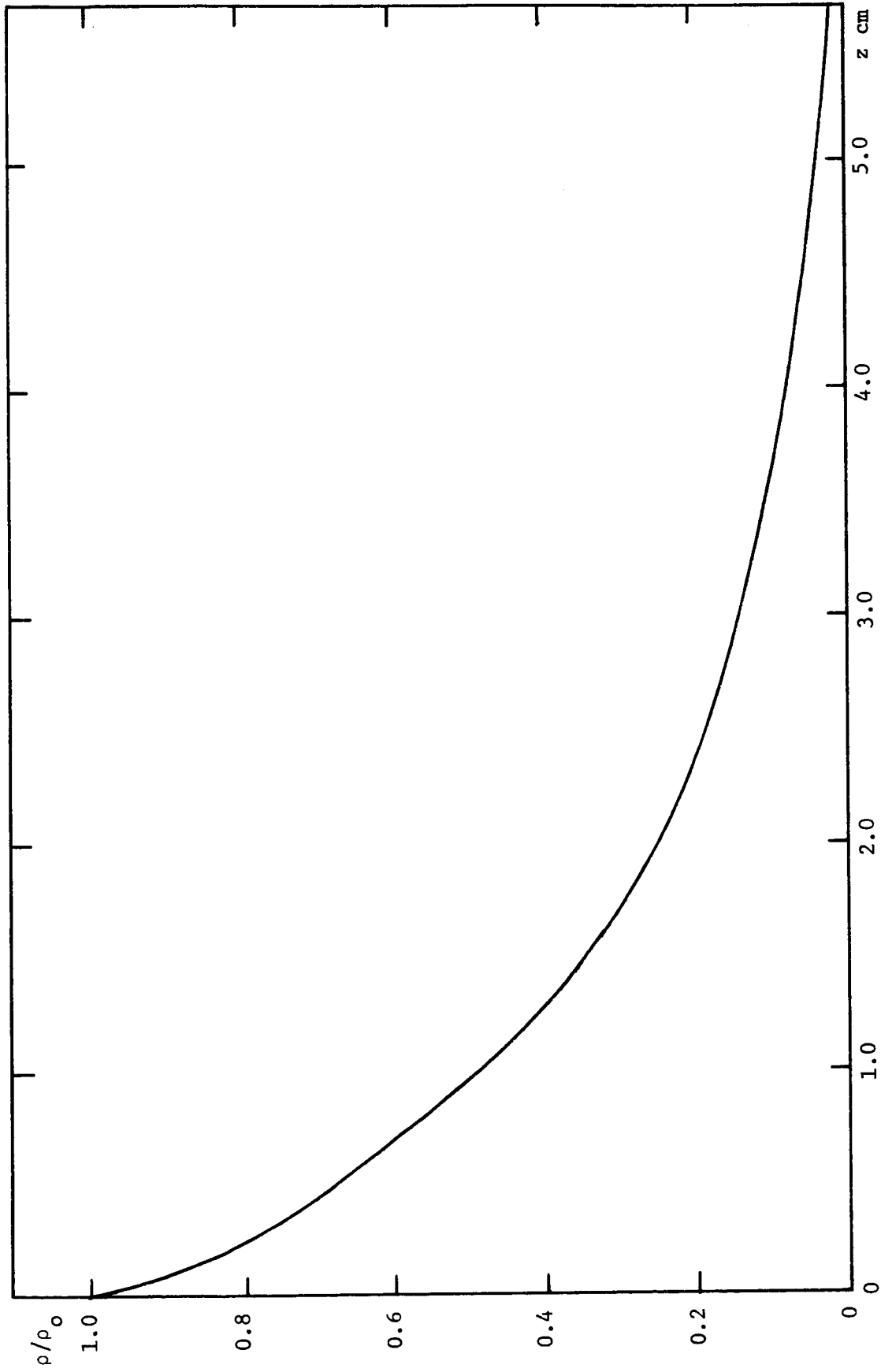
Figure 5.2 Plot of ρ/ρ_o as a function of z for the General Model, equation (5.35)

$$\rho \approx \rho_0 (1 - 2\sqrt{5} \, b_6 \, z/3) \qquad , \qquad (5.54)$$

where the constant b_6 is given in equation (5.43).

V. Surface Charge Density and Half Height

From equations (3.49) and (4.21) the surface charge density is given by

$$\sigma = -(1/4\pi e) \int_0^\infty d^2V/dz^2 \; dz = -(1/4\pi e) \; dV/dz \; \Big|_{z=0}^{z \to \infty} \qquad . \qquad (5.55)$$

At infinity the field vanishes or $dV/dz = 0$. At the surface $z = 0$, $V = 0$ by definition (5.45). From equation (5.38) the surface charge density then becomes

$$\sigma = (5b_5/4)^{1/2} / 4\pi e \qquad , \qquad (5.56)$$

where b_5 is specified by equation (5.39).

From equation (3.51) and (4.21) the half height h, is specified by

$$\sigma/2 = -(1/4\pi e) \int_0^h (d^2V/dz^2) dz = -(1/4\pi e) \; dV/dz \; \Big|_{z=0}^{z=h} \qquad . \qquad (5.57)$$

From equation (5.38) this yields the condition

$$\sigma/2 = (1/4\pi e) \left\{ (5b_5/4)^{1/2} \right.$$

$$\left. - [b_5(5/4 + V/2\lambda kT)]^{1/2} \varepsilon^{-V/2\lambda kT} \right\} \Big|_{z = h} \qquad . \qquad (5.58)$$

Using equation (5.56) this then yields

$$[(5/2 + V/\lambda kT)\varepsilon^{-V/\lambda kT}] \Big|_{z = h} = 5/8 \qquad . \qquad (5.59)$$

The value of h may now be found by solving equation (5.59) numerically

for $V/\lambda kT$ and substituting the result into equation (5.45).

VI. Numerical Estimates of the Parameters

Using the values for k, λ, T, W, and f as presented in Table 3.1,

the value for ρ_0, equation (5.36), is found to be

$$\rho_0 = 1.1 \times 10^5 \text{ cm}^{-3} \qquad . \qquad (5.60)$$

The total surface charge density is found from equation (5.56) and is

$$\sigma = 10 \times 10^{-5} \text{ statcoul cm}^{-2} \qquad . \qquad (5.61)$$

The value of h as found from equations (5.58) and (5.45) is

$$h = 1.6 \text{ cm} \qquad . \qquad (5.62)$$

These results are tabulated in Table 5.1.

Table 5.1 Parameters for the General Model

(all parameters which remain the same are given in Table 3.1)

Quantity	Symbol	Value
Electron density	ρ_o	11×10^4 cm^{-3}
Half height	h	1.6 cm
Surface charge density	σ	10×10^{-5} statcoul cm^{-2}

CHAPTER VI

ADIABATIC GAS MODEL

I. Determination of the Electron Density

A completely different approach may be made to the problem of a lunar atmosphere consisting of photoelectrons if it is assumed that the adiabatic condition holds for the electron gas. With this assumption the pressure of the gas is proportional to ρ^γ, or

$$p = A \rho^\gamma \quad , \tag{6.1}$$

where A and γ are constants which must be determined. In order for equilibrium to exist, the net force on a unit volume of the photoelectron gas must be zero. This equilibrium condition is satisfied if

$$- dp/dz + F = 0 \quad , \tag{6.2}$$

where F is the body force on a unit volume of the gas at the point z above the surface. The body force is given by

$$F = -e \rho \bar{E} \quad , \tag{6.3}$$

where ρ is the electron density, $\bar{E}$ is the electrostatic field produced by the electron distribution. The gravitational force is assumed to be

negligible as compared to the force due to the electrostatic field. Expressing the electrostatic field $\bar{E}$, as the negative gradient of a potential ϕ, equation (6.2), using equation (6.3), becomes

$$dp/dz = e\rho d\phi/dz \quad . \qquad (6.4)$$

Substituting equation (6.1) into equation (6.4) yields

$$\gamma A\rho^{\gamma-1}d\rho/dz = e\rho d\phi/dz \quad . \qquad (6.5)$$

Equation (6.5) may be rearranged and integrated yielding

$$[A\gamma/e(\gamma - 1)] \, \rho^{\gamma-1} = \phi \quad , \qquad (6.6)$$

where the constant of integration has been chosen to satisfy the condition that ρ approaches zero as ϕ approaches zero in the region of $z \to \infty$.

II. The Differential Equation and Its Solution

The desired differential equation may now be obtained by solving equation (6.6) for ρ and substituting into Poisson's equation,

$$d^2\phi/dz^2 = 4\pi e\rho \quad , \qquad (6.7)$$

which yields

$$d^2\phi/dz^2 = c_o\phi^{1/(\gamma-1)} \quad , \tag{6.8}$$

where the constant c_o is defined as

$$c_o = 4\pi e \ [(\gamma - 1)e/\gamma A]^{1/(\gamma-1)} \quad . \tag{6.9}$$

Multiplying both sides of equation (6.9) by $d\phi/dz$ and integrating over the z coordinate yields

$$(d\phi/dz)^2 = [2(\gamma - 1)/\gamma] \ c_o\phi^{\gamma/(\gamma-1)} \quad , \tag{6.10}$$

where the constant of integration has been chosen to satisfy the boundary condition that $d\phi/dz$ approaches zero as ϕ becomes zero in the region of $z \to \infty$.

Taking the minus sign with the square root of equation (6.10) and integrating again yields

$$[2(\gamma - 1)/(2 - \gamma)]\phi^{(\gamma-2)/2(\gamma-1)} = [2(\gamma - 1)c_o/\gamma]^{1/2}z + \text{a constant} . \tag{6.11}$$

This yields the desired relation between the electrostatic potential ϕ and the height above the surface z. Using the boundary condition $\phi = \phi_o$ at z = 0, the constant may be evaluated and equation (6.11) reduces to

$$\phi = [c_1 + c_2z]^{2(\gamma-1)/(\gamma-2)} \quad , \tag{6.12}$$

where the constant c_1 is

$$c_1 = \phi_o^{(\gamma-2)/2(\gamma-1)} \quad , \tag{6.13}$$

and the constant c_2 is

$$c_2 = [2\pi e \ (2 - \gamma)^2/\gamma(\gamma - 1)]^{1/2}[e(\gamma - 1)/\gamma A]^{1/2(\gamma-1)} \quad . \tag{6.14}$$

The density ρ may now be obtained as a function of z by taking the second derivative of ϕ with respect to z, equation (6.12), and substituting the result into Poisson's equation, equation (6.7) or from equations (6.12) and (6.6); this yields

$$\rho = \rho_o(1 + c_3 z)^{2/(\gamma-2)} \quad , \tag{6.15}$$

where the constant ρ_o is

$$\rho_o = [e(\gamma - 1)\phi_o/\gamma A]^{1/(\gamma-1)} \tag{6.16}$$

and c_3 is given by

$$c_3 = c_2/c_1 = [2\pi e(2 - \gamma)^2/\gamma(\gamma - 1)]^{1/2}[e(\gamma - 1)/\gamma A\phi_o^{(\gamma-2)}]^{1/2(\gamma-1)} . \tag{6.17}$$

The constants ϕ_o and A remain to be determined.

III. The Electron Density at the Surface, ρ_o

The current passing through a unit area at the z = 0 plane is given by

$$j_o = e\rho_o \langle v_{zo} \rangle /2 \quad , \qquad (6.18)$$

where the zero subscript indicates values at the z = 0 plane and the brackets indicate the average velocity of the ejected photoelectrons as they pass through the z = 0 plane. The current, j_o, may also be given by

$$j_o = -en_o \quad , \qquad (6.19)$$

where n_o is the electron flux as defined in equation (3.45). Combining equations (6.18) and (6.19) yields

$$\rho_o = 2n_o / \langle v_{zo} \rangle \quad . \qquad (6.20)$$

The average kinetic energy of the photoelectrons as they are ejected from the surface ignoring straggling may be obtained from the effective mean energy E, equation (3.47). Using equation (3.11) the kinetic energy of the photoelectrons as they leave the surface is

$$m \langle v_o^2 \rangle /2 = 3\lambda kT \quad , \qquad (6.21)$$

where T is the temperature of the Sun's surface. By assuming that the
electrons achieve an isotropic velocity distribution above the surface
and by using the principal of equipartition of energy, the average z
component of the velocity squared $\langle v_{zo}^2 \rangle$, equation (6.21) becomes

$$\langle v_{zo}^2 \rangle = 2\lambda kT/m \quad . \tag{6.22}$$

The average upward velocity may now be approximated, using equation
(6.22), by

$$\langle v_{zo} \rangle \approx (2\lambda kT/m)^{1/2} \quad . \tag{6.23}$$

The total flux, n_o, of the electrons passing upward through the
z = 0 plane is given by equation (3.43). The density ρ_o may now be
found by combining equation (3.43), (6.20), and (6.23); which yields

$$\rho_o = [10fR_o(6 + 4x + x^2)/\pi 4] (m\lambda/2k^3T^3)^{1/2}\epsilon^{-x} \quad , \tag{6.24}$$

where x = W/kT, which may be compared with equation (5.36).

IV. Evaluation of the Constants A and ϕ_o

According to the kinetic theory of gases the pressure of a mona-
tomic gas is proportional to the average kinetic energy of each molecule
(Sears, 1953)(i.e., each electron for this case), the relationship being

$$P_o = \rho_o m \langle v_o^2 \rangle /3 \quad , \tag{6.25}$$

where the zero subscript designates values at the surface. Combining equations (6.25) and (6.21) yields

$$P_o = 2\lambda kT\rho_o \quad , \tag{6.26}$$

where ρ_o is given by equation (6.24). From equation (6.1) the constant A may now be obtained; thus, using equation (6.26),

$$A = P_o/\rho_o^\gamma = 2\lambda kT\rho_o^{(1-\gamma)} \quad . \tag{6.27}$$

Substituting this value of A into equation (6.6) and evaluating the equation at the $z = 0$ surface the following expression for ϕ_o may be obtained:

$$\phi_o = 2\gamma\lambda kT/e(\gamma - 1) \quad . \tag{6.28}$$

V. Surface Charge Density and Half Height

From equation (3.49) and (6.15), the surface charge density becomes

$$\sigma = e\rho_o \int_0^\infty (1 + c_3 z)^{2/(\gamma-2)} dz \quad . \tag{6.29}$$

Upon integration equation (6.29) yields

$$\sigma = (2 - \gamma) e \rho_o / \gamma c_3 \quad , \tag{6.30}$$

where ρ_o and c_3 may be found from equations (6.24) and (6.17).

The half height h, of the charge distribution may be found from equation (3.51) upon substituting equation (6.15) and integrating; thus,

$$h = c_3^{-1} [2^{(2-\gamma)/\gamma} - 1] \quad . \tag{6.31}$$

VI. Numerical Estimates of the Parameters

For a monatomic gas the ratio of the specific heats γ, has the value 5/3 (Sears, 1953). Using the values for λ, k, T, and W from Chapter III (see Table 3.1), the value of ρ_o is found from equation (6.24) to be

$$\rho_o = 8.2 \times 10^4 \text{ cm}^{-3} \quad . \tag{6.32}$$

The expression for the electron density, equation (6.15) and (6.17), becomes

$$\rho = \rho_o (1 + 0.0845 \ z)^{-6} \quad , \tag{6.33}$$

which may be compared with equations (3.22), (4.28), and (5.35). A plot

of ρ/ρ_o versus z is shown in Figure 6.1. The surface charge density is found from equation (6.30), using equation (6.32),

$$\sigma = 9.3 \times 10^{-5} \text{ statcoulombs cm}^{-2} \quad . \tag{6.34}$$

The half height above the surface h, from equation (6.31) is

$$h = 1.8 \text{ cm} \quad . \tag{6.35}$$

The constants are tabulated in Table 6.1.

Table 6.1 Parameters for the Adiabatic Gas Model
(other parameters that remain the same are presented in Table 3.1)

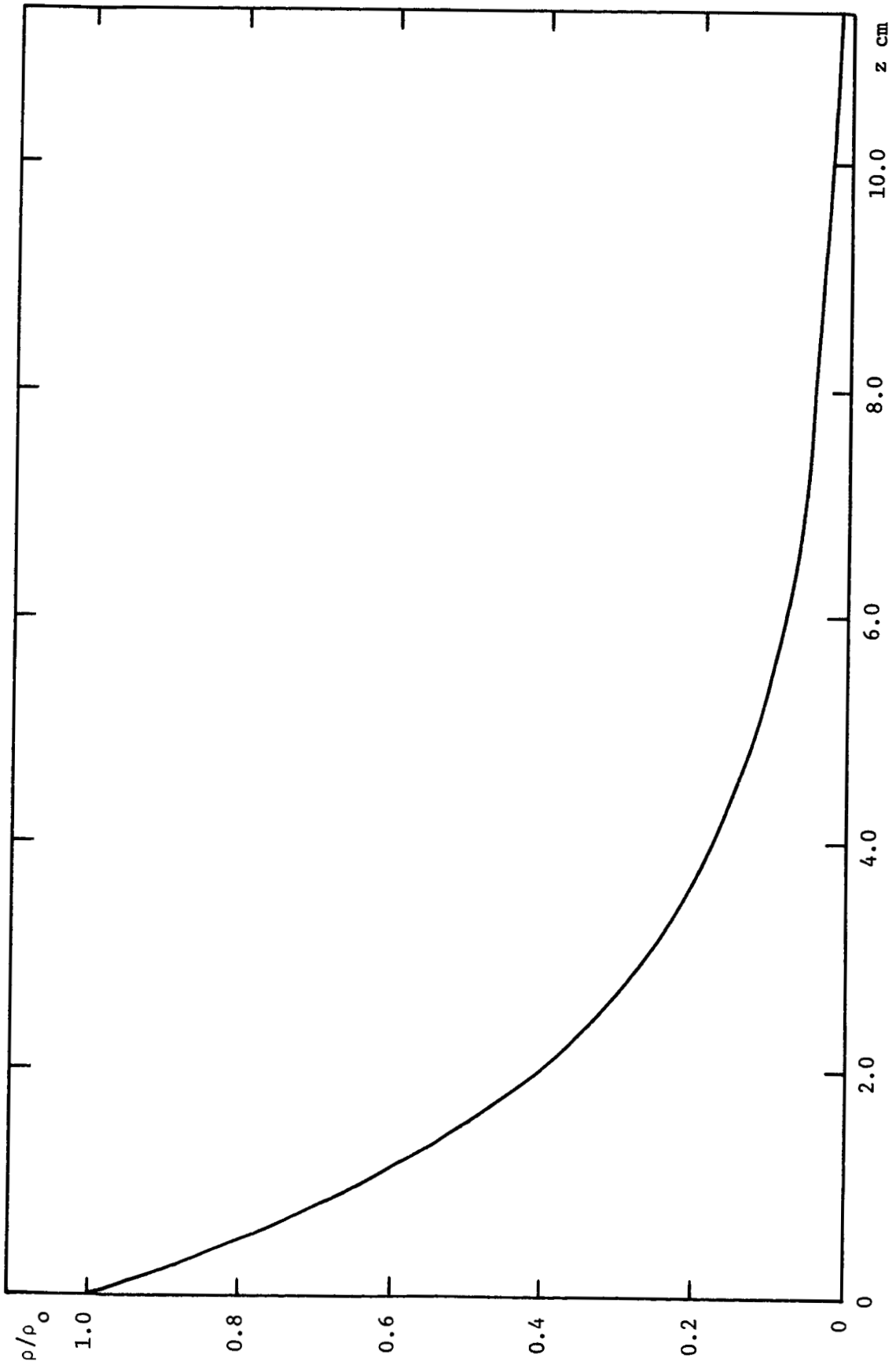| Quantity	Symbol	Value
Electron density	ρ_o	$8.2 \times 10^4 \text{ cm}^{-3}$
Half height	h	1.8 cm
Surface charge density	σ	9.3×10^{-5} statcoul cm

Figure 6.1 Plot of ρ/ρ_o as a function of z for the Adiabatic Gas Model, equation (6.34)

CHAPTER VII

DISCUSSION OF THE VARIOUS MODELS AND OBSERVATIONS

The four different models considered in the previous chapters
may now be compared. The numerical results are collected in Table 7.1.
By examining Table 7.1, it may be seen that the average parameters, sur-
face charge density, σ, the half height, h, and the mean charge density
near the surface $\bar{\rho} = \sigma/2eh$ are all within a reasonable range of each
other for the various models. However, only the numerical values for
the more General Model and the Adiabatic Gas Model are of sufficient
accuracy to warrant any serious consideration. The other models were
discussed to provide a stepwise development of the theory and to pro-
vide a convenient rough check on the more accurate results.

The agreement between the various models in Table 7.1 indicates
that the results do not depend strongly upon the particular model chosen
(to within the many approximations made). It is, thus, possible to
have some degree of confidence in the results. The particular choices
for the parameters, λ, W and f (which affect all of the models) are sub-
ject to considerable doubt. Any discrepancy between theory and observa-
tion may therefore be properly assigned to the uncertainty in these
parameters rather than an uncertainty in the theory (either the General
Model or the Adiabatic Gas Model).

Of the parameters σ, h, and $\bar{\rho}$ only $\bar{\rho}$ has been estimated by obser-
vation. Elsmore and Whitfield (1955) and Costain, Elsmore and Whitfield

(1956) estimate a free electron density of the lunar atmosphere of the order of 10^3 or 10^4 cm^{-3}. These values are considerably lower than the value of $\bar{\rho}$ presented in Table 7.1. This is not surprising, however, since $\bar{\rho}$ measures the average density near the surface and does not indicate the much lower density which trails off to infinity. It may also be noted that any gas molecules present will tend to expand the photoelectron atmosphere by providing some positive charge above the surface. The low values of the electron density observed can probably be accounted for in this way. A lower emission rate from the surface due to a larger W, smaller f or smaller λ would also raise h, lower σ, and lower $\bar{\rho}$.

The functional relationship indicating how the charge density varies with height above the lunar surface is indicated in Figures 3.1, 4.1, 5.2, and 6.1. The curve for the Simplest Model is clearly unrealistic, even though the Simplest Model can provide a rough estimate of the average parameters involved. The General Model and the Adiabatic Gas Model give an electron atmosphere which drops off rapidly with height in a manner which appears to be reasonable.

Table 7.1 Parameters for the Various Models

Quantity	Symbol	Value			
		Simple Model	Modified Simple Model	General Model	Adiabatic Gas Model
Solar constant	R_0	1.33×10^6 ergs cm^{-2} sec^{-1}			
Solar temperature	T	$6000°$ K			
Approximate work function	W	6.6×10^{-12} ergs			
Efficiency factor	f	10^{-3}			
Electron density at z = 0	ρ_0	4.8×10^4	4.8×10^4	11×10^4	8.2×10^4 cm^{-3}
Half height	h	1.6	1.4	1.6	1.8 cm
Surface charge density	σ	1.3×10^{-4}	5.0×10^{-5}	10×10^{-5}	9.3×10^{-5} statcoul cm^{-2}
Average density	$\bar{\rho}$	9.0×10^4	0.8×10^4	6.6×10^4	5.5×10^4 cm^{-3}
Maximum height	z_m	1.8	6.6	∞	∞ cm

CHAPTER VIII

SOME CONSEQUENCES OF A PHOTOELECTRON ATMOSPHERE

The effects of the photoelectron atmosphere on the Moon may now
be estimated using the theoretical models developed in Chapters III, IV,
V, and VI. The precise nature of the Moon's surface remains unknown.
Consequently the yield factor, f, and the work function, W, for the
lunar surface remain unknown. A decrease in the yield factor or an
increase in the work function W, would raise the half height h, decrease
the surface charge density σ, and decrease the average electron den-
sity $\bar{\rho}$. The presence of positively charged ions or particles in the
atmosphere could greatly increase the height of the distribution.

I. Levitation of Charged Dust Particles

The solar wind, solar radiation, and cosmic rays will charge
small dust particles near the lunar surface. A positively charged
particle will experience an upward force in the electrostatic field
produced by the photoelectron space-charge distribution above the
surface. If the electrostatic force equals the gravitational force,
the particle is levitated above the surface. Any lateral variation
in the charge density will cause a lateral movement of the positively
charged particle. Positively charged particles will tend to return to
the lunar surface at points where the electrostatic field vanishes or
is small, i.e., areas of shadow or depressions. This provides an

explanation of how cosmic dust or dust kicked up by meteorites might become distributed over the lunar surface in such a way as to smooth the surface on the scale of 10 centimeters. The consolidation of the surface after the dust particles have been deposited may be assumed to occur under the bombardment of the solar wind and cosmic rays.

The mass of the particles that can be levitated may be estimated by considering the force exerted on a charged particle; thus,

$$F = -q \, d\phi/dz \quad , \tag{8.1}$$

where ϕ is the electric potential at the point z above the surface and q is the charge on the particle. The mass M, which may be supported by the electrostatic field at the point z above the lunar surface then becomes

$$M = -(q/a)d\phi/dz \quad , \tag{8.2}$$

where a is the acceleration of gravity at the lunar surface.

The maximum mass that may be levitated at a height z for the Simple Model may be found from the known value of ϕ as given by equation (3.21) and (3.2). Taking the first derivative of equation (3.21), using equations (3.2), (3.17), (3.25), (3.48), and (3.55), and substituting the result into equation (8.2) yields

$$M = M_0(1 - z/z_m)^{1/3} \quad , \tag{8.3}$$

where M_o, the maximum mass that may be levitated at $z = 0$, is given by

$$M_o = (4q/a)(\pi n_o)^{1/2}(6m\lambda^3/kT)^{1/4}\epsilon^{-x/2}$$

$$= (4q/a)[5fR_o(6 + 4x + x^2)/\pi^3]^{1/2}(6m\lambda^3/kT)^{1/4}\epsilon^{-x/2} \quad , \qquad (8.4)$$

where x is defined by equation (3.40).

The Modified Simple Model yields the maximum mass that can be levitated as a function of z from equation (4.25) and (4.16). Taking a first derivative of equation (4.25), using equations (4.16), (4.23), (3.48) and (3.2), and substituting the result into equation (8.2) yields

$$M = M_o(1 - z/z_m)^3 \quad , \qquad (8.5)$$

where M_o, the maximum mass that can be levitated at the surface $z = 0$, is given by

$$M_o = (8q/a)(\pi n_o)^{1/2}(m\lambda kT/6)^{1/4}$$

$$= (8q/a)[5fR_o(6 + 4x + x^2)/\pi^3]^{1/2}(m\lambda^3/6kT)^{1/4}\epsilon^{-x/2} \quad , \qquad (8.6)$$

where x is defined by equation (3.40). This result may be compared with equations (8.3) and (8.4).

The maximum mass levitated as a function of V for the General Model may be found from equation (5.38). Taking the square root of

equation (5.38), using equations (5.39) and (3.40), and substituting
the result into equation (8.2) yields

$$M = M_0(1 + 2V/5\lambda kT)^{1/2} \epsilon^{-V/2\lambda kT} \quad , \qquad (8.7)$$

where M_0 is given by

$$M_0 = (5q/a) \ [2fR_0(6 + 4x + x^2)]^{1/2}(2m\lambda^3/\pi^5 kT)^{1/4}\epsilon^{-x/2} \quad . \qquad (8.8)$$

where x is defined by equation (3.40). This result may be compared with
equations (8.4) and (8.6). The mass M as a function of z may be obtained
by using equation (5.45) to obtain V as a function of z (see Figure 5.1).

The maximum mass levitated as a function of z for the Adiabatic
Gas Model may be found from equation (6.10). Taking the first deriva-
tive of equation (6.12) and substituting into equation (8.2) with the
appropriate change of sign

$$M = M_0(1 + c_3 z)^{\gamma/(\gamma-2)} \quad , \qquad (8.9)$$

where c_3 is a constant defined by equation (6.17) and M_0, the maximum
mass that can be levitated at the surface z = 0, is given by

$$M_0 = [2q(\gamma - 1)c_2/a(2 - \gamma)] \ c_1^{\gamma/(\gamma-2)} \quad , \qquad (8.10)$$

where c_1 and c_2 are defined by equations (6.13) and (6.14). Using

equations (6.27) for A and (6.28) for ϕ_o, and equation (6.24) for ρ_o, M_o becomes

$$M_o = (4q/a) \ [5fR_o(6 + 4x + x^2)/\pi^3]^{1/2}(2m\lambda^3/kT)^{1/4} \quad , \qquad (8.11)$$

which may be compared with equations (8.4), (8.6) and (8.8). From equations (6.17), (6.27), (6.28), and (6.24),

$$c_3 = [e(2 - \gamma)/\gamma\pi^2kT] \ [10\pi fR_o(6 + 4x + x^2)]^{1/2}(m\lambda/2kT)^{1/4} \quad , \qquad (8.12)$$

where x is defined by equation (3.40).

Assuming that a particle can have a positive charge of approximately 10 electron charges; M_o for the various models, using q = 10e and using a = 167 cm sec^{-2} (Handbook of Chemistry and Physics, 1964) for the Moon, equations (8.4), (8.6), (8.8) and (8.11), is given in Table 8.1.

The density of a lunar electron atmosphere is of such a small magnitude that it produces no effect on the lunar features other than through the electrostatic forces exerted on charged particles. Although the maximum mass which may be levitated is very small, the net effect over a very long period of time could be considerable.

The size of the dust particle which may be levitated assuming a density of 2 is approximately

$$d = (3M_o/4\pi\rho)^{1/3} \approx 1.6 \times 10^{-5} \text{ cm} \quad , \qquad (8.13)$$

which compares with the optical estimate of 0.03 centimeter for grains observed on the moon.

II. Communication Through Photoelectron Atmosphere

The presence of an electron atmosphere near the lunar surface may provide a means for communicating over the lunar horizon, beyond the line of sight. A longitudinal plasma wave can be propagated in the atmosphere. The velocity of propagation of a longitudinal wave in a plasma is given (Stix, 1962) by

$$v = \omega/k = \beta/\sqrt{1 - \omega_p^2/\omega^2} \quad , \tag{8.14}$$

where β is the velocity of sound in the medium, ω_p is the plasma frequency; which is

$$\omega_p^2 = 4\pi e^2 \rho/m \quad , \tag{8.15}$$

and ω is the impressed frequency. The value of $\bar{\rho}$ for the two most realistic models falls at approximately 6×10^4 per cubic centimeter. Using this average value for $\bar{\rho}$, ω_p, equation (8.14), yields the value

$$\omega_p = 1.3 \times 10^7 \text{ sec}^{-1} \quad . \tag{8.16}$$

This is a frequency of about 10 megacycles. This plasma frequency ω_p is

the cut off frequency, or lowest frequency, which will propagate in the plasma. The value of β is given (Sears and Zemansky, 1955) by

$$\beta = \sqrt{\gamma p / m \rho} \quad , \qquad (8.17)$$

where p is the pressure and ρ is the density. Using the Adiabatic Model as an example, the ratio of p to ρ is given by equation (6.26). The value of β then is, using γ = 5/3,

$$\beta = 2.4 \times 10^7 \text{ cm sec}^{-1} \quad . \qquad (8.18)$$

This is a lower limit for the velocity of propagation of a longitudinal wave in the plasma, the actual velocity being governed by equation (8.14).

Due to the lack of sources which produce perturbations of the space-charge near the lunar surface, it should be possible to detect waves of very low energy density, the lower limit being determined by the design of the detector. A suitable generating and detecting system would have to be designed for this purpose.

III. Occultations of Radio Stars

An electron atmosphere confined to within a few meters of the lunar surface will probably not produce any measurable occultation of radio stars prior to the occultation by the solid lunar surface. However, as mentioned previously, there are several mechanisms which might

act to raise the height to a distance at which the charge distribution might produce occultations of radio stars before the solid surface. Residual gases retained near the lunar surface by the Moon's gravitational field, and positively charged particles supported by the electrostatic field, will have a considerable effect on the height of the photoelectron atmosphere. The actual effect of such residual gases and dust particles would be hard to predict, but it is conceivable that they could expand the atmosphere to a height such that it might be able to produce the occultations of radio stars as observed experimentally.

Table 8.1 Values of M_o for the Various Models

(Using q = 10e; other parameters that remain

the same are presented in Table 7.1)

Simplest Model	Modified Simple Model	General Model	Adiabatic Gas Model
4.4×10^{-14}	3.6×10^{-14}	3.5×10^{-14}	3.3×10^{-14} grams

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