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BY

Lee Heflinger, Stuart Ridgway, and Allan Schaffer

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9610 Bellanca Avenue
Los Angeles, California

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FINAL REPORT

MAGNETIC INDUCTION PLASMA ENGINE

BY

Lee Heflinger, Stuart Ridgway, and Allan Schaffer

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

April 5, 1965

CONTRACT NAS 3-5912

Technical Management
NASA Lewis Research Center
Cleveland, Ohio
Spacecraft Technology Division
H. G. Kosmahl

GENERAL TECHNOLOGY CORPORATION
9610 Bellanca Avenue
Los Angeles, California

ABSTRACT

This report describes work performed at General Technology Corporation under NASA Contract NAS-3-5912, which covers the final phase in a magnetic induction plasma accelerator development program. The aim of this development program has been to develop an engine that accelerated a plasma to a velocity corresponding to a specific impulse of 3000 seconds, or more, with good efficiency. Good accelerative coupling had been demonstrated in the previous work between helium plasma and a traveling transverse 450 gauss magnetic field, but with excessive wall interaction. The wall interaction was not significantly reduced by the changes in magnetic field geometry investigated. A theoretical study of the interaction suggests potential improvement by shortening the accelerator.

AUTHOR

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SUMMARY

At the conclusion of the previous years work good accelerative coupling had been demonstrated to a helium plasma, with specific impulses of 6000 seconds observed, and thrusts of the order of several newtons under pulsed conditions. However 50% of the energy and momentum transferred to the plasma was found to be lost at the walls of the propulsion tube.

The motivating force that directed the efforts of the final phase of the work was to understand the wall interactions, and hopefully to mitigate them. Two modifications were made in the geometry of the magnetic field. The first was to replace the system of individual drive coils that formed the magnetic field by a set of coils in which adjacent coils overlapped. The original method gave a moving field on the median plane of the apparatus, but at the pole tips the field was oscillating in time and stationary in space. The revised method gave a moving magnetic field throughout the volume of the accelerator tube. The anticipated improvement in the reduction of the wall interactions was not forthcoming, although there was a significant improvement in thrust.

In the transverse geometry for helium a maximum specific impulse of 7500 seconds was observed at a centerline magnetic field of 780 gauss, and a mass loading of 5×10^{-8} kg. Data for helium, nitrogen, and argon at centerline magnetic fields of 540 gauss gave a specific impulse that reached a maximum of 6400 seconds for helium at the 5×10^{-8} kg mass loading, and decreased with increasing mass loading for helium, and the other gases. The data was consistent with specific impulse being a unique function of mass loading independent of the gas chosen. Typical good values of transfer efficiency were 50%, and of calorimetric efficiency were also 50%. At the higher mass loadings the thrust reached 4.5 newtons.

The other geometrical variation tried was a change to cylindrically symmetric geometry. A sequence of four coils with their axis coincident with the axis of the propulsion tube was used. In this arrangement the acceleration comes from the radial component of the magnetic field, and it is hoped that the longitudinal component of field will give

some containment of the plasma. The radial field is zero on the axis, and in general is appreciably smaller than the fields available in the transverse geometry, and one therefore expects poorer accelerative coupling to the plasma. These expectations were borne out.

For helium in the best case the portion of energy absorbed by the plasma that appeared as directed kinetic energy was about one third, while two thirds appeared as heat. There was also somewhat better isolation of the plasma from the wall, for about two thirds the energy was in the exhaust, and one third went to the tube walls as heat. The best transfer efficiency was 28%. The maximum specific impulse was 4200 seconds.

An analytical discussion of the acceleration process, and of the motion of the plasma in a direction perpendicular to the acceleration direction is conducted in Appendix 2. It gives some indication that there might exist a regime of operation that would give good acceleration without wall interaction. A future program might profitably extend the analysis with the help of more powerful computing methods, and experimentally test the more promising configurations.

I. Introduction

The ships man lifts off his planet to explore the void, earth's satellites, and the other planets must be propelled by the rearward ejection of mass. This burden must be carried from the start, and its necessity limits the useful loads and the possible destinations of space ships. His present attainments have been made with chemical fuels used both as the ejected mass and the source of energy. A quality measure for a space propulsion system is the specific impulse, the thrust available per unit mass expenditure rate, usually expressed in seconds. Present chemical propellants are capable of 300 seconds, but many interesting missions could well use a specific impulse ten times as great.

To the question - does the vast store of energy in the nucleus hold the key to the desired far ranging engines - we must recognize two partial answers. If thermal forces are used to accelerate the propellant, a specific impulse of perhaps 1000 seconds is available. If electrical forces are used, then very high specific impulses may be had, but with the limitation of low thrust. An electrical propulsion system envisages nuclear energy transformed to electrical energy, and the electrical energy used to accelerate the propellant to a useful velocity.

The electrical acceleration may use either electrostatic or electro magnetic forces. The first gives good efficiency and reasonable impulse but suffers from very low thrust. The magnetic engine promises a much higher thrust (although still small compared to chemical rockets), and some forms avoid the use of electrodes that may erode and waste away in the course of a long mission.

The object of the presently reported development program was to develop an engine that provided a specific impulse in the range of 2000 - 10,000 seconds with good electrical efficiency and mass utilization using electromagnetic forces for propellant acceleration.

II. Design of the Accelerator

A desirable accelerator would have the characteristics of high thrust, low erosion of walls, high electrical efficiency, suitable specific impulse, desirable propellant, high mass utilization, and a power demand within the capabilities of projected space nuclear power plants. To orient oneself to the magnitudes of the essential quantities, let us apply some elementary analysis to an ideal accelerator that delivers one-half kilogram force (4.9 newtons) of thrust at a specific impulse of 3000 seconds. The mass flow rate $\dot{m}$ would be $\frac{1}{3000} = 1.66 \times 10^{-4}$ kilogram/second. The velocity of the accelerated mass is 9.8 meters/seconds, or 29,400 meters/second. The kinetic energy rate in the beam is $\frac{1}{2} \dot{m} v^2 = 1.43 \times 10^5$ watts. Let us further imagine a nuclear plant with a specific power of 0.1 kw/kg, thus weighing 1430 kg, and assign equal masses for the propellant, the structure and the payload. The initial acceleration of the 4290 kilogram vehicle would be 1.17×10^{-4} gravities. The duration of powered flight would be 8.6×10^6 seconds, closely 100 days, and the total velocity change in field free space would be $29,400 \ln 3/2$ or 11,900 meters/second, sufficient for many interesting missions that would start from earth's orbit.

It is not the purpose here to examine how this ideal performance will be degraded as realistic weights of power plants, attainable efficiencies, and the consequences of maneuver in a gravitational field are considered. We need only remark that at constant mass flow rate the energy price of acceleration goes as the square of the exhaust velocity, while the thrust is only linear with velocity: in general there will be an optimum specific impulse for missions that must be executed in a finite time even if no limit is placed on the total amount of energy available from the power plant.

The velocity of the exhaust stream from an engine with a specific impulse ten fold greater than that from a chemical rocket will be ten times as great, and the energy density will be one hundred times as great. It is thus a challenge to the designer of such an accelerator

to achieve such a high energy density. Any thoughts of accelerating solid nonsuperconducting macroscopic matter in some kind of motor must fail. A dissipative loss of only 1% of the accelerating energy into the armature will volatilize it. The low mass flow rate and the high energy density lead to the conclusion that the propellant that is to be accelerated will be a gas at low density, ionized to provide electrical conductivity, a plasma.

In this program it was decided to accelerate plasma with a density of the order of 10^{16} particles/cm³ by means of a moving transverse magnetic field. The choice of magnetic forces is to gain the large thrusts available. The choice of the density was made as the result of two considerations. The first is that other programs were conducting the exploration of the acceleration problem at lower densities. The second is that at a high density the collision frequency is so high that an electron will make many collisions in the time that it would take it to execute a cyclotron orbit in the magnetic field. Thus the behavior of the plasma would be that of a continuous conducting fluid, and the Hall effect would be negligible, that is the current in the plasma would be in the direction of the electric field vector. The transverse geometry of the magnetic field was chosen for its difference from other efforts, and the expectation of strong coupling between the magnetic field and the plasma.

The acceleration method can be viewed in two ways. If the conductivity of the plasma is very high and the magnetic pressure is greater than the plasma pressure, then one may consider the moving magnetic field as an impenetrable piston that moves down the propulsion tube driving the plasma before it. The behavior of the first engine built may be interpreted fairly well on this model. The second point of view is to look at the plasma as the armature in an induction motor. Currents are induced in it by the slip of the magnetic field past the plasma. These currents give rise to $\mathbf{J} \times \mathbf{B}$ forces on the plasma that accelerate it forward. This picture is more appropriate to the second engine that was built and tested.

III. Description of First Engine

The first engine built formed its moving magnetic field by the ringing discharge of a group of capacitors. The coils were rectangular loops of copper strap with holes through a pair of opposite sides of the loop for the propulsion tube. Capacitors were fired through these coils in controlled sequence to give about a four cycle ringing discharge. The propellant was originally in the propulsion tube at pressures varying from 50 microns to 5000 microns. The velocity to which the propellant was accelerated and other characteristics of the motion were determined by a fast rotating mirror framing camera that was developed for the purpose. The prominent results of the first years work with this engine were:

1. It is possible to get good coupling to a plasma with a transverse magnetic field and to accelerate it to a velocity close to the phase velocity of the magnetic field.
2. For good coupling it was desirable that the magnetic pressure be about equal to the plasma pressure.
3. Good acceleration required a relative phase of the current in adjacent coils of 90° .
4. Good acceleration required that the fields of adjacent coils overlap to the extent that the maximum of the magnetic field moved smoothly down the tube.
5. Minimization of the wall interaction required the same conditions as good acceleration.

IV. Description of Second Engine

Since the first engine allowed no efficiency or thrust measurements, the second stage engine was built to operate for 500 cycles of oscillation of the magnetic field. All further work was done with this engine. It consisted of:

1. A megawatt oscillator at 480 kc.
2. A pulsed gas valve that would introduce a constant flow of propellant for the 1.25 milliseconds of operation of the system.
3. Timing circuits to operate the oscillator and valve in controlled sequence.
4. A set of drive coils mounted on a pendulum for the measurement of back reaction.
5. A vacuum chamber for the engine to exhaust into.
6. Instrumentation to measure the electrical energy delivered to the plasma.
7. Calorimeters for the measurement of the plasma energy appearing at the wall and delivered to the vacuum chamber.

The work of the second year consisted mainly in the design and construction of this engine, and some preliminary tests. With the oscillator it was possible to achieve moving magnetic fields that had a peak value of 700 gauss on the axis. The time of operation of the oscillator was 1.25 milliseconds, and the frequency was 480 kc, giving a phase velocity of the magnetic field of about 54,800 meters/second. It was possible to achieve a 90° phase relation between the currents in adjacent coils, and this phase relation was not significantly perturbed by the presence of plasma.

V. Description of Third Year's Work

With the apparatus essentially completed, the third year's work consisted in evaluating various propellants and conditions of operation. The best propellant was found to be helium, with nitrogen second. Electromagnetic specific impulses of 6000 seconds were attained. During this work it became apparent that the interaction of the plasma with the walls of the propulsion tube were considerably more severe than would have been anticipated from the first year's observations. A combination of calorimetric and thrust measurements yielded the conclusions that of the energy absorbed by the plasma in a favorable regime of operation, half went into the kinetic energy of the plasma as indicated by the electromagnetic back reaction, and the other half appeared as heat. Measurements were made of the energy transfer to the propulsion tube walls, and to a copper bag calorimeter into which the propulsion tube exhausted. Half the energy input to the plasma appeared at the tube walls, and the other half appeared in the bag calorimeter. The only thermodynamically reasonable interpretation of these measurements is that these two energy transfers have the same ratio of heat to kinetic energy, so one must conclude that half of the kinetic energy given to the plasma is lost at the walls of the propulsion tube. This interpretation was verified by tests made using a propulsion tube with a moveable inner wall. This wall was suspended as a pendulum, and the drag forces exerted upon it were measured by observing its deflections.

The loss of useful thrust, and the erosion of the walls consequent upon such wall losses is intolerable. A major effort of the third year was to attempt to understand them and reduce them to an acceptable value. The flow field of the plasma was explored with small glass spherical pendulum bobs in the flow field whose quarter cycle deflections were photographed. These measurements showed that down the propulsion tube and off the axis the flow was partly outward to the wall. It seems clear that this flow is due to electromagnetic forces. Gas pressure forces are insufficient to explain the observed divergence of the flow. A calculation of the wall drag

based on the laws of turbulent heat and momentum transfer also yields a magnitude much too small to explain the observed results. The detailed results of these studies are reported in the annual report for the third year.

VI . Experimental Results in Final Phase of the Work.

In this section we shall present the experimental results of the concluding phases of the program. Two new geometries of the accelerating magnetic field were investigated. In the first the ferrite core with its windings on pole pieces was replaced with a set of four coils on each side of the propulsion tube that were overlapped in such a way as to provide a field that translated uniformly down the propulsion tube at the walls as well as in the center. The original configuration gave a running field at the center, but at the walls gave a field that was a superposition of fields that were running in both directions. To construct a running field requires the overlapping of time periodic fields of different phase. The overlapping of the coil windings provided for overlap at the walls of the tube as well as in the center. The second field geometry tested was a running axially symmetric field that was formed by winding the coils with their axes coincident with the propulsion tube axis. In this geometry the accelerative coupling to the plasma is provided by the radial component of the field, which tends to be appreciably smaller than the transverse fields available in the previous case. A large axial component of the magnetic field is present, and it is hoped that this would restrain the plasma from the wall.

Diagnostic Method

The quantities of interest for the evaluation of the performance of the engine are the thrust, the mass utilized, the specific impulse, the energy input to the plasma, the energy that appears as translational kinetic energy, the heat energy content of the plasma, and the momentum and energy loss to the wall. The methods used for determining these quantities have been described in previous reports, but for completeness we shall recapitulate them here, and discuss their uncertainties.

The mass used is determined by an ion gauge that determines the jump in pressure in the main vacuum tank after a shot. Accurate and reproducible readings are obtained after several preliminary shots scrub the propulsion tube wall and the bag calorimeter. There was usually no significant difference in mass used between oscillator on and oscillator off except in the case of neon, where it was observed that the discharge was penetrating up the duct that led the gas into the propulsion tube. This restricted the propellant flow, probably by $\vec{J} \times \vec{B}$ forces in the upstream direction, since the thrust and specific impulse were anomalously low in the case of clean neon. The thrust was measured by determining the back impulse transferred to the pendulously mounted drive coils. Combining the impulse with the mass utilized gives the specific impulse. The translational kinetic energy may also be computed from these measurements if the assumption is made that all the plasma is accelerated to the same velocity, and this assumption is made in the data analysis to be presented. The impulse was measured by observing the deflection of the drive coil pendulum with a commercial differential transformer instrument. Its calibration was not as steady as could be desired, and frequent recalibration was necessary. In general the impulse measurements were correct to a few percent, but occasional errors as large as 10% may exist in some points.

The energy input to the drive coils from the oscillator was measured with Hall effect multipliers that determined the integral of $V \cdot I$ over the millisecond operation of the oscillator. The coil copper losses were determined by integrating I^2 over the same time by hot wire detectors. In this device the temperature rise in a short length of resistance wire that carried a portion of the coil current was measured with a thermocouple. The system was calibrated using no plasma load to determine the calibration ratio between the two energy instruments, and a graphite load whose temperature rise gave the calibration of the energy scale. A consistency check was made in which it was asked that the energies be proportional to the square of the charging voltage. When the energy absorbed by the plasma was half the total energy input

to the coils, the measurements seem to be good to 2 to 3%. At very low mass loadings of the accelerator, when the plasma absorbs 10% of the energy input to the drive coils or less, the calculated energy absorbed is the difference of two fairly large numbers, and is subject to considerably greater error. In general it was not possible to get consistent data on the energy absorption if the valve plenum pressure was at or below 5 torr. Thrust measurements were also difficult at low mass loadings.

A copper bag calorimeter was placed downstream of the propulsion tube to catch the exhaust from the accelerator. It was instrumented with thermocouples to determine the temperature rise. Thermocouples were attached to twelve positions on the propulsion tube wall to determine the heat transfer to the wall. From reproducibility and energy balance comparisons with the electrical energy measurements the errors are not believed to exceed 5%.

Performance With Distributed Coils and Transverse Magnetic Field

Measurements were made of thrust, heat transfer, and energy transfer for helium, and of thrust and energy transfer for nitrogen, argon, and neon. There was definite evidence of air contamination of the neon so the results are omitted except for the remark that they were not much different from those for nitrogen. In Figure 1 is presented the specific impulse as a function of the valve plenum pressure for helium, nitrogen, and argon at 25 kv charging voltage, and for helium at 30 kv charging voltage. As the mass loading of the accelerator increases, the exit velocity falls off. At 10 torr the helium values rise significantly above synchronous velocity. We are uncertain as to what interpretation to place on this result. In Figure 2 is plotted the same data as a function of the mass loading instead of the valve plenum pressure, and it is clear the results for the different gases fair into each other quite well. This indicates that the electrical conductivity of the different gases is not much different over the range of loadings for which data was taken, and that the significant parameter controlling

the specific impulse is the mass loading. An elementary theory of the acceleration process predicts that the exit velocity exponentially converges to the phase velocity as the reciprocal of the mass loading. In Figure 3 we have plotted the specific impulse as a function of the reciprocal of the mass loading, and the empirically determined fit of the theoretically predicted form, $I_{sp} = 220 + 6900 (1 - e^{-.1208/m})$ where the mass loading m is in milligrams. The small constant term in the formula chosen to fit the data is without theoretical foundation, but was included for the slight improvement that it gave in the fit for the high mass loading part of the curve. The elementary theory predicts that the coefficient of $1/m$ in the exponent should be $-10^6 \sigma B_0^2 LA t / 2$ where σ is the plasma electrical conductivity, B_0 the magnetic field amplitude, L the accelerator length, A the propulsion tube area, t the time duration of the gas admission, and the factor 10^6 is due to the fact that m has been expressed in milligrams. Substituting actual values in this formula yields a coefficient 0.635 which is five times the observed value. This discrepancy comes about from the fact that the theory used assumes that the accelerator extends indefinitely in the direction which is perpendicular to the field and the acceleration direction. The current pattern in the plasma in the simplified model consists of currents alternately plus and minus in the x direction which form closed loops by connection at infinity. In the actual finite accelerator the loop closing currents must run along the wall. An approximate calculation of the effect of the increased non-thrust productive path for currents to flow reduces the computed coefficient by a factor 4.

In Figure 4 are plotted the total and the kinetic energy as a function of the mass loading for the three gases. The kinetic energy of the plasma for the three gases lies reasonably well along a single curve. The trend line was computed from the previously obtained fit to the specific impulse data. The total energy, however, is appreciably different for each gas. In Appendix A of the previous Annual Report the total energy is determined to be equal to mvv_p with v the phase velocity and v_p the plasma velocity. This quantity was

computed from the fit to the specific impulse data, and the result lay above all the total energy data, but did follow the helium results fairly well. The theory depended upon the hypothesis of inelastic collision between the magnetic field and the plasma, or equivalently that the inductance of the plasma being negligible compared to the resistance. This sheds no light on the distinct differences between the gases. The dependence of ionization potential with degree of ionization is very similar for nitrogen and argon, and cannot be invoked to explain the nitrogen argon difference. One suspects that the differences in molecular weight must be in some way accountable, but if so it becomes somewhat of a mystery as to why the kinetic energy and specific impulse for the three gases fit so well together.

Wall Interactions

For the transverse geometry of the magnetic field, the wall interaction was studied for helium. There was no particular difference from the results obtained earlier in the program with the field provided by the discrete poles. The calorimetric efficiency which is taken as the ratio of the heat observed in the bag calorimeter and the part of the accelerator tube wall downstream of the last accelerating coil to the total energy input, ran about 50% with no particular dependence upon the mass loading. The calorimetric results are given in Figure 5. The interesting features are that the heat out of the accelerator closely follows at reduced scale the shape of the total energy input curve; and that the wall transfer closely follows the kinetic energy determined from the electromagnetic back reaction on the drive coils.

If the gas driven to the wall has the same proportion of kinetic energy to heat energy as the gas exiting from the accelerator, then the calorimetric efficiency measures the factor by which the back reaction thrust must be multiplied to get the net useful thrust. The product of the transfer efficiency and the calorimetric efficiency gives a net efficiency of 25%. This net efficiency is the ratio of the thrust obtained in the actual engine to that for an ideal engine that converted all the electrical input energy into directed motion of the plasma without

wall interaction and without heating the plasma.

In Figures 6 and 7 are shown the detailed distributions of the energy transfer to the walls of the propulsion tube. The direction of the magnetic field is horizontal. The energy transfer is somewhat greater perpendicular to the direction of the field than parallel to it. The energy transfer is very nearly the same in both magnitude and distribution to that observed with the discrete poles in the earlier work. If the accelerator were shortened, and the importance of the transfer in the direction perpendicular to the field and the acceleration direction reduced by significantly heightening the accelerator, a possibly very significant improvement in the efficiency might be obtained.

Field Penetration Measurements

Measurements were made of the amplitude of the transverse magnetic field in the median plane as a function of the mass loading. These measurements show the extent to which the magnetic field penetrates the plasma. The results show that the shielding increases with the mass loading, but is not so great as to make completely inapplicable a model that neglects the contribution of the currents in the plasma to the magnetic field. The results are plotted in Figure 8. The axial position of the measurement was at the midpoint of the accelerator. Since the electrical conductivity of the plasma is fairly independent of both mass loading, and of position in the tube, we must interpret the decrease in field penetration with increasing mass loading. As the plasma velocity decreases, the slip velocity increases, and the currents induced in the plasma increase. It might be possible to exploit this effect to study in a more detailed manner the convergence of the plasma velocity to synchronism.

Experimental Results in Solenoidal Geometry

Measurements similar to those made in the transverse geometry were made with the solenoidal accelerating coils. The coils were overlapped in order to give a moving magnetic field throughout the volume of the propulsion tube. The phase velocity was the same as in the transverse case. The mutual inductance between the 0° and the 90° sets of coils was considerably larger than in the transverse case. A considerable modification of the balancing circuit was necessary to remove this interaction. The gases studied were helium, nitrogen, nitric oxide, and neon.

Nitric oxide was studied to test whether the ratio of the ionization potential to the dissociation potential of diatomic gases was a significant factor in the performance in the accelerator. This possibility was suspected from the failure of hydrogen to give good performance. Hydrogen has a low dissociation potential, 4.4 volts, and a relatively high ionization potential, 15.6 volts. Thus the discharge could be quenched by the absorption of energy in dissociation, which yields no current carriers. Nitrogen is much better in this respect, the dissociation potential being 9.1 volts, and the ionization potential being 15.5 volts. Nitric oxide has a dissociation potential of 6.1 volts, and an ionization potential of 9.5 volts. The ratio of these potentials is 10% more favorable than for nitrogen, and the ionization potential is 50% lower, so it would be expected that less energy would be consumed in ionization. This argument would have more force for an accelerator in which the gas was more gently ionized and accelerated than the present device. In the present accelerator the energy deposition in the plasma is so large that multiple ionization is the usual case, and the energy cost of first ionization is not great significance. The dissociation quenches the starting of the discharge in hydrogen, but for the other gases that escape the starting troubles the effect must be small. The performance of nitric oxide in the accelerator was slightly inferior to nitrogen.

The performance of neon was greatly inferior to that obtained for the other gases. It was suspected that this was due to contamination by inleakage of air into the gas handling system. The handling procedure was such that a small leak was of no consequence for gases available in cylinders, since there was a steady flow of the gas through the system at all times. Neon, being expensive, was attached to the system in liter flasks, and a small leak could build up contamination. Considerable effort was spent to make the gas handling system truly tight, both to allow neon tests, and to assure that the leaks that perturbed the neon results were sufficiently small that the previous results were not in question. The result - pure neon was not as good as air contaminated neon!

For both clean neon, and for helium it was observed that the mass of gas admitted to the system was not proportional to the valve plenum pressure. At low pressures the admission was deficient. This we believe is due to the discharge penetrating the duct that admitted the gas to the propulsion tube. This effect occurred very occasionally in the transverse geometry. The solenoidal geometry has clear closed paths for the induced currents, circles about the axis, that are free of the intervention of the wall. In the confined space of the entrance duct only in the solenoidal geometry were conditions proper for the existence of a discharge.

In Figure 9 is plotted the specific impulse of the three gases as a function of the mass loading. The data for the different gases does not fit so well together as it did in the transverse geometry case. The trend line is drawn through the helium data. The maximum specific impulse, 4150 seconds for helium, is about $2/3$ of that obtained in the transverse geometry. The analysis presented in the appendix on magnetic field structure indicates that the forces to be expected on the plasma should be appreciably less.

In Figure 10 is presented the data on the kinetic energy and the total energy. The kinetic energy is a considerably smaller fraction of the electrical energy supplied than in the transverse case. The

transfer efficiency has a maximum of 28% at a mass loading of .165mg of helium. The trend of the total energy for nitrogen and nitric oxide is below that for helium at corresponding mass loading.

Calorimetry in Solenoidal Geometry

In Figure 11 is presented the distribution of the heat transfer to the wall for helium at 30 torr valve plenum pressure, and in Figure 12 is presented the distribution at 50 torr plenum pressure. One expects the side and top heat transfers to be identical because of the cylindrical symmetry of the magnetic field. Stray fields from the leads from the coils may be the cause of the modest deviations from symmetry. Differences in the axial distribution of the heat transfer is observed for the two mass loadings studied and for the transverse geometry. One would be surprised at the lack of a difference between the two geometries. One may conclude that the motion of the plasma to the walls is magnetic field structure sensitive, which is reassuring to observe although it surprises no one. In the transverse case the heat transfer drops abruptly beyond the drive coil area, while in the solenoidal case the transfer holds up to the end of the propulsion tube. At the higher mass loading in the solenoidal case the transfer shows a small minimum in the region of the third coil, and a small drop at the end, compared to a steady climb in the heat transfer in the lesser loading case with distance down the propulsion tube.

The following table summarizes the results of the calorimetric studies in solenoidal geometry. All energies are in joules.

<u>Pressure</u>	<u>Electrical Energy Input</u>	<u>Heat Energy Output</u>	<u>Kinetic Energy</u>	<u>Heat to Wall</u>	<u>Heat to Exit</u>	<u>Calorimetric Efficiency</u>
50 torr	300	313	85	115	198	63%
30 torr	146	131	41	56	75	58%

Appendix 1

Magnetic Field Structure

It is the intention in the design of the magnetic accelerating structure to provide a magnetic field from a set of coils carrying alternating currents of suitably chosen phase that translates in the acceleration direction. The following derivation is intended to show what may be attained in the way of accelerating and focusing forces in plane and cylindrical geometry. The equations must be considerably simplified to make them tractable, but it is hoped that it will provide a useful first approximation to the true state of affairs.

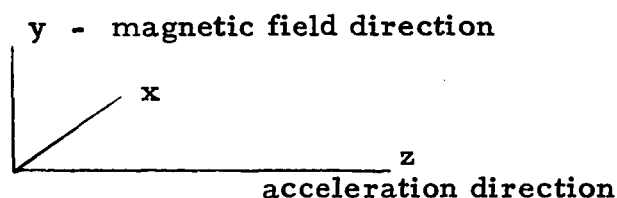
The first assumption that will be made is that the induced currents in the plasma are sufficiently small compared to the field producing currents in the drive coils that they may be neglected. Field penetration measurements show that this simplification should not be grossly misleading. Secondly, since the dimensions of the apparatus are very small compared to the 480 kc radio wave length, displacement currents are of no consequence.

Therefore outside the driving coils we have $\nabla \times \underline{B} = 0$ and $\nabla \cdot \underline{B} = 0$. Since $\nabla \times \underline{B} = 0$, there exists a scalar potential from which $\underline{B}$ may be derived. Then:

$$\underline{B} = \text{grad } \Omega$$

$$\nabla^2 \Omega = 0 \quad (\text{Laplace's equation})$$

Let us first consider the transverse geometry case



with a magnetic field normal to the median plane, of infinite extent in the x direction and translating in the z direction at a velocity v . We desire that

$B_y = B_0 \cos 2\pi (x - vt)/\lambda$, $B_x = 0$, $B_z = 0$ in the median plane. This suffices to determine the field. Separating Laplace's equation:

$$\begin{aligned}\Omega &= YZ & \nabla^2 \Omega &= Y''Z + YZ'' \\ \frac{\nabla^2 \Omega}{\Omega} &= \frac{Y''}{Y} + \frac{Z''}{Z} = 0 \\ \frac{Y''}{Y} &= \frac{Z''}{Z} = -\Gamma^2\end{aligned}$$

Γ^2 being the familiar separation constant. Γ we will choose real, so that the z dependence of the solution will be trigonometric and the y dependence hyperbolic.

The z equation is:

$$Z'' + \Gamma^2 Z = 0.$$

The general solution is $Z = Ce^{i\Gamma z} + De^{-i\Gamma z}$
Similarly the solution of the y equation is

$$Y = Ee^{+\Gamma y} + Fe^{-\Gamma y}$$

It is apparent that the choice $\Gamma^2 = 4\pi^2/\lambda^2$ will give the correct field variation along the z direction in the median plane. We wish:

$$Z = Ce^{2\pi i z/\lambda} + D e^{-2\pi i z/\lambda} = \cos \frac{2\pi}{\lambda} (z - vt)$$

to get the correct z and time dependence. This is accomplished

when:

$$2C = e^{2\pi i v t / \lambda} ; 2D = e^{-2\pi i v t / \lambda}$$

$$\Omega = (E e^{2\pi y / \lambda} + F e^{-2\pi y / \lambda}) \cos 2\pi(z - vt) / \lambda$$

$$B_y = \frac{\partial \Omega}{\partial y} = \left(\frac{2\pi}{\lambda} E e^{2\pi y / \lambda} - \frac{2\pi}{\lambda} F e^{-2\pi y / \lambda} \right) \cos 2\pi(z - vt) / \lambda$$

Since the desired solution for B is symmetric about the median plane

$$B_y(y) = B_y(-y), \quad F = -E, \quad E = B_0 \lambda / 4\pi$$

so our final results for the potential and the y and z components of the field are:

$$\Omega = (B_0 / 2\pi) \sinh(2\pi y / \lambda) \cos 2\pi(z - vt) / \lambda$$

$$B_x = 0$$

$$B_y = B_0 \cosh(2\pi y / \lambda) \cos 2\pi(z - vt) / \lambda$$

$$B_z = -B_0 \sinh(2\pi y / \lambda) \sin 2\pi(z - vt) / \lambda$$

It will be shown that the induced current J in the plasma is in the x direction. The forces on the plasma are given by $\underline{F} = \underline{J} \times \underline{B}$ or

$$F_x = 0$$

$$F_y = -J_x B_z$$

$$F_z = J_x B_y \quad (J_y = 0, J_z = 0)$$

The currents are most easily found in a coordinate system moving with the field. The force on a charge q is $\underline{F} = q(\underline{E} + (\underline{v}_p - \underline{v}) \times \underline{B})$ where

$\underline{v}_p$ is the plasma velocity in the laboratory coordinate system. Since $\underline{B}$ is constant in the moving coordinate system, $\underline{E} = 0$, and the E.M.F. $(\underline{v}_p - \underline{v}) \times \underline{B}$ drives the currents. This vector is in the x direction, and we have thus:

$$\underline{J}_x = (-v_p + v_z) B_y, \text{ and defining } (\underline{v} - \underline{v}_p) = \underline{v}_s,$$

the slip velocity and writing the result in the laboratory coordinate system:

$$J_x = \sigma v_s B_0 \cosh 2\pi y/\lambda \cos 2\pi(z - vt)/\lambda$$

The force density $\underline{F} = \underline{J} \times \underline{B}$

$$\begin{aligned} F_x &= 0 \\ F_y &= \frac{\sigma v_s}{4} B_0^2 \sinh 4\pi y/\lambda \sin 4\pi(z - vt)/\lambda \\ F_z &= \sigma v_s B_0^2 \cosh^2 2\pi y/\lambda \cos^2 2\pi(z - vt)/\lambda \end{aligned}$$

We have the interesting result that the accelerating force is always in the forward direction, whereas the force perpendicular to the direction of acceleration oscillates toward and away from the axis at twice the relative frequency. This latter force is zero at the median plane, and grows rapidly away from the median plane.

In a very similar way the field structure and accelerating forces may be solved for a solenoidal geometry. Laplace's equation is now solved in cylindrical coordinates, yielding solutions that are the product of trigonometric functions of $z - vt$ and Bessel functions of r . Writing:

$$\begin{aligned} \underline{B} &= \text{grad } \Omega \\ \nabla^2 \Omega &= 0 \\ \Omega &= R(r) Z(z) \end{aligned}$$

The separated equations are:

$$\frac{d^2 Z}{dz^2} + \mu^2 Z = 0$$

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} - \mu^2 R = 0$$

The solutions are:

$$\Omega = \frac{\lambda B_0}{2\pi} J_0(2\pi r/\lambda) \sin 2\pi(z-vt)/\lambda$$

$$B_z = B_0 J_0(2\pi r/\lambda) \cos 2\pi(z-vt)/\lambda$$

$$B_r = -i B_0 J_1(2\pi r/\lambda) \sin 2\pi(z-vt)/\lambda$$

In a way similar to the transverse magnetic field as the induced electromotive force of interest, E_θ , can be found to be:

$$E_\theta = v_s B_0 i J_1(2\pi r/\lambda) \sin 2\pi(z-vt)/\lambda$$

From this and the magnetic field, the force density is:

$$F_r = \frac{\sigma v_s}{2} B_0^2 i J_0(2\pi r/\lambda) J_1(2\pi r/\lambda) \sin 4\pi(z-vt)/\lambda$$

$$F_\theta = 0$$

$$F_z = \sigma v_s B_0^2 [-i J_1(2\pi r/\lambda)]^2 \sin^2 2\pi(z-vt)/\lambda$$

We see that again the accelerating force is always downstream oscillating between zero and peak at double the relative frequency, and that the radial force is alternately inward and outward at twice the relative frequency. One difference lies in that the downstream force is zero on the axis, and increases approximately linearly with radius. The radial force is also zero on the axis and goes approximately as the square of the radius.

There is another difference between the transverse and the cylindrical situation. The relative phase between the accelerating force and the radial or transverse force is different in the two cases. This might be invoked to explain the somewhat better isolation of the plasma

from the wall in the cylindrical case. If the argument of the $\sin^2$ term is between $\pi/2$ and π , or $3\pi/2$ and 2π , the accelerating force increases as the plasma slips back on the magnetic field. One would then expect that these quadrants would be regions of stable acceleration, and therefore preferred by the plasma in conditions for good acceleration. For these quadrants the radial force is to the axis. In the transverse case the accelerating force is proportional to $\cos^2 2\pi (z - vt)/\lambda$ and the first and third quadrants are those of "stable" acceleration, and the y force, $+ \sin 4\pi (z - vt)/\lambda$ is away from the median plane in these quadrants.

In this derivation of the forces there has been no restriction to a constant slip velocity.

APPENDIX 2

The Acceleration Process

The discussion of the magnetic field structure has given an approximate expression for the force density as a function of time and of position in the accelerating region, the conductivity of the plasma, and the strength of the moving magnetic field.

It would be desirable to analyze the accelerative process in more detail in order to understand the impulse performance, and the strength of the wall interaction.

The first step is to estimate the physical state of the plasma at the exit of the accelerator. We do this for shot T-97. The basic parameters and the results of the diagnostics for this shot are:

Specific impulse	5600 seconds
Phase velocity	54,000 meters/sec
Mass used	9.7×10^{-8} kg
Energy input	280 joules
Kinetic energy	148 joules
Internal energy	132 joules
Flow area	1.6×10^{-3} meters ²
Mass flow rate	.0467 kilograms/meter ² sec
Exit mass density	8.53×10^{-7} kilograms/meter ³
Exit molar density	2.13×10^{-4} gram mols/meter ³
Exit number density	1.28×10^{20} atoms/meter ³
Total atoms	1.46×10^{19} atoms
Internal energy/atom	56.5 ev/atom

Internal energy/kg	1.36×10^9 joules/kg
Entrance velocity	883 meters/sec

To evaluate the force on the plasma one needs to know the slip velocity and the electrical conductivity. The slip velocity will appear as a result of an integration of the equation of motion. The electrical conductivity will depend upon the energy deposited in the plasma during the course of the acceleration. We will show that the conductivity will have, for practical purposes, at most two values, depending upon whether the energy deposited is enough for single or for double ionization of the helium propellant gas.

It will turn out, for the regimes of interest in the accelerator, that the conductivity is controlled by scattering of the drifting electrons by the ions. As the ionization increases the number of scattering centers increases counterbalancing the effect of the increased number of electrons for carrying current. Spitzer* gives for the resistivity of an ionized gas

$$N = \frac{6.53 \times 10^3}{T^{3/2}} \ln \Lambda \text{ ohm} - \text{cm}$$

where Λ is proportional to the Debye length. $\ln \Lambda$ varies quite slowly with temperature and electron density. Temperature of the plasma is the most important factor controlling the resistivity of the plasma.

From shot T-97 we have found an internal energy of 56.5 ev/atom, which is sufficient to singly ionize all the gas, and doubly ionize an appreciable fraction of it. A relation between the fractional ionization and the energy input was computed in the following manner from the assumption of thermodynamic equilibrium and the validity of the Saha equation:

$$\log_{10} (p_+ p_+ / p_n) = -6.180 - 5037 \Delta U_+^+ / T + \log_{10} g_+ / g_n + \frac{5}{2} \log_{10} T$$

*Physics of fully ionized gases, Lyman Spitzer, Jr., Interscience, New York, 1956

$$\log_{10}(p_{++}p_-^2/p_n) = -12.360 - 5037\Delta U_{+}^{++}/T + \log_{10} g_{++}/g_n + 5 \log_{10} T$$

Where p_+ , p_- , p_n , p_{++} are the pressures in atmospheres of positive ions, electrons, neutrals, and doubly positive ions respectively, g_+ , g_{++} , and g_n are the statistical weights of the respective species, ΔU_{+}^{++} the ionization potential in electron volts, and T is in degrees Kelvin. These equations were transformed to exhibit number density, and fractional ionization, and the ionization was computed as a function of temperature. From the ionization and temperature the energy investment in ionization, and in kinetic energy of the free particles is quickly evaluated, and compared with the internal energy available. Since most of the energy goes into ionization, and only 10 to 20% into heat motion of the particles, the calculation converged rapidly. The ionization varies extremely rapidly with the temperature, so that the temperature effectively remains constant at 15,000°K as single ionization proceeds, then increased abruptly to 35,000°K as double ionization sets in.

At low ionization collisions with neutral atoms begins to become important. In this region the conductivity is proportional to the fractional ionization. The resistivity was evaluated in this region by using the published data on the drift velocity of electrons in helium at room temperature, and multiplying by $(T/300)^{1/2}$ to obtain an estimate of the resistivity at 15,000°K. This result was added to the resistivity estimate from the fully ionized theory to get the total resistivity. The result is plotted in Figure 13. The calculation is reproduced in Table 1.

Table 1

% Ionized	$\ln A$	η_i (ohm-cm) ionized theory	η_n (ohm-cm) neutral theory	η , Total (ohm-cm)	σ (mho/cm)
0.1	10.5	.04	2.11	2.15	.465
1.0	9.44	.034	.211	.245	4.08
5.0	8.73	.031	.0424	.074	13.5
10.0	8.05	.029	.0106	.040	25.2

The Starting of the Ionization

As the discharge starts in the gas the energy input is proportional to the conductivity, and so the conductivity will grow exponentially with time.

$$\frac{D\sigma}{Dt} = \frac{K}{f} E^2 \sigma \quad K = 4.87 \times 10^{-5} \frac{\text{mho kg}}{\text{meter joule}}$$

$$E^2 = \frac{v^2 B_o^2}{Z} = 3 \times 10^6 \text{ volt}^2/\text{meter}^2$$

The flow is introduced to the tube at a velocity of 883 m/sec^2 and a density of $5.3 \times 10^{-5} \text{ kg/meter}^3$. Inserting these values the equation becomes:

$$\frac{D\sigma}{Dt} = 2.9 \times 10^6 \sigma, \quad \sigma = \sigma_o e^{2.9 \times 10^6 t}$$

so the conductivity can increase more than an order of magnitude in a microsecond. However the initial conductivity of the gas is zero, and it is not immediately clear how long it takes for the discharge to start.

Back diffusion of electrons and ions against the stream is a method for the maintenance of a steady discharge.

In a coordinate system moving with the introduced but unaccelerated gas,

$$\frac{\partial \sigma}{\partial t} = 2.9 \times 10^6 \sigma + \Gamma \frac{\partial^2 \sigma}{\partial z^2}$$

where Γ is the ambipolar diffusion constant. In the laboratory system the equation becomes:

$$\frac{\partial \sigma}{\partial t} + u \frac{\partial \sigma}{\partial z} = 2.9 \times 10^6 \sigma + \Gamma \frac{\partial^2 \sigma}{\partial z^2}$$

where u is the entrance velocity of the gas. We desire a steady state solution, so dropping $\frac{\partial \sigma}{\partial t}$, we may write:

$$\frac{d^2 \sigma}{dz^2} - \frac{u}{\Gamma} \frac{d\sigma}{dz} + \frac{2.9 \times 10^6 \sigma}{\Gamma} = 0$$

The solution is:

$$= e^{\alpha z} (A e^{+\beta z} + B e^{-\beta z})$$

where α is $u/2\Gamma$, and β is $u(1 - 1.16 \times 10^7 \Gamma/u^2)/2$

The ambipolar diffusion constant for He is given as $540 \text{ cm}^2/\text{sec}$ at room temperature and 1 torr, and using $15,000^\circ \text{K}$ as the temperature we compute:

$$\begin{aligned} \Gamma &= 540 \text{ torr cm}^2/\text{sec} \times 10^{-4} \frac{\text{meter}^2}{\text{cm}^2} \times \frac{(50)^{1/2}}{8.19 \times 10^{-4} \text{ torr}} \\ &= 468 \text{ meter}^2/\text{sec} \end{aligned}$$

and obtain for the solution:

$$\sigma = e^{.942z} (A \cos 78.6z + B \sin 78.6z)$$

We need to match this solution to that which obtains in the region of constant conductivity. In this region we write:

$$u \frac{d\sigma}{dz} = 2.9 \times 10^6 \sigma_t + \Gamma \frac{d^2\sigma}{dz^2}$$

where σ is now no longer the electrical conductivity, but is proportional to the ionization, and would be the conductivity if electron ion encounters were of no consequence. σ_t is the constant conductivity in this second region. At the junction $\sigma = \sigma_t$, $\frac{d\sigma}{dz} = 2.9 \times 10^6 / u = 3280 \sigma_t$

$$\frac{1}{\sigma} \frac{d\sigma}{dz} = 3280$$

Applying the boundary condition that $\sigma = 0$ at entrance, and $\frac{1}{\sigma} \frac{d\sigma}{dz} = 3200$

at the transition point,

$$\frac{1}{\sigma} \frac{d\sigma}{dz} = \frac{78.6 \cos 78.6z + .942 \sin 78.6z}{\sin 78.6z} = 78.6 \cot 78.6z + .942$$

$$\cot 78.6z = \frac{3279}{78.6} = 41.7$$

$$z_t = 3.04 \times 10^{-4} \text{ meters}$$

Thus the initiation region is extremely small.

The choice of the boundary condition $\sigma(0) = 0$ requires that the current of electrons and ions diffusing to the left be annulled by recombination on the back end of the propulsion tube. If it is assumed that this face reflects this diffusion current, it can be shown that there is no growth region at all, and the gas begins with essentially the full

constant conductivity of the singly ionized region.

For practical purposes the conductivity may be taken as constant during the acceleration process at a value of 3000 - 3500 mho/meter until about 30 electron volts/atom of heat energy have been absorbed by the plasma, then it rises to about 14,000 mho/meter.

The Velocity Field.

In the section on magnetic field structure, we obtained the expression

$$F = (v - w) \sigma B_o^2 \cos 2\pi (z - vt)/\lambda$$

for the force density on the plasma. At a slip velocity of 1/2 the phase velocity for the plasma conditions of T-97 we calculate:

$$F = (27,400 \text{ m/sec}) (3000 \text{ mho/meter}) (2.04 \times 10^{-3} \text{ webers/meter}^2) (1/2) = 84,000 \text{ newtons/meter}^3$$

FV, an estimate of the total force, = 15.8 newtons which is 3 to 4 times the actual back reaction observed, but shows that the estimated body force is of the right order of magnitude. It does show that a conductivity 1/10 as great would not allow the observed acceleration.

Let us estimate the gas pressure at 50% ionization and double the exit density

$$p = n(1 + x)RT = (4.26 \times 10^{-10} \text{ mols/cc}) (1.5) (8.3 \times 10^7) (1.5 \times 10^4) = 800 \text{ dynes/cm}^2 = 80 \text{ newtons/m}^2$$

To compare this with the electromagnetic force density we must divide it by some characteristic length in order to get a force per unit volume,

which for the acceleration should be the length of the accelerator, and for divergence toward the wall should be the propulsion tube radius. For the length of the accelerator, 0.1 meter, we get 800 newtons/m³, small compared to the body force, and for the radius of the propulsion tube, .023 m, we get 3,500 newtons/m³.

At full single ionization the mean time to reach equilibrium between ions and electrons is 2×10^{-7} seconds at the exit density. At 10% ionization the time is 2×10^{-6} seconds. The lag of the ion temperature behind the electron temperature should have some effect, and cause the plasma pressure to be somewhat lower than estimated here.

To examine the field plasma interaction from another point of view, let us reproduce from Cowling* the fundamental equation for the behavior of a magnetic field in a fluid of constant conductivity:

$$\frac{\partial \underline{H}}{\partial t} = \text{curl}(\underline{v} \times \underline{H}) + \eta \nabla^2 \underline{H}$$

$$\eta = \frac{1}{4\pi \gamma} \quad \eta \text{ is a magnetic diffusivity.}$$

$$\text{and for our plasma } \eta = \frac{1}{4\pi (3 \times 10^{-8} \text{ abmho/cm})} = 2.66 \times 10^6 \text{ cm}^2/\text{sec}$$

The second term of the equation represents the tendency of the magnetic field to diffuse through the plasma, and the first term the tendency to transport the field as though it were frozen into the material. When both terms are significant the lines of force tend to be carried with the material, and at the same time they leak through it.

The condition for the transport to dominate the leak is that the magnetic Reynolds number

$$R_m = \frac{LV}{\eta}$$

be large compared with unity. L is a characteristic length, and V a

*Magnetohydrodynamics, T. G. Cowling, Interscience, New York, 1956

velocity comparable with velocities actually present. For our case

$$L \cong \frac{11.3}{2\pi} = 1.8 \text{ cm}, V \cong 2.7 \times 10^6 \text{ cm/sec}$$

$$R_m = 1.82$$

so the leak is very important, and the calculation of the fields in the plasma by neglecting the effects of the induced currents has fair validity. The externally applied fields quickly and fully establish themselves in the plasma.

Cowling gives for the time of decay of a motion of material across lines of force, when resistance is important, the formula

$$t = \rho / \mu H^2$$

$$t = \frac{8.53 \times 10^{-2} \text{ g/cm}^3}{3 \times 1 \times 2.04 \times 10^5} = 1.39 \times 10^{-7} \text{ sec}$$

At the entrance ρ is nearly 100 fold greater, so

$$t = 10^{-5} \text{ seconds.}$$

This means that except in the early stages of acceleration, the plasma motion is fairly quick to establish itself in synchronization with the field. At a field of 450 gauss, the magnetic pressure is $\frac{H^2}{8\pi} = \frac{2.04 \times 10^5}{8\pi}$
 $= 8200 \text{ dynes/cm}^2$, which is an order of magnitude greater than the gas pressure.

Estimated Electromagnetic Confinement

If the central pressure is 800 dynes/cm^2 , and the wall is at zero pressure, the slip velocity toward the wall will be

$$\frac{P/d}{\sigma H^2} = \frac{800 \text{ dynes/cm}^2 / 2.3 \text{ cm}}{3 \times 10^{-8} \text{ abmho/cm} \times 2.04 \times 10^5 \text{ gauss}^2} = 5.7 \times 10^4 \text{ cm/sec}$$

Since the pressure is small compared to the electromagnetic body forces, we may write the Eulerian equation of motion for the plasma with only the body force as:

$$\rho \left(\frac{\partial w}{\partial t} + w \frac{\partial w}{\partial z} \right) = (v - w) B_0^2 \cos^2 \frac{2\pi}{\lambda} (z - vt)$$

and the equation of continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho w)}{\partial z} = 0$$

We will derive an approximate solution of these equations by replacing $\cos^2 \frac{2\pi}{\lambda} (z - vt)$ by its average value of $1/2$, in the force term. We then focus our interest on a steady flow solution, drop the time derivatives, and integrate the equations. Using the density derived from the first approximation, we place it in the first equation, restore the time deviative of w , and the time dependence of the force, in order to obtain a second approximation. A third approximation that would involve solving the time dependent equation of continuity was attempted, but proved to be beyond our analytical powers.

$$\text{If } \frac{\partial \rho}{\partial t} = 0, \rho w = \rho_0 w_0 = m \text{ (mass rate/unit area)}$$

so the equation becomes

$$\frac{\rho_0 w_0}{w} \left(1 + w \frac{\partial w}{\partial z} \right) = \sigma B_0^2 (1/2),$$

$$\frac{dw}{dz} = \frac{\sigma B_0^2}{2m_0} (v - w)$$

$$\int_{w_0}^w \frac{dw}{v-w} = \int_0^z \frac{\sigma B_0^2}{2m_0} dz$$

$$\text{Ln} \frac{v-w}{v-w_0} = - \frac{\sigma B_0^2}{2m_0} z$$

$$w = v - (v - w_0) e^{-\sigma B_0^2 z / 2m_0}$$

This states that the velocity converges upon the phase velocity of the moving magnetic field. The characteristic length of the accelerator in which the slip decreases by a factor of e being $\frac{2m_0}{\sigma B_0^2} = .0152$ meters. The length of the accelerator being .113 meters, this gives 7.45 characteristic lengths as the acceleration distance, so we find that the plasma should come up to phase velocity very quickly, and coast through the latter part of the system. This characteristic length is proportional to the mass accelerated, and so one should expect a falling off in exit velocity as the propellant mass per shot is increased. We proceed to the next approximation.

Restoring the time dependence to the equation of motion we have:

$$\frac{1}{w} \frac{dw}{dt} + \frac{dw}{dz} = \frac{\sigma B_0^2}{2m_0} (v-w) \left(1 + \cos \frac{4\pi}{\lambda} (z - vt) \right)$$

Introducing new independent variables

$$z' = \frac{4\pi}{\lambda} z$$

$$t' = \frac{4\pi}{\lambda} vt$$

$$w' = \frac{w}{v}$$

The equation simplifies to:

$$\frac{1}{w'} \frac{dw'}{dt'} + \frac{dw'}{dz'} = \frac{\sigma B_0^2 \lambda}{8\pi} (1 - w') (1 + \cos (z' - t'))$$

Next we transform to a coordinate system moving with velocity v . This has the purpose of simplifying the force term,

$$\begin{aligned} \text{Put } z'' &= z' - t' & z' &= z'' + t'' \\ t'' &= t' & t' &= t'' \end{aligned}$$

and using the general relations for any function f

$$\frac{\partial f}{\partial z'} = \frac{\partial f}{\partial z''} \frac{\partial z''}{\partial z'} + \frac{\partial f}{\partial t''} \frac{\partial t''}{\partial z'} = \frac{\partial f}{\partial z''}$$

$$\frac{\partial f}{\partial t'} = \frac{\partial f}{\partial t''} \frac{\partial t''}{\partial t'} + \frac{\partial f}{\partial z''} \frac{\partial z''}{\partial t'} = \frac{\partial f}{\partial t''} - \frac{\partial f}{\partial z''}$$

we obtain:

$$\frac{\partial w'}{\partial t''} + (w - 1) \frac{\partial w'}{\partial z''} = \alpha(w) (1 - w) (1 + \cos z'')$$

where we have replaced $\frac{e B_0^2 \lambda}{8\pi m_0}$ by α .

This equation may be attacked by the method of Lagrange*. The first step is to solve the related system of ordinary equations

$$\frac{dt}{1} = \frac{dz}{w - 1} = \frac{dw}{\alpha(w) (1 - w) (1 + \cos z)}$$

The second pair may be integrated to give:

$$z + \sin z + \frac{1}{\alpha} \ln w = c_1$$

and $w = e^{\alpha(c_1 - z - \sin z)}$. Inserting this into the first pair we obtain a second relation by integration

$$t - \int \frac{dz}{e^{\alpha(c_1 - z - \sin z)} - 1} = c_2$$

*Differential Equations, Abraham Cohen, D.C. Heath & Co., 1933, pg 250 ff.

The general solution of the partial differential equation is then:

$$\Phi \left(t - \int \frac{dz}{e^{\alpha}(c_1 - z - \sin z) - 1}, z + \sin z + \frac{1}{\alpha} \ln w \right) = 0$$

Where Φ is an arbitrary function, which in principle may be solved for w as a function of z and t . The constant c_1 is a parameter in the term $\int \frac{dz}{e^{\alpha}(c_1 - z - \sin z) - 1}$ for the purposes of the indicated

integration, but then should be replaced by the quantity $z + \sin z + \frac{1}{\alpha} \ln w$ after the integration is completed. The solution may be verified by direct substitution. It remains to transform this to a form suitable for fitting the boundary conditions.

We may write the solution in the form:

$$t - \int_0^{z-t} \frac{d\theta}{\exp[\alpha(z-t + \sin(z-t) + \frac{1}{\alpha} \ln w - \theta - \sin \theta)] - 1} = 0$$

$$= G(z-t + \sin(z-t) + \frac{1}{\alpha} \ln w)$$

Where G is an arbitrary function of its argument, and we have transformed back to the laboratory coordinate system.

If the initial velocity w_0 at $z = 0$ is substituted in this equation, both sides become a function of t which can be solved to obtain the dependance of G on its argument. Let c_1 be the argument of G . We obtain

$$G(c_1) = t - \int_0^{-t} \frac{d\theta}{\exp[\alpha(c_1 - \theta - \sin \theta)] - 1} \quad (a)$$

$$\text{with } c_1 = -t - \sin t + \frac{1}{\alpha} \ln w_0 \quad (b)$$

Solving (b) for t we write $t = \gamma \left(\frac{1}{\alpha} \ln w_0 - c_1 \right)$ where the function γ is determined by the equation

$$\gamma(x + \sin x) = x$$

$$G(c_1) = \gamma \left(\frac{1}{\alpha} \ln w_0 - c_1 \right) - \int_0^{\gamma \left(\frac{1}{\alpha} \ln w_0 - c_1 \right)} \frac{d\theta}{\exp [\alpha (c_1 - \theta - \sin \theta)] - 1}$$

so finally we obtain for our solution

$$t - \gamma \left(\frac{1}{\alpha} \ln w_0 - c_1 \right) - \int_{\gamma \left(\frac{1}{\alpha} \ln w_0 - c_1 \right)}^{z - t} \frac{d\theta}{\exp [\alpha (c_1 - \theta - \sin \theta)] - 1} = 0$$

with $c_1 = (z - t) + \sin(z - t) + \frac{1}{\alpha} \ln w$

We will cast this result in a slightly more convenient form:

Putting $\phi = -\theta$

$$t - \gamma \left(\frac{1}{\alpha} \ln w_0 - c_1 \right) = \int_{\gamma}^{t - z} \frac{d\phi}{1 - e^{\alpha (c_1 + \phi + \sin \phi)}}$$

and denoting the starting phase by $\phi_0 = t_0$

$$\phi_0 + \sin \phi_0 = \frac{1}{\alpha} \ln w_0 - c_1, \quad c_1 = \frac{1}{\alpha} \ln w_0 - \phi_0 - \sin \phi_0$$

The solution appears in the form:

$$t - \phi_0 = \int_{\phi_0}^{t - z} \frac{d\phi}{1 - w_0 e^{\alpha (\phi + \sin \phi - \phi_0 - \sin \phi_0)}}$$

One recognizes that the integrand is $\frac{1}{1 - w_0}$ at the lower limit, and $\frac{1}{1 - w}$ at the upper limit. Each substitution of ϕ_0 in the solution gives a trajectory in the t, z plane that is followed by a particular fluid element. This may be shown in the following way. Differentiation of the solution yields:

$$dt = \frac{dt - dz}{1 - w_0 e^{\chi(t - z + \sin(t - z) - \phi_0 - \sin \phi_0)}}$$

$$-w dt = -dz \quad \frac{dz}{dt} = w \quad \text{which is the equation for the}$$

trajectory of a fluid element. A knowledge of the starting phase and the instantaneous phase determines the velocity from

$$c_1 = \frac{1}{\chi} \ln w_0 - \phi_0 - \sin \phi_0 = \frac{1}{\chi} \ln w - \phi - \sin \phi$$

$$\text{or } w = w_0 e^{\chi(\phi + \sin \phi) - \chi(\phi_0 + \sin \phi_0)}$$

The evaluation of the integral allows one to determine the values of t and z corresponding to the values of phase ϕ and of the velocity w attained.

The integration was done numerically for two values of starting phase, $\phi_0 = 0$, and $\phi_0 = -3.2$, corresponding to the force being a maximum and zero respectively at the start of the motion. The results are given in Table 2. A plot is made of z vs t derived from these two solutions, and plotted in Figure 14. The $\phi_0 = 0$ curve shows the velocity increasing slowly at the start, remaining steady for a while in the region $3 < t < 3.5$ corresponding to the vicinity of the zero of the force at $\phi = \pi$ and then rapidly accelerating as ϕ increases toward 2π . Essentially synchronous velocity is reached at $\phi = 2\pi$, and a z value of 1.5. From then on the plasma moves forward at synchronous velocity, and the phase remains constant. In the units of this discussion, the active length of the accelerator is 12 units long, so synchronous velocity is reached very early.

For the trajectory starting earlier at $\phi_0 = -3.2$, at the zero of the force function, the velocity remains small until the phase passes through zero. Then the force has built up and the velocity

accelerates to a value of 0.65 of synchronous during the progress of the phase to $+\pi$, where the force is again zero. The plasma waits for quite a while, for the phase is changing slowly with time, since the velocity is not too far from synchronous. Eventually the phase again becomes favorable for acceleration, and the velocity comes up to essential synchronism at a z value of 10, and a phase close to that of the $\phi_0 = 0$ solution. One observes some compression in the phase difference between these two trajectories. For the second trajectory nearly the whole length of the accelerator is used in reaching synchronous velocity.

The fluid velocity is available as function of the starting phase and the change of phase of the fluid element relative to the moving magnetic field. It is plotted as a function of $\Delta\phi$ for the two cases $\phi_0 = 0$ and $\phi_0 = -3.2$ in Figure 15, and as a function of the distance attained down the accelerator in Figure 16. On that graph is also shown the velocity as a function of distance if the modulated force is replaced by its average value.

The abscissa is expanded near the origin to present the details of the early acceleration. One may see that if α , which is a measure of the ratio of the strength of the accelerating force to the mass loading were much smaller than its actual value, the final velocity would fall significantly below synchronism.

The next problem is that of the motion of the plasma toward the walls. If the plasma reaches synchronous velocity, it is a simple matter to determine the phase of the plasma at output as a function of the starting phase. The results are plotted in Figure 17. One observes that the output phase is constant at 4.8 for a fairly wide range of input phase in the neighborhood of -3.2. Since we have already integrated the trajectory starting at -3.2 to obtain z and t as a function of phase, and since the phase bunching observed indicates that a significant portion of the plasma will follow this trajectory, we will determine the motion of the plasma in the y direction, transverse to

the direction of acceleration for that trajectory. With v being the velocity in the y direction, we write

$$\frac{1}{w} \frac{Dv}{Dt} = - \frac{\alpha y}{2} (1 - w) \sin (z - t)$$

where $\frac{D}{Dt}$ is the comoving derivative $\frac{d}{dt} + w \frac{d}{dz}$

For convenience in the numerical integration, this equation is written with ϕ as the independent variable, since it was used in the other integrations. The new equation is:

$$(1 - w) \frac{D^2 y}{D\phi^2} = - \frac{\alpha}{2} w y \sin \phi + w (1 + \cos \phi) \frac{Dy}{D\phi}$$

This equation is linear. It contains the assumption that the y velocity is small compared to the slip velocity, so that the $u \cdot B_z$ term in $\underline{v} \times \underline{B}$ does not contribute significantly to the E.M.F. that drives the induced current in the plasma.

With the above restriction in mind, the equation was integrated by Milnes' method, and the solution for starting values $y = 1$, $\frac{dy}{d\phi} = 0$ is presented in Figure 18. The climbing of y toward infinity at $\phi = 4.84$ is to be expected, since this limiting value of ϕ is reached as z and t reach infinity. The solution is plotted as a function of the more interesting variable z in Figure 19. One observes that at the start the plasma is moved very slightly outward, and then driven strongly toward the axis. At the time it reaches the axis it has a high velocity toward the axis, and the force is changing sign from focusing to defocusing. The equation predicts that the plasma would be driven through the median plane, and strongly outward on the opposite side. In reality we expect gas pressure forces, which have been so far neglected, to come into play, and reflect the motion about the median plane. It seems reasonable that the bounce should be elastic, and the true state of affairs obtained by continuing the solution from its image in the median plane. The solution shows that a

large portion of the plasma will be driven strongly into the wall before exiting from the accelerator. An inspection of the equation indicates that probably only a small range of starting phase will yield solutions that do not diverge strongly from the center during the course of the transit of the plasma down the tube.

This analysis has assumed that the velocity in the y direction is small compared to the slip velocity, so that it is correct to neglect the forces due to the motion in the y direction across the field lines. The inclusion of these forces will cause the ejection of the plasma toward the wall to be less vigorous than predicted. However, it cannot prevent the motion toward the wall, for if it did the y velocity would be sufficiently small that the hypothesis behind the equation would be fully valid.

If one examines the details of the acceleration to synchronous velocity and the motion toward the wall, one notices that if the accelerator were half as long the acceleration performance would not be too much degraded, but the amount of wall interaction might be greatly reduced. It may not be out of place to remark here that the accelerator studied in the first year's work was considerably shorter, and this difference may explain the absence of wall interactions in that case.

TABLE II A

<u>ϕ</u>	<u>w</u>	<u>t</u>	<u>z</u>
0.0	.01600	00000	0
.4	.02549	.40834	.00834
.8	.03917	.82157	.02157
1.2	.05629	1.24156	.04156
1.6	.07433	1.66956	.06956
2.0	.08904	2.10532	.10532
2.4	.09820	2.54689	.14689
2.8	.10171	2.99152	.19152
3.2	.10211	3.43697	.23697
3.6	.10308	3.88276	.28276
4.0	.10842	4.32981	.32981
4.4	.12236	4.78152	.38152
4.8	.15092	5.24405	.44405
5.2	.20422	5.72902	.52902
5.6	.30005	6.26138	.66138
6.0	.46755	6.90683	.90683
6.2	.59067	7.33175	1.13175
6.4	.74776	7.94303	1.54303
6.5	.84086	8.43600	1.93600
6.6	.94400	9.18335	2.58335
6.64	.98861	10.6769	4.0369

TABLE II B

<u>ϕ</u>	<u>w</u>	<u>t</u>	<u>z</u>
-2.8	.016062	.40651	.00651
-2.4	.016636	.81313	.01313
-1.6	.021980	1.62835	.02835
- .8	.041712	2.45309	.05309
.0	.10211	3.31058	.11058
.8	.24997	4.27380	.27380
1.2	.35922	4.84938	.44938
1.6	.47435	5.53778	.73778
2.0	.56823	6.37994	1.17994
2.4	.62673	7.38379	1.78379
2.8	.64912	8.49675	2.49675
3.2	.65168	9.64343	3.24343
3.6	.65782	10.79757	3.99757
4.4	.78092	13.51585	5.91585
4.6	.85721	14.6256	6.8256
4.7	.90604	15.4794	7.5794
4.8	.96316	17.1109	9.1109
4.84	.98875	18.986	10.982

CONCLUSIONS

It is possible to obtain strong coupling between a moving transverse magnetic field and a plasma of density 10^{16} particles/cm³. Exit velocities of the order of the phase velocities of the magnetic field could be achieved at an appropriate mass loading, corresponding to specific impulses of 6000 to 7000 seconds. Wall interactions were a serious problem that was not resolved in the course of the program.

It was possible to gain a fair understanding of the performance of the accelerator. The theory, which took account only of the $\vec{J} \times \vec{B}$ body forces on the plasma, gave a good explanation of the dependence of the specific impulse upon mass loading, and the relative independence of specific impulse on propellant choice; and a semiquantitative explanation of the strength of the wall interactions.

The problems that need to be solved before the engine could be considered for development for a mission are several. First, the wall interactions are excessive. It seems possible to reduce them by flattening the cross section of the accelerator and shortening it.

The second problem is that of the theoretical limit to the efficiency of 50%, which is due to the average slip velocity of the plasma relative to the magnetic field being 50%. One must have a variable phase velocity to improve this situation so that the average slip is held to perhaps 20%. This conflicts with the requirement that the accelerator be short for small wall interactions. This points to the potential desirability of a two or three stage engine, probably with modulation of the feed so the plasma enters at phases such that the electromagnetic forces are mostly focusing.

The third problem is that of the large internal energy investment in ionization. At present the 50% input energy that appears as internal energy goes almost entirely into ionization. Complete single and some double ionization is achieved. The reduction in the inelasticity of the interaction between the magnetic field and the plasma that could be gained by the use of variable phase velocity would reduce the ionization. It could be reduced to as low as 10% without seriously affecting the strength of the coupling. There is no need to pursue easily ionized propellants in this specific impulse region.

The fourth problem is that of the drive coil losses. Approximately 50% of the output power of the oscillator was lost in the drive coils. The frequency of operation, 480 kc, is in a region where litz wire techniques are quite helpful, and a reduction in the losses by a factor of three, and possible ten, may be expected by the use of litz wire for the drive coils.

Let us somewhat arbitrarily estimate ultimately attainable efficiencies at each stage of the acceleration process:

Power conditioning	95%
R. F. Power Generation	85%
Drive Coil Efficiency	90%
Acceleration Efficiency	80%
Transport Efficiency	90%
(100% Wall Loss)	

The combination of these efficiencies predicts a possible overall efficiency of 52%.

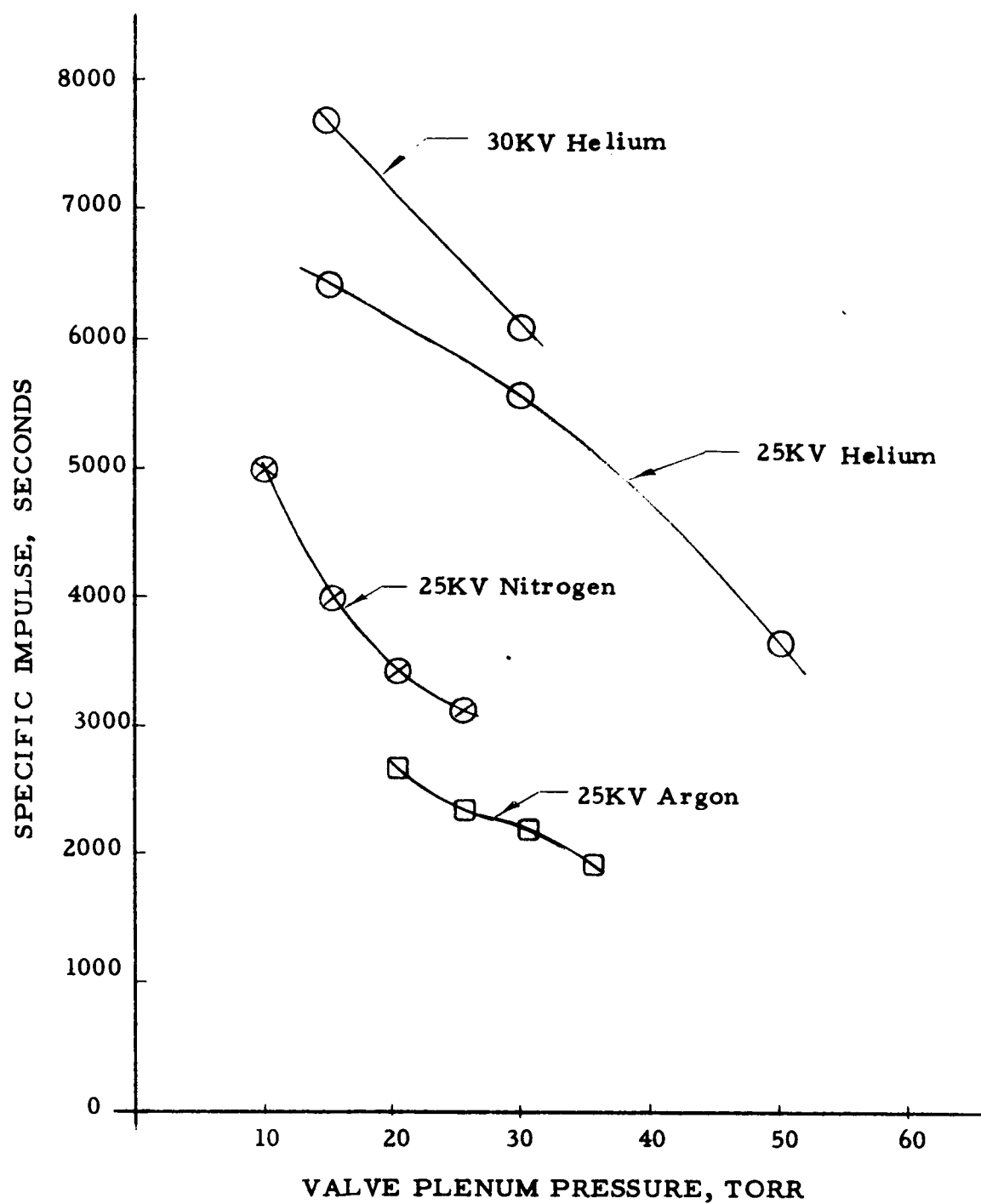


Figure 1. Specific impulse as a function of valve plenum pressure for various propellants and transverse magnetic field.

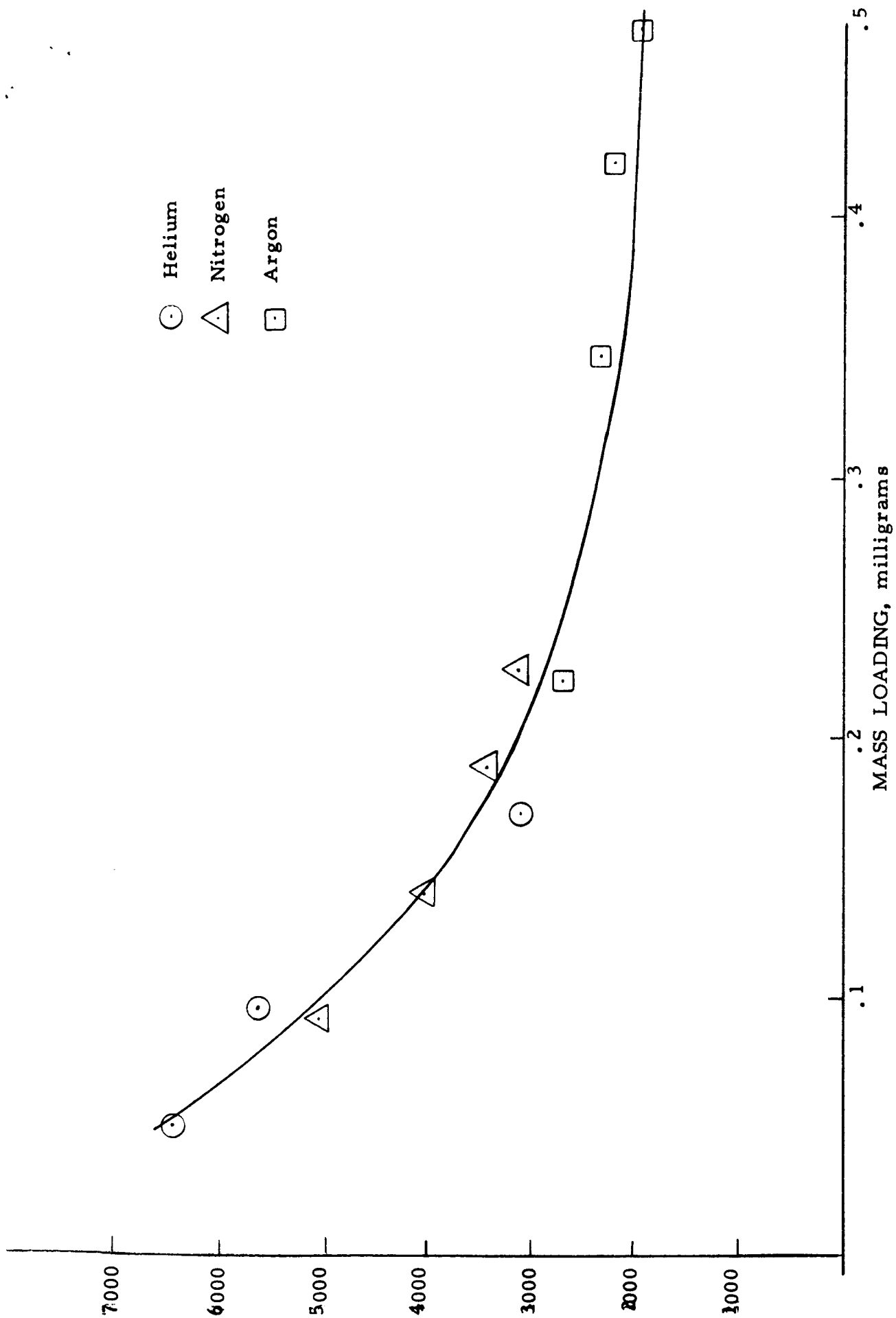


Figure 2. Specific impulse as a function of mass loading for various propellants and transverse magnetic field.

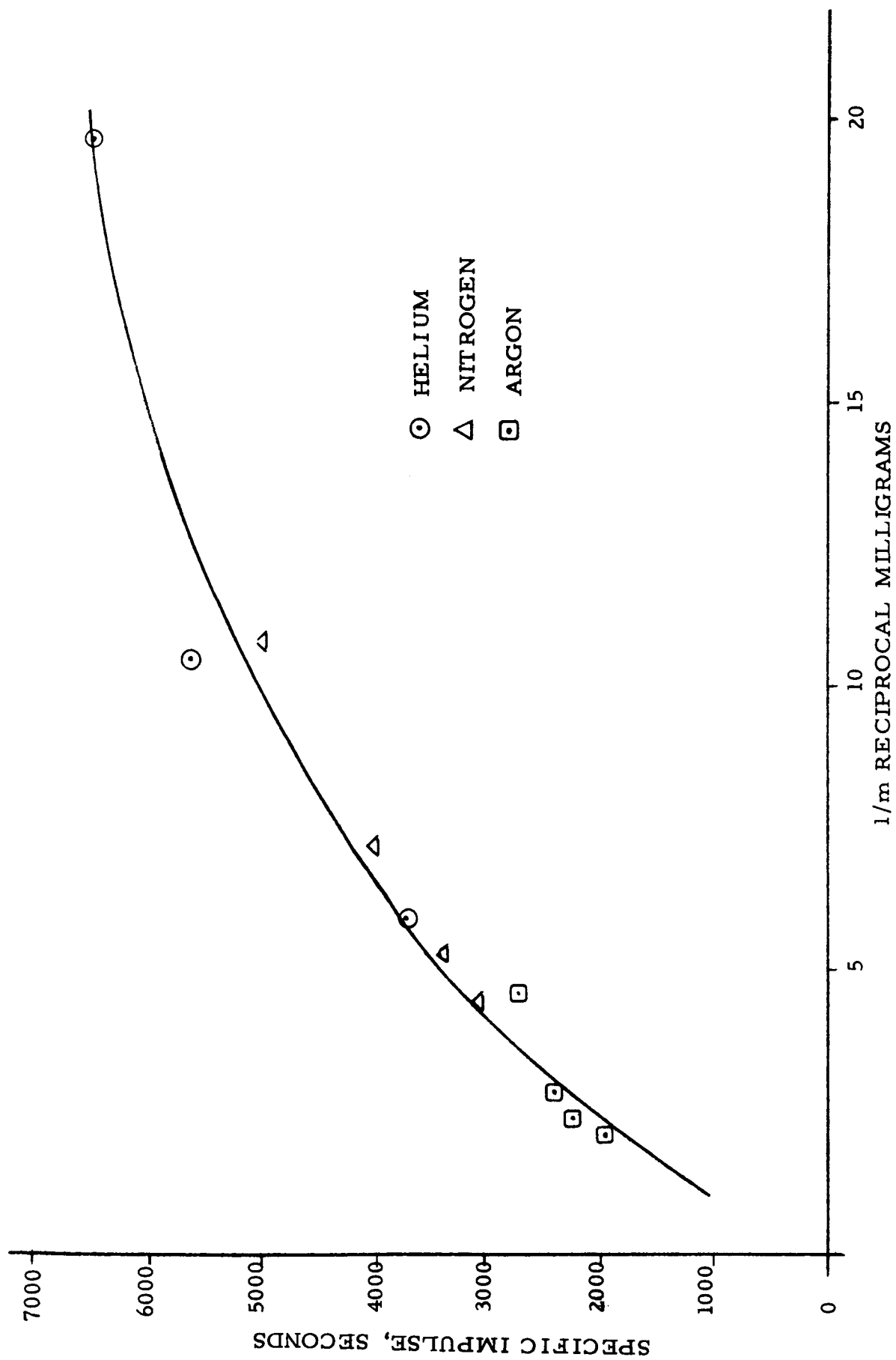


Figure 3. Specific impulse as a function of the reciprocal of the mass loading for various propellants and transverse magnetic field.

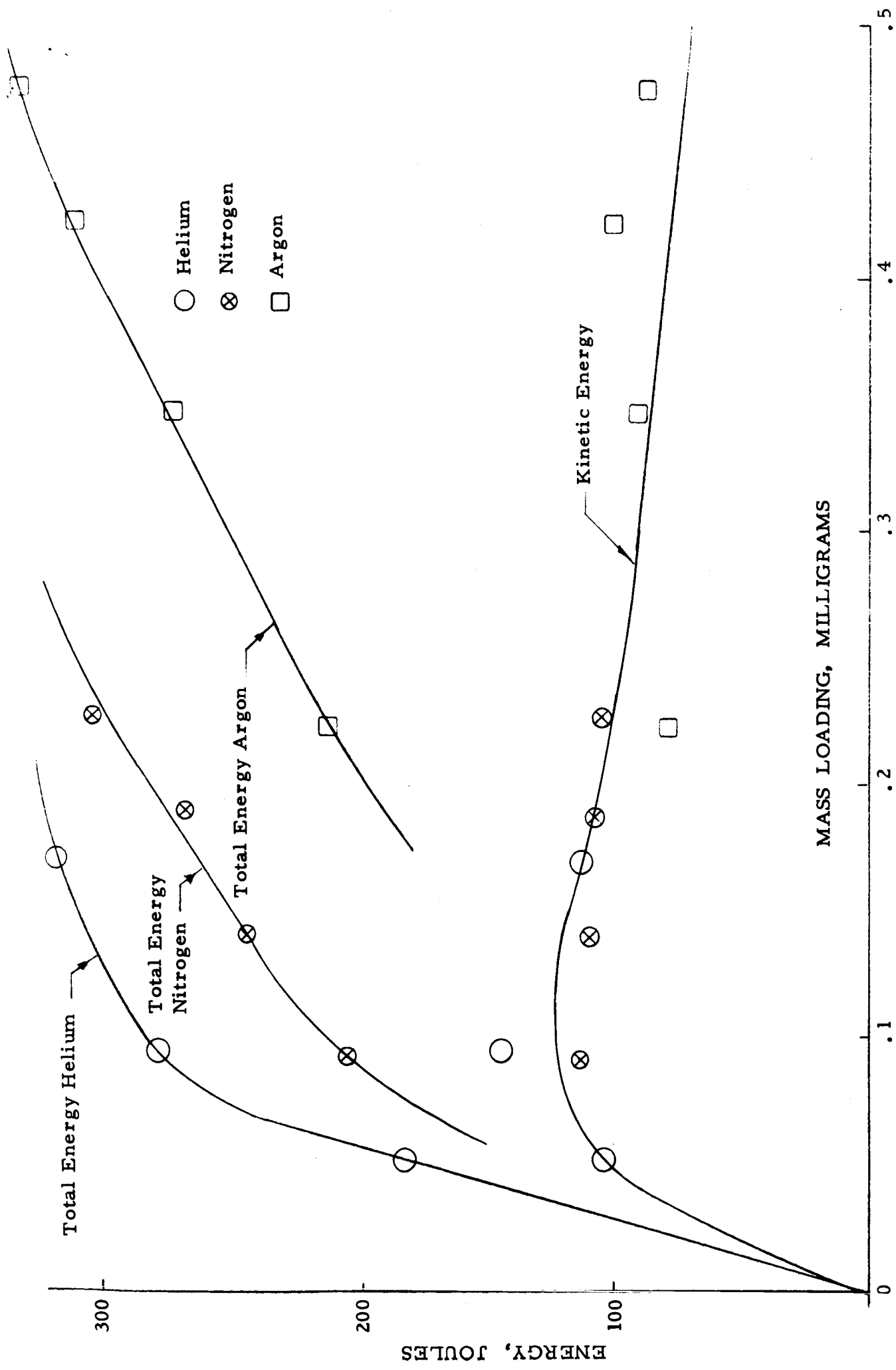


Figure 4. Total and kinetic energy as a function of mass loading for various propellants and transverse magnetic field.

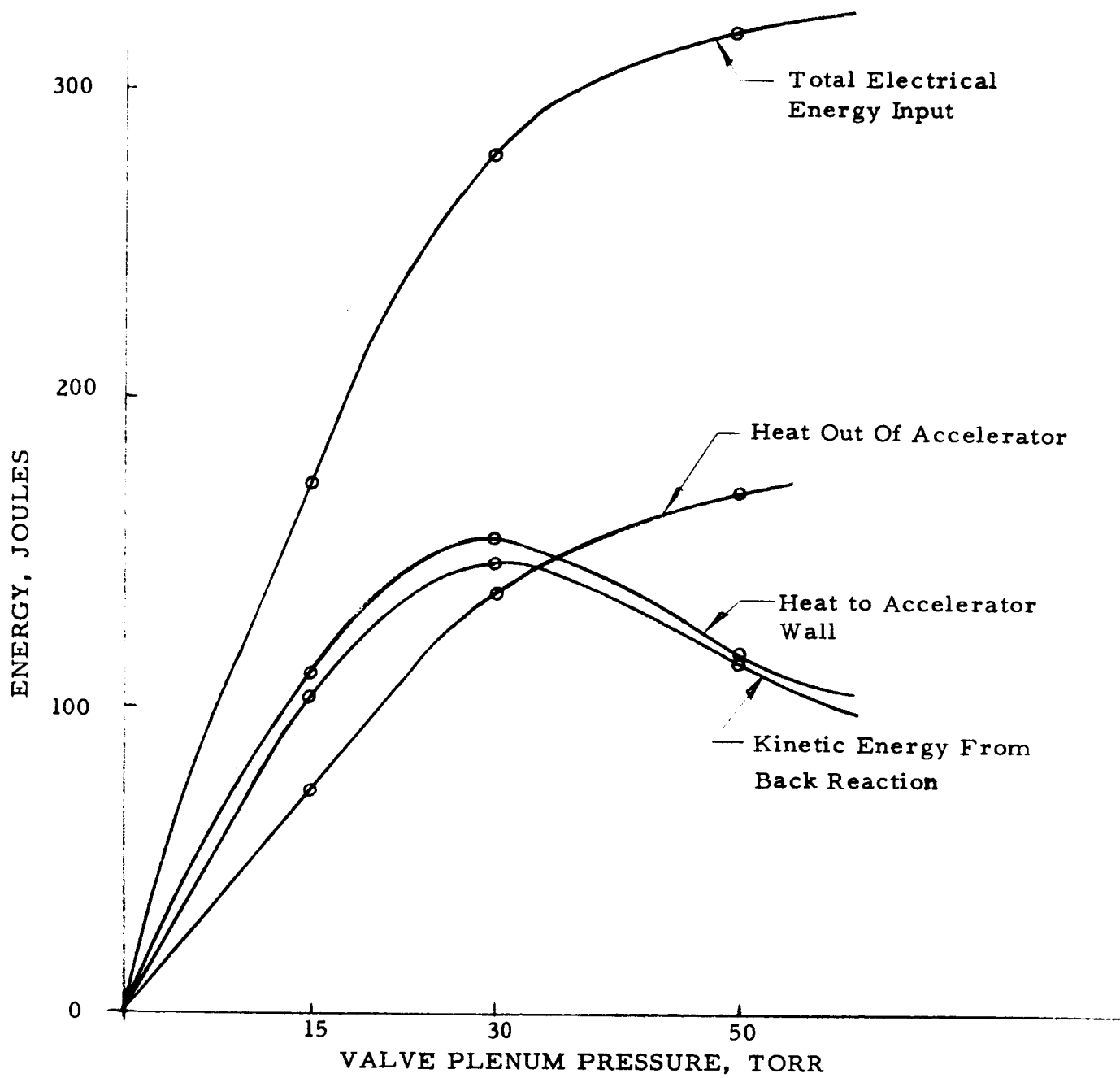


Figure 5. Calorimetric results for helium and transverse magnetic field.

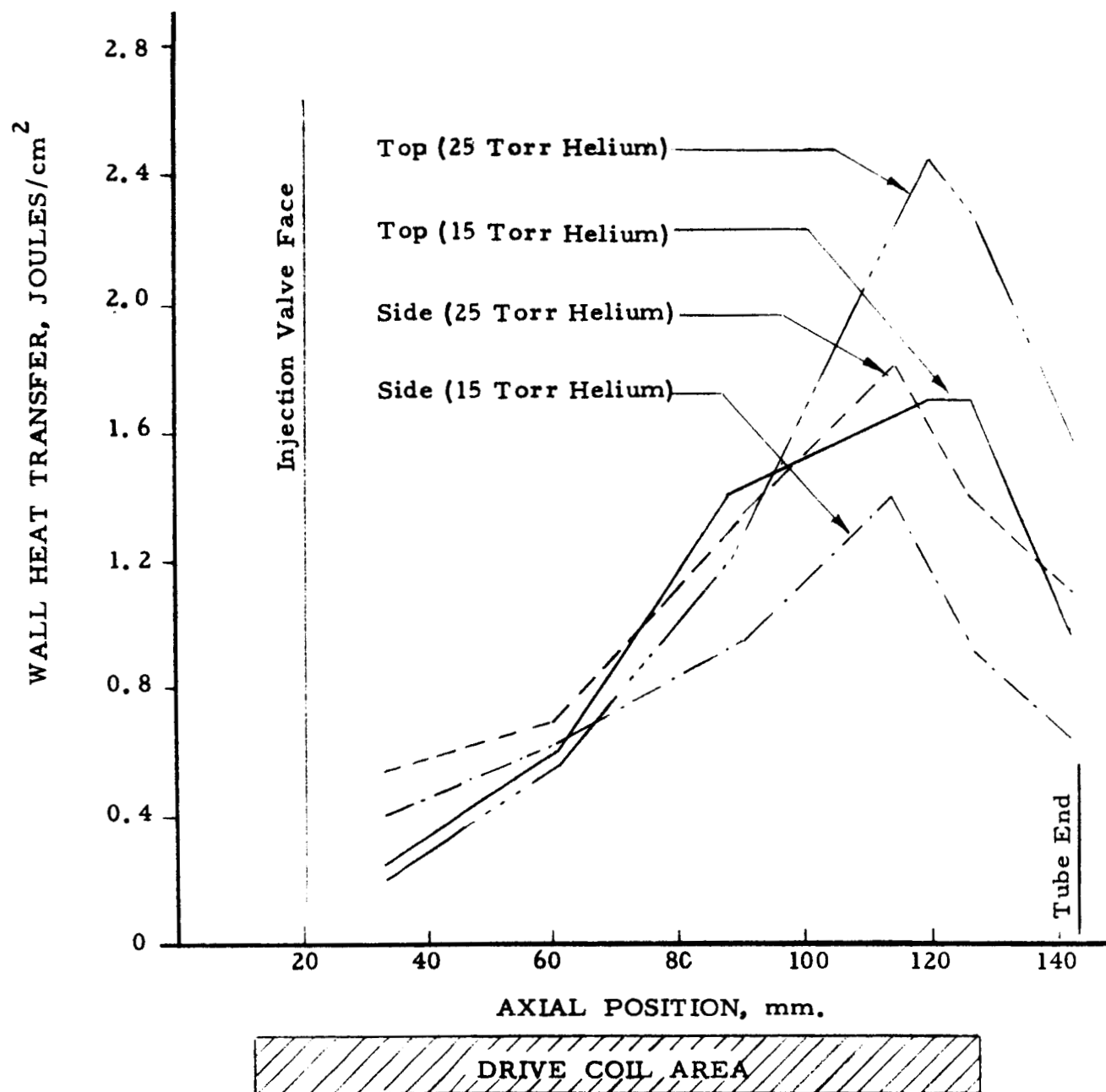


Figure 6. Distribution of heat transfer to propulsion tube walls for transverse magnetic field. Helium at 15 and 25 torr valve plenum pressure.

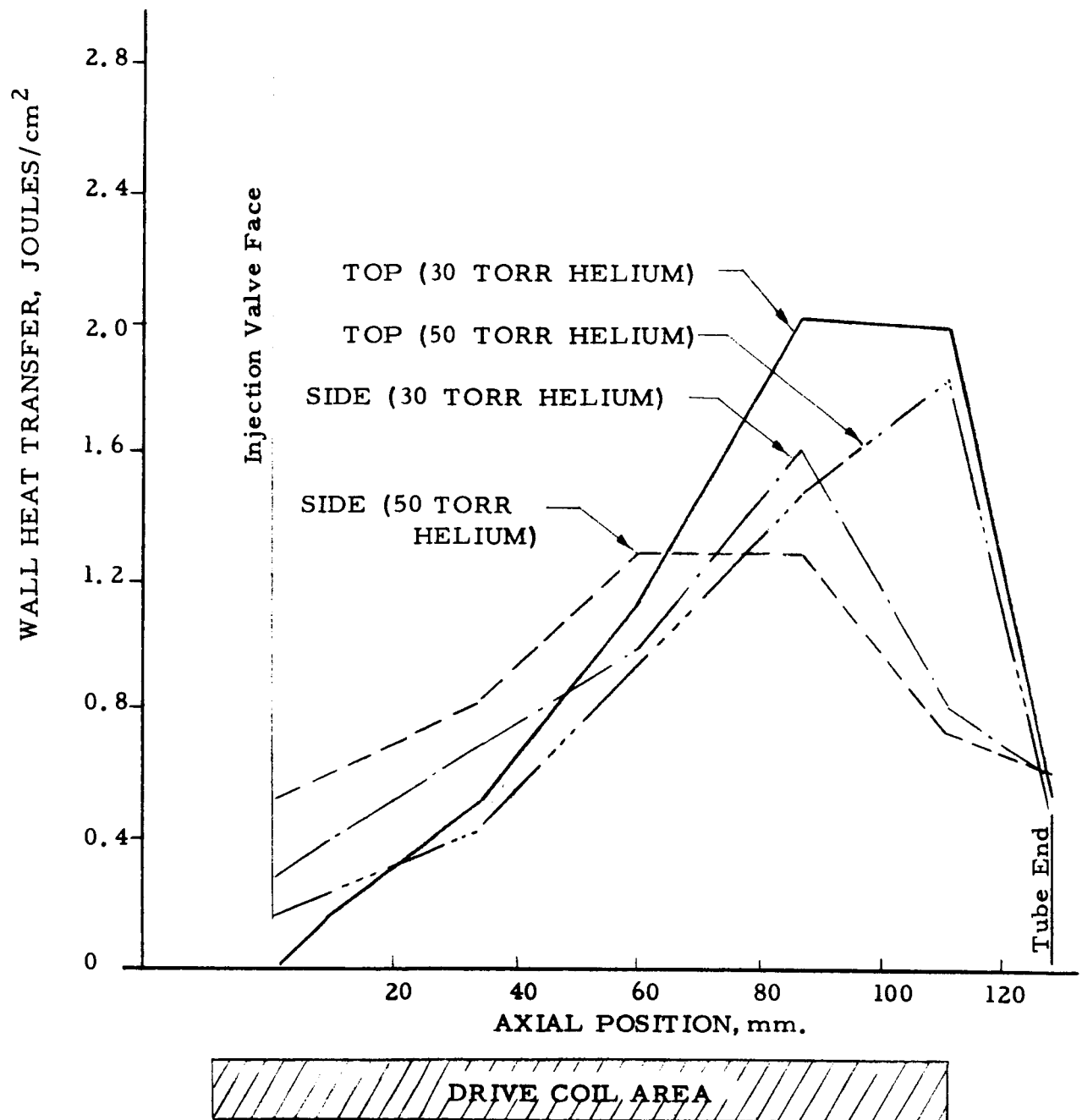


Figure 7. Distribution of heat transfer to propulsion tube walls for transverse magnetic field. Helium at 30 and 50 torr valve plenum pressure.

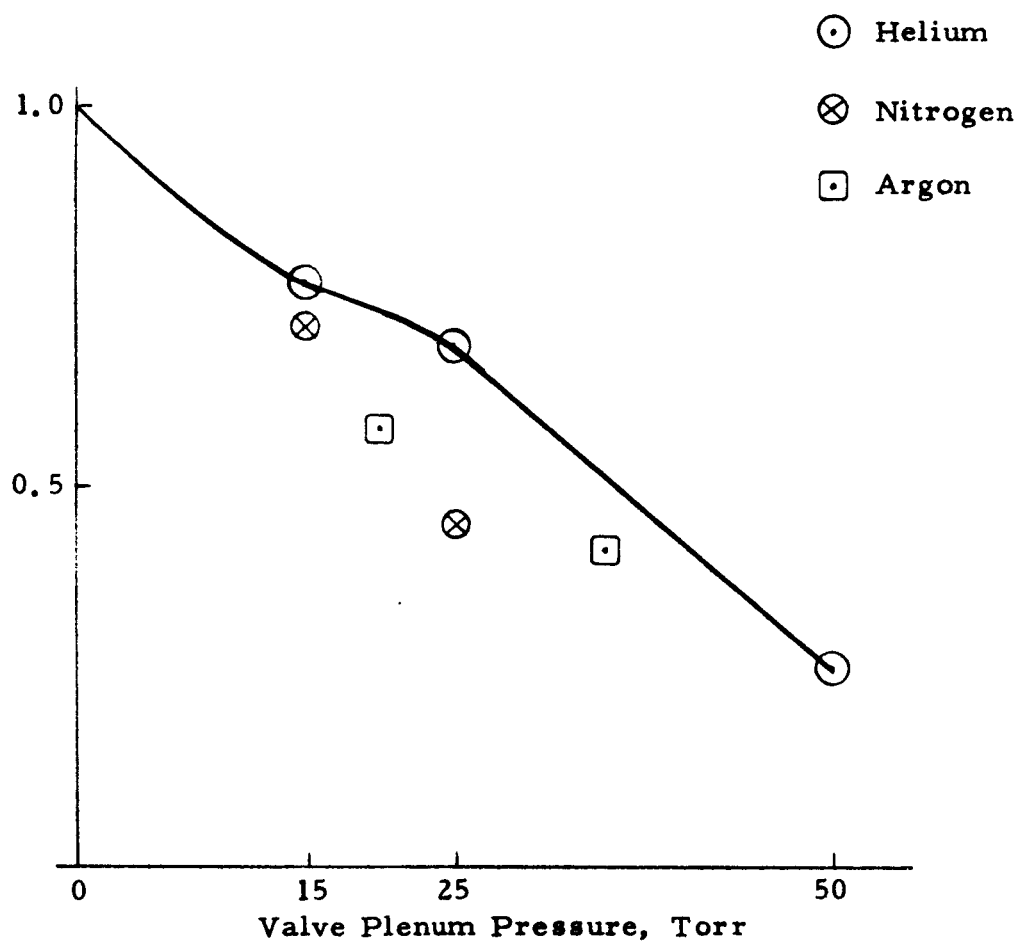


Figure 8. Penetration of transverse magnetic field into plasma.

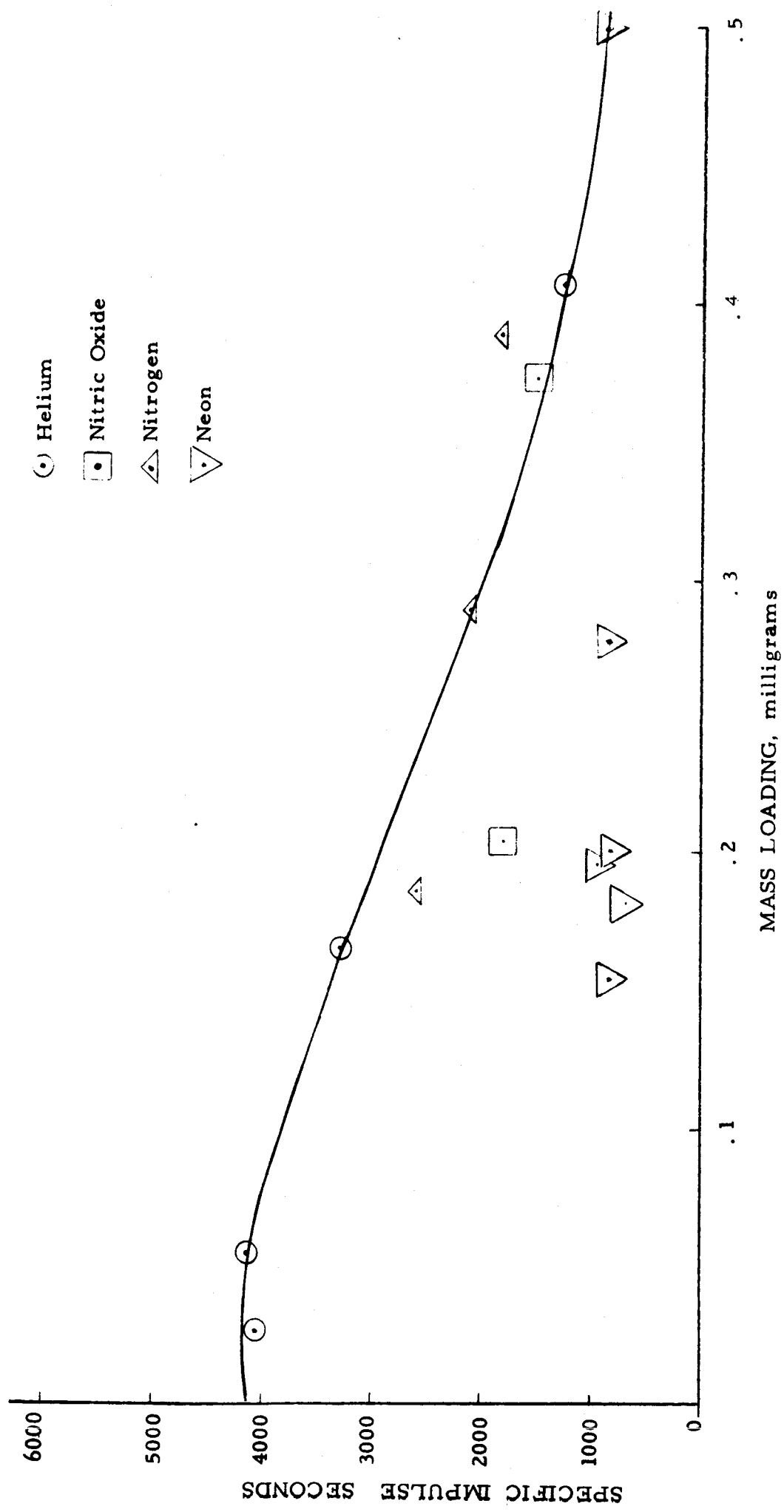


Figure 9. Specific impulse for various propellants and cylindrically symmetric magnetic field.

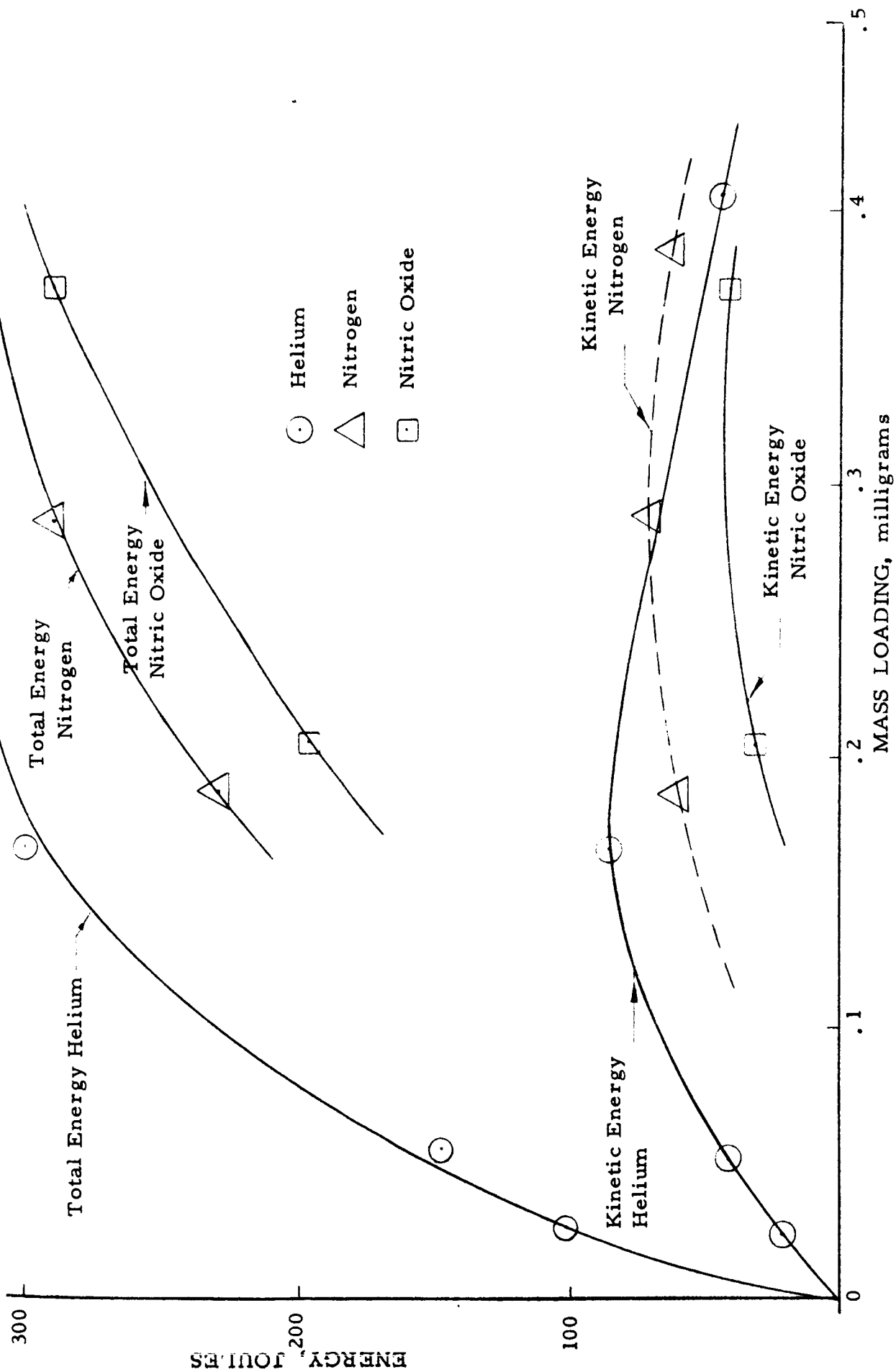


Figure 10. Kinetic energy and total energy as a function of mass loading for various propellants and cylindrically symmetric magnetic field.

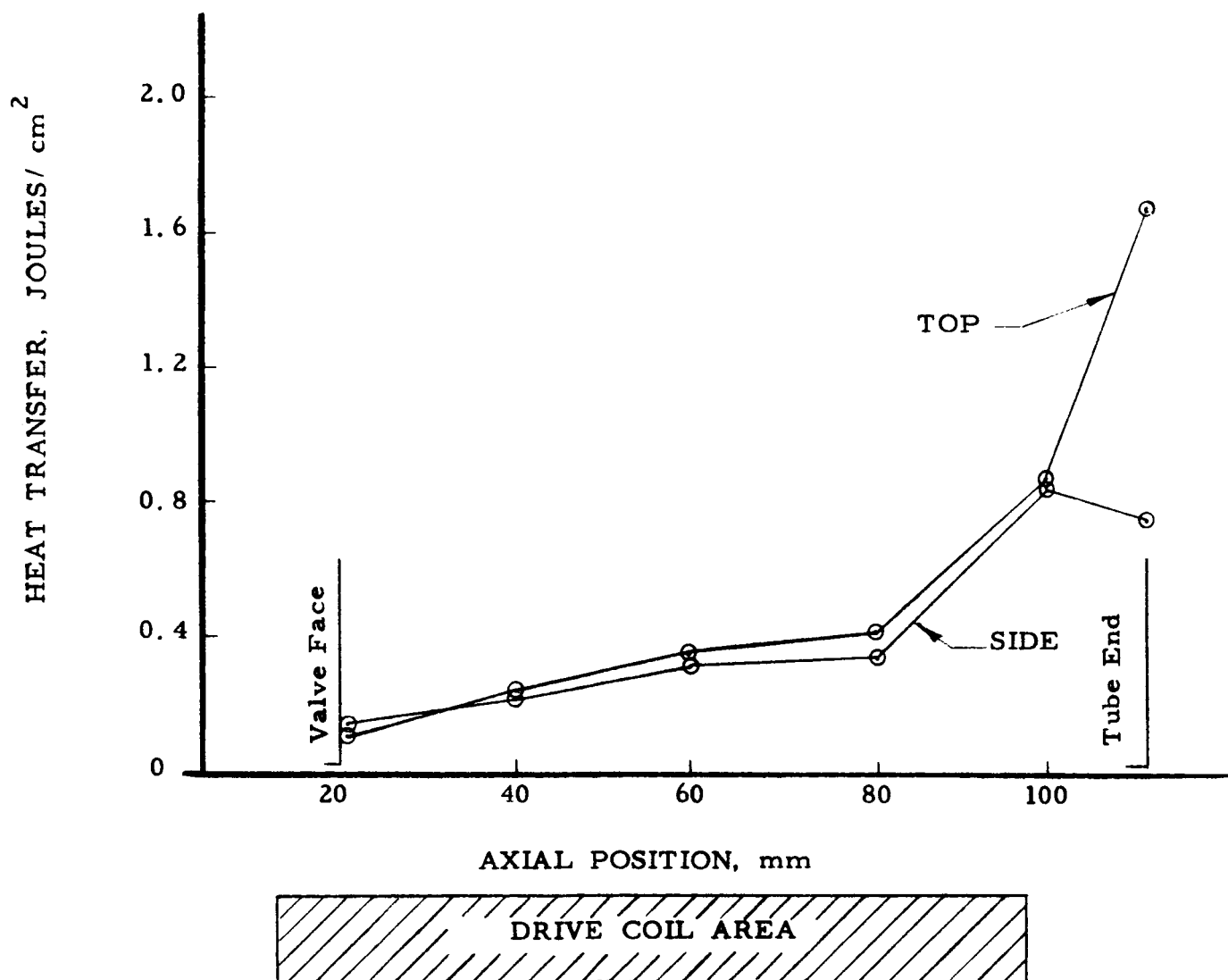


Figure 11. Distribution of heat loss to propulsion tube wall at 30 torr helium valve plenum pressure for cylindrically symmetric magnetic field.

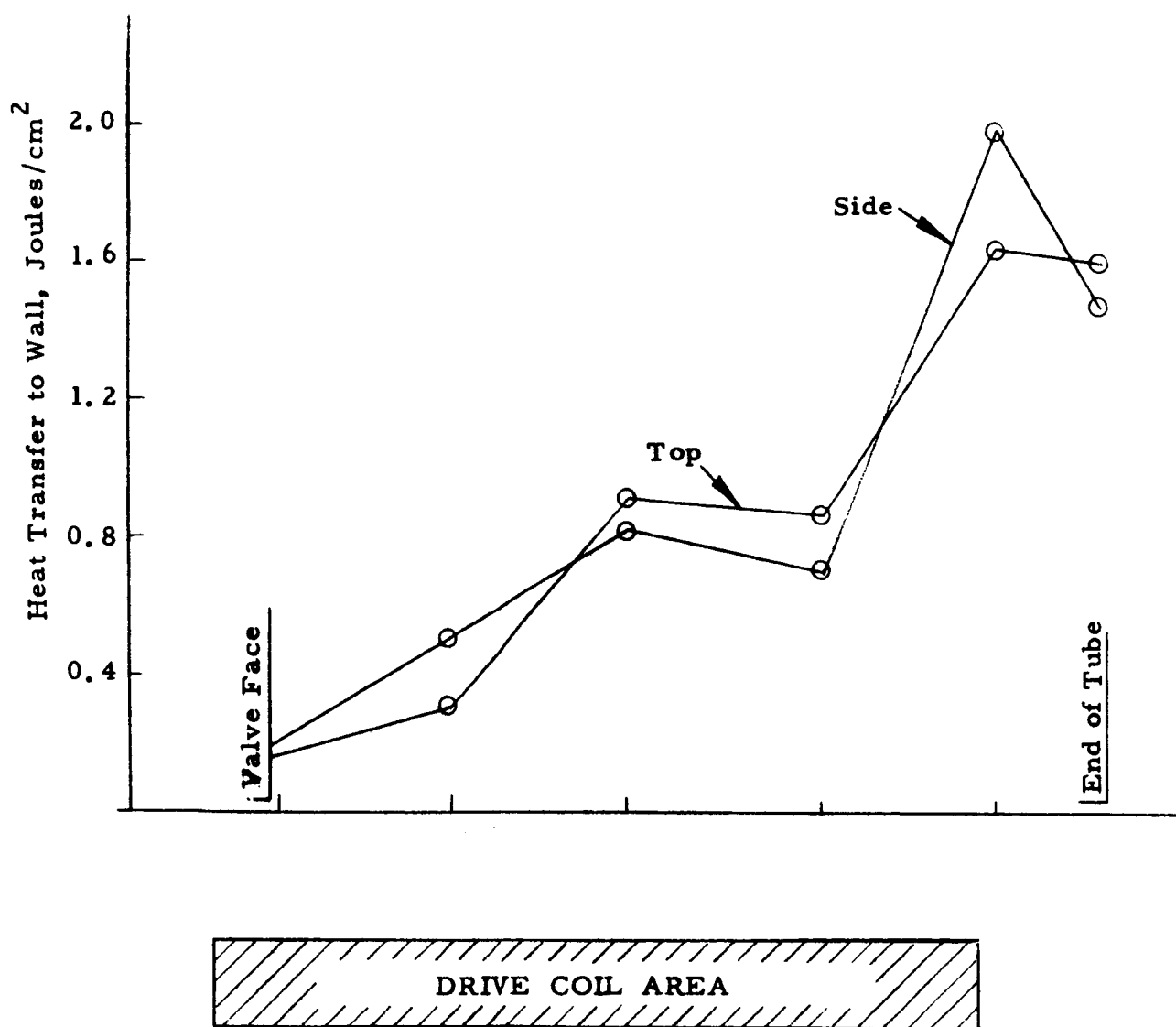


Figure 12. Distribution of heat loss to propulsion tube wall at 50 torr helium valve plenum pressure for cylindrically symmetric magnetic field.

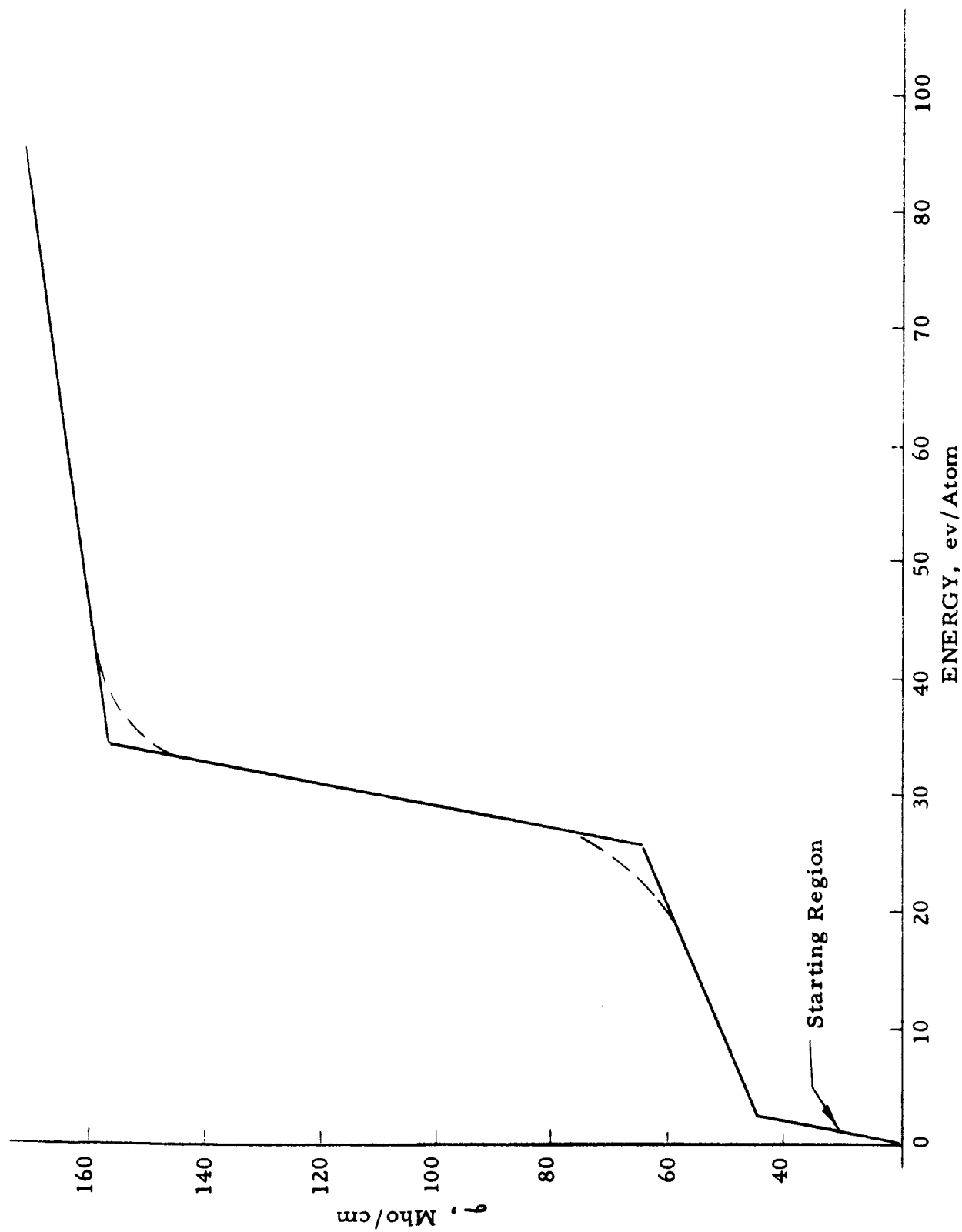


Figure 13. Electrical conductivity of helium plasma as function of the internal energy for pressures typical of the plasma accelerator.

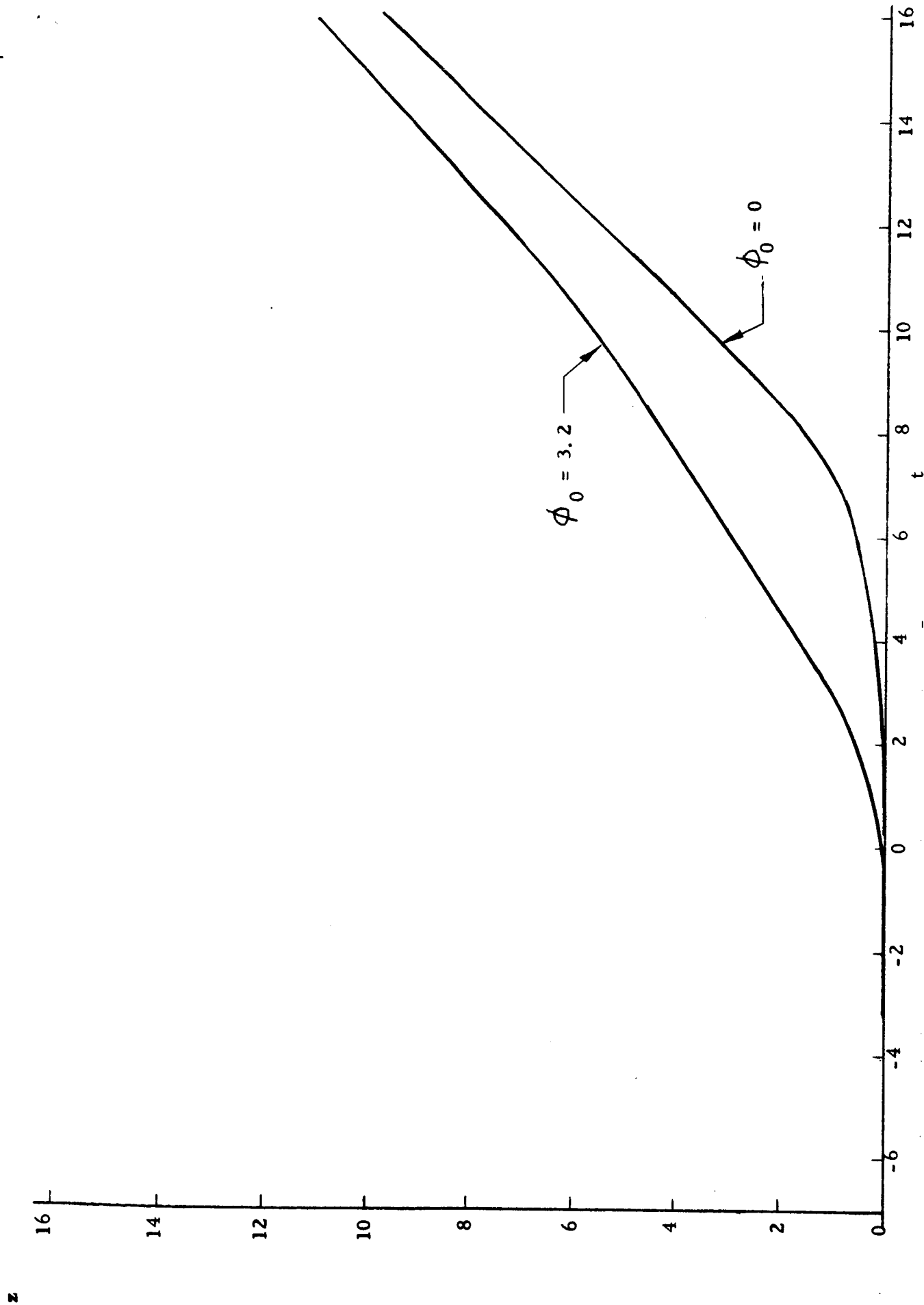


Figure 14. Trajectories in the t, z plane for two fluid elements.

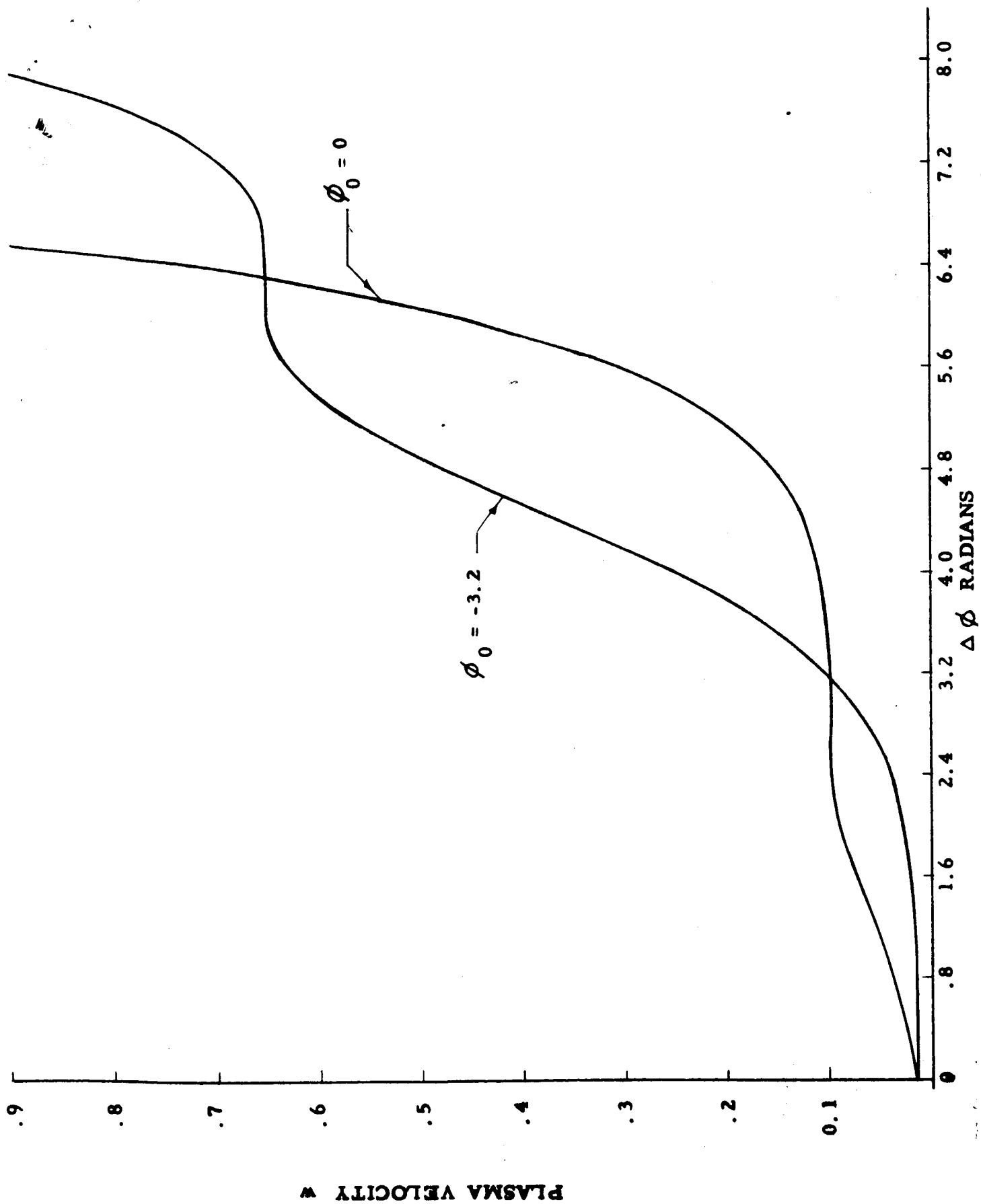


Figure 15. Plasma velocity as a function of the phase of the plasma element relative to the moving magnetic field.

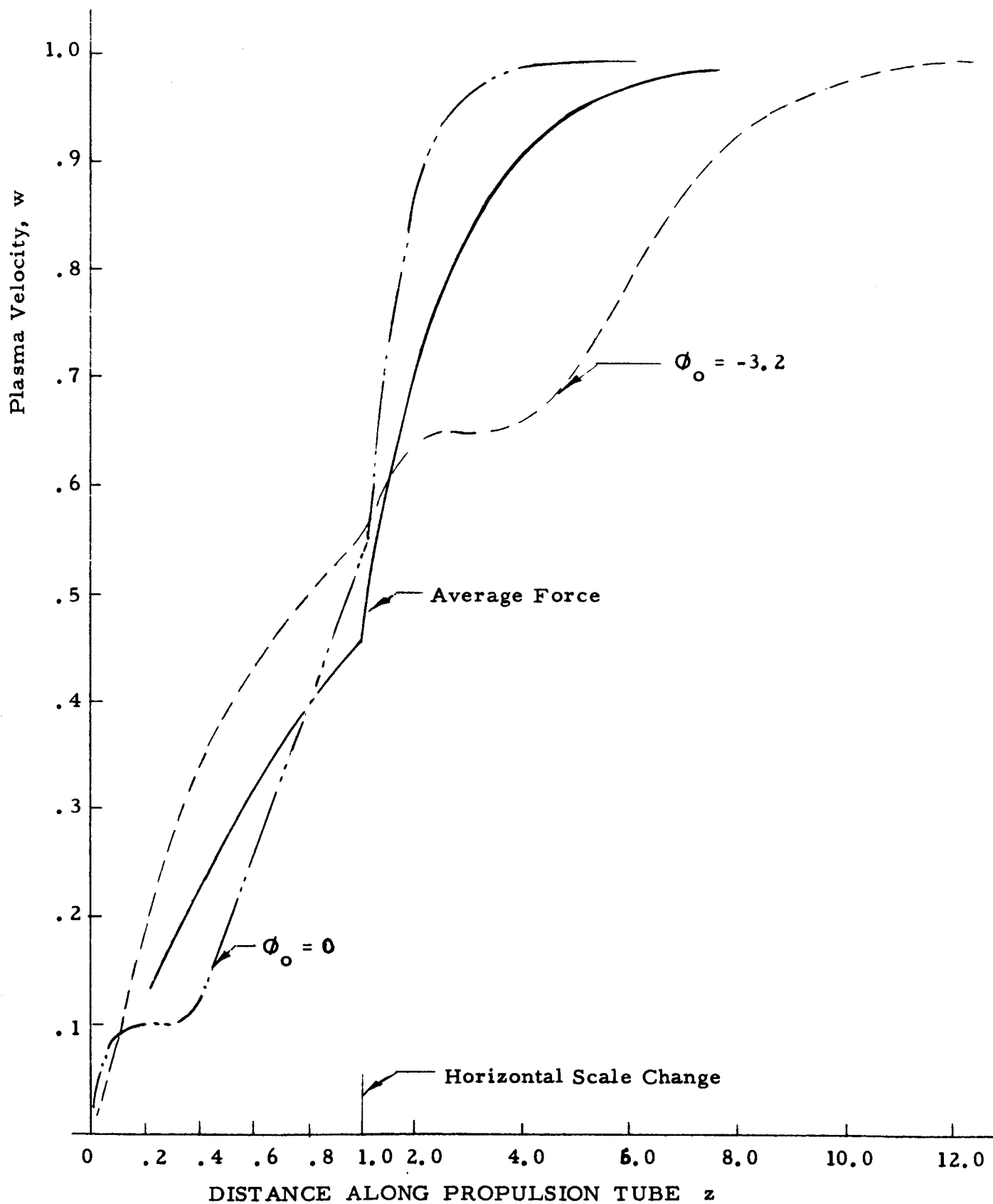


Figure 16. Plasma velocity as a function of distance down the accelerator tube.

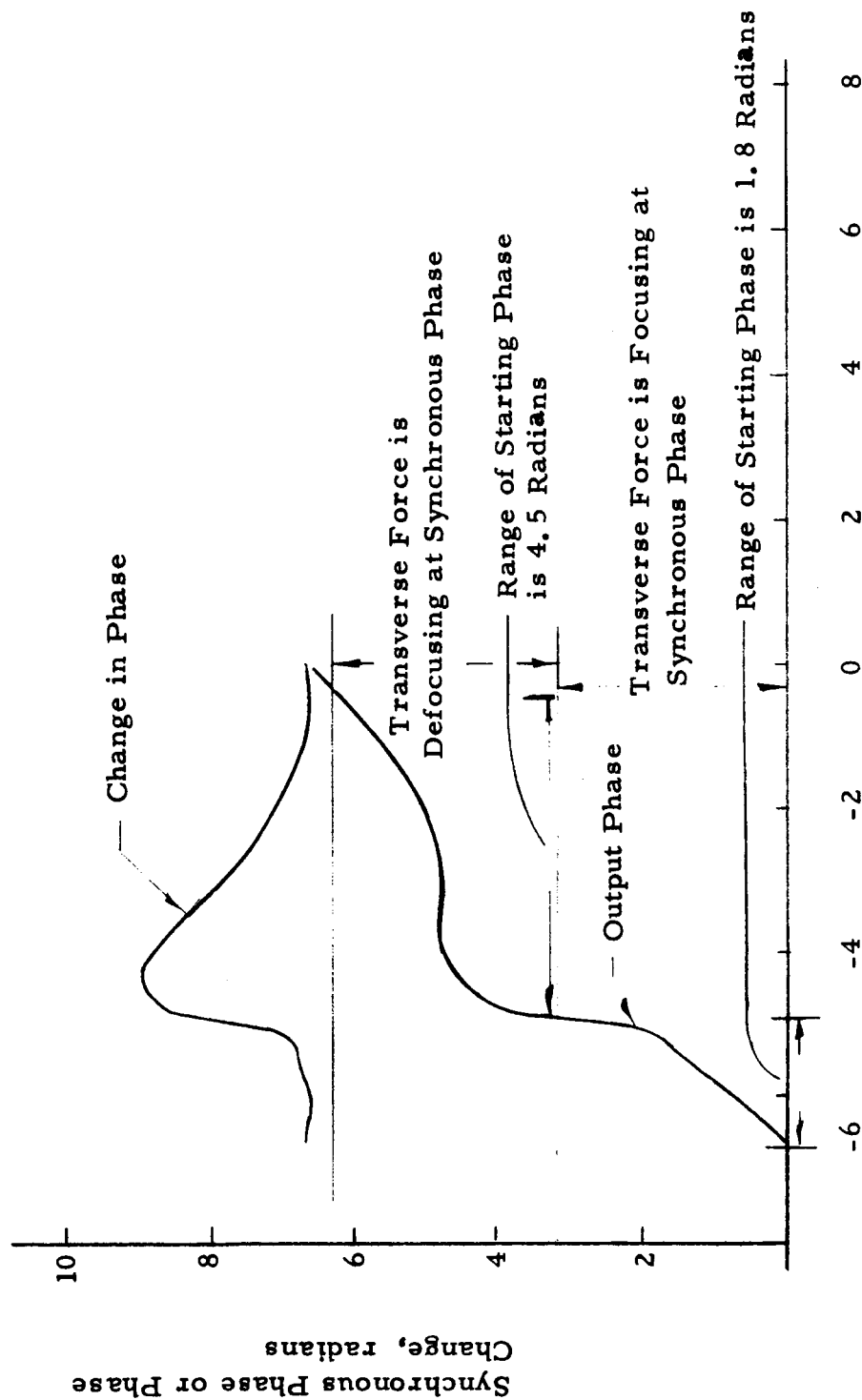


Figure 17. The final phase of a plasma element relative to the moving magnetic field as a function of the starting phase. Phase compression into a transverse direction defocusing region is shown.

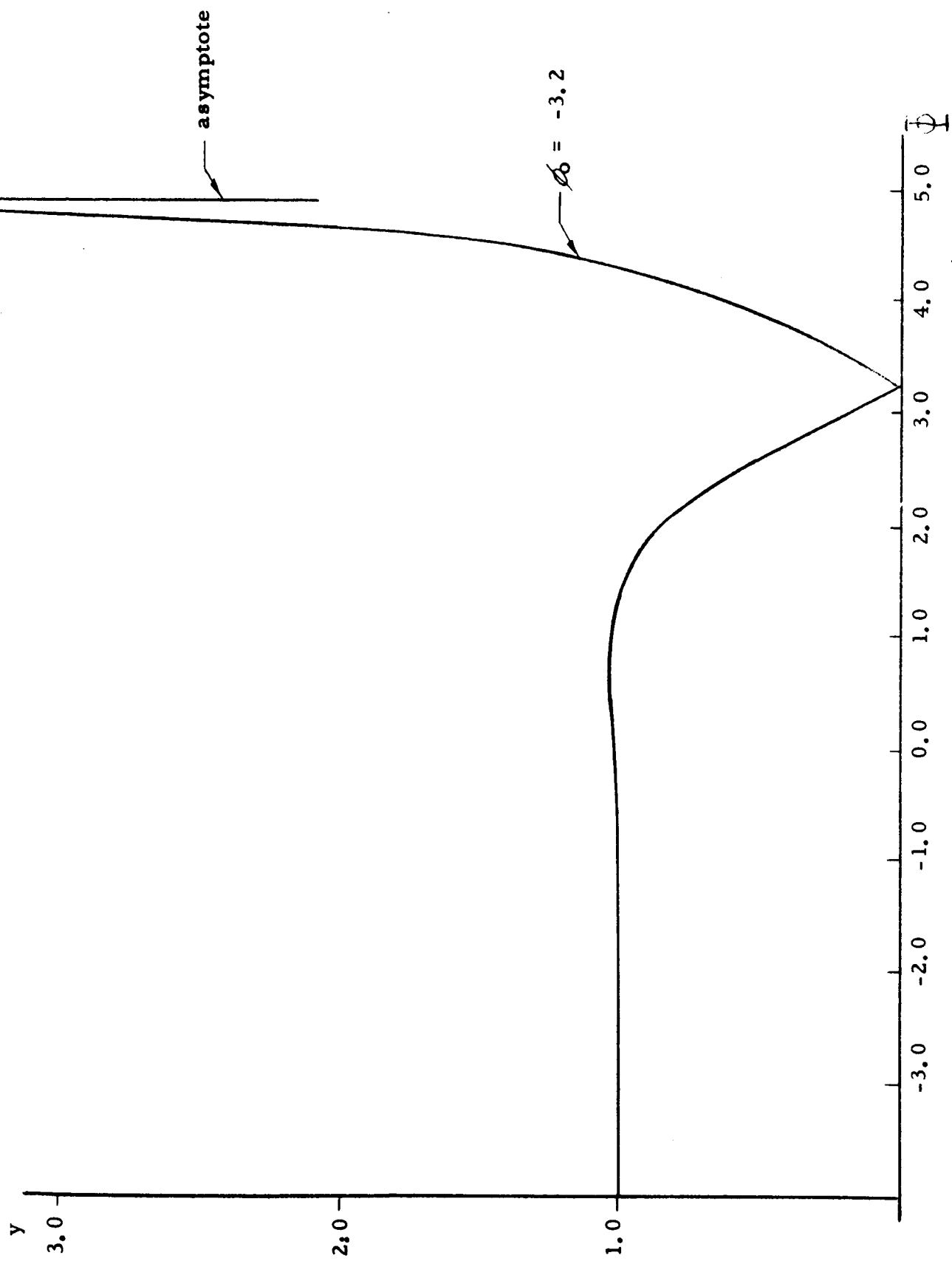


Figure 18. Computed motion of the plasma perpendicular to the direction of acceleration. Displacement as a function of phase relative to the field.

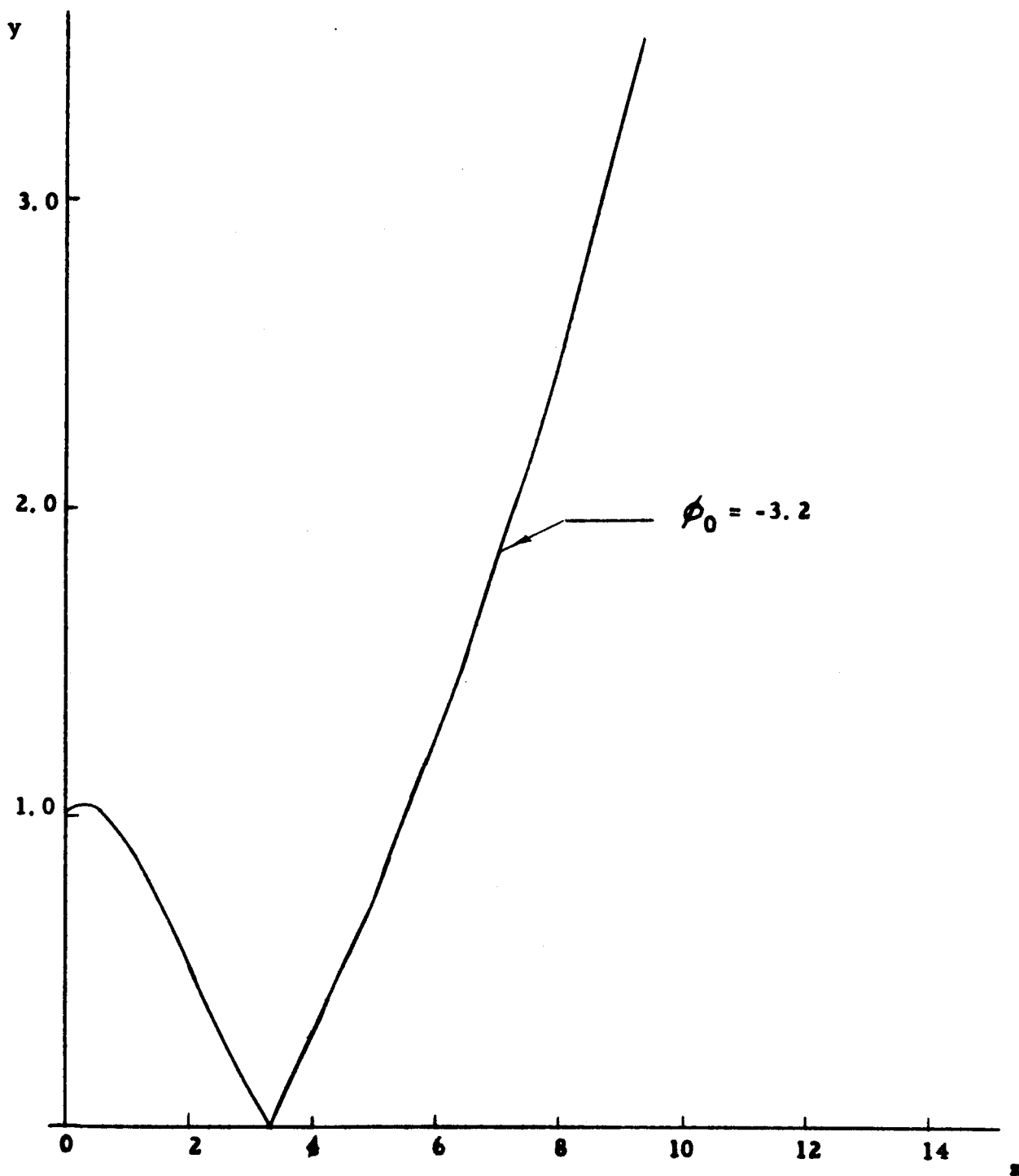


Figure 19. Computed motion of the plasma perpendicular to the direction of acceleration. Displacement as a function of distance down the propulsion tube.

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