THEORY OF

PHASELOCK TECHNIQUES

AS APPLIED TO

AEROSPACE TRANSPONDERS

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Chapter 2

MATHEMATICAL REVIEW

2-1. Laplace Transforms. (Ref: M. F. Gardner and J. L. Barnes, Transients in Linear Systems, Wiley, New York, 1942.)

1. Complex "Frequency" $s = \sigma + j\omega$

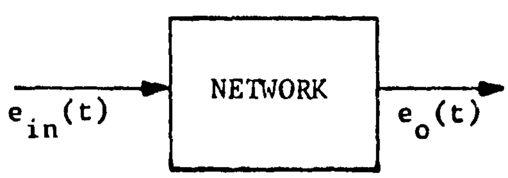
$$\mathcal{L}\left[x(t)\right] = X(s)$$

$$\mathcal{L}^{-1}\left[X(s)\right] = x(t)$$

2. Useful Transforms $x(t) \equiv 0$ for $t < 0$

	$x(t)$	$X(s)$
	1 (unit step)	$\frac{1}{s}$
	t (unit ramp)	$\frac{1}{s^2}$
	e^{-at}	$\frac{1}{s+a}$
derivative	$\frac{dy(t)}{dt}$	$sY(s) - y(0+)$
definite integral	$\int_0^t y(t)dt$	$\frac{1}{s}Y(s)$
Final Value	$\lim_{t\to\infty} y(t)$	$\lim_{s\to 0} sY(s)$

3. Transfer Function



Transfer function defined as

$$F(s) = \frac{E_o(s)}{E_{in}(s)}$$

For present purposes, F(s) will be ratio of polynomials in s.

$$F(s) = \frac{a_m s_m + a_{m-1} s^{m-1} + \cdots \cdots a_o}{b_n s^n + b_{n-1} s^{n-1} + \cdots \cdots b_o}$$

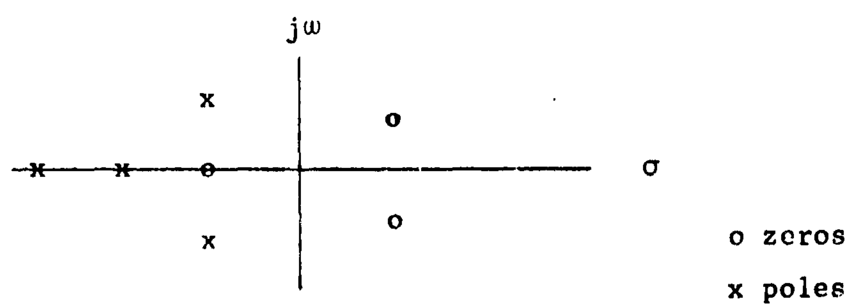
$m < n$ for a realizable network.

Polynomials may be factored and expressed as products of roots.

$$F(s) = \frac{a_m(s - z_m)(s - z_{m-1}) \cdots \cdots (s - z_1)}{b_n(s - p_n)(s - p_{n-1}) \cdots \cdots (s - p_1)}$$

Roots of numerator are called zeros. Roots of denominator are called poles. There are m zeros and n poles.

Roots may be plotted in complex s-plane



Complex roots must appear in conjugate pairs. Real roots may appear alone.

Poles must appear only in left half plane (LHP) if network is stable (realizable). Zeros may appear any place but we will only be concerned with networks where zeros are in LHP. (Minimum phase networks).

4. Frequency Response

If

$$e_{in}(t) = \sin \omega t$$

$$e_o(t) = A(\omega) \sin\left[\omega t + \emptyset(\omega)\right]$$

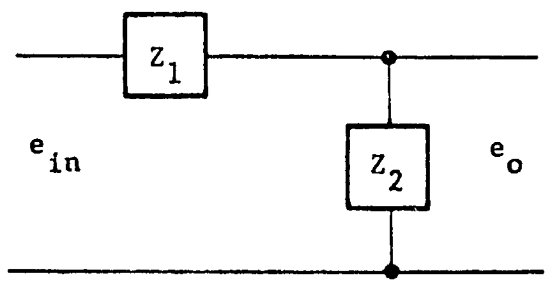
$$F(j\omega) = \frac{E_o(j\omega)}{E_{in}(j\omega)} = A(\omega) e^{j\emptyset(\omega)}$$

A is the amplitude of the frequency response and $\emptyset$ is the phase. Both are functions of frequency.

$$A(\omega) = |F(j\omega)| \; ; \; \emptyset(\omega) = \mathrm{Arg}\ F(j\omega)$$

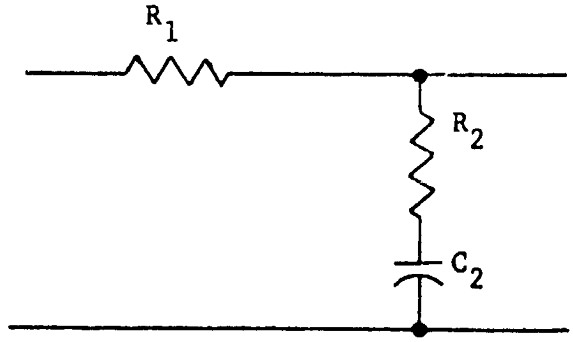
5. Computation of Transfer Functions

a. Typical L network



$$F(s) = \frac{E_o}{E_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

Consider (will be used often)



$$Z_1 = R_1 \qquad Z_2 = R_2 + \frac{1}{sC_2}$$

$$F(s) = \frac{R_2 + \frac{1}{sC_2}}{R_1 + R_2 + \frac{1}{sC_2}}$$

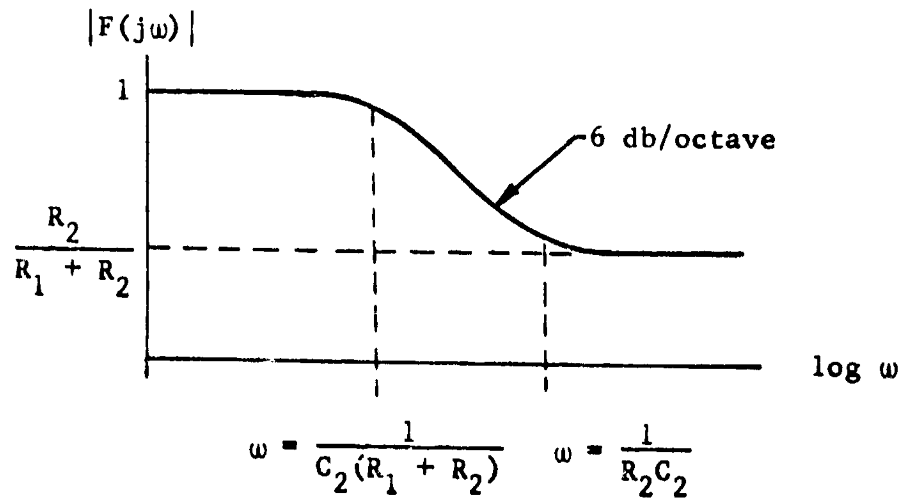
$$= \frac{sC_2R_2 + 1}{sC_2(R_1 + R_2) + 1}$$

$$F(j\omega) = \frac{1 + j\omega R_2C_2}{1 + j\omega C_2(R_1 + R_2)}$$

$$|F(j\omega)| = \sqrt{\frac{(1 + j\omega R_2C_2)(1 - j\omega R_2C_2)}{[1 + j\omega C_2(R_1 + R_2)][1 - j\omega C_2(R_1 + R_2)]}}$$

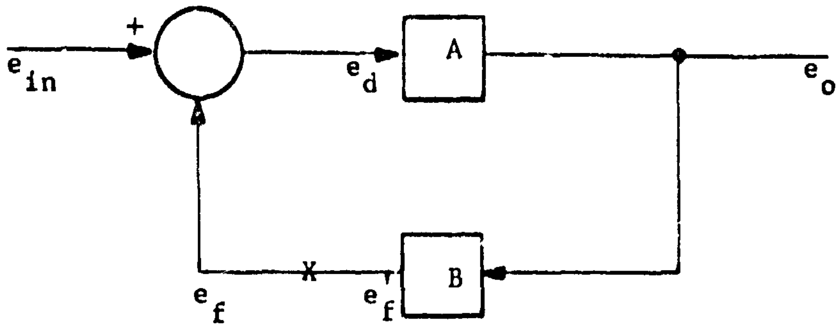
$$|F(j\omega)| = \sqrt{\frac{1 + \omega^2 R_2^2 C_2^2}{1 + \omega^2 C_2^2 (R_1 + R_2)^2}}$$

$$\phi(j\omega) = \tan^{-1} \omega R_2C_2 - \tan^{-1} \omega C_2(R_1 + R_2)$$



2-2. Feedback

1. Basic Feedback Loop Equations



A and B are complex transfer functions of kind discussed earlier.

Error voltage $e_d(t)$ is difference between input voltage e_{in} and the feedback voltage e_f.

$$e_d(t) = e_{in}(t) - e_f(t)$$

$$E_d(s) = E_{in}(s) - E_f(s)$$

$$E_o(s) = A(s)E_d(s)$$

$$E_f(s) = B(s)E_o(s)$$

$$E_d(s) = E_{in}(s) - B(s)E_o(s)$$

$$E_o(s) = A(s)\left[E_{in}(s) - B(s)E_o(s)\right]$$

$$E_o(s) = \frac{A(s)E_{in}(s)}{1 + A(s)B(s)}$$

$$\frac{E_o}{E_{in}} = \frac{A}{1 + AB} = G_c \text{ , Closed Loop Gain}$$

If B = 1 (common situation)

$$\frac{E_o}{E_{in}} = \frac{A}{1 + A}$$

$$E_d = E_{in} - BE_o$$

$$\frac{E_d}{E_{in}} = 1 - \frac{BE_o}{E_{in}}$$

$$= 1 - \frac{BA}{1 + AB} = \frac{1}{1 + AB}$$

Break loop at any point, say "x". Set $e_{in} = 0$. Apply test voltage e_f at input side of break. Compute resulting voltage e_f at output side of break.

$$e_d = -e_f \text{ (since } e_{in} = 0)$$

$$E_o = AE_d$$

$$E_f = BE_o = ABE_d$$

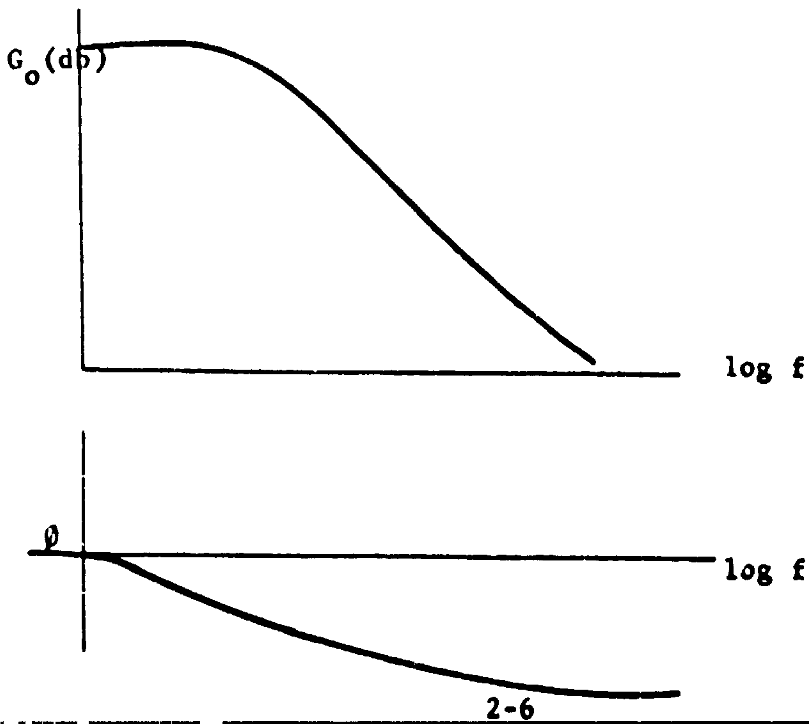
$$E_f' = -ABE_f$$

$$\frac{E_f'}{E_f} = -AB = G_o \text{ defined as cpen loop gain}$$

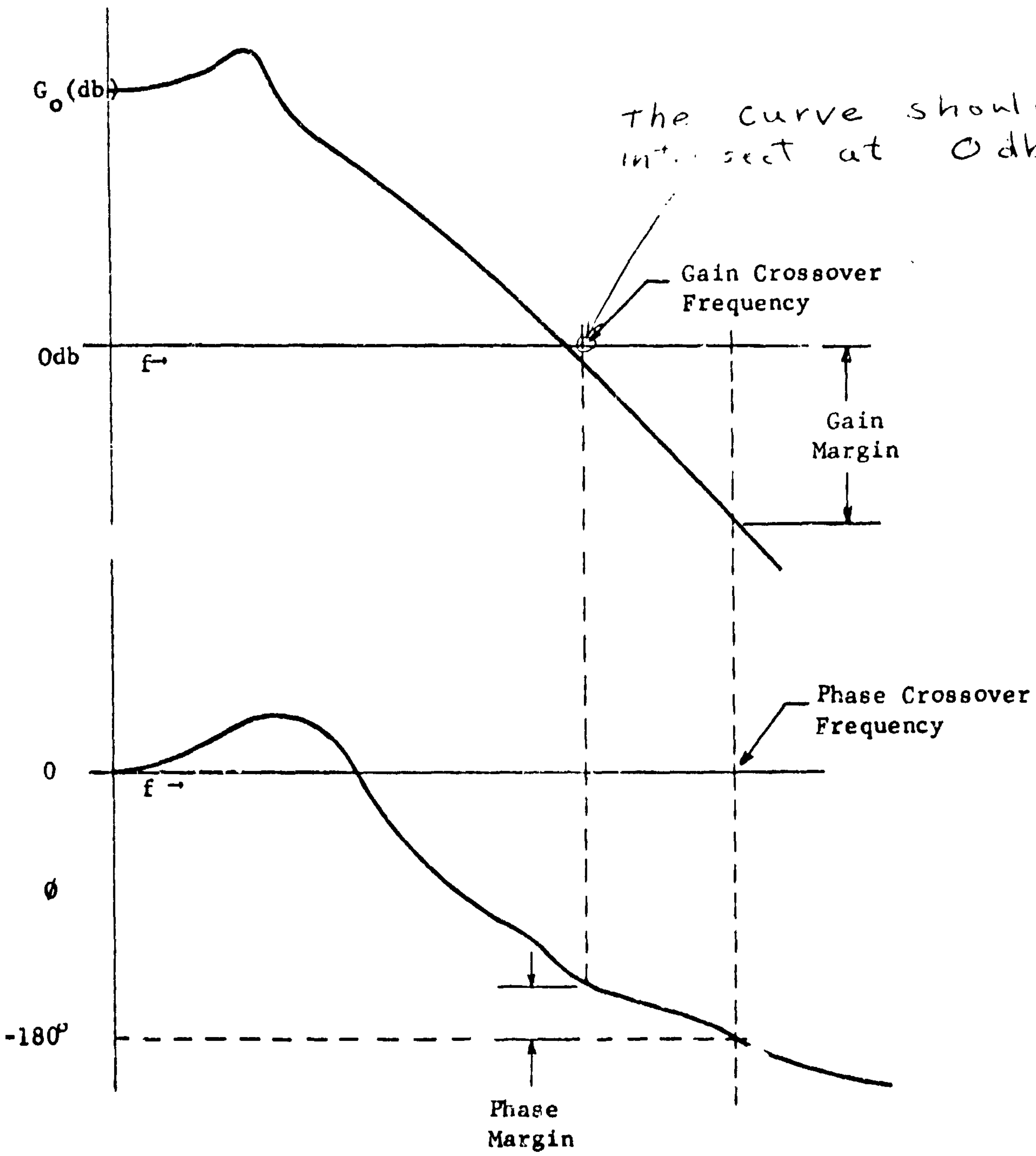
2-3. Stability

Closed loop can oscillate. (Poles in RHP).

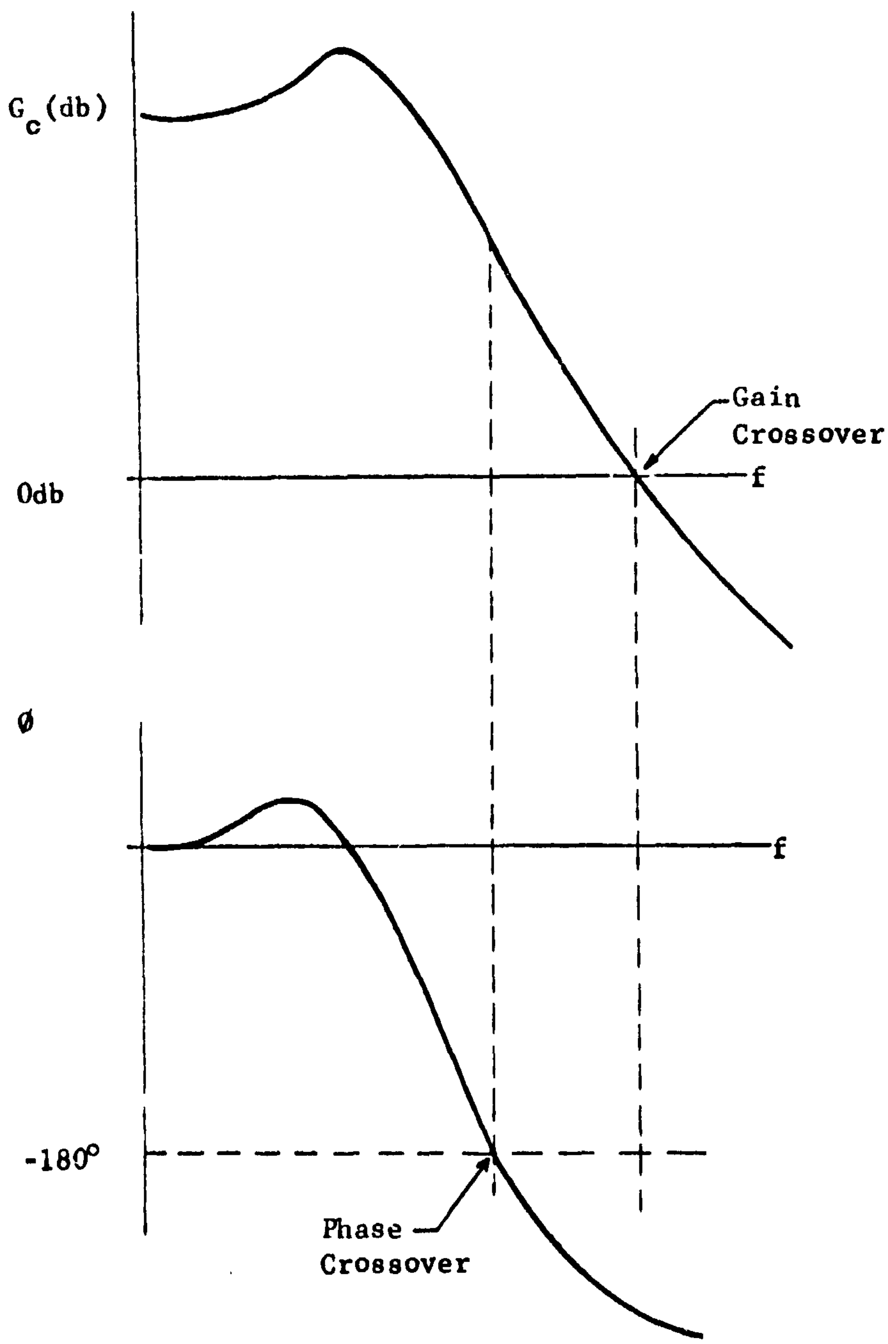
For this course, loop analyzed by Bode Diagrams, i.e.: plot of log G_o versus log f.



Criterion of Stability: Gain must fall below 0-db before phase reaches -180°.



Stable Case



Unstable Case

Any loop component causing phase lag is likely to be damaging to stability.

2-4. Noise Fundamentals. (Ref: W. B. Davenport and W. L. Root, Random Signals and Noise, McGraw-Hill, New York, 1958.)

Consider noise voltage n(t); how may it be described for analytical purposes?

For random noise, the actual waveshape is unpredictable so the function n(t) only has value as a concept; it is not generally possible to write an explicit expression for n(t). Only statistical quantities are available.

Assume n(t) is stationary (i.e.: all statistical properties constant over all time). Consider some useful statistical properties.

1. Mean Value (DC value, average value)

$$\overline{n(t)} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n(t)\, dt$$

We will usually be concerned with noise voltages having zero mean.

2. Mean Square Value

$$\sigma_n^2 = \overline{n^2(t)} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n^2(t)\, dt$$

3. Probability density function is denoted as p(n).

$\int_{n_1}^{n_2} p(n)\, dn$ is the probability that the amplitude of a sample of n(t) will lie in the range of n_1 to n_2.

In other words, probability density is a statistical statement describing, in part, the amplitude and, in some degree, the waveshape of the noise.

For any function to be a probability density

$$p(n) \geq 0 \text{ for all } n$$

$$\int_{-\infty}^{\infty} p(n)\, dn = 1$$

Previous averages (mean and mean square) were time averages. Using probability density, they may be expressed as ensemble averages.

$$\overline{n} = \int_{-\infty}^{\infty} n\ p(n)\ dn$$

$$\overline{n^2} = \int_{-\infty}^{\infty} n^2\ p(n)\ dn = \sigma_n^2$$

For stationary noise, the ensemble and time averages are equal.

A very commonly encountered density is the <u>Gaussian</u> or <u>Normal</u> function. It is given by

$$p(n) = \frac{1}{\sqrt{2\pi}\ \sigma_n} \exp\left[\frac{-(n - \overline{n})^2}{2\sigma_n^2}\right]$$

4. Autocorrelation Function

$$R(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n(t)n(t + \tau)\ dt$$

Some properties:

$$R(\tau) = R(-\tau)$$

$$R(0) \geq R(\tau) \text{ for all } \tau$$

$$R(0) = \sigma_n^2$$

5. Spectral density

Defined as Fourier transform of autocorrelation function.

$$W(f) = \int_{-\infty}^{\infty} R(\tau)e^{-j\omega\tau}\ d\tau\ ,\ (\omega = 2\pi f)$$

It is also true that

$$R(\tau) = \int_{-\infty}^{\infty} W(f)\ e^{j\omega\tau}\ df$$

Spectral density (power spectrum) is a very useful describer of the noise.

Properties of Spectral Density

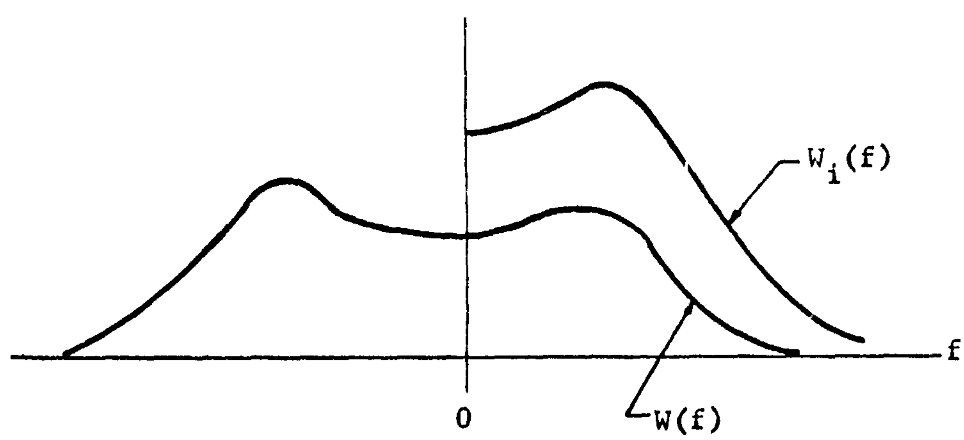
$$\int_{-\infty}^{\infty} W(f)\, df = \sigma_n^2$$

$$W(f) = W(-f)$$

A complete description of W(f) may be obtained solely from its values at positive values of f. Thus, although mathematical definition of W results in a "two-sided density", it is also possible to speak of a "one-sided density" $W_1(f)$ which involves only positive frequencies; that is

$$W_1(f) = 2\, W(f) \qquad (f \geq 0)$$

$$= 0 \qquad (f < 0)$$



$W_1(f)$

As derived here, dimensions of W are in $(\text{volts})^2/\text{cps}$ and is therefore proportional to power. Thus, W is often called "power spectral density". It could be defined slightly differently and dimensions would be watts/cps.

Noise is often passed through filters. If input spectral density is $W_{in}(f)$ and filter transfer function is $H(f)$, then output spectrum is

$$W_{out}(f) = W_{in}(f)\ |H(f)|^2$$

A convenient fiction often employed is the concept of "white noise". For this case $W(f)$ is constant for all frequencies. No physical process can be truly white since that would imply infinite power. A practical definition of whiteness is that the noise spectral density is constant at all frequencies of interest.

A white noise spectrum is completely specified by a single number; the spectral density at any frequency. It is necessary to state one-sided or two-sided spectrum.

> Caution: Noise is very commonly specified as white, gaussian noise. These are independent statements and neither one implies the other. Noise can be non-gaussian or non-white, or both.

2-5. Narrow-band Noise

If a noise voltage $n(t)$ has associated with it a relatively narrow band-pass spectrum, it is permissible and often convenient to write

$$n(t) = n_c(t) \cos \omega_1 t - n_s(t) \sin \omega_1 t$$

where ω_1 is any arbitrary frequency whatever, but most convenient results are usually obtained if it is selected as being in the center of the narrow pass-band.

Some properties of this expansion are as follows:

1. Spectrum. The spectra of n_c and n_s will be low-pass in nature.
2. Gaussian. If $n(t)$ is gaussian, n_c and n_s are also gaussian.
3. Mean. If $n(t)$ has zero mean, then n_c and n_s will also have zero mean value.
4. Variance.

$$\overline{n^2(t)} = \overline{n_c^{\,2}(t)} = \overline{n_s^{\,2}(t)}$$

5. Independence. The functions n_c and n_s are independent. That is

$$\overline{n_c(t)\, n_s(t)} = \overline{n_c(t)}\ \overline{n_s(t)}$$

$$= 0 \text{ if } \overline{n(t)} = 0$$

6. Spectrum. Consider n(t) to have a spectrum W(f) defined as

$$W(f) = W_o, \left(\frac{\omega_1}{2\pi} - \frac{B}{2}\right) \leq f \leq \left(\frac{\omega_1}{2\pi} + \frac{B}{2}\right)$$

$$= 0 \qquad \text{otherwise}$$

That is, the spectrum of n(t) is a bandpass rectangle of width B and height W_o, centered at $f_1 = \omega_1/2\pi$.

For this case $n_c(t)$ and $n_s(t)$ will have spectra defined as

$$W_c(f) = W_s(f) = 2W_o \qquad f < B/2$$

$$= 0 \qquad f > B/2$$

or, in other words, n_c and n_s have low-pass spectra of bandwidth B/2 and spectral density $2W_o$.

These results will be used later.

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Chapter 3
LOOP FUNDAMENTALS

3-1. Basic Loop Equations

Consider an elementary loop consisting of a phase detector, a low-pass loop-filter, and a VCO.

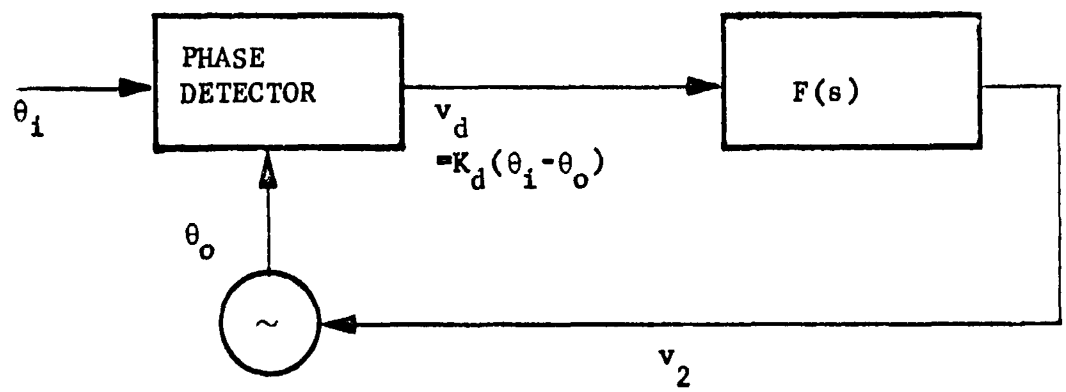


Fig. 3-1
Basic Loop Block Diagram

The input signal has a phase of $\theta_i(t)$ and the VCO output has a phase $\theta_o(t)$.

For the present, it will be assumed that the phase detector is linear (this assumption is justified and qualified in Chapter 4) and that its output voltage is proportional to the difference in phase between its inputs. That is

$$v_d = K_d(\theta_i - \theta_o) \tag{3-1}$$

where K_d will be called the "phase detector gain factor" and has dimensions of volts per radian.

Phase error voltage is filtered by the low-pass loop filter. Noise and high frequency signal components are suppressed; also, the filter helps determine dynamic performance of the loop. Filter transfer function is given by F(s).

Frequency of the voltage-controlled oscillator (VCO) is controlled by the filtered error voltage v_2. Deviation of the VCO from its center frequency is $\Delta\omega = K_o v_2$ where K_o is the "VCO gain constant" and has dimensions of radians per second per volt.

Since frequency is the derivative of phase, the VCO operation may be described as $d\theta_o/dt = K_o v_2$.

Taking La Place transforms

$$\frac{d\theta_o(t)}{dt} = s\theta_o(s) = K_oV_2(s) \tag{3-2}$$

s.

$$\theta_o(s) = \frac{K_oV_2(s)}{s}$$

In other words, the phase of the VCO output will be proportional to the integral of the input control voltage.

Using La Place notation, the following equations are applicable:

$$V_d(s) = K_d\left[\theta_i(s) - \theta_o(s)\right] \tag{3-1}$$

$$V_2(s) = F(s)V_d(s) \tag{3-2}$$

$$\theta_o(s) = \frac{K_oV_2(s)}{s} \tag{3-3}$$

Combining these equations results in the basic loop equations:

$$\boxed{\frac{\theta_o(s)}{\theta_i(s)} = H(s) = \frac{K_oK_dF(s)}{s + K_oK_dF(s)}} \tag{3-4}$$

and

$$\boxed{\frac{\theta_i(s) - \theta_o(s)}{\theta_i(s)} = \frac{\theta_e(s)}{\theta_i(s)} = \frac{s}{s + K_oK_dF(s)}} \tag{3-5}$$

Before proceeding further, it is necessary to specify the loop filter, F(s).

3-2. Second-Order Loop

Two widely-used loop filters are shown with their respective transfer functions in Figure 3-2. The passive filter is quite simple and often is satisfactory for many purposes. The active filter requires a high-gain DC amplifier but yields better tracking performance, as will be seen in Chapter 5.

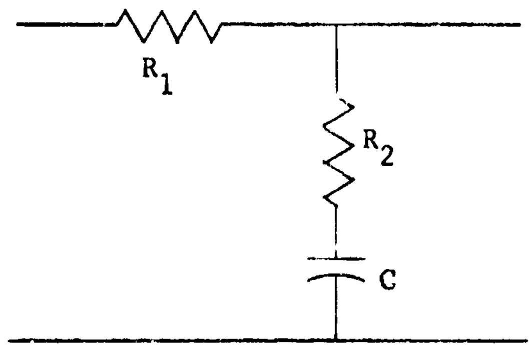
For the passive filter, the loop transfer function is

$$H_1(s) = \frac{K_oK_d(s\tau_2 + 1)/(\tau_1 + \tau_2)}{s^2 + \dfrac{s(1 + K_oK_d\tau_2)}{\tau_1 + \tau_2} + \dfrac{K_oK_d}{\tau_1 + \tau_2}}$$

For the active filter

$$H_2(s) = \frac{K_oK_d(s\tau_2 + 1)/\tau_1}{s^2 + s\,\dfrac{K_oK_d\tau_2}{\tau_1} + \dfrac{K_oK_d}{\tau_1}}$$

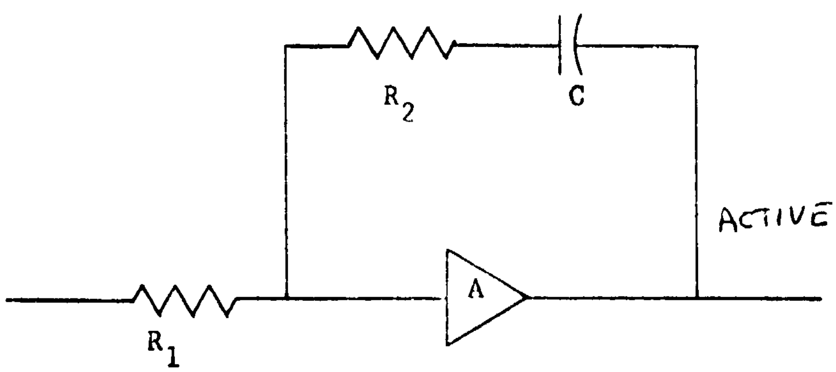
provided that gain of the amplifier is very large.



Passive Filter

$$F_1(s) = \frac{sCR_2 + 1}{sC(R_1 + R_2) + 1} = \frac{s\tau_2 + 1}{s(\tau_1 + \tau_2) + 1}$$

$$\tau_1 = R_1C \qquad \tau_2 = R_2C$$



ACTIVE FILTER

$$F_2(s) = \frac{A(sCR_2 + 1)}{sCR_2+1 + (1-A)(sCR_1)}$$

$$\approx \frac{sCR_2 + 1}{sCR_1} = \frac{s\tau_2 + 1}{s\tau_1}$$

For large (A)

Fig 3-2

Filters used in Second-Order Loops

These transfer functions may be rewritten as

$$H_1(s) = \frac{s\left(2\zeta\omega_n - \frac{\omega_n^2}{K_oK_d}\right) + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{s\omega_n\left(2\zeta - \frac{\omega_n}{K_oK_d}\right) + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{3-6}$$

$$H_2(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{3-7}$$

where, drawing upon servo terminology, ω_n is the "natural frequency" of the loop and ζ is the "damping factor".

Passive Filter	Active Filter
$\omega_n = \sqrt{\frac{K_oK_d}{\tau_1 + \tau_2}}$	$\omega_n = \sqrt{\frac{K_oK_d}{\tau_1}}$
$\zeta = \frac{1}{2}\sqrt{\frac{K_oK_d}{\tau_1 + \tau_2}}\left(\tau_2 + \frac{1}{K_oK_d}\right)$	$\zeta = \frac{\tau_2}{2}\sqrt{\frac{K_oK_d}{\tau_1}}$

(3-8)

Observe that the two transfer functions are the same if $\omega_n/K_oK_d \ll 2\zeta$ in the passive loop.

Because the highest power of "s" in the denominator is two, the loop is known as a "second-order loop". This form of second-order loop is very widely applied because of simplicity and good performance.

The frequency response of a high gain loop is plotted in Fig. 3-3 for several values of damping factor. It can be seen that the loop performs a low pass filtering operation on phase inputs.

The transfer function H(s) has a well-defined 3 db bandwidth which we shall label "ω_{3db}". There is generally very little reason to be interested in ω_{3db} but its relation to ω_n is presented here so as to provide a comparison to a familiar concept of bandwidth.

By setting $|H(j\omega)|^2 = 1/2$ and solving for ω, it is found that

$$\omega_{3db} = \omega_n\left[2\zeta^2 + 1 + \sqrt{(2\zeta^2 + 1)^2 + 1}\right]^{1/2}$$

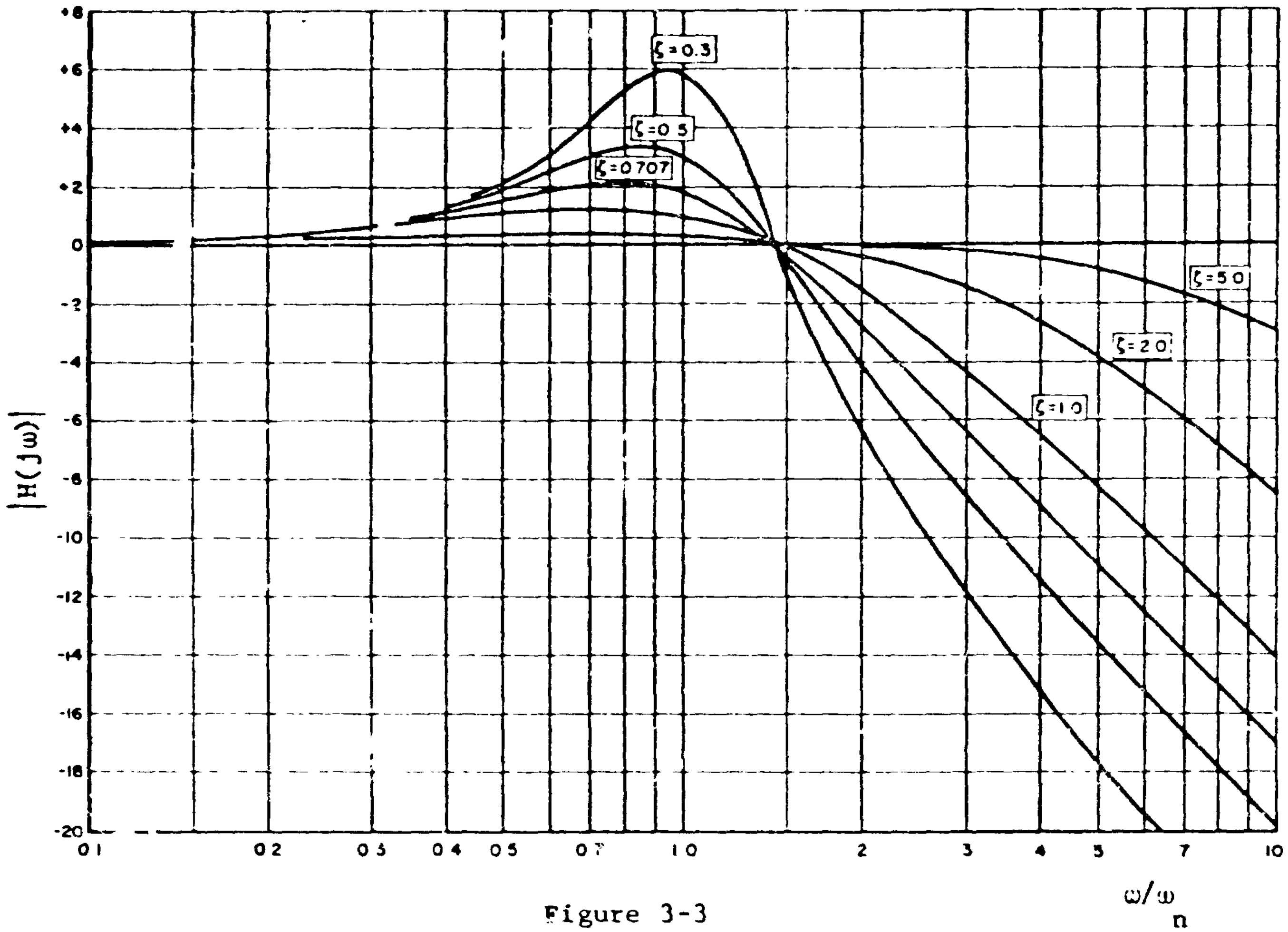


Figure 3-3

Frequency Response; High-Gain, Second-Order Loop

Typical values are shown below for a high-gain loop.

ζ	ω_{3db}/ω_n
0.500	1.82
0.707	2.06
1.000	2.48

Error response of the loop is also of interest. For a high-gain, second-order loop the error response is

$$\frac{\theta_e(s)}{\theta_i(s)} = 1 - H(s) = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{3-9}$$

whereas, for a low-gain loop

$$\frac{\theta_e(s)}{\theta_i(s)} = \frac{s\left(s + \frac{\omega_n^2}{K_o K_d}\right)}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{s\left(s + \frac{1}{\tau_1 + \tau_2}\right)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{3-10}$$

Error response is plotted in figure 3-4 for a high-gain loop with $\zeta = 0.707$. A high-pass characteristic is obtained, which is to say. the loop tracks low-frequency changes but cannot track high frequencies.

3-3. Other Loop Types

A first order loop is obtained if the filter is omitted entirely, that is, $F(s) = 1$. The loop transfer function is of the form

$$H(s) = \frac{K_o K_d}{s + K_o K_d} \tag{3-11}$$

so that loop gain ($K_o K_d$) is the only parameter available to the designer for adjustment. If it is necessary to have large loop gain (often needed to insure good tracking) the bandwidth must also be large. Therefore, narrow bandwidth and good tracking are incompatible in the first-order loop; for this reason it is not often used.

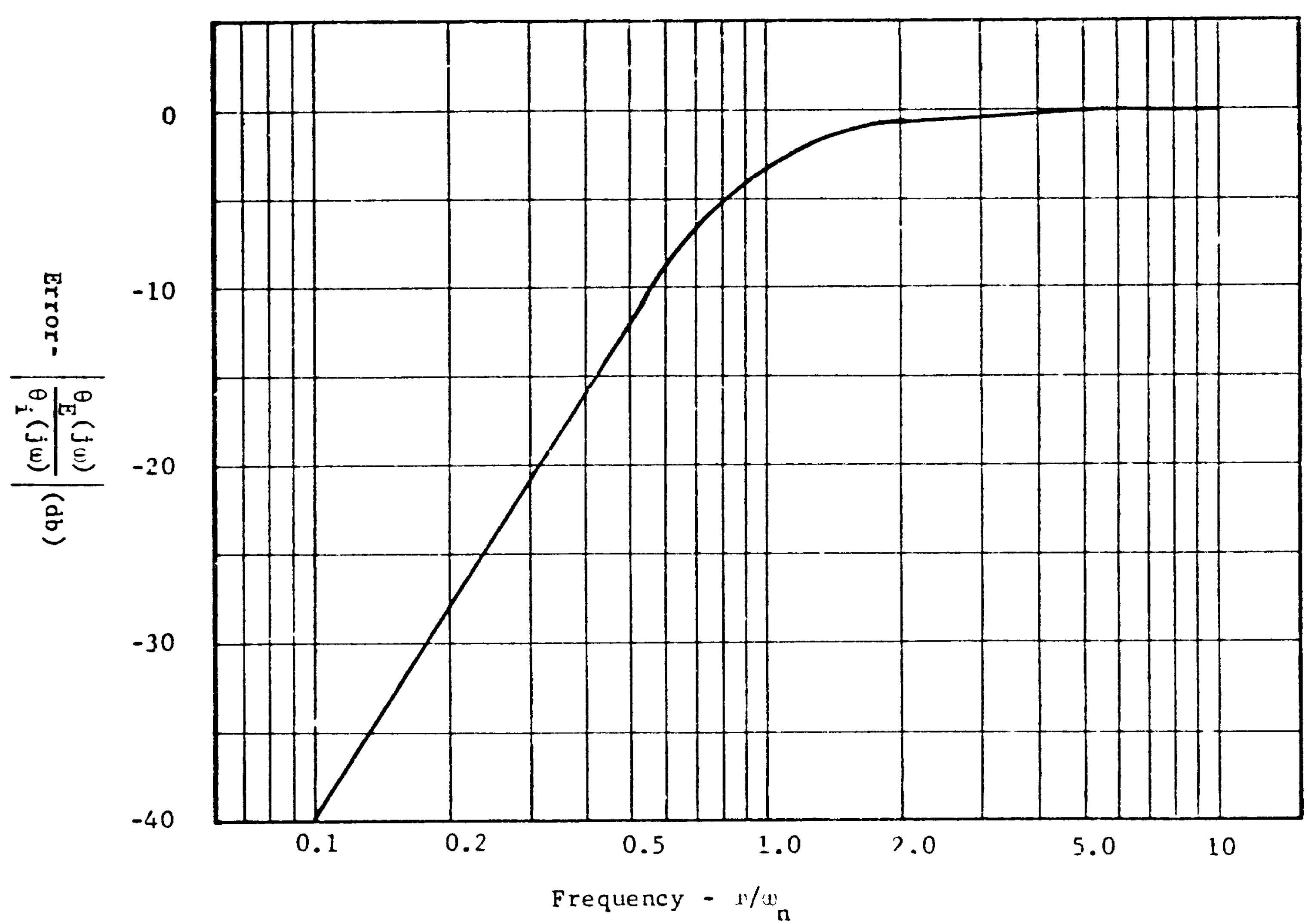


Figure 3-4

Error Response of High Gain Loop

$\zeta = \sqrt{2}/2$

A second-order loop results if a filter

$$F(s) = \frac{1}{s\tau + 1} \tag{3-12}$$

is used. The loop transfer function is therefore

$$H(s) = \frac{\frac{K_o K_d}{\tau}}{s^2 + \frac{s}{\tau} + \frac{K_o K_d}{\tau}} \tag{3-13}$$

whence

$$\omega_n = \sqrt{\frac{K_o K_d}{\tau}}$$

$$\zeta = 1/2\sqrt{\frac{1}{\tau K_o K_d}} \tag{3-14}$$

There are two circuit parameters available (τ and $K_o K_d$) whereas there are usually three loop parameter specifications to be met (ω_n, ζ, and $K_o K_d$). Obviously, the three loop parameters cannot be chosen independently. If it is necessary to have large gain and small bandwidth, the loop will be badly underdamped and transient response will be poor.

A very similar condition is found in servomechanisms; in the simplest servos damping becomes very small as gain increases. The solution to the servo problem is to employ tachometer feedback or to use lag-lead compensation. The latter expedient is commonly used in phase-lock loops and results in the filters of Fig. 3-2 which have already been analysed.

Since the lag-lead filter has two independent time constants, the natural frequency and damping can be chosen independently. Furthermore, loop gain can be made as large as may be necessary for good tracking.

There are situations in which a third-order loop provides useful performance characteristics not obtainable with a simpler loop. Accordingly, it is sometimes used in special applications. Further short discussion of the third-order loop may be found in Chapters 5 and 6.

To our knowledge, there has never been a loop constructed with a higher-order than third. One reason would seem to be that there has been no need for higher-order loops in the situations where phase lock-techniques are most commonly applied. Also, the closed loop parameters of high order, active networks tend to be overly sensitive to changes of gain and circuit components. Finally, it is more difficult to stabilize a high-order loop whereas the second-order loop (as commonly built) is unconditionally stable.

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Chapter 4

Noise Performance

4-1. Linear Analysis

Consider the phase detector to be a perfect multiplier having two inputs $e_1(t)$ and $e_2(t)$. Its output will be $K_m e_1 e_2$.

Now suppose the signal input is

$$v_i(t) = V_s \sin (\omega_i t + \theta_i)$$

and the VCO waveform* is

$$v_o(t) = V_o \cos (\omega_i t + \theta_o)$$

Output of the phase detector (neglecting double-frequency terms which will be removed by the loop filter) is

$$v_d = \frac{V_s V_o K_m}{2} \sin (\theta_i - \theta_o) \tag{4-1}$$

The linearizing approximation invariably made is to require that $(\theta_i - \theta_o)$ be small and then use the relation

$$\sin (\theta_i - \theta_o) \approx (\theta_i - \theta_o)$$

For this approximation, phase detector output is

$$v_d \approx \frac{K_m V_s V_o}{2} (\theta_i - \theta_o) \tag{4-2}$$

In terms of earlier notation, the phase detector gain constant is

$$K_d = \frac{K_m V_s V_o}{2} \tag{4-3}$$

---

*Note that v_i and v_o are really 90^o out of phase with one another. The input has been written as a sine and the VCO output has been written as a cosine. The two phases θ_i and θ_o are referred to these quadrature references.

which is a function of input signal level. Therefore, if the input signal amplitude varies, K_d and all loop parameters dependent upon loop gain will also vary.

Suppose the input to the loop consists of a sinusoidal signal plus narrowband gaussian noise.

$$v_i(t) = V_s \sin(\omega_i t + \theta_i) + n(t) \tag{4-4}$$

As shown in Chapter 2, the noise may be expanded as

$$n(t) = n_c(t)\cos \omega_i t - n_s(t)\sin \omega_i(t) \tag{4-5}$$

This noise is then multiplied in the phase detector by the VCO waveform and the noise output of the phase detector will be (discarding double-frequency terms)

$$v_{dn}(t) = \frac{V_o}{2}\left[n_c(t)\cos\theta + n_s(t)\sin\theta_o\right] \tag{4-6}$$

To obtain simple results, the approximation is made that θ_o is independent of n(t). This approximation is reasonable if θ_o is changing slowly compared to the input noise. Such conditions obtain if phase error due to noise is small and loop bandwidth is narrow compared to input bandwidth.

This assumption of independence cannot be strictly true. Nonetheless, it proves to be a useful approximation and will be applied here.

Applying the approximation, the rms noise output from the phase detector is found to be

$$\left(\overline{v_{dn}^2}\right)^{1/2} = \frac{K_m V_o}{2}\left(\overline{n^2(t)}\right)^{1/2} \tag{4-7}$$

Let us now determine the equivalent phase jitter in the input signal that would give the same rms noise output from the phase detector. Denote the mean square input phase jitter as $\overline{\theta^2_{ni}}$ and consider θ_{ni} to be additive to θ_i. Then rms phase detector output would be

$\frac{1}{2} K_m V_s V_o \left(\overline{\theta_{ni}^2}\right)^{1/2}$.

Equating this expression to Eq. (4-7) and solving gives an equivalent input phase variance of

$$\boxed{\overline{\theta_{ni}^2} = \frac{\overline{n^2(t)}}{V_s^2} = \frac{P_n}{2P_s} \ (\text{radians})^2} \tag{4-8}$$

where P_s is the input signal power and P_n is the total noise power in the input.

Consider that the loop is preceded by an input bandpass filter with a rectangular shape of bandwidth B_i cps and center frequency $f_i = \omega_i/2\pi$. (All pass-bands and spectra will be taken as one-sided). If the spectrum of n(t) is flat within the input bandwidth, the input spectral density is

$$N_o = \frac{\overline{n^2(t)}}{B_i} \ (\text{volts})^2/\text{cps} \tag{4-9}$$

Spectrum of the equivalent input phase noise θ_{ni} is a low-pass rectangle with bandwidth $B_i/2$ and a density of

$$\Phi = \frac{\overline{\theta_{ni}^2}}{B_i/2} = \frac{2N_o}{V_s^2} \ (\text{rad})^2/\text{cps} \tag{4-10}$$

If the input power spectral density is W_o watts/cps, the phase spectral density is

$$\Phi = \frac{W_o}{P_s} \ (\text{rad})^2/\text{cps} \tag{4-10a}$$

Mean square output phase jitter is given by

$$\overline{\theta_{no}^2} = \int_0^\infty \Phi \; |H(j\omega)|^2 \; df$$

$$= \frac{\Phi}{2\pi j} \left| \int_{jo}^{j\infty} \frac{2\zeta\,\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \right|^2 ds \qquad (4\text{-}11)$$

$$= \frac{\Phi}{2\pi} \int_o^\infty \frac{\omega_n^2\,(4\zeta^2\omega^2 + \omega_n^2)\, d\omega}{\omega_n^4 + 2\,\omega_n^2\omega^2(2\,\zeta^2 - 1) + \omega^4}$$

This integral may be evaluated by reference to published tables (e.g.: G. Petit Bois, Tables of Indefinite Integrals, Dover, New York, 1961). The result is the "loop noise bandwidth"

$$\boxed{B_L = \int_0^\infty |H(j\omega)|^2 \; df = \frac{\omega_n}{2}(\zeta + \frac{1}{4\zeta}) \text{ cps}} \qquad (4\text{-}12)$$

which has dimensions of cycles per second, despite the fact that dimensions of ω_n are in radians per second.

The loop noise bandwidth, as used here, is a one-sided bandwidth. It is very common, however, to find references to a "two-sided loop noise bandwidth"; this quantity is simply $2B_L$.

From the conventional definition of noise bandwidth it may be stated that the amount of phase noise in the loop output is identical to that which would emerge from a rectangular low-pass filter with cutoff at B_L cps and unity transmission from DC to B_L cps.

Loop noise bandwidth B_L as a function of damping is plotted in Fig. 4-1. There is a minimum for $\zeta = 1/2$; in that case, $B_L/\omega_n = 1/2$. For the very common damping of $\zeta = 0.707$, $B_L/\omega_n = \frac{3}{4\sqrt{2}} = 0.53$. Between the limits of $0.25 < \zeta < 1.0$, the loop bandwidth never exceeds its minimum value by more than 25% (equivalent to 1-db noise power).

When the integral of Eq. 4-11 is replaced by B_L, mean square output phase jitter is found to be

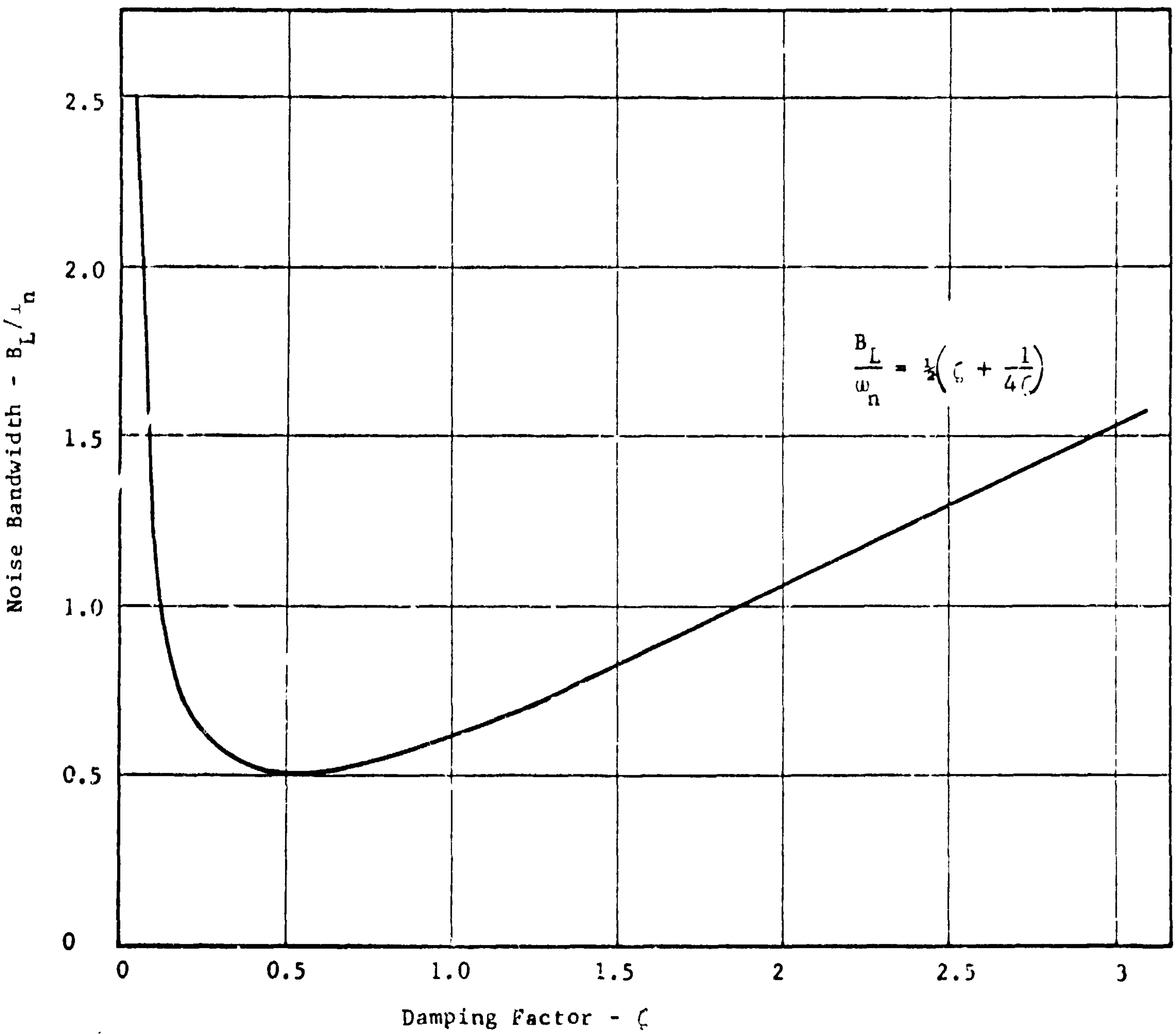

Figure 4-1

Loop Noise Bandwidth

(for high-gain, second-order loop)

$$\overline{\theta_{no}^2} = \Phi B_L = 2\, \overline{\theta_{ni}^2}\, \frac{B_L}{B_i}$$

$$= \frac{P_n}{P_s} \frac{B_L}{B_i} = \frac{W_o B_L}{P_s} \tag{4-13}$$

This expression is valid if the rms phase jitter of the output is less than approximately 13° (MAR-3). Non-linear behavior at high noise conditions will be treated later.

Signal to noise ratio in the input bandwidth is

$$(SNR)_i = \frac{P_s}{P_n} = \frac{1}{2\, \overline{\theta_{ni}^2}} \tag{4-14}$$

Analogously, we would like a similar relation between output phase and loop signal to noise ratio, viz.:

$$\overline{\theta_{no}^2} = \frac{1}{2(SNR)_L} \tag{4-15}$$

which leads to the <u>definition</u>

$$\boxed{(SNR)_L = \frac{(SNR)_i\, B_i}{2B_L} = \frac{P_s}{2B_L W_o}} \tag{4-16}$$

A logical oddity arises from this definition; Eq. 4-16 is the <u>definition</u> of $(SNR)_L$ for any value of signal-to-noise ratio, high or low. However, 4-15, which was used in arriving at the definition, is an approximation that is valid only for $(SNR)_L > 10$. There will be further discussion of the relation between phase jitter and $(SNR)_L$ later in this chapter.

It must also be observed that Eq. 4-16 is a somewhat arbitrary definition and is not unique.* Therefore, due caution should be

---

*An alternative definition, sometimes encountered, is that $(SNR) = P_s/B_L W_o$. For this definition, the expression for output phase jitter must be changed accordingly.

exercized in attempting to assign physical meaning to $(SNR)_L$. Signal to noise ratio in a loop does not have the same clearly discernible meaning as it would in, say, an IF amplifier.

4-2. Noise Threshold

Output phase jitter increases as the noise-to-signal ratio increases. A phase detector has only a limited range of operation; if the phase error exceeds this range, the loop will drop out of lock.

Phase error is a fluctuating statistical quantity. It is described by its rms value but noise peaks can greatly exceed the rms. For this reason, there is always some probability that the phase detector limits will be exceeded, no matter how small the noise. This probability is negligible for strong signals but becomes progressively larger as noise increases.

For sufficiently large noise, the probability of exceeding the phase detector limits will be large and it will be found to be nearly impossible to hold the loop in lock.

It has been found from practical experience (MAR-3) that lock cannot be held below 0-db signal-to-noise ratio in the loop. Actually, at this SNR the loop is in lock only part of the time and any additional disturbance will tend to cause complete loss of lock.

It is very nearly impossible to acquire a signal if $(SNR)_L = 0$ db. In general, $(SNR)_L$ of 6 db is needed for acquisition. If the frequency of the incoming signal is well-known, Martin (MAR-3) indicates that acquisition at $(SNR)_L = 3$ db is practicable. The question of acquisition behavior will be considered in more detail later.

If modulation or transient phase error is present, a higher signal-to-noise ratio is needed to acquire and hold lock.

It is often convenient to introduce the concept of loop threshold. The most general definition of threshold is: "that value of loop signal-to-noise ratio, below which desired performance cannot be obtained." Threshold is not defined until the criterion of performance is defined first.

The most obvious performance criterion to choose is loss of lock. However, as noted previously, "holding lock" can only be defined in a statistical sense inasmuch as a loop remains "in-lock" for some short period of time, even for high noise conditions.

If satisfactory statistical criteria were to be defined, there remains a more formidable barrier to analytical derivation of a threshold criterion. Loop behavior is non-linear and mathematical tools for non-linear analysis are generally inadequate for the phase lock loop. Nonetheless, there has been work that sheds considerable light on the problem.

Develet (DEV-1, 2) has derived an "absolute" unlock threshold. He assumed that the phase detector non-linearity can be approximated by considering effective phase detector gain to be dependent upon the loop signal-to-noise ratio. His conclusion is that the loop unlocks if loop SNR falls below + 1.34 db. At this threshold level, the rms phase jitter is calculated to be 1.0 radian. This result shows reasonably good agreement with Martin's empirical approximation (MAR-3) that threshold is close to 0 db, at which condition phase jitter is 1 radian rms.

A different threshold criterion might be taken as that value of loop SNR for which the output phase fluctuation exceeds some prescribed value. In order to make use of this criterion, it is necessary to know the behavior of $\overline{\theta_{no}^2}$ as a function of $(SNR)_L$.

If $(SNR)_L$ is large (+10 db or more) the linear approximation is valid, i.e.:

$$\overline{\theta_{no}^2} = \frac{1}{2(SNR)_L} \tag{4-15}$$

For small $(SNR)_L$, the approximation fails.

In the general case, there has been no solution for phase fluctuation versus $(SNR)_L$. However, for the special case of the first-order loop (loop filter transfer function F(s) = 1) Tikhonov (TIK-1, 2) and Viterbi (VIT-2, 5) have devised an exact solution of the problem. The asymptotes of $\overline{\theta_{no}^2}$ of the solution reduce to the linear case for large SNR and to $\pi^2/3$ for small SNR.

The value of $\pi^2/3$ arises because a random noise with phase uniformly distributed in the range of $-\pi$ to $+\pi$ can be shown to have a mean square phase of $\pi^2/3$ radians. (RMS phase fluctuation $\approx 104^o$)

The results for the first-order loop are very instructive but not many first-order loops are encountered in practice. As exact analysis has yet to be discovered for the much-used second-order loop.

Van Trees (VAN-2) obtains a quadratic approximation of

$$\overline{\theta_{no}^2} = \frac{1}{2(SNR)_L} + \frac{1}{6(SNR)_L^2} \qquad (4\text{-}17)$$

for the second order loop which is probably valid if $(SNR)_L > 1/2$. For unity signal to noise ratio it yields 0.82 radians rms phase error.

Viterbi's method, leading to a limiting variance of $\pi^2/3$ is very disturbing to the intuition since experience would seem to indicate that jitter should increase at least in proportion to increasing noise. His definition of phase sheds some light on the meaning of the asymptotic phase, $\pi^2/3$. He considers phase modulo 2π; that is if actual phase is ψ, he instead considers a phase of $\emptyset = \psi - 2n\pi$ where n is an integer such that $-\pi \leq \emptyset \leq + \pi$. Then, in order to take account of the fact that ψ can exceed $\pm \pi$ radians, he obtains the probability of skipping cycles. Thus, although $\overline{\theta_{no}^2}$ approaches $\pi^2/3$ (modulo 2π), the loop is continually slipping cycles.

The reason for this unusual definition of phase lies in the unfortunate mathematical properties of θ_{no}. Because there is some finite, if very small, probability of skipping cycles if any (gaussian) noise at all is present, an infinite number of cycles will have been skipped after an infinite time. Therefore, since the averaging interval for determining mean square jitter must be infinite to be mathematically correct, the rigorous application of the conventional definition of phase jitter leads to an infinite answer*.

*An alternative point of view may be obtained once it is recognized that the loop phase jitter - like the random walk - is not a stationary process. The conventional statistics of stationary processes therefore are not directly applicable.

Viterbi's redefinition of phase (modulo 2π) avoids the mathematical difficulty. Furthermore, almost any laboratory phase meter will have a range of no more than 2π radians; its measurements will be modulo 2π and determination of larger variations must be made by counting skipped cycles.

From these considerations, it appears that phase jitter is not a good criterion of threshold and that some other quantity might be preferable.

One such quantity might be the probability of skipping cycles. Viterbi has computed this quantity for the first order loop (VIT-5). No exact solution exists for the second order loop but Sanneman and Rowbotham (SAN-1) have performed a computer simulation and obtained approximate results. They considered a high-gain loop with damping of 0.707 and obtained the average elapsed time to skip one cycle, for various noise conditions. The investigation included several initial conditions but the result for zero initial error is sufficiently representative and is the only case presented here*.

Sanneman and Rowbotham's results are shown in Fig. 4-2. The straight-line fit to the data on semi-log paper suggests that mean time to unlock may be represented by

$$\boxed{T_{av} = \frac{2}{\omega_n} \exp\left[\pi\ (SNR)_L\right]} \tag{4-18}$$

at least for the range of SNR covered in Fig. 4-2. One is encouraged to accept this equation inasmuch as Viterbi (VIT-5), in his exact analysis of the first order loop also arrives at a simple exponential approximation at sufficiently high SNR. It would be of considerable interest to know whether Eq. (4-18) is valid at large SNR also.

Sanneman and Rowbotham obtained their results by many independent trials on the computer and were able to observe the statistical behavior of the experiment. They found that an exponential of the form

*As might be expected, any phase error (due, for example, to modulation) increases the probability of skipping cycles.

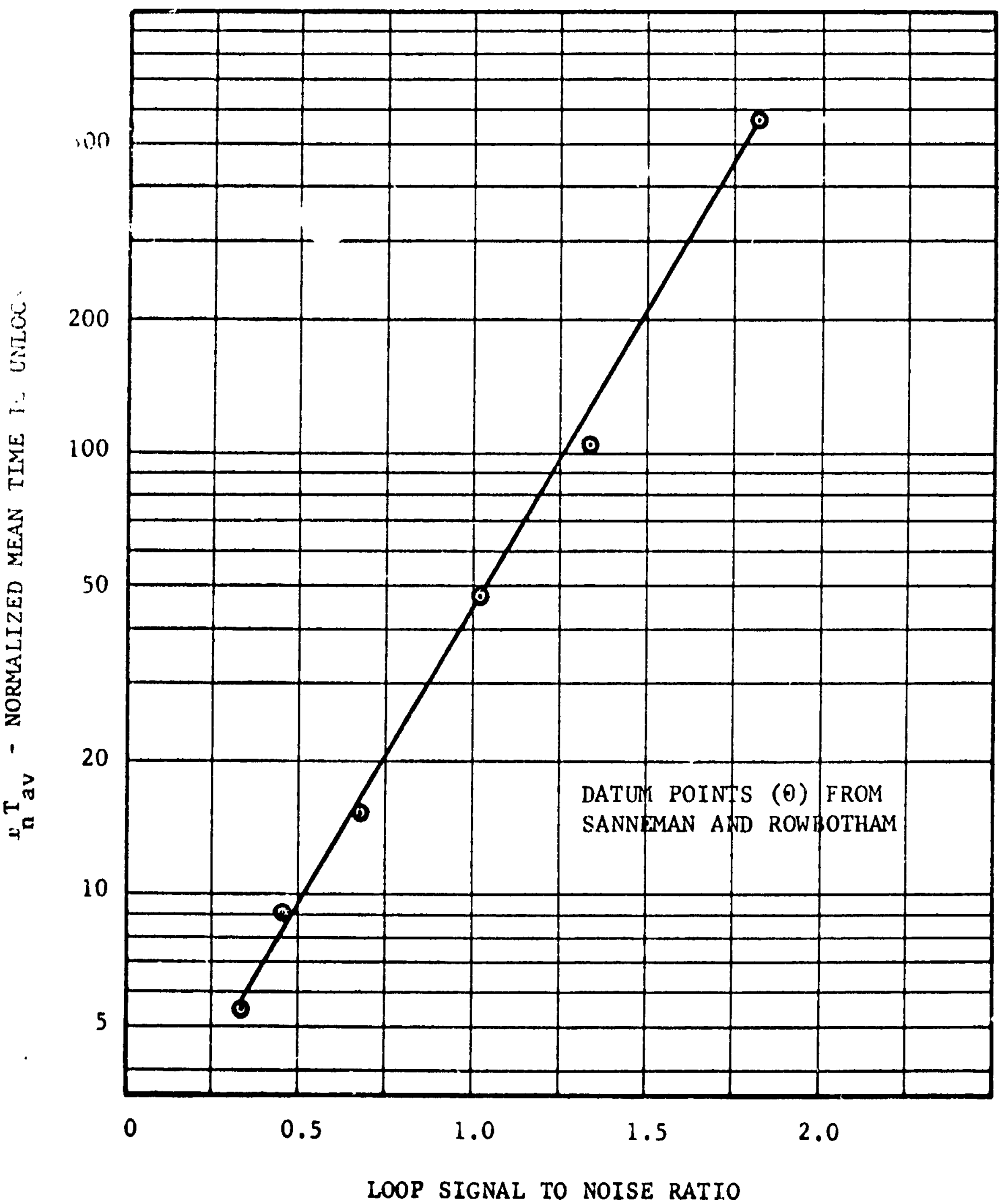

Fig. 4-2

Unlock Behavior of High-Gain, Second-Order Loop, $\zeta = 0.707$

$$P(T) = 1 - e^{-T/T_{av}}$$

provided a good empirical fit to their data. The quantity P(T) is the probability that the loop has skipped a cycle (unlocked) after time T has elapsed, starting from a zero-error initial condition.

It should be pointed out that these results are obtained only for a special case; a second-order constant-bandwidth high-gain loop with damping of 0.707. Although intuition might suggest that the results can be applied to other situations, caution should be exercised. In particular, if a limiter is used in the loop, damping and bandwidth are not constant but are functions of the input signal-to-noise ratio. (Effects of limiters are considered further in Chapter 6).

Furthermore, these results give no indication of loop behavior after the first skipped cycle. We are not told whether the loop settles down (temporarily, of course) in its new phase or if it falls completely out of lock and proceeds to skip cycles at an ever-increasing rate. There is probably no simple answer to this question; at high SNR, one would expect occasional skipping of individual cycles whereas the catastrophic behavior is more likely to be found at low SNR.

All these conditions and restrictions notwithstanding, the meaning of Fig. 4-2 is clear; loop performance is poor at unity signal-to-noise ratio in the loop and improves rapidly as SNR increases. If an approximate definition of threshold is desired, $(SNR)_L = 1.0$ will do very well.

Chapter 5

TRACKING AND ACQUISITION

5-1. Linear Tracking

To study tracking, we examine the phase error θ_e that results from a specified input, θ_i. A small phase error is usually desired and is considered to be the criterion of good tracking performance.*

Phase error (in the frequency domain) is given by

$$\theta_e(s) = \frac{s\theta_i(s)}{s + K_o K_d F(s)} \tag{5-1}$$

The simplest errors to analyze are the steady state errors remaining after any transients have died away. These errors are readily evaluated by means of the Final Value Theorem of La Place Transforms (Chapter 2).

Phase error is studied because, in a locked loop, there is no average frequency error. For each cycle of the input, there must be a corresponding cycle of the output. If the VCO skips cycles, the loop is considered to have lost lock, even if only monentarily. The problems of unlock behavior will be considered in a later section. Here, the concern is with tracking of a locked loop.

The final value theorem states

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) \tag{5-2}$$

which is to say, the steady state value of a function in the time domain is readily determined from inspection of its transform in the frequency domain.

Applying the final value theorem to the phase error equation yields

$$\lim_{t \to \infty} \theta_e(t) = \lim_{s \to 0} \frac{s^2\theta_i(s)}{s + K_o K_d F(s)} \tag{5-3}$$

*Discriminators are a special case where the "phase error" is the useful output and, therefore, close tracking is not necessarily useful.

As a useful example, consider the steady-state error resulting from a step change of frequency, of magnitude $\Delta\omega$. Input phase is a ramp so $\theta_i(s) = \Delta\omega/s^2$. Substituting this value for θ_i into (5-3) results in

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} \frac{\Delta\omega}{s + K_o K_d F(s)} = \frac{\Delta\omega}{K_o K_d F(0)} \tag{5-4}$$

The product $K_o K_d F(0)$ is often called the "velocity constant" or "DC loop gain" and is denoted by the symbol K_v. Those familiar with servos will recognize it as the velocity error coefficient. Note that K_v has the dimensions of frequency.

It is not to be expected that the incoming signal frequency will agree exactly with the free-running (zero control voltage) frequency of the VCO. There will generally be a frequency difference $\Delta\omega$ between the two. The frequency difference may be due to an actual difference between the transmitter and receiver or it may be due to a Doppler shift. In either case, the resulting phase error is often called the "velocity error" or, simply, "static phase error" and is given by

$$\boxed{\theta_v = \frac{\Delta\omega}{K_v}} \tag{5-4a}$$

Let us now evaluate K_v for the second-order loop. Two types of loop filters were considered in Chapter 3: a passive filter and an active filter. For the passive filter $F(0) = 1$ whereas, for the active filter, $F(0) = A$, where A is the DC gain of an operational amplifier. Assuming $K_o K_d$ the same in both cases, it may be seen that K_v will be much larger, and θ_v much smaller, if an active filter is used. (Voltage gains of 10^2 to 10^7 are typical.) As a practical matter, it is not difficult, in most cases, to make A sufficiently large so that θ_v is no more than a few degrees for the maximum frequency displacement encountered.

Next, let us suppose that the input frequency is linearly changing with time at a rate of $\Delta\dot{\omega}$ radians per second^2. Such input behavior might arise from accelerated motion between transmitter and receiver, from change of Doppler frequency during an overhead pass of a satellite, or from sweep-frequency modulation.

Input phase is $\theta_i(s) = \Delta\dot{\omega}/s^3$ and it can be shown that phase error will grow without bound if K_v is finite. It is of interest to calculate this rate of growth.

By the final value theorem, the steady state rate of change of phase would be

$$\lim_{t \to \infty} \frac{d\theta_e(t)}{dt} = \lim_{s \to 0} s \left[s\theta_e(s) \right]$$

$$= \lim_{s \to 0} \frac{s^3 \theta_e(s)}{s + K_o K_d F(s)} = \lim_{s \to 0} \frac{\Delta\dot{\omega}}{s + K_o K_d F(s)}$$

$$= \frac{\Delta\dot{\omega}}{K_v} \text{ radians per second.} \qquad (5\text{-}5)$$

and the accumulated phase error after an elapsed time t is $\Delta\dot{\omega} t/K_v$. This expression will be recognized as nothing more than the previously derived velocity error and can be neglected for sufficiently large K_v.

Suppose that the gain of the operational amplifier is infinite so that phase error may be written as

$$\theta_e(s) = \frac{s^2 \theta_i(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (5\text{-}6)$$

This leads to the "acceleration error" (sometimes called "dynamic tracking error").

$$\theta_a = \lim_{t \to \infty} \theta_e(t) = \lim_{s \to 0} \frac{\Delta\dot{\omega}}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (5\text{-}7)$$

$$\boxed{\theta_a = \frac{\Delta\dot{\omega}}{\omega_n^2} \text{ radians}} \qquad (5\text{-}7a)$$

It is sometimes necessary to track an accelerating transmitter without steady-state tracking error. Let us determine the form of F(s) needed to reduce θ_a to zero.

The expression for final value acceleration error is

$$\theta_a = \lim_{s \to 0} \frac{\Delta\dot{\omega}}{s\left[s + K_o K_d F(s)\right]} \qquad (5\text{-}8)$$

In order for θ_a to be zero, it is necessary that F(s) have the form $G(s)/s^2$, where $G(0) \neq 0$. The factor $1/s^2$ implies that the loop filter must contain two cascaded integrators. Closed loop response then has a polynomial of third degree in its denominator and we speak of a third-order loop. Because of this property of eliminating the steady-state acceleration error, a third-order loop is occasionally used in tracking of satellites.

Next, let us investigate loop behavior in the presence of a modulated input. For sinusoidal phase modulation

$$\theta_i(t) = \Delta\theta \sin \omega_m t \tag{5-9}$$

and for sinusoidal frequency modulation

$$\theta_i(t) = \frac{\Delta\omega}{\omega_m} \cos \omega_m t \tag{5-10}$$

where $\Delta\theta$ is peak phase deviation, $\Delta\omega$ is peak frequency deviation and ω_m is modulating frequency.

The phase error in each case is then

$$\theta_e\Big]_{PM} = \frac{\omega_m \sqrt{\omega_m^2 + \dfrac{\omega_n^4}{\left(K_o K_d\right)^2}}}{\sqrt{\left(\omega_n^2 - \omega_m^2\right)^2 + 4\zeta^2\omega_n^2\omega_m^2}} \Delta\theta \sin(\omega_m t + \psi) \tag{5-11}$$

$$\theta_e\Big]_{FM} = \frac{\omega_m \sqrt{\omega_m^2 + \dfrac{\omega_n^4}{(K_o K_d)^2}}}{\sqrt{\left(\omega_n^2 - \omega_m^2\right)^2 + 4\zeta^2\omega_n^2\omega_m^2}} \frac{\Delta\omega}{\omega_m} \cos(\omega_m t + \psi) \tag{5-12}$$

where

$$\psi = \pi/2 + \tan^{-1}(\omega_m K_o K_d/\omega_n^2) - \tan^{-1}\left[2\zeta\omega_n\omega_m/(\omega_n^2-\omega_m^2)\right] \tag{5-13}$$

These expressions are simply the steady-state frequency response of the loop (MAR-3).

(Note: Eqs. 5-14, 15, 16 have been deleted.)

For PM, with a fixed phase deviation $\Delta\theta$, the phase error is small for low modulating frequencies, rises at 12 db per octave, and eventually levels out at high frequencies to be equal to the deviation. This behavior is the "error response" as plotted in Figure 3-4.

For FM, with fixed deviation $\Delta\omega$, the phase error is small at low modulation frequencies, rises to a maximum at $\omega_m = \omega_n$, and falls off at higher frequencies. Asymptotes at low and high frequencies are 6 db per octave. Response is plotted in Figure 5-1 for several values of damping factors.

Finally, let us consider the transient behavior of loop error for various inputs consisting of

1. A step of phase, $\Delta\theta$ radians.
2. A step of frequency (phase ramp), $\Delta\omega$ radians per second.
3. A step of acceleration (frequency ramp), $\Delta\dot{\omega}$ radians per second^2.

For each of these inputs, the $\mathcal{L}$-Transformed input phase is $\Delta\theta/s$, $\Delta\omega/s^2$, and $\Delta\dot{\omega}/s^3$, respectively. To compute phase errors, each input is substituted into Eq.(5-6) and inverse $\mathcal{L}$-Transforms are then computed (or looked up in tables) to determine time response. The results for the special but important case of a high gain second-order loop are shown in Table 5-1 (HOF-1).

These expressions are not unduly complex but are nonetheless quite tedious to evaluate without a computer. The chore of computation has already been performed by Hoffman (HOF-1) and his plots of transient error versus time are shown in Figs. 5-2, 3, and 4, for various damping factors ζ.

5-2. Hold-in Performance

All of the previous material on tracking and phase error is based upon the assumption that the error is sufficiently small that the loop can be considered to be linear in its operation. This assumption becomes progressively worse as error increases until, finally, the loop drops out of lock and the assumption becomes worthless. In this section, the linear assumption is discarded and the limiting conditions for which a loop holds lock are investigated.

The most commonly encountered phase detector* is one whose output voltage e_d is related to phase error by

$$e_d = K_d \sin \theta_e \tag{5-17}$$

For sufficiently small error, $\theta_e \approx \sin \theta_e$ and the linear approximation is usable. In this section, no linear approximation will be made.

The first topic considered will be the input frequency range over which the loop will hold lock. In Eq. 5-4a, the linear approximation of phase error due to a frequency offset was shown to be $\theta_v = \Delta\omega/K_v$.

However, for a sinusoidal characteri .. phase detector, the true expression should be (GRU-1)

$$\sin \theta_V = \frac{\Delta\omega}{K_V}$$

The sine function cannot exceed unit magnitude; therefore, if $\Delta\omega > K_v$, there is no solution to this equation. Instead, the loop falls out of lock and the phase detector voltage becomes a beat-note rather then a DC level. The hold-in range of a loop may therefore be defined as

$$\Delta\omega_H = \pm K_v \tag{5-18}$$

Equation 5-18 states that the hold-in range can be made arbitrarily large, simply by using very high loop gain. Of course, this cannot be entirely

*For discussion of various types of phase detectors, see Chapter 6.

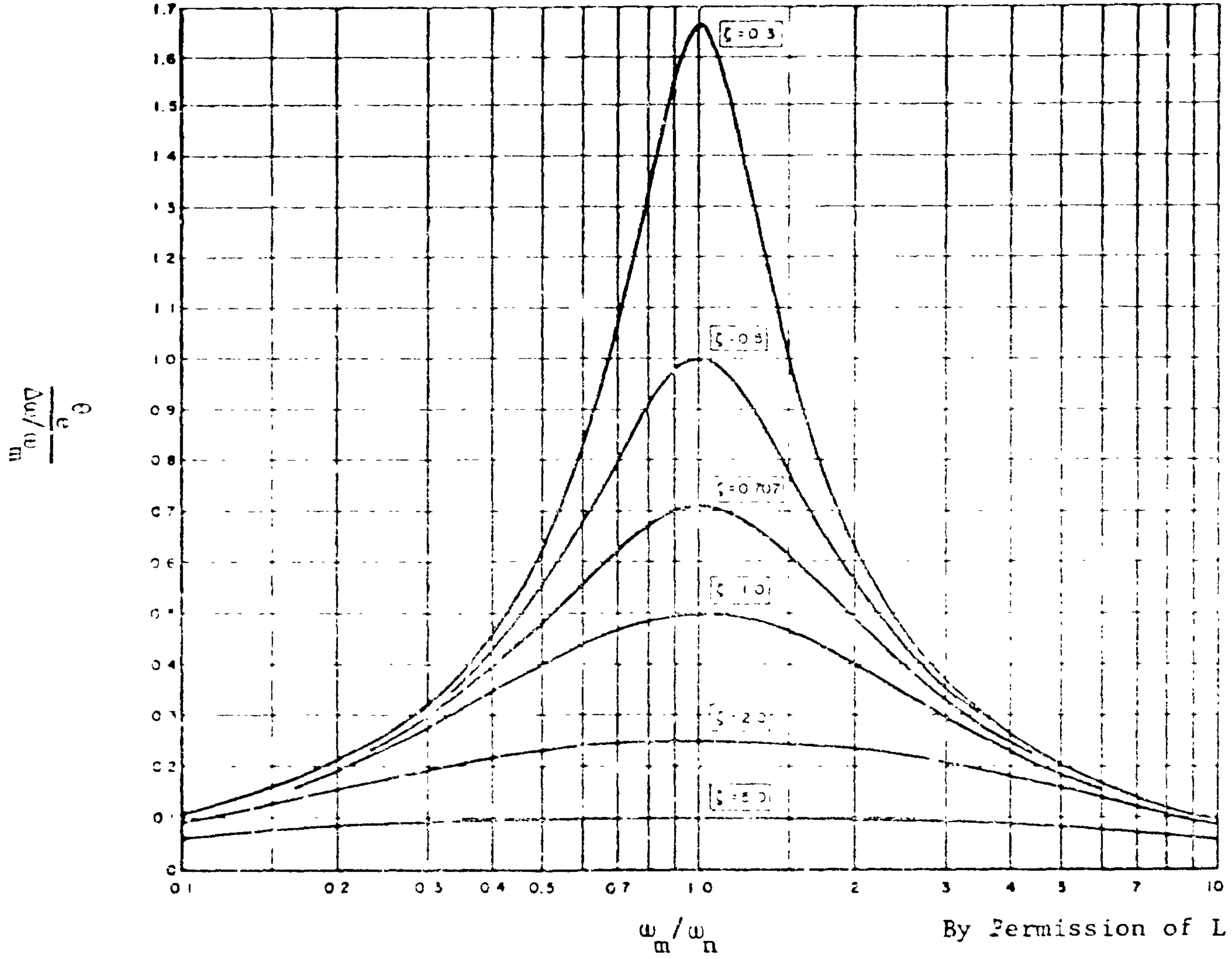

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Fig. 5-1

Steady-State Peak Phase-Error Due to Sinusoidal FM

Input / Damping	Phase Step $\Delta\theta$ Radians	Frequency Step $\Delta\omega$ Radians/Sec	Frequency Ramp $\Delta\dot{\omega}$ Radians/Sec^2
$\zeta < 1$	$\Delta\theta\left[\cos\sqrt{1-\zeta^2}\,\omega_n t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\sqrt{1-\zeta^2}\,\omega_n t\right]e^{-\zeta\omega_n t}$	$\frac{\Delta\omega}{\omega_n}\left[\frac{1}{\sqrt{1-\zeta^2}}\sin\sqrt{1-\zeta^2}\,\omega_n t\right]e^{-\zeta\omega_n t}$	$\frac{\Delta\dot{\omega}t}{K_v} + \frac{\Delta\dot{\omega}}{\omega_n^2} - \frac{\Delta\dot{\omega}}{\omega_n^2}\left[\cos\sqrt{1-\zeta^2}\,\omega_n t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\sqrt{1-\zeta^2}\,\omega_n t\right]e^{-\zeta\omega_n t}$
$\zeta = 1$	$\Delta\theta\left[1-\omega_n t\right]e^{-\omega_n t}$	$\frac{\Delta\omega}{\omega_n}(\omega_n t)e^{-\omega_n t}$	$\frac{\Delta\dot{\omega}t}{K_v} + \frac{\Delta\dot{\omega}}{\omega_n^2} - \frac{\Delta\dot{\omega}}{\omega_n^2}\left[1 + \omega_n t\right]e^{-\omega_n t}$
$\zeta > 1$	$\Delta\theta\left[\cosh\sqrt{\zeta^2-1}\,\omega_n t - \frac{\zeta}{\sqrt{\zeta^2-1}}\sinh\sqrt{\zeta^2-1}\,\omega_n t\right]e^{-\zeta\omega_n t}$	$\frac{\Delta\omega}{\omega_n}\left[\frac{1}{\sqrt{\zeta^2-1}}\sinh\sqrt{\zeta^2-1}\,\omega_n t\right]e^{-\zeta\omega_n t}$	$\frac{\Delta\dot{\omega}t}{K_v} + \frac{\Delta\dot{\omega}}{\omega_n^2} - \frac{\Delta\dot{\omega}}{\omega_n^2}\left[\cosh\sqrt{\zeta^2-1}\,\omega_n t + \frac{\zeta}{\sqrt{\zeta^2-1}}\sinh\sqrt{\zeta^2-1}\,\omega_n t\right]e^{-\zeta\omega_n t}$
	Steady State Error = 0	Steady State Error = $\frac{\Delta\omega}{K_v}$ (Not included above)	Steady State Error = $\frac{\Delta\dot{\omega}t}{K_v} + \frac{\Delta\dot{\omega}}{\omega_n^2}$ (Included above)

Table 5-1

Transient Phase Error of Second-Order Loop, $\theta_e(t)$ in radians

(High Loop Gain; $K_o K_d \gg \omega_n$)

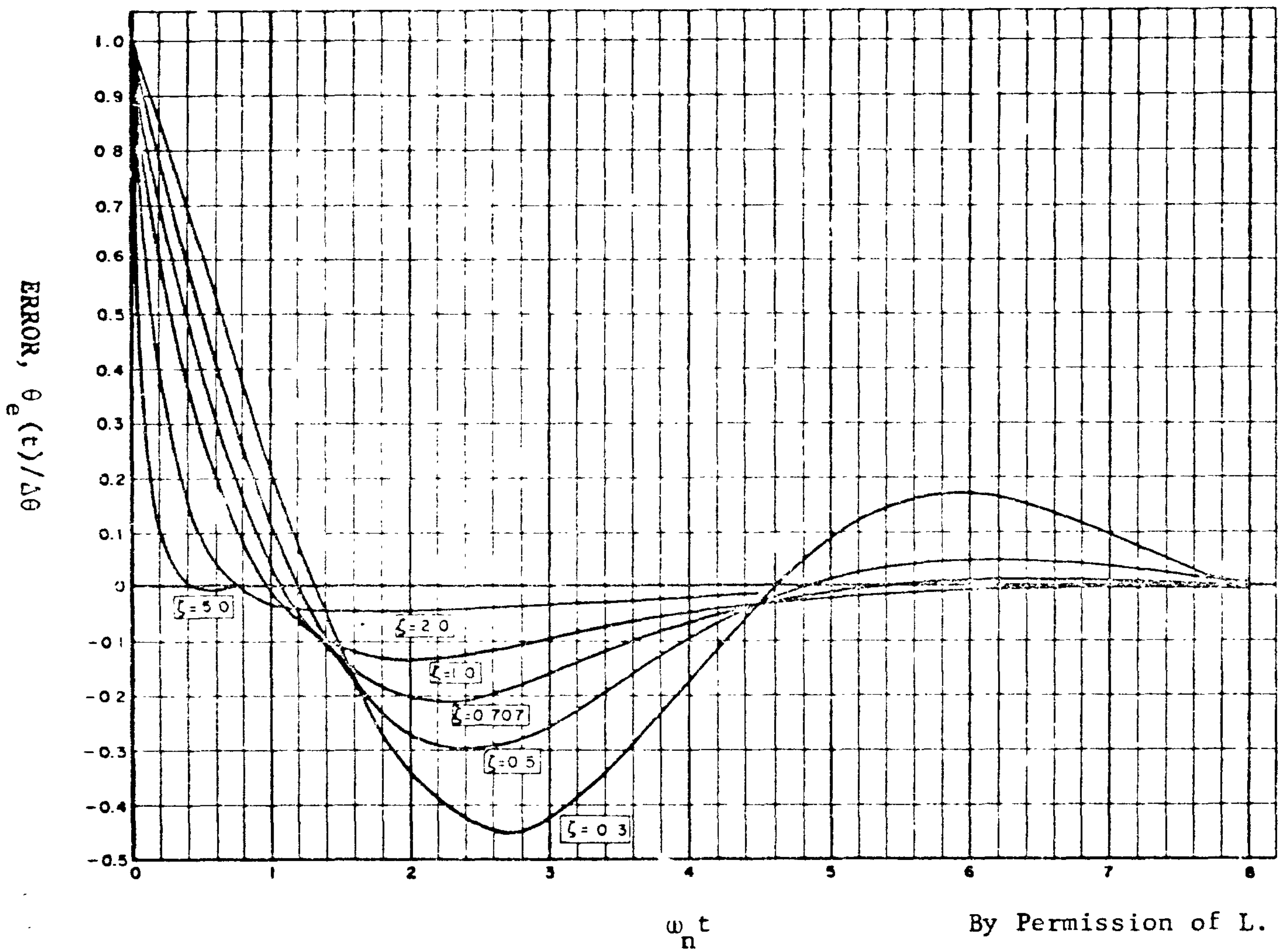

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Fig. 5-2

Phase-Error $\theta_e(t)$ Due to a Step in Phase $\Delta\theta$

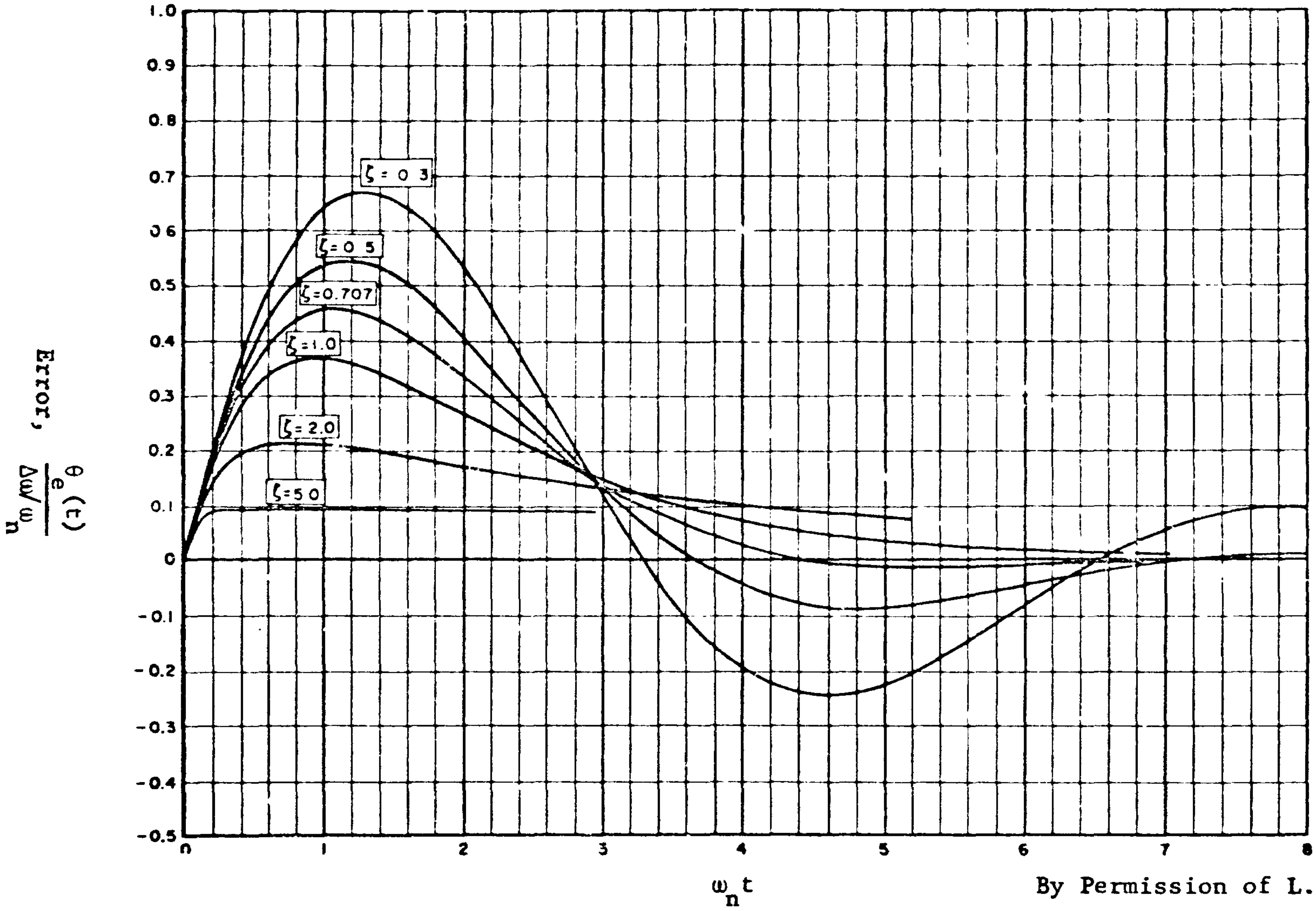

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Fig. 5-3

Transient Phase-Error $\theta_e(t)$ Due to a Step in Frequency $\Delta\omega$

(Steady-State Velocity Error, $\Delta\omega/K_v$, neglected

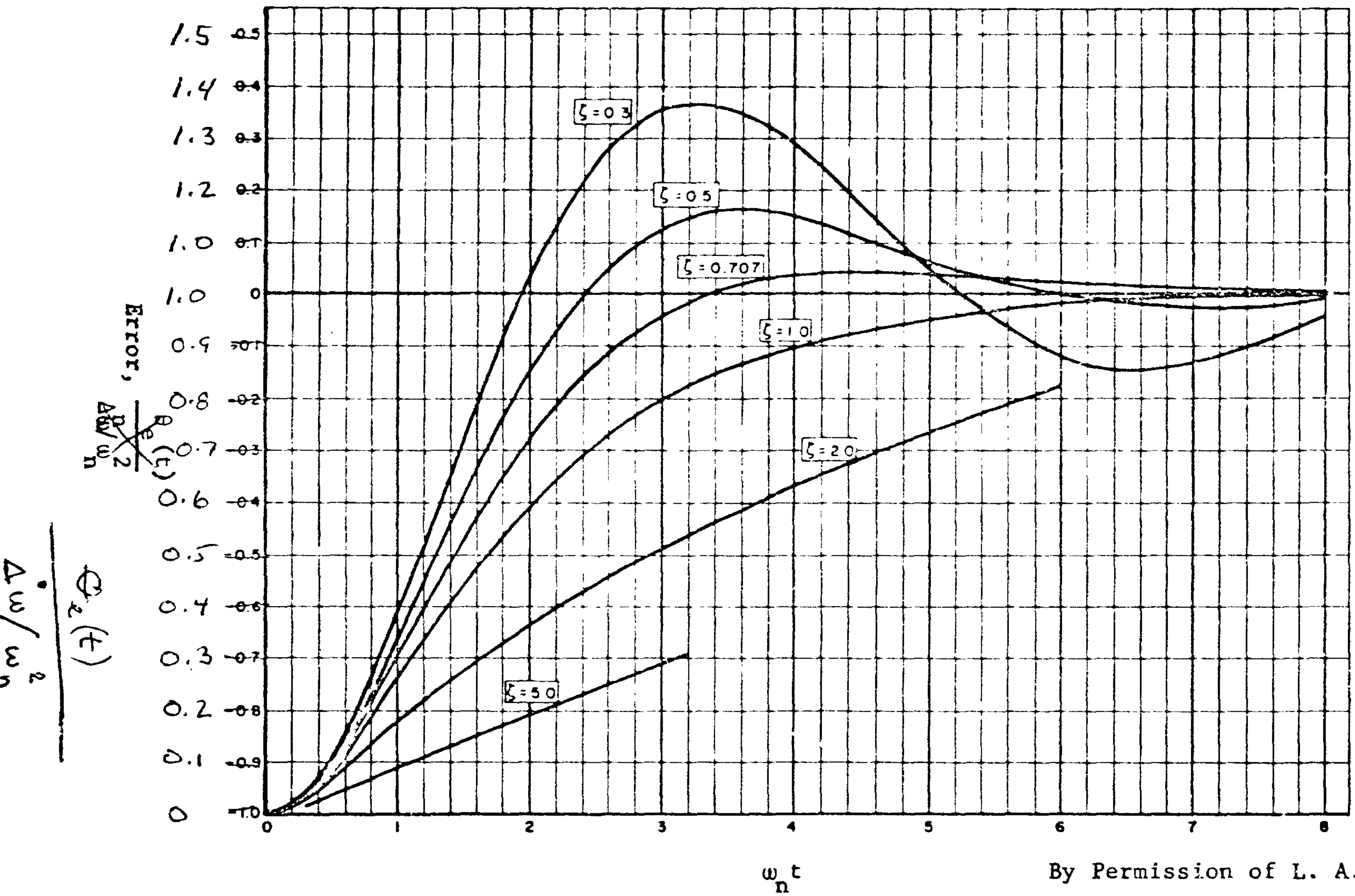

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Fig. 5-4

Transient Phase-Error $\theta_e(t)$ Ramp in Frequency $\Delta\dot{\omega}$ (Steady-State Acceleration Error, $\Delta\dot{\omega}/\omega_n^2$, included. Velocity Error, $\Delta\dot{\omega}t/K_v$, neglected) correct because some other component in the loop will then saturate before the phase detector. That is to say, to achieve any given frequency deviation of the VCO, some definite control voltage is needed. However, the loop amplifier (if one is used) has some maximum voltage it can deliver and the VCO has some maximum voltage it can accept. If either of these limits is exceeded, the loop unlocks. It is not uncommon to find active loops with such high gain that the amplifier saturates when static phase error is only a few degrees.

Dynamic error in a second-order loop was previously (Eq. 5-7a) approximated as $\theta_a = \Delta\dot{\omega}/\omega_n^2$. The correct expression should be

$$\sin \theta_a = \frac{\Delta\dot{\omega}}{\omega_n^2} \qquad (5\text{-}19)$$

from which it may be deduced that the maxin.. permissible rate of change of input frequency is (VIT-1)

$$\Delta\dot{\omega} = \omega_n^2 \qquad (5\text{-}20)$$

If the input rate should exceed this amount, the loop falls out of lock. (To anticipate matters covered in the next section, acquisition of lock at sweep rates approaching ω_n^2 is very difficult or impossible.)

In the case of a step of frequency, Figure 5-3 shows that the transient phase error greatly exceeds the static error. One might well ask, can the transient error pull the loop out of lock, even if the static error is within the hold-in range? The answer is not simple; it depends upon circumstances and is, perhaps, not fully established yet in the literature. A summary of published results is presented in the following paragraphs.

First, consider the infinite-gain, second-order loop. Rue and Lux (RUE-1) point out that, in principle, this type of loop can never lose lock permanently. If a large frequency step is applied, the loop unlocks, skips cycles for a while, and then locks up once again. The phase error is a ringing oscillation for a number of cycles corresponding to the number of cycles skipped.

There is some frequency step limit below which the loop does not skip cycles but remains in lock. Viterbi (VIT-1) shows phase-plane trajectories for different values of damping factor. From these plots it is possible to determine the pull-out frequency; results are shown in Figure 5-5. The

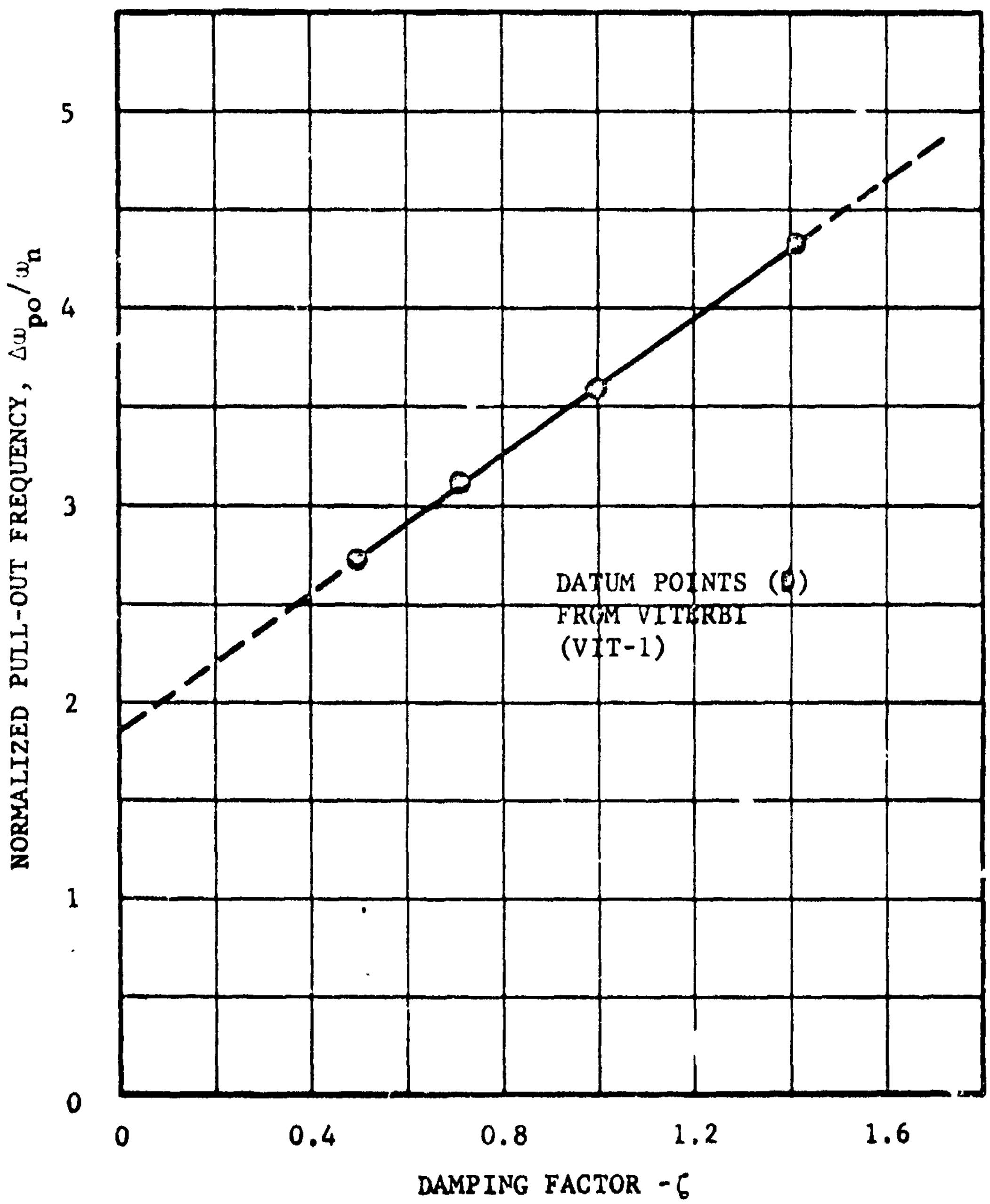


Fig. 5-5

Pull-out Frequency of High-Gain, Second-Order Loop data are well-fitted by a straight line with the equation

$$\Delta\omega_{PO} = \omega_n(\zeta\sqrt{\pi} + \ln 2\pi) \tag{5-21}$$

at least for the range of ζ covered by Viterbi.

To reiterate, if a step of frequency is less than $\Delta\omega_{P}$ the transient error is such that the loop remains in lock. If $\Delta\omega > \Delta\omega_{PO}$, the loop skips cycles before settling into lock once again.

For $\Delta\omega = \Delta\omega_{PO}$, the peak phase error is 180°, not 90° as might be supposed. However, the error increases very rapidly once it exceeds 90° so that the frequency step causing 90° peak error is only slightly less than $\Delta\omega_{PO}$. Figure 5-6 illustrates the situation for the special case of $\zeta = 0.707$.

(The fit to the true data provided by the linear approximation is of considerable interest. Phase error predicted by the approximation is seen to be within 5% of being correct for errors as large as 50°. Furthermore, the extrapolated linear error reaches 90° at a frequency only 8% higher than the actual pull-out frequency. These results suggest that loose application of the linear approximation will often lead to results that are not grossly incorrect.)

This discussion of pull-out frequency and peak error (below pull-out) has been restricted to the case of the high-gain second-order loop. Other-order loops (REY-1, VIT-1) have very different performance.

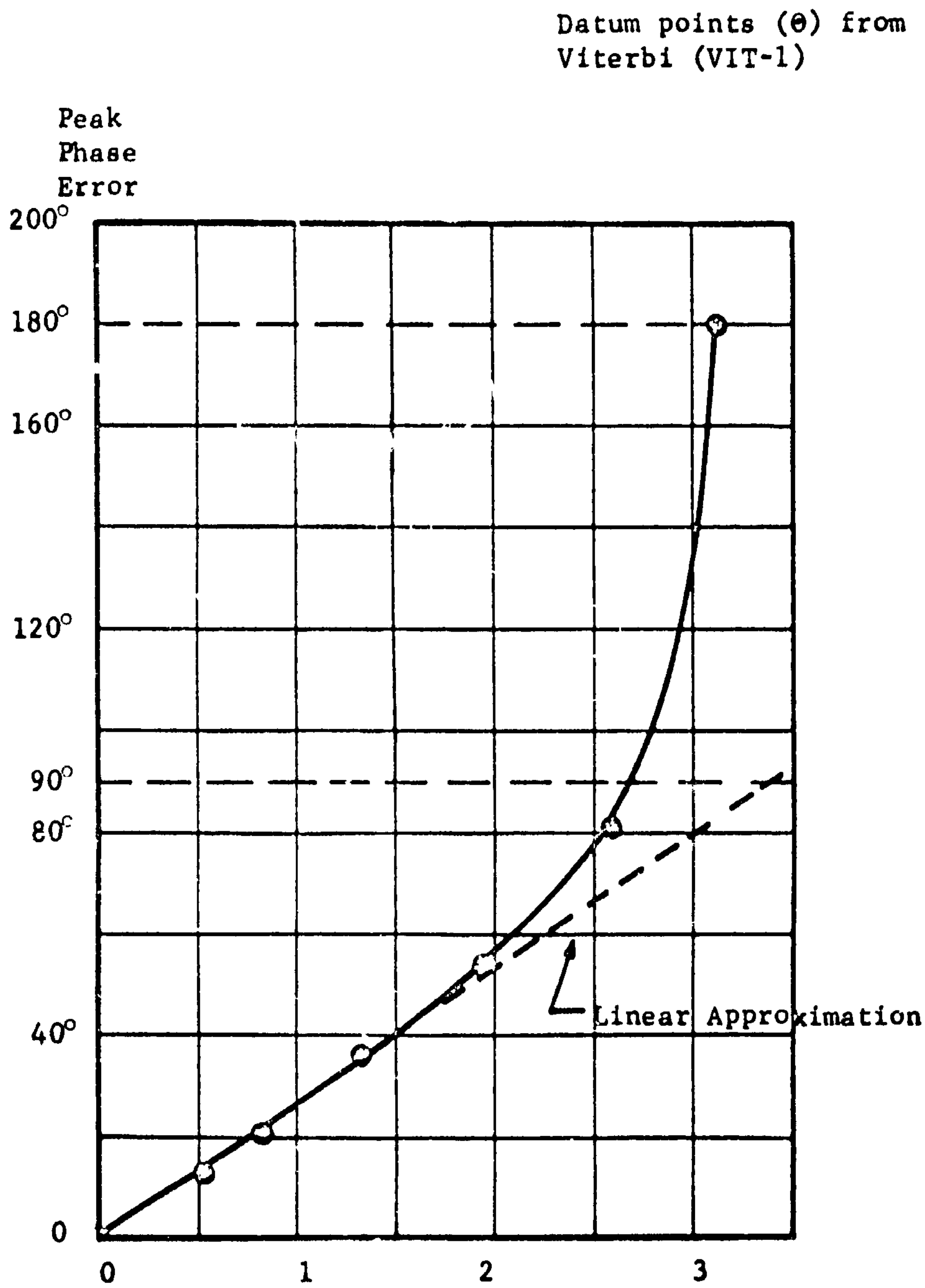
For example, the first-order loop (no loop filter) has a hold-in frequency equal to its 3-db frequency, which is also equal to its pull-out frequency, and, as will be discussed later, is also equal to its pull-in frequency. That is,

$$\Delta\omega = K_v \tag{5-22}$$

where $\Delta\omega$ has the meaning of any of the above frequencies, and K_v is the loop gain. Maximum possible phase error cannot exceed 90°.

For a second-order loop of moderate gain, one would expect performance to be degraded from the high-gain case. It is to be expected that the pull-out frequency, as previously defined, would probably be reduced*, but no

*Viterbi (VIT-1) has generated some phase-plane trajectories of a moderate-gain loop. These data indeed show a slightly reduced pull-out frequency.



NORMALIZED FREQUENCY STEP

$\Delta\omega/\omega_n$

Figure 5-6

Peak Phase Error Due to Frequency Step

High-Gain, Second-Order Loop

$\zeta = 0.707$

quantitative determination of the reduction has been made. Furthermore, if the step is sufficiently large, the loop will drop out of lock and stay out. Let us call this limit the "drop-out" frequency. (It is more commonly (REY-1) called the pull-out frequency but that name has already been used here for another quantity. Available terminology is becoming scarce.)

It should be clear that the drop-out frequency $\Delta\omega_{DO} < K_v$.

There are indications (REY-1) that $0.72\ K_v < \Delta\omega_{DO} < K_v$ but this is not at all certain. Intuitively, it would seem that the drop-out frequency should equal the pull-in frequency (which will be discussed in the next section). This topic bears further investigation.

Third-order loops are discussed by Viterbi (VIT-1) and Gupta (GUP-1). Briefly, the third-order loop exhibits significant improvement in tracking performance over a second-order loop when the input is a frequency ramp.

One must also be concerned with loop hold-in problems when the input signal is angle-modulated (i.e., phase-lock loop used as a discriminator in FM-FM telemetry). Following Martin (MAR-3) we distinguish between "Carrier Tracking Loops" in which the modulation is entirely outside the loop bandwidth and "Modulation Tracking Loops" in which the modulation spectrum is primarily within the loop bandwidth. The first type of loop is used for PM demodulation while the second is used as an FM discriminator.

In the Carrier Tracking Loop, the modulation must be restricted so that there actually is a carrier to track. If sinusoidal phase modulation of peak deviation θ is applied, the carrier strength is proportional to the zero-order Bessel function $J_o(\theta)$. This function passes through its first zero for $\theta = 2.4$ radians (137^o). Moreover, to avoid severe distortion of the recovered modulation, the deviation has to be less than 90^o. In other words, where a phase-lock loop is used as a PM demodulator, the modulation index must be limited to relatively small values (certainly less than 2.4).

The situation for FM is not so restrictuve. Since the loop tracks the modulation, it is possible to have an arbitrarily large modulation index. It is only necessary that the loop bandwidth be wide enough to track the modulation sufficiently closely.

Runyan (RUN-2) defines "sufficiently closely" as meaning that the loop phase error remains within the linear range of the phase detector.* This

*He also provides some dramatic laboratory examples illustrating the effects of over-modulation.

restraint will avoid distortion but is conservative with respect to hold-in capabilities. The curves of Figure 3-3 show peak sinusoidal phase error versus modulating frequency for fixed frequency deviation. It is apparent that the greatest error occurs when the modulating frequency is equal to the loop natural frequency.

As a reasonable rule of thumb, the loop should remain locked if this peak error always remains less than 90°.

In all of these discussions of hold-in and pull-out behavior, it has been tacitly assumed that the loop was essentially noise-free. If noise is present it can be expected that performance will be degraded.

Where quantitative results have been given, a sinusoidal-characteristic phase detector has been assumed. If a triangular or saw-tooth characteristic (see Chapter 6) were used instead, it is likely that improved hold-in performance could be obtained.

Finally, the phase detector has been assumed to be the only non-linear element in the loop. The analyses would have to be revised drastically if saturation of the loop amplifier or VCO were a significant problem.

5-3. Acquisition

For all of the topics so far discussed in this and previous chapters, it has been tacitly assumed that the loop was initially in lock. The purpose of this section is to examine an out-of-lock loop and explain how it may be brought into lock.

There are a number of methods by which lock can be acquired:

1. If, for some reason, the frequency difference between input and VCO is less than the loop bandwidth, the loop will lock up almost instantaneously without slipping cycles. The maximum frequency difference for which this fast acquisition is possible will be called the lock-in frequency, $\Delta\omega_L$.
2. There are loop types (including the most common second-order loop) in which the VCO frequency will slowly walk in towards the input frequency, despite the fact that the initial difference frequency may greatly exceed the loop bandwidth. The maximum difference frequency from which the loop will eventually lock itself is called the pull-in frequency $\Delta\omega_P$.
3. The loop could be outside pull-in range, or pull-in might require too long a time. In that case the VCO can be swept at a suitable rate in order to search for the signals.

4. If noise level is sufficiently low, faster acquisition is possible if the loop bandwidth is widened.
5. A frequency discriminator can be used to adjust the VCO to within lock-in range of the input frequency in order to acquire rapidly.

The remainder of this chapter is devoted to consideration of all of these topics.

It is instructive to begin the discussion with an analysis of a first-order loop. There is no filter in this loop $\left[F(s) = 1\right]$ so the linearized loop transfer function is

$$H(s) = \frac{K_v}{s + K_v} \tag{5-23}$$

The 3-db frequency (loop bandwidth) is K_v radians per second and it was earlier shown (Eq. 5-22) that the hold-in frequency limit is also K_v.

To show lock-in performance, we will derive the non-linear differential equation of the loop and analyze its meaning. For this purpose, let ω_i be the input frequency (assumed constant) and ω_o equal to the center frequency of the VCO so that the instantaneous frequency of the VCO is $\omega_o + K_o e_d$. Voltage $e_d = K_d \sin \theta_e$ is the error voltage out of the phase detector

Input phase is $\omega_i t$ and output phase is

$$\theta_o = \omega_o t + \int_o^t K_o e_d dt \tag{5-24}$$

$$= \omega_o t + \int_o^t K_o K_d \sin \theta_e dt$$

Phase error is

$$\theta_e = \theta_i - \theta_o = \omega_i t - \omega_o t - \int_o^t K_v \sin \theta_e dt \tag{5-25}$$

Let $\omega_i - \omega_o = \Delta\omega$ and differentiate to obtain

$$\dot{\theta}_e = \Delta\omega - K_v \sin \theta_e \tag{5-26}$$

This is the non-linear differential equation of the first-order phase-lock loop.

The loop is locked only if $\dot{\theta}_e$ is zero, by definition of lock. However, we must show that the converse is true; that if $\dot{\theta}_e = 0$, the loop is necessarily locked.

From the first condition, the hold-in limit is obtained. That is, if $\dot{\theta}_e = 0$, then $\sin \theta_e = \Delta\omega/K_v$. Since $\sin \theta_e$ cannot exceed unity, the loop can lock only if $\Delta\omega/K_v < 1$.

To examine the second question, first observe that the values of θ_e for which $\dot{\theta}_e = 0$ are given by

$$\theta_{e1} = \sin^{-1} \frac{\Delta\omega}{K_v} + 2n\pi$$

and

$$\theta_{e2} = (2n - 1)\pi - \sin^{-1} \frac{\Delta\omega}{K_v}$$

where n could be any integer. These nulls may be seen to alternate with one-another along the θ_e axis.

Next consider the nature of $\dot{\theta}_e$ if θ_e is slightly displaced from one of the nulls. To do this, differentiate $\dot{\theta}_e$ with respect to θ_e and obtain

$$\delta\dot{\theta}_e = -K_v (\delta\theta_e) \cos \theta_e$$

$$\delta\dot{\theta}_{e1} = -\delta\theta_{e1} K_v \cos\left[\sin^{-1} \frac{\Delta\omega}{K_v} + 2n\pi\right]$$

or

$$\delta\dot{\theta}_{e2} = -\delta\theta_{e2} K_v \cos\left[-\sin^{-1} \frac{\Delta\omega}{K_v} + (2n - 1)\pi\right]$$

By applying standard trigonometric identities and manuplating one obtains

$$\delta\dot{\theta}_{e1} = -\delta\theta_{e1}\sqrt{K_v^2 - \Delta\omega^2}$$

or

$$\delta\dot{\theta}_{e2} = \delta\theta_{e2}\sqrt{K_v^2 - \Delta\omega^2}$$

At the first set of nulls, the sign of $\delta\dot{\theta}_{e1}$ is opposite that of $\delta\theta_{e1}$ so that any change in θ_e must be in the direction of the null. Thus, the first set of nulls are stable; if the loop reaches any one of them it locks up.

At the second set of nulls, $\delta\dot{\theta}_{e2}$ and $\delta\theta_{e2}$ have the same sign; any change in θ_e must be away from the null and the null is therefore unstable.

Prior to lock $\dot{\theta}_e$ is non-zero which means that θ_e must change (increase or decrease) monotonically. For this reason, θ_e must eventually take on the value of one of the stable nulls (provided, of course, that $\Delta\omega < K_v$). When θ_e reaches a stable null, the loop is locked and θ_e remains fixed at the static error. From this argument, it may be concluded that the lock-in and pull-in frequencies are both equal to K_v radians per second in the first-order loop.

Since every cycle has a stable null, θ_e cannot change by more than a half cycle before locking. Thus, there is no cycle-skipping in the lock-up process. The time required to lock-up depends upon the initial values of phase and frequency but, as a rough rule of thumb, will be on the order of $1/K_v$ seconds.

Because a first-order loop is so rarely found in practice, its analysis is of interest only for the light it sheds on high-order loops. In particular, the second-order loop is of greatest concern because of its widespread usage. We will first obtain an expression for lock-in frequency and then discuss the pull-in phenomenon.

The frequency response of the loop filter of a second-order loop is shown in Fig. 5-7. At high frequencies, the gain of the filter is $\tau_2/(\tau_1 + \tau_2)$ for a passive filter, or just τ_2/τ_1 for an active filter. (Note that the gain of the amplifier does not enter into the high-frequency gain.) Total loop gain at high frequencies is therefore $K_o K_d \tau_2/(\tau_1 + \tau_2)$ or $K_o K_d \tau_2/\tau_1$.

At high frequencies, this loop is indistinguishable from a first-order loop with the same gain. However, for the first-order loop it was shown that the lock-in frequency was equal to the loop gain. The same should be true for the second-order loop (RIC-2); the lock-in frequency is equal to the high-frequency loop gain.

$$\begin{aligned} \Delta\omega_L &= K_o K_d \tau_2/(\tau_1 + \tau_2) && \text{(passive filter)} \\ &= K_o K_d \tau_2/\tau_1 && \text{(active filter)} \end{aligned} \qquad (5\text{-}27)$$

By making use of equation (3-8) the lock-in frequency can be expressed in terms of the loop parameters as

$$\boxed{\Delta\omega_L \approx 2\zeta\omega_n} \qquad (5\text{-}28)$$

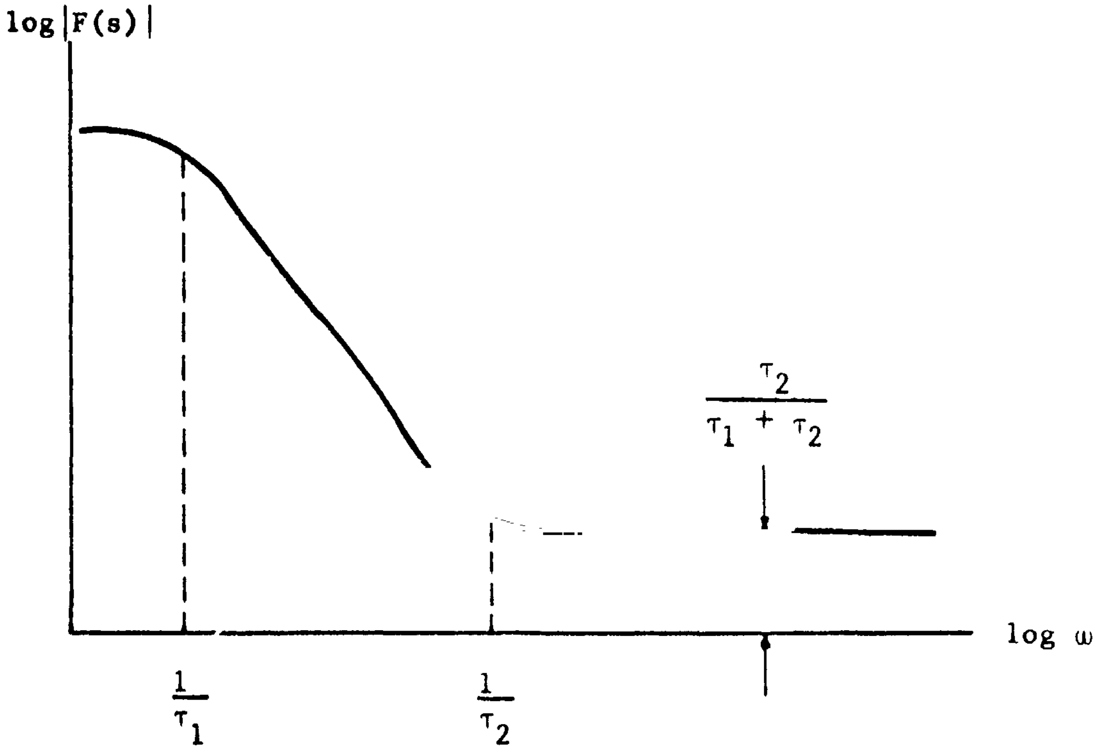


Fig. 5-7
Frequency Response of Loop Filter

In other words, the lock-in performance of the second-order loop is similar to that of the first-order loop. If the signal appears within the loop bandwidth (approximately), the loop locks on immediately without skipping cycles. The lock-up transient occupies a time on the order of $1/\omega_n$ seconds.

Earlier it was shown that the hold-in range of any loop was K_v. In the usual second-order loop, K_v is much larger than ω_n so that the hold-in range is much larger than the lock-in range.

There is also a frequency interval called the pull-in range. If the initial frequency difference (between VCO and input) is within the pull-in range, the VCO frequency will slowly change in a direction to reduce the difference and, if not interrupted, will eventually lock up.

Pull-in behavior may be understood by recognizing that the phase detector output, in the unlocked condition, consists of a beat-note at the difference frequency between the input and the VCO. The beat note is reduced in amplitude by the factor $\tau/(\tau_1 + \tau_2)$ by the loop filter but it is not suppressed completely.

The portion of the beat note that passes through the filter will frequency-modulate the VCO at the difference frequency. Therefore, the phase detector output is the product of a sine wave and a frequency-modulated wave. Since the modulating frequency is equal to the beat frequency, the beat-note could hardly be sinusoidal.

It is a simple matter to select some arbitrary numbers and to compute a waveform of the beat-note. Figure 5-8 shows an approximate plot of a typical beat-note waveshape. (Initial frequency difference was taken as 1.5 times the lock-in frequency.)

The non-sinusoidal character of the beat-note is glaringly evident. Moreover, and vitally important, the positive and negative excursions are manifestly unequal in area; therefore, the phase detector output must contain a DC component.

It is the presence of this DC component that allows pull-in to occur; polarity of the DC is such as to reduce the difference frequency. Once the existence of a DC component is recognized, an alternative explanation of its presence aids understanding. That is, the VCO frequency ω_o is frequency-modulated by the beat note $\Delta\omega$ to form sidebands at $\Delta\omega_o \pm n\Delta\omega$ where n takes on all integral values. This composite signal is multiplied in the phase detector by the input signal and the resulting difference signal is of frequency content $\Delta\omega = \omega_i - \omega_o$ so the frequency corresponding to n = +1 is zero frequency--DC.

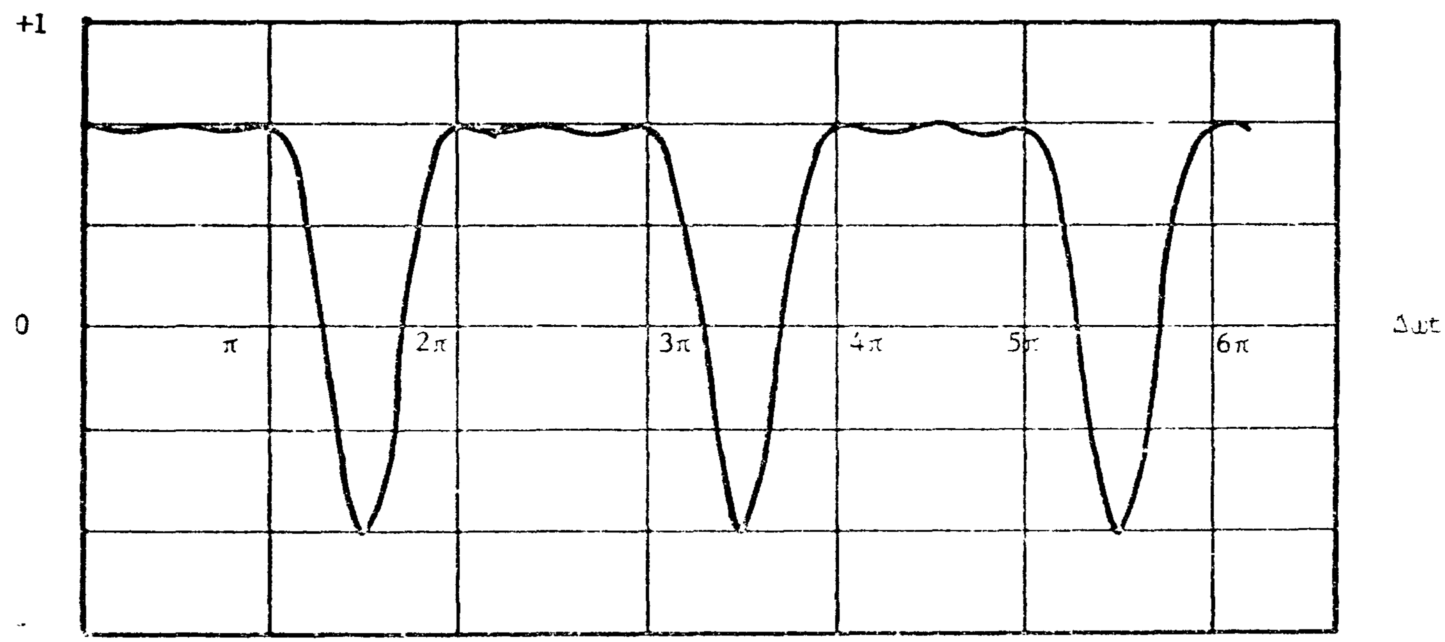


Fig. 5-8

Beat Note Waveshape

In a first-order loop the effect is not of much value; if the initial difference frequency exceeds the lock-in frequency the magnitude of the DC component is insufficient to pull into lock. However, the average difference frequency is reduced. That is, even the first-order loop will tend to pull towards lock, despite the fact that it won't reach lock.

The second-order loop includes an integrator in its loop filter. This integrator builds up an increasing output in response to a DC input; the accumulated output (delivered to the VCO) can greatly exceed the magnitude of the filter beat-note that modulates the VCO. As the integrator output builds up, the VCO frequency is adjusted towards the direction of lock. If the initial difference frequency is not too great, the loop will eventually lock-up.

A number of authors (GRU-1, REY-1, RIC-2, VIT-1) have attempted to obtain explicit formulas for the pull-in ranges of a second-order loop. They all were forced to make approximations and, since each has taken a different approach, they all arrive at somewhat different results. The algebraic forms of the individual results (except for REY) are fairly similar and any one of the forms could probably be used to obtain a rough approximation for pull-in frequency

Fortunately, Gruen has provided experimental data which indicate that Richman's derivation best fits reality, at least for high-gain loops. Richman's formula for pull-in frequency is

$$\boxed{\Delta\omega_p \approx \sqrt{2}\,(2\zeta\omega_n K_v - \omega_n^2)^{\frac{1}{2}}} \qquad (5\text{-}29)$$

This formula fits Gruen's data very well for moderate to high gain ($\omega_n/K_v < 0.4$) but is very poor for low gain ($\omega_n/K_v > 0.5$). For a very high gain loop (active filter) the equation reduces to

$$\boxed{\Delta\omega_p = 2\sqrt{\zeta\omega_n K_v}} \qquad (5\text{-}30)$$

To reiterate the meaning of pull-in: if initial frequency difference $|\Delta\omega|$ between input and VCO is less than $\Delta\omega_p$, the loop will eventually pull into lock, unaided (provided it is not disturbed).

Viterbi (VIT-1) and Richman (RIC-2) both derive approximate values for the time required for a loop to pull into lock for some initial frequency offset, $\Delta\omega$. Viterbi's answer is

$$\boxed{T_p \approx \frac{(\Delta\omega)^2}{2\zeta\omega_n^3}} \qquad (5\text{-}31)$$

Because of the approximations, this formula should not be applied if $\Delta\omega$ is very large (near $\Delta\omega_p$) or very small (near $\Delta\omega_L$). It is best applied in the mid-range and should be considered as the time required to pull-in from the initial offset to a beat frequency equal to $\Delta\omega_L$ (at which time, of course, the loop quickly locks in).

For the special case of a high-gain loop with $\zeta = 0.707$, the pull-in time is

$$T_p = \frac{27(\Delta\omega)^2}{256B_L^3} \approx \frac{4.2(\Delta f)^2}{B_L^3} \text{ sec} \tag{5-32}$$

A narrow-band loop can take a very long time to pull-in. For example, consider a situation where $\Delta f = 1$kc and $B_L = 10$ cps. Pull-in time would be an hour and ten minutes, which is intolerably long, even for deep space applications.

Because of long pull-in time, it is very often necessary to use some other method in order to acquire lock much more rapidly.

One expedient very commonly used is to apply a sweep voltage to the VCO and search for the input frequency. If done properly, the loop will lock up as the VCO frequency sweeps into the input frequency.

From the earlier discussion on the question of hold-in in the presence of a frequency ramp, it should be evident that the sweep rate must not be allowed to become excessive. We have already shown that the loop cannot hold lock if the sweep rate $\Delta\dot{\omega}$ exceeds ω_n^2. If a loop cannot <u>hold</u> lock on a signal it certainly will be unable to <u>acquire</u> lock. Therefore, an absolute maximum limit on the allowable sweep rate is $\Delta\omega < \omega_n^2$.

Viterbi has investigated acquisition problems by means of phase plane trajectories (VIT-1). He discovered that acquisition is not certain, even if $\Delta\dot{\omega} < \omega_n^2$ and the loop is noise-free. If $\Delta\dot{\omega}$ becomes somewhat larger than $\omega_n^2/2$, there is a possibility that the VCO can sweep right through the input frequency without locking. The chance of locking or non-locking depends upon the random initial conditions of frequency and phase. Using Viterbi's phase-plane trajectories, the probability of locking was computed graphically and is plotted against sweep rate in Fig. 5-9. These results apply directly only to the special case of a high-gain second-order loop with $\zeta = 0.707$. However, qualitatively-similar behavior should be expected for other damping factors.

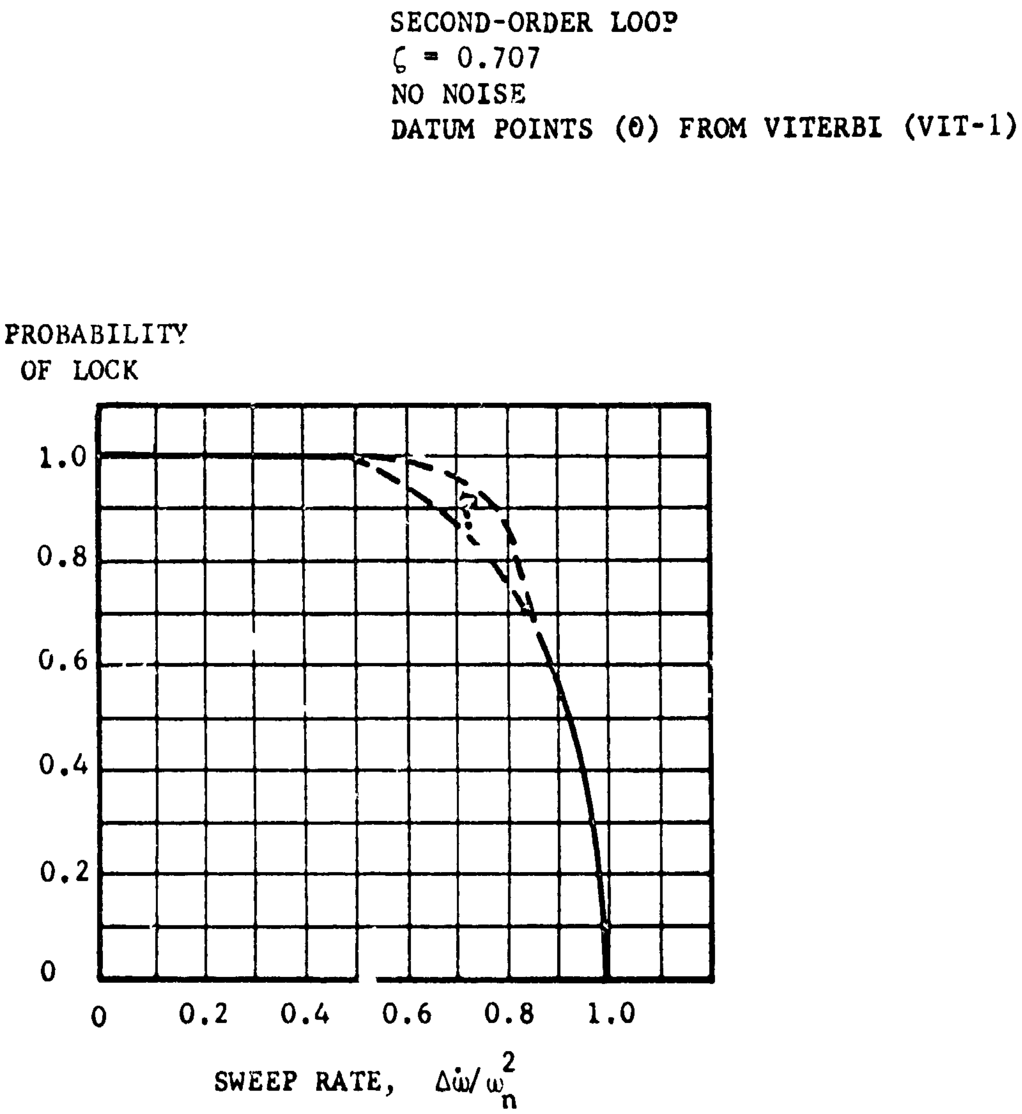


Fig. 5-9
Probability of Sweep Acquisition

A popular rule of thumb (evidently unpublished) is that sweep rate may be as large as

$$\boxed{\dot{\Delta f} = \frac{B_L^2}{3} \text{ cps/sec}} \qquad (5\text{-}33)$$

which, for $\zeta = 0.707$, can also be written as $\Delta\dot{\omega} = 0.59\ \omega_n^2$. Comparison with Fig. 5-9 indicates that this rule is very close to the maximum rate that will guarantee acquisition in a noise-free loop. It is of some interest to observe that the steady-state tracking error, once lock has been achieved, is 36° for this sweep rate.

Further qualitative information on sweep acquisition behavior is available from the simulation study by Frazier and Page (FRA-1)*. Their paper indicates that, for fixed natural frequency and sweep rate, the probability of lock is lowered as damping decreases. See Fig. 5-10 for a sketch to illustrate the performance. This Figure would seem to imply that the loop should be heavily damped, at least until it is locked.

Such a conclusion is unwarranted; loop noise bandwidth varies with damping even though natural frequency is fixed (Refer to Fig. 4-1). On the basis of fixed noise bandwidth, it would appear that the best acquisition performance is obtained for $\zeta = 0.707$. The exact number is not certain, but there appears to be no question that the best performance lies someplace in the range of damping factors between 0.5 and 1.0.

So far, we have assumed that the loop is essentially noise-free. In real life, noise is always with us and must be taken into account. Intuitively, it is to be expected that noise will make it more difficult to acquire a signal; it would be useful if this difficulty could be expressed by a number.

Frazier and Page's experiments provide empirical data which suggest that sweep rate should be reduced by a factor of $\left[1 - (\text{SNR})_L^{-1/2}\right]$ if an acceptably high probability of acquisition is to be maintained in the presence of noise. This expression predicts that acquisition becomes impossible at 0-db signal-to-noise ratio in the loop.

*There appears to be an underlying error in this paper that makes the interpretation of almost all quantitative results open to question. However, the paper is useful for providing insight into the qualitative behavior of loops.

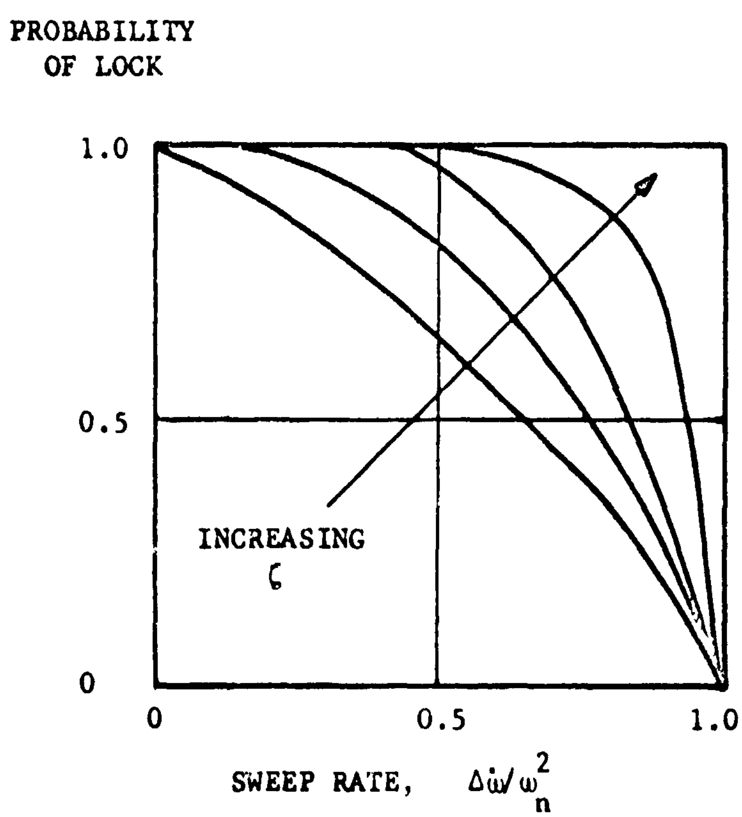


Fig. 5-10

Probability of Sweep Acquisition
Showing Effect of Damping

This result is based upon the assumption that loop bandwidth remains constant under all conditions. However, it is very common practice to employ limiters in phase-lock equipments. When a limiter is used, the gain of the loop--and therefore, the damping and bandwidth--are functions of the signal-to-noise ratio at the input to the limiter. The effect is such that the bandwidth becomes narrower as the SNR decreases. (The subject of limiters is extensively covered in the next chapter). Therefore, where a limiter is used, the acquisition sweep rate must be considerably reduced from the no-noise condition. Reduction is necessary both because of the presence of noise and because of bandwidth narrowing.

These same facts can be restated in a more optimistic manner. If two phase-lock loops are to have the same threshold tracking ability, they must have the same bandwidth under low-signal conditions. A loop containing a limiter will widen its bandwidth as the signal-to-noise ratio improves and thereby will be capable of accommodating much faster sweep rates. If the loop contains no limiter, the bandwidth remains constant at the low-signal value and any permissible increase of sweep rate is due entirely to the reduction of noise in the loop.

Frazier and Page have obtained an empirical equation which predicts the sweep rate that will provide 90% probability of acquisition, while taking account of noise and the effect of limiting. Their results may be adapted* to be

$$\boxed{\Delta\dot{\omega}_{max} = \frac{\left[1 - (SNR)_L^{-1/2}\right] \frac{\alpha}{\alpha_o}\omega_{no}^2}{1 + d}} \qquad (5\text{-}34)$$

where α is the signal suppression factor due to the limiter (see next Chapter), α_o is the signal suppression factor measured at some arbitrary input SNR (usually threshold), ω_{no} is loop natural frequency measured at the same input SNR, and d is a factor depending upon damping.

*Actually, their equation is greater than that shown here by a factor of $\sqrt{2}$. It is believed that there is a consistent error of 1.4 to 1.5 in the value of loop gain they used which leads to incorrect numerical interpretation of many of their results.

If $\zeta < 1$, $d = \exp(-\zeta\pi/\sqrt{1-\zeta^2})$ whereas, if $\zeta \geq 1$, $d = 0$. If, for the minimum signal to be tracked, $\zeta = 0.707$ (a very common condition), d will be less than 0.05 for all larger signals and therefore may be neglected compared to unity.

This equation can be utilized to obtain an approximate upper limit on allowable sweep rate. Considering its empirical antecedents and that it is supposed to predict 90% probability of acquisition, a somewhat lower rate ought to be used in any physical equipment in order to provide a safety margin.

When using this equation, it is important to remember that $(SNR)_L$ is not directly proportional to input SNR. Since loop bandwidth changes due to the presence of the limiter, loop signal-to-noise ratio is also a function of α.

5-3. Techniques of Acquisition

Frequency sweep is obtained by applying a ramp voltage to the VCO input. This may be derived from an independent sweep generator but, in a second-order loop, a simpler method is available. The loop filter contains an integrator; if a step function is applied to the filter input, the output will contain the desired ramp. Slope of the ramp (and, therefore, sweep rate) may be controlled by adjusting the magnitude of the input step. One may consider that the VCO is being slewed by the step voltage.

Some portion of the step (approximately τ_2/τ_1) appears directly in the output of the filter, causing a corresponding jump in VCO frequency just as the sweep begins. The particular application must be able to tolerate such a jump. If it cannot, an independent sweep circuit, without a jump, must be used.

Suppose that the sweep voltage (however derived) continued to be applied, even after the loop locked up. If that were to happen, there would be a static phase error of such a sign and magnitude as to exactly cancel the sweep voltage and the VCO would be held at the proper frequency.

To a first approximation, the sweep voltage could be allowed to continue and simply could be ignored. The phase error it causes represents a loop disturbance, but this might well be of tolerable magnitude.

In some cases, the sweep voltage might reach so large a value as to saturate the filter amplifier or the VCO. However, this eventuality can be avoided simply by not sweeping outside the linear limit of the loop components. Ordinarily, direction of sweep is periodically reversed as the sweep reaches some predetermined limits.

Now suppose that the loop has been locked for a while and that the signal fades out for a short time. Fading causes unlock and the sweep immediately carries the VCO frequency off from the signal frequency. When the signal returns, the VCO will have been carried off to some distance and very likely will be receding further. Obviously, presence of the sweep voltage makes reacquisition more difficult than it need be. For this reason, it is good practice to turn off the sweep voltage once lock has been acquired.

Turn-off need not be very rapid; the previous arguments have shown that sweep can often be tolerated during lock. There should be adequate time allowed to verify, with a high degree of certainty, that lock actually has been obtained.

In the absence of sweep, the VCO of a second-order loop will tend to remain close to its locked frequency in the event of signal dropout. When the signal returns, reacquisition by lock-in or pull-in should be very rapid. Thus the loop has a velocity (frequency) memory.

Frequency information is stored in the form of charge in the integrator. When signal drops out, the loop opens and the discharge time constant of the integrator is $|A|R_1C$. (See Figure 3-2 for nomenclature). Gain, A, is unity in a passive loop so the memory evaporates fairly quickly. However, in an active loop, A can be very large and one would expect long holding times.

This expectation is only partly met in actual equipment. Any real DC amplifier will have some offset and drift and any real phase detector will have some small DC output (due, for example, to imperfect balance) particularly if there is a noise input. These drifts, unbalances, offsets, and rectified noise all combine to form a small slewing voltage that is integrated and which drives the VCO away from its proper frequency.

Presumably, there is some optimum DC gain which balances the effects of integrator discharge against those of unwanted slewing and thereby achieves a maximum memory time. Obviously, any expedient that reduces offset will permit a higher gain and longer memory.

Another approach is sometimes taken when operator intervention is allowable. In this situation, control voltage to the VCO is monitored and the VCO is manually tuned to keep the voltage at zero. In this way, the correct frequency is represented by zero charge on the integrator and there can be no evaporation of memory. The offset problem is handled by adjusting the amplifier DC gain so that amplified offset, after the integrator has reached its final value, is small enough that the VCO is still within easy pull-in range of the signal frequency.

Memory capability of other loop types is of some interest. A first-order loop can have no memory; if the signal fades out, the VCO immediately reverts to its center value. On the other hand, a third-order loop has an acceleration memory; if an accelerating signal--such as the Doppler signal from a satellite--should fade out, the loop will keep tracking at the same rate of change of frequency. This feature is particularly attractive in a tumbling satellite which exhibits periodic and frequent fading.

Closely associated with the subject of acquisition is the question of how to tell if the loop is in lock or not. If loop signal-to-noise ratio is moderately good and the input signal does not jump around overly much, it is not too difficult to detect lock. However, near threshold conditions, lock may not be very easy to detect and, in fact, the very definition of lock may become hazy (as was discussed in Chapter 4).

Even if the signal is good, the locked condition cannot be detected instantaneously. Instead, it is necessary to filter the indication for some appropriate length of time (generally comparable to loop bandwidth) to reduce the confusion caused by noise. For this reason, there must necessarily be some delay between the time a loop locks up and the time that the lock is positively indicated.

A method of lock indication that is employed almost universally is the "quadrature" or "auxiliary" phase detector. The typical arrangement is shown in Figure 5-11. The quadrature phase detector has the received signal applied as one input and a 90° phase-shifted version of the VCO as its other input. The main phase detector has an output voltage proportional to $\sin \theta_e$ whereas the quadrature output will be $\cos \theta_e$. In the locked condition, θ_e is small so $\cos \theta_e \approx 1$. When the loop is unlocked, the outputs from both phase detectors are beat-notes at the difference frequency and the DC output is almost zero.

Thus, the filtered output of the quadrature detector provides a useful indication of lock. The magnitude of the output voltage, relative to that obtained from a noise-free stable input, provides a measure of the quality of lock. (If θ_e jitters, the average of $\cos \theta_e$ is less than unity). When usedin this manner, the smoothed voltage is sometimes known as the "correlatic " output.

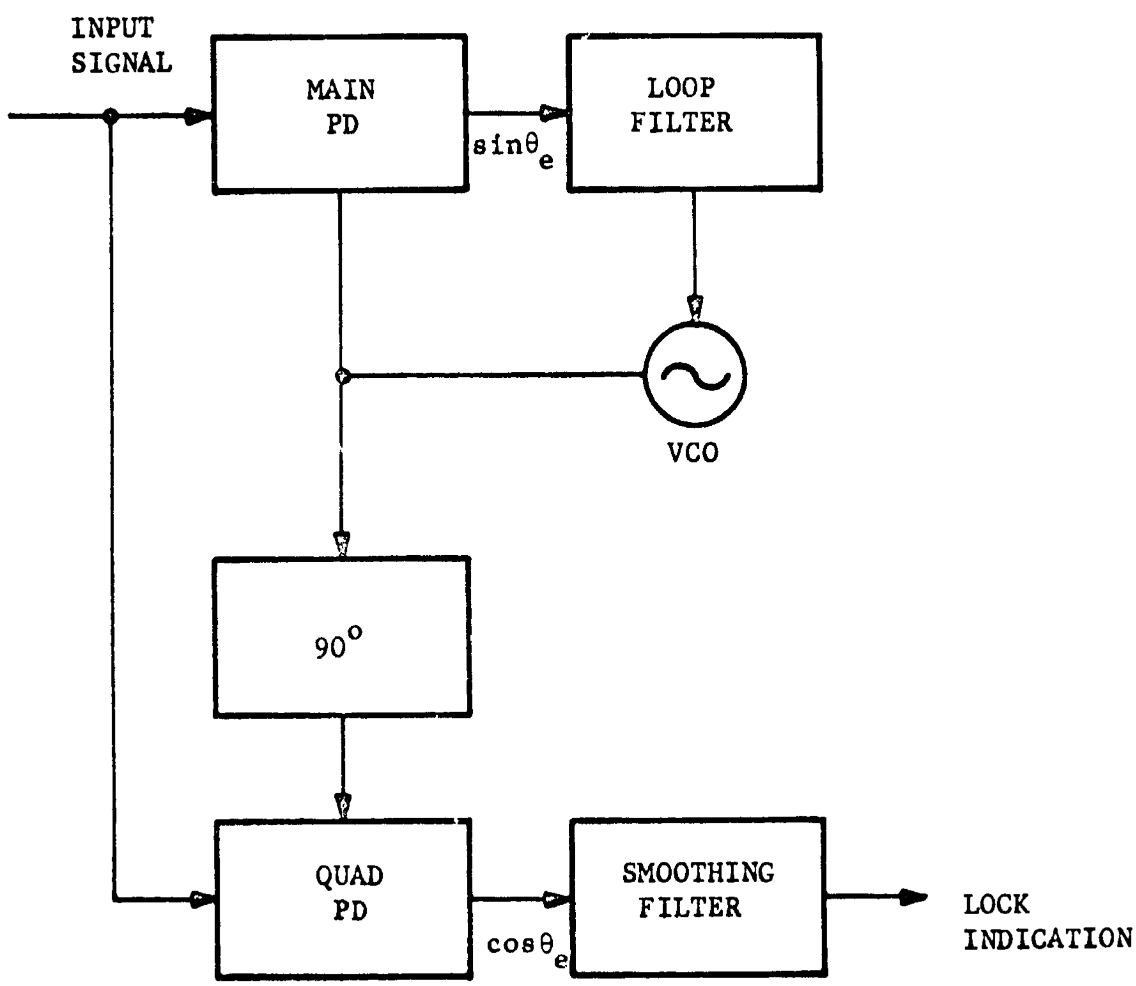


Fig. 5-11

Typical Lock Indicator

It is also possible to use the same voltage as a source of AGC control voltage. This topic has been extensively analyzed by Victor and Brockman (VIC-2). (Note that if AGC is to be obtained from the quadrature detector, the signal applied to it must not be limited.)

Rapid acqusition is possible by means other than sweeping. One method often used is to employ two different bandwidths. For acquisition, the loop would have a wide bandwidth but, for tracking, the loop would be considerably narrowed. From the formulas presented earlier (Eqs. 5-29, 30, 31) it may be seen that the pull-in range would be increased modestly while the pull-in time would be dramatically reduced (inversely proportional to ω_n^3).

It should be apparent that increase of bandwidth can be successful only if signal-to-noise ratio is sufficiently large. If the bandwidth change brings the loop close to threshold, acquisition is not very likely.

Bandwidth may be changed by any of several methods. A straight forward approach is to switch loop filter components. (It is usually advisable to switch the resistors only; if a new capacitor is switched in, the integrator charge is disturbed and the switching process might cause loss of lock).

It is also feasible to switch the gain of the loop and thereby change bandwidth. Richman (RIC-3) has examined both filter-switching and gain-switching and has devised some useful points-of-view in considering the problem. The interested reader is referred to his article.

The switching command signal would be the lock indication voltage from the quadrature phase detector. When the loop is out of lock, the absence of indication voltage would permit the switches to be in their wideband position. When the loop locked, the indication voltage would appear and force the switches into their narrow band position.

If coherent AGC is employed, the same effect can be obtained without switches. In the unlocked condition, there would be no AGC voltage and the signal level at the phase detector would be very large. When the loop locked, AGC voltage would appear and would reduce the applied signal voltage. Since phase detector gain--and therefore loop gain--is proportional to signal level, the loop bandwidth and damping will both decrease automatically when the loop locks: no switches are needed.

One other method, sometimes used, is to employ a frequency discriminator in a conventional AFC arrangement as in Fig. 5-12. If the initial frequency difference is large, the discriminator pulls the VCO towards the direction of lock. When the difference frequency comes within the grasp of the phase-lock loop, the phase detector takes over and locks the loop. A conventional discriminator may be used where the locking frequency is fixed (as in a superheterodyne receiver). Otherwise a device known as a quadricorrelator (RIC-2) can be used as a frequency-difference detector. Signal-to-noise ratio in the discriminator bandwidth (which is at least as wide as the desired acquisition bandwidth and is ordinarily many times greater than the phase-lock loop bandwidth) must be fairly high-- + 10-db or so. This is a severe restriction and renders the method useless for acquiring signals buried in the noise.

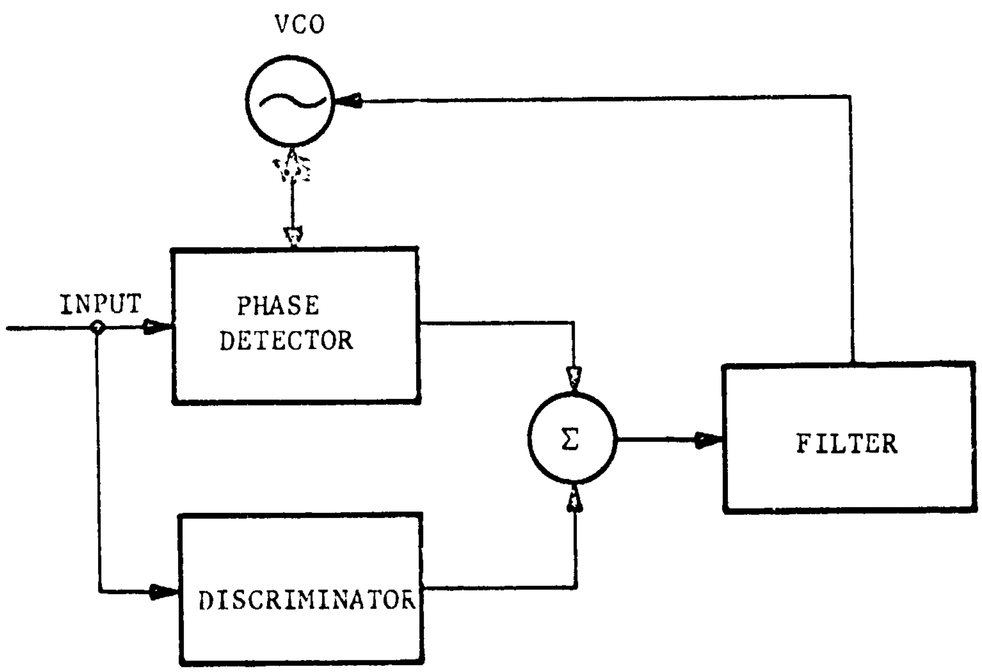


Fig. 5-12

Discriminator-aided Acquisition

Chapter 6

OPERATION OF LOOP COMPONENTS

6-1. Introduction

In this chapter, we discuss the operation and analysis of limiters, phase detectors, and VCO's. Loop filters, IF amplifiers, IF filters, and AGC are considered elsewhere.

6-2. Limiter Performance

It is common practice in present-day phase-lock receiver design to place a bandpass limiter in front of the phase detector. The intent of the practice is not simply to limit the power delivered to the phase detector (although, obviously this function is served) but to cause the receiver to adapt itself to varying signal-to-noise input conditions. This section will describe the properties of a bandpass limiter and show how these properties lead to useful adaptive behavior.

Davenport (DAV-1) has performed the classic analysis of limiters. His major result is that a bandpass limiter degrades signal-to-noise ratio only slightly (1.06 db) for signals deeply embedded in the noise. This is extremely important because if limiters were to cause significant degradation of SNR (as, for example, envelope detectors do below their threshold), they could not be used.

Davenport obtains exact expressions for output signal and noise as a function of input signal-to-noise ratio. These expressions contain infinite sums of confluent hyper-geometric functions and are not much help to the practicing engineer. However, the relations are reasonably well approximated by

$$\boxed{P_s \approx \frac{2L^2}{\pi} \left[\frac{\frac{4}{\pi}(SNR)_i}{\frac{4}{\pi} + (SNR)_i} \right]} \qquad (6\text{-}1)$$

and

$$P_n \approx \frac{2L^2}{\pi} \left[\frac{\frac{4}{\pi}}{1 + 2\,(SNR)_i} \right] \qquad (6\text{-}2)$$

where P_s is the limiter output signal power, P_n is the output noise, $(SNR)_i$ is the signal-to-noise ratio at the input to the limiter, and L is the peak limiter output voltage before filtering.* Using these expressions, the total bandpass limiter output power, $P_s + P_n$, remains constant, within ± 1/2 db, over the full range of $(SNR)_i$.

Output signal-to-noise ratio is easily determined to be (approximately)

$$\boxed{\frac{P_s}{P_n} = (SNR)_o \approx (SNR)_i \left[\frac{1 + 2\,(SNR)_i}{\frac{4}{\pi} + (SNR)_i}\right]} \qquad (6\text{-}3)$$

Fig. 6-1 is a plot of SNR performance. At low $(SNR)_i$, the signal is degraded only by a factor of $\pi/4$ and is actually enhanced by 3-db at high $(SNR)_i$. (Phase-lock loops are normally used to recover small signals from large noise so the enhancement feature is likely to be of only academic interest.)

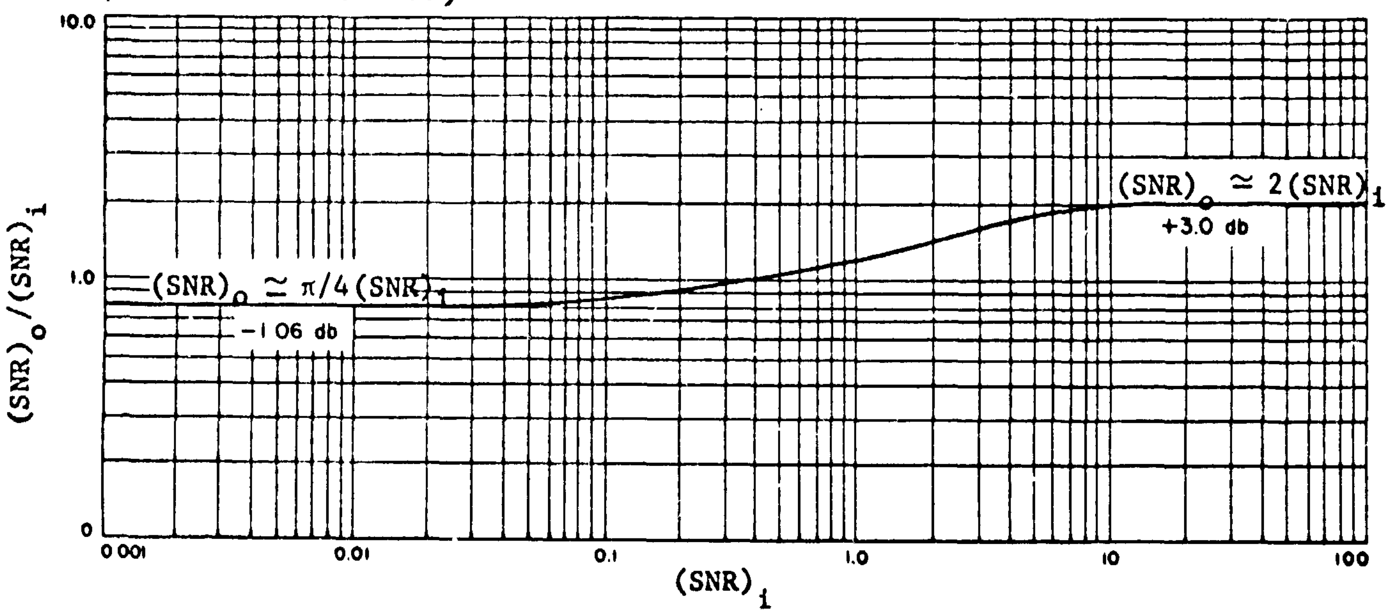


Fig. 6-1

SNR Performance of an ideal bandpass limiter

*An ideal, "snap-action" limiter is assumed. If input voltage is positive, the output voltage is +L; if instantaneous input voltage is negative, the output voltage is -L. Thus, the output of the limiter itself is a square wave. This output is then filtered in a bandpass filter centered at the input frequency. The expressions for S_o and N_o are for the signal and noise at the output of the filter.

If the input is noise-free, the limiter delivers a signal power proportional to $8L^2/\pi^2$ to the phase detector. For this case, the peak sinusoidal signal delivered to the phase detector is $4L/\pi$ volts (which is the peak of the fundamental component of a square wave of amplitude L). This voltage is taken into account in the computation of detector gain factor, K_d. (See Eqs. 4-2, 3).

Signal voltage delivered to the phase detector will be reduced as noise increases. This reduction of signal voltage reduces phase detector gain and, therefore, loop gain; in turn, loop bandwidth and damping are affected. The signal voltage will vary according to the "limiter signal suppression factor"

$$\boxed{\alpha = \sqrt{\frac{(SNR)_i}{\frac{4}{\pi} + (SNR)_i}}} \qquad (6\text{-}4)$$

It may be seen that $\alpha \le 1$.

In all of the previous material, wherever loop gain (K_oK_d) appears, it must be multiplied by α if a limiter is used. For example, the DC loop gain (See Chapter 3) is $K_v = \alpha K_o K_d F(0)$. Most of the other quantities derived in earlier chapters also have a simple dependence upon α. In particular, ω_n and ζ are both proportional to $\sqrt{\alpha}$ so that bandwidth widens out and damping increases as input signal-to-noise ratio improves.

Noise bandwidth (Eq. 4-12) is a function of ω_n and ζ; it also increases as $(SNR)_i$ increases. Minimum noise bandwidth will occur at minimum signal level in any particular loop. This minimum signal is usually specified as the "threshold" and will be designated here with the subscript T. Noise bandwidth may therefore be written as

$$\boxed{B_L = B_{LT}\frac{\left(\frac{\alpha}{\alpha_T} + \frac{1}{4\,\zeta_T^2}\right)}{\left(1 + \frac{1}{4\,\zeta_T^2}\right)}} \qquad (6\text{-}5)$$

In the common situation of $\zeta_T = 0.707$, the noise bandwidth becomes

$$B_L = \frac{B_{LT}}{3}\left(2\,\frac{\alpha}{\alpha_T} + 1\right) \tag{6-5a}$$

Hoffman (HOF-1) has plotted noise bandwidth as a function of input signal-to-noise ratio for $\zeta_T = 0.707$; his curves are reproduced here as Figure 6-2.

6-3. Phase Detectors

Chapter 4 has shown that an ideal analog multiplier behaves as a phase detector and has a sinusoidal output characteristic. Any book on analog computers will provide analyses and circuits of multipliers. There has been some small use made of these devices, particularly at low frequencies. However, the typical multiplier is only useful at low frequencies and most phase-lock work is done at higher frequencies. As a result, although a multiplier is a convenient mathematical model of a phase detector, the actual hardware used is more likely to have a different underlying mechanism.

(In very recent years, field effect transistors have appeared which can be used as simple, effective multipliers (HIG-1, MAR-6). There has been no report of their use as phase detectors but it is reasonable to expect to be able to obtain good performance at higher frequencies than multipliers have been used hitherto.)

One of the most popular phase detector circuits for receivers uses balanced diode peak detectors as shown in Fig. 6-3.

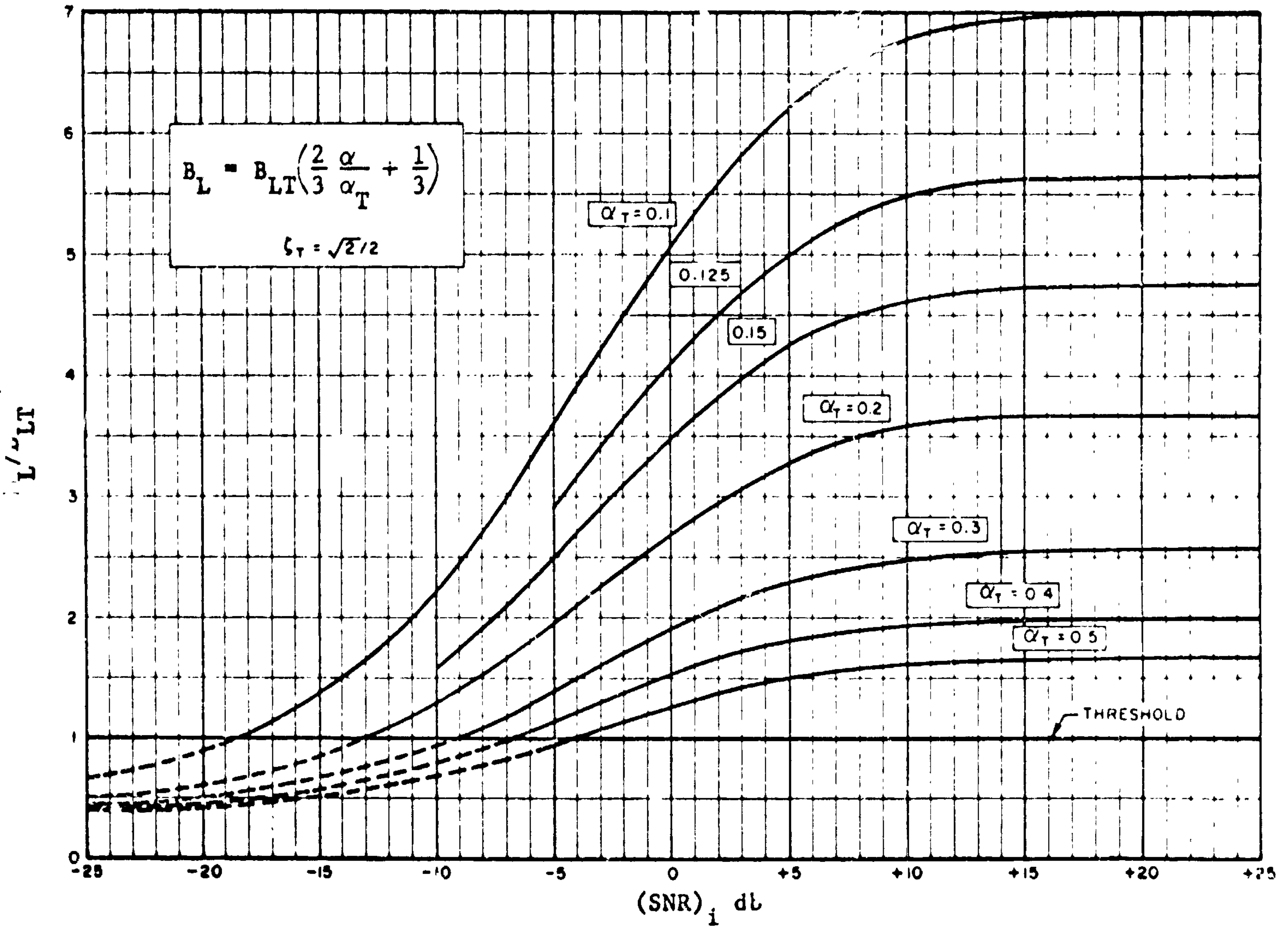


Fig. 6-2

Loop Noise Bandwidth Variation with Input Signal to Noise Ratio

"By Permission of L. A. Hoffman"

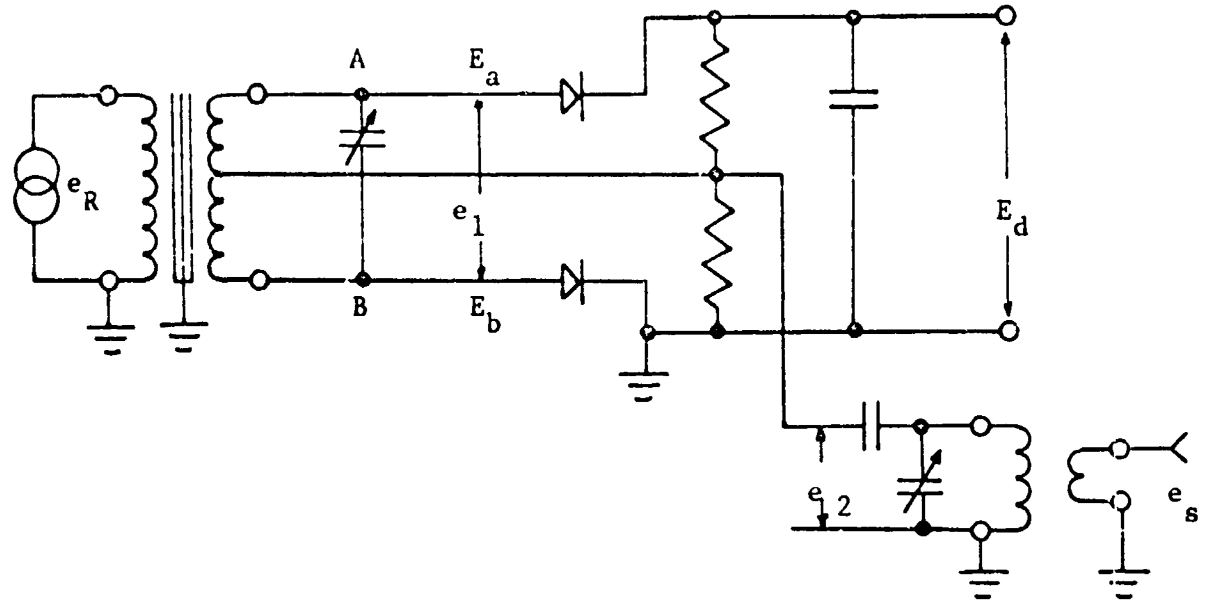


Fig. 6-3
Diode Phase Detector

For this circuit, the reference and signal voltages can be represented by

$$e_R = E_R \cos \omega t \tag{6-6}$$

$$e_s = E_S \sin (\omega t + \theta) \tag{6-7}$$

where θ = phase difference between e_R and e_S

Due t. the 90^o phase shift of the transformers,

$$e_1 = E_1 \cos (\omega t + \pi/2) = E_1 \sin \omega t \tag{6-8}$$

$$e_2 = E_2 \sin (\omega t + \frac{\pi}{2} + \theta) = E_2 \cos (\omega t + \theta) \tag{6-9}$$

Voltages e_1 and e_2 are summed at points A and B to produce E_A and E_B; the vector sums may be represented as shown in Fig. 6-4. (GRU-1)

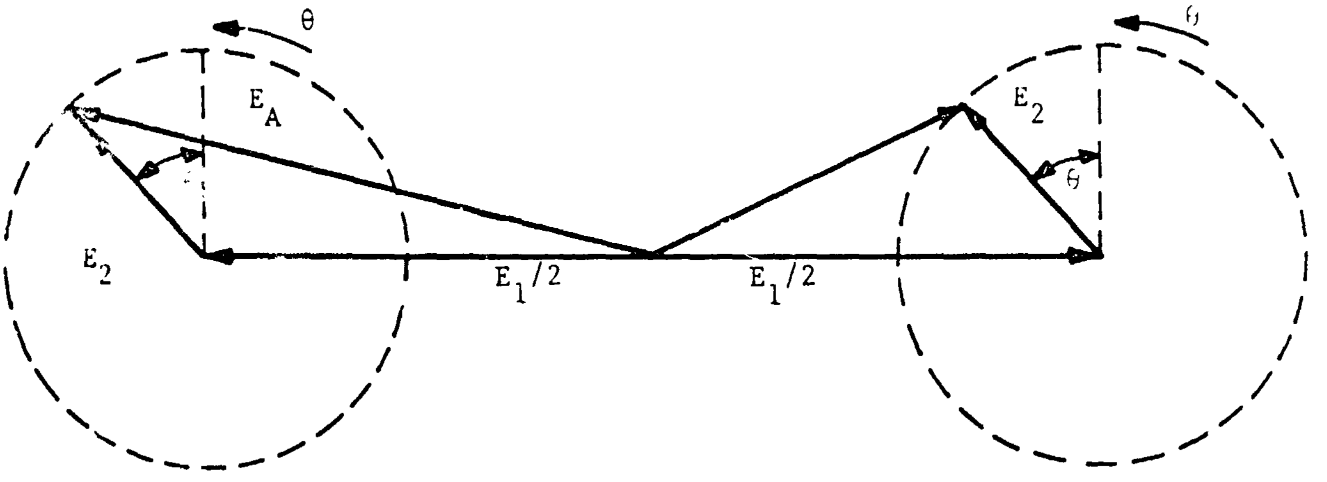


Fig. 6-4
Vector Diagram, Phase Detector

The two circles of radius $r = E_2$ describe the path of voltages E_A and E_B as the phase difference between e_1 and e_2 varies from 0 to 360°.

Using the law of cosines and Figure 6-4

$$E_A^2 = \frac{E_1^2}{4} + E_2^2 + E_1E_2 \sin\theta \tag{6-10}$$

$$E_B^2 = \frac{E_1^2}{4} + E_2^2 - E_1E_2 \sin\theta \tag{6-11}$$

The phase detector output voltage, E_d, is equal to the difference of the two rectified voltages so that

$$E_d = E_A - E_B \tag{6-12}$$

Subtracting equation 11 from 10 gives

$$E_A^2 - E_B^2 = 2E_1E_2 \sin\theta \tag{6-13}$$

factoring,

$$E_A - E_B = \frac{2E_1E_2 \sin\theta}{E_A + E_B} \tag{6-14}$$

or,

$$\boxed{E_d = \frac{2E_1E_2 \sin \theta}{E_A + E_B}} \tag{6-15}$$

Now, if $\frac{E_1}{2} \gg E_2$, then

$$E_A + E_B \approx E_1 \tag{6-16}$$

and,

$$E_d \approx \frac{2E_1 \, E_2 \sin \theta}{E_1} \tag{6-17}$$

the detector output voltage then becomes

$$\boxed{E_d \approx 2E_2 \sin \theta} \tag{6-18}$$

Equation 18 shows that E_d is directly proportional to E_2 but is independent of E_1 when $E_1/2 \gg E_2$.* Also, from equation 18, it can be seen that $E_d = 0$ when $\theta = 0^o$ and $E_d = 2E_2$ when $\theta = \pm 90^o$.

Several important conclusions can be drawn from the preceding derivation of equation 18:

1. If $E_1/2 \gg E_2$, then the diode operation can be likened to a switch that is turned on and off by E_1, allowing E_2 to charge C_1. The larger E_1 becomes (without causing diode break-down or saturation), the more accurate equation 18 becomes.
2. If E_d is to be independent of signal amplitude variation, E_2 must be held constant.
3. The phase detector can be used as an amplitude sensitive device (or AGC detector) by maintaining $\theta = 90^o$ and allowing E_2 to vary as a function of input signal level.

*An analysis that does not make this approximation and which takes non-ideal diode characteristics into account has been performed by Dishington (DIS-1).

4. In order for E_d to have zero output with no signal input, the phase detector must be carefully balanced with respect to the reference input.

Ideally, the phase detector is perfectly balanced and, in the presence of a noisy signal, has a DC output proportional to the signal phase only. In practice, of course, this is not the case. There is a minimum SNR below which the output of the phase detector is not usable. The generally accepted minimum SNR is -30 db for a diode circuit of this type. However, most designers try to maintain SNR ≥ -20 db at the input to the phase detector by placing a narrow-band filter in the I F amplifier.

The maximum frequency limit on this type of phase detector is set by the reverse recovery time of the diodes. At sufficiently high frequencies, the reverse recovery is a significant portion of a cycle period and rectifier performance deteriorates. For diodes of the 1N914 class, precision phase detectors (those used in narrow-band loops with high noise levels) have been built at frequencies as high as 10 mc. The same quality of diodes can be used in circuits up to 30 mc where very narrow bandwidths and high noise levels are not encountered.

A few diode types have appreciably faster response than the 1N914; presumably, they could be used in higher-frequency phase-detectors. Precision circuits capable of operating at 90 to 100 mc would be very convenient.

(To avoid the reverse recovery problem, there have been suggestions to use varactor diodes in balanced circuits. Diodes would always be reverse-biased so that the frequency limitation could be in the microwave region. The non-linear capacitance of the varactor would be used to obtain a multiplier characteristic. If such a concept can be made to work, it would represent a major advance in the phase detector art).

Another common type is the switching phase detector which, in essence, consists of practically nothing but a switch. The device that functions as the switch may be a transistor, a diode quad, or even a mechanical switch or chopper. The switch is driven synchronously with the input signal; on alternate half-cycles it either allows the input to pass or not to pass.

Figure 6-5 illustrates the nomenclature and typical waveforms. If the input is $E_s \cos(\omega t + \theta)$ and the switch changes state at the zero crossings of $\sin \omega t$, then the output is $E_s \cos(\omega t + \theta)$ for $0 < \omega t < \pi$ and zero for $\pi < \omega t < 2\pi$. The DC output of the detector is

$$
\begin{aligned}
E_d &= \frac{E_s}{2\pi} \int_0^{\pi} \cos(\omega t + \theta)\, d\omega t \\
&= -\frac{E_s}{\pi} \sin \theta
\end{aligned}
\tag{6-19}
$$

Figure 6-5 illustrates a half-wave detector; if a full-wave detector were used instead, the DC output would be doubled (which is of no great consequence) and the ripple frequency would also be doubled. In very wide-band loops there will often be problems of phase detector ripple getting to the VCO and causing phase jitter. In such cases, additional filtering cannot be used without narrowing (and possibly unstabilizing) the loop. Only ingenious design of the phase detector can relieve the problem and full-wave operation is a first step in the proper direction.

Several commonly used, switching-type phase-detector circuits are shown in Fig. 6-6.

We have analyzed three different types of phase detectors and in each case have found a sinusoidal characteristic. It can readily be shown that the form of the characteristic is due to the sine wave input and not to the circuit itself. For example, if square wave inputs* were to be applied to any of the three types of circuits, the output characteristic would be triangular rather than sinusoidal. (See Fig. 6-7b).

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*When both inputs are square, binary, digital operation is approached. A phase detector degenerates to an Exclusive-OR gate whose error output is the time average of its two logic states.

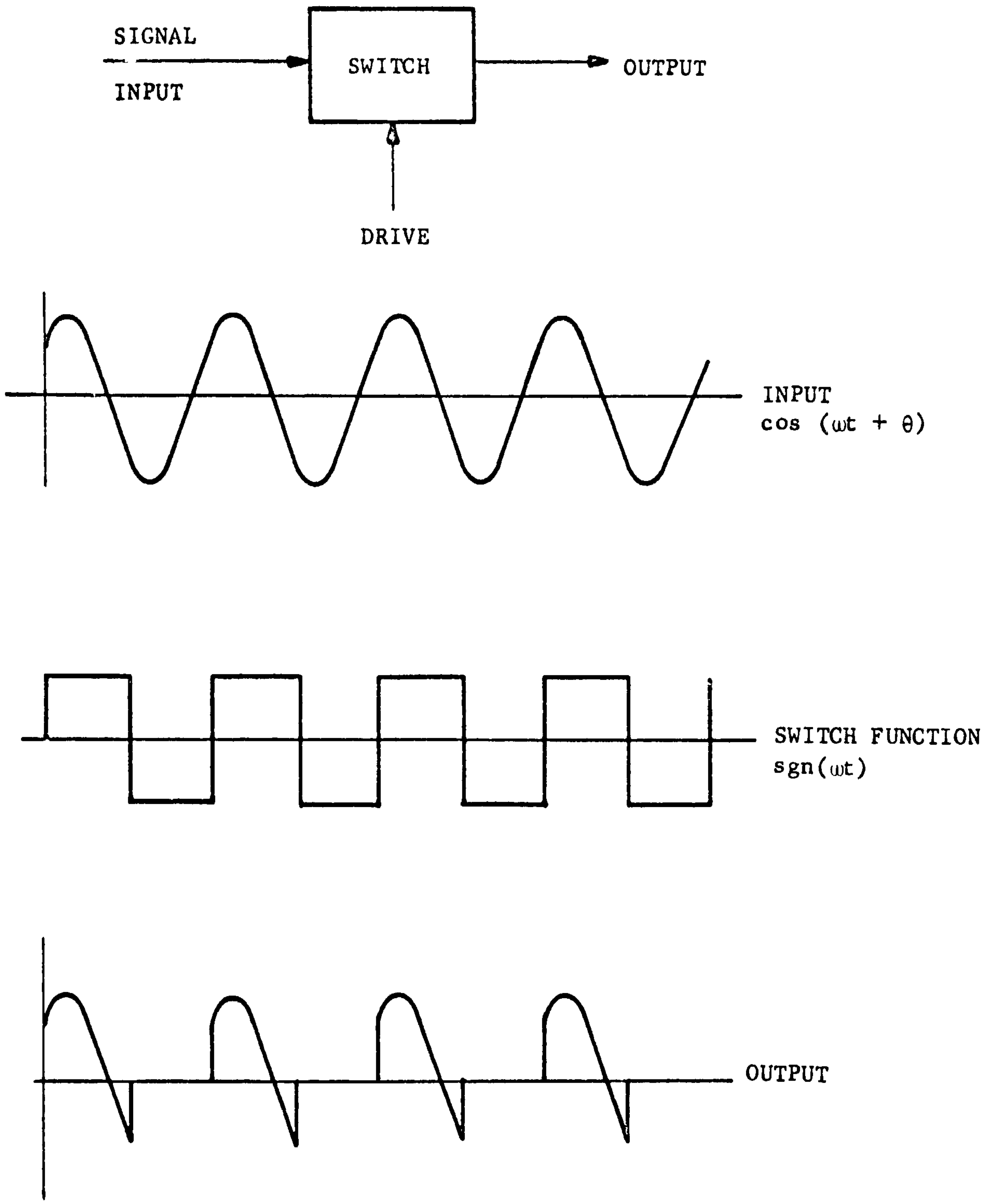


Figure 6-5

Operation of Switching-Type Phase-Detector

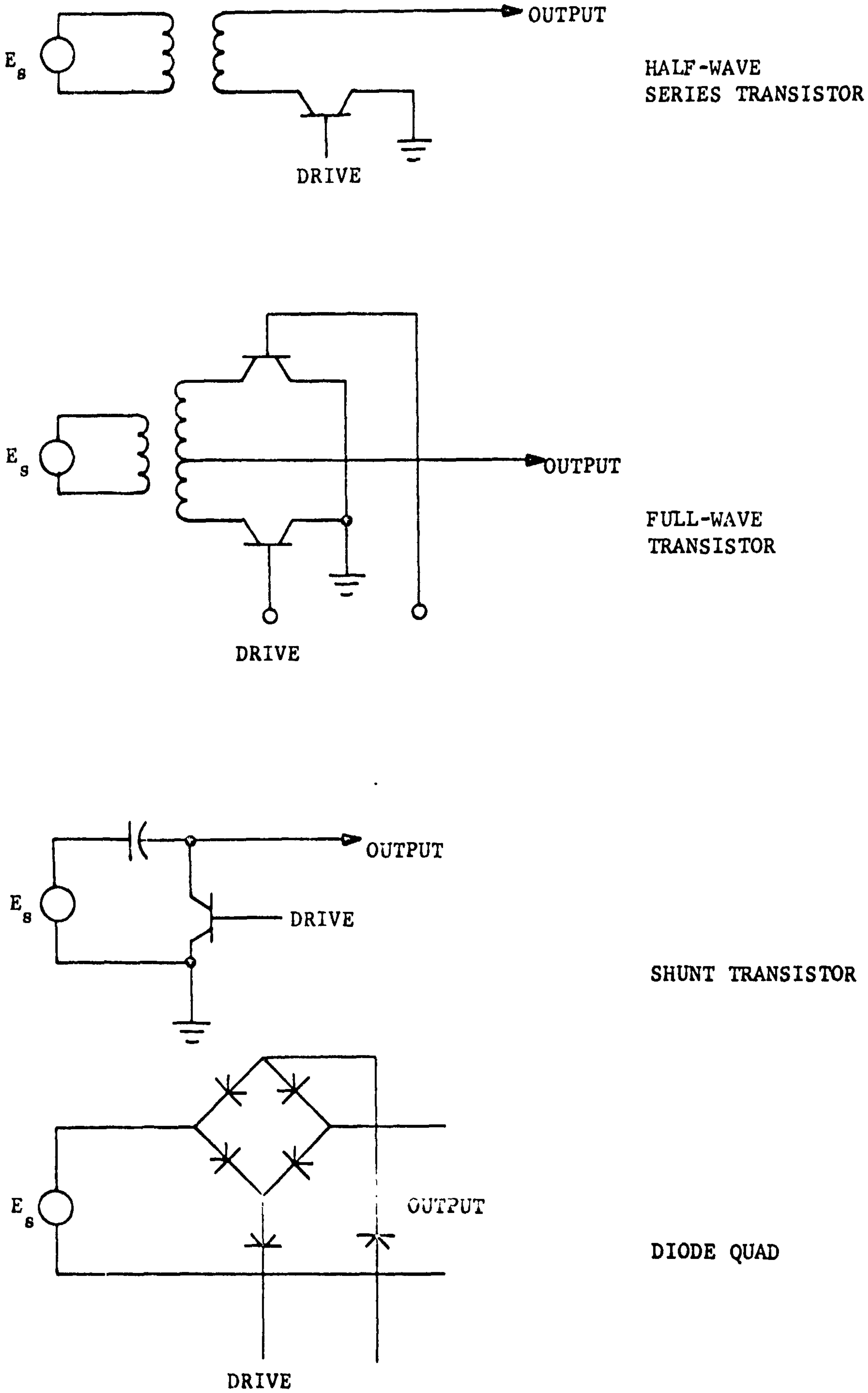
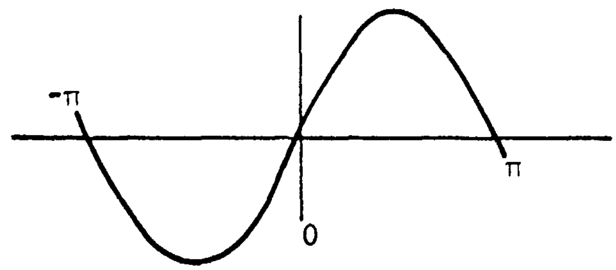
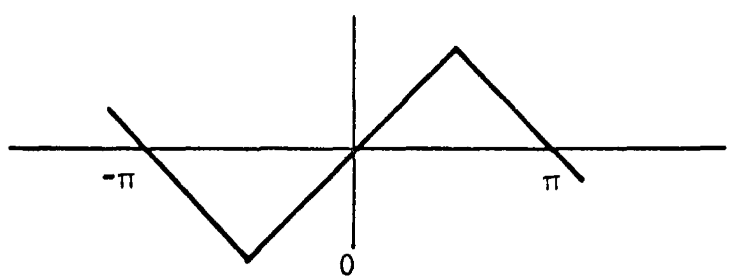


Figure 6-6

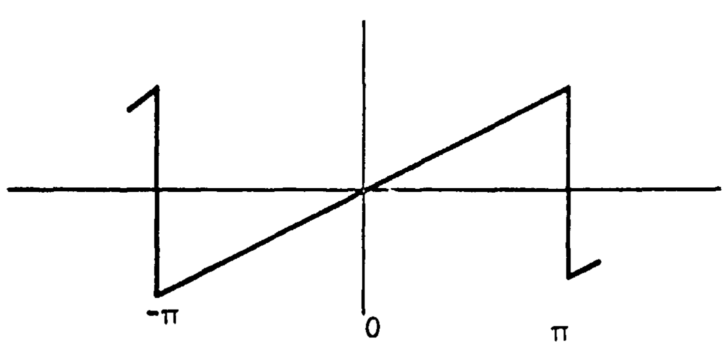
Phase Detector Circuits



a. Sinusoidal



b. Triangular



c. Sawtooth

Figure 6-7

Phase Detector Characteristics

Square waves may be obtained by passing both inputs to a phase detector through wide band limiters. This situation is often closely approached in receivers that use limiters for bandwidth adaptation.

Linearity in the triangular case is near-perfect for phase angles as large as 90° -- a significant improvement over the sinusoidal case. Where a loop is intended as an FM discriminator, linearity is an important feature and the triangular characteristic is widely used.

It would be desirable to extend the linear range even beyond 90°, if possible. A phase-detection scheme known as "Tanlock" (BAL-1, ROB-2) provides a measure of improvement. In this method the control voltage is of the form

$$E_d = \frac{(1 + x) \sin \theta_e}{1 + x \cos \theta_e}$$

which can be shown to have a greater linear range than $\sin \theta_e$ for proper choice of x. The functions $\sin \theta_e$ and $\cos \theta_e$ are obtained from conventional phase detectors driven in quadrature and the quotient is obtained from an analog divider (a multiplier in a feedback loop). The greater linear range not only reduces distortion of the recovered modulation, but, from experimental results, claims have been made for improvements in noise threshold, hold-in range, and pull-out frequency.

There are special conditions for which a sawtooth characteristic (Fig. 6-7c) is possible. A phase detector that provides such a characteristic can be nothing more complicated than a flip-flop (BYR-1, GOL-2). For such a detector, the signal input sets the FF once each cycle and the VCO toggles (changes the state) of the FF once each cycle. Output error voltage is the average of the output of the FF. Analysis (or reference to Byrne) will show that a sawtooth characteristic is obtained.

Besides the obvious advantage of linearity, this type of phase detector will also have improved tracking, hold-in and pull-in characteristics (BAR-1, GOL-3). Unfortunately, the two signals must both be of such quality as to be able to trigger a flip-flop reliably. Input signal-to-noise ratio must be high which means that threshold will be high. Such a phase detector is of no value if signal must be recovered from a larger noise.

6-4. Voltage-Controlled Oscillators

There are many requirements placed upon VCO's in different applications. These requirements are usually in conflict with one another and a compromise is therefore needed. Some of the more important requirements include:

1. Large electrical tuning range
2. Phase stability
3. Linearity of frequency versus control voltage
4. Reasonably large gain factor (K_o)
5. Capability for accepting wideband modulation

The requirement for phase stability is in direct opposition to all of the other four requirements. To obtain any of the wideband features, one must inevitably sacrifice phase stability.

Three types of VCO are in common use; in order of decreasing stability they are:

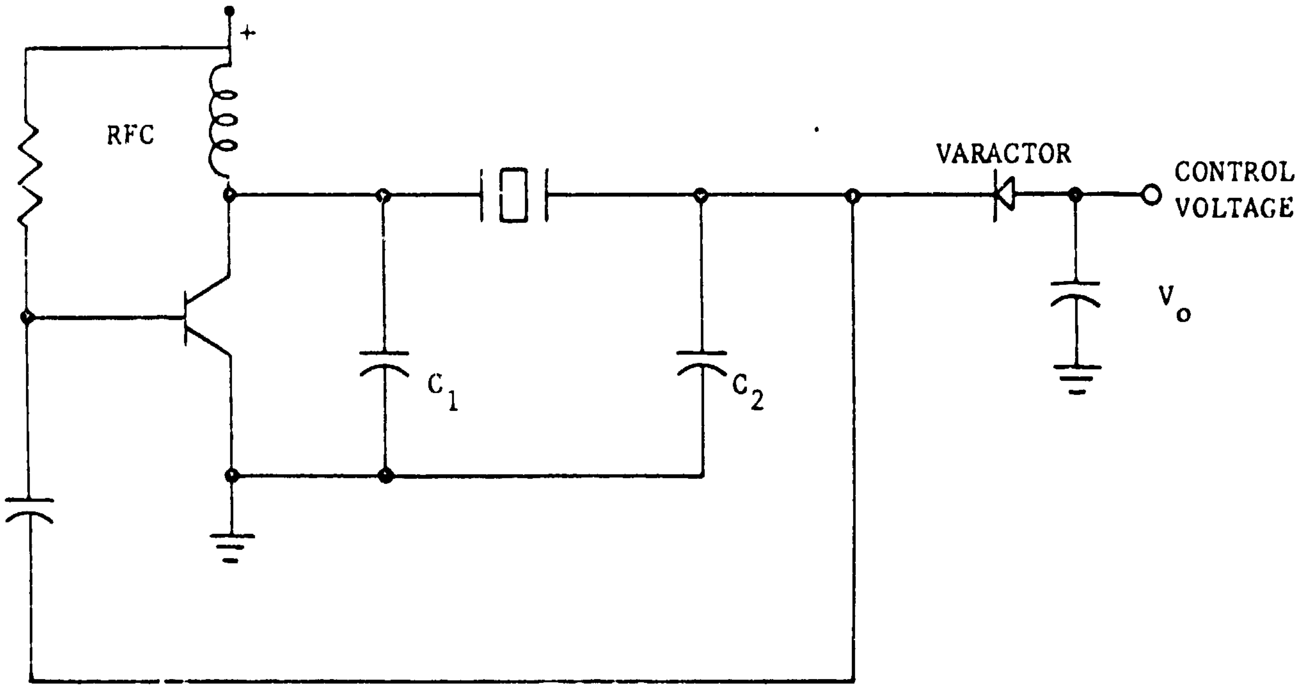
1. Crystal oscillators (VCXO)
2. LC oscillators
3. RC multivibrators

In today's technology, the most stable crystal oscillators are those using high-Q, vacuum mounted, 2.5 or 5.0 mc, fifth-overtone, AT-cut crystals. (WAR-1, SYK-1, AND-1, JPL-6, JPL-7).

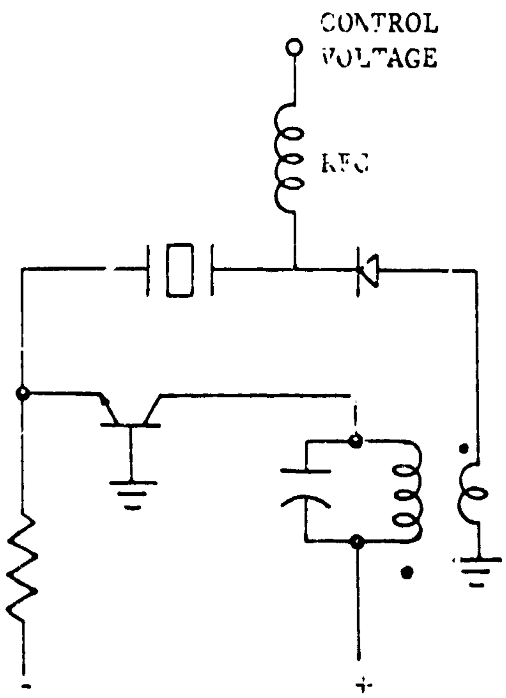
A circuit commonly used (Fig. 6-8a) is a variation on the familiar Pierce crystal oscillator (FEL-1, JPL-7, SMI-1). The crystal is operated in its series mode and capacitors C_1 and C_2 adjust the amount of feedback. A varactor diode provides a small variation of C_2 and results in a pulling of the oscillation frequency.

The tuning range of this circuit is very small when using high-Q crystals. To obtain a greater range, it is common practice to use ordinary AT-cut crystals in their fundamental mode* in a circuit as

*Overtone crystals have a narrower pulling range than fundamental crystals.



a) Modified Pierce Oscillator



b) Grounded-Base Oscillator

Fig. 6-8

VCXO Circuits

shown in Figure 6-8b. The crystal here is also operated in its series mode. The varactor is in series with the crystal and effectively varies the resonant frequency over some range greater than the first circuit.

Phase-stability is enhanced by a number of factors:

1. High-Q in the crystal and circuit
2. Low noise in the amplifier portion
3. Temperature stability
4. Mechanical stability

The precision 5 mc crystals mentioned above have an unloaded Q of approximately 2×10^6. Other crystals can be expected to have unloaded Q's in the range of 10,000 to 200,000.

Circuit losses will inevitably degrade the intrinsic Q of the crystal alone; these losses must be minimized for best performance. In a series-mode crystal, the driving and load impedances should be as small as possible in order to avoid degradation of Q.

Much of the phase jitter of an oscillator arises from noise in the associated amplifier. The transistor (or other device) should be operated in a low noise condition and, of course, a low-noise transistor should be used. It is plain that high-frequency thermal and shot noise contribute significantly to the jitter; moreover, there is considerable evidence that low-frequency, 1/f (flicker) noise is also important. (FEY-1, ATT-1, GRI-1). (This latter consideration suggests that improved operation might be obtained if field effect transistors, which have low 1/f noise, were used instead of conventional bipolar devices).

To obtain good signal-to-noise ratio in the oscillator (and therefore low jitter), it seems reasonable to operate the circuit at a high RF power level. There is a competing effect, however; excessive vibration of the crystal drives it into non-linear modes of mechanical damping and the Q is reduced thereby. As a result, there is an optimum drive level for any crystal. Powers of 10 to 500 microwatts are typical; these levels are usually much smaller than the maximum rated power which is established on heat dissipation limitations.

Crystal parameters are temperature sensitive; to obtain best phase stability the VCO would ordinarily be enclosed in a double proportional-control oven. Temperature transients and fluctuations are especially to be avoided.

There is a considerable literature extant on the subject of noise in oscillators (ATT-1, BAR-2, EDS-1, ESP-1, GOL-5, GRI-1, MAL-1, MUL-1, SAN-2); the detailed theory is beyond the scope of this book. Rules for designing a low-jitter oscillator have been presented here but the considerable art of building VCXO's is also beyond the scope of this book (and, furthermore, tends to be in the nature of trade secrets).

We are concerned with the behavior of a VCO in a phase-locked loop. Suppose a loop is receiving a perfectly stable, noise-free signal, but the VCO has some inherent jitter ϕ_o. The feedback action of the loop causes the VCO to track the input so the actual phase error between input and VCO will be less than ϕ_o (full phase jitter ϕ_o will only appear when the loop is open).

It may be shown readily that the actual loop jitter θ_p is given by

$$\frac{\theta_p}{\phi_o}(s) = -\left[1 - H(s)\right] \tag{6-20}$$

which, in essence, is the same as the loop error response (Eq. 3-9 and Fig. 3-4).

Loop phase fluctuation is

$$\overline{\theta_p^{\,2}} = \frac{1}{2\pi}\int_0^\infty \Phi_o(\omega)\,\left|1 - H(\omega)\right|^2 d\omega \tag{6-21}$$

where $\Phi_o(\omega)$ is the spectral density of the oscillator phase jitter in $(\text{radian})^2$ per cps.

It is evident that loop jitter will be zero if $H(\omega) \equiv 1$: that is, if the loop tracks the input perfectly there is no error. This condition requires that loop bandwidth be infinite. For the practical finite-bandwidth loop, the error will not be zero; there will be an inverse relationship between bandwidth and loop jitter of the form

$$\overline{\theta_p^{\,2}} = \frac{J}{(B_L)^\gamma} \tag{6-22}$$

where J is a measure of the noisiness of the particular oscillator and γ is a constant depending upon the noise spectrum of the oscillator jitter. Fragmentary evidence (JPL-6) suggests that $\gamma = 2.4$ for bandwidths in the range of 1/3 cps to 50 cps for the types of crystals and circuits mentioned above. The same reference reports that rms phase jitter as low as 0.005^{o} in a bandwidth of $B_L = 1.5$ cps has been achieved with a precision 5 mc crystal.

As loop bandwidth is reduced further and further, the loop phase jitter continually increases. If bandwidth is made too narrow, the jitter becomes excessive (tracking is too sluggish) and the loop will not be able to maintain lock. A measure of the quality of an oscillator is the minimum bandwidth for which it still remains locked.

When wide tuning range becomes more important than stability, other oscillator types must be used. It is understood that X-cut crystals in parallel-mode circuits have been employed in very wide range VCXO's but, as far as is known, extreme tuning limits of 0.25 to 0.5% of oscillator frequency are all that have been achieved.

If a wider range is needed, an LC oscillator must be used. In this application, the standard Hartley, Colpitts, and Clapp circuits make their appearance. Tuning may be accomplished by means of a varactor although saturable inductors have also been used. Some early loops made use of "reactance tubes" but this method became obsolete with the disappearance of tubes from low power circuits. (With the recent advent of the field effect transistor the reactance modulator might conceivably make a limited comeback. However, the convenience of varactors would make this event unlikely).

Finally, where stability is of little importance, where large tuning range is needed, (and where low cost is a factor) relaxation oscillators such as multivibrators and blocking oscillators are used. The operating frequency of practical relaxation oscillators has been limited to a few megacycles. Linearity of frequency versus control voltage (or current) is generally excellent.

To measure phase jitter, it is necessary to compare two oscillators against one another. Both will have jitter and it is impossible to determine which of the two oscillators is responsible. If both oscillators are identical, half of the mean-square jitter can be assigned to each.

To avoid problems with frequency differences, one oscillator must be locked to the other by means of a narrow band, phase-lock loop. Phase jitter is then measured at the output of the loop phase detector.

Good quality oscillators will exhibit very little jitter at their fundamental frequencies in loops of reasonable bandwidth. In order to magnify the oscillator jitter, their frequencies can be multiplied up to the microwave region and comparison is performed there (VIC-1).

Chapter 7

OPTIMIZATION OF LOOP PERFORMANCE

7-1. Introduction

Two general principles may be abstracted from the preceding chapters:

1. To minimize output phase jitter due to external noise, the loop bandwidth should be made as narrow as possible.
2. To minimize transient error due to signal modulation, or to minimize output jitter due to internal oscillator noise, or to obtain best tracking and acquisition properties, the loop bandwidth should be made as wide as possible.

These principles are directly opposed to one another; improvement in one type of performance can only come at the expense of degrading the other. Some compromise between the two is always necessary. Almost always there is a compromise that is "best" in some sense; this compromise is called "optimum."

It must be recognized that there is no unique optimum result that applies under all conditions. On the contrary, there are many possible results, depending upon the criteria of performance, the nature of the input signal, and any restrictions placed upon loop configuration.

7-2. Optimization

The best-known optimization is that derived by Jaffe and Rechtin (JAF-1) following the Wiener* method. Their criterion of loop performance is the mean square loop error

$$\Sigma^2 = \overline{\theta_{no}^{\;2}} + \lambda^2 E_T^{\;2} \tag{7-1}$$

*Details of the Wiener method are far beyond the scope of this book. For an extensive exposition of the subject, see Y. W. Lee, Statistical Theory of Communication, Wiley, New York, 1960, Chapters 14 through 17. A more directly applicable explanation will be found in Rechtin's notes (REC-1).

where $\overline{\theta_{no}^{2}}$ is the phase jitter due to noise (Eq. 4-13) and E_T^2 is a measure of the total transient error:

$$E_T^2 = \int_0^\infty \theta_e^2 (t)dt \tag{7-2}$$

where θ_e (t) is the instantaneous phase error in the loop due to transients. The quantity λ is a multiplier which establishes the relative proportions of noise and transient error that are to be permitted. (Notice that λ^2 has dimensions of time^{-1} -- that is, frequency.)

In the Wiener optimization method, the known quantities are the spectra of the signal and noise while the criterion of performance is the mean square error Σ^2. The result of the method is a description of an "optimum" filter whose output provides a minimum mean square error.

Jaffe and Rechtin have assumed white noise and three different types of modulation at the input: phase step, frequency step, and frequency ramp. For each condition, they arrive at an optimum loop transfer function H(s) and the corresponding transfer function for the loop filter F(s). Results are summarized in Table 7-1.

For the three different types of input, the optimum filter types are first-, second-, and third-order loops, respectively. That is, the Wiener method specifies optimum filter shape as well as bandwidth. In the optimum second-order loop (of greatest interest because of its widespread usage) damping factor is $\zeta = 0.707$.

It will be noted that optimum bandwidth is a function of the input signal-to-noise ratio. In order to minimize the total error, the loop should be capable of measuring SNR and readjusting its bandwidth for optimum performance. To perform this optimum adaptation exactly would be a complex and difficult task; as far as is known, there has never been a serious attempt at perfect adaptation.

One reason for the lack of effort is that Jaffe and Rechtin discovered near-optimum adaptation may be achieved by very simple means: namely, use of a bandpass limiter prior to the phase detector. In Chapter 6, we found that the presence of a limiter causes loop bandwidth and damping to vary as a function of input SNR. This variation is not optimum (damping should remain constant and the variation of ω_n should have a different form).

INPUT	OPTIMUM H(s)	F(s)	Optimum "Bandwidth"
Phase Step $\theta_i(t) = \Delta\,\theta$	$\dfrac{\omega_1}{s + \omega_1}$	$\dfrac{\omega_1}{K_o K_d}$	$\omega_1 = \Delta\theta\lambda\sqrt{\dfrac{2\,P_s}{W_o}}$
Frequency Step $\theta_i(t) = \Delta\omega t$	$\dfrac{\omega_n^2 + \sqrt{2}\,\omega_n s}{\omega_n^2 + \sqrt{2}\,\omega_n s + s^2}$	$\dfrac{\omega_n^2 + \sqrt{2}\,\omega_n s}{K_o\,K_d\,s}$	$\omega_n^2 = \Delta\omega\lambda\sqrt{\dfrac{2\,P_s}{W_o}}$
Frequency Ramp $\theta_i(t) = \dfrac{\Delta\dot{\omega}t^2}{2}$	$\dfrac{\omega_n^3 + 2\,\omega_n^2 s + 2\,\omega_n s^2}{\omega_n^3 + 2\,\omega_n^2 s + 2\,\omega_n s^2 + s^3}$	$\dfrac{\omega_n^3 + 2\,\omega_n^2 s + 2\,\omega_n s^2}{K_o\,K_d\,s^2}$	$\omega_n^3 = \Delta\dot{\omega}\lambda\sqrt{\dfrac{2\,P_s}{W_o}}$

P_s = Input Signal Power

W_o = Input noise spectral density (one-sided)

TABLE 7-1

WIENER - OPTIMIZED LOOPS

but it is sufficiently close to optimum to be very useful. Limiters are very widely used in sensitive phase-lock receivers.*

It is of interest to observe that the definition used here of E_T^2 is such that steady-state error must be zero. If this were not true, E_T^2 would be infinite. If some other definition of transient error were to be used (e.g.: peak error), it is probable that different optimum results would be obtained.

The Wiener analysis is strictly applicable only to linear systems; to apply it to the phase-lock loop requires that the linear approximation be made. Furthermore, Jaffe and Rechtin's exact results are applicable only if noise is white (see NIS-1 for an approach to correlated noise input), when the input is one of the three specific types listed here, and when the error criterion is as in Eq. 7-1 (see GOL-4 as an example of different input and different error criterion). All of which is to say that we have so far only shown <u>an</u> optimum (or, rather, three optimum) loops and not <u>the</u> optimum loop, even in the restricted category of Wiener filters.

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* It should be possible to obtain similar performance from wideband (non-coherent) AGC since the same phenomenon of signal suppression occurs. There would superficially appear to be a 1-db advantage to AGC for low SNR because the limiter causes 1-db SNR degradation and the AGC does not.

Coherent AGC on the other hand, maintains signal level constant at the phase detector and therefore has no adaptive-bandwidth properties. There are situations where coherent AGC and limiting are used simultaneously (BRO-1). In that case, the limiter provides bandwidth adaptation. Purpose of the AGC might be to prevent limiting at places in the receiver other than the limiter, to standardize signal level so as to be able to recover and measure amplitude modulation, to measure signal level, or to standardize bandwidth of auxiliary channels (e.g., antenna angle tracking loops).

In practice, where narrow bandwidth is needed, a second-order loop is the type most commonly used. A first-order loop necessitates a major sacrifice of hold-in range whereas a third-order loop is more complicated, harder to analyze and can become unstable if not treated properly. (However, both first- and third-order loops have their uses in which they will substantially out-perform the second-order loop.) For the remainder of this chapter, we will restrict ourselves to the second-order loop and give examples of different optimizations that are possible.

Suppose the natural frequency is determined by some well-defined dynamic feature of the input signal. For example, a satellite will exhibit a very definite rate of change of Doppler frequency; if a limit is placed upon the permissible acceleration error, ω_n is immediately fixed. Given this value of ω_n, what value of damping factor results in the least phase jitter due to noise? The answer, referring to Fig. 4-1, is obviously $\zeta = 0.5$ since this is the value that minimizes noise bandwidth.

For another possibility, suppose that noise bandwidth is fixed by, let us say, restrictions on the maximum allowable phase noise jitter. What value of damping will permit the largest frequency step $\Delta\omega$ without the loop being pulled out of lock, even temporarily? In Eq. 4-12, the noise bandwidth was found to be

$$B_L = \frac{\omega_n}{2}\left(\zeta + \frac{1}{4\zeta}\right)$$

and Eq. 5-2 approximates pullout frequency as

$$\Delta\omega_{po} = \omega_n\left(\zeta\sqrt{\pi} + \ln 2\pi\right)$$

Eliminating ω_n between these equations yields

$$\Delta\omega_{po} = \frac{2B_L\left(\zeta\sqrt{\pi} + \ln 2\pi\right)}{\zeta + \frac{1}{4\zeta}} \tag{7-3}$$

Differentiating $\Delta\omega_{po}$ with respect to ζ, setting the derivative equal to zero, and solving gives $\zeta = 0.75$ as the damping that maximizes pullout frequency. This maximum value is $\Delta\omega_{po} \approx 5.87\ B_L$ radians per second. Pullout frequency at $\zeta = 0.707$ would be $5.85\ B_L$ so that it is hardly worthwhile to bother to optimize pullout as such.

This finding tends to illustrate a common property of optima; the performance criterion quantity tends to change very slowly near the optimum so that there is no need to adjust the loop so as to attain exactly the best performance. The extremum will usually be quite broad.

Hoffman (HOF-1) has derived another optimization that appears to have greater value than the previous one. A phase-lock receiver is often required to track accelerating transmitter (either true acceleration of a missile or apparent acceleration of a satellite) with a second-order loop. What acceleration error -- and, therefore, what loop bandwidth -- should be used to achieve "optimum" performance?

First it is necessary to arrive at a criterion of performance. Rechtin's criterion cannot be applied because the non-zero steady-state acceleration error would lead to an infinite integrated-square transient error. Hoffman used noise threshold as his criterion. His definition of threshold is an empirical relation taken from Martin (MAR-3) which states that, at threshold

$$\theta_a + \sigma\theta_{no} = \pi/2 \tag{7-4}$$

where θ_a is the acceleration error (Eq. 5-7a), θ_{no} is the rms noise jitter in the loop (Eq. 4-13), and σ is a confidence factor that takes account of the fact that peak noise considerably exceeds the rms value. Equation 7-4 states that threshold error is exceeded if the sum of the individual errors exceeds 90^o.

The quantity to be optimized is the input signal power, P_s. From the discussion of behavior of θ_{no} in Chapter 4, and Eq. 4-15, an expression of

$$\overline{\theta_{no}^{\ 2}} = \xi^2/(SNR)_L \tag{7-5}$$

may be deduced. (For $(SNR)_L > 10$, $\xi^2 = \frac{1}{2}$; for $(SNR)_L = 1$, $\xi^2 \approx 1$. The factor ξ is itself a function of $(SNR)_L$ but we shall regard it as essentially constant.)

From Eq. 4-16, $(SNR)_L = P_s/2B_LW_o$ where W_o is the input noise density. Eq. 7-4 may now be written as

$$\theta_a + \sigma\xi\sqrt{\frac{2B_L W_o}{P_s}} = \frac{\pi}{2} \tag{7-6}$$

Using Eq. 4-12 and 5-7a, B_L may be eliminated from Eq. 7-6 leaving

$$\theta_a + \sigma\xi \sqrt{\frac{W_o}{P_s} \left(\zeta + \frac{1}{4\zeta}\right) \sqrt{\frac{\Delta\dot{\omega}}{\theta_a}}} = \frac{\pi}{2} \tag{7-7}$$

Solving for signal power required at threshold yields

$$P_s = \frac{\sigma^2 \xi^2 \left(\zeta + \frac{1}{4\zeta}\right) \sqrt{\frac{\Delta\dot{\omega}}{\theta_a}}\; W_o}{\left(\frac{\pi}{2} - \theta_a\right)^2} \tag{7-8}$$

When P_s is minimized with respect to θ_a, the surprising result is that $\theta_a = \pi/10$ (that is, 18^o), independently of σ, ξ, ζ, or W_o. This exact result is dependent upon two approximations: using Eq. 7-4 as the definition of threshold and assuming ξ to be constant. An exact analysis, if one should ever be discovered, would probably yield a somewhat different result but, presumably, not much different.

Calculation of the minimum P_s still requires that a confidence factor σ be specified and a suitable value for ξ deduced. The latter might require an iterative process and is complicated by the fact that the functional dependence of ξ upon $(SNR)_L$ is not known within limits closer than about ± 1 db. Also, refer to Chapter 4 for a discussion of fundamental difficulties in defining θ_{no}.

From Eq. 7-8, it may be seen that P_s can also be minimized with respect to damping factor; the optimum value is clearly $\zeta = 0.5$. Hoffman arbitrarily uses $\zeta = 0.707$ and thereby obtains a threshold power that is higher than optimum by 0.26 db.

Hoffman's approach suggests another possible optimization to be used where acceleration error must be considered. Suppose that $(SNR)_L$ is reasonably large (> 10) and let the criterion of performance be

$$\Sigma^2 = {\theta_a}^2 + {\theta_{no}}^2 = \frac{(\Delta\dot{\omega})^2}{{\omega_n}^4} + \frac{B_L W_o}{P_s}$$

$$= \frac{(\Delta\dot{\omega})^2 \left(\zeta + \frac{1}{4\zeta}\right)^4}{16\, {B_L}^4} + \frac{B_L W_o}{P_s} \tag{7-9}$$

which is to be minimized with respect to B_L and ζ. It is immediately evident that the optimum damping is $\zeta = 0.5$ and the usual differentiation will yield

$$B_L^{\,5} = \frac{P_s\,(\Delta\dot{\omega})^2}{4W_o} \tag{7-10}$$

for optimum loop noise bandwidth.

We will end the chapter with one more example that may be useful. Suppose the signal transmitter is essentially stationary with respect to the receiver so that dynamic phase errors may be neglected. This situation could arise in tracking a synchronous satellite. Also, if a vehicle is on a ballistic trajectory, its apparent acceleration can be predicted with great accuracy. The VCO can be externally programmed to follow this prediction very closely and the loop is only required to track the error between predicted and actual trajectories.

The criterion of performance will be taken as the total mean square phase jitter in the loop which, of course, is to be minimized. Jitter is composed of a part due to external noise (Eq. 4-13) and a part due to inherent VCO jitter (Eq. 6-22). The total mean square jitter is

$$\Sigma^2 = \frac{W_o B_L}{P_s} + \frac{J}{(B_L)^\gamma} \tag{7-11}$$

from which the optimum bandwidth may be found to be

$$B_L^{\,\gamma+1} = \frac{\gamma J P_S}{W_o} \tag{7-12}$$

and the minimum mean square error is

$$\Sigma^2 = (\gamma J)^{\frac{1}{\gamma+1}} \left(\frac{W_o}{P_s}\right)^{\frac{\gamma}{\gamma+1}} \left(1 + \frac{1}{\gamma}\right) \tag{7-13}$$

To summarize, consider the following points:

1. There is no uniquely optimum loop nor is there a unique optimization procedure.
2. A criterion of performance must be defined. This criterion depends upon the conditions of operation of the loop and the requirements placed upon it. From the examples given here, it may be seen that no general rule can be used in establishing the criterion.
3. Once an optimum is found, it is not usually necessary to adjust the loop parameters exactly to their optimum values. It is very common for an extremum to be quite broad to an extent that moderate departure from optimum parameters has little adverse effect on loop performance.

Chapter 8

TYPICAL TRANSPONDER DESIGN

8-1. Introduction

In an attempt to clarify any questions concerning the use of the equations previously derived, this section describes the procedure followed in determining the loop parameters for a typical transponder.

Certain parameters must be specified before the designer can proceed, while other parameters must be set by the designer himself. Decisions must be made, early in the design, as to what type of system configurations will be used and how the gain is to be distributed.

8-2. Selection of Frequencies

Although the designer of a phase-lock system does not generally have a free choice of signal input or output frequencies, it is essential that he have an understanding of the relationships between all of the signals in the system. Therefore, this discussion will derive (in general form) the frequencies involved in typical transponder phase-lock loops.

8-2.1 Configurations for Phase-lock Loops

In general, there are two configurations of the phase-lock loop for transponder applications. Simplified block diagrams of these are shown in Figs. 8-1 and 8-2. Fig 8-1 is the simplified block diagram of the general form of a phase-lock transponder. Fig. 8-2 is a special case of Fig. 8-1 where $N_1 = N_4$.

The general equation for the sum of the frequencies around the loop can be written

$$\Sigma(f_r, Nf_o) \equiv 0 \qquad (8\text{-}1)$$

The output frequency (f_t) is

$$f_t = N_4 f_o \qquad (8\text{-}2)$$

Examination of the block diagram shows there are four possible combinations for Eq. (8-1); i.e., either one or both of the two mixing frequencies (N_1f_o, N_2f_o) can be above or below the signal input frequency (Note: N_2f_o always equals the 2nd I-F frequency).

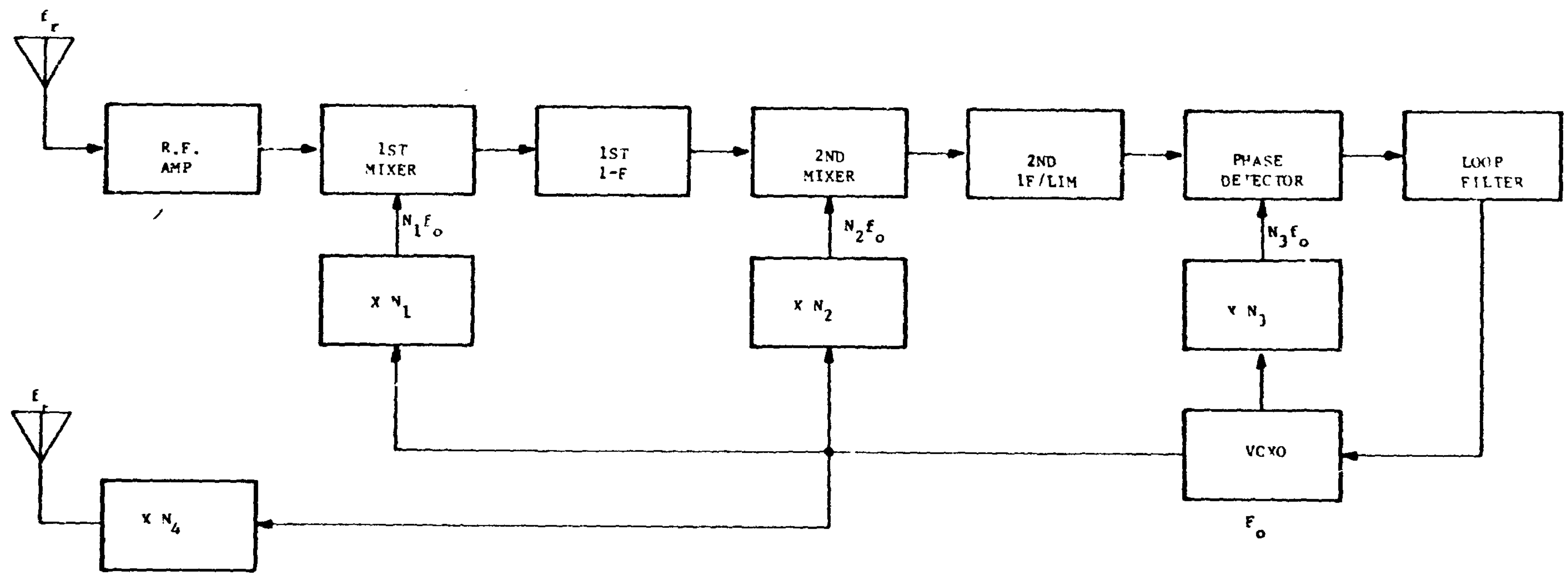

Figure 8-1

Phase-Lock Transponder

Configuration No. 1

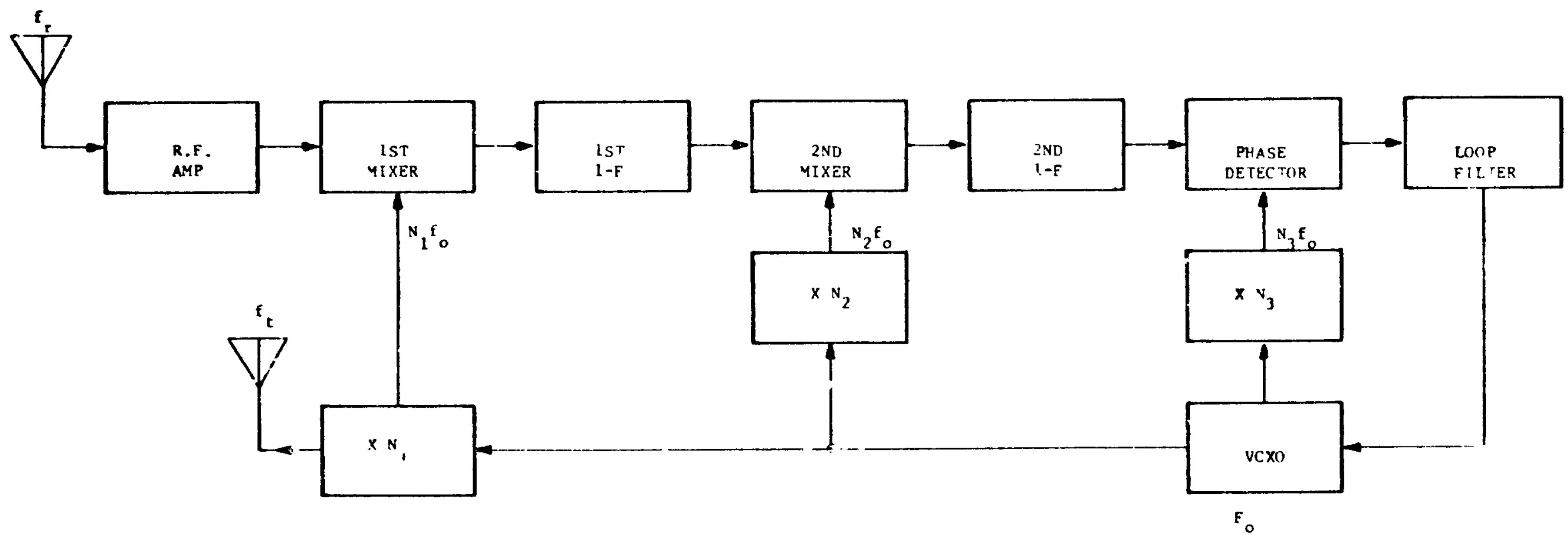

Figure 8-2

Phase-Lock Transponder

Configuration No. 2

For the configuration of Fig. 8-1, the four cases are:

Case 1. (N_1f_o and N_2f_o below the signal)

$$f_r - N_1f_o - N_2f_o - N_3f_o = 0 \tag{8-3}$$

$$\frac{f_t}{f_r} = \frac{N_4}{N_1 + N_2 + N_3} \tag{8-4}$$

Case 2. (N_1f_o above, N_2f_o below)

$$f_r - N_1f_o + N_2f_o + N_3f_o = 0 \tag{8-5}$$

$$\frac{f_t}{f_r} = \frac{N_4}{N_1 - N_2 - N_3} \tag{8-6}$$

Case 3. (N_1f_o below, N_2f_o above)

$$f_r - N_1f_o - N_2f_o + N_3f_o = 0 \tag{8-7}$$

$$\frac{f_t}{f_r} = \frac{N_4}{N_1 + N_2 - N_3} \tag{8-8}$$

Case 4. (N_1f_o above, N_2f_o above)

$$f_r - N_1f_o + N_2f_o - N_3f_o = 0 \tag{8-9}$$

$$\frac{f_t}{f_r} = \frac{N_4}{N_1 - N_2 + N_3} \tag{8-10}$$

The use of the configuration shown in Fig. 8-1 allows almost any ratio of input to output frequencies to be obtained. It can be used as an up- or down-converter simply by changing N_4 appropriately.

The advantage of the use of the configuration shown in Fig. 8-2 is that it requires less components and enjoys all attendant benefits therefrom. The disadvantage is that the f_t/f_r ratio is rather limited. Also, the flexibility in the choice of multipliers is considerably reduced.

Although there are four cases of Fig. 8-2, as before, two of them are up-convertors and the other two are down-convertors (assuming: $N_1 > N_2 > N_3$).

The equations become:

A. Down-Convertors

Case 1.

$$\frac{f_t}{f_r} = \frac{N_1}{N_1 + N_2 + N_3} \tag{8-11}$$

Case 3.

$$\frac{f_t}{f_r} = \frac{N_1}{N_1 + N_2 - N_3} \tag{8-12}$$

B. Up-Convertors

Case 2.

$$\frac{f_t}{f_r} = \frac{N_1}{N_1 - N_2 - N_3} \tag{8-13}$$

Case 4.

$$\frac{f_t}{f_r} = \frac{N_1}{N_1 - N_2 + N_3} \tag{8-14}$$

8-2.2 Choice of Multipliers (N)

The system designer can select any frequency combinations he chooses within the constraints of equations (8-1) through (8-14) and, of course, within the realm of practicality. The choice of the multipliers (N_1 through N_4) are pretty much determined by the voltage-controlled oscillator requirements. Once the frequency of the VCO is selected, the multiplier N_4 is determined directly.

The choice of N_3 is not completely free due to system oscillation or false-lock considerations. In most practical designs, the gain of the second I-F/Lim. amplifier module is fairly high (>50-db). As a result, it is unwise to choose $N_3 \geq 1$ because of feed-through problems in the second mixer. For this reason, N_3 is generally selected to be 1/2 or 1/3 in most designs.

N_1 is generally chosen to produce a convenient first I-F frequency at which practical high-gain amplifiers can be constructed. The only freedom of choice on N_2, then, is whether to make it above or below the first I-F frequency.

8-3. Specifications

It should be quite apparent to the reader that phase-lock receivers are special pieces of equipment intended to perform special functions: there is no such thing as a "general purpose" phaselock receiver. Each receiver is designed to meet a particular set of specifications in order to perform a certain task. If the receiver is used for other purposes, it should be expected that its performance will be far from optimum. For this reason, the design specifications must be drawn up very carefully if desired performance is to be achieved. Before any specifications can be written or designed to, it is necessary to decide what functions are to be tracked and their effect on the phaselock threshold.

The specification writer should be as familiar with the performance of phase-lock loops as the designer. If the desired goal if for maximum sensitivity, then a narrowband loop must be used; if the goal is for high modulation tracking rates, then a wideband loop must be used.

There are many requirements that a transponder or receiver must meet before they can be considered as practical, operational units. The scope of these requirements ranges from electrical to mechanical to environmental. To properly discuss the whole problem concerning "specifications" would require several complete texts and the writer does not presume to even attempt such a formidable task. This discussion will be limited to the bare minimum requirements in order to proceed with the design of a phase-lock transponder.

A typical set of simplified specifications for a missile-borne carrier-tracking phase-locked transponder is as follows:

Input frequency	2113 5/16 mc
Output frequency	2295 mc
Output/input coherent frequency ratio	240/221
Input tuning range	± 5 mc
Input/output impedance	50 ohms
Input/output VSWR	1.5:1
Threshold sensitivity	-120 dbm

Maximum input signal level	0 dbm
Tracking bandwidth	± 250 KC
Maximum missile acceleration	260 ft/sec^2
Signal acquisition time	≤ 0.5 seconds
Transmitter output power	≥ 1.0 watt
Primary input voltage	25 to 31 volts dc
Primary input power	≤ 85 watts
Maximum temperature	+ 100°C
Volume	≤ 250 in^3
Weight	≤ 11 lbs

The remaining portions of this section will discuss some of the procedures to be followed in designing this transponder and some of the problems encountered.

8-4. Design Procedure

Before proceeding with design of the transponder, it is convenient to collect all of the pertinent loop equations for handy reference. These are listed below:

$$\zeta_T = 0.707 \tag{8-15}$$

$$\theta_e + 2\theta_N = \frac{\pi}{2} \text{ radians (peak)}$$

$$\theta_e = \theta_v + \theta_a \tag{8-16a}$$

$$\theta_v = 0.86 \text{ radians} \tag{8-16b}$$

$$\theta_a = \frac{\pi}{10} \text{ radian} \tag{8-16c}$$

$$B_{LT} = 0.945\sqrt{\dot{\omega}_D} \text{ cps} \tag{8-17}$$

$$10 \log B_{LT} = 10 \log \overline{\theta_n^2} + P_S(\text{dbm}) - \text{N.F.(db)} - \text{KT(dbm)} \tag{8-19}$$

$$B_{LT} = 0.53\ \omega_{nT} \text{ radians/sec} \tag{8-20}$$

$$K_o = \frac{\omega_D}{\theta_V} \sec^{-1} \tag{8-21}$$

$$\tau_1 = \frac{K_T}{\omega_{nT}^2} = (R_1 + R_2)C \text{ sec.} \tag{8-22}$$

$$\tau_2 = \frac{\sqrt{2}}{\omega_{nT}} = R_2C \text{ sec.} \tag{8-23}$$

$$K_T = \alpha_T K_d K_f K_{dc} K_o M \text{ sec}^{-1} \tag{8-24}$$

$$\alpha_T = \sqrt{\frac{1}{1 + \frac{4}{\pi}\left(\frac{N}{S}\right)_T}} \tag{8-25}$$

$$K_d = \text{gain of phase detector} = \cancel{\frac{3E_p}{\pi}} = E_p \tag{8-26}$$

E_p = peak phase detector output for SNR >> 1.0

$$K_f = 1.0 \text{ (passive filter)} \tag{8-27}$$

$$K_d = \text{gain of dc amplifier, if used} \tag{8-28}$$

$$K_o = \text{VCO gain, radians/volt-sec} \tag{8-29}$$

$$M = \text{Loop multiplier constant} \tag{8-30}$$

$$B_L = \frac{B_{LT}}{3}\left(2\,\frac{\alpha}{\alpha_T} + 1\right) \text{ cps} \tag{8-31}$$

$$\omega_n = \sqrt{\frac{\alpha}{\alpha_T}}\,\omega_{nT} \text{ radians per sec} \tag{8-32}$$

$$\zeta = 0.707\sqrt{\frac{\alpha}{\alpha_T}} \tag{8-33}$$

8-4.1 Determination of B_{LT}

For this particular design the threshold has been specified at $\leq$ -120 dbm. The question that must be answered then is: "What threshold loop bandwidth is required to handle the maximum rate-of-change of frequency?" Examination of the specifications indicates there are two different rates involved:

Missile acceleration = 260 ft/sec^2

Automatic acquisition time $\leq$ 0.5 seconds

Intuition tells us that the acquisition sweep rate will far exceed the missile acceleration rate. In order to acquire on 0.5 seconds, the actual sweep time will have to be 0.25 seconds. The two rates are:

$$\dot{f}_a = \frac{260}{\lambda} = 556 \text{ cycles/sec}^2 \tag{8-34}$$

where λ = 0.468 ft/cycles at 2113 mc

$$\dot{f}_{SW} = \frac{500 \times 10^3}{0.25} = 2 \times 10^6 \text{ cycles/sec}^2 \qquad (8\text{-}35)$$

If the loop is to track this rate-of-change of frequency at threshold, the loop bandwidth, from Eq. (8-17), must be:

$$B_{LT} = 0.945 \sqrt{2\pi \times 2 \times 10^6} = 3350 \text{ cps} \qquad (8\text{-}36)$$

Since the specifications do not call out any Noise Figure requirements, the above value for B_{LT} can be inserted into Eq. (8-19) and the maximum allowable Noise Figure determined as follows:

$$\text{max. N. F.} = 10 \log \overline{\theta_n^2} + P_s(\text{dbm}) - KT\ (\text{dbm}) - 10 \log B_{LT}$$

$$= 12.31 \text{ db} \qquad (8\text{-}37)$$

where P_s = -120 dbm

KT = -173 dbm (100°C)

θ_n = 0.535 radians

With the low-noise amplifiers available, this value of N.F. can certainly be achieved, but not without some sacrifices in terms of cost, reliability, size and weight. The use of a tunnel diode amplifier was rejected for the preceding reasons plus the temperature problems.

Based on best engineering compromises, it was decided to use a balanced mixer/bandpass filter combination for the input of this transponder. In surveying the market for these items, the best Noise Figure that could be obtained was 16 db after taking into consideration the loss of the bandpass filter and the Noise Figure of the 1st I-F amplifier.

Faced with this problem, it is obvious that a design trade-off must be made. Either we must increase the bandwidth or reduce the tracking requirements. The threshold requirement of -120 dbm is firm, so the only alternative is to reduce the tracking requirements.

With a N.F. = 16-db and a threshold of -120-dbm, the maximum B_{LT} can be determined from Eq. (8-19) as:

$$B_{LT} = \underline{\underline{1400}} \text{ cps} \tag{8-38}$$

Substitution of this value into Eq. (8-17) and calculating the maximum rate the loop can track gives:

$$\dot{f} = 350,000 \text{ cycles/sec}^2$$

which is far below the desired value of 2×10^6 cycles/sec^2. Thus, it can be concluded that the signal acquisition time of 0.5 seconds cannot be obtained at the -120 dbm threshold with any reasonable degree of reliability.

All is not completely lost, however, when one considers the practical aspects of the overall tracking system. From an operational standpoint, it is extremely unlikely that the system will be required to acquire lock at threshold. For most applications (particularly deep space probes) the transponder will be receding so that signal levels will stay below threshold, once it has been reached.

Experience has shown that the main cause for loss of signal, during missile tracking is due to missile staging operations. During these periods of flight, the signal level fluctuates quite rapidly and can vary by as much as 30- to 40-db. The normal signal levels at the transponder are usually greater than 10- to 30-db above threshold when the last staging occurs. Thus, from a practical point of view, the "acquisition threshold" can be set 10-db above the "drop-out threshold" without degrading the overall system performance.

Remembering that the loop bandwidth increases with signal level (due to the limiter action), we can calculate the increase in B_L for a signal-to-noise increase of 10-db. From Eq. (8-25), we can write

$$\left(\frac{\alpha}{\alpha_T}\right)^2 = \frac{1 + \frac{4}{\pi}\left(\frac{N}{S}\right)_T}{1 + \frac{4}{10\pi}\left(\frac{N}{S}\right)_T}$$

re-arranging

$$\left(\frac{\alpha}{\alpha_T}\right)^2 = \frac{\left(\frac{S}{N}\right)_T + \frac{4}{\pi}}{\left(\frac{S}{N}\right)_T + \frac{4}{10\pi}}$$

if

$$\left(\frac{S}{N}\right)_T << \frac{4}{10\pi} \text{ (usual case)}$$

Then $\frac{\alpha}{\alpha_T} \approx \sqrt{10} = 3.16$

And from Eq. (8-31)

$$B_L = \frac{B_{LT}}{3}\left(2\frac{\alpha}{\alpha_T} + 1\right) \approx 3400 \text{ cps}$$

which is approximately equal to the 2 sigma value of 3350 cps.

Thus, it is possible to obtain the threshold of -120 dbm if we are willing to sacrifice the acquisition threshold a very reasonable amount (10-db).

The new loop specifications then become:

$$\zeta = 0.707$$

$$P_s = -120 \text{ dbm}$$

$$B_{LT} = 1400 \text{ cps}$$

Acquisition threshold $\geq$ -110 dbm

N.F. $\leq$ 16-db

θ_N = 0.535 radians, rms

θ_a = 0.314 radians, peak

$$\theta_v = 0.186 \text{ radians, peak}$$

$$\text{tracking range} = \pm 250{,}000 \text{ cycles/sec}$$

8-4.2. Calculation of Total Loop Gain

Since the maximum frequency lock range (ω_D) is ±250KC, the total loop gain required can be determined from Eq. (8-21).

$$K_T = \frac{2\pi \times 250{,}000}{0.186} = 8.45 \times 10^6 \text{ sec}^{-1}$$

8-4.3. Calculation of ω_n

From Eq. (8-20)

$$\omega_{nT} = \frac{1400}{0.53} = 2640 \text{ radians/sec}$$

8-4.4. Lead-lag Filter

The passive filter time constants become, from (8-22) and (8-23)

$$\tau_1 = \frac{8.45 \times 10^6}{(2.64)^2 \times 10^6} = 1.21 \text{ sec.}$$

$$\tau_2 = \frac{\sqrt{2}}{2.64 \times 10^3} = 0.535 \times 10^{-3} \text{ sec.}$$

Selecting a convenient value of 1.0 μfd for the filter capacitor requires from Eq. (8-22) and (8-23).

$$R_1 = 1.21 \text{ megohms}$$

$$R_2 = 535 \text{ ohms}$$

8-4.5. System Configuration

Paragraphs 8-4.1 through 8-4.4 define all of the pertinent parameters of the loop with the exception of the distribution of the loop gain, the I-F bandwidth, and the limiter suppression factor. Before these parameters can be determined, it is necessary to decide on the system configuration to be employed and to choose the various frequencies throughout the system.

In this particular design, the need for practical I-F and VCO frequencies dictated the use of the general system configuration shown in Fig. 8-1; with the first local oscillator frequency below the incoming signal and the second local oscillator signal above the incoming signal. The choice of multipliers then is covered by the Case 3 equation:

$$\frac{f_t}{f_r} = \frac{240}{221} = \frac{N_4}{N_1 + N_2 - N_3} \qquad (8\text{-}40)$$

Under this condition then

$$N_4 = 120$$

$$N_1 + N_2 = 111$$

The VCO frequency, f_o, becomes

$$f_o = \frac{2295 \text{ mc}}{120} = 19\ 1/8 \text{ mc}$$

which is acceptable.

If the condition is made that the first I-F frequency must be less than 100 mc, then:

$$f_r - N_1 f_o < 100 \text{ mc}$$

and

$$N_1 > \frac{2113\ 5/16 - 100}{19\ 1/8}$$

or

$$N_1 > 105.2$$

The factors of 108 are 2^2 and 3^3; which are very practical multiplier combinations. Therefore, the best choice of multipliers and frequencies are:

$$N_1 = 108$$

$$N_2 = 3$$

$$N_3 = 1/2$$

$$M = 110.5$$

$$f_o = 19\ 1/8 \text{ mc}$$

$$\text{1st I-F} = 47\ 13/16 \text{ mc}$$

$$\text{2nd I-F} = 9\ 9/16 \text{ mc}$$

8-4.6. Distribution of Loop Gain

The total loop gain required (at threshold) for the system is 8.45×10^6 sec^{-1} (paragraph 8-4.2), and consists of the following parameters:

$$K_T = \alpha_T K_d K_f K_{dc} K_o M$$

where

α_T = limiter suppression factor

K_d = phase detector gain, volts/radian

K_f = filter gain

K_{dc} = d.c. amplifier, if required

K_0 = VCO gain, radians/volts-sec

M = multiplier (following VCO)

This system is concerned with the accuracy of Doppler cycle count, so it is important that the short-term phase jitter be held to a minimum. This requirement dictates the use of a "stiff VCO" and a high output phase detector. Typical solid-state phase detectors are capable of producing 30 volts peak-to-peak output for maximum phase error inputs on strong signals. On noisy signals, however, this output is reduced considerably due to signal suppression in the limiter amplifier.

Since the signal suppression factor, α, is a function of the noise-to-signal ratio at the input to the limiter, the I-F bandwidth should be as narrow as possible without introducing excessive phase-errors due to Doppler frequency changes. The minimum bandwidth required to pass the Doppler information is equal to 1/2 of the required VCO pulling range. The VCO range is calculated as follows:

$$VCO_R = \frac{\pm f_d}{M} = \frac{\pm 250KC}{110.5} = \pm 2.26KC$$

Thus, the minimum I-F bandwidth must be at least 2.3 KC. However, for maximum Doppler excursions the phase shift would be ±45°, which is excessive for this design (the goal is < 10°). To hold the phase shift to small values, the bandwidth should be at least 10 times that calculated. To make a filter at approximately 10 mc with a 25 KC bandwidth is not feasible without the use of a crystal lattice, which in turn is not practical from a phase-shift standpoint.

A filter bandwidth of 200 KC can easily be attained by using L-C components. This results in a suppression factor of (at threshold).

$$\alpha_T = \sqrt{\frac{1}{1 + \frac{4}{\pi}\left(\frac{N}{S}\right)_T}} = 0.139$$

where

$$(N/S)_T = 16 \text{ db } (+100°C)$$

The maximum phase detector output at threshold (±90°) is then

$$E_d = 15 \times 0.139 = 2.0 \text{ volts peak.}$$

For a steady-state velocity error of $\Theta_v = 0.186$ radians, the maximum voltage available out of the phase detector becomes

$$V_d = \pm 0.186 \times 2.0 = \pm 0.372 \text{ volts}$$

Typical VCO sensitivities, in the frequency range of 10-20 mc are in the order of 3,000 cycles per volt-second. The VCO used in this design had a pulling range of ±12.5 KC and a gain of

$$V_{vco} = \frac{\pm 2.26}{3.0} = \pm 0.753 \text{ volts}$$

In order to obtain this voltage, it is necessary to provide an additional gain of approximately 2.1 through the use of a d.c. amplifier. Since the passive loop filter requires a high impedance load, the amplifier can be used following the filter to hold the filter gain to unity. An additional advantage of the d.c. amplifier is that by making its gain variable, the loop bandwidth can be adjusted in the final alignment to provide the correct B_{LT}. Using these values results in a total loop gain of

$$K_T = -0.139 \times 14.3 \times 1.0 \times 2.1 \times 3{,}000 \times 110.5 \times 2\pi$$

$$K_T = 8.7 \times 10^6 \text{ sec.}^{-1}$$

which is more than the total required. Thus, there is some room for adjustment to take care of system tolerances.

It should be noted that the gain of the phase detector is 14.3 volts/radian instead of the 12.6 volts/radians obtained by assuming a sine wave output. The reason for this is that linear output is assumed and the proper K_d is obtained by correcting the peak output as follows:

$$K_d = \frac{\pi E_p}{\pi} \text{ volts/radian (strong signal).}$$

8-4.7 I-F Gain Considerations

The amount of signal gain required in the receiver is determined by the input requirements of the loop phase detector and the I-F bandwidth. Sufficient gain must be provided so that the noise power out of the limiter (no signal input) is at the desired level for driving the phase detector. Most systems require from 0 dbm to +15 dbm (50 ohms) at the input to the phase detector.

Once the phase detector input level is determined the gain can be calculated on the basis of the noise power contained in the I-F bandwidth as follows:

$$N_{if}(\text{dbm}) = 10 \log KT + 10 \log B_{if} + NF(\text{db})$$

and

$$G \geq P_d - N_{if}(\text{db})$$

where

G = required gain

P_d = input power of phase detector, dbm

B_{if} = noise bandwidth of the I-F amplifier

NF = Noise Figure

$10 \log KT = -174$ dbm/cycle $(T = 290^\circ K)$

In order to insure "solid" limiting on noise, the available gain should be at least 10-db more than is required for the system. In other words, the limiter/amplifier should be driven at least 10-db harder than is required to produce a "just limiting" condition.

For example, the receiver described in this section required +15 dbm into the phase detector and has an I-F noise bandwidth of 200 KC. The I-F noise power is

$$N_{if} = -174 \text{ dbm} + 53 \text{ db} + 16 \text{ db} = -105 \text{ dbm}$$

and

$$G \geq +15 \text{ dbm} - (-105 \text{ dbm}) \geq 120 \text{ db}.$$

To insure solid limiting, the gain was set so it equalled 135-db in the absence of noise. Thus, the system has "15 db of limiting" on noise alone.

8-4.8 Phase-Stable I-F Amplifiers

It is self-evident that in order for any phaselock receiver to faithfully follow the incoming signal, the receiver should introduce no incremental phase shifts over the entire dynamic range of signal input and environmental variations. Phase shift variations in the receiver are one of the most difficult problems the designer must solve. Circuit designers and component manufacturers are continually working to reduce phase shift problems and much has been accomplished with solid-state designs in recent months.

The main causes of phase shift in the I-F amplifiers are:

1. Internal feedback in the amplifier.
2. AGC variations
3. Temperature variations
4. Frequency changes due to Doppler excursions
5. Saturation on strong signal levels

Causes of phase-shift 2, 3 and 4 can be reduced considerably by simply employing wide-band tuned circuits with a low L/C ratio. This technique is almost universally used in all phaselock equipments and has provc' very effective. Internal feedback effects are overcome by the use of mismatching techniques. The necessary narrow-bandwidth is obtained though the use of passive filters.

Because the incremental phase-shift requirements are in the order of 10°, the above techniques are not satisfactory in themselves. Additional techniques must be used to meet these stringent requirements. Many types of AGC circuits have been tried in order to minimize phase shift over dynamic ranges of 80 to 100 db. The types have ranged from various combinations of "forward" and "reverse" AGC to the use of diode-type attenuators between stages. Each of these methods has had some success but has not really been completely satisfactory. One of the latest methods developed employs two transistors in a differential amplifier form as shown in Fig. 8-3

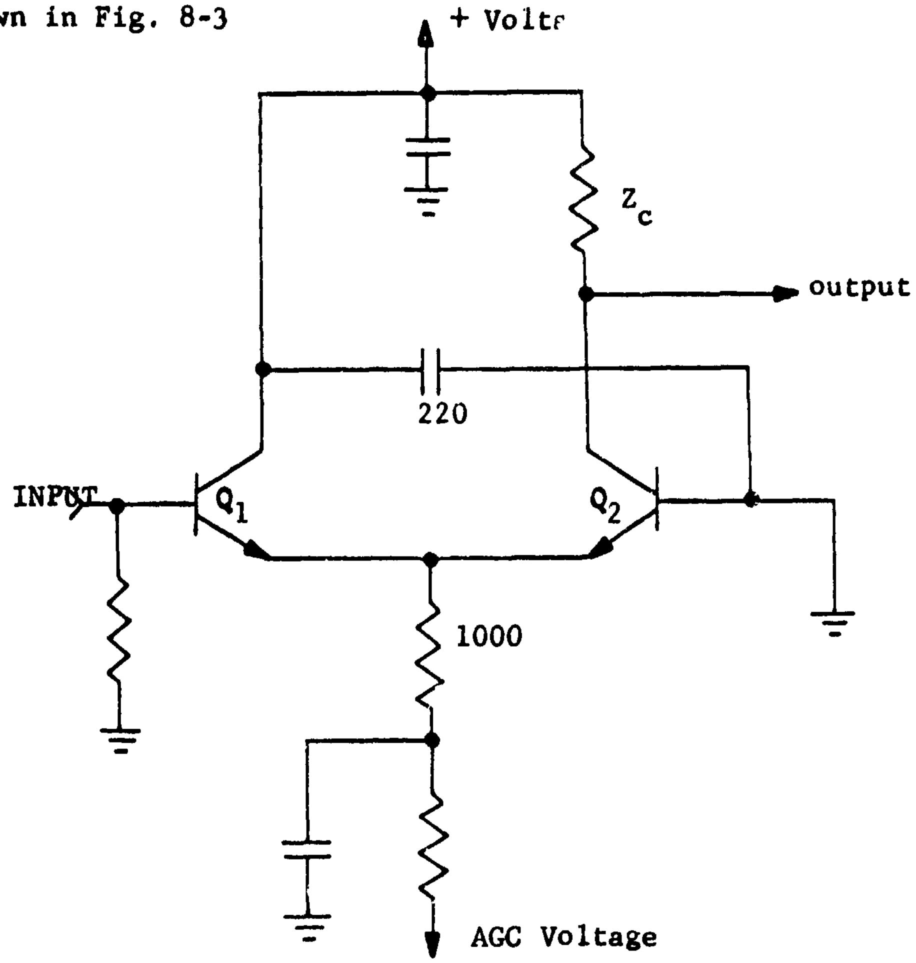

Fig. 8-3
I-F Amplifier Stage With AGC Applied

This configuration provides a relatively high input impedance, good isolation from input to output, low base to emitter capacitance, plus capacitance cancellation with current changes. The stage operates similar to any differential amplifier. Gain control is accomplished by varying the emitter current with the AGC voltage.

Tests conducted on this type of configuration have shown the phase shift to be less than 5° over a dynamic range of 30-db.

Incremental phase shift due to temperature changes are also minimized through the use of the circuitry shown. In addition, temperature-stable components (especially capacitors) must be used. Some temperature compensation devices may also be required. Incremental phase shift due to Doppler frequency excursions are reduced by using broadbanded tuned circuits with a small L/C ratio. Large capacity values are used to improve phase shift due to temperature and AGC variations.

Saturation of amplifier stages causes signal distortion with attendent phase shifts. In a system that employs AGC in the I-F amplifiers, saturation (or limiting) on noise in the I-F string is undesirable because it results in signal suppression due to the limiting action. The designer must take the necessary precautions to eliminate this particular problem (this is discussed further in paragraph 8-5.1.2).

8-4.9 AGC versus Limiting

Jaffe and Rechtin (JAF-1) have shown that the use of a limiter provides a near optimum phase-lock loop because the system self-compensates for signal level changes near threshold. This effect has been proved both in the laboratory and in the field. Thus, as far as theoretical operation is concerned, AGC is not a system necessity.

Besides the incremental phase-shift problems associated with the use of AGC, there are several other good reasons for eliminating AGC, if possible. The problems associated with interaction between the AGC loop and the phase-lock loop are not clearly understood because of second-order effects that do not lend themselves to easy analysis. Since the problem of "threshold" is not clearly understood, it would seem advisable not to complicate it further by the inclusion of the AGC loop.

A third problem, associated with the use of coherent AGC, is the generation of spurious signals within the equipment due to receiver overload prior to locking on strong signals. This problem can be solved, but it generally requires the use of two additional non-coherent AGC detectors - one before and one after the I-F bandpass filter.

All of the above problems can be eliminated if an amplifier can be designed that will limit, as the signal increases, without causing phase-shift. Transistor circuitry has recently been developed that will limit without introducing undersirable phase shift. The circuit diagram for such a single stage amplifier is shown in Fig. 8-5.

This circuit is identical to Fig. 8-4 except the emitter resistor is returned to a fixed voltage.

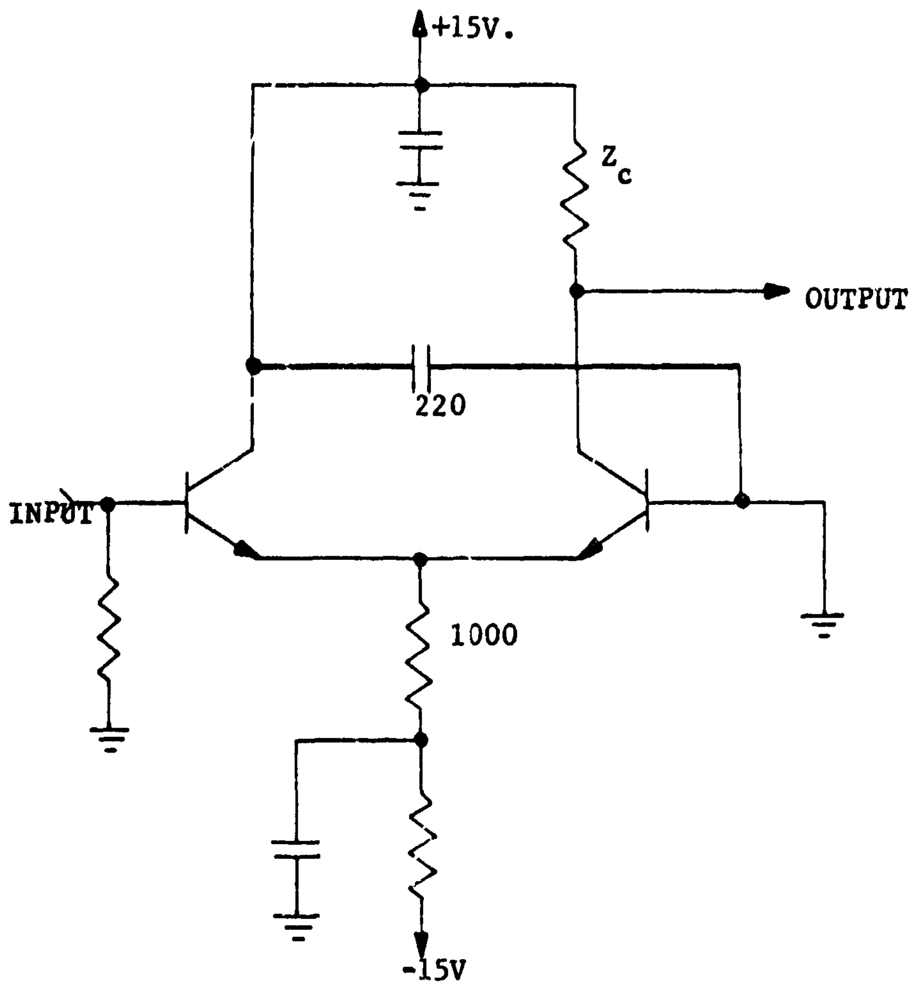

Fig. 8-5
Limiter/Amplifier Stage

A simplified explanation of the operation is as follows: For low level inputs the stage operates as a linear differential amplifier. As the signal level increases a point is reached where the output stage is not conducting any current during a portion of the input signal. Beyond this point then, no more energy can be transferred to the output circuit - the output power is limited by the maximum current available in the output stage. In effect, then, the output stage is switched on and off by the incoming signal.

The circuitry shown has been assembled in a four stage amplifier and, from low-level to full saturation of all four stages, the total measurable phase-shift is in the order of 5°. This is equal to, or better, than most of the AGC circuits developed to date. The amplifier shown provides 20 db of gain, at 50 mc, and can safely handle up to 0-dbm (50 ohm) input levels without affecting the operation.

8-4.10 Bandpass Filter

As mentioned previously, the bandpass filter is used to provide the narrow bandpass required for proper system operation. Its main purposes are to reduce the noise-to-signal ratio at the input to the phase detector, and to reduce the noise power level in the I-F amplifier to prevent undesired noise limiting.

The type of filter employed is naturally dependent on system requirements and the choice is up to the designer. However, most phase-lock receivers use either conventional L-C filters, or crystal filters. (Some receivers, with low I-F frequencies, use "mechanical" filters). L-C filters are generally used until the "loaded-Q" requirements exceed 50 to 100. Above this value, it becomes impractical to construct L-C filters and crystal filters are generally used.

The most important characteristics of the filter (outside of noise bandwidth) is that it have low phase-shift across the pass-band with no rapid phase-fluctuations or reversals anywhere in the receiver pass-band. Phase fluctuations or reversals in the filter will prevent the loop from acquiring the incoming signal. For this reason, the crystal filter generally employs only a single element that is constructed so the series and parallel resonances are separated as much as is practical.

8-5. Construction Precautions

Extreme care must be exercised in the design and construction of a phase-lock receiver in order to eliminate the possibilities of false-locks, self-oscillations, and susceptibility to spurious signals from any source. The phase-lock receiver is more susceptible to interference than most electronic equipments because of the high signal gains required and the small amount of phase error permissible. Signal input levels as low as -160 dbm are not unusual for narrow-band phase-lock receivers.

There is no "one" construction technique that will eliminate all of the practical problems to be encountered in receiver designs. Each unit has its own problems that can only be solved by the individual designers drawing upon their own wealth of knowledge and experience. Certain problems are peculiar to most phase-lock units -- these problems will be discussed briefly in the following paragraphs.

8-5.1 False-Lock Problems

Possibly the worst thing that can happen to a phase-lock receiver is to lock to an undesirable signal. This is especially true in the case of a transponder that is beyond reach of human help. Therefore, it is imperative that only signals coming from the antenna get into the unit, and only those within the receiver tracking band be tracked.

The main causes of false-locks are internally generated spurious signals, receiver saturation due to excessively large signals, and external spurious signals due to poor shielding or power line filtering. The internally generated signals are the most troublesome because they are usually an undesirable by-product of the VCO and/or the necessary multiplier/mixer combinations.

8-5.1.1 VCO Frequency and Harmonics

Examination of Figs. 8-1 and 8-2 and the four Case equations show there are several frequency combinations that can lead to false-lock problems. The most obvious is that the reference frequency into the phase detector is exactly the same as the 2nd I-F frequency. This, of course, is necessary in order to achieve lock (this is true for any phase-lock receiver).

Should the reference signal get into the Signal channel, of course, the loop could then lock on itself. There are several ways that this condition can come about:

1. Direct radiation into the 2nd I-F
2. Coupling through common power supply
3. Coupling through common ground currents
4. Coupling through the various multiplier/mixer combinations

The way to minimize this problem is:

1. Use just sufficient I-F gains to insure proper operating levels under all operating conditions. (Excessive I-F gains only complicate matters.)
2. Maintain the reference power at a practical minimum usable level
3. Provide physical isolation between modules and use "double" r-f shielding where possible

4. Provide filtering to reduce conducted interference. This filtering should provide attenuation to all frequencies of possible concern (i.e., incoming as well as outgoing).
5. Provide sufficient isolation in the amplifiers to reduce any reverse feed-through signals to negligible levels. A specific point in case is the reference signal in the AGC phase detector feeding back through the I-F signal input to the I-F amplifier and then into the signal input of the loop phase detector.

Harmonics of the VCO and various mixing combinations of these harmonics can be very troublesome in the unit. It is possible for a given combination to occur at the 1st I-F frequency, within the tracking band. From the case equations, the 1st I-F frequency (f_1) is related to the VCO frequency (f_o) in the following manner:

Cases 1 and 2: $f_1 = (N_2 + N_3)\ f_o$

Cases 3 and 4: $f_1 = (N_2 - N_3)\ f_o$

These equations can be written:

Cases 1 and 2: $$\frac{f_1}{N_3 f_o} = \frac{N_2}{N_3} + 1 \qquad (8\text{-}41)$$

Cases 3 and 4: $$\frac{f_1}{N_3 f_o} = \frac{N_2}{N_3} - 1 \qquad (8\text{-}42)$$

Thus the larger N_2 is with respect to N_3, the easier it will be to eliminate this type of false-lock. There are, of course, practical limits as to how high this ratio can be set.

Due to the many cross-product combinations possible as a result of the various frequency multiplications, extreme care must be exercised in the design of these circuits and special emphasis must be placed on filtering and shielding.

8-5.1.2 Large Signal Saturation

In a transponder (or receiver) that employs coherent AGC (i.e., AGC is applied only when the loop is in lock, and it is derived from the quadrature phase detector), false signals can be produced due to saturation occurring on large input signals prior to the loop acquiring lock. Signal clipping can produce harmonics of the I-F frequency which in turn can mix with harmonics of the local oscillator frequencies.

One method of preventing this situation is to use a non-coherent AGC that is over-ridden when the loop goes into a locked condition. This can be accomplished quite easily without degrading the performance, by simply using a conventional detector and setting its operating level below saturation but sufficiently above the noise level to prevent loss of gain on weak signals. The output of this detector can be fed into the AGC amplifier that is used for the coherent AGC. If the AGC voltage is also used as a lock indication (or automatic sweep control), it will be necessary to accomplish these functions in a different way.

It should be pointed out that there are two possibilities of saturation occurring in a typical narrow-banded receiver. The overload can occur either before or after the narrow bandpass filter in the I-F amplifier. For example, if the strong signal is within the passband of the narrow I-F, the overload will occur after the filter; if the signal is outside the passband, the overload can occur in the 1st I-F or 2nd mixer. To account for this likely event, it may be necessary to provide AGC detectors both before and after the bandpass filter.

8-5.1.3 Spurious Signals

Spurious signals can get into the receiver in three ways:

1. Direct radiation into the antenna
2. Radiation into the unit through the housing (poor shielding)
3. Power line coupling into the unit

The standard precautions of high gain receivers applies in so far as image rejection and 2nd I-F rejection at the antenna input are concerned. In addition, it is wise to reduce the r-f input bandwidth to a satisfactory acceptable minimum. If a wide r-f input range is desired, it can be accomplished through the use of tuneable band-pass filters. The reasoning behind this approach is: "the fewer signals that can enter the receiver, the fewer problems to be encountered."

Because of the many frequencies that are produced in the receiver (due to the large frequency multiplications necessary, it is generally considered necessary to have sufficient r-f shielding to reduce radiated signals from within the receiver to the level of -180 to -200 dbm. This type of shielding will reduce most radiation susceptibility problems to negligible proportions.

Conducted interference requirements are usually in accordance with MIL-I-6181 or equivalent. Here again, a goal to aim for is the -200 dbm level.

8-6 Internal Shielding and Filtering

Most of the radiation and conduction interference problems will come from within the unit itself. The combination of high density packaging, extreme sensitivity, and relatively high internal power levels (up to +20 dbm in the local oscillator chains) makes it extremely difficult to isolate signals within the unit.

The designer must take the approach to "isolate the signals at their source". In other words, do not allow the undesired radiated or conducted signals to get away from their point of origin. In the case of radiated signals, a good engineering practice is to use common ground points to reduce the ground current paths which can be very troublesome. Double-shielding is very effective in reducing radiated signals.

Conducted filtering is usually accomplished through the use of π-section L-C filters. R-F shielding is generally required between sections to eliminate mutual coupling between coils. Because of the many frequencies involved in a "super-het" phase-lock receiver, the π-filters should provide attenuation to all incoming, as well as outgoing, signals that can possibly cause problems. An example of this is: if the 2nd I-F frequency gets into the 1st I-F, it can get back into the 2nd I-F through the 2nd mixer.

A technique that has been used successfully to attenuate frequencies widely separated (i.e., 10 mc and 60 mc) is to use the π-section as shown in Fig. 8-6.

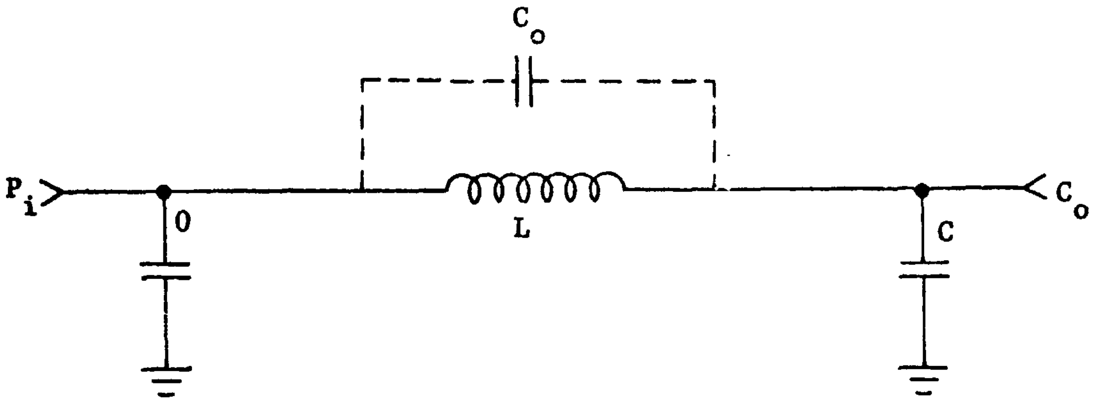

Fig. 8-6
π-Section Filter

If the filter coil is selected so that it is parallel resonant at the lower frequency, the L-C_o combination will look capacitive at the higher frequency of interest. If the capacitor "C" is chosen to be large compared to C_o, the filter will effectively attentuate both the high and low frequencies of interest. Care must be taken to prevent any possibility of series resonance occurring at any of the frequencies of interest.

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Chapter 9

OTHER APPLICATIONS OF PHASE-LOCK

9-1. Introduction

There are many applications of phase-lock techniques besides the receivers and coherent transponders of the last chapter. Discussed briefly here will be tracking filters, stabilization of oscillators, clean-up functions, frequency translation loops (including frequency multipliers and dividers), discriminators, and PCM bit synchronizers. Other applications, not covered, include automatic frequency control, television synchronization, (RIC-1, 2; SCH-6, WEN-1), and automatic steering of antenna arrays (BRE-1, BIC-1).

9-2. Tracking Filters

The term "tracking filter" or "audio-tracking filter" has come to describe a phase-locked loop that is used at the output of a receiver. Thus, the entire receiver is outside the loop in contrast to the previous chapter where most of the receiver (beginning at the first mixer) was inside the loop.

There are some decided advantages to this tail-end approach. When phase-lock was in its infancy, use of a separate tracking filter permitted a conventional receiver to be used without modification (DEB-1, GAR-1) as in Fig. 9-1. A very weak signal (from a satellite, for example) would be added to a much stronger, fixed, local reference signal at the receiver input. The reference is required to be much stronger than any noise so that the receiver detector operates well above its threshold. Output of the detector is then a beat-note (in the early satellites using a 108 mc transmission frequency, the beat-note was in the audio range) between the received signal and the local reference.

The beat-note would be expected to be deeply embedded in the noise so a narrow bandwidth filter is needed to recover it. Frequency of the beat-note changes as the Doppler frequency varies, so the filter must track the beat-note frequency. A phase-lock loop is an obvious and logical method of building the tracking filter.

Frequency of the local standard would be close to the expected carrier frequency of the input signal. In practice, an offset in

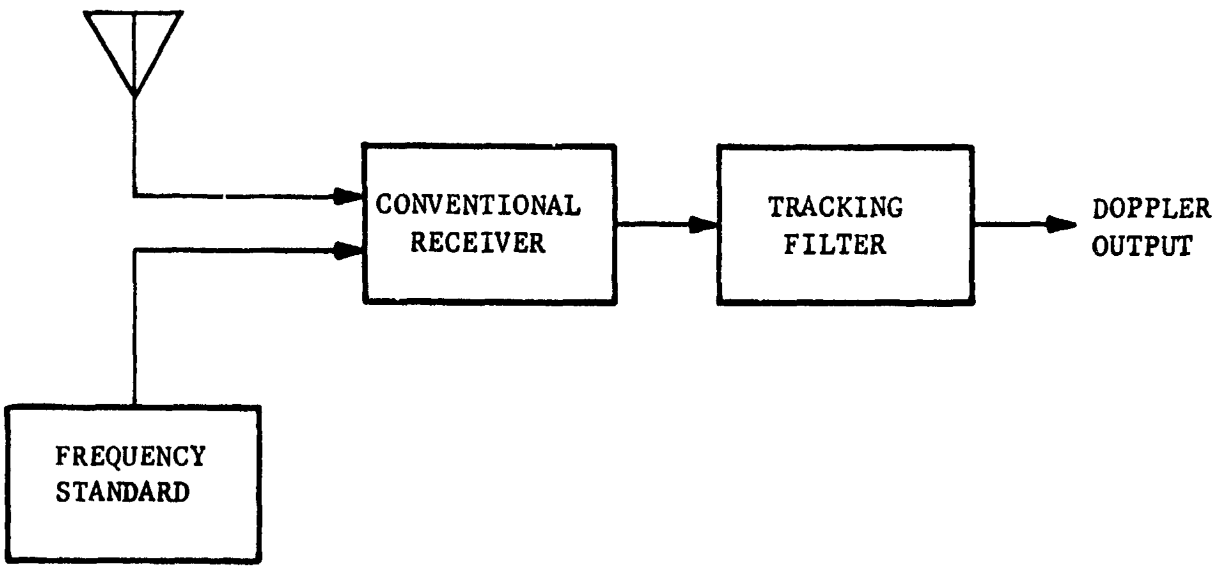


Fig. 9-1
Use of Tracking Filter

excess of the maximum Doppler frequency would be introduced so that the recovered beat-note would never pass through zero frequency.

Stability of the frequency standard is limited only by the state-of-the-art of precision, fixed oscillators and the multipliers and synthesizer needed to obtain the desired injection frequency. To a first order, there need be no concern over stability of oscillators within the receiver itself, since any such instability will affect both signal and reference identically. The only VCO is within the tracking filter and operates at low frequencies.

It is possible to obtain much better phase stability from a fixed frequency standard (particularly some of the atomic standards) than it is from a VCO. Using a tail-end, tracking-filter approach, the only precision high-frequency oscillator needed may be fixed in frequency. If a phase-locked receiver is used with the entire receiver (following the first mixer) included within the loop, it is necessary to derive the first injection frequency from a VCO.

From these considerations, it is sometimes possible to utilize improved oscillator stability, and therefore, potentially narrower bandwidth, if a tracking filter approach is employed.

There are, of course, disadvantages to the approach. Receiver bandwidth must exceed the entire Doppler shift that is to be accommodated. This is in contrast to the phase-locked receiver wherein the bandwidth may be much narrower than the Doppler range and the receiver is only required to tune over the range.

Another problem arises from the inevitable change of phase shift as a function of frequency of the fixed receiver. Since input frequency will be changing, the net phase shift through the receiver changes accordingly and appears as a small frequency error.

Considered in another manner, the shift in receiver phase from maximum Doppler frequency to minimum appears to add additional Doppler cycles into the record. An error in velocity is necessarily incurred.

The effect can be minimized only by using an extremely wide band receiver so that the phase change over the Doppler range is negligible. By contrast, since a phase-locked receiver exactly tracks the input frequency, the only components contributing to a phase slope are the antenna, preselector and any RF amplifiers. These are normally very wideband circuits (by comparison to a Doppler shift) and therefore do not usually have any significant phase slope.

9-3. Oscillator Stabilization and Clean-Up

Crystal oscillators used as frequency standards have their best long-term stability if they are operated at extremely low RF power levels. (Crystal aging is slower at the low levels). However, as was noted in Chapter 6, best short term phase stability is obtained at an intermediate power level where the RF signal is much greater than the circuit noise.

The best results are obtained if two separate oscillators are used: a very low-level one for good long-term stability, and a second oscillator, phase-locked to the first, operated at a higher level for good short-term stability. Bandwidth of the loop would be as narrow as possible consistent with maintaining reliable lock. Output would be taken from the locked oscillator.

Using the loop is equivalent to suppressing the amplitude fluctuations of the first oscillator almost completely, and passing the phase noise through an extremely narrow filter so as to reduce it substantially.

The same technique is useful for cleaning up the output of frequency synthesizers where harmonics and multiplier products are often present.

Another use of phase-lock arises in the stabilization of microwave oscillators (BEN-1, BEN-2, BUR-1, PET-1, POY-1, STR-1, STR-2). There are a number of oscillator types (Klystrons, voltage-tuned magnetrons, BWO's, and even triodes) which are capable of providing moderate power outputs (50 mw to several watts) at microwave frequencies. Besides the power capability, these devices are generally rather simple and easy to adjust. They have the common drawback of poor frequency- and phase-stability.

To overcome the inferior stability, such devices may be phase-locked to a harmonic of a stable oscillator at much lower frequency. With suitable design of the loop configuration, the harmonic power requirements can be very small -- fractional microwatts -- and good locking will still be achieved.

On the assumption that the reference signal -- even after repeated multiplication -- has far superior phase stability* to the microwave oscillator, it should be clear that loop bandwidth ought to be made as wide as possible in order to obtain the best tracking and greatest reduction of phase jitter. Any low-pass loop filter will only restrict bandwidth so it appears reasonable to use a first-order loop with no filter at all.** Bandwidth then becomes equal to loop gain K_oK_d. (Some filtering may be needed to prevent phase detector ripple from modulating the oscillator.)

*If this assumption is not valid, there is little or no advantage to locking the microwave oscillator.

**This argument can be carried one step further and the loop filter becomes a differentiator for some conditions (See GOL-4).

If the reference frequency can be changed, the locked oscillator may be tuned over some useful range (PET-1, POY-1).

9-4. Translation Loops, Multipliers and Dividers

An oscillator can be locked to one of its harmonics or subharmonics so as to constitute a frequency divider or multiplier, respectively. One application of this effect has been for obtaining harmonics of a frequency standard (CLA-1).

A related application would be to use a loop at the output end of a chain of multipliers to suppress unwanted subharmonics that are difficult to remove by means of passive filters.

Ordinary switching-type detectors will operate only with odd harmonic relationships between input frequencies, but unusual circuits have been devised (PED-1) so that even harmonics can also be used.

In either case, the phase detector itself may be regarded as generating harmonics of its lower-frequency input and comparing one of these against the higher frequency input. An ideal multiplier-type phase detector generates no harmonics and therefore cannot be used in a multiplier or divider (if the inputs are sinusoidal). Phase detector gain factor K_d is greatly reduced when used in harmonic service.

For any multiplier or divider application, the lock range is $\pm 90^o$ of the higher-frequency input.

Harmonic loops have no outstanding advantage in their favor and therefore are not widely used. A translation loop, on the other hand, can be extremely useful as may be seen from an example. Suppose it is desired to offset a 30 mc signal by 1 kc. One way to accomplish this would be by means of conventional single-sideband techniques but good suppression of carrier and rejected sideband would depend upon critical circuit adjustments.

A phase-lock offset could be completely non-critical if obtained as in Fig. 9-2. In this technique, a VCO whose uncontrolled frequency is close to the desired output is heterodyned with the incoming frequency; their beat-note is close to the desired offset. This beat is compared against an oscillator having exactly the correct offset frequency and the loop is closed back to the VCO so that the beat-note is locked to the offset oscillator. If the input frequency is f_1 and the

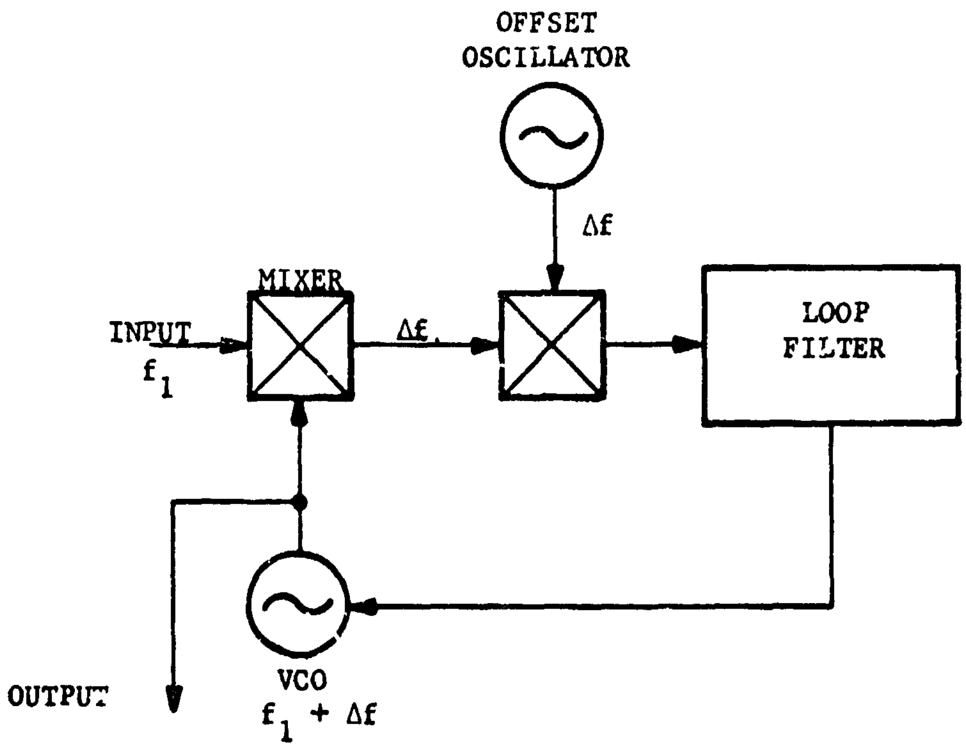


Figure 9-2

Basic Translation Loop

output is to be at $f_1 + \Delta f$, the offset oscillator must have a frequency Δf.

At first appearance, it would seem that phase lock has completely eliminated the residual carrier and unwanted sidebands that remain in conventional SSB techniques. Such perfection is not obtainable in the real world; any phase detector ripple will modulate the VCO and produce unwanted sidebands in the output. If a full-wave phase detector is used, the carrier, in principle, will not appear and the dominant sidebands will be at $f_1 - \Delta f$, and $f_1 + 3\Delta f$. If a half-wave phase detector is used, the first-order sidebands will be at f_1 and $f_1 + 2\Delta f$; the undesired sideband at $f_1 - \Delta f$ is dependent upon the second-order Bessel function.

Ripple may be reduced to any desired extent by means of brute force, non-critical low-pass filtering in the loop filter. It is to be expected that such filtering will usually require a narrowing of loop bandwidth.

There is no inherent reason why the offset Δf must be obtained from a separate oscillator. Instead, it could very well be derived from f_1 by means of mixers, multipliers and other offset loops. In this manner, it is possible for the output to be coherent with the input. In Chapter 8, coherent transponders were described; they may now be seen to be a form of coherent translation loop. Such loops are also widely used in complex receivers where coherent Doppler must be recovered.

9-5. Discriminators

Phase-lock loops are widely used as frequency discriminators for FM-FM telemetry. In this service, they provide a somewhat improved threshold over conventional discriminators but can be troublesome if they should drop out of lock.

To understand operation of a loop as a discriminator, it is useful to begin with the phase error response Eq. (3-5).

$$\theta_e(s) = \left[1 - H(s)\right] \theta_i(s) = \frac{s\theta_i(s)}{s + K_o K_d F(s)}$$

As a practical matter, attention will be restricted to the passive-filter, second-order loop. For that case, Eq. (3-5) becomes

$$\theta_e(s) = \frac{\left[s(\tau_1 + \tau_2) + 1\right] s\theta_i(s)}{\left[s^2(\tau_1 + \tau_2) + s(1 + K_oK_d\tau_2) + K_oK_d\right]} \tag{9-1}$$

and the phase detector output voltage is $V_d = K_d\theta_e$.

The term $s\theta_i(s)$ in Eq. (9-1) represents the frequency modulation of the input signal; output voltage from the phase detector therefore is recovered modulation as filtered by the bracketed terms.

Direct use of the phase detector output is unsatisfactory for two reasons: the output would be very noisy and the equivalent filter is undesirable. The noise difficulty may be appreciated from inspection of Fig. 3-4 from which it can be seen that practically all of the input noise will appear at the phase detector output.

These difficulties are circumvented by taking the demodulated signal from the output of the loop filter (V_r in Fig. 9-3).* It is readily determined that

$$\frac{V_r}{V_d} = \frac{1}{s(\tau_1 + \tau_2) + 1} \tag{9-2}$$

so that the output voltage is

$$V_r = \frac{s\theta_i(s)K_d}{s^2(\tau_1 + \tau_2) + s(1 + K_oK_d\tau_2) + K_oK_d}$$

$$= \frac{K_d s\theta_i(s)/(\tau_1 + \tau_2)}{s^2 + s(1 + K_oK_d\tau_2)/(\tau_1 + \tau_2) + K_oK_d/(\tau_1 + \tau_2)}$$

$$= s\theta_i(s)\left[\frac{1}{K_o}\right]\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{9-3}$$

*Sometimes the VCO control voltage (V_c in Fig. 9-3) is used as the FM output. In that case, an RC filter (with time constant R_2C) should be used to obtain the best filtering. However, the external filter is superfluous if the output is V_r.

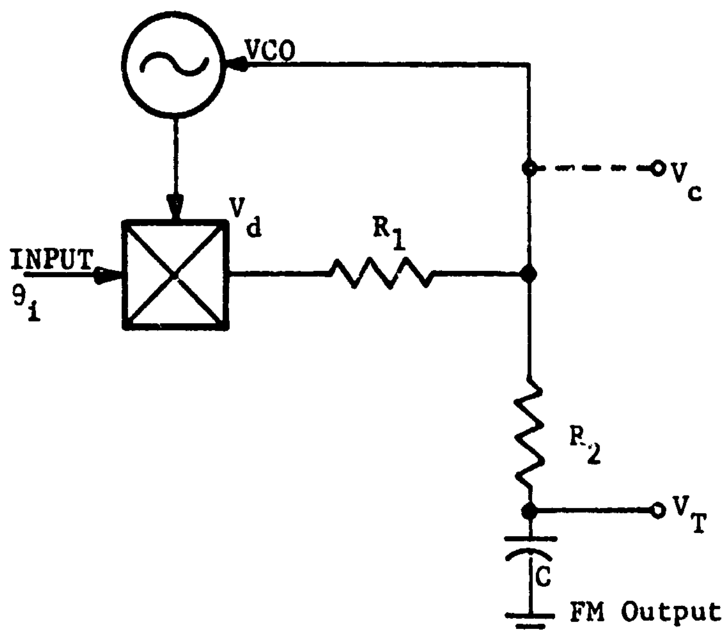


Figure 9-3

Loop Used as Discriminator

$\tau_1 = R_1C, \ \tau_2 = R_2C$

The first factor of this product, $s\theta_i(s)$, is the frequency modulation of the input signal, the second term is a gain factor, and the third term represents a second-order, low-pass filter.

(It is common practice to employ a post detection filter after the discriminator; five- and six-pole Butterworth and Bessel characteristics are often encountered. The 2-pole filtering of the loop is conveniently incorporated into the total post-detection filter thereby reducing the complexity of the external filter. For the remainder of this discussion the existence of external post-filtering will not be considered).

Noise spectrum at the FM output is also described by Eq. (9-3). Input phase noise is typically white so the shape of the noise power spectrum is the same as the squared magnitude of the transfer function: that is, $|V_r(\omega)/\theta_i(\omega)|^2$. The transfer function has the familiar triangular shape associated with the output noise spectrum of conventional FM detectors. (See sketch in Fig. 9-4).

Equation 9-3 is obtained on the basis of a linear approximation to the phase detector characteristic. Linearity is very important in a discriminator since any non-linearity will probably be interpreted as a data error. In order to obtain good linearity, it is common practice to make use of the triangular-characteristic phase detector (Fig. 6-7b) which is linear in the range of $\pm 90^{\circ}$. By contrast, a sinusoidal characteristic departs from a straight line by almost 5% at $\pm 30^{\circ}$. Useful range of the triangular detector is therefore almost tripled.

A triangular characteristic is obtained by applying square waves to both inputs of the phase detector. A limiter may be used to obtain a square signal input.

In Chapter 6 the behavior of a bandpass limiter was described. A bandpass limiter has a filter in its output that suppresses all harmonics, but the limiter used ahead of a phase-locked discriminator cannot have such a filter if a square-wave is to be delivered to the phase detector. All of the properties of the bandpass limiter were based upon the use of an output filter. If the filter is absent there is no assurance that the properties remain unchanged or even similar. Nevertheless, for lack of better information (no analysis of the wideband limiter could be found), it will be assumed that the wideband limiter has the same properties as outlined in Chapter 6.

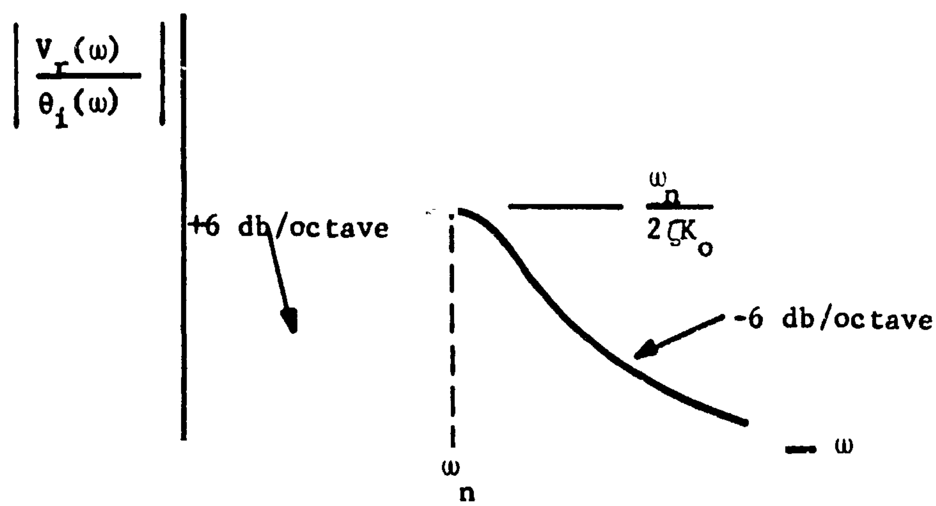


Figure 9-4

Discriminator Phase-Noise Transfer Function

When a limiter is used, the signal suppression factor α must be taken into account at low signal-to-noise ratios. In Eq. (9-3), natural frequency ω_n and damping ζ are both dependent upon α. The filtering action (loop bandwidth) is a function of SNR. Bandwidth (ω_n) reduction is 3% at $(SNR)_i$ of +10 db and 18% at 0 db (Eq. (6-4)).

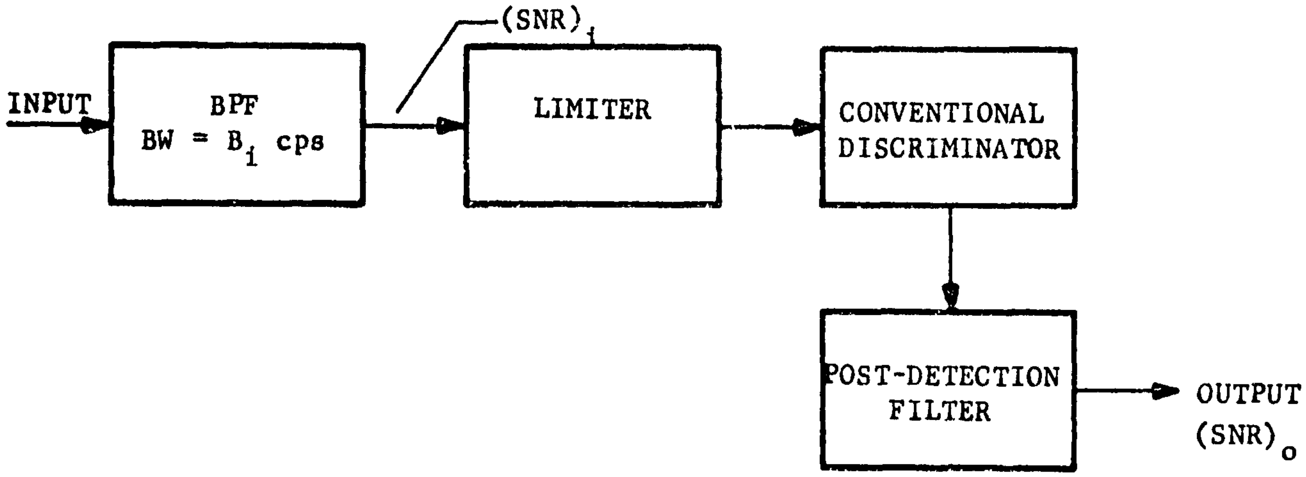
Gilchriest (GIL-1) points out that this suppression effect also exists in conventional discriminators. However, in that case, the bandwidth is fixed but the gain of the discriminator is reduced. Therefore, if the discriminator is calibrated at high SNR, it will be out of calibration at low SNR. Gain will be proportional to α and is therefore reduced by 6% at the +10 db threshold SNR of conventional discriminators. A phase-lock discriminator exhibits this change of calibration only to the extent that the change of bandwidth affects the signal. DC calibration remains unchanged.

(Ordinarily, no effort is expended to compensate for the change in loop bandwidth. However, coherent AGC, derived and applied after the limiter, would appear to offer a method of keeping bandwidth constant. No mechanization of this idea has come to light in the literature).

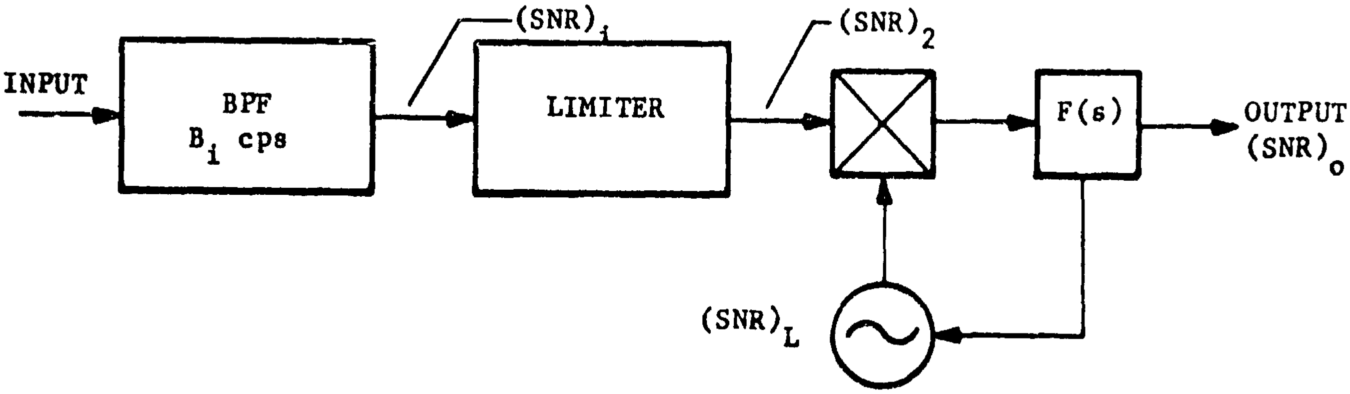
Above threshold, a phase-locked discriminator has an output SNR identical to that of a conventional discriminator with the same input and output filtering. Threshold of a conventional discriminator is considered to be +10 db SNR at the input to the limiter; we will derive approximate threshold values for the phase-lock loop in order to determine the improvement that can be gained.

The block diagrams of Fig. 9-5 will apply to the analysis. Comparison between thresholds will be based upon $(SNR)_i$, the signal-to-noise ratio in the input bandwidth B_i. This is also the signal-to-noise ratio applied to the limiter.

Phase-lock noise threshold will be assumed to occur when the signal-to-noise ratio in the loop is 0 db, that is, $(SNR)_L = 1$. Using Eq. 4-16, the signal-to-noise ratio at the input to the loop in a bandwidth B_i can be found to be



A. Conventional



B. Phase-Locked

Figure 9-5

Discriminator Configurations

$$(SNR)_2 = \frac{2B_L}{B_i}(SNR)_L$$

$$= \frac{2B_L}{B_i} \text{ at threshold} \qquad (9\text{-}4)$$

Having found $(SNR)_2$, Eq. (9-3) may be used to determine input signal-to-noise ratio as

$$(SNR)_i = \tfrac{1}{4}\left[(SNR)_2 - 1 + \sqrt{\left[(SNR)_2 - 1\right]^2 + \frac{32}{\pi}(SNR)_2}\right] \qquad (9\text{-}5)$$

Substituting 9-4 yields the phase-lock threshold as

$$(SNR)_{iT} = \tfrac{1}{4}\left[\frac{2B_L}{B_i} - 1 + \sqrt{\left(\frac{2B_L}{B_i} - 1\right)^2 + \frac{64B_L}{\pi B_i}}\right] \qquad (9\text{-}6)$$

It now remains only to specify B_L/B_i.

Two forms of input frequency modulation will be considered: a step of magnitude $\Delta\omega = 2\pi B_i$ and sinusoidal modulation with excursion $\Delta\omega$ and maximum modulating frequency ω_m.

For the step input, we will specify the minimum allowable bandwidth to be such that the resulting peak phase error is 90°. (We also assume high gain in the loop so that static error is much smaller.) Peak phase error may be obtained from the curves of Fig. 5-3 and minimum ω_n determined as a function of damping. Loop bandwidth B_L is then calculated from Eq. (4-12) and the numerical results are shown in Table 9-1. These values may be substituted into 9-6 and threshold determined therefrom.

Table 9-1

Minimum Allowable Loop Bandwidth B_L for Frequency Step $\Delta\omega = 2\pi B_i$

ζ	B_L/B_i
0.5	1.08
0.707	0.98
1	0.93
2	0.89
5	1.01

Peak transient phase error = 90°

From the table, it may be seen that $B_L/B_i \approx 1$ is within ½ db of being correct for all values of damping shown. Using $B_L/B_i = 1$, the threshold is calculated as

$$\boxed{(SNR)_{iT} = 1.4, \text{ (about 1.5 db)}} \tag{9-7}$$

Therefore, if the loop must accommodate a full-bandwidth step of frequency, the threshold improvement over a conventional discriminator is approximately 8.5 db.

Gilchriest (GIL-1), on the basis of different criteria, arrives at an improvement of 10 to 20 db. If the non-linear behavior of the loop is taken into account, his 10 db estimate seems to be the most reasonable since to obtain 20 db improvement on his criteria would imply that the loop is capable of holding lock at -7.5 db $(SNR)_L$. The two independent results of 8.5 db and 10 db are sufficiently close as to suggest that the approximate magnitude of the true value has been found.

A full-bandwidth step input is a rather drastic requirement to impose upon a loop. If PAM-FM-FM is the modulation form, then a discriminator must accommodate the step but if ordinary FM-FM is used, the situation changes and a narrower loop bandwidth is allowable.

Consider that sinusoidal frequency modulation with deviation $\Delta\omega$ and maximum modulating frequency ω_m has been applied to the incoming signal. Modulation index is $\Delta\omega/\omega_m$ and will be denoted by the symbol

D. According to Black* the RF bandwidth occupied by an FM signal is approximately $2(\Delta\omega + \omega_m) = 2\ (D + 1)\ \omega_m$. We shall assume that the bandwidth $2\pi B_i$ of the input filter is set equal to this minimum permissible bandwidth. Based on this assumption, it is possible to determine $\Delta\omega$ and ω_m if D and B_i are specified.

Equation 5-12 gives the loop phase error for sinusoidal FM input. From this equation, we determine the value of ω_n that causes the peak error to be 90^o; this value defines the minimum allowable bandwidth of the loop. In terms of D and B_i (rather than $\Delta\omega$ and ω_m) the minimum natural frequency is

$$\omega_{n_{min}} = \frac{\pi B_i}{D + 1}\left[1 - 2\zeta^2 + \sqrt{(1 - 2\zeta^2)^2 + 1 + \frac{4\ D^2}{\pi^2}}\right]^{1/2} \qquad (9\text{-}8)$$

where it has been assumed that loop gain is large. (To be precise, $\Delta\omega/K_oK_d \ll \pi/2$. If this assumption is incorrect, a wider bandwidth is needed).

If damping and natural frequency are known, loop noise bandwidth may be calculated from Eq. (4-12). Figure 9-6 shows plots of normalized minimum noise bandwidth versus modulation index for different values of damping.

It may be seen from the figure that if $D > 5$ damping of $\zeta = 0.5$ permits the smallest bandwidth. (This finding is in harmony with Spilker's (SPI-3) conclusion that $\zeta = 0.5$ is optimum for large modulation indices). For small D, it is evident that heavy damping is needed if a small bandwidth is to be obtained.

Values of B_L/B_i may be taken from Fig. 9-6 and substituted into Eq. (9-6) to obtain the loop threshold for various dampings and modulation indices. Results are shown in Fig. 9-7. The following conclusions may be drawn:

1. There is always some threshold improvement over the +10 db of a conventional discriminator.
2. Improvement is greatest for large modulation index.
3. If modulation index is large, a damping of $\zeta = 0.5$ appears optimum.

*Harold S. Black, Modulation Theory, Van Nostrand, New York, 1953.

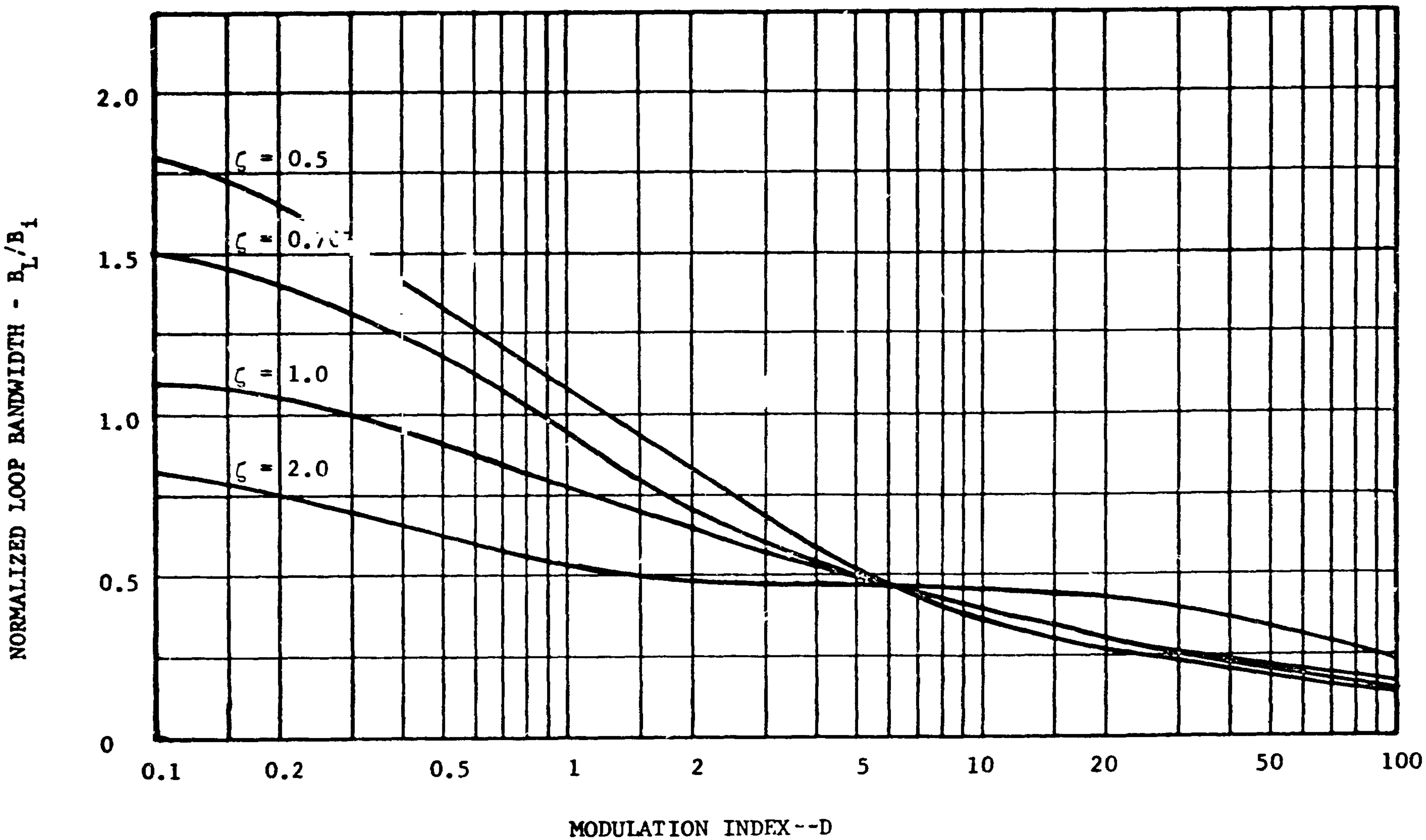


Figure 9-6

Minimum Loop Bandwidth for Sinusoidal FM

$$2\pi B_i = 2\ (\Delta\omega + \omega_m) = 2\ (D + 1)\omega_m$$

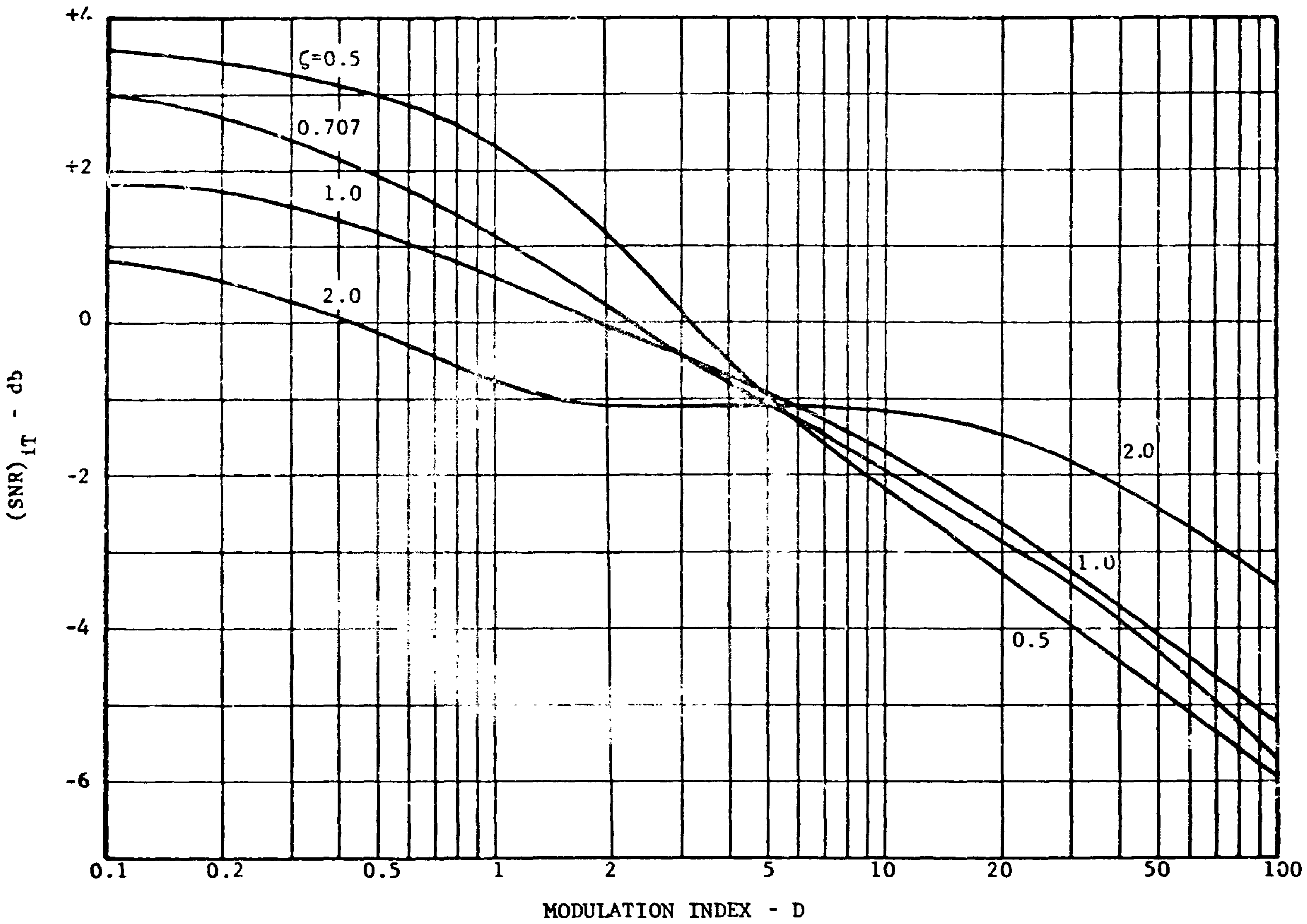


Fig. 9-7

Threshold $(SNR)_i$ for Sinusoidal FM

4. If modulation index is small, the loop should be heavily damped.
5. For $D \approx 5$, the threshold is virtually independent of ζ. The IRIG FM-FM standard is D=5; therefore, the common practice of utilizing the two poles of the loop as part of a more complex post-detection filter has no adverse effect upon threshold.

The foregoing analyses have been entirely heuristic in nature and have been chosen more for ease of explanation than for rigor. A critique of the approach is given in the following paragraphs.

One great simplification in each analysis has been to ignore the effect of the input filter. This neglect is justified for $B_L << B_i$ (large deviation) but cannot be expected to be correct if the two bandwidths are comparable. In the case of step modulation, the input filter alters the modulation so that the signal actually reaching the loop will have a rise time of approximately $0.7/B_i$. The finite rise time would imply that a somewhat narrower loop bandwidth could be tolerated.

For either type of modulation, the filter will eliminate some of the incoming noise whereas the analyses have assumed full noise reaching the loop. In this respect, the results obtained will tend to be somewhat pessimistic.

A much more important question is the arbitrary definition of threshold that has been assumed: 0 db signal-to-noise ratio in the loop and, independently, 90^o peak modulation error. If either of these conditions represents a threshold by itself, they certainly cannot occur simultaneously if the loop is to remain locked. To that extent the threshold criterion has been decidedly optimistic.

Spilker (SPI-3) has performed an analysis for sinusoidal FM and damping of $\zeta = 0.5$. His input threshold is 4 to 6 db higher than has been derived here; moreover he presents experimental data which indicate that even his result is slightly optimistic. The criterion of loop threshold he uses is that total rms phase error in the loop not exceed 1/2 radian. If there were no modulation, this criterion would be equivalent to 3 db signal-to-noise ratio in the loop compared to the 0 db assumption used here.

It would appear, therefore, that the thresholds derived here represent extreme lower limits on performance -- limits that can never be reached with real loops.

Modulation spectrum is another item to consider. A step input can reasonably be expected if PAM or PDM data are to be handled but sinusoidal modulation is usually only a convenient fiction. Lindsey (LIN-1) has considered the case in which the modulation is equivalent to white noise passed through a simple RC filter with a transfer function of $\omega_m/(s + \omega_m)$. If we still define modulation index as $D = \Delta\omega/\omega_m$, it is still reasonable to require $2\pi B_i = 2\ (D + 1)\ \omega_m$ as the minimum bandwidth of the input filter.

Lindsey computes input threshold using the criterion of total rms error equal to 1 radian as loop threshold. His results* are plotted in Fig. 9-8. (There is no mention of damping because the Wiener optimum filter has been used for each modulation index). It is evident that a filtered random modulation is not as severe a constraint as sine wave FM since Lindsey's threshold is some 4 to 6 db less than the best in Fig. 9-7.

The following conclusions may be drawn regarding discriminators:

1. At high input SNR's there is no appreciable difference between the various types.
2. A phase-locked loop will have a lower threshold than the +10 db of a conventional discriminator.
3. The improvement that can be gained depends upon the modulation of the input signal. No one number or one rule will cover all situations.
4. Even when modulation has been specified, there is still some uncertainty over the obtainable improvement because of the arbitrariness of any definition of phase lock threshold.
5. For best results, the loop should be specifically designed for the modulation actually present.
6. Premodulation filtering can provide better performance.

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*In a later article (LIN-3) he points out that threshold is strongly dependent upon modulation spectrum and that suitable pre-modulation filtering can enhance system performance in considerable degree.

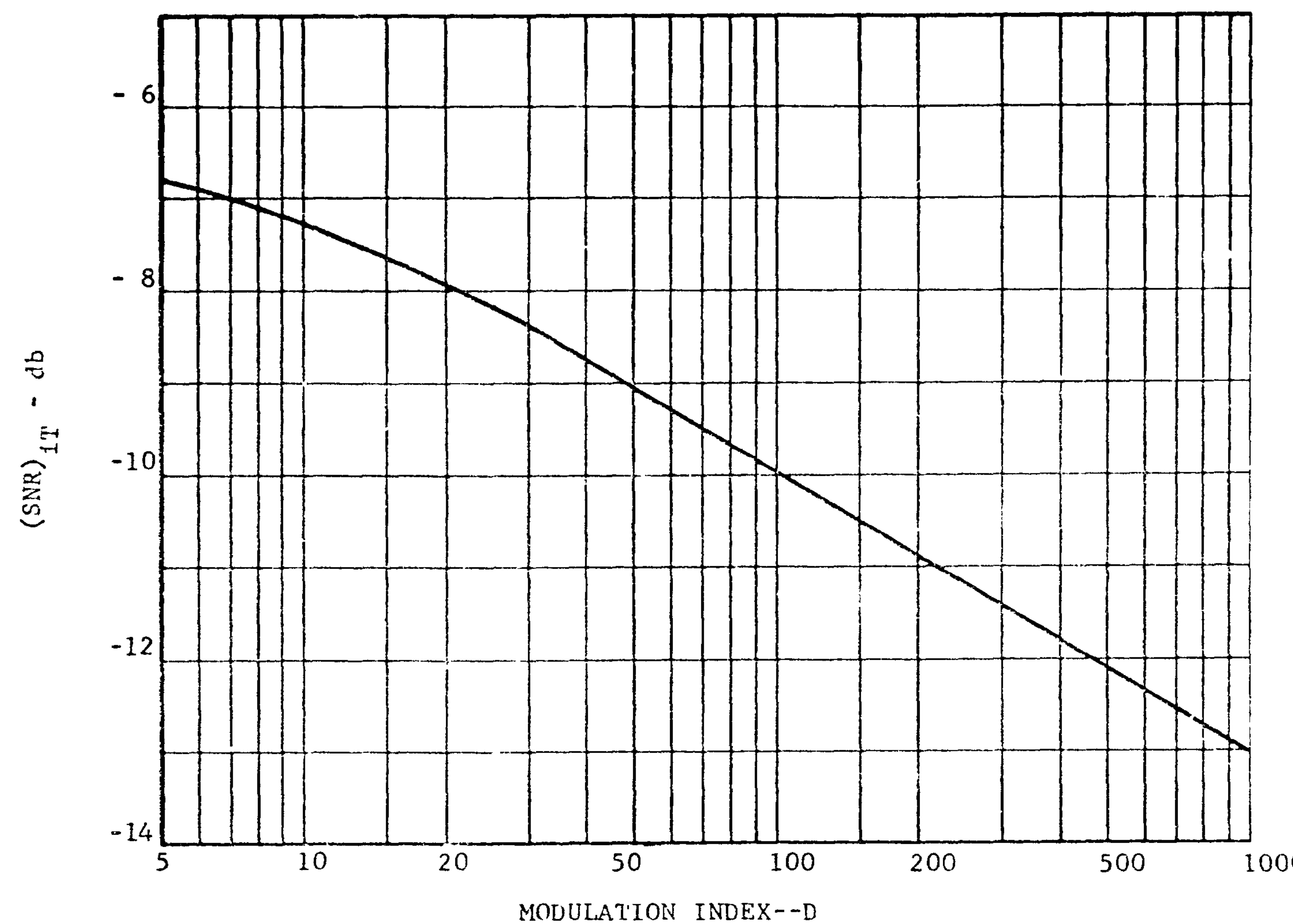

Figure 9-8

Threshold $(SNR)_i$ for Random Modulation (After Lindsey)

9-6. PCM Bit Synchronization

A PCM signal consists of a series of binary digits (bits) occurring at a periodic rate. The weight of each bit ("zero" or "one") is random but the duration of each bit, and therefore the periodic "bit rate" is constant (or essentially so). For detection and further processing of the digits, it is necessary to have a "clock" that is coherent with the bit rate. This clock must ordinarily be derived from the incoming data stream.* Phase-lock techniques are widely used to recover the clock from the data.

Some form of Non Return to Zero (NRZ) modulation is almost always used in order to maximize data rate in a given transmission bandwidth. In a truly random NRZ bit stream, there are no discrete frequency components present; specifically, there is no component at the the bit rate.** In fact, the continuous spectrum of an NRZ wave has a null at the bit frequency.

A helpful analogy is found in double-sideband, suppressed-carrier modulation. In this case, the carrier is not present (it has been balanced out at the modulator) but a local carrier is needed for proper demodulation. It has been demonstrated (COS-1) that a DSB signal has sufficient information in the sidebands to permit carrier reconstruction at the receiver. A modified form of phase-lock is used for the reconstruction.

Similarly, an NRZ signal may be regarded as lacking a "carrier" which must be reconstructed from information contained within the signal. It is impossible to recover the clock merely by applying the input signal to the phase lock loop; there is nothing for the loop to lock on to.

Timing information in a PCM signal is carried in the data transitions; the time of a transition marks one boundary of an individual bit. Transitions can be of either positive or negative direction,

---

*Sometimes a separate pilot signal is transmitted for synchronization purposes. This is rare and is contrary to IRIG PCM standards. Moreover, Stiffler (STI-1) has shown that best use of transmitter power is obtained by devoting all power to the data and none to a pilot.

**W. R. Bennett, "Statistics of Regenerative Digital Transmission", BSTJ, Vol. 37, pp. 1501-1542, November, 1958.

but either polarity has the same meaning for timing recovery. If a series of undirectional pulses is generated to mark transition times, there will be a discrete component of the bit frequency in the pulse train and a loop can be locked to it.

Figure 9-9 illustrates one method of timing recovery and Fig. 9-10 shows typical waveforms. In this illustration, pretransmission filtering of the bit stream has been assumed. The received signal is first differentiated in order to mark the locations of the data transitions. A rounded pulse of corresponding polarity is obtained for each positive and negative transition.

A rectifier converts all pulses to the same polarity. A full-wave rectifier has been shown but half-wave is possible. On the average, half of the available information is discarded by a half-wave rectifier.

The rectified pulses can be shown to contain a discrete spectral component at the bit frequency that the loop can track. For convenience of understanding, the rectifier output may be considered to be a coherent signal, periodic at the bit rate, that is randomly keyed on and off by a keying signal whose transitions are synchronous with the bit rate. During the "on" intervals, the loop tracks the coherent signal; during the "off" intervals, the loop remembers the last frequency present and still provides a clock output.

The apparatus of Fig. 9-9 can and has been used as shown. There are other methods (such as variations of early-late gates) that are also encountered frequently. Whatever the actual details may be, all systems must have two properties in common:

1. A method of locating the data transitions. This is normally performed by some kind of linear differentiating or differencing operation.
2. A form of rectification that converts the transition information to a usable form. This operation is necessarily a second-order (or higher even-order) non-linearity.

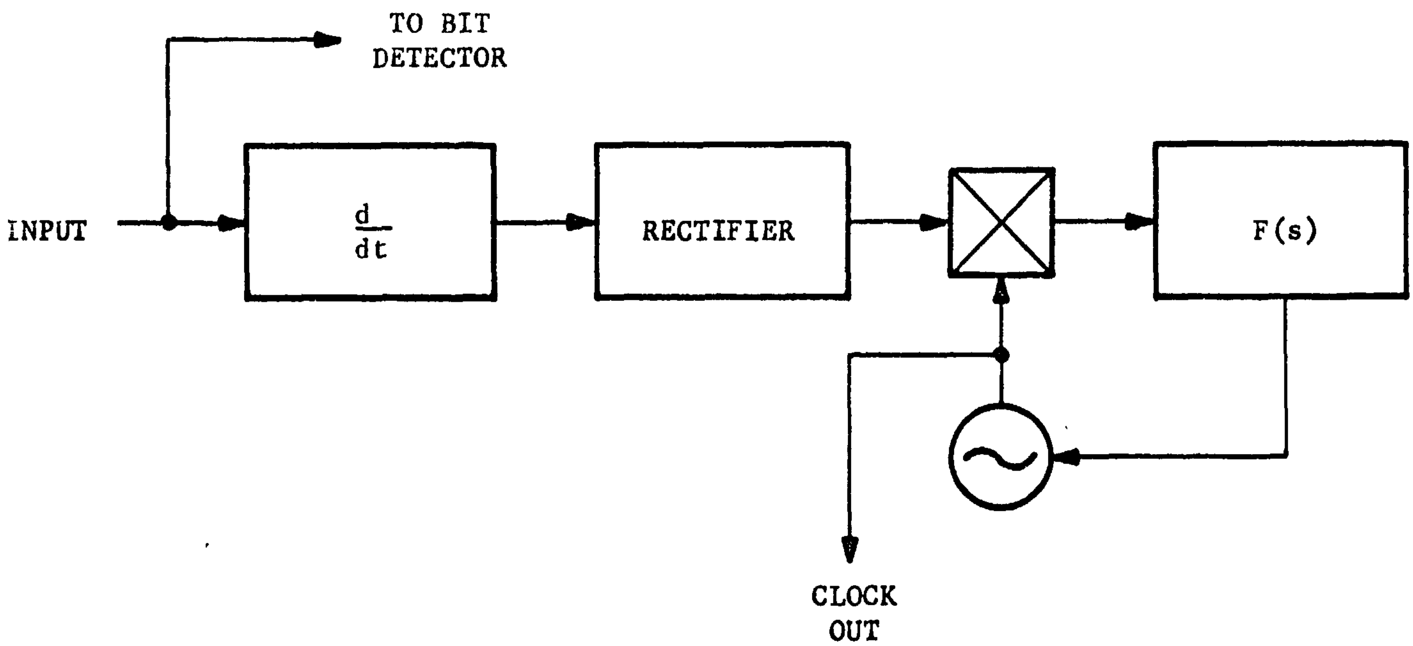

Fig. 9-9

Timing Recovery for NRZ Digital Data

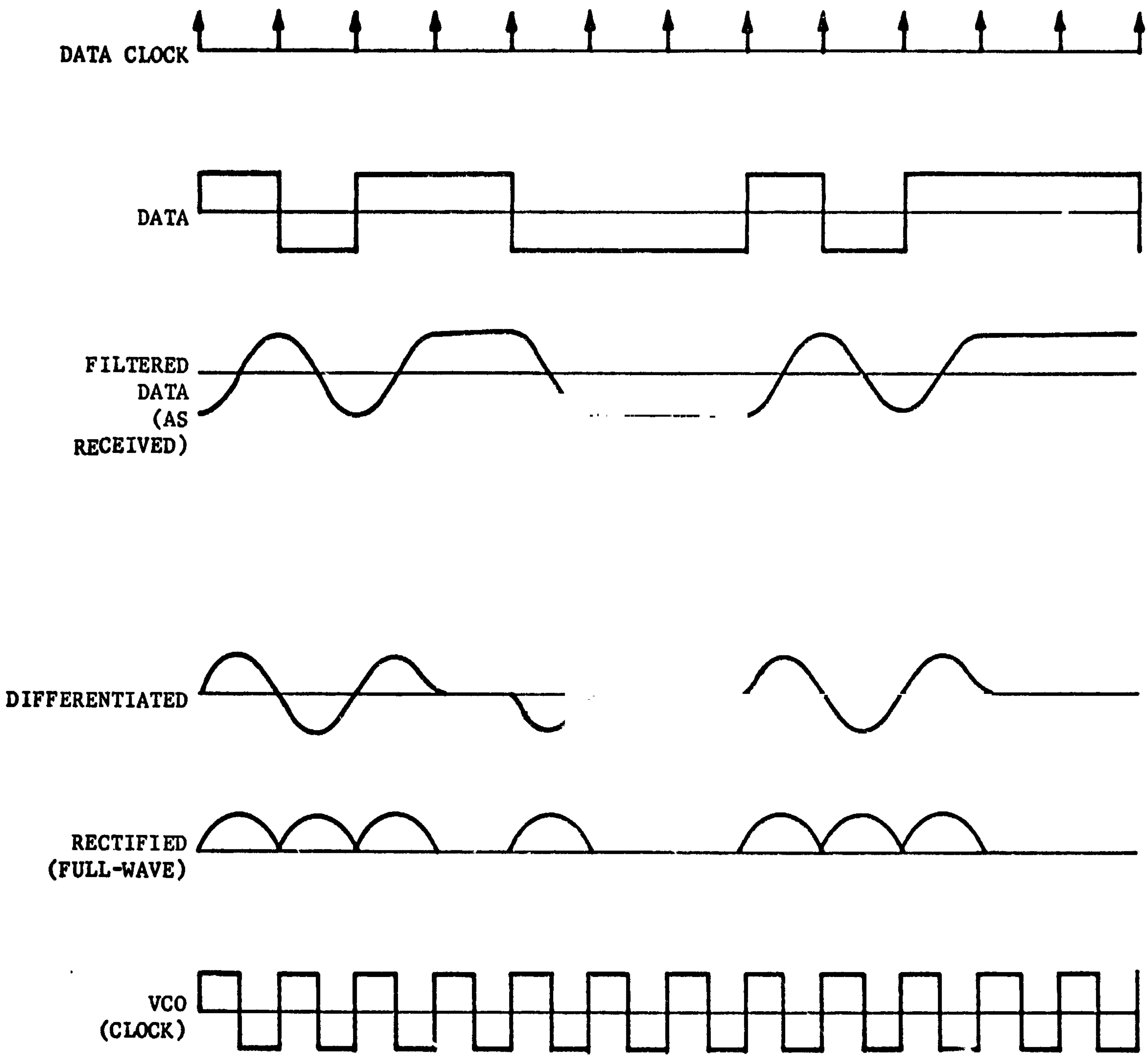

Figure 9-10

Timing Waveforms

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Chapter 10

TESTING THE PHASELOCK LOOP

10-1. Introduction

The measurement of system parameters at threshold is seriously hampered by the presence of noise in the loop. The usual procedures employed are: determine the parameters on strong signal levels and project them to their threshold values by means of the mathematics; or, measure the parameters in a low-noise environment, simulating threshold signal levels. A simulated operational test can be conducted to determine threshold through the use of test equipment capable of simulating threshold conditions. For field checkout, the simulated operational test is the preferred method.

10-2. Simulated Operational Test

The best test of whether a phaselocked transponder (or receiver) is performing to specifications is to actually simulate the specified threshold conditions and determine experimentally if the observed performance is acceptable. Although this test requires special test equipment, it is generally considered the most acceptable measure of the systems performance and is one of the simplest to conduct. This test is a measure of the threshold acceleration and velocity tracking capabilities.

Simulation of the threshold conditions requires the use of a signal generator with a continuously variable power output that can be adjusted to at least 10-db below the specified threshold. In addition, the frequency of the generator must be variable to simulate anticipated frequency excursions due to velocity and acceleration. Generally, the frequency variations (modulation) are accomplished electronically, while the signal level is manually adjusted.

The input test signal for the typical CW Doppler tracking loop is a frequency ramp generated by modulating the signal generator oscillator with a triangular-shaped waveform. The peaks of the modulation are adjusted to simulate maximum Doppler deviation, while the slope is adjusted to simulate maximum acceleration. Once the proper modulation is determined, the signal level is reduced to specified threshold and operation of the unit is observed to determine "satisfactory performance."

"Satisfactory performance" at threshold has been defined in Chapter 4 as: "that value of loop signal-to-noise ratio, below which desired performance cannot be obtained." The most obvious criterion of performance (from Chapter 4) is "loss of lock," which can only be defined in a statistical sense; thus, determination of satisfactory performance becomes a matter of definition for threshold conditions.

~~Since the design criteria is based on the 2σ confidence factor ($\theta_e + 2\theta_n \leq \pi/2$), the output of the loop phase detector can be observed on an oscilloscope (Calibrated for ±90° phase deviation) and the peak phase error noted. If the system is performing properly, only a small percentage (≈ 5%) of the phase jitter peaks will reach or exceed ±90° and the loop should remain in lock "most of the time."~~

10-3. Threshold Sensitivity (no modulation)

The threshold sensitivity of a phaselocked loop in the absence of signal modulation is of considerable interest (i.e., such a condition arises when the missile is traveling radially away from the transmitter at a constant velocity; as in deep-space tracking). In this case, the phase error due to acceleration is reduced to zero and only the velocity (θ_v) and noise (θ_n) errors are present in the loop. Minimum sensitivity occurs when θ_v is a maximum, while the maximum sensitivity occurs when θ_v is a minimum.

Determination of the threshold, without modulation, is again done by definition. The usual procedure is to reduce the input signal level until the loop is out of lock approximately 50% of the time. The quadrature phase detector output voltage can be used to determine the lock condition of the loop. The observation time interval is in the order of one minute (i.e., the voltage out of the quadrature phase detector is greater than some absolute value approximately 50% of the time).

10-4. Loop Bandwidth (by use of an input frequency ramp)

The threshold loop bandwidth (B_{LT}) can be indirectly determined from measurements of the dynamic phase error (θ_a) in the loop under low noise conditions. A frequency ramp is used at the input and the dynamic phase error is measured at the output of the loop phase detector. Knowing the peak phase error (θ_a) and the rate-of-change of frequency

($\Delta\dot{\omega}$), the loop bandwidth (B_{LT}) can be calculated from the following relationships:

$$\omega_n = \sqrt{\frac{\Delta\dot{\omega}}{\theta_a}} \qquad \text{radians/sec.}$$

$$\omega_{nT} = \omega_n \sqrt{\frac{\alpha_T}{\alpha}} \qquad \text{radians/sec.}$$

$$B_{LT} = 0.53\ \omega_{nT} = 0.53\ \omega_n \sqrt{\frac{\alpha_T}{\alpha}} \qquad \text{cycles/sec.}$$

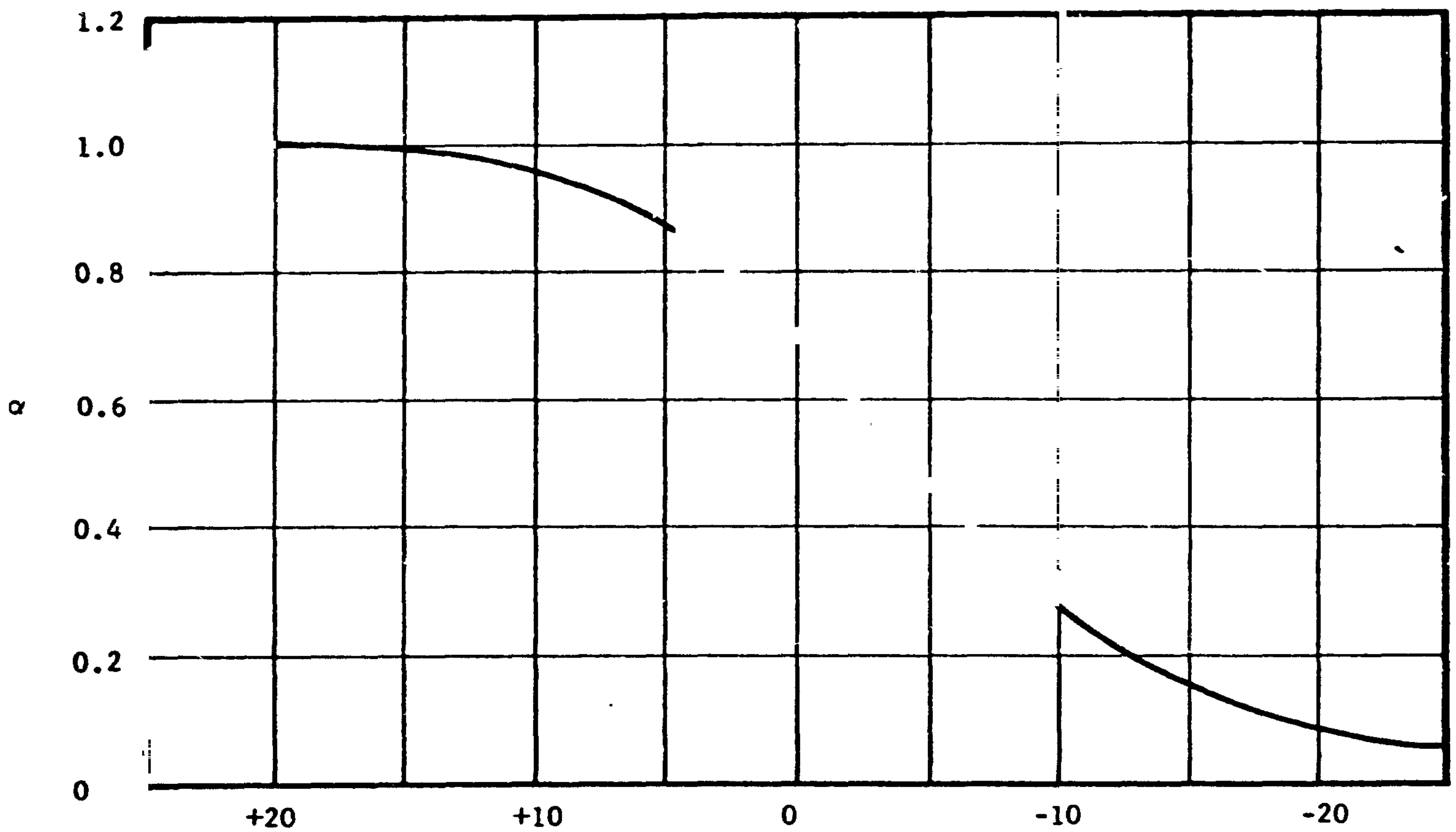
where

ω_n = natural resonant frequency, at the input SNR
ω_{nT} = natural resonant frequency at loop threshold
α = limiter suppression factor at the input SNR
α_T = limiter suppression factor at threshold

The suppression factor (α) can be determined from the curve of Fig. 10-1, once the input signal-to-noise ratio is determined.

There are two generally acceptable methods of performing the above bandwidth measurement. The first method is to use a large input signal level (SNR $\geq$20-db) so that the noise contribution to the loop error is negligible. The limiter suppression factor under this condition is 1.0. The second method also employs a large SNR, but, in addition, an attenuator is inserted between the limiter output and the loop phase detector input. The attenuator output level is set to simulate the signal level into the phase detector that would occur at threshold (in effect, $\alpha = \alpha_T$). Thus, in the second method, no correction factor is necessary in the calculations.

The first method described above can also be used to measure the Doppler tracking rate under strong signal conditions. In this case, the input rate-of-change of frequency ($\Delta\dot{\omega}$) is increased until the peak dynamic phase error (θ_a) is equal to 30°. This acceleration error (30°) is generally considered the maximum allowable for acceptable tracking of the phase-locked loop.



By permission of L. A. Hoffman

Fig. 10-1

α Versus Input S/N Ratio

10-5. Loop Bandwidth (using input sinusoidal phase modulation)

The loop bandwidth can be determined (again, indirectly) by making a plot of the normalized loop error function, measured at the output of the loop phase detector. The equation of the error function is: (Chapter 3, Eq. 3-9)

$$\theta_e(s) = \frac{\theta_i(s) - \theta_o(s)}{\theta_i(s)} = 1 - H(s)$$

or

$$\frac{\theta_e(s)}{\theta_i(s)} = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Using the value of $1/\sqrt{2}$ for ζ, replacing s with $j\omega$ and rearranging terms,

$$\frac{\theta_e(j\omega)}{\theta_i(j\omega)} = \frac{\left(\frac{-\omega^2}{\omega_n^2}\right)}{1 + \frac{\sqrt{2}\, j\omega}{\omega_n} - \frac{\omega^2}{\omega_n^2}}$$

When

$$\omega \gg \omega_n \gg \sqrt{2}$$

$$\frac{\theta_e}{\theta_i}(j\omega) = 1.0$$

For the special case of $\omega = \omega_n$, the normalized error reduces to $\frac{1}{2}$, and is shifted in phase by 90°, that is,

$$\frac{\theta_e(j\omega)}{\theta_i(j\omega)} = \frac{j}{\sqrt{2}}$$

Thus, if the normalized error function is plotted, on a db scale, the curve of Fig. 10-2 results, where the -3db point occurs when $\omega = \omega_n$ (Note: Fig. 10-2 is actually Fig. 3-4 repeated here for convenience.)

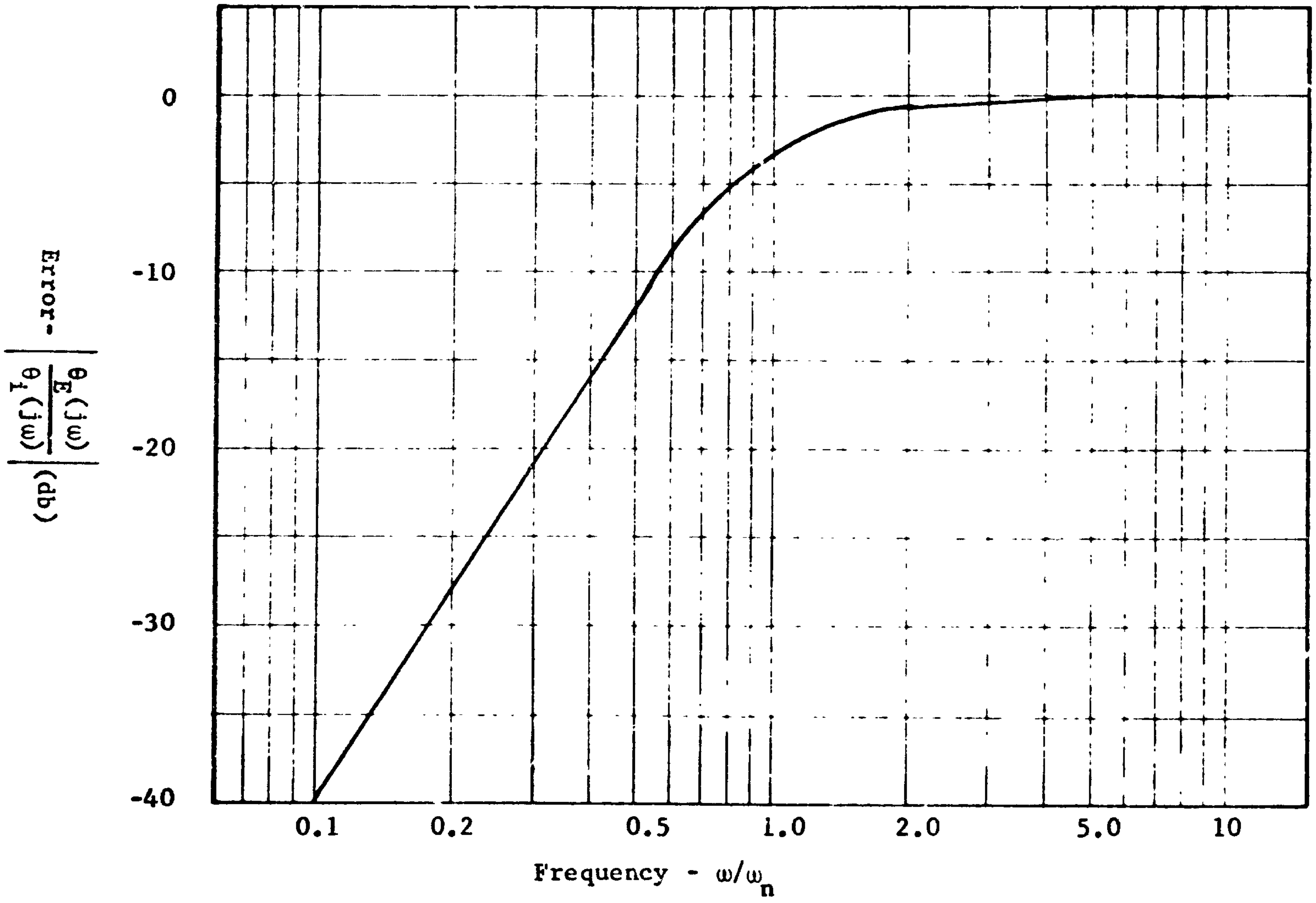


Figure 10-2

Error Response of High Gain Loop

$\zeta = \sqrt{2}/2$

If the measured data are taken under the conditions described in the second method above ($\alpha = \alpha_T$), the -3db point will occur at $\omega = \omega_{nT}$; B_L can then be calculated from the relationship,

$$B_{LT} = 0.53\ \omega_{nT}$$

The procedure to be followed is to sinusoidally phase modulate the input signal at a much higher rate (ω_m) than the loop can follow. Adjust the deviation so that it is approximately 60° peak-to-peak at the output of the loop phase detector (as observed on an oscilloscope). Adjust the gain of the scope for any convenient amplitude reference. While maintaining the phase deviation constant at the signal generator, plot the curve of the normalized peak-to-peak phase error as a function of modulation frequency (ω_m).

10-6. AGC Loop

If an AGC loop is employed in the system, it is necessary to determine its characteristics. The data recorded are AGC voltage versus input signal level and the frequency response of the AGC loop. The noise bandwidth is determined from the frequency response.

10-6.1 Noise Bandwidth

The usual procedure employed to determine noise bandwidth is to amplitude modulate the incoming signal and observe the AGC voltage on an oscilloscope. The data recorded is the modulation amplitude appearing on the AGC voltage versus modulation frequency. The square of the normalized AGC output is plotted versus frequency on linear graph paper. The area under the squared curve is measured, either by counting squares or a planimeter. The width of a rectangle of equal area and unity height, then, is the AGC noise bandwidth in cycles per second. A typical example is shown in Fig. 10-3.

10-7. Phase Shift with Signal Level Changes

Since any incremental phase-shift introduced by the system dilutes the desired data, the system should be calibrated for this error. Unfortunately, this measurement is not easy to perform and requires a special setup. The following procedure describes an exact method of performing this measurement. This procedure is valid for systems employing either AGC or limiting type I-F amplifiers.

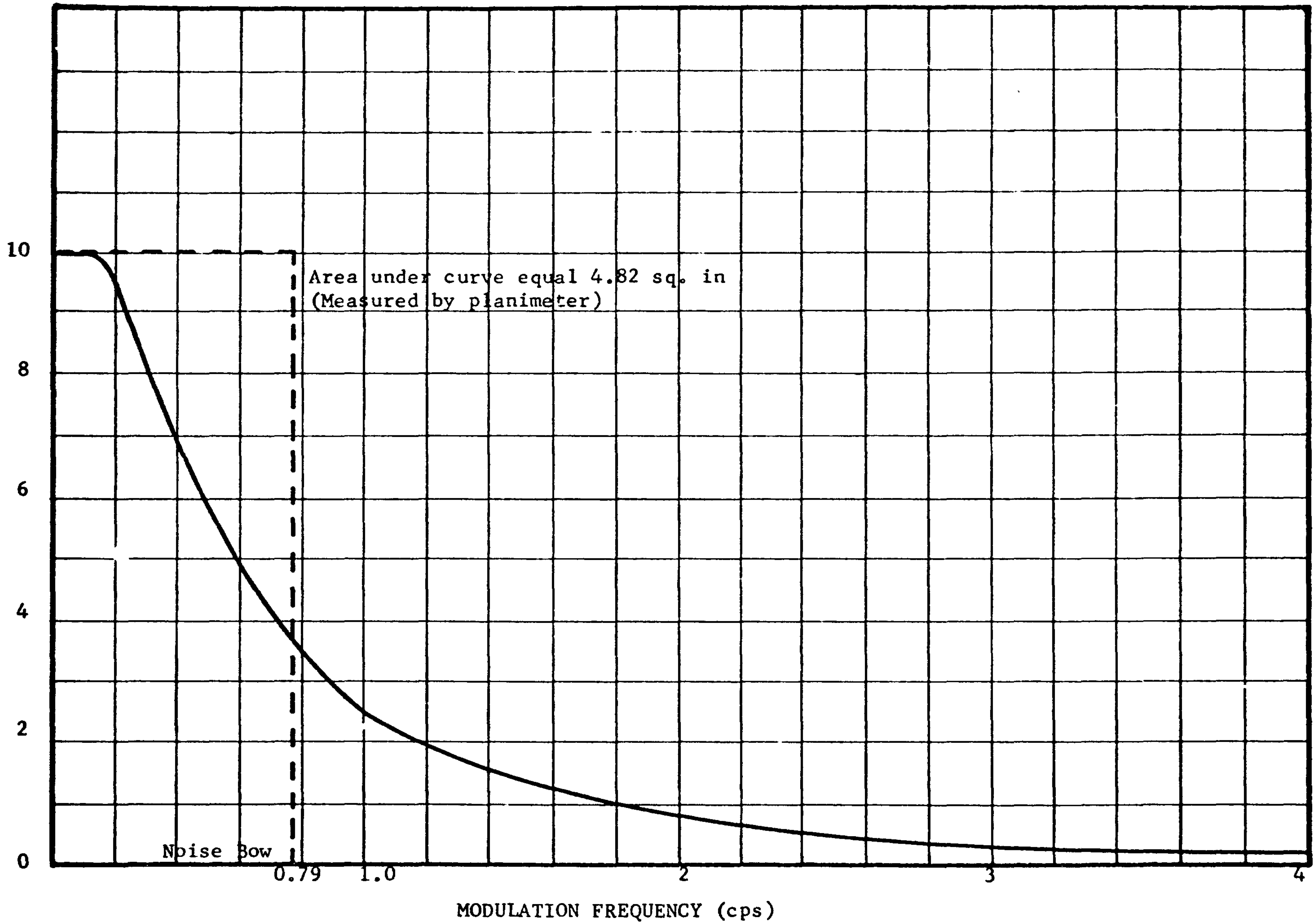


Figure 10-3

Typical Curve for Determining AGC Noise Bandwidth

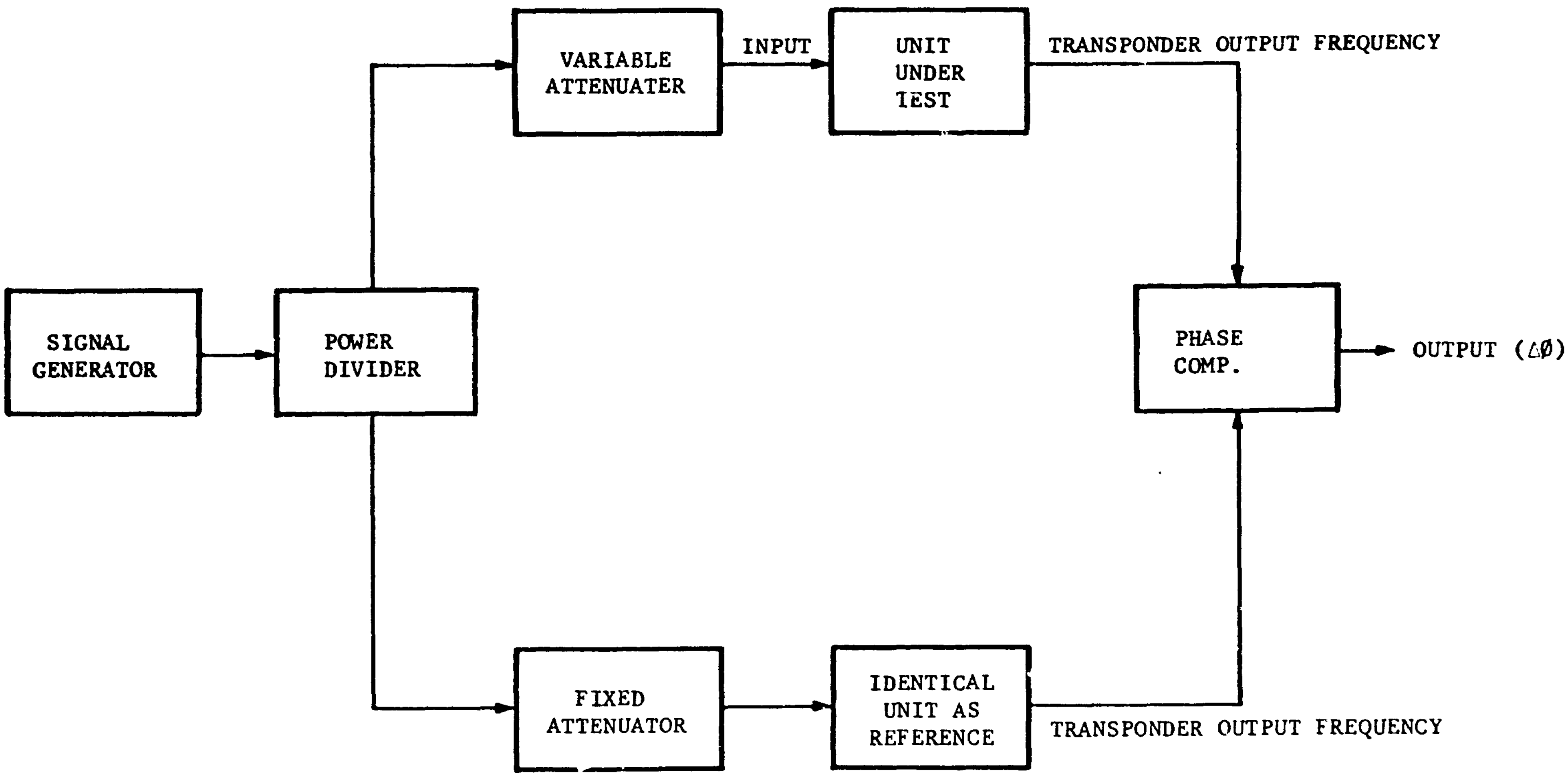


Fig. 10-5

Block Diagram of Test Set-up for Phase Shift Versus Signal Level Measurements

The test set-up required is shown in block diagram form in Fig. 10-5. The measurement involves the use of two transponders and a phase comparison device. Both transponders are phaselocked to the incoming signal at a strong signal level (the phase comparator output is calibrated for ±90° phase difference). The signal level into the unit under test is then varied over its entire operating range and the phase variations are plotted as a function of signal level.

~~A simpler method, requiring only one transponder can be used for coarse measurements. The test set-up is shown in Fig. 10-5. In this test, the slotted line is terminated in a short-circuit to provide large standing waves on the line. The probe of the line is then adjusted for a minimum voltage position on the line. The input signal level is varied over its operating range and the shift of the voltage minimum is plotted in degrees.~~

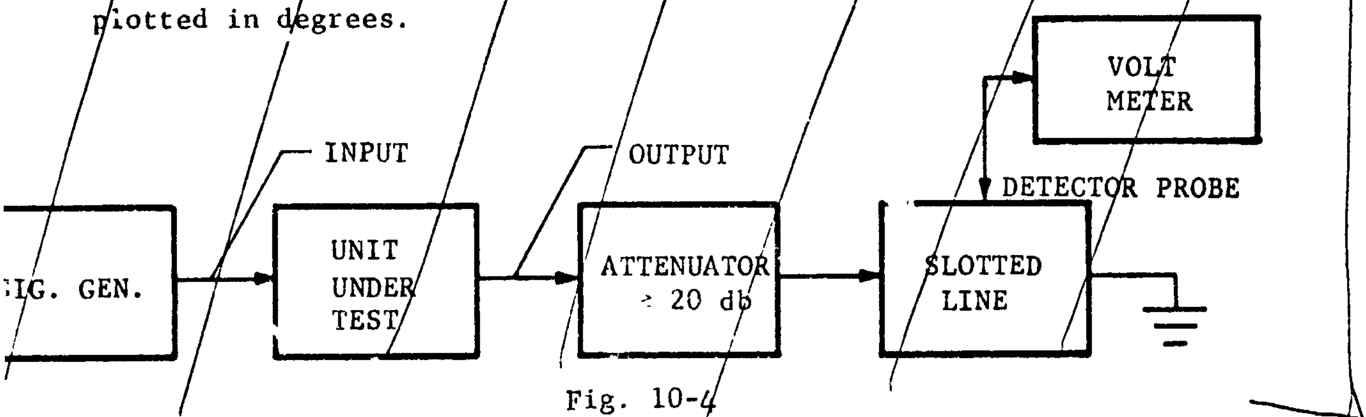


Fig. 10-4

Phase-Shift vs Signal Level; Slotted Line Technique

10-8. Phase Jitter Measurement

The phase jitter inherent in the phase-locked loop can be measured by observing the loop phase detector output under low noise conditions. The measurement can be done by the use of an oscilloscope or a colorimetric power meter.

In order to perform this measurement, it is necessary to simulate threshold signal level with low noise input to the phase detector. This involves inserting an attenuator between the limiter output and the phase detector input. A high level signal is then fed into the unit (to produce a large SNR) and the attenuator is adjusted to simulate threshold signal level into the phase detector ($\alpha = \alpha_T$).

The procedure to be following is as follows:

1. Set the attenuator to 0-db insertion loss and unlock the loop, keeping the phase detector beat frequency fairly low.

2. Observe the phase detector output on the oscilloscope and adjust the gain of the scope so that full scale deflection is equal to 180 degrees peak-to-peak (±90°).
3. Connect the power meter to the output and record the measured output power as W_1.
4. Lock the loop and adjust the attenuator to simulate threshold signal level. Measure the noise jitter power and record as W_2.

Calculate the rms noise jitter as follows:

$$\theta_n = \text{noise jitter (rms)} = 42.4 \sqrt{\frac{W_2}{W_1}} \text{ degrees} \qquad (10\text{-}1)$$

The above equation is derived in the following manner: the total VCO jitter, θ_N, was assumed to be $\leq 30°$ rms (by linearization assumptions). Therefore, to approximate the non-linear decrease from the 180° p-p to 60° p-p, W_1 must be divided by 4 (sin 30° = 1/2), or

$$\frac{W_1}{4} \cong 60^{\circ} \text{ p-p} \cong 21.2^{\circ} \text{ rms} \qquad (10\text{-}2)$$

Any decrease below this level is considered linear, that is, $\sin \theta = \theta$. Since we are measuring power and want the jitter in degrees rms, the square root of the ratio of the two powers (W_2 and $W_1/4$) is desired.

10-9. Additional Measurements

Many other measurements should be conducted on any phaselocked system to insure compliance with specifications and proper performance. Most of these measurements are straight forward and need not be described herein; however, the more important ones are listed below:

1. System input noise figure.
2. I-F noise bandwidth.
3. Limiter output power level.
4. Phase detector gain and balance.
5. AGC detector input level.
6. VCO gain, linearity, and pulling range.
7. Loop filter time constant (τ_1 and τ_2).
8. R-F and I-F total gain.
9. Input VSWR.
10. Spurious signal response.

11. False-lock.

12. Acquisition time.

13. Acquisition threshold.

14. Drop-out threshold.

15. D. C. Amplifier gain (if used).

10-10. Maintenance

The amount and type of maintenance necessary on a system employing phaselock techniques is naturally dependent on the system itself. In this day and age of all solid-state components, maintenance has changed considerably--no longer are we much concerned with replacement of components (preventive-maintenance) every 1000 hours or so to insure proper operation. Maintenance, today, is more a matter of periodic checks to determine that the alignment has not drifted, that accidental damage has not occurred, and that environmental conditions have not deteriorated the systems performance.

The tests and measurements discussed in the previous sections will determine the extent of maintenance necessary for proper operation. Probably the most informative test (to check alignment) is threshold sensitivity without modulation. If the unit passes this test successfuly it is reasonable to assume it will perform to specifications (of course, this assumes the unit was completely acceptable when first received). This test should be performed at three discrete frequencies; i.e., zero Doppler, maximum Doppler, and minimum Doppler. Also, observation of the AGC voltage is a reliable indication of the total R-F and I-F gains.

If the unit fails to perform properly on the above tests, then the more detailed checks listed in paragraph 10-9 should be performed to locate the cause of the failure. After successful repair, the system should be completely recalibrated for future reference.

NOMENCLATURE

A	Gain of DC amplifier
B_i	Bandwidth of filter preceding phase-lock loop, cps
B_L	Loop noise bandwidth, cps
B_{LT}	Threshold value of B_L, cps
D	Modulation index of angle-modulated signal
f	Frequency, cps
$F(s)$	Transfer function of loop filter
$H(s)$	Phase transfer function of loop, θ_o/θ_i
J	An oscillator noise parameter
K_d	Phase detector gain factor, volts/radian
K_o	VCO gain, radians/(sec)(volt)
K_v	DC loop gain, $K_o K_d$ F(o), dimensions of (seconds)$^{-1}$
L	Limiter output voltage
N_o	Noise spectral density, (volts)^2/cps
$n(t)$	Noise voltage
P_s	Signal power, watts
P_n	Noise power, watts
s	LaPlace complex variable
SNR	Power signal-to-noise ratio
$(SNR)_i$	SNR at input to loop
$(SNR)_L$	SNR in loop
T_{av}	Average time between skipping cycles, sec.
T_p	Pull-in time, sec.
v_d, V_d	Phase detector output voltage
W_o	Noise spectral density watts/cps
α	Limiter signal suppression factor
α_T	Threshold value of α
γ	Oscillator noise spectral exponent
ζ	Damping factor
θ_a	Steady state phase error due to frequency ramp input, rad.
$\theta_e = (\theta_i - \theta_o)$	Phase error, rad.
θ_i	Input phase, rad.

$\overline{\theta_{ni}^2}$	Equivalent mean square input phase variance, $(rad)^2$
θ_o	Output (VCO) phase, rad.
$\overline{\theta_{no}^2}$	Mean square VCO phase error due to input noise, $(rad)^2$
θ_v	Steady-state phase error due to input frequency offset, rad.
τ_1, τ_2	Time constants of loop filter, sec
ω	Radian frequency, rad/sec
ω_m	Modulation frequency, rad/sec
ω_n	Loop natural frequency, radians/sec
$\Delta\omega$	Frequency offset or deviation, rad/sec
$\Delta\omega_H$	Hold-in frequency, rad/sec
$\Delta\omega_L$	Lock-in frequency, rad/sec
$\Delta\omega_p$	Pull-in frequency, rad/sec
$\Delta\omega_{po}$	Pull-out frequency, rad/sec
$\Delta\dot{\omega}$	Rate of change of input frequency, rad/sec^2

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Applications
Acquisition
Basic Operation
Circuits
Discriminators
Oscillators
Optimization
Phase Detectors
Receivers
Tracking
Threshold

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