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ON THE ADIABATIC MOTION OF ENERGETIC PARTICLES IN A MODEL MAGNETOSPHERE

by

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ABSTRACT

The motion of charged particles in a model magnetosphere is studied using the three adiabatic invariants. The particle shell geometry is determined, and drift velocities. bounce periods and equatorial pitch angles are computed as a function of local time. The following conclusions were reached:

1. Shell splitting in the outer magnetosphere becomes important beyond 5 earth radii; dipole-type descriptions of the radiation belt become invalid.

2. Equatorial pitch angles tend to align along field lines on the night side of the magnetosphere, and perpendicularly to the field. on the day side.

3. There are regions in the magnetosphere. where only pseudo-trapped particles can mirror. i.e. particles which will leave the magnetosphere before completing a 180° drift.

4. Longitudinal drift velocities depart considerably from the dipole values beyond 5 R_e. and can be as much as 2-3 times greater on the night side than on the day side. Thus a given particle spends 2-3 times more time in the day side than in the night side.

5. The action of a pitch angle scattering mechanism will lead to a radial diffusion of particles. The loss mechanism will be greatly enhanced by scattering of mirror points into the pseudo-trapping regions.

6. After recovery from a prototype magnetic storm, particles which were in the day side during the sudden commencement will have higher energies. their shells having moved radially inwards. Particles caught in the night side will have moved outwards. with their energies decreased.

7. The repeated action of magnetic storms will result in a net inward diffusion of particles. with a net increase of their energy.

ON THE ADIABATIC MOTION OF ENERGETIC PARTICLES IN A MODEL MAGNETOSPHERE

I. INTRODUCTION

Recent experimental results on trapped particle flux behavior in the outer magnetosphere indicate that physical processes governing particle diffusion and acceleration, are strongly influenced by the trapping field itself and by the time-changes of its configuration. The main evidence comes from the observed strong correlations between particle flux and energy spectra variations beyond 2-3 earth radii, with geomagnetic perturbations such as the sudden commencement of a geomagnetic storm or the ring current during the main phase [Frank, 1966; McIlwain, 1965; McIlwain, 1966]. It seems therefore useful to attempt a detailed theoretical description of the behavior of a flux of trapped particles using a reasonably accurate magnetospheric field model, and simulating prototype time variations of the field configuration. The first detailed studies of this type were done by Hones [1963] for auroral particles, and by Fairfield [1964] for energetic particles.

There are several sources of the field in the geomagnetic cavity: the magnetization of the earth's interior, the currents flowing on the surface of the magnetopause, the currents in the "neutral sheet" of the tail of the magnetosphere and, eventually, diamagnetic ring currents originating in trapped particle density gradients at 2-4 earth radii. At geocentric distances of less than, say, 4 earth radii, the internal geomagnetic field dominates; beyond $4R_e$, the currents in the magnetopause (and in the neutral sheet) perturb the dipole-type internal field, and introduce a strong noon-midnight asymmetry. Any model must take these sources into account.

Before adopting de facto a given field model, let us list our requirements. First of all, we are mainly interested in particles trapped on field lines reaching out beyond, say, 5 earth radii, on the equatorial plane. This means that we can safely ignore all higher multipoles of the internal field, and replace it by a centered dipole. Second, we shall also ignore the effect of a ring current. Third, we shall consider the dipole axis perpendicular to the sun-earth line, which of course is a very substantial limitation. However, a "wobbling" dipole would make our calculations immensely more complicated, without, however, adding much to the general results, at least within the scope of this paper. Finally, electric fields will be ignored; this means that we are restricting ourselves to particles of high enough energy to ensure that the gradient drift always prevails over the $E \times B$ drift.

A model which satisfies these requirements and which has already predicted or explained experimental results with good quantitative agreement, is that given by [Mead, 1964; Williams and Mead, 1965; Mead, 1965]. This model considers two sources, in addition to the internal dipole; currents in the magnetopause, and currents in the tail of the magnetosphere. Four adjustable parameters determine the field in Mead's model; (1) The distance R_s from the center of the earth to the magnetopause, in the solar direction; (2) and (3), the distances R_{min}, R_{max} from the center of the earth to the close and far limit of the neutral sheet in the anti-solar direction, respectively, and (4) the field intensity B_T near the neutral sheet. Most of the typical variations of these parameters. See Section IV for choice of parameters actually used.

Fig. 1 shows field lines of this model in the noon-midnight meridians, corresponding to the parameters which we shall adopt as describing the quiet-time

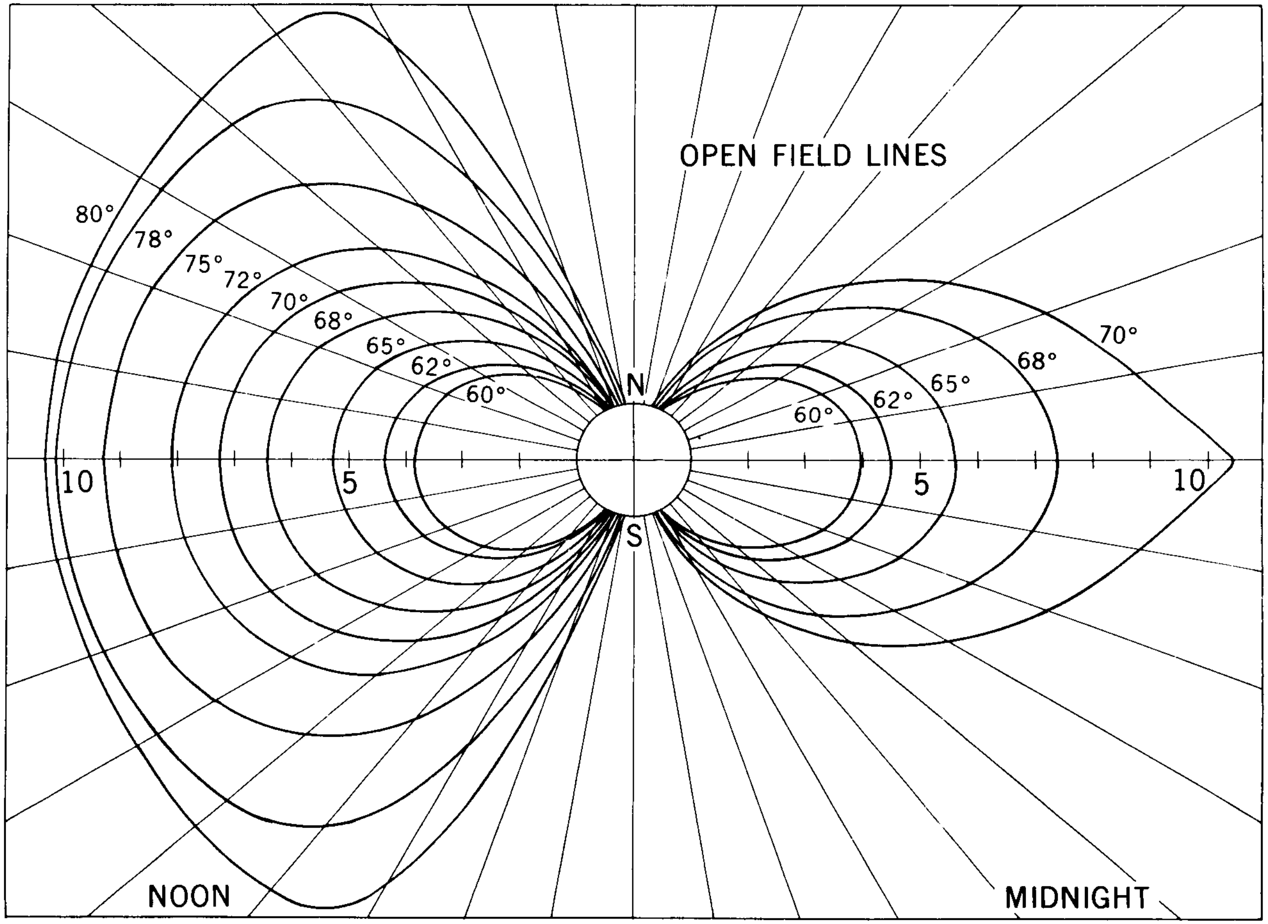

Figure 1—Closed field lines in the model magnetosphere, for parameter values given in the text (page 18). Field lines labeled with the geomagnetic latitude of their intersection with the earth's surface.

state of the magnetosphere. In this figure, all field lines start at equally spaced latitudes on the earth's surface. In order to save time and space, we shall call Mead's model of the magnetospheric field "the meadosphere."

II. ADIABATIC INVARIANTS

The motion of charged particles in a trapping field geometry can be described by means of three adiabatic constants of motion [Northrop, 1963]. Although the the invariants yield incomplete information about the actual position of a particle as a function of time, they do lead to the determination of the two dimensional manifolds or "particle shells," on which the guiding centers of the particles are confined. They further provide general information on the energy changes of the particles, although, again, no "microscopic" time-history would be available.

The three adiabatic invariants are the following: (1) the magnetic moment M of the particle, generated by its cyclotron motion around a field line; (2) the second invariant J, associated with its bounce motion along a field line between mirror points; (3) the flux invariant, associated with its azimuthal or longitudinal drift motion. The definition of these quantities are:

$$M = \frac{p_\perp^2}{2m_0 B} \tag{1}$$

$$J = \oint p_{\parallel}\, ds \tag{2}$$

$$\Phi = \oint \vec{A} \cdot \vec{dx} \tag{3}$$

$p_{\perp}$ and $p_{\parallel}$ are components of the momentum perpendicular and parallel to the magnetic field vector, respectively; B is the absolute magnetic field at the instantaneous position of the guiding center and m_0 the rest mass. In (2), the integration is extended along the field line for a complete bounce oscillation; ds is the element of arc of the field line. (3) represents the magnetic flux enclosed by the particle shell; $\vec{A}$ is the magnetic vector potential, and the integral is extended along any closed path passing around the particle shell and lying in it. These quantities are adiabatic constants, i.e. conserved only under certain conditions. Each of the invariants has an associated characteristic period of time and a characteristic length. These are respectively; (1) cyclotron period and gyroradius; (2) bounce period and arc length between conjugate mirror points; (3) azimuthal drift period and arc length of the equatorial ring of the shell. In most magnetic field configurations, these characteristic quantities differ by several orders of magnitude from one another. Adiabaticity requires that the field configuration should not change appreciably during a characteristic period. If this condition is violated, the corresponding value of the invariant will no longer be conserved. The other two may still remain unaffected.

In place of (1) and (2) we shall introduce two other expressions which are much more convenient for numerical computations. According to (1), we introduce the mirror point field intensity B_m:

$$B_m = \frac{p^2}{2m_0M} \tag{4}$$

p is the total momentum of the particle. Further, we introduce the geometric integral

$$I = \int \sqrt{1 - \frac{B(s)}{B_m}}\, ds = \frac{J}{2p} \tag{5}$$

extended along the field line between one mirror point and its conjugate. B_m and I uniquely determine a particle shell and will be used in what follows as the two identifying parameters for such a shell. Notice at once that for a time-independent magnetic field, in absence of electric fields, (4) and (5) are adiabatic invariants, too. The advantage of (4) and (5) is that they only depend on the field geometry. For time dependent fields, care has to be taken regarding relation (5). If p changes appreciably during one bounce, we have:

$$\frac{J}{2\overline{p}} = \frac{1}{2\overline{p}} \oint p(s) \sqrt{1 - \frac{B(s)}{B_m}}\; ds \neq I \tag{5'}$$

Only for slowly varing fields, (5) holds at all times.

Introducing the usual relativistic factor $\gamma = m / m_0$, we can combine (4) and (5) with (1) and (2) to obtain

$$(\gamma^2 - 1)\, I^2 = K_1 = \text{const.} \tag{6}$$

$$I^2 B_m = K_2 = \text{const.} \tag{7}$$

To these, we must add (3), which we write in the form:

$$\Phi = \Phi\,(B_m, I) = K_3 = \text{const.} \tag{8}$$

These are the adiabatic constants which will be used henceforth. They uniquely determine γ, I and B_m for any static field configuration. For time dependent fields, K_1 and K_2 in general are not constant <u>during</u> the interval of change (see (5')). However, if this interval is transient, i.e. if it is preceded and followed by time independent states of the field, the constancy of K_1 and K_2 <u>does</u> hold when (6) and (7) are evaluated for the initial and the final states.

For a field constant in time, (8) is no longer needed, and γ, I and B_m are conserved individually. A special case is that of particles mirroring on or close to the equator ($I = 0$, $B_m = B_e$). In that case we replace (6) by the following, derived from (4):

$$\frac{1}{\gamma^2 - 1} B_e = \text{const.} \tag{6a}$$

This holds all the time, even in non-static fields. In this special case, (8) is a function of B_e only.

We have to add Liouville's Theorem, to relations (6), (7) and (8), in order to complete our description of trapped particle dynamics. We introduce the directional, differential flux of particles populating a given I, B_m shell:

$$j = j\,(\gamma,\ I,\ B_m) \qquad \text{Particles/steradian} \times \text{energy} \times \text{sec} \tag{9}$$

Assume particles distributed on shell in such a way that there is a steady state. For each point of the shell, there is a unique cone along whose elements the given group of particles is streaming. This direction is the particle's pitch angle, which at the equatorial points of the shell is given by

$$\alpha_e = \text{arc sin}\sqrt{\frac{B_e}{B_m}} \tag{10}$$

$B_e = B_e(I, B_m; \text{azimuth})$ is the minimum or equatorial B value of a particular field line of the shell. We must point out at once that even for time-independent fields, α_e is <u>not</u> an adiabatic invariant like the mirror point field B_m; in general it will depend on azimuth (longitude or local time) through B_e.

The particle density in phase space, f, associated with the flux (9) is given by $f = j/p^2$. Liouville's Theorem states that this density in phase space remains constant along the dynamical path of a particle. This means that, as long as all particles always remain on a common shell*, even if the shell itself changes with time, the following conservation theorem holds:

$$\frac{j(\gamma, I, B_m)}{\gamma^2 - 1} = K_4 = \text{const.} \tag{11}$$

(6), (7), (8), and (11) are the basic expressions to be used for the study of the evolution in space and time of trapped particles in the outer magnetosphere. The first three lead to the determination of the actual shell of a given group of particles, and their actual energy; equation (11) gives the actual value of the directional differential flux of this group of particles.

As was shown by [Anderson, Crane, Francis, Newkirk and Walt, 1964] the average equatorial azimuthal drift velocity u of the particles populating and I, B_m shell can be obtained as a direct consequence of Liouville's Theorem. The value is, with our notation:

$$u = \frac{m_0 c^2}{eB_e} \frac{\gamma^2 - 1}{\gamma} \frac{\nabla I}{S_b} \tag{12}$$

∇I is the limit of $\delta I/\delta y$ where δy is the equatorial distance between two neighboring shells, each one characterized by I and $I + \delta I$, respectively, and by the same B_m-value. S_b is the half-bounce path, i.e. the rectified path of the particle between one mirror point and its conjugate:

$$S_b = \int \frac{ds}{\sqrt{1 - \frac{B(s)}{B_m}}}$$

*This condition will be fulfilled if all invariants (6) – (8) are conserved throughout the evolution of the system.

S_b is related to the bounce period by $\tau_b = 2 S_b / v$ where v = particle velocity. We can obtain an expression for S_b by taking the derivative of (5) with respect to the mirror point field intensity B_m along a given field line. It can be shown by simple algebra that

$$S_b = I - 2 B_m \frac{\partial I}{\partial B_m} \tag{13}$$

The derivative has to be taken along the given field line. This expression has general validity for any trapping field geometry, and is very useful for computational purposes, for it only requires the calculation of I on two neighboring points of a field line.

III. QUALITATIVE DISCUSSION

Before getting into the discussion of numerical computations for particle shell geometry and time variations of the meadosphere, it is useful to present a qualitative analysis of the consequences of the previous section. First of all, let us envisage a trapping magnetic field constant in time. In this case, (4) and (5) are conserved. We can assign to each point in space a pair of values I, B_m such that a particle mirroring there has the value I for the integral (5), B_m being simply the field intensity at that point. In this way, I and B_m become uniform functions of space. As the particle drifts to other field lines, it must keep these values constant, i.e., it will cover a shell of field lines which pass through

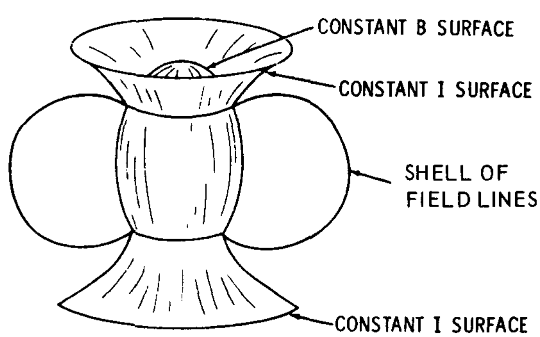

Figure 2–Geometric definition of a particle shell

the intersections of two given constant-I and constant-B_m surfaces (Fig. 2). Notice carefully that the surface I = const. is not the particle shell.

Let us consider the geomagnetic field. Take a particle which starts at a given longitude ϕ, circling around a given field line and mirroring at a value B_m. The integral (5) computed along the field line between the two mirror points, has a value I. This means that when drifting through any other longitude, say 180° away, this particle will be bouncing along a field line which passes through the intersection of the corresponding I = const. and B_m = const. surfaces. Now take a particle which starts on the same initial field line, but which mirrors at a lower value $B'_m < B_m$ (Fig. 3). Its integral (5) will also be smaller, $I' < I$. After a 180°

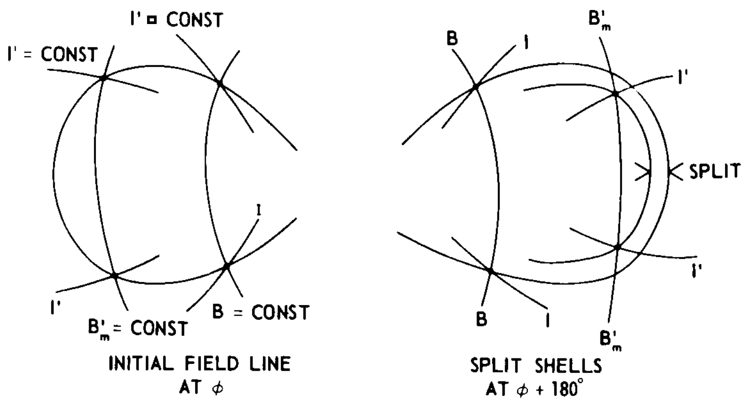

Figure 3–Shell splitting in asymmetric fields.

longitudinal drift, this second particle will be traveling along a field line which passes through the intersection of the surface I' = const. and B'_m = const. Only in case of perfect azimuthal symmetry (as in the pure dipole), will these surfaces intersect exactly on the same line as that of the first particle. In the general case, particles starting on the same field line at a given longitude will populate different shells, according to their initial mirror point fields, or, what is equivalent, according to their initial equatorial pitch angles (10) (of course, all these different shells would be tangent to each other at the initial field line).

For the case of the real geomagnetic field in absence of external perturbations, i.e. within about 3-4 earth radii, it can be shown that the distance between split shells is only very small, a fraction of 1% of the distance of the equatorial point of a field line to the center of the earth. In other words, with a very good approximation, one can say that all particles initially on the same field line, will mirror on a common field line at any other longitude. This has an important consequence: it enables a two-dimensional description of the three-dimensional radiation belts, at least up to distances of about $4R_e$. Indeed, if particles do populate the same shell irrespective of their initial mirror points, the omnidirectional flux of these particles will be the same on all points of the shell having the same B value, irrespective of the longitude (i.e. local time)(provided of course, no appreciable injections or losses occur during the drift). In order to describe omnidirectional particle fluxes in the inner magnetosphere, we therefore need only two "space" parameters: the value of the magnetic field intensity at the point of measurement and a parameter which characterizes the (unique) shell which goes through that point. This latter is McIlwain's L-parameter [McIlwain, 1961]. L is a particular relation between I and B_m which remains constant (within $\lesssim 1\%$) on a given field line, and, therfore, on the whole shell generated by particles starting on that field line. Numerically, L gives the average distance of the equatorial points of a shell to the magnetic center.

But what happens in the outer magnetosphere, where the azimuthal symmetry is brutally removed? Particles starting on the same field line, say in the noon meridional plane, will now populate different shells, depending on their initial mirror points or equatorial pitch angles. For instance, they will cross the midnight meridian on <u>different</u> lines.

Let us start with a particle mirroring at or near the equator, on a line in the noon meridian, close to the boundary. For this particle, $I \simeq 0$; according to (6a) it will drift around the earth on the equator following a constant-B path. This constant-B path comes considerably closer to the earth at the night side, because the field is weaker there (less compression), and we must go to lower altitudes in order to find a given B value. On the other hand, a particle which starts on the same field line on the noon meridian, but which is mirroring at high latitudes, will have a high I value. Under these circumstances, Mead has shown [Mead, 1965] that the value of I is not much different from the arc length of the field line between mirror points. On the midnight meridian, the particle will therefore be found on a line which has nearly the same length than the initial one, i.e. stretching out to roughly the same equatorial distance. In summary, all particles initially on the same noon-line, will cross the midnight plane on line portions sketched in Fig. 4. Furthermore, it is easy to realize that particles mirroring

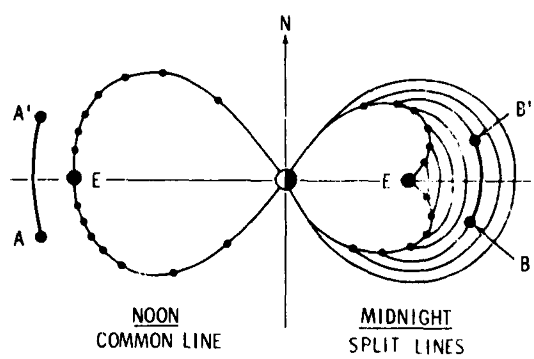

Figure 4–Qualitative picture of shell splitting in a model magnetosphere for particles starting on a common field line in the noon meridian.

inside that area (BB′), will cross the noon meridian outside (AA′) of the initial line. If this noon-line is very close to the boundary, no stably trapped particle could be found mirroring inside the hatched area in the midnight meridional plane. Any particle doing this would not be able to complete a drift around the earth: it would abandon the magnetosphere before reaching the noon meridian. We shall call this a "pseudo-trapped" particle [11] (only transiently trapped). Notice finally that a sharp trapping boundary in the noon side would not result in a sharp boundary in the back side.

On the other hand, for a given field line in the midnight meridian, all particles mirroring anywhere on this line, will cross the noon meridian in an area like the one shown in Fig. 5. All particles mirroring outside that area (BB′) will cross the midnight-meridian outside (AA′) of the given line. If now there

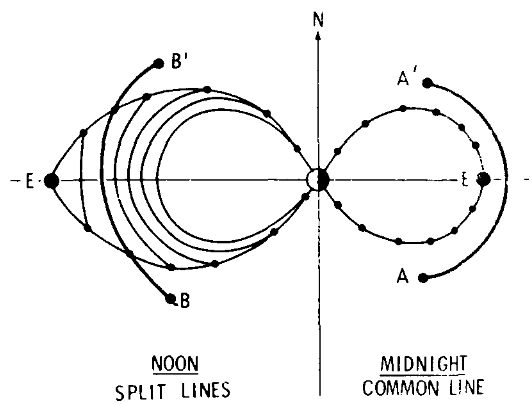

Figure 5 – Qualitative picture of shell splitting for particles starting on a common line in the midnight meridian.

is an "obstacle" behind that line (like for instance the neutral sheet), no stably trapped particle could be found outside the hatched area in the noon meridian. Any particle injected there, would be lost into the "obstacle" before reaching the midnight meridian: in this high latitude noon region, only pseudo-trapped particles could exist.

We can also make some qualitative predictions regarding equatorial pitch angles and longitudinal drift velocities of particles trapped in the meadosphere. Take again a group of particles starting on a common noon-side line. It is easy to see from Fig. 4, that the equatorial B-value of a given shell must decrease towards the midnight meridian except for particles mirroring close to the equator. This means that, according to (10), the equatorial pitch angle must also decrease: particles align along field lines, in the midnight meridian. On the other hand, inspection of Fig. 5 reveals that the opposite is true for particles starting on a common field line in the midnight meridian: they align perpendicularly to the field line on the day side of the meadosphere.

Regarding the drift velocity (12), we can qualitatively say that the field near the day-side boundary is much more homogeneous than on the night side, or than in the case of a pure dipole (Fig. 1). Therefore, ∇I will be relatively smaller, and S_b greater. Both facts indicate that the particles will drift slowest on the noon meridian, spending therefore a greater fractional time in the day side of the meadosphere. This may have very important consequences for outer belt dynamics.

Turning now to a qualitative discussion of the effects of a time-dependent magnetic field, we first have to point out that slow changes, which conserve all three invariants (6), (7) and (8), are reversible. This means that whenever the field is back to its initial configuration, all particles will be back on their initial shells with no net change in energy or directional flux. During the change, itself, a given shell will be deformed, its particles being accelerated or decelerated and their directional flux changing accordingly. In order to have a net change in shell, energy and flux after a complete cycle, it is necessary to have a non-adiabatic change in the field, which violates, say, the third invariant. In this case, the time history of a particle depends on where in longitudinal position on the shell the particles was surprised by the non-adiabatic field change. It is easy to realize that due to this, the end-effect will always be a diffusion, even if the cause itself (the non-adiabatic change in field configuration) is not at all a stochastic process. Non-adiabatic compressions and distentions of the magnetosphere very likely are the main cause for radial diffusion and acceleration of trapped protons [Nakada, Dungey and Hess, 1965].

There is, however, another possible radial diffusion process, associated with shell splitting in the meadosphere. Suppose the action of an elastic pitch angle scattering process, such as interactions with electromagnetic or hydromagnetic waves. This is a very short-time process which violates all three invariants. However, the fact that the particle remains on the same field line during the event (within one cyclotron radius) predetermines the new shell on which the particle will drift after the interaction. Inspection of Figs. 4 and 5 reveals that for instance, a scattering process which, occuring on the noon side, increases the pitch angle (lowers B_m), will bring the particle to a shell which on the average gets closer to the earth. The same scattering process, occuring on the night side, would situate the particle on a shell extending further out. Any type of pitch angle diffusion process will therefore be accompanied by a radial diffusion, which will be inwards or outwards according to where in longitude the particles are more likely caught by the individual processes. If the original pitch angle scattering process is elastic, there would be no change in energy in this type of radial diffusion. Such a diffusion mechanism was experimentally observed in the Laboratory, and studied theoretically, by [Gibson, Jordan and Lauer, 1963].

IV. NUMERICAL RESULTS

A computer code was set up to determine particle shells in the meadosphere, in order to study shell splitting and longitude dependence of drift velocities and equatorial pitch angles for the static case, and in order to analyze the evolution of a system of particle shells in a time-dependent case. The computer program consists of four main parts, which perform more or less independent operations.

I. Field line geometry. This part furnishes complete geometric information about a given field line, computes I and B_m values for a set of particles mirroring on this field line with prefixed equatorial pitch angles [relations (10), (4) and (5)] and determines their drift velocities and bounce periods [relations (12) and (13)].

II. Shell geometry. Given the particles defined in part (1), this section of the program finds the points of prefixed I, B_m values at other longitudes, and traces the corresponding field lines, computing for each case, drift velocity and bounce period. In this way, the complete shell for each particle is generated.

III. Third invariant. This part computes the magnetic flux (3) enclosed by the shell generated by a given particle. It is a combination of (1) and (2), and a subroutine which computes the integral (3) along the equatorial ring of the shell.

IV. Non-adiabatic compression. Given a particle on a field line, this part changes the field configuration simulating a sudden commencement, finds the new field line (going through the same intersection with the earth's surface), and locates the particle's mirror points supposing conservation of the first two invariants.

V. Shell deformation during adiabatic time variations. This part finds the new I and B_m values, as well as the energy, of a particle after a slow adiabatic change in the trapping magnetic field. Conservation theorems (6), (7) and (8) are used in this computation.

Part (1) is mainly based on McIlwain's INVAR code, conveniently implemented for our purposes. Part (2) contains a key subroutine called SEARCH, which at a given longitude finds the point of prescribed I and B_m values (within prefixed tolerances), by an iteration method. This subroutine is quite fast (a fraction of a second on an IBM 7094, for a relative tolerance of 10^{-4} in I and B). Part (5) contains subroutine LOOK, which starts with an approximate value of I, finds B_m through (7) for prescribed values of K_1 and K_2 (determined by the initial state), and computes the flux (8). The value of I is then corrected and the procedure iterated until the prefixed value K_3 for Φ is approached within the wanted tolerance; γ is then computed through (6). Remember that each step, i.e. each evaluation of flux requires the complete determination of a particle shell. Even so, this program is quite fast (10-20 seconds, to find a prefixed K_1, K_2, K_3 shell) with a relative tolerance for Φ of 10^{-3}).

This program was applied using Mead's model of the magnetosphere. The numerical values of the four intervening parameters (Section I) for the quiet meadosphere were taken from [Ness and Williams, 1966]: $R_s = 10 R_e$, $R_{min} = 8 R_e$, $R_{max} = 200 R_e$ and $B_T = 15$ gammas. Typical closed field lines are shown in Fig. 1. The kink of the field lines reaching out beyond 8 R_e in the night side is caused by the assumption in Mead's model of a two dimensional neutral sheet (recent measurements, however, suggest a finite thickness of several earth

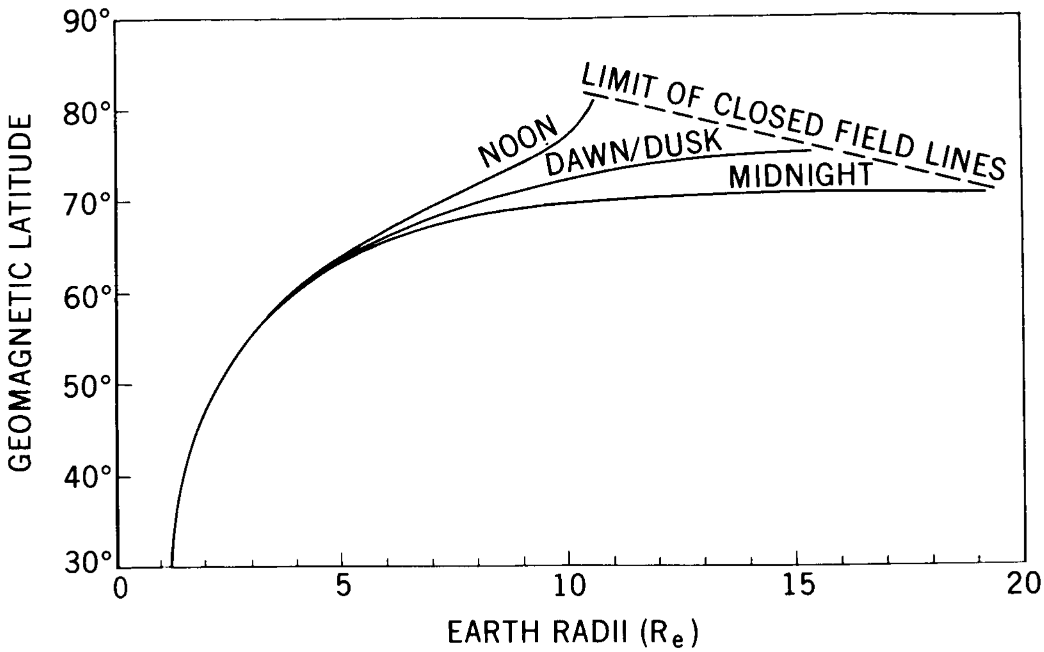

Figure 6–Relationship between geomagnetic latitude of the intersection of a field line with the earth's surface, and the radial distance to its equatorial point, for the noon, midnight and dawn/dusk meridians.

radii [Bame, Esbridge, Felthauser, Olson and Strong, 1966]. Figure 6 shows the relationship between geomagnetic latitude of the intersection of a field line with the earth's surface, and the radial distance to its equatorial point, for the noon, midnight and dawn/dusk meridians.

Parts I and II of the code were applied to this quiet time field configuration, to obtain magnetic shells for particles initially mirroring on a common field line, and having equatorial pitch angles with cosines 0.2, 0.4, 0.6, 0.8 and nearly 1 (mirroring close to the earth's surface). Fig. 7 shows how particles, starting on a common line in the noon meridian, do indeed drift on different shells, which intersect the midnight meridian along the field lines shown in the figure. The dots represent particles' mirror points. Curves giving the position of mirror points for constant equatorial pitch angles are traced for comparison

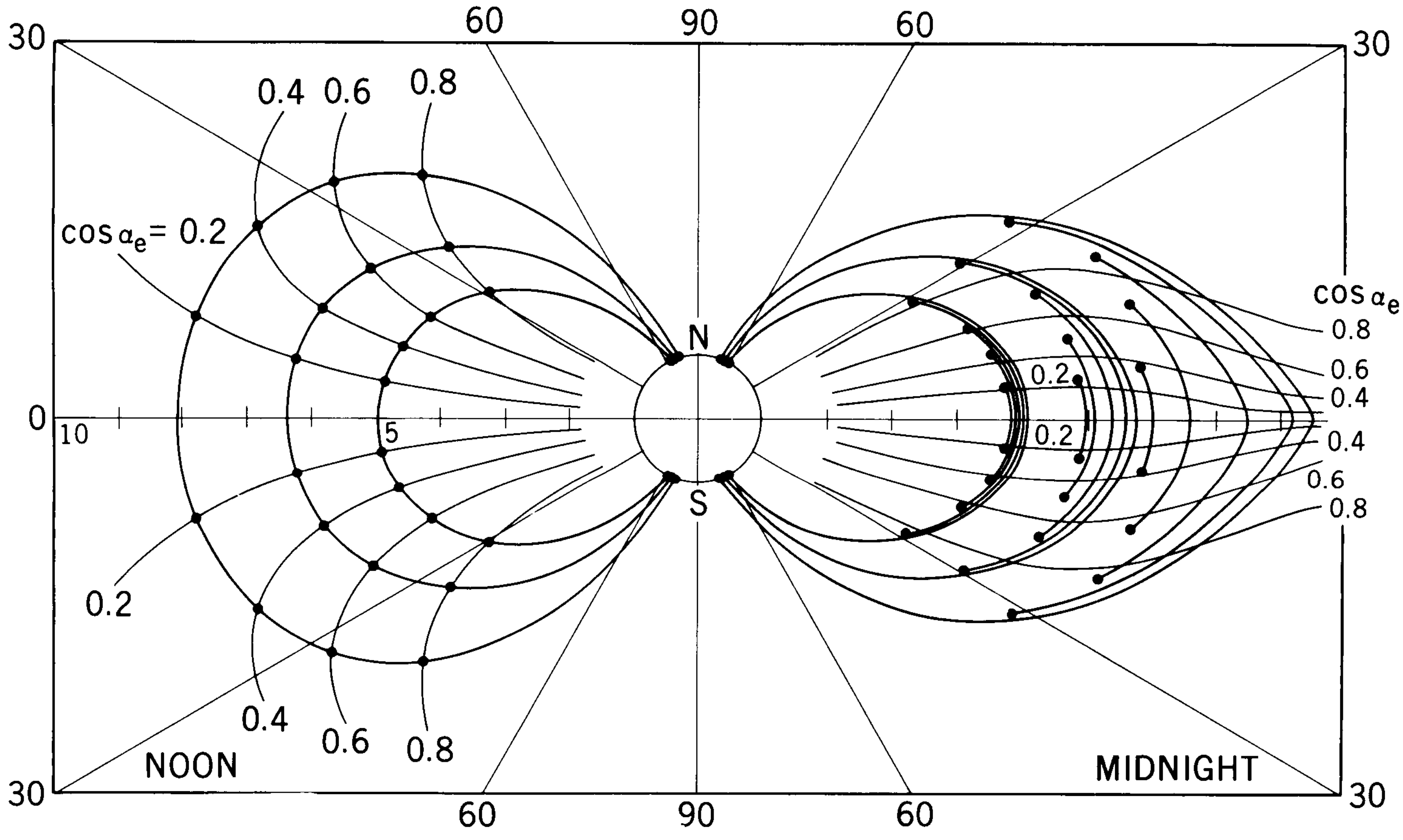

Figure 7–Computed shell splitting for particles starting on common field lines in the noon meridian. Dots represent particles' mirror points. Curves giving the position of mirror points for constant equatorial pitch angle α_e are shown.

(in a dipole field, they are constant latitude lines). Notice the change (decrease) in equatorial pitch angle for the same particle, when it drifts from noon to midnight. Fig. 7 clearly confirms our qualitative predictions given in Section II: shell splitting becomes considerable beyond 5 earth radii, and completely invalidates the use of "L-values" or any other dipole-type description of the outer radiation belt. Fig. 8 depicts the same features for particles starting on a common field line in the midnight meridian. In this case, again, the pitch angle changes considerably when the particle drifts to the opposite meridian (increasing at noon).

Notice from Figs. 7 and 8 that as equatorial pitch angles increase, shell splitting is directed radially inwards for particles starting on the same field line at noon, and radially outwards for particles starting on a common field line at midnight. Furthermore, shell splitting is maximum for particles mirroring close to the equator: for a given change in pitch angle (in degrees), the radial displacement of the shell will be greater for equatorial particles.

When the initial field line has its equatorial point beyond about 8 R_e, a fraction of the particles mirroring on it can only be pseudo-trapped, being lost before completing a 180° drift. In particular, the computations reveal that particles mirroring at low latitudes in the back side, abandon the meadosphere through the boundary 30-40 degrees before reaching the noon meridian. On the other hand, particles mirroring at high latitudes on the day side, run into the tail (open field lines) 10-20 degrees before reaching the midnight meridian. Fig. 9 shows computed limits between stable trapping and pseudo-trapping regions in the meadosphere, on the noon-midnight plane. At other local times, both regions approach more closely the meadospheric boundary; going from noon to midnight,

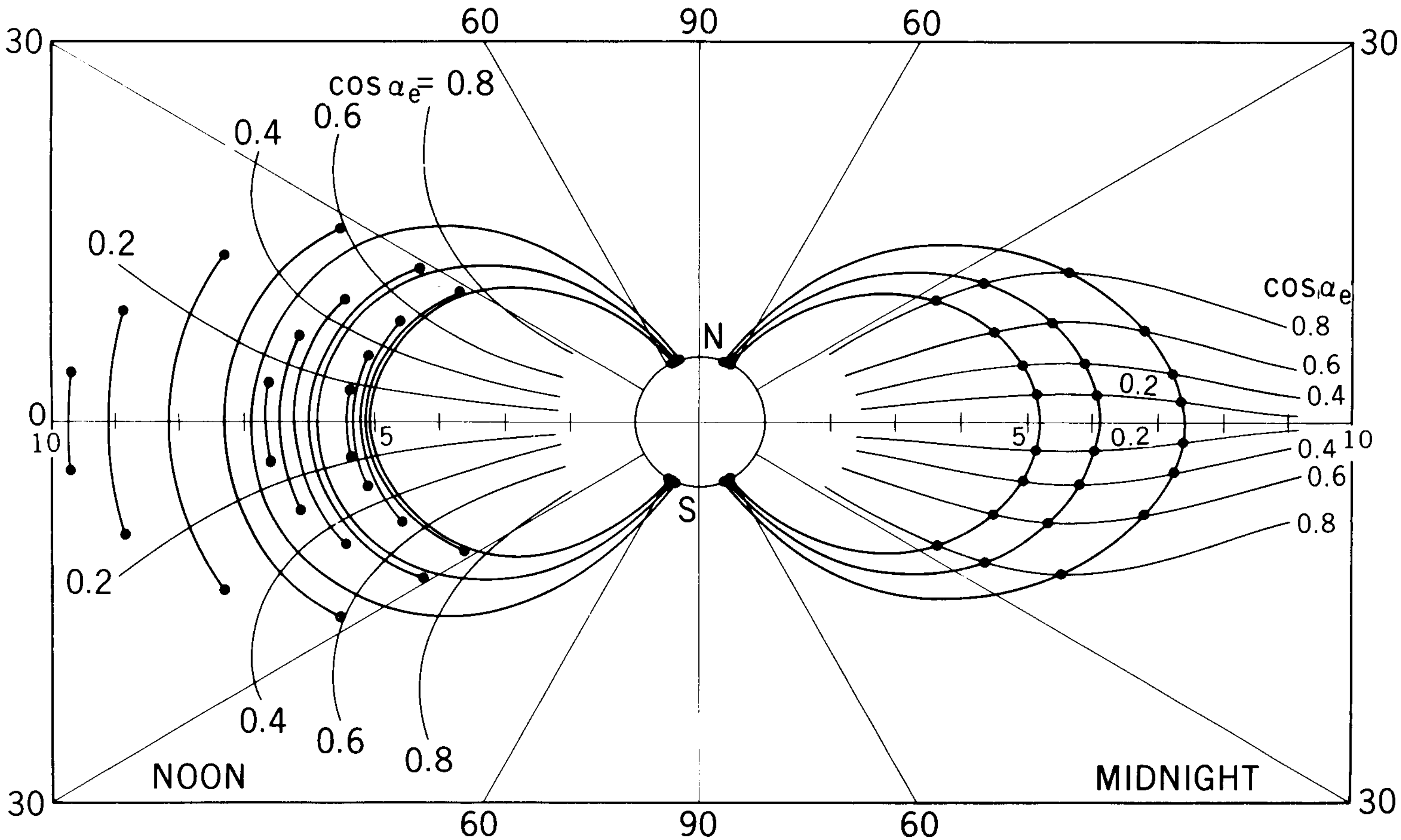

Figure 8–Computed shell splitting for particles starting on common field lines in the midnight meridian. Dots represent particles' mirror points. Curves giving the position of mirror points for constant equatorial pitch angle α_e are shown.

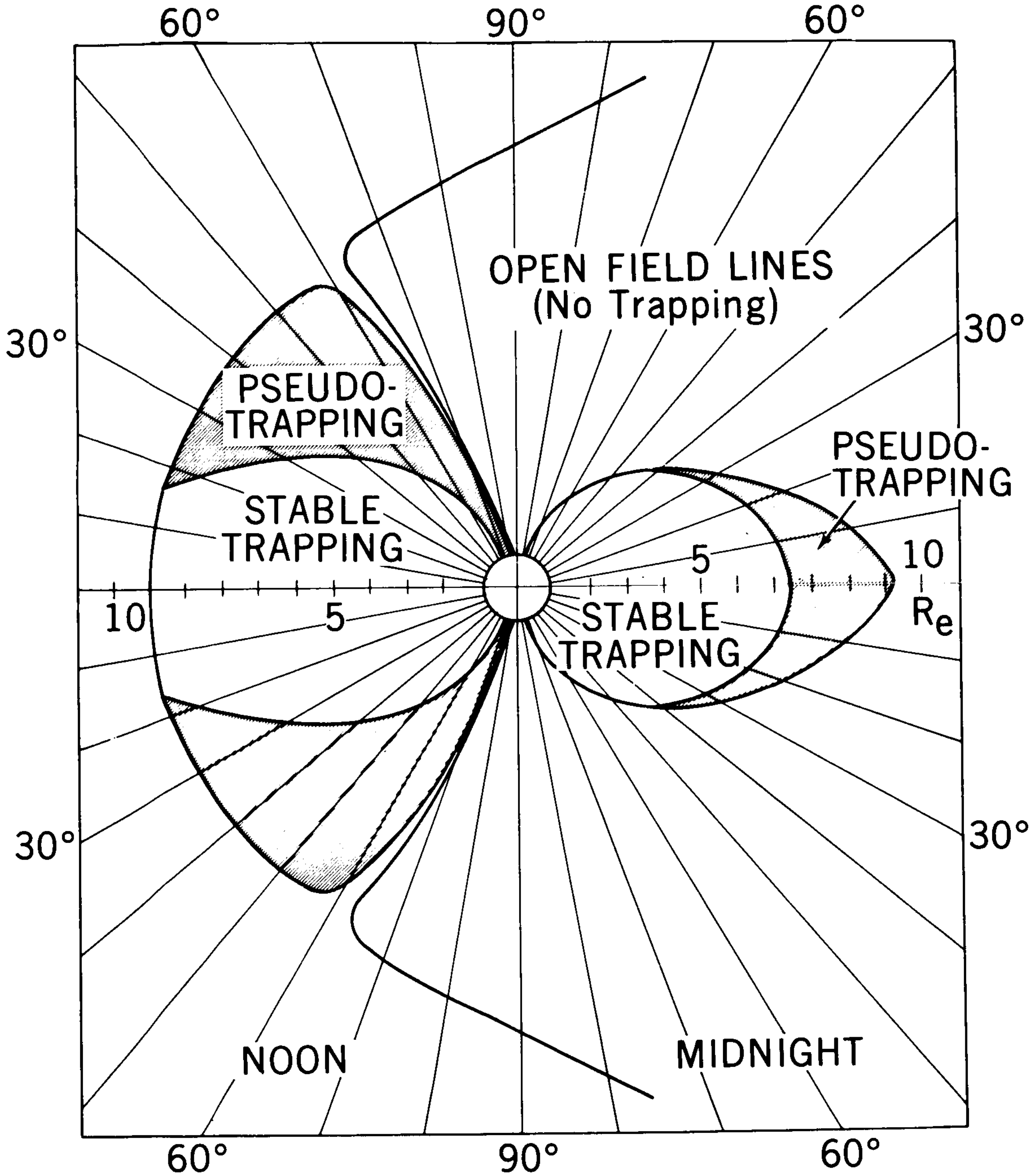

Figure 9—Location of the "pseudo-trapping" regions in the magnetosphere. Particles mirroring inside those regions are unable to complete a 180° drift around the earth. Those injected into the left side will be lost into the tail; those injected into the right portion will abandon the magnetosphere through the boundary, on the day side.

one disappears at the expense of the growth of the other. These results indicate the existence of a quite considerable loss cone in the day side of the meado-sphere.

Figs. 10a and 10b summarize the information about the shell splitting effect. In both figures, the relation between noon and midnight radial distances to the equatorial points of a particle shell is given. In Fig. 10a, particles start on a common field line at noon, reaching out to R_{noon} earth radii; in Fig. 10b, particles start on a line at midnight, reaching out to $R_{midn.}$. Curves are labeled with the cosines of the initial pitch angles. Figs. 11a and 11b show how these pitch angles change when the particles drift to the opposite meridian. Notice again the marked tendency of particles to align along field lines on the night side, and to "squeeze" transverse to the field on the day side.

The numerical calculations also confirm our predictions for local time dependence of the equatorial drift velocity (Section II). The geometrical factors appearing in (12) were computed, and represented in Figs. 12a and 12b as a function of the equatorial distance of the corresponding field line, for different pitch angles, and for noon and midnight, respectively. For a better understanding, angular drift factors are shown. For radial distances $< 3 R_e$, we observe a dipole-like dependence. Beyond $3 R_e$, there is a considerable departure. Drift velocities on the night side are indeed appreciably higher than on the day side. The peculiar inversion of the pitch angle dependence, occuring on the day side (Fig. 12a), is due to a shift in relative importance of curvature drift versus gradient drift (particles mirroring at high latitude experience more of a dipole-type field, and drift faster).

In Figs. 13a and 13b we have represented the percentage change of the linear drift velocity of a given particle, when it drifts to the opposite meridian. A

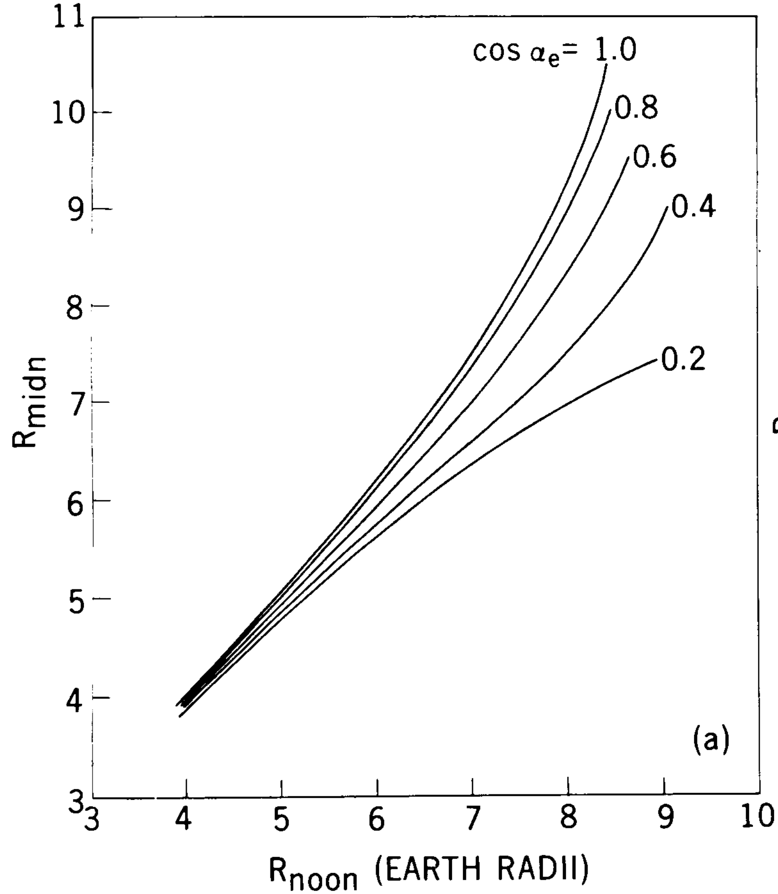

Figure 10a–Particles starting in the noon meridian at a field line reaching out to R_{noon} will cross the midnight meridian on a field line reaching out to R_{midn}. Curves are labeled by the cosine of the particles' equatorial pitch angles.

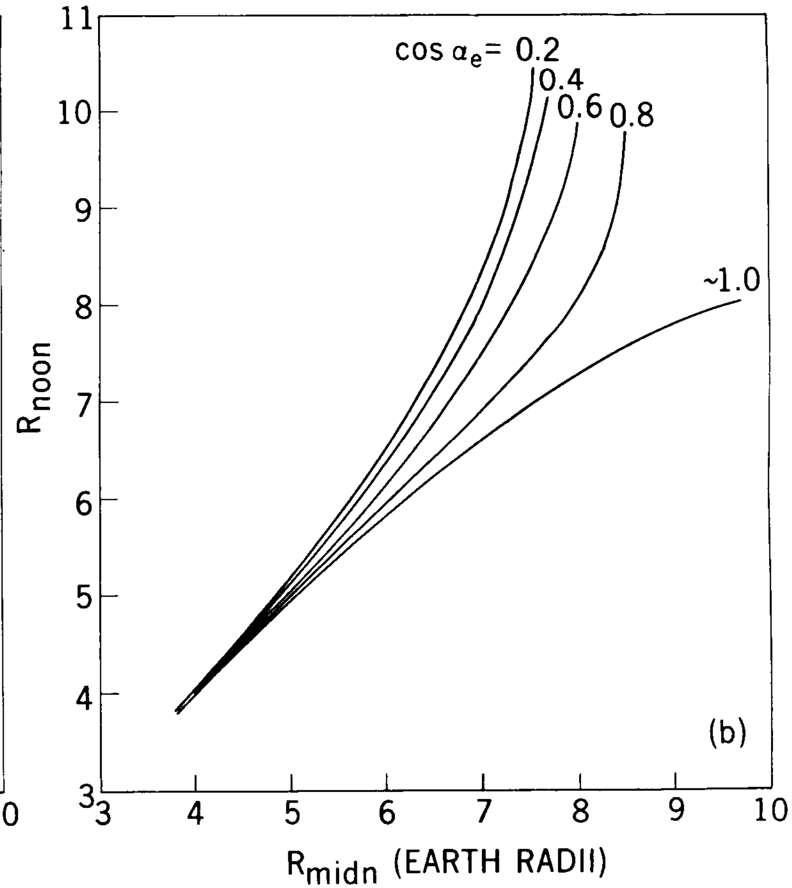

Figure 10b–Particles starting in the midnight meridian at a field line reaching out to R_{midn} will cross the noon meridian on a field line reaching out to R_{noon}.

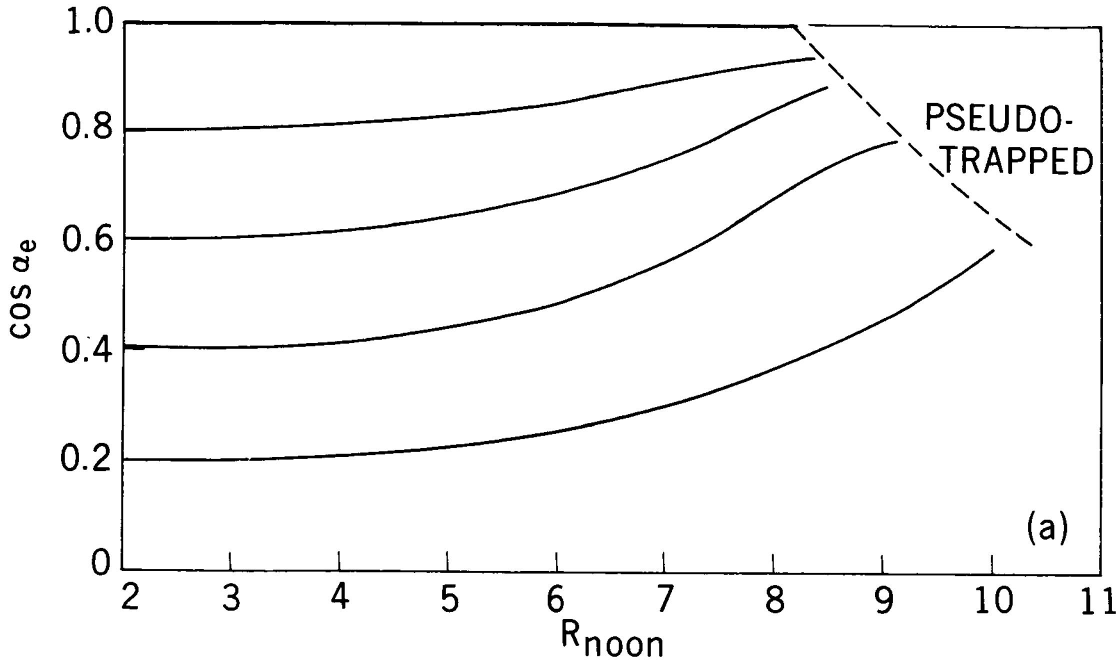

Figure 11a–Particles starting in the noon meridian at a field line reaching out to R_{noon} and initially having equatorial pitch angles of cosines 0.2, 0.4, 0.6, 0.8 and 1.0 respectively, will appear on the midnight meridian with cosines of pitch angles given by the curves. Notice the effect of alignment along the field lines (occuring at midnight), for radial distances $\gtrsim$ 6.

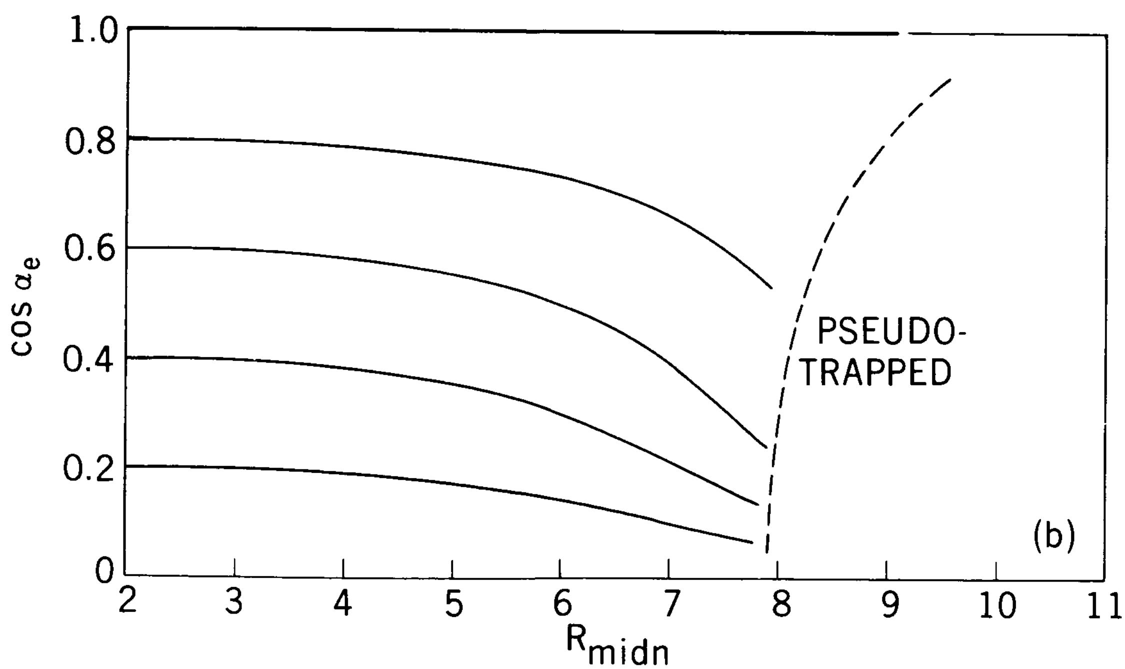

Figure 11b–Particles starting in the midnight meridian at a field line reaching out to R_{noon} and initially having equatorial pitch angles of cosines 0.2, 0.4, 0.6, 0.8 and 1.0 respectively, will appear on the midnight meridian with cosines of pitch angles given by the curves. Notice the effect of alignment along the field lines (occurring at midnight), for radial distances $\gtrsim$ 5.

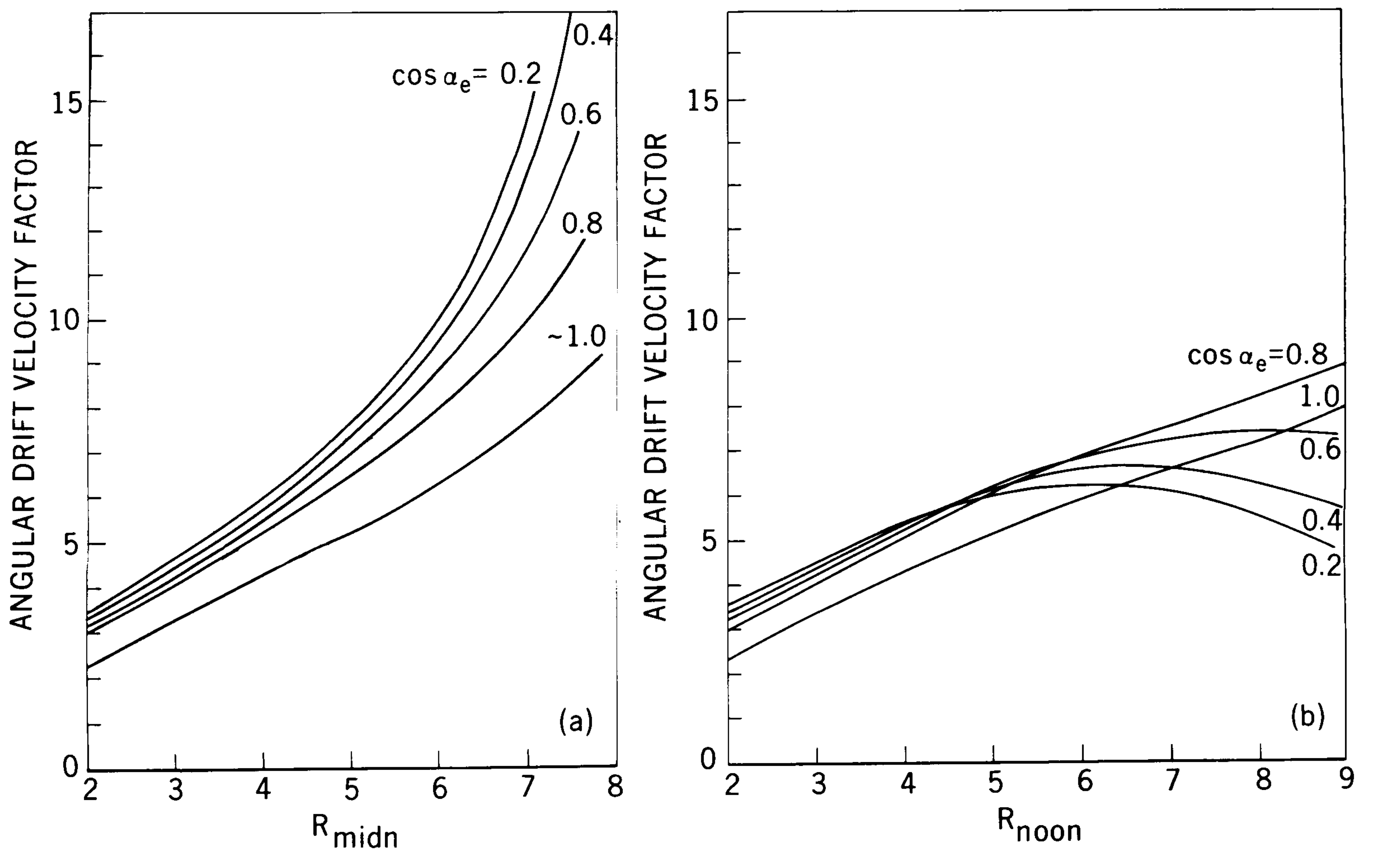

Figure 12a and b—Value of the geometric factors intervening in the angular drift velocity, as a function of the equatorial distance of the corresponding field line, for different pitch angles, and for noon (12a) and midnight (12b), respectively. In order to obtain drift velocities in degrees/sec, multiply the above values by the factor $2.321 \times 10^{-2}(\gamma^2 - 1)/\gamma \times$ (rest mass in electron masses).

close inspection of the local time dependence of the drift velocity (not shown here) leads to the important conclusion that a given particle trapped in the outer meadosphere ($\gtrsim 6\,R_e$) spends up to 2/3 - 3/4 of its total lifetime in the day side. In other words, there is always a higher probability to find a particle in the day side than in the night side. It can be shown that, as a consequence, particle volume densities can be about two times greater on the noon meridian, than at midnight, for a given class of particles. This represents an additional important asymmetry for trapped particle fluxes in the outer meadosphere.

As discussed in Section II, any pitch angle scattering mechanism will lead to radial diffusion, due to shell splitting. According to the preceding results, this radial diffusion will be most effective for equatorial particles. This mechanism would tend to mix and blurr energy spectra of particles at different radial distances. Furthermore, the existence of large pseudo-trapping regions, especially on the day side, implies an efficient particle sink for any pitch angle scattering mechanism (enhanced loss cone). On the other hand, the reverse could also be true: particles which happened to enter the pseudo-trapping regions from outside, could be scattered into stably trapped orbits by any pitch angle scattering mechanism.

We now turn to the numerical results for a time-dependent magnetic field configuration. The purpose is to study the trapped particle behaviour during a simulated magnetic storm. Following steps were adopted as a "model" storm:

1. sudden, non-adiabatic compression, simulated by a decrease ΔR_s of the parameter R_s (inward displacement of the magnetopause).
2. optional sudden increase ΔB_T of the tail field.
3. gradual, adiabatic recovery to the initial field configuration.

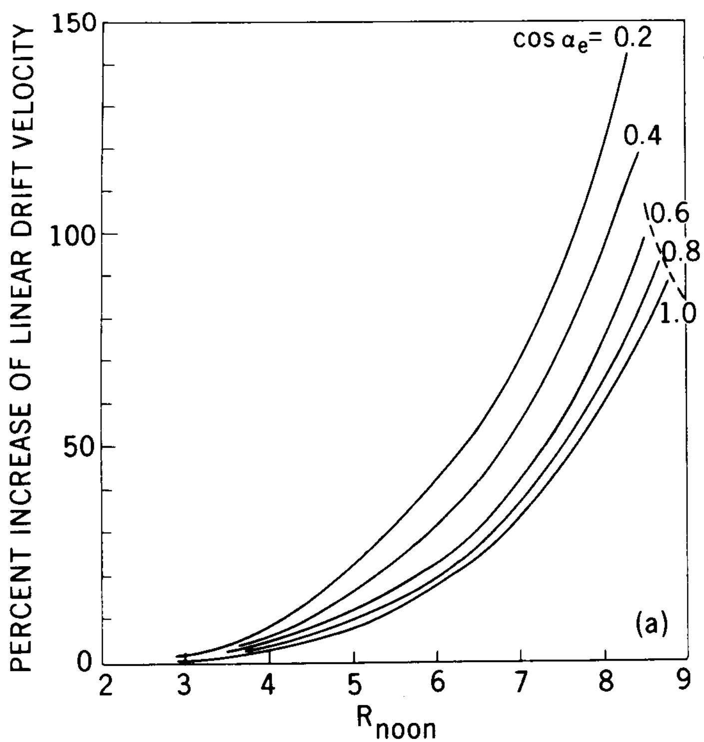

Figure 13a–Maximum percentage increase of the linear drift velocity for particles starting at noon on field lines reaching out to R_{noon} when they drift to the midnight meridian.

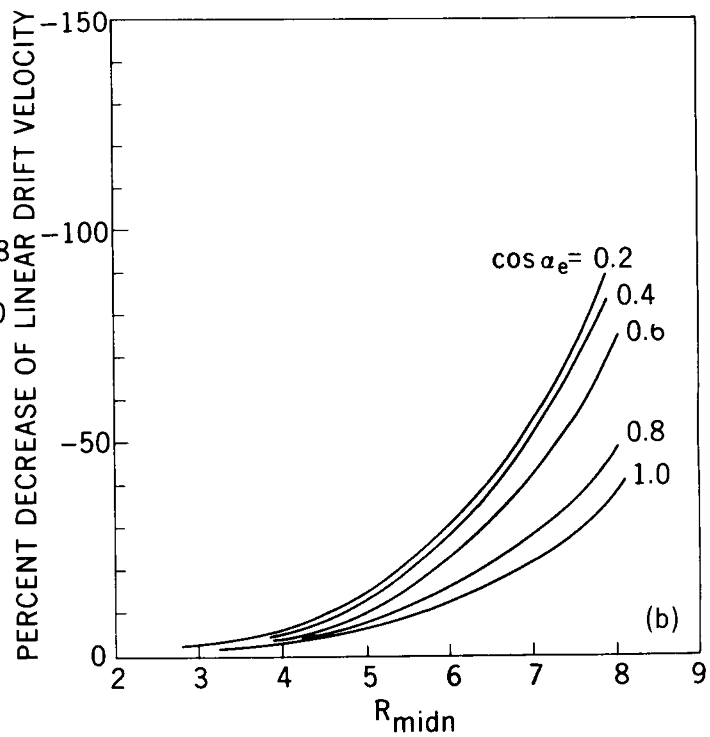

Figure 13b–Maximum percentage decrease of linear drift velocity for particles starting at midnight on a field line reaching out to R_{midn} when they drift to the noon meridian.

For (1) and (2) it is assumed that particles, while violating the third invariant (8), stick to their original field line driven by the dominating E×B drift, still conserving the first two invariants (this "original" field line is supposed to be rigidly rooted in the ionosphere, during the sudden compression). Part (4) of the computer code is applied for this calculation. The gradual recovery in (3) is assumed to be flux-conserving; part (5) of the computer code was used here. Some of the results are summarized in Fig. 14, in which the percentage change of kinetic energy of a particle is represented as a function of equatorial distance of the initial field line, for different pitch angles, and for particles caught by the compression at noon (upper curves), and at midnight (lower curves). This energy variation is independent of the initial energy, and increases with the amount of compression ΔR_s (taken = 2 R_e for the curves in Fig. 14). An inward motion of the boundary of only 1 R_e would yield energy changes roughly 0.45 times those shown in the figure. The curves given in Fig. 14 were derived for a sudden commencement without a sudden increase of the tail field. If one adds a typical increase ΔB_T of 15 gammas, acceleration, deceleration and radial displacements become greater by a factor of about 2, the effect being considerably enhanced for particles which were in the night side during the sudden commencement.

One clearly sees in Fig. 14 that the final effect of a storm depends on where in local time the particle was caught during the non-adiabatic phase: particles which happened to be in the day side of the meadosphere will have a higher energy when the field recovers to the initial state; particles surprised in the night side will be decelerated. Furthermore, the first group of particles will have moved radially inwards, whereas the second group will be found on shells "inflated" outwards (Fig. 15). It should be pointed out that during the first phase of

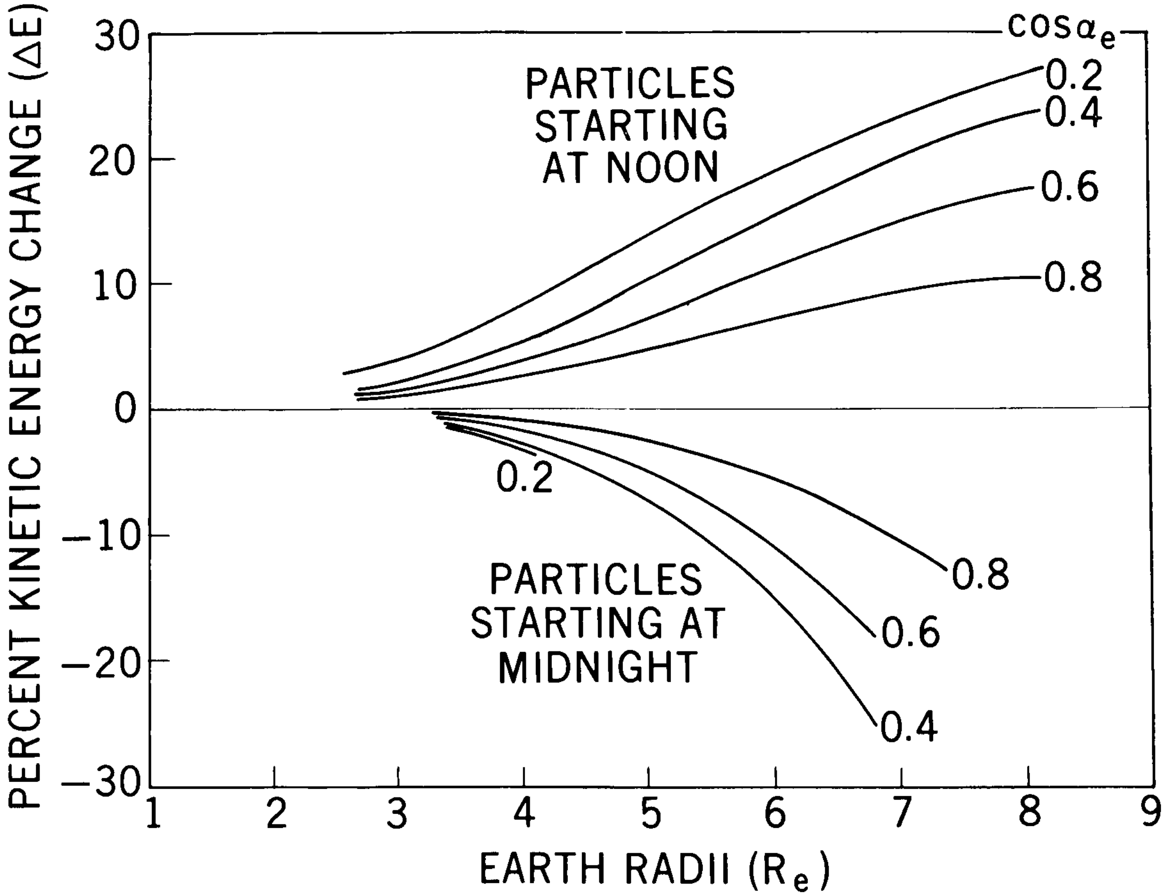

Figure 14—Percentage kinetic energy change after a prototype storm, for particles caught by the sudden commencement in the noon meridian, and at midnight, respectively, as a function of the radial distance to the equatorial point of the initial field line, and for different initial pitch angles. The inward displacement of the boundary was taken as $\Delta R_s = 2$ earth radii.

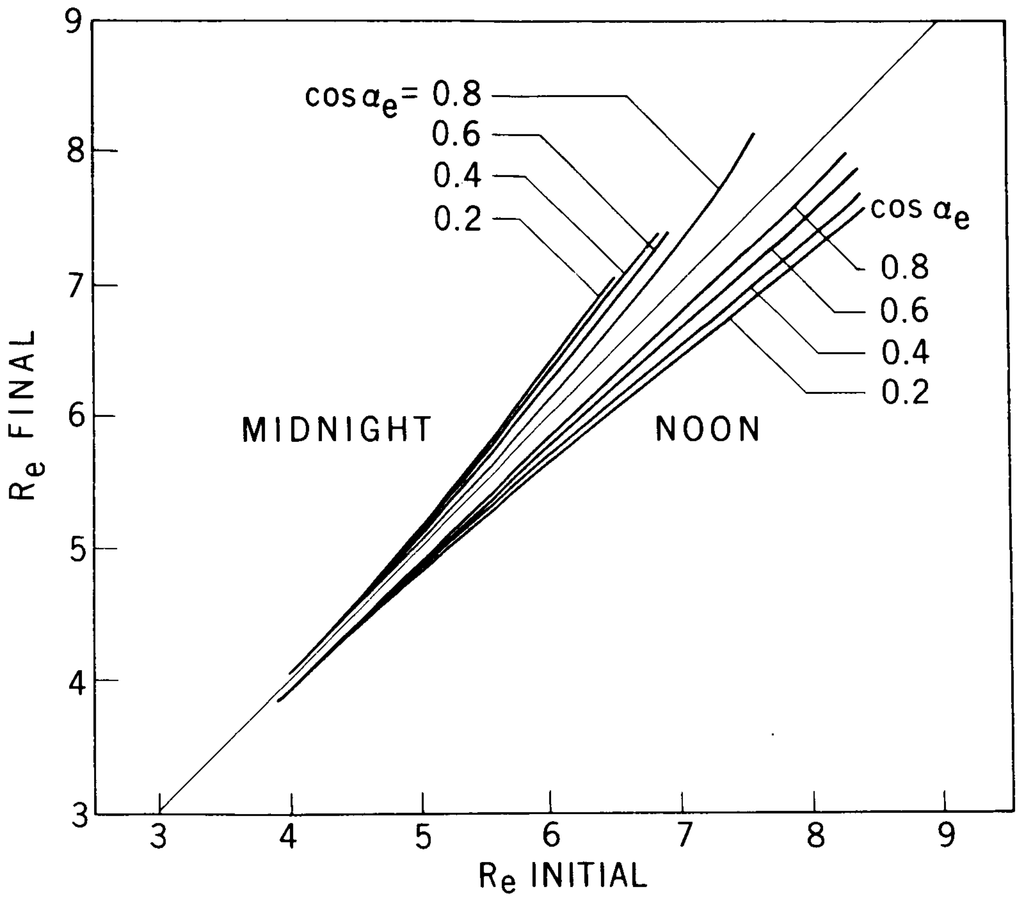

Figure 15–Relation between initial and final position of the equatorial point of a shell at a given meridian (noon and midnight), after a prototype storm, for particles caught by the sudden commencement at noon, and at midnight, respectively ($\Delta R_s = 2$).

compression, both groups of particles attain higher energies; it is during the adiabatic recovery, that this peculiar asymmetry arises. The fact that a given particle always spends more time in the day side, leads to the conclusion that a magnetic storm should always have a net effect of inward diffusion and acceleration of trapped particles, after the field has recovered to the same initial configuration. We finally must mention that during a magnetic storm a considerable fraction of particles can be driven into pseudo-trapping regions of the meadosphere, and therefore be lost through the boundary, or into the tail. This effect is particularly important for storms with increases of the tail field, which mainly leads to losses through the boundary on the day side. Likewise, particles which for some reason happened to be injected into pseudo-trapping regions during recovery, can become stably trapped, under favorable circumstances of injection.

The repeated action of magnetic storms will therefore cause acceleration and radial diffusion towards lower altitudes, of particles trapped in the outer meadosphere. This mechanism leads to energy spectra which depend on radial distance, hardening towards lower R values. This is indeed observed for protons [Davis and Williamson, 1963; Vernov, Vakulov, Kuznetsov, Logatchev, Nikolaev, Sosnovets and Stolpovsky, 1966], and was explained theoretically by [Nakada, Dungey and Hess, 1965]. Electrons, however, should in addition be subject to pitch angle scattering by electromagnetic waves; as discussed above, the radial diffusion which accompanies pitch angle scattering when shell splitting is considerable, should greatly blurr the radial dependence of energy spectra for electrons.

IV. CONCLUSIONS

1. Shell splitting in the outer magnetosphere becomes important beyond 5 earth radii; dipole-type descriptions of the radiation belt become invalid.

2. Equatorial pitch angles tend to align along field lines on the night side of the magnetosphere, and perpendicularly to the field, on the day side.

3. There are regions in the magnetosphere, where only pseudo-trapped particles can mirror, i.e. particles which will leave the magnetosphere before completing a 180° drift.

4. Longitudinal drift velocities depart considerably from the dipole values beyond 5 R_e, and can be as much as 2-3 times greater on the night side than on the day side. Thus a given particle spends 2-3 times more time in the day side than in the night side.

5. The action of a pitch angle scattering mechanism will lead to a radial diffusion of particles. The loss mechanism will be greatly enhanced by scattering of mirror points into the pseudo-trapping regions.

6. After recovery from a prototype magnetic storm, particles which were in the day side during the sudden commencement will have higher energies, their shells having moved radially inwards. Particles caught in the night side will have moved outwards, with their energies decreased.

7. The repeated action of magnetic storms will result in a net inward diffusion of particles, with a net increase of their energy.

ACKNOWLEDGEMENTS

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REFERENCES

Anderson, A. D., C. E. Crane, W. E. Francis, L. L. Newkirk and M. Walt, "Theoretical Investigation of Geomagnetically Trapped Electrons from High Altitude Nuclear Explosion," Lockheed Missiles and Space Company Report LMSC-895355 (1964)

Bame, S. J., J. R. Esbridge, H. E. Felthauser, R. A. Olson and I. B. Strong, "Electrons in the Plasma Sheet of the Earth's Magnetic Tail," Phys. Rev. Letters 16, 138-142 (1966)

Davis, L. R. and J. M. Williamson, "Low Energy Trapped Protons," Space Research III, 365, North Holland Publ. Co. (1963)

Fairfield, D. H., "Trapped Particles in a Distorted Dipole Field," Journal of Geophys. Res. 69, 3919-3926 (1964)

Frank, L. A., "Explorer 12 Observations of the Temporal Variations of Low Energy Electron Intensities in the Outer Radiation Zone During Geomagnetic Storms," State University of Iowa Report 66-8 (1966)

Gibson, G., W. C. Jordan and E. J. Lauer, Phys. of Fluids, 6, 133, (1963)

Hones, E. W., "Motion of Charged Particles Trapped in the Earth's Magnetosphere," Journal of Geophys. Res. 68, 1209-1219 (1963)

McIlwain, C. E., "Redistribution of Trapped Protons During a Magnetic Storm," Space Research V, 347-391, North Holland Publ. Co. (1965)

McIlwain, C. E., "Ring Current Effects on Trapped Particles," University of California, San Diego, Report USCD-SP-66-1 (1966)

McIlwain, C. E., "Coordinates for Mapping the Distribution of Magnetically Trapped Particles," Journal of Geophys. Res. 66, 3681-3691 (1961)

Mead, G. D., "Deformation of the Geomagnetic Field by the Solar Winds," Journal of Geophys. Res. 69, 1181-1196 (1964)

Mead, G. D., "The Motion of Trapped Particles in a Distorted Field," Proceedings of the Advanced Study Institute "Radiation Trapped in the Earth's Magnetic Field," August 1965, Bergen, Norway, Edited by B. M. McCormac, and D. Reidel, Holland, 1966

Nakada, M. P., J. W. Dungey and W. N. Hess, "Inward Radial Diffusion of Outer Belt Protons," Journal of Geophys. Res. 70, 3529-3532 (1965)

Ness, N. F. and D. J. Williams, "Correlated Magnetic Tail and Radiation Belt Observations," Journal of Geophys. Res. 71, 322-325 (1966)

Northrop, T. G., "The Adiabatic Motion of Charged Particles," Interscience Publishers (1963)

Roederer, J. G., "Magnetospheric Phenomena," presented at the International Conference on Cosmic Rays, London 1965, to be published

Vernov, S. N., P. V. Vakulov, S. N. Kuznetsov, Yu. I. Logatchev, A. G. Nikolaev, E. N. Sosnovets and V. G. Stolpovsky, "The Structure of the Earth's

Radiation Belts according to the Data of the "Electron" Series of Satellites," presented at the Sixth Space Science Symposium, COSPAR, Vienna 1966, to be published.

Williams, D. J. and G. D. Mead, Nightside Magnetosphere Configuration as Obtained from Trapped Electrons at 1100 kilometers, J. Geophys. Res., 70, 3017-3029, 1965

APPENDIX I

Listing of the program briefly described in the text. For more details, see the comment cards and the list of output which follows the listing. The deck is available on request.

LIST OF OUTPUT

Initial field line:

VN 1: Radial distance in earth radii to field line points

OLAT: latitude of field line points

OLONG: longitude of field line points

B: field intensity in gauss, at field line points

SPITCH: cosines of equatorial pitch angles for a group of particles on the field line

BDIM: mirror point field intensities for these particles

FIDIM: corresponding values of second invariant (in earth radii)

SALT, SLAT, SLONG: coordinates of mirror points

SPATH: half-bounce path (expression 13 in the text) corresponding to these particles (in earth radii)

SDRIFT: geometric factors giving drift velocities of these particles (for conversion into cm/sec, see comment card)

EALT: radial distance to the equatorial point of the field line

EQB: equatorial B-value

ENERGY: kinetic energy of the particles, in units of rest energy

FLX: magnetic flux subtended by the shell generated by the particles (in gauss (earth radii)^2)

New field line (at a different longitude (splitting), or at the same longitude after the simulation of a storm).

VN1, OLAT, OLONG, B: same as above

RPITCH, RALT, RLAT, RLONG, RPATH, RDRIFT; same as SPITCH, SALT, etc.

EQALT: radial distance to the equatorial points

EQBB: equatorial B-values

SPLIT

```
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
      DIMENSION V(3), BDIM(9), FIDIM(9), EQALT(9), EQLONG(9)
      DIMENSION SPITCH(9), RPITCH(9)
      DIMENSION SALT(9), SLAT(9), SLONG(9), RALT(9), RLAT(9), RLONG(9)
      DIMENSION SDRIFT(9), RDRIFT(9), SPATH(9), RPATH(9), EQBB(9), RLOST
     1T(9)
      DIMENSION ENERGY(9), FLX(9), QV2(9), QV3(9)
      DIMENSION BEG(200), BEND(200), ECO(200), BLOG(200)
      ERR=0.0001
      ERRB=0.004
      ERRI=0.008
      ERRF=0.001
C
C     FRONT = DISTANCE TO MAGNETOPAUSE AT STAGNATION POINT IN EARTH RADI
C     SHEET1 = MINIMUM DISTANCE TO NEUTRAL SHEET IN EARTH RADII
C     SHEET2 = MAXIMUM RADIAL EXTENSION OF NEUTRAL SHEET IN MIDNIGHT
C     MERIDIAN IN EARTH RADII
C     FSHEET = FIELD STRENGTH PARALLEL TO NEUTRAL SHEET IN GAMMA
C
      READ (5,39) FRONT,SHEET1,SHEET2,FSHEET
C
C     KMAX = NUMBER OF MIRROR POINTS AND PITCH ANGLES WANTED (.LE.9)
C     DELTA = STEP SIZE IN LONGITUDE (DEGREES)
C     START = LONGITUDE INTERVAL TO BE SKIPPED BEFORE STARTING STEPS
C     TERM IS LONGITUDE WHERE SHELL TRACING SHOULD BE TERMINATED
C
      READ (5,40) KMAX,DELTA,START,TERM
C
C     ISTORM = 1 MEANS THAT AFTER FIRST SPLIT ANALYSIS, A GEOMAGNETIC
C     STORM SHOULD BE TURNED ON. WIND IS THE INWARD DISPLACEMENT
C     OF THE FRONT SIDE OF THE MAGNETOSPHERE. STORM IS THE INWARD
C     DISPLACEMENT OF THE EDGE OF THE NEUTRAL SHEET
C     HAIL IS THE INCREASE OF THE TAIL FIELD
C     IF NO STORM EFFECT IS WANTED, SET ISTORM=0
C
      READ (5,40) ISTORM,WIND,STORM,HAIL
C
C     IF NO TRACING AT THE OPPOSITE MERIDIAN IS WANTED, SET NOSPLT=1
C     OTHERWISE SET NOSPLT=0
C
      READ (5,40) NOSPLT
C
C     GAMMA IS TOTAL ENERGY IN UNITS OF REST MASSES
C
      READ (5,39) GAMMA
C
C     FLONG=180. IS NOON MERIDIAN
C     ALT MUST BE GIVEN IN EARTH RADII FROM CENTER OF EARTH
C
1     READ (5,39) ALT,FLAT,FLONG
      IF (ALT.LT.0.5) STOP
      KONTRL=0
      COSPIT=1.
```

```
      KMIN=1
      GSQ=GAMMA*GAMMA
      GSQM1=GSQ-1.
      ORLONG=FLONG+START
      IF (ISTORM.EQ.1) ORLONG=FLONG+180.
      AALT=ALT
      EKMAX=KMAX
      DCOS=1./EKMAX
      DO 2 K=1,KMAX
      ENERGY(K)=GAMMA-1.
      RLOSTT(K)=0.
    2 SDRIFT(K)=0.
      V(1)=ALT
      V(2)=(90.-FLAT)/57.2957795
      V(3)=FLONG/57.2957795
      DUM1=V(1)
      DUM2=V(2)
      DUM3=V(3)
C
C     DEFINE INITIAL FIELD LINE
C
    3 CONTINUE
      CALL INVAR (V(1),V(2),V(3),ERR,BDIM(1),FIDIM(1))
      IF (KONTRL.EQ.0) GO TO 4
      WRITE (6,41) FLONG,FLAT,ALT
      GO TO 5
    4 WRITE (6,42) FLONG,FLAT,ALT
    5 DO 6 J=2,JUP
      OLAT=90.-VN2(J)*57.2957795
      OLONG=VN3(J)*57.2957795
      IF (OLONG.GT.180.) OLONG=OLONG-360.
      IF (OLONG.LT.(-180.)) OLONG=OLONG+360.
      WRITE (6,43) J,VN1(J),OLAT,OLONG,B(J)
      IF (VN1(J).GT.12.) GO TO 1
    6 CONTINUE
      ERR1=0.001*ERR
      IF (KONTRL.EQ.1) GO TO 29
      N=4
      IF (JUP.LE.4) N=3
      CALL INVAR (VN1(N),VN2(N),VN3(N),ERR,B2,FI2)
      SPATH(1)=FIDIM(1)+(FIDIM(1)-FI2)*2.*BDIM(1)/(BDIM(1)-B2)
      CALL EQUAT (VNEAR(1),VNEAR(2),VNEAR(3),EQB,EALT,ELAT,ELONG,ERR1)
      IF (EALT.GT.12.) GO TO 1
      EQALT=EALT
      VN11=VNEAR(1)
      VN13=VNEAR(3)
      RDIPOL=VN11
      Q=0.31/(RDIPOL**3.
      QQ=Q/EQB
      SPITCH(1)=SQRT(1.-EQB/BDIM(1))
      SALT(1)=ALT
      SLAT(1)=FLAT
      SLONG(1)=FLONG
C
C       LOOP TO DETERMINE ALL I-BM POINTS ON THE INITIAL COMMON FIELD LIN
C       CORRESPONDING TO EQUALLY SPACED VALUES OF COS(PITCH ANGLE)
```

```
C
      IF (KMAX.EQ.1) GO TO 8
      DO 7 K=2,KMAX
      COSPIT=COSPIT-DCCS
      SSQ=1.-COSPIT*COSPIT
      IF (SPITCH(1).LT.COSPIT) SSQ=1.-SPITCH(1)*SPITCH(1)
      BO=EQB/SSQ
      CALL BESECT (DUM1,DUM2,DUM3,BALT,BLAT,BLONG,ERR1)
      JJ=JUP-1
      DUM1=VN1(JJ)
      DUM2=VN2(JJ)
      DUM3=VN3(JJ)
      SALT(K)=VBO(1)
      SLAT(K)=90.-VBO(2)*57.2957795
      SLONG(K)=VBO(3)*57.2957795
      IF (SLONG(K).GT.180.) SLONG(K)=SLONG(K)-360.
      IF (SLONG(K).LT.(-180.)) SLONG(K)=SLONG(K)+360.
      CALL INVAR (VBO(1),VBO(2),VBO(3),ERR,BDIM(K),FIDIM(K))
      N=4
      IF (JUP.LE.4) N=3
      CALL INVAR (VN1(N),VN2(N),VN3(N),ERR,B2,FI2)
      SPATH(K)=FIDIM(K)+(FIDIM(K)-FI2)*2.*BDIM(K)/(BDIM(K)-B2)
7     SPITCH(K)=SQRT(1.-EQB/BDIM(K))
8     CONTINUE
      ERR=ERR*40.
      ERR1=ERR*0.00125
      ERR2=ERR*0.025
C
C      LOOP TO DETERMINE DRIFT VELOCITY ON THE INITIAL LINE
C     TO DETERMINE EQUATORIAL DRIFT VELOCITY IN CM/SEC MULTIPLY DRIFT BY
C     2.5766 E 05 • (RDIPOL**3) • (MASS IN ELECTRON MASSES) • (GAMMA) •
C     • (BETA**2) . BETA = V/C.
C
      DO 9 K=1,KMAX
      FISTAR=FIDIM(K)*0.98
      OALT=SALT(K)
      OLAT=SLAT(K)
      OLONG=SLONG(K)
      CALL SEARCH (OALT,OLAT,OLONG,BDIM(K),FISTAR,BB,FI,ERR,ERRB,ERRI,RR
     1)
      IF (RR.GT.0.5) GO TO 9
      CALL EQUAT (VNEAR(1),VNEAR(2),VNEAR(3),EB,EALT,ELAT,ELONG,ERR1)
      DIST=ABS(VNEAR(1)-VN11)
      SDRIFT(K)=-(FISTAR-FIDIM(K))*QQ/(DIST*SPATH(K))
9     CONTINUE
      DO 10 K=1,KMAX
      IF (SDRIFT(K).LT.1.E-05) SPATH(K)=0.
10    CONTINUE
      WRITE (6,44)
      WRITE (6,45) (K,BDIM(K),FIDIM(K),SPITCH(K),SDRIFT(K),SPATH(K),EQB,
     1SALT(K),SLAT(K),K=KMIN,KMAX)
      IF (NOSPLT.EQ.1) GO TO 24
11    CONTINUE
      FLONG=ORLONG
12    KASE=1
      DO 13 K=1,KMAX
```

```
      RDRIFT(K)=0.
      EQALT(K)=1111.111
      EQLONG(K)=1111.11
      RPITCH(K)=0.
13    EQBB(K)=0.
C
C      LOOP TO DETERMINE I-BM POINTS AND SPLIT FIELD LINES AT NEW
C      LONGITUDE ORLONG
C
      DO 17 K=KMIN,KMAX
      IF (RLOSTT(K).GT.0.5) GO TO 16
      CALL SEARCH (ALT,FLAT,FLONG,BDIM(K),FIDIM(K),BB,FI,ERR,ERRB,ERRI,R
     1LOSTT(K))
      IF (RLOSTT(K).GT.0.5) GO TO 16
      WRITE (6,46) K,BDIM(K)
      IF ((ABS(FLONG).GT.10.).AND.((180.-ABS(FLONG)).GT.10.)) GO TO 15
      DO 14 J=2,JUP
      OLAT=90.-VN2(J)*57.2957795
      OLONG=VN3(J)*57.2957795
      IF (OLONG.GT.180.) OLONG=OLONG-360.
      IF (OLONG.LT.(-180.)) OLONG=OLONG+360.
      WRITE (6,43) J,VN1(J),OLAT,OLONG,B(J)
14    CONTINUE
15    CONTINUE
      N=4
      IF (JUP.LE.4) N=3
      CALL INVAR (VN1(N),VN2(N),VN3(N),ERR2,B2,FI2)
      RPATH(K)=FIDIM(K)+(FIDIM(K)-FI2)*2.*BDIM(K)/(BDIM(K)-B2)
      CALL EQUAT (VNEAR(1),VNEAR(2),VNEAR(3),EQBB(K),EQALT(K),ELAT,EQLON
     1G(K),ERR1)
      RALT(K)=VSAVE(1)
      RLAT(K)=FLAT
      RLONG(K)=FLONG
      RPITCH(K)=SQRT(1.-EQBB(K)/BDIM(K))
      IF (K.EQ.KMAX) GO TO 17
      BO=BDIM(K+1)
      IF (EQBB(K).GT.BO) BO=(BDIM(K)+EQBB(K))*0.5
      CALL BESECT (VSAVE(1),VSAVE(2),VSAVE(3),BALT,BLAT,BLONG,ERR1)
      ALT=VBO(1)
      FLAT=BLAT
      FLONG=BLONG
      GO TO 17
16    IF (K.EQ.KMIN) KASE=1
      KK=K
      IF (KASE.EQ.1) GO TO 19
      IF (K.EQ.KMAX) GO TO 17
      K1=K+1
      ALT=SALT(K1)-1.
      IF (ALT.LT.1.) ALT=1.
      IF (ALT.GT.5.) ALT=5.
      FLAT=ABS(SLAT(K1))-5.
      IF (ABS(SLAT(K1)).LT.5.) FLAT=5.
17    CONTINUE
      DO 18 K=KMIN,KMAX
      IF (RLOSTT(K).LT.0.5) GO TO 21
18    CONTINUE
```

```
      GO TO 1
19    DO 20 K=KK,KMAX
20    RLOSTT(K)=1.
21    CONTINUE
C
C      LOOP TO DETERMINE DRIFT VELOCITIES AT SPLIT FIELD LINES
C
      DO 22 K=KMIN,KMAX
      IF (RPATH(K).LE.1.E-10) GO TO 22
      IF (RLOSTT(K).GT.0.5) GO TO 22
      VN11=EQALT(K)
      VN13=EQLONG(K)/57.2957795
      QQ=Q/EQBB(K)
      FISTAR=FIDIM(K)*0.98
      OALT=RALT(K)
      OLAT=RLAT(K)
      OLONG=RLONG(K)
      CALL SEARCH (OALT,OLAT,OLONG,BDIM(K),FISTAR,BB,FI,ERR,ERRB,ERRI,RR
     1)
      IF (RR.GT.0.5) GO TO 22
      CALL EQUAT (VNEAR(1),VNEAR(2),VNEAR(3),EB,EALT,ELAT,ELONG,ERR1)
      DIST=ABS(VNEAR(1)-VN11)
C      RDRIFT AND SDRIFT HAVE AS COMMON FACTOR 0.31/(RDIPOL**3)
      RDRIFT(K)=-(FISTAR-FIDIM(K))*QQ/(DIST*RPATH(K))
22    CONTINUE
      DO 23 K=KMIN,KMAX
      IF (RDRIFT(K).LT.1.E-05) RPATH(K)=0.
23    CONTINUE
      WRITE (6,47) AALT,FRONT,SHEET1,SHEET2,FSHEET
      WRITE (6,48)
      WRITE (6,49) (K,BDIM(K),FIDIM(K),RPITCH(K),RDRIFT(K),RPATH(K),EQBB
     1(K),RALT(K),RLAT(K),K=KMIN,KMAX)
      WRITE (6,49)
      WRITE (6,50)
      WRITE (6,51) (K,EQLONG(K),EQALT,EQALT(K),SPITCH(K),RPITCH(K),SDRIF
     1T(K),RDRIFT(K),ENERGY(K),K=KMIN,KMAX)
      IF (ISTORM.EQ.0) GO TO 28
      IF (KONTRL.EQ.0) GO TO 24
      GO TO (25,27,30), KONTRL
24    FRONT=FRONT-WIND
      SHEET1=SHEET1-STORM
      FSHEET=FSHEET+HAIL
      KONTRL=1
      ERR=ERR*0.25
      WRITE (6,52) FRONT,SHEET1,FSHEET
      GO TO 3
25    FRONT=FRONT+WIND
      SHEET1=SHEET1+STORM
      FSHEET=FSHEET-HAIL
      ORLONG=ORLONG+180.
      KONTRL=2
      DO 26 K=KMIN,KMAX
      IF (RLOSTT(K).GT.0.5) GO TO 26
      FISQ=FIDIM(K)*FIDIM(K)
      CC=ENERGY(K)+1.
      CONST=(CC*CC-1.)*FISQ
```

```
      SQBI=   BDIM(K)*FISQ
C
C     LOOK SEARCHES FOR THE SHELL HAVING A PREFIXED VALUE OF THE
C     THIRD INVARIANT, COMPATIBLE WITH CONSERVATION OF THE OTHER TWO
C
      CALL LOOK (SALT(K),SLAT(K),SLONG(K),SQBI,      FLX(K),BDIM(K),FIDI
     1M(K),ERR,ERRI,ERRB,ERRF,PHI,RLOSTT(K))
      FISQ=FIDIM(K)*FIDIM(K)
      GAMNEW=SQRT(1.+CONST/FISQ)
      ENERGY(K)=GAMNEW-1.
26    CONTINUE
      WRITE (6,53) FRONT,SHEET1,FSHEET
      IF (RLOSTT(2).GT.0.5) GO TO 11
      ALT=SALT(2)
      FLAT=SLAT(2)
      GO TO 11
27    ORLONG=ORLONG+180.
      KONTRL=3
      IF (NOSPLT.EQ.1) GO TO 38
      GO TO 11
28    CONTINUE
      ORLONG=ORLONG+DELTA
      IF (ORLONG.GT.TERM) GO TO 38
      FLONG=ORLONG
      IF (RLOSTT(1).GT.0.5) GO TO 12
      ALT=RALT(1)
      FLAT=RLAT(1)
      GO TO 12
C
C     POINTS HAVING THE PREFIXED VALUES OF CONST=(I**2)*(GAMMA**2-1.)
C     AND (I**2)*B ARE DETERMINED ON THE DISTORTED
C     FIELD LINE
C
29    FI=FIDIM(1)
      BB=BDIM(1)
      FISQ=FI*FI
      CONST=FISQ*GSQM1
      COMP=FISQ*BB
      KMIN=2
      K=KMIN
30    FISQ=FIDIM(K)*FIDIM(K)
      CONST=FISQ*GSQM1
      SQBI=BDIM(K)*FISQ
31    IF (COMP.LT.SQBI) GO TO 36
      DO 32 J=4,JUP
      IF (B(J).GT.B(3)) GO TO 33
32    CONTINUE
33    JUP=J-1
      IF (JUP.LE.4) STOP1
      S1=VN1(2)
      S2=VN2(2)
      S3=VN3(2)
      SI=FI
      SCOMP=COMP
      DO 34 J=1,JUP
      VN1(J)=VN1(J+1)
```

```
      VN2(J)=VN2(J+1)
      VN3(J)=VN3(J+1)
      B(J)=B(J+1)
      BLOG(J)=ALOG(B(J))
34    ARC(J)=ABS(ARC(J+1))
      JEP=JUP-1
      DO 35 J=2,JEP
      ASUM=ARC(J)+ARC(J+1)
      DX=BLOG(J-1)-BLOG(J)
      DN=ASUM*ARC(J)*ARC(J+1)
      BCO=((BLOG(J-1)-BLOG(J+1))*ARC(J)**2-DX*ASUM**2)/DN
      CCO=(DX*ARC(J+1)-(BLOG(J)-BLOG(J+1))*ARC(J))/DN
      SA=.75*ARC(J)
      SC=SA+.25*ASUM
      DCO=BLOG(J-1)-CCO*SA*SC
      ECO(J)=BCO+CCO*(SA+SC)
      BEG(J)=EXP(DCO+ECO(J)*.5*ARC(J))
35    BEND(J)=EXP(DCO+ECO(J)*.5*(ASUM+ARC(J)))
      BEG(JUP)=BEND(JEP)
      BEND(JUP)=B(JUP)
      ECO(JUP)=(2.0/ARC(JUP))*ALOG(BEND(JUP)/BEG(JUP))
      CALL INTEG (ARC,BEG,BEND,B,JEP,ECO,FLINT)
      FI=FLINT
      BB=B(2)
      FISQ=FI*FI
      COMP=FISQ*BB
      GO TO 31
36    SLOPE=(SQBI-SCOMP)/(COMP-SCOMP)
      SALT(K)=S1+(VN1(2)-S1)*SLOPE
      QV2(K)=S2+(VN2(2)-S2)*SLOPE
      QV3(K)=S3+(VN3(2)-S3)*SLOPE
      SI=SI+(FI-SI)*SLOPE
      SLAT(K)=90.-QV2(K)*57.2957795
      SLONG(K)=QV3(K)*57.2957795
      IF (SLONG(K).GE.180.) SLONG(K)=SLONG(K)-360.
      IF (SLONG(K).LT.-180.) SLONG(K)=SLONG(K)+180.
      FISQ=SI*SI
      GAMNEW=SQRT(1.+CONST/FISQ)
      ENERGY(K)=GAMNEW-1.
      FIDIM(K)=SI
      SIT=ABS(SIN(QV2(K)))
      CALL MODMAG (SALT(K),SIT,QV3(K),BR,BT,BP,BBB,QV2(K))
      BDIM(K)=BBB
      K=K+1
      IF (K.LE.KMAX) GO TO 30
      CALL EQUAT (VNEAR(1),VNEAR(2),VNEAR(3),EQB,EALT,ELAT,ELONG,ERR1)
      DO 37 K=KMIN,KMAX
      RLOSTT(K)=0.
      CALL FLUX (EALT,ELAT,ELONG,SALT(K),SLAT(K),SLONG(K),BDIM(K),FIDIM(
     1K),FLX(K),ERR,ERRB,ERRI,RLOSTT(K))
37    SPITCH(K)=SQRT(1.-EQB/BDIM(K))
      WRITE (6,54)
      WRITE (6,45) (K,BDIM(K),FIDIM(K),SPITCH(K),FLX(K),ENERGY(K),EALT,S
     1ALT(K),SLAT(K),K=KMIN,KMAX)
      ERR=ERR*40.
      ERR1=ERR*0.00025
```

```
      ERR2=ERR*0.025
      ALT=SALT(2)
      FLAT=SLAT(2)
      FLONG=SLONG(2)
      IF (NCSPLT.EQ.1) GO TO 25
      GO TO 11
38    ERR=ERR*0.025
      GO TO 1
C
C
39    FORMAT (4F10.4)
40    FORMAT (I5,3F10.2)
41    FORMAT (29H  DISTORTED COMMON FIELD LINE/40X,3F15.2//)
42    FORMAT (1X////27H  INITIAL COMMON FIELD LINE/40X,3F15.2//)
43    FORMAT (I5,F15.4,F10.2,F15.2,E20.5)
44    FORMAT (1X///2X,1HK,9X,4HBDIM,12X,5HFIDIM,9X,6HSPITCH,9X,6HSDRIFT,
     19X,5HSPATH,11X,4HEQB ,9X,4HSALT,4X,4HSLAT/)
45    FORMAT (I3,F15.6,4F15.4,F15.6,5X,2F7.2///)
46    FORMAT (/20H   SPLIT FIELD LINES/I20,F20.8/)
47    FORMAT (1X//50X,5F10.2//)
48    FORMAT (2X,1HK,9X,4HBDIM,12X,5HFIDIM,9X,6HRPITCH,9X,6HRDRIFT,9X,5H
     1RPATH,11X,4HEQBB,9X,4HRALT,4X,4HRLAT/)
49    FORMAT (28X,26HSHELL SPLITTING PARAMETERS//)
50    FORMAT (2X,1HK,4X,6HEQLONG,12X,11HEARTH RADII,16X,9HCOS PITCH,13X,
     114HDRIFT VELOCITY,13X,6HENERGY/)
51    FORMAT (I3,F10.2,5X,2F10.3,5X,2F10.3,2E15.3,F14.6///)
52    FORMAT (2X//27H  WIND AND STORM ARE RAGING/50X,3F20.2/)
53    FORMAT (2X//21H  SPACE IS CALM AGAIN/50X,3F20.2/)
54    FORMAT (1X///2X,1HK,9X,4HBDIM,12X,5HFIDIM,9X,6HSPITCH,9X,6H FLUX ,
     19X,5HENERG,11X,4HEALT,9X,4HSALT,4X,4HSLAT/)
      END
```

```
      SUBROUTINE SEARCH (ALT,FLAT,FLONG,SB,SI,BB,FI,ERR,ERRB,ERRI,RLOST)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VB0(3),
     1VSAVE(3),B0,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
C
C     SUBROUTINE DEFINES FIELD LINE GOING THROUGH POINT OF PREFIXED
C     B AND SECOND INVARIANT I , AT A GIVEN LONGITUDE
C
      DIMENSION V(3), V1(3), V2(3)
      DV=0.02
      RLOST=0.
      MCHECK=0
      ICHECK=0
      SERR=ERR
      SERRB=ERRB
      SERRI=ERRI
      V(1)=ALT
      IF (V(1).LT.0.5) GO TO 11
      V(2)=(90.-FLAT)/57.2957795
      V(3)=FLONG/57.2957795
      DCLT=1.5708
      ICON=1
1     ILIT=1
      DELV2=DV
      ICHECK=ICHECK+1
      IF (ICHECK.GT.20) GO TO 12
2     SIT=ABS(SIN(V(2)))
      CALL MODMAG (V(1),SIT,V(3),BR,BT,BP,BB,V(2))
3     FAC=1.-(SB-BB)/(3.*SB)
      IF (FAC.GT.1.5) FAC=1.5
      IF (FAC.LT.0.666) FAC=0.666
      V(1)=V(1)*FAC
      IF ((V(1).GT.100.).OR.(V(1).LT.0.5)) GO TO 11
      V1(1)=V(1)
      V1(2)=V(2)
      MCHECK=MCHECK+1
      IF (MCHECK.GT.15) GO TO 13
      CALL MODMAG (V(1),SIT,V(3),BR,BT,BP,BB,V(2))
      IF (ABS((BB-SB)/SB).GT.ERRB) GO TO 3
      MCHECK=0
      IF (ILIT.NE.1) GO TO 7
      ILIT=2
      CALL INVAR (V(1),V(2),V(3),ERR,BB,FI)
      IF (JUP.LT.0) GO TO 14
      V2(1)=V(1)
      V2(2)=V(2)
      B2=BB
      FI2=FI
      IF (ABS((FI-SI)/SI).LE.ERRI) GO TO 8
      IF (ABS(V(2)-DCLT).LT.0.1) GO TO 6
      SGN=SIGN(1.,(FI-SI))
      IF (V(2).LT.DCLT) GO TO 4
      DELV2=-SGN*DELV2
      GO TO 5
4     DELV2=SGN*DELV2
```

```
5       V(2)=V(2)+DELV2
        GO TO 1
6       V(2)=V(2)+DELV2
        CALL INVAR (V(1),V(2),V(3),ERR,BB,FI)
        IF (JUP.LT.0) GO TO 14
        IF ((FI-SI)*(FI-FI2).LE.0.) GO TO 2
        V(2)=V(2)-2.*DELV2
        GO TO 2
7       B1=BB
        CALL INVAR (V(1),V(2),V(3),ERR,BB,FI)
        IF (JUP.LT.0) GO TO 14
        IF (ABS((FI-SI)/SI).LE.ERRI) GO TO 8
        FACT=(SI-FI)/(FI2-FI)
        IF (ABS(FACT).GT.3.) FACT=3.*SIGN(1.,FACT)
        V(1)=V1(1)+(V2(1)-V1(1))*FACT
        V(2)=V1(2)+(V2(2)-V1(2))*FACT
        Y=AMIN1(ABS(V(2)-V1(2)),ABS(V(2)-V2(2)))
        IF (Y.GT.ABS(V1(2)-V2(2))) GO TO 1
        DV=Y
        GO TO 1
8       CONTINUE
        IF (ICON.EQ.2) GO TO 9
        ICON=2
        DV=DV*0.1
        ERR=ERR*0.025
        ERRB=ERRB*0.05
        ERRI=ERRI*0.04
        GO TO 1
9       ALT=V(1)
        FLAT=90.-V(2)*57.2957795
        FLONG=V(3)*57.2957795
        IF (FLONG.GT.180.) FLONG=FLONG-360.
        IF (FLONG.LT.(-180.)) FLONG=FLONG+360.
        DO 10 I=1,3
10      VSAVE(I)=V(I)
        GO TO 16
11      WRITE (6,17) V(1),FLAT,FLONG
        GO TO 15
12      WRITE (6,18) ICHECK
        GO TO 15
13      WRITE (6,19) ICHECK,MCHECK
        GO TO 15
14      WRITE (6,20) ICHECK,MCHECK,ALT,FLAT,FLONG
        JUP=1
15      RLCST=1.
16      ERR=SERR
        ERRB=SERRB
        ERRI=SERRI
        RETURN
C
C
17      FORMAT (19H  ALT OUT OF LIMITS/3F10.3)
18      FORMAT (51H  SORRY,BUT I CANNOT FIND THAT DAMN POINT IN ICHECK/2I1
       10)
19      FORMAT (51H  SORRY,BUT I CANNOT FIND THAT DAMN POINT IN MCHECK/2I1
       10)
20      FORMAT (40H  SORRY,BUT POINT IS IN THAT DAMN POCKET/2I10,3E15.5)
        END
```

```
      SUBROUTINE LOOK (ALT,FLAT,FLONG,SQBI,       THIRD,BMIR,FIMIR,ERR,ER
     1RI,ERRB,ERRF,PHI,CUTL)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
      IF (THIRD.LE.0.) GO TO 7
      ERRI=ERR*0.01
      CUTL=0.
      ICOUNT=0
1     CONTINUE
      CALL SEARCH (ALT,FLAT,FLONG,BMIR,FIMIR,BB,FI,ERR,ERRI,ERRB,RR)
      IF (RR.GT.0.5) GO TO 7
      CALL EQUAT (VNEAR(1),VNEAR(2),VNEAR(3),EQB,EALT,ELAT,ELONG,ERRI)
      CALL FLUX (EALT,ELAT,ELONG,ALT,FLAT,FLONG,BB,FI,PHI,ERR,ERRB,ERRI,
     1CUT)
      IF (CUT.LT.0.5) GO TO 2
      IF (ICOUNT.EQ.0) GO TO 7
      FAC=FAC*0.5
      FIMIR=FOLD
      GO TO 5
2     DPHI=THIRD-PHI
      IF (ABS(DPHI/THIRD).LE.ERRF) GO TO 9
      IF (ICOUNT.GT.0) GO TO 3
      FAC=-1./(12.56*EALT*EQB)
      GO TO 4
3     FAC=(FOLD-FIMIR)/(POLD-PHI)
4     FOLD=FIMIR
      POLD=PHI
5     SUM=DPHI*FAC
      IF (ABS(SUM/FIMIR).LT.0.1) GO TO 6
      IF (ABS(SUM/FIMIR).GT.10.) FAC=FAC*0.1
      FAC=FAC*0.5
      GO TO 5
6     FIMIR=FIMIR+SUM
      FISQ=FIMIR*FIMIR
      BMIR=SQBI/                       FISQ)
      ICOUNT=ICOUNT+1
      IF (ICOUNT.EQ.9) GO TO 8
      GO TO 1
7     WRITE (6,10)
      CUTL=1.
      GO TO 9
8     WRITE (6,11)
      CUTL=1.
9     RETURN
C
10    FORMAT (24H  NOT ACCESSIBLE IN LOOK)
11    FORMAT (39H  I CANNOT FIND THAT DAMN SHELL IN LOOK)
      END
```

```
      SUBROUTINE FLUX (XEALT,XELAT,XELONG,XALT,XLAT,XLONG,BMIR,FIMIR,THI
     1RD,ERR,ERRB,ERRI,CUT)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
      CUT=0.
      THIRD=0.
      KOUNT=0
      EALT=XEALT
      ELAT=XELAT
      ELONG=XELONG
      SALT=XALT
      SLAT=XLAT
      SLONG=XLONG
      ERR1=ERR*0.01
      DL=30.
      CALL VECPOT (EALT,ELAT,ELONG,APHI)
      S1=EALT*APHI
      SD=SLONG/57.2957795
    1 IF (SD) 2,3,3
    2 SD=SD+6.283185307
      GO TO 1
    3 IF (SD-6.283185307) 5,5,4
    4 SD=SD-6.283185307
      GO TO 3
    5 CONTINUE
    6 SLONG=SLONG+DL
      KOUNT=KOUNT+1
      IF (KOUNT.GT.6) GO TO 13
      CALL SEARCH (SALT,SLAT,SLONG,BMIR,FIMIR,BB,FI,ERR,ERRB,ERRI,RR)
      IF (RR.GT.0.5) GO TO 12
      CALL EQUAT (VNEAR(1),VNEAR(2),VNEAR(3),EQB,EALT,ELAT,ELONG,ERR1)
      CALL VECPOT (EALT,ELAT,ELONG,APHI)
      S2=EALT*APHI
      SSD=VNEAR(3)
    7 IF (SSD) 8,9,9
    8 SSD=SSD+6.283185307
      GO TO 7
    9 IF (SSD-6.283185307) 11,11,10
   10 SSD=SSD-6.283185307
      GO TO 9
   11 CONTINUE
      DEL=ABS(SSD-SD)
      SD=SSD
      THIRD=THIRD+(S1+S2)*DEL*0.5
      S1=S2
      GO TO 6
   12 WRITE (6,14)
      CUT=1.
   13 RETURN
C
   14 FORMAT (22H  INACCESSIBLE IN FLUX)
      END
```

```
      SUBROUTINE VECPOT (RO,ELAT,ELONG,A)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
      A=0.31
      ONG=ELONG/57.2957795
 1    IF (ONG) 2,3,3
 2    ONG=ONG+6.283185307
      GO TO 1
 3    IF (ONG-6.283185307) 5,5,4
 4    ONG=ONG-6.283185307
      GO TO 3
 5    CONTINUE
      R=1.
      DV=(RO-1.)*0.005
      R2=R+DV*0.5
      VLAT=(90.-ELAT)/57.2957795
      SIT=ABS(SIN(VLAT))
 6    CALL MODMAG (R2,SIT,ONG,BR,BT,BP,BB,VLAT)
      A=A-ABS(BT)*R2*DV
      R2=R2+DV
      IF (R2.GT.RO) GO TO 7
      GO TO 6
 7    A=A/RO
      RETURN
      END
```

```
      SUBROUTINE EQUAT (DUM1,DUM2,DUM3,EB,EALT,ELAT,ELONG,ERR)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
C
C     SUBROUTINE TRACES FIELD LINE FROM A GIVEN POINT TO MINIMUM B
C     MINIMUM B POCKETS AT HIGH LATITUDES ON NOON SIDE ARE IGNORED
C
      DIMENSION V(3,3), VN(3), VP(3), R1(3), R2(3), R3(3)
      ERR1=ERR
      MMM=1
      JUP=1
      V(1,2)=DUM1
      V(2,2)=DUM2
      V(3,2)=DUM3
      ARC(1)=0.
      DCLT=1.5708
1     ARC(2)=V(1,2)*SQRT(ERR)*0.3
      IF (V(2,2)-DCLT) 2,3,3
2     ARC(2)=-ARC(2)
3     CALL START (R1,R2,R3,B,ARC,ERR,V)
      IF (JUP.LT.0) GO TO 5
      DO 4 I=1,3
      VP(I)=V(I,2)
4     VN(I)=V(I,3)
      CALL LINES (R1,R2,R3,B,ARC,ERR,J,VP,VN)
      IF (J.LT.200) GO TO 7
      ERR=4.*ERR
      GO TO 6
5     JUP=1
      ERR=ERR*0.1
6     CONTINUE
      WRITE (6,8) ERR
      GO TO 1
7     ERR=ERR1
      EB=BNEAR
      EALT=VNEAR(1)
      ELAT=90.-VNEAR(2)*57.2957795
      ELONG=VNEAR(3)*57.2957795
      IF (ELONG.GT.180.) ELONG=ELONG-360.
      IF (ELONG.LT.(-180.)) ELONG=ELONG+360.
      RETURN
C
C
8     FORMAT (24H ERROR CHANGED  IN EQUAT,E15.4)
      END
```

```
      SUBROUTINE BESECT (RUM1,RUM2,RUM3,BALT,BLAT,BLONG,ERR)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
C
C     SUBROUTINE TRACES FIELD LINE UPWARDS FROM A GIVEN POINT UNTIL
C     A PREFIXED B-VALUE IS REACHED
C
      DIMENSION V(3,3), VN(3), VP(3), R1(3), R2(3), R3(3)
      ERR1=ERR
      MMM=3
      JUP=1
      V(1,1)=RUM1
      V(2,1)=RUM2
      V(3,1)=RUM3
      ARC(1)=0.
      DCLT=1.5708
1     ARC(2)=V(1,2)*SQRT(ERR)*0.3
      IF (V(2,2)-DCLT) 3,3,2
2     ARC(2)=-ARC(2)
3     CALL START (R1,R2,R3,B,ARC,ERR,V)
      IF (JUP.LT.0) GO TO 5
      DO 4 I=1,3
      VP(I)=V(I,2)
4     VN(I)=V(I,3)
      CALL LINES (R1,R2,R3,B,ARC,ERR,J,VP,VN)
      IF (J.LT.200) GO TO 7
      ERR=4.*ERR
      GO TO 6
5     JUP=1
      ERR=ERR*0.1
6     CONTINUE
      WRITE (6,8) ERR
      GO TO 1
7     ERR=ERR1
      JUP=J
      BALT=VBO(1)*6371.2
      BLAT=90.-VBO(2)*57.2957795
      BLONG=VBO(3)*57.2957795
      IF (BLONG.GT.180.) BLONG=BLONG-360.
      IF (BLONG.LT.(-180.)) BLONG=BLONG+360.
      RETURN
C
C
8     FORMAT (24H ERROR CHANGED IN BESECT,E15.4)
      END
```

```
      SUBROUTINE INVAR (DUM1,DUM2,DUM3,ERR,BB,FI)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
C
C     SUBROUTINES INVAR,START,LINES AND INTEG ARE BASED ON MCILWAINS
C     INVAR CODE
C
      DIMENSION V(3,3), VN(3), VP(3), BEG(200), BEND(200), BLOG(200), EC
     1O(200), R1(3), R2(3), R3(3)
      MMM=0
      JUP=1
      V(1,2)=DUM1
      V(2,2)=DUM2
      V(3,2)=DUM3
      ERR1=ERR
      ARC(1)=0.
    1 ARC(2)=V(1,2)*SQRT(ERR)*0.3
      DCLT=1.5708
      IF (V(2,2)-DCLT) 2,3,3
    2 ARC(2)=-ARC(2)
    3 CALL START (R1,R2,R3,B,ARC,ERR,V)
      IF (JUP.LT.0) GO TO 8
      DO 4 I=1,3
      VP(I)=V(I,2)
    4 VN(I)=V(I,3)
      CALL LINES (R1,R2,R3,B,ARC,ERR,J,VP,VN)
      IF (J.LT.200) GO TO 5
      ERR=ERR*4.
      WRITE (6,9) ERR
      GO TO 1
    5 ERR=ERR1
      JUP=J
      DO 6 J=1,JUP
      ARC(J)=ABS(ARC(J))
    6 BLOG(J)=ALOG(B(J))
      JEP=JUP-1
      DO 7 J=2,JEP
      ASUM=ARC(J)+ARC(J+1)
      DX=BLOG(J-1)-BLOG(J)
      DN=ASUM*ARC(J)*ARC(J+1)
      BCO=((BLOG(J-1)-BLOG(J+1))*ARC(J)**2-DX*ASUM**2)/DN
      CCO=(DX*ARC(J+1)-(BLOG(J)-BLOG(J+1))*ARC(J))/DN
      SA=.75*ARC(J)
      SC=SA+.25*ASUM
      DCO=BLOG(J-1)-CCO*SA*SC
      ECO(J)=BCO+CCO*(SA+SC)
      BEG(J)=EXP(DCO+ECO(J)*.5*ARC(J))
    7 BEND(J)=EXP(DCO+ECO(J)*.5*(ASUM+ARC(J)))
      BEG(JUP)=BEND(JEP)
      BEND(JUP)=B(JUP)
      ECO(JUP)=(2.0/ARC(JUP))*ALOG(BEND(JUP)/BEG(JUP))
      CALL INTEG (ARC,BEG,BEND,B,JEP,ECO,FLINT)
      FI=FLINT
      BB=B(1)
    8 CONTINUE
      RETURN
C
C
    9 FORMAT (26H ERROR INCREASED IN  INVAR,E15.4)
      END
```

```
      SUBROUTINE START (R1,R2,R3,B,ARC,ERR,V)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
      DIMENSION V(3,3), R1(3), R2(3), R3(3)
      LOOP=1
      SIT=ABS(SIN(V(2,2)))
1     IF (V(3,2)) 2,3,3
2     V(3,2)=V(3,2)+6.283185307
      GO TO 3
3     CALL MODMAG (V(1,2),SIT,V(3,2),BR,BT,BP,B(2),V(2,2))
      R2(1)=BR/B(2)
      DN=B(2)*V(1,2)
      R2(2)=BT/DN
      R2(3)=BP/(DN*SIT)
      IS=1
4     DO 5 I=1,3
5     V(I,1)=V(I,2)-ARC(2)*R2(I)
      SIT=ABS(SIN(V(2,1)))
6     CALL MODMAG (V,SIT,V(3,1),BR,BT,BP,B(1),V(2,1))
      IF (B(1)-B(2)) 7,8,8
7     ARC(2)=-ARC(2)
      IF (LOOP.EQ.2) GO TO 12
      LOOP=2
      GO TO 4
8     R1(1)=BR/B(1)
      ARC(3)=ARC(2)
      DN=B(1)*V(1,1)
      R1(2)=BT/DN
      R1(3)=BP/(DN*SIT)
      DO 9 I=1,3
9     V(I,1)=V(I,2)-ARC(2)*(R1(I)+R2(I))/2.
      SIT=ABS(SIN(V(2,1)))
      IS=IS+1
      GO TO (6,10), IS
10    DO 11 I=1,3
11    V(I,3)=V(I,2)+ARC(3)*((1.5)*R2(I)-.5*R1(I))
      GO TO 13
12    JUP=-1
13    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LINES (R1,R2,R3,B,ARC,ERR,J,VP,VN)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
      INTEGER FLAG1,FLAG2
      DIMENSION R1(3), R2(3), R3(3), VN(3), VP(3), RA(3)
      M=MMM
      FLAG1=0
      DEL=0.01
      CRE=0.15
      IF (ERR-0.15625) 1,2,2
1     CRE=(ERR**0.333333333)
2     A3=ARC(3)
      AAB=ABS(A3)
      SNA=A3/AAB
      A1=ARC(1)
      A2=ARC(2)
      AO6=A3*A3/6.0
      VN1(2)=VP(1)
      VN2(2)=VP(2)
      VN3(2)=VP(3)
      J=3
      ILP=1
      IS=1
      GO TO 8
3     IS=1
      J=J+1
      AO6=A3*A3/6.0
      ARCJ=A1+A2+A3
      AD=(ASUM+A1)/AA
      BD=ASUM/BB
      CD=A1/CC
4     DO 7 I=1,3
      DD=R1(I)/AA-R2(I)/BB+R3(I)/CC
      GO TO (5,6), IS
5     RT=R1(I)-(AD*R1(I)-BD*R2(I)+CD*R3(I)-DD*ARCJ)*ARCJ
      RA(I)=R1(I)
      R1(I)=R2(I)
      R2(I)=R3(I)
      R3(I)=RT
      VP(I)=VN(I)
6     RBAR=(R2(I)+R3(I))/2.-DD*AO6
7     VN(I)=VP(I)+A3*RBAR
8     IF (VN(2)) 9,10,10
9     VN(2)=-VN(2)
10    IF (VN(2)-3.141592653) 12,12,11
11    VN(2)=6.283185307-VN(2)
      GO TO 10
12    IF (VN(3)) 13,14,14
13    VN(3)=VN(3)+6.283185307
      GO TO 12
14    IF (VN(3)-6.283185307) 16,16,15
15    VN(3)=VN(3)-6.283185307
      GO TO 14
16    GO TO (17,18), IS
```

```
17    SIT=ABS(SIN(VN(2)))
      PRE1=VN(1)
      PRE2=PRE1*VN(2)
      PRE3=PRE1*SIT*VN(3)
      CALL MODMAG (VN,SIT,VN(3),BR,BT,BP,B(J),VN(2))
      R3(1)=BR/B(J)
      DN=B(J)*VN(1)
      R3(2)=BT/DN
      R3(3)=BP/(DN*SIT)
      ASUM=A3+A2
      AA=ASUM*A2
      BB=A3*A2
      CC=ASUM*A3
      IS=2
      GO TO 4
18    SIT=ABS(SIN(VN(2)))
      IF (VN(1).GT.8.) GO TO 19
      B(J)=B(J)*((PRE1/VN(1))**3)
19    IF (M.EQ.1) GO TO 23
      QRT=.5*ABS(R3(1))/(.1+ABS(R3(2)*VN(1)))
      X=(ABS(VN(1)-PRE1)+QRT*ABS(VN(1)*VN(2)-PRE2)+ABS(VN(1)*SIT*VN(3)-P
     1RE3))/(AAB*ERR*SQRT(1.+QRT*QRT))
      GO TO (23,20,23), ILP
20    IF (X-3.3) 23,21,21
21    A3=A3*0.2*(8.0+X)/(0.8+X)
      J=J-1
      ILP=3
      ASUM=A2+A1
      AA=ASUM*A1
      BB=A2*A1
      CC=ASUM*A2
      DO 22 I=1,3
      VN(I)=VP(I)
      R3(I)=R2(I)
      R2(I)=R1(I)
22    R1(I)=RA(I)
      GO TO 35
23    VN1(J)=VN(1)
      VN2(J)=VN(2)
      VN3(J)=VN(3)
      IF (M.EQ.3) GO TO 27
      IF (B(J-1).GT.B(J)) GO TO 29
      IF (ABS(VN2(J)-1.57).GT.0.26) GO TO 29
      IF (M.EQ.1) GO TO 25
      IF (FLAG1.EQ.1) GO TO 29
      FLAG1=1
      DO 24 I=1,3
24    VNEAR(I)=VN(I)
      BNEAR=B(J)
      GO TO 29
25    BNEAR=B(J-1)
      DO 26 I=1,3
26    VNEAR(I)=VP(I)
      GO TO 37
27    IF (B(J).GT.BO) GO TO 29
      FAC=(BO-B(J-1))/(B(J)-B(J-1))
```

```
      DO 28 I=1,3
28    VBO(I)=VP(I)+(VN(I)-VP(I))*FAC
      ARC(J)=ARC(J)*FAC
      B(J)=BO
      VN1(J)=VBO(1)
      VN2(J)=VBO(2)
      VN3(J)=VBO(3)
      GO TO 37
29    IF (J.GE.200) GO TO 37
      A1=A2
      IF (M.NE.1) GO TO 30
      IF (B(J)-B(2)) 30,30,36
30    ILP=2
      A2=A3
      IF (M.EQ.1) GO TO 35
      A3=A3*.2*(6.+X)/(.8+X)
      AM=(2.-R3(2)*VN(1))*VN(1)*CRE
      IF (ABS(A3)-AM) 32,32,31
31    A3=SNA*AM
32    IF (SNA*R3(1)+.5) 33,33,35
33    AM=-.5*SNA*VN(1)/R3(1)
      IF (ABS(A3)-AM) 35,35,34
34    A3=SNA*AM
35    ARC(J+1)=A3
      AAB=ABS(A3)
      GO TO 3
36    CONTINUE
37    RETURN
      END
```

```
      SUBROUTINE INTEG (ARC,BEG,BEND,B,JEP,ECO,FI)
      COMMON B(200),VN1(200),VN2(200),VN3(200),ARC(200),VNEAR(3),VBO(3),
     1VSAVE(3),BO,BNEAR,JUP,MMM
      COMMON FRONT,SHEET1,SHEET2,FSHEET
      DIMENSION BEG(200), BEND(200), ECO(200)
1     KK=JEP
      IF (KK-4) 3,2,4
2     KK=KK-1
3     A=B(KK-1)/B(2)
      X2=B(KK)/B(2)
      X3=B(KK+1)/B(2)
      ASUM=ARC(KK)+ARC(KK+1)
      DN=ARC(KK)*ARC(KK+1)*ASUM
      BB=(-A*ARC(KK+1)*(ARC(KK)+ASUM)+X2*ASUM**2-X3*ARC(KK)**2)/DN
      C=(A*ARC(KK+1)-X2*ASUM+X3*ARC(KK))/DN
      FI=1.570796326*(1.-A+BB*BB/(4.*C))/SQRT(ABS(C))
      RETURN
4     T=SQRT(1.-BEND(2)/B(2))
      FI=(2.*T-ALOG((1.+T)/(1.-T)))/ECO(2)
      IF (B(2)-BEND(KK)) 6,6,5
5     KK=KK+1
6     T=SQRT(ABS(1.0-BEG(KK)/B(2)))
      FI=FI-(2.*T-ALOG((1.+T)/(1.-T)))/ECO(KK)
      KK=KK-1
      DO 15 I=3,KK
      ARG1=1.-BEND(I)/B(2)
      IF (ARG1) 7,7,8
7     TE=1.E-5
      GO TO 9
8     TE=SQRT(ARG1)
9     ARG1=1.-BEG(I)/B(2)
      IF (ARG1) 11,11,10
10    TB=SQRT(ARG1)
      GO TO 12
11    TB=1.E-5
12    IF (ABS(ECO(I))-2.E-5) 13,13,14
13    FI=FI+((TE+TB)*(ARC(I)+ARC(I+1)))/4.
      GO TO 15
14    FI=FI+(2.*(TE-TB)-ALOG((1.+TE)*(1.-TB)/((1.-TE)*(1.+TB))))/ECO(I)
15    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MODMAG (RR,SINTH,PPHI,BR,BTHETA,BPHI,BB,THET)
      COMMON FRONT,SHEET1,SHEET2,FSHEET
C
C     SUBROUTINE ASSEMBLES MAGNETIC FIELD FROM TAIL, MAGNETOPAUSE AND
C     INTERNAL DIPOLE - FOR ENQUIRIES WRITE TO GILBERT MEAD, GODDARD
C     SPACE FLIGHT CENTER, GREENBELT MARYLAND 20771
C
      DIMENSION GG(7,7)
      DIMENSION G(7,7), CONST(7,7), P(7,7), DP(7,7), SP(7), CP(7)
      COSTH=COS(THET)
      SINPHI=-SIN(PPHI)
      COSPHI=-COS(PPHI)
      RO=FRONT
      R1=SHEET1
      R2=SHEET2
      BCS=FSHEET
      IF (JFIRST-13) 1,5,1
1     JFIRST=13
C
C     SET UP INITIAL CONSTANTS THE FIRST TIME AROUND
C
      DO 2 N=1,7
      DO 2 M=1,7
2     GG(N,M)=0.
      NMAX=7
C
C     THE FOLLOWING COEFFICIENTS ARE SCHMIDT-NORMALIZED
C
      GG(2,1)=-0.25111E5
      GG(3,2)=-0.12424E5
      GG(4,1)=-0.00716E5
      GG(4,3)=-0.02333E5
      GG(5,2)=-0.02397E5
      GG(5,4)=-0.00163E5
      GG(6,1)=0.00569E5
      GG(6,3)=-0.01078E5
      GG(6,5)=-0.00103E5
      GG(7,2)=0.00126E5
      GG(7,4)=-0.00187E5
      GG(7,6)=-0.00041E5
      P(1,1)=1.0
      DP(1,1)=0.
      SP(1)=0.
      CP(1)=1.
      DO 3 N=3,NMAX
      FN=N
      N2=N-2
      DO 3 M=1,N2
      FM=M
3     CONST(N,M)=((FN-2.)**2-(FM-1.)**2)/((2.*FN-3.)*(2.*FN-5.))
      DIMENSION SHMIDT(7,7)
      SHMIDT(1,1)=1.0
      DO 4 N=2,7
      FN=N
      SHMIDT(N,1)=SHMIDT(N-1,1)*(FN+FN-3.0)/(FN-1.0)
      FACT=2.0
      DO 4 M=2,N
      FM=M
      SHMIDT(N,M)=SHMIDT(N,M-1)*SQRT((FN-FM+1.0)*FACT/(FN+FM-2.0))
```

```
4     FACT=1.0
5     IF (RO-ROOLD) 6,9,6
6     ROOLD=RO
      DIMENSION FAC(7)
      FAC(2)=RO**3
      DO 7 N=3,NMAX
7     FAC(N)=RO*FAC(N-1)
      DO 8 N=2,NMAX
      DO 8 M=1,N
8     G(N,M)=SHMIDT(N,M)*G3(N,M)/FAC(N)
9     CONTINUE
C     BEGIN CALCULATION FOR SPECIFIED INPUT
C
      CT=COSTH
      ST=SINTH
      SP(2)=SINPHI
      CP(2)=COSPHI
C
C     CALCULATE SIN(M*PHI) AND COS(M*PHI)
C
      DO 10 M=3,NMAX
      SP(M)=SP(2)*CP(M-1)+CP(2)*SP(M-1)
10    CP(M)=CP(2)*CP(M-1)-SP(2)*SP(M-1)
      R=RR
      A=1.
      RA=R/A
      ROA=1.
      BR=0.
      BTHETA=0.
      BPHI=0.
      FN=1.
C
C     CALCULATE SPHERICAL HARMONICS FOR CAVITY FIELD
C
      DO 16 N=2,NMAX
      SUMR=0.
      SUMT=0.
      SUMP=0.
      FM=0.
C
C     DEVELOP LEGENDRE FUNCTIONS AND THEIR DERIVATIVES BY RECURSION FORM
C
      DO 15 M=1,N
      IF (N-M-1) 13,12,11
11    P(N,M)=CT*P(N-1,M)-CONST(N,M)*P(N-2,M)
      DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)-CONST(N,M)*DP(N-2,M)
      GO TO 14
12    P(N,M)=CT*P(N-1,M)
      DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)
      GO TO 14
13    P(N,N)=ST*P(N-1,N-1)
      DP(N,N)=ST*DP(N-1,N-1)+CT*P(N-1,N-1)
14    CONTINUE
      TS=G(N,M)*CP(M)
      SUMR=SUMR+P(N,M)*TS
      SUMT=SUMT+DP(N,M)*TS
      SUMP=SUMP+FM*P(N,M)*G(N,M)*SP(M)
```

```
      FM=FM+1.
15    CONTINUE
      BR=BR-ROA*FN*SUMR
      BTHETA=BTHETA-ROA*SUMT
      BPHI=BPHI+ROA*SUMP
      ROA=ROA*RA
      FN=FN+1.
15    CONTINUE
      BPHI=BPHI/ST
C
C     CALCULATE TAIL FIELD
C
      RCT=R*CT
      RCT2=RCT**2
      RSC=R*ST*CP(2)
      TOP=R2+RSC
      BOT=R1+RSC
      BX=-BCS*(ATAN(TOP/RCT)-ATAN(BOT/RCT))/3.14159265
      BPHI=BPHI+BX*SP(2)
      BRHO=-BX*CP(2)
      BY=BCS*ALOG((RCT2+TOP**2)/(RCT2+BOT**2))/6.28318531
      BR=BR+BRHO*ST-BY*CT
      BTHETA=BTHETA+BRHO*CT+BY*ST
C
C     ADD DIPOLE FIELD TO CAVITY FIELD
C
      R3=R**3
      BR=BR-62000.*COSTH/R3
      BTHETA=BTHETA-31000.*SINTH/R3
      BR=BR*1.E-05
      BTHETA=BTHETA*1.E-05
      BPHI=BPHI*1.E-05
      BB=SQRT(BR*BR+BTHETA*BTHETA+BPHI*BPHI)
      RETURN
      END
```