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FINAL STUDY REPORT
(July through December 1965)

A STUDY OF
A GRAVITY-GRADIENT-STABILIZED
NIMBUS SPACECRAFT

CONTRACT NO.: NAS 5-9611

Prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

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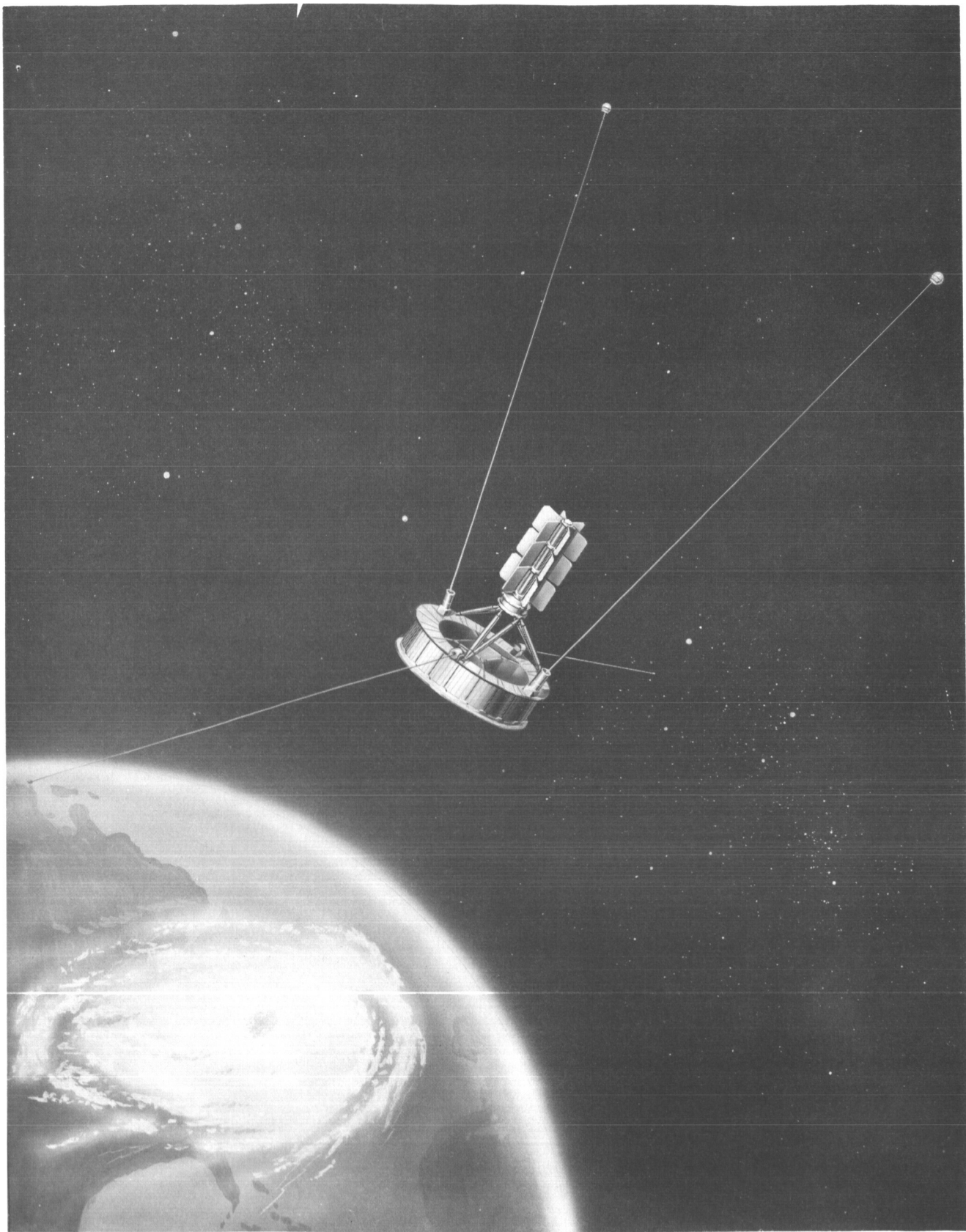
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Greenbelt, Maryland

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ABSTRACT

The General Electric Company has conducted a six-month, fixed-price study of a gravity-gradient stabilized NIMBUS spacecraft for NASA-GSFC under Contract NAS 5-9611. The objective of this study was to provide a preliminary spacecraft design capable of meeting specified performance requirements for a nominal 750-nautical-mile, "high-noon," sun-synchronous orbit. Additionally, performance of the spacecraft was to be determined under conditions of uncompensated internal momentum, orbital-plane drift from nominal conditions, and a range in orbital eccentricities. The spacecraft features the NIMBUS sensory ring, with its present thermal control subsystem, three SNAP-19 radioisotope thermoelectric generators (RTGs), and a gravity-gradient torqued, passively-damped, three-axis control subsystem. The scope of the study, reported in this documentation, has been limited to the selection and preliminary design of the spacecraft configuration and the selection and performance analysis of the passive attitude control subsystem.

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SECTION 1

INTRODUCTION

The application of passive attitude control and radioisotope thermal electric generator technologies to the NIMBUS program offers the potential of significant advantages in spacecraft operational flexibility, increased life, and reduced operational cost. The successful orbital flight experience with passive attitude control subsystems has shown them to be operationally practical. These subsystems require no electrical power after deployment, and because of their extreme simplicity, they are inherently long-lived. Radioisotope power generation subsystems have been under development for many years and are now reaching operational usage. These devices are independent of external environment, require no attitude control and have long-life capabilities. The Spacecraft Department of the General Electric Company has conducted a six-month design study of a meteorological spacecraft for NASA-GSFC under contract NAS 5-9611. The spacecraft features the NIMBUS sensory ring, with its present thermal control subsystem, three SNAP-19 radioisotope thermoelectric generators (RTGs), and a gravity-gradient torqued, passively-damped, three-axis control subsystem. The scope of the study, reported in this documentation, has been limited to the selection and preliminary design of the spacecraft configuration and the selection and performance analysis of the passive attitude control subsystems.

The selected spacecraft configuration, referred to in this report as NIMPAC, for NIMBUS Passive Attitude Control, is completely compliant with the study specification established by NASA-GSFC. The salient characteristics of this design are:

- a. A spacecraft configuration which is compatible with the NIMBUS fairing and adapter and which does not require modification to the NIMBUS sensory ring or thermal control subsystem.
- b. An RTG support structure which is similar to the NIMBUS active control truss structure and which affords a simple interface with the spacecraft for handling and testing and accommodates nuclear safety ejection requirements.

- c. Attitude performance of 1.6 degrees P, 0.3 degrees R, and 1.6 degrees Y under design-reference-orbit conditions and capability of operating over a wide range of off-design-orbit conditions with minimal deviation in design performance.
- d. Capture from specified initial separation conditions to required steady-state operating performance in 186 hours.
- e. A passive attitude control subsystem configured so as not to obstruct the field of view of potential payloads.

The following sections of this report are divided into three major areas. Section 2 contains a summary of the study requirements as specified by NASA-GSFC. Section 3 presents the description and performance analysis of the selected spacecraft configuration. The information presented in Section 3 meets all of the study requirements specified in Section 2. Section 4 contains the rationale for the selected design. Following these major sections is a descriptive summary of the computer programs used in the study and a bibliography.

SECTION 2

STUDY REQUIREMENTS

2.1 GENERAL

The major objectives of this six-month study are (1) the analytical performance evaluation of a three-axis, gravity-gradient-stabilized configuration of a NIMBUS sensory ring and radioisotope power supplies and (2) the selection of a recommended configuration within the design requirements and constraints defined by NASA-GSFC. This section summarizes the study requirements and constraints as presented in the NASA-GSFC Technical Specification and as modified by the Contract Schedule.

2.2 CONFIGURATION DESIGN REQUIREMENTS

2.2.1 CONFIGURATION

The spacecraft configuration consists of the NIMBUS sensory ring with its active thermal control subsystem, a three-axis passive attitude control subsystem, three SNAP-19 radioisotope thermoelectric generator (RTG) power supplies, and appropriate supporting and connecting structure. The configuration must be dimensionally within the requirements of the existing NIMBUS fairing. No part of the system is to extend into a 70-degree half-angle cone whose surface contains the outer lower circumference of the sensory ring and whose apex points away from the earth.

The total weight of the spacecraft, excluding the launch vehicle adapter, is not to exceed 600 pounds. The weight of the NIMBUS sensory ring, including all contained subsystems is assumed to be 400 pounds, uniformly distributed angularly and radially. The RTG units are identified as SNAP-19 generators, manufactured by The Martin Company, with an assumed weight of 30 pounds per unit, exclusive of the power converter (included in the sensory subsystem weight). The passive attitude control subsystem is to weigh approximately 40 pounds.

2.2.2 THERMAL CONTROL

The sensory ring components are assumed to dissipate 100 watts uniformly distributed, and each of the SNAP-19 RTG units are assumed to dissipate 650 watts. Thermal shielding techniques are to be analyzed, and recommendations are to be made for minimizing the transmission of heat from the RTGs to the sensory ring. The temperature of each bay of the sensory ring is to be determined for the assumed dissipation, based on the existing NIMBUS sensory ring thermal-control subsystem, for the nominal high-noon sun-synchronous orbit. Heating of attitude control components by the dissipation of RTGs and sensor ring is to be determined.

2.3 PERFORMANCE REQUIREMENTS

2.3.1 DESIGN-REFERENCE-ORBIT PERFORMANCE

The operational design point is a high-noon sun-synchronous orbit with a mean altitude of 750 ± 25 nautical miles and a maximum apogee-minus-perigee of 75 nautical miles. The spacecraft is assumed to have a residual magnetic dipole of 100 pole-centimeters, with worst-case phasing, and no internally generated disturbance torques. The passive attitude control subsystem must be capable of maintaining pitch and roll attitude within ± 5 degrees and yaw attitude within ± 10 degrees of the corresponding orbit axis. Body rates about all three axes must be no greater than 0.05 degrees per second.

2.3.2 OFF-DESIGN-ORBIT PERFORMANCE

An analysis of off-design-orbit performance is required for apogee-perigee differences as great as 150 nautical miles and for orbit plane offsets from the earth-sun line of 30 degrees, 60 degrees, and 90 degrees. In addition, performance is required under conditions of an internally generated uncompensated momentum of 25×10^{-5} foot-pound-seconds along the pitch axis, throughout the orbit, and an additional 25×10^{-4} foot-pound-seconds uncompensated momentum for 4 minutes per orbit, with worst-case phasing.

2.3.3 CAPTURE PERFORMANCE

Initial stabilization of the spacecraft is required with respect to any of the four stable operating points, from maximum separation conditions of ± 5 degrees position error, and with initial separation rates of 1 degree per second, about each axis relative to the respective orbital axis.

2.4 REQUIREMENTS SUMMARY

The following summarizes the NIMPAC System Requirements.

Functional Spacecraft

Basic Structure	NIMBUS sensory ring with active thermal control subsystem.
Power	3 SNAP-19 RTGs, 100 watts
Attitude Control	3-axis gravity-gradient torqued, passively damped
Attitude Reference	Not within study scope
Inverting Technique	Not within study scope
Residual Magnetic Dipole	100 pole-cms, worst-case phasing
Payload	Uniformly distributed mass and heat dissipation (100 watts)
Antennas	Not within study scope
Spacecraft weight	600 pounds
<u>Operational: Design-Reference-Orbit</u>	
Orbit Altitude	750 $\pm$ 25 nautical miles
Orbit Inclination	78.7 degrees

Orbit Type	High-noon sun-synchronous
Orbit Eccentricity	0.0089
Attitude Stabilization	± 5 degrees, pitch and roll and ± 10 degrees yaw
Attitude Rates	0.05 degree/second
<u>Operational: Initial Stabilization</u>	
Initial Position Error	± 5 degrees each axis
Initial Rate	1 degree/second each axis
Stabilization Attitude	Not specified
Time to Steady State	Not specified

SECTION 3

SPACECRAFT PRELIMINARY DESIGN AND PERFORMANCE

3.1 GENERAL

The application of passive attitude control and radioisotope thermoelectric generator technologies to the NIMBUS program has been the subject of a six-month preliminary design and performance study conducted by General Electric under Contract to NASA-GSFC. The study requirements as specified by NASA-GSFC have been summarized in Section 2 of this report. This section documents the preliminary design of the selected spacecraft configuration and its performance under reference design and off-design orbital conditions. Rationale for the configuration selection and detailed discussions of passive attitude control analysis are presented in Section 4.

The selected spacecraft configuration, illustrated in Figure 3-1, consists of three SNAP-19 RTG power supplies mounted in series on a truss structure which is mated to the NIMBUS sensory ring. The passive attitude control configuration consists of two magnetically-anchored eddy-current dampers and four extendable gravity-gradient rods mounted on the upper surface of the sensory ring. The upper gravity-gradient rods are mounted in the plane of the orbit at an angle of $18-1/2$ degrees to the yaw axis and support the two dampers. These rods are 60 feet long, when erected. The lower gravity-gradient rods are 80 feet long, when erected, and are in a plane normal to the orbit plane, at an angle of $19-1/2$ degrees below the horizontal. The lower rods do not have tip weights. A motor-driven erection unit is used for each of the gravity-gradient rods.

The RTG supporting truss structure is similar to the NIMBUS active control subsystem support truss and uses the same sensory ring mating technique. The height of the truss structure has been selected to minimize the thermal effects of the RTGs on the sensory ring without sacrificing the structural integrity of the design.

Figure 3-1. Passively Stabilized NIMBUS - NIMPAC

A weight summary of the selected configuration is presented in Table 3-1. The weight of each subsystem and the weight of the total spacecraft are within the weight specifications of the study requirements. Table 3-1 also includes the moments of inertia and cg location of the selected configuration prior to the erection of the gravity-gradient rods.

Structural and dynamic analyses have been conducted to determine the structural suitability of the design and to size the RTG support truss and passive-attitude-control components support structure for accurate weight estimates. It can be concluded from these analyses that the selected design is structurally adequate and that the standard NIMBUS sensory ring is structurally compatible with the anticipated loadings. In addition, the RTG support truss acts as a vibration absorber and limits RTG dynamic loading to values less than those already experienced by the SNAP-19 RTGs during component testing for the NIMBUS B application. Because of this, it can be concluded that the SNAP-19 RTGs will survive the anticipated structural environment for the selected configuration.

Thermal analyses, conducted during the study, indicate that the NIMBUS sensory ring thermal control control subsystem, operating in a high-noon sun-synchronous orbit, will be capable of maintaining payload temperatures within desired limits (NIMBUS A temperature limits have been selected as a reference), while rejecting the specified payload dissipative heating and the thermal conduction from the RTGs. In addition, thermal design techniques have been specified for the selected configuration. These techniques will minimize the thermal effect of the RTGs on the sensory ring. Analyses of the heating of attitude control components by the dissipation of the RTGs and the sensory ring indicate that these will produce negligible effects in comparison to solar heating.

The performance of the passive attitude control subsystem has been determined for the reference-design orbital conditions and the off-design orbital conditions. A summary of this performance is presented in Table 3-2. Where applicable, performance requirement values from Section 2 are included for comparison.

Table 3-1. Weight Summary and Properties of Selected Configuration

WEIGHT SUMMARY

NIMBUS Sensory Ring		400.0 lb.
(Including active thermal control subsystem and payload subsystems)		
SNAP-19 RTGs (3)		90.0 lb.
Attitude Control Subsystem		40.0 lb.
(2) Magnetically - anchored eddy-current dampers	24.0	
(4) Gravity-gradient rods and erection units	<u>16.0</u>	
	40.0 lb.	
Support Structure		56.5 lb.
(1) RTG Support Truss	9.9	
(2) Damper and upper rod mounting structure	13.9	
(2) Lower rod mounting structure	5.2	
(1) SNAP - 19 vibration isolator and ejector	<u>27.5</u>	
	56.5 lb.	
TOTAL WEIGHT		<u>586.5 lb.</u>

CENTERS OF GRAVITY

X = 0 (Along roll axis)
 Z = 204.30 (Along yaw axis. Separation plane=220.5)
 Y = 0 (Along pitch axis)

MOMENTS OF INERTIA (Prior to rod extention)

I_{ox} (roll) = 64.66 slug - ft²

I_{oz} (yaw) = 57.37 slug - ft²

I_{oy} (pitch) = 69.31 slug - ft²

PRODUCTS OF INERTIA ARE ZERO

Table 3-2. Summary of Passive Attitude Control Performance

	<u>Pitch</u>	<u>Roll</u>	<u>Yaw</u>
<u>Steady-State Performance</u> (degrees)			
Specification Requirement	± 5	± 5	± 10
Reference-Orbit Conditions	± 1.6	± 0.3	± 1.6
<u>Steady-State Body Rates</u> (degree/sec)			
Specification Requirement	0.05	0.05	0.05
Reference-Orbit Conditions	0.004	0.0016	0.0010
<u>Off-Reference Conditions</u> (degrees)			
Eccentricity $\epsilon = 0$	± 0.9	± 0.3	± 1.6
$\epsilon = 0.0089$	± 1.6	± 0.3	± 1.6
$\epsilon = 0.0179$	± 2.5	± 0.3	± 1.6
$\epsilon = 0.0385$	± 4.5	± 0.3	± 1.6
Orbit Plane Drift 30 degrees	± 1.6	± 0.3	± 1.6
60 degrees	± 1.6	± 0.6	± 1.6
90 degrees	± 1.6	± 0.3	± 1.6
Internal Momentum	Negligible Effect		
<u>Capture Performance</u>	Last pitch tumble within 6 hours after separation		
<u>Damping Time to Steady State</u>	All errors within specified values 186 hours after separation.		

3.2 SPACECRAFT PRELIMINARY DESIGN

3.2.1 GENERAL

The basic spacecraft arrangement, illustrated in Figure 3-1, is a natural evolution of the NIMBUS spacecraft, utilizing the NIMBUS sensory ring for payload accommodation. Three SNAP-19 RTGs are mounted in series on top of a space structure, 18 inches high, which is mated to the NIMBUS sensory ring. Consolidation of the RTGs in this way (with no other structure in the area) allows them to be easily and reliably ejected and presents a clean interface for satellite handling and testing. Passive stabilization is accomplished with four extendable gravity-gradient rod units mounted to the upper surface of the sensory ring, leaving all eighteen modular compartments available for equipment. The gravity-gradient rod units are detuned from the sensory ring, to avoid dynamic coupling, by means of stiff mounting bases. In addition, they are oriented to meet the requirement that no part projects into a 70 degree half-angle cone whose surface contains the outer lower circumference of the sensory ring and whose apex points away from the earth.

3.2.2 CONFIGURATION DESCRIPTION

The 18-bay NIMBUS sensory ring is the foundation of NIMPAC, with six 2-inch diameter tubes forming a shore dimensional truss that supports the RTGs 18 inches above the sensory ring. The tubes are standard stock aluminum parts (2-inch O.D. with a 0.035 wall thickness) similar to those used on NIMBUS. They mate with machined end fittings, into which standard mono-ball-bearing eye bolts are threaded, one with a right-hand thread and the other with a left-hand thread. In effect, then, each truss tube is a turnbuckle, thereby permitting the RTGs to be accurately positioned and aligned to maintain the desired vehicle weight and balance properties. Slight misalignment of the tubes from their theoretical position is accommodated by the mono-ball end bearings. Fittings on the sensory ring and RTG base, to which the truss tubes are connected, are designed so that lines of force of tubes, sensory ring castings, and RTG base centroids are coincident to minimize eccentricity effects (see Figure 3-2). The rationale for selecting an 18-inch truss height is presented in Section 4.3.

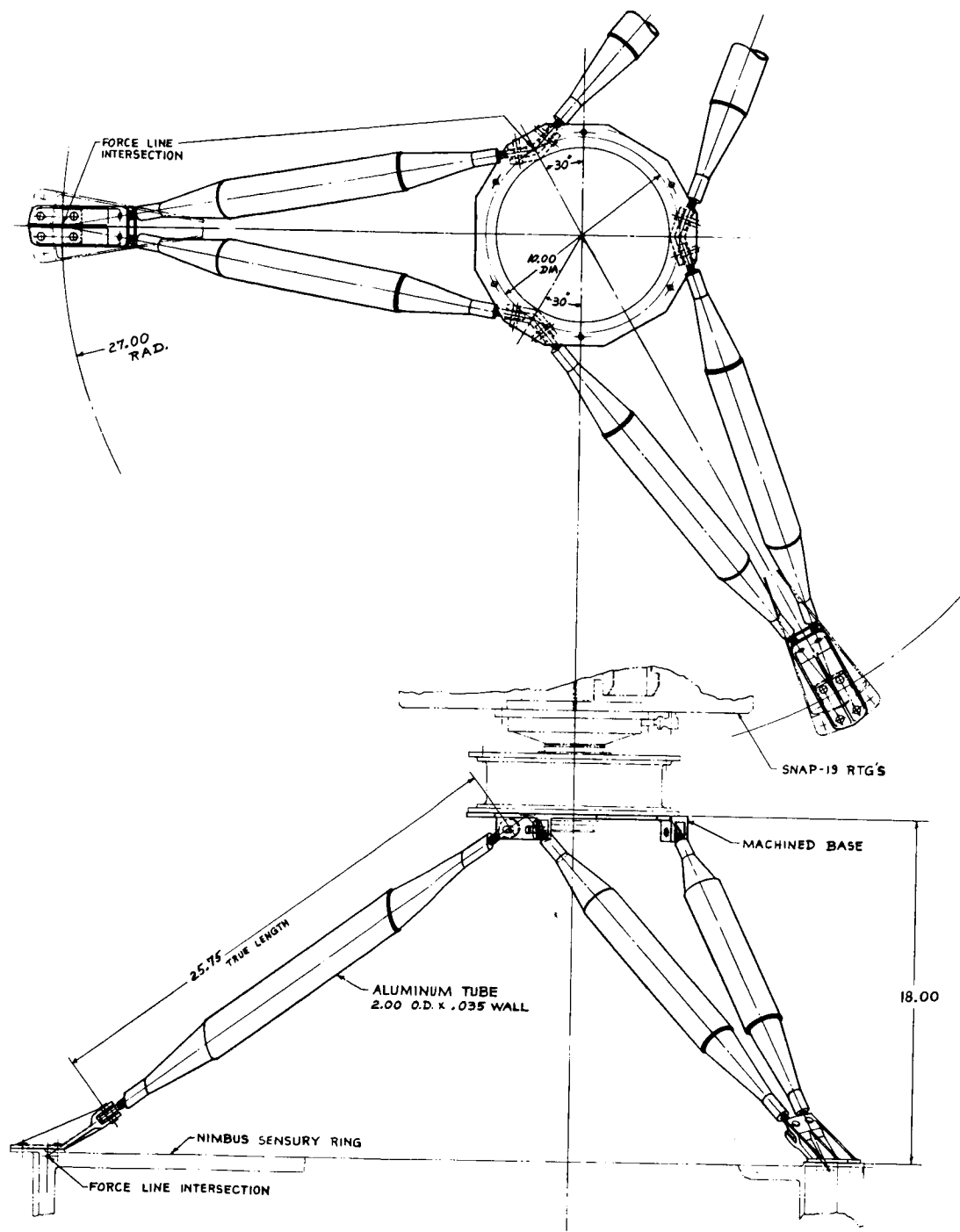


Figure 3-2. SNAP-19 RTG Support Truss Structure

The design concept for the RTG base (Figure 3-2) is a simple flat aluminum plate with three machined truss tube lugs, a center cutout large enough to clear the ejection mechanism, and straight outside edge cuts to simplify machining. Dynamic design aspects are considered in subsection 3.2.3. The truss tube lugs are also formed by straight cuts and can be an integral part of the base or separate detail parts that are attached to the base with fasteners. Bolting patterns for mounting the RTGs can be drilled into the base by use of coordinated tooling to insure an accurate interface fit.

The NIMBUS sensory ring basically consists of 18 modular compartments (6 inches tangentially, 8 inches radially, and 13 inches deep) formed by 18 separator castings. They are tied together on their outboard surfaces by an 18-sided upper ring and a similar lower ring. The inboard surfaces are spaced by individual channel sections. Mounted to the sides of separator castings are active thermal control units which sense the temperature of the modular equipment and actuate externally mounted venetian-blind-type insulation shutters, to maintain a uniform structure temperature. The gravity-gradient rod erection units are each supported by four tabs that are located very near the center of gravity of the units. Each unit is encased by a two-section container. In the case of the damper rod units (Figure 3-3), the can is structural and serves as the support for the damper, with a Marmon clamp separation interface. For the other two units (Figure 3-4), the can is of thin material and serves only as a dust and handling shield. Each unit is mounted on an all-welded channel section base that is designed to be stiff enough to detune the unit from the sensory ring. These bases are similar to those used on NIMBUS B for sensors such as IRIS, HRIR, and MRIR. Bases have mounting bolt patterns, drilled by use of coordinated tooling, to match the erection unit lugs. Therefore, installation can proceed in steps: the base is mounted to the sensory ring first, and then the units are mounted to the respective base. Textolite washers are used under the lugs to align the Marmon ring interface with the damper, in addition to serving as a thermal isolator to reduce heat leakage to and from the erection unit. Similarly, textolite washers and bushings are used to mount the bases to the sensory ring to reduce heat leakage and to provide alignment capability of the entire erection unit to the spacecraft.

Figure 3-4. Gravity-Gradient Rod Mounting Structure

Bases are fabricated by welding 2-inch aluminum channels into a frame large enough to span over two separator castings and to accept the erection units. Bases are spaced about 1/2 inch away from the sensory ring surface to clear equipment mounting tabs and harnesses. This also brings the surface of the base to the level of the undersurface of the sensory ring insulation, thus permitting a level insulation. Cutouts are made in the insulation, around the damper unit, the edges of which are held down with hook and pile nylon strips. Hook strips are sewn to the underside of the insulation around the cutout, and pile strips are bonded to the damper unit base. Insulation elsewhere on the sensory ring is supported by light metal brackets very similar to those used on the NIMBUS program.

The mounting base for the erection unit over bay 5 required a slight modification on one mounting leg in order to clear the truss tube fitting as shown in Figure 3-4 (view D). Essentially, the channel is cut short at the fitting, and an extension block is welded to the channel. This block is small enough to pass under the fitting and is structurally adequate. The mounting bolt at this leg is installed from the inside of the separator casting, because access from the upper surface is blocked by the truss tube fitting.

All structural parts will have appropriate corrosion protective finishes, such as anodizing for aluminum, Dow treatments for magnesium, and fasteners installed wet with zinc chromate. Electrical grounding will be provided for by using the electrically conductive types of the above finishes.

3.2.3 STRUCTURAL AND DYNAMIC CONSIDERATIONS

3.2.3.1 General

The selected configuration was analyzed for structural adequacy; the truss tubes and sensory ring were found to be adequate to withstand the imposed loads. The inertial g loads were obtained for the two major masses by first performing a dynamic analysis of the system response to a vibration input to the base of the adapter. The NIMBUS A adapter was used in the analysis. The greatest inertial load for any member was applied to that member as though it were a static load, and a margin of safety was computed.

3.2.3.2 Dynamic Analysis

In the dynamic analysis, the spacecraft was idealized by a two-mass (representing the RTGs and sensory ring) mathematical model, in which the masses are connected to each other and to the input base by springs (truss and spacecraft adapter). A two-degree of freedom model was taken for thrust direction loads, and RTG rocking freedom was added for lateral loads. See Figure 3-5.

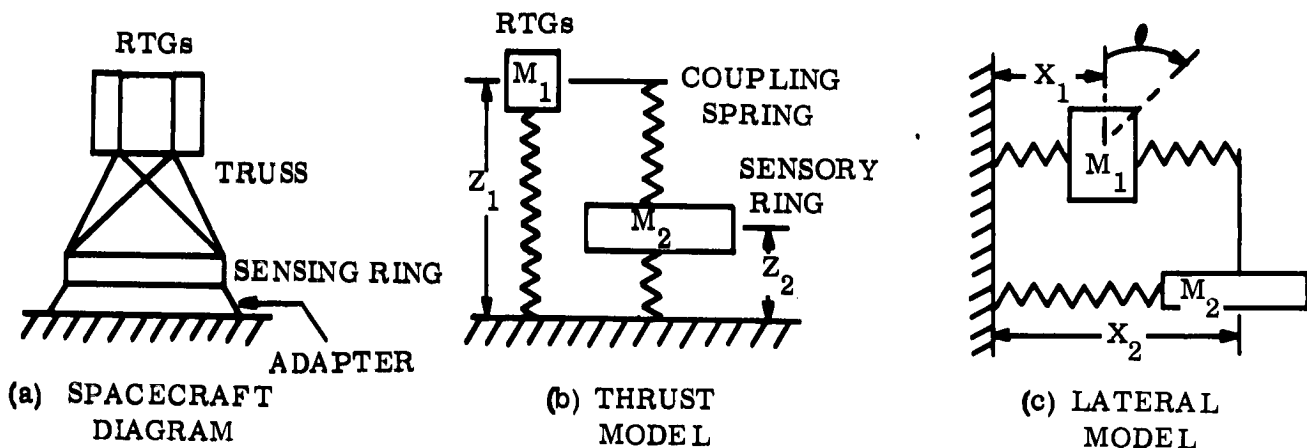


Figure 3-5. NIMPAC Dynamic Models

Flexibilities were obtained from spacecraft deflections, which were computed using a program developed for NIMBUS A, and include deflections of the sensory ring, as well as of the truss tubes. Mode shapes, natural frequencies, and forced responses were obtained by subsequent matrix operation routines. The damping factors used were typical values required to produce agreement on the NIMBUS program between analysis and test results.

This approach was necessary, since it is impractical to evaluate the distribution and magnitude of energy absorption within the structural system. Table 3-3 summarizes the inertial load response at spacecraft modal resonant frequencies for NIMBUS A prototype level sinusoidal input to the adapter. These are the loads used for the structural analysis described in Section 3.2.3.3. Good agreement exists between the mode frequencies of NIMPAC and NIMBUS A, as shown in Table 3-4, in spite of the different truss height and masses. As the truss height diminishes, flexibility due to the sensory ring increases and the outward thrust of the truss tubes is greater. This compensates for the increase in axial stiffness of the shorter tubes. The same type of compensating effect exists also in the lateral direction.

Determination of what type of RTG vibration isolation mount would be required, was considered. The above analyses imply that an isolator is not needed. This conclusion is reached by comparing the response at the RTGs of this analysis, which does not consider an isolation mount, with those responses obtained from NIMBUS B RTG testing at a 5- g hard mount component input specification. The NIMBUS B units are mounted on an isolation spring mount to tune the component to an 80-cps thrust resonance, and a 20-to 30-cps lateral resonance. Table 3-5 shows this comparison, from which it is apparent that the NIMPAC RTGs experience a lesser inertial loading than the NIMBUS B units, and, since the latter survived, the former will survive also. Another viewpoint is that the truss tubes are in themselves a "spring mount" which is tuned such that the RTG response is acceptable. The preceding conclusion cannot be considered final, since the NIMBUS B RTGs are lighter and shorter than the NIMPAC units, and the RTG ejection technique selected for this application might require a flexible, single-point mounting. The design of the RTG ejection and isolation mechanism is beyond the scope of this study. For these reasons, the NIMBUS-B SNAP-19 isolator is used in the illustrations in this document.

Mounting bases for the gravity-gradient rod units were designed to be stiff enough so that their natural frequencies are removed from the sensory ring modes. This avoids dynamic coupling which would cause high inertial loads. Therefore, the mounting bases are relatively large and stiff and resulting stress levels are low. The design criterion of detuning was successfully used in NIMBUS A and C mounting base designs for the HRIR and MRIR sensors.

Table 3-3. Frequency and Dynamic Response of Selected Configuration
(NIMBUS A Prototype Sinusoidal Vibration Specification)

<u>Thrust Axis Input:</u> ⁽¹⁾ 1.5 g over 22-150 cps; and 3.0 g over 150-400 cps.			
	(2)		
	1st Mode		2nd Mode
	<u>59 cps</u>		<u>208 cps</u>
	(3)		
RTGs	1800 lb (15.6 g)		-65 lb (-0.56 g)
Sensory Ring and Payload	67.5 lb (0.15 g)		6675 lb (14.8 g)
 <u>Pitch Axis Input:</u> 0.75 g over 5-150 cps			
	1st Mode	2nd Mode	3rd Mode
	<u>22 cps</u>	<u>50.4 cps</u>	<u>97.9 cps</u>
RTGs	755 lb (6.65g)	93 lb (0.80g)	-87.7 lb (0.77g)
RTG Rock Moment (cg)	4090 in-lb	2650 in-lb	1200 in-lb
Sensory Ring and Payload	169 lb (0.375 g)	1540 lb (3.43 g)	121 lb (0.27 g)
 <u>Roll Axis Input:</u> 0.75 g over 5-150 cps			
	1st Mode	2nd Mode	3rd Mode
	<u>20.8 cps</u>	<u>50.4 cps</u>	<u>104.7 cps</u>
RTGs	739 lb (6.44g)	100 lb (0.87g)	-75.7 lb (0.66g)
RTG Rock Moment (cg)	4060 in-lb	-2545 in-lb	1020 in-lb
Sensory Ring and Payload	142 lb (0.315g)	1565 lb (3.5g)	101 lb (0.225g)

Notes:

- (1) In each case the dynamic loading is an input to the sensory ring adapter.
- (2) Modes are defined as load peaks which occur at the spacecraft resonance frequencies.
- (3) Numbers in parentheses are equivalent g's obtained by dividing the inertial loads by the mass of either the RTGs (≈ 115 lb) or the sensory ring (≈ 450 lb).

Table 3-4. Comparison of NIMPAC and NIMBUS A Configurations

	<u>NIMPAC</u>	<u>NIMBUS A</u>
Top Mass	115 lb (RTG and Mount)	176 lb (NIMCO)
Sensory Ring Mass	450 lb	570 lb
Truss height, top sensory ring to top truss fitting	18.0 in.	47.5 in.
<u>Mode Frequencies:</u>		
1st Mode, Vertical	59 cps	56 cps
2nd Mode, Vertical	208 cps	140 cps
1st Mode, Roll	22 cps	25 cps
2nd Mode, Roll	50.4 cps	48 cps
1st Mode, Pitch	20.8 cps	27 cps
2nd Mode, Pitch	50.4 cps	52 cps

Table 3-5. Peak RTG Response

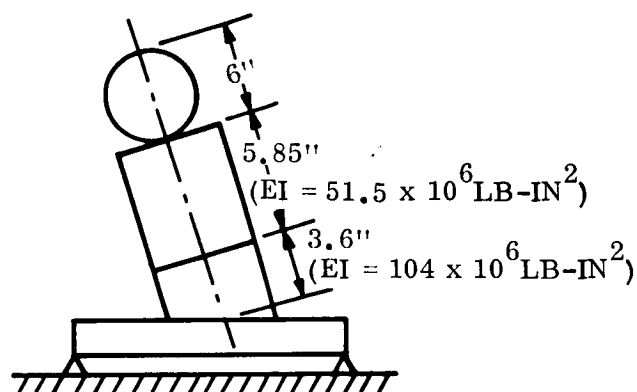
<u>Component Test of RTG with Isolation (NIMBUS B; 5-g Input)</u>			<u>Analysis of Selected Configuration (NIMBUS A Prototype Level)</u>	
	<u>Frequency (cps)</u>	<u>Loading (g)</u>	<u>Frequency (cps)</u>	<u>Loading (g)</u>
Vertical	80	37	59	15.6
Radial	20-25	40	22	6.65
Tangential	25-30	40	21	6.44

Computations were made of the damper response at sensory ring resonances, and it was found that a stiff damper support is required due to the damper weight and its relatively long moment arm from the mounting base. Therefore the erection unit case, which serves also as the damper support, was stiffened by using thicker gage material and adding webs (see Figure 3-3).

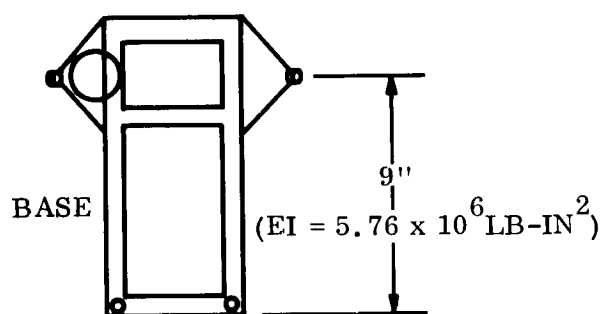
The following basic assumptions were made in the damper mount analysis:

- a. All weight is concentrated in the damper.
- b. All secondary bending is neglected (e. g. , flanges and joints).
- c. Conservative dimensions are used.

DAMPER WEIGHT
= 12 POUNDS



(A) DAMPER - SIDE VIEW



(B) DAMPER - TOP VIEW

Figure 3-6. Damper and Base Model

Results of the analysis indicate that the damper response at the sensory ring resonance is 11.7 g (at 208 cps) and that the damper response at the damper resonance is 15.7 g (at 243 cps). These are reasonable levels, typical of components of this sort on NIMBUS A, and indicate the magnitude of component vibration qualification levels.

3.2.3.3 Structural Analysis

Loads used in the structural analysis were obtained from the dynamic analysis and are shown in Table 3-2. Spacecraft geometry and load definition is shown in Figure 3-7. First, a unit load solution was performed resulting in truss tube loads, corresponding to each of the six loads applied to the RTG centroid as indicated in Figure 3-7. Then, actual loads were obtained for the critical conditions per the dynamic solution. These are all summarized in Table 3-6. Using the most severe truss tube load of 1323 pounds compression in a column analysis results in a minimum margin of safety of +2.11, based on buckling allowables at 300^oF tube temperature.

The sensory ring structure was checked for the loads of Table 3-6 by using a NIMBUS B structural analysis program, resulting in a relatively refined analysis, in spite of the basic difference in crossbeam configuration. For the NIMBUS A sensory ring, the crossbeam is H-shaped, whereas in NIMBUS B, it is A-shaped. This difference does not significantly influence the truss tube loading effects, but does play a significant part in the separation clamp load distribution. However, the values obtained are representative and illustrate the magnitude of RTG effects. Figure 3-8 shows diagrammatically the panel and member designation used in the computer program. Table 3-7 summarizes the computer output and shows, in the right-hand column, a ratio between member loads due to truss tube effects and the total capacity of the member. The most severe effect on the sensory ring is in the outer web, where the RTG loading in the thrust direction accounts for 44% of the web capacity. Other elements have much greater capacities above that portion required for RTG effects. Based on NIMBUS A and C experience, it is a reasonable assumption that truss tube loading will constitute the major portion of the total loading in all sensory ring areas except the lower plane. In this area, truss tube loading has a small effect, whereas the separation band clamping loads produce high member loads. Clamping effects are shown separately in Table 3-7 to permit comparison with RTG effects.

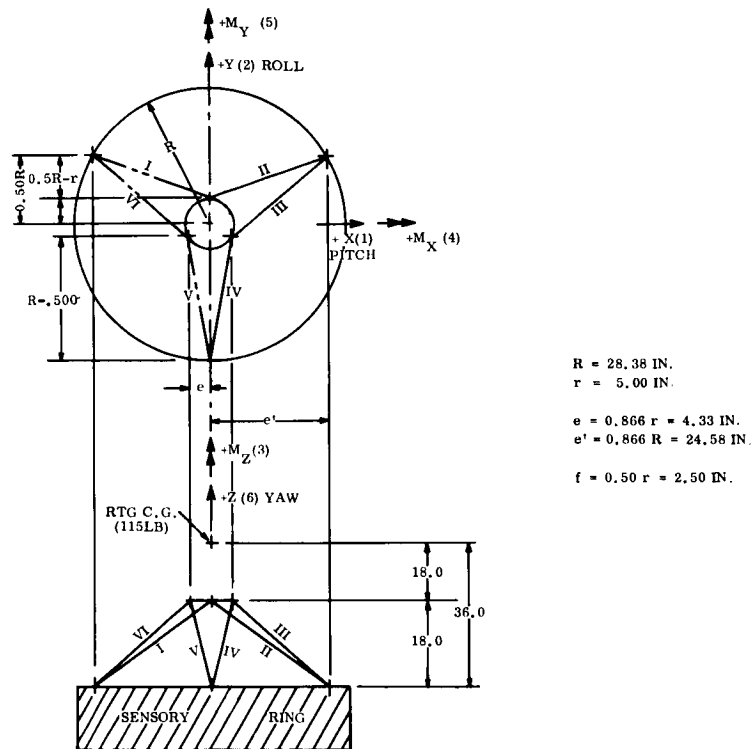


Figure 3-7. NIMPAC Geometry and Load Definition

Table 3-6. RTG Support Truss Loads

Member	Unit Loading						Conditions						
							Thrust	Pitch			Roll		
	F_x = 1000 lb	F_y = 1000 lb	F_z = 1000 lb	M_x = 1000in lb	M_y = 1000in lb	M_z = 1000in lb	F_z = Total (1800 lb)	F_x (755 lb)	M_y (-4090 in-lb)	Total	F_y (739 lb)	M_x (4060 in-lb)	Total
I	-1414	- 241	295	118	- 56	-43	531	-1068	229	- 839	-178	479	301
II	-1414	241	295	118	56	43	531	-1068	-229	-1297	178	479	657
III	888	-1224	337	-103	- 85	-40	607	670	347	1017	-905	-418	-1323
IV	517	-1278	255	- 8	-127	48	459	390	519	909	-944	- 32	- 976
V	517	1278	255	- 8	127	-48	459	390	-519	- 129	944	- 32	912
VI	888	1224	337	-103	85	40	607	670	-347	323	905	-418	487

Summary of Critical Conditions Per Dynamic Solution

Condition	Load (LB)					
	F_x	F_y	F_z	M_x	M_y	M_z
Thrust	0	0	1800	0	0	0
Pitch	755	0	0	0	-4090	0
Roll	0	739	0	4060	0	0

Note: Loads are applied at the RTG CG
Height = 18.0 in.
 $r = 5.00 \text{ in.}$
 $R = 28.38 \text{ in.}$

Table 3-7. RTG Support Loading of the Sensory Ring

Edge Rings

Location	Member	Condition	Allowable Compression Stress - F_c (PSI)	RTG Effect Limit (PSI)	RTG ⁽¹⁾ Effect Ultimate (PSI)	$\frac{RTG (Ultimate)}{F_c}$
INBD Upper	23-24	Pitch	42,000	5466	8200	0.195
	28-29	Roll	42,000	6066	9100	0.217
	22-23	Yaw	42,000	10660	15990	0.381
INBD Lower	53-54	Pitch	42,000	2300	3450	0.082
	57-58	Roll	42,000	2783	4170	0.099
	58-59	Yaw	42,000	3098	4650	0.111
OUTBD Upper ⁽²⁾	11-13	Pitch	52,000	5630	8450	0.163
	1- 3	Roll	52,000	5340	8010	0.154
	13-15	Yaw	52,000	8405	12610	0.243
OUTBD Lower ⁽³⁾	41-43	Pitch	60,000	4231	6350	0.106
	33-35	Roll	60,000	4234	6350	0.106
	33-35	Yaw	60,000	4581	6870	0.115

⁽¹⁾ Ult = 1.5 limit

⁽²⁾ Marmon clamp effect (58-59) = 0.330
RTG + Marmon clamp = .111 + 0.330 = 0.440 (max)

⁽³⁾ Clamp effect (41-43) = 0.170; RTG + clamp = 0.106 + 0.170 = 0.276

Outer Webs

Panel	Load Point	Allowable Load in Pounds (Ultimate)	Condition	RTG Effect in Pounds (Limit)	RTG Effect in Pounds (Limit)	$\frac{RTG (Ultimate)}{Allowable}$
18	19	468	Pitch	72	108	0.231
2	3	468	Roll	104	158	0.333
14	15	468	Yaw	137	206	0.440

Thermal Panels

Panel	Load Point	Allowable Load in Pounds (Ultimate)	Condition	RTG Effect in Pounds (Limit)	RTG Effect in Pounds (Limit)	$\frac{RTG (Ultimate)}{Allowable}$
42	13	579	Pitch	86	129	0.223
42	13	579	Roll	52	78	0.135
42	13	579	Yaw	119	179	0.309

Separator Casting

Location	Member	Condition	Allowable F_c (PSI)	RTG Effect in PSI (Limit)	Ultimate (PSI)	$\frac{RTG (Ultimate)}{F_c}$
Outboard Post	(Typical)	Max.	11,500	1040	1560	0.136
Inboard Post		Max.	14,600	440	660	0.045
Upper Plane	11-24	Pitch	23,750	4106	6160	0.259
	3-28	Roll	23,750	4104	6156	0.259
	3-28	Yaw	23,750	4494	6741	0.284
Lower Plane Without Clamping	41-54	Pitch	27,750	2804	4206	0.177
	43-53	Pitch	27,750	2299	3449	0.145
	33-58	Roll	27,750	2798	4197	0.177
	43-53	Roll	27,750	716	1074	0.045
	33-58	Yaw	27,750	1093	1640	0.069
	43-53	Yaw	27,750	136	204	0.009
Lower Plane Clamping Effect, Only	41-54	Pitch	27,750	1467	2200	0.093
	43-53	Pitch	27,750	4533	6800	0.286
	33-58	Roll	27,750	2008	3012	0.127
	43-53	Roll	27,750	4533	6800	0.286
	33-58	Yaw	27,750	1490	2985	0.126
	43-53	Yaw	27,750	4533	6800	0.286
Lower Plane Total	41-54	Pitch	27,750			0.270
	43-53	Pitch	27,750			0.431
	33-58	Roll	27,750			0.304
	43-53	Roll	27,750			0.331
	33-58	Yaw	27,750			0.195
	43-53	Yaw	27,750			0.295

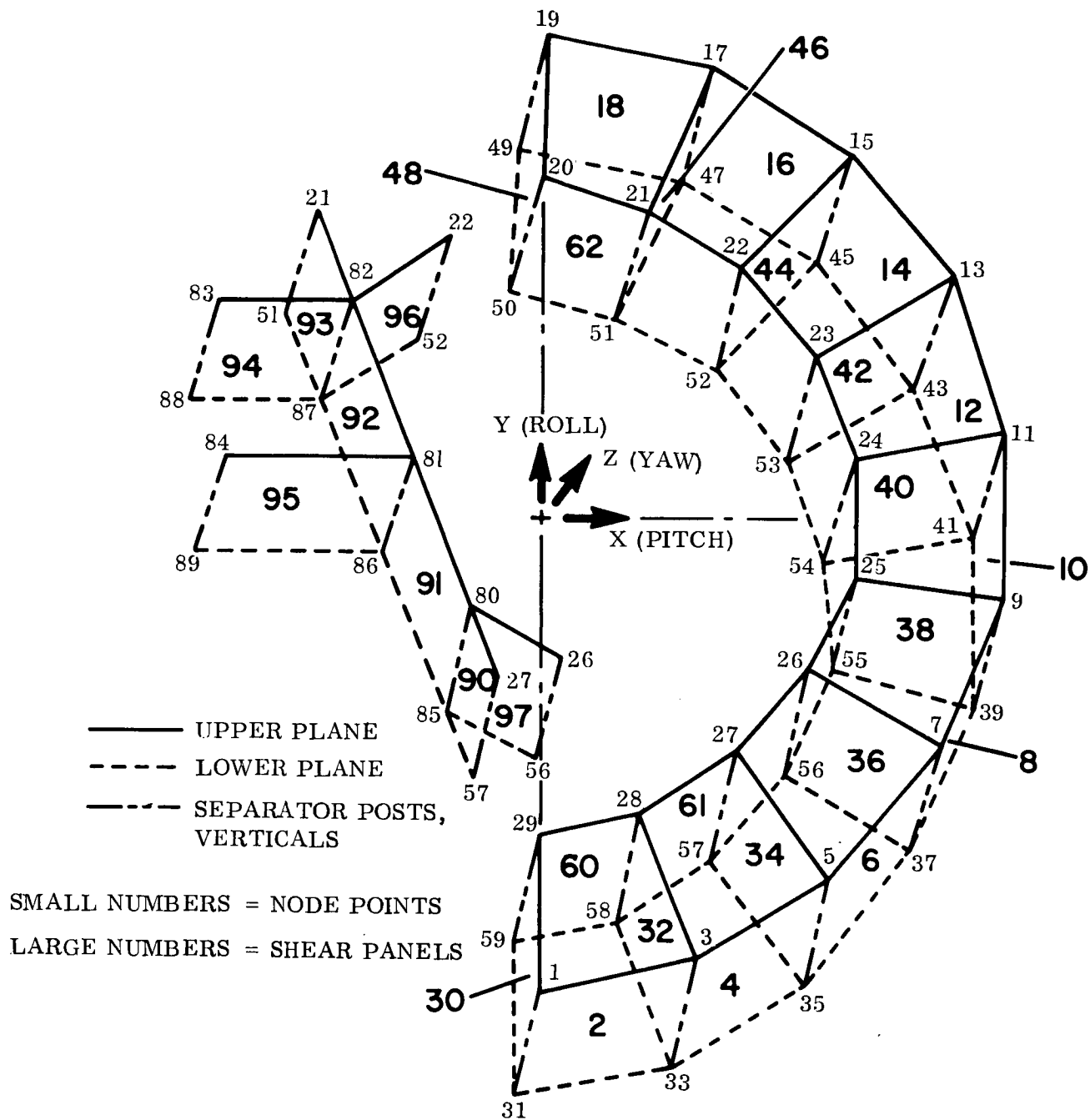


Figure 3-8. Sensory Ring Structural Members

3.3 THERMAL DESIGN CONSIDERATIONS

3.3.1 GENERAL

The SNAP-19 RTGs mounted above the NIMBUS sensory ring have two distinct thermal effects on spacecraft subsystems. First, the gravity-gradient rods will be subject to radiant heating from the RTGs, resulting in circumferential gradients superimposed on those caused by solar radiation. Second, the thermal performance of the sensory ring can be affected by both radiative and conductive heating from the RTGs. Thermal analyses conducted during the study indicate that, under worst conditions, local circumferential gradients on the gravity-gradient rods, due to RTG thermal radiation, are small compared to those caused by the solar radiation. Further, the heat leakage to the sensory ring from the RTGs, together with the expected heat generated in each bay, will be within the ring heat rejection capability. RTG thermal effects were minimized by separating the RTG from the sensory ring on a support truss which was thermally optimized in length.

Sensory ring dissipative heating of attitude control components is negligible in comparison to RTG heating, and RTG heating is small in comparison to solar heating. The attitude control rod ejection motor mechanisms have been thermally isolated from the sensory ring to prevent heat leakage into and out of the sensory ring. The technique used is identical to that employed with externally mounted NIMBUS payloads and is illustrated in Figures 3-3 and 3-4.

3.3.2 THERMAL ANALYSIS OF RTG HEATING OF GRAVITY-GRADIENT RODS

Circumferential thermal gradients in the gravity-gradient rods caused by the RTG heat flux will result in local distortion of the rods. A thermal analysis was conducted to evaluate the extent of this effect in comparison to normal solar heating.

The geometrical relationship of the RTGs and the rods used in the analysis is shown on Figure 3-9. The RTGs were conservatively assumed to be a 21-inch diameter cylindrical heat source; the base of this cylinder facing the NIMBUS ring is uninsulated. A 0.9 infrared

emittance characteristic was chosen for the RTG surfaces. For all cases studied, a silver-plated beryllium-copper gravity-gradient rod of 0.55 inch diameter and 2 mil thickness was assumed, with an infrared emittance (and absorptance) of 0.1. The solar absorptance of the rod was assumed to range between 0.2 and 0.3, which represents the range normally encountered with silver plating. The rod thermal gradients were investigated at nine different locations over an approximate 65-inch span from the root of the rod. This length was determined to be the practical limit of RTG influence. These locations are shown on Figure 3-9. At each position along the length of the rod, a differential area was chosen on the rod periphery in such a way as to obtain the maximum RTG heat flux arriving locally; further, differential areas were assumed to be parallel to the RTG axis. This last assumption will result in conservative estimates of temperature. A closed-form analytical solution to steady-state temperature distributions in a discontinuous cylindrical tube experiencing solar flux has been determined during a previous study and is described in Reference 5. This analytical technique was used during the study, where the maximum RTG heat flux incident on the rod is assumed to behave similarly to a solar flux impinging normally on the rod periphery.

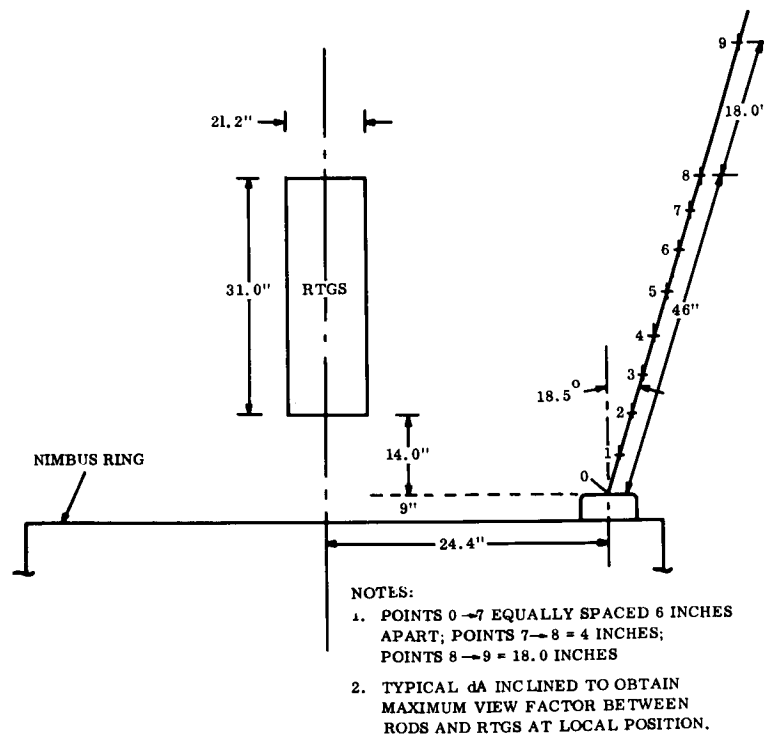


Figure 3-9. NIMPAC Thermal Model

In the analysis in Reference 5, the radiation exchange within the rod has been regarded as having a relatively insignificant effect on the overall temperature gradient. Also, the contact conductance along the rod overlap was considered to be zero. Both of these will result in conservative values of temperature differentials. The average steady-state rod temperature due to RTG heating along the rod length is shown in Figure 3-10 and was calculated assuming a worst-case orientation of the rod seam to the normal RTG flux and neglecting axial conduction. Although an appreciable temperature gradient may be observed in Figure 3-10 along the rod span under study, temperature levels remain nevertheless low, ranging from -187 to -55°F. Figure 3-11 illustrates the nodal-average steady-state rod temperatures under a combined RTG and solar load as compared to the temperature induced by a normal solar flux. This result is to be expected, as the RTG flux contribution represents a small addition to the normal solar flux. In the case of combined heat fluxes, a 22-degree solar offset has been assumed, since the RTGs will shadow solar flux at smaller angles. The absorbed RTG load is 18 percent of the absorbed solar load at position (6), which has the largest geometric view factor to the RTGs. In the case of a rod with a solar absorptance greater than 0.2, the gradients along the rod would be reduced further, although steady-state temperature would increase to a higher level.

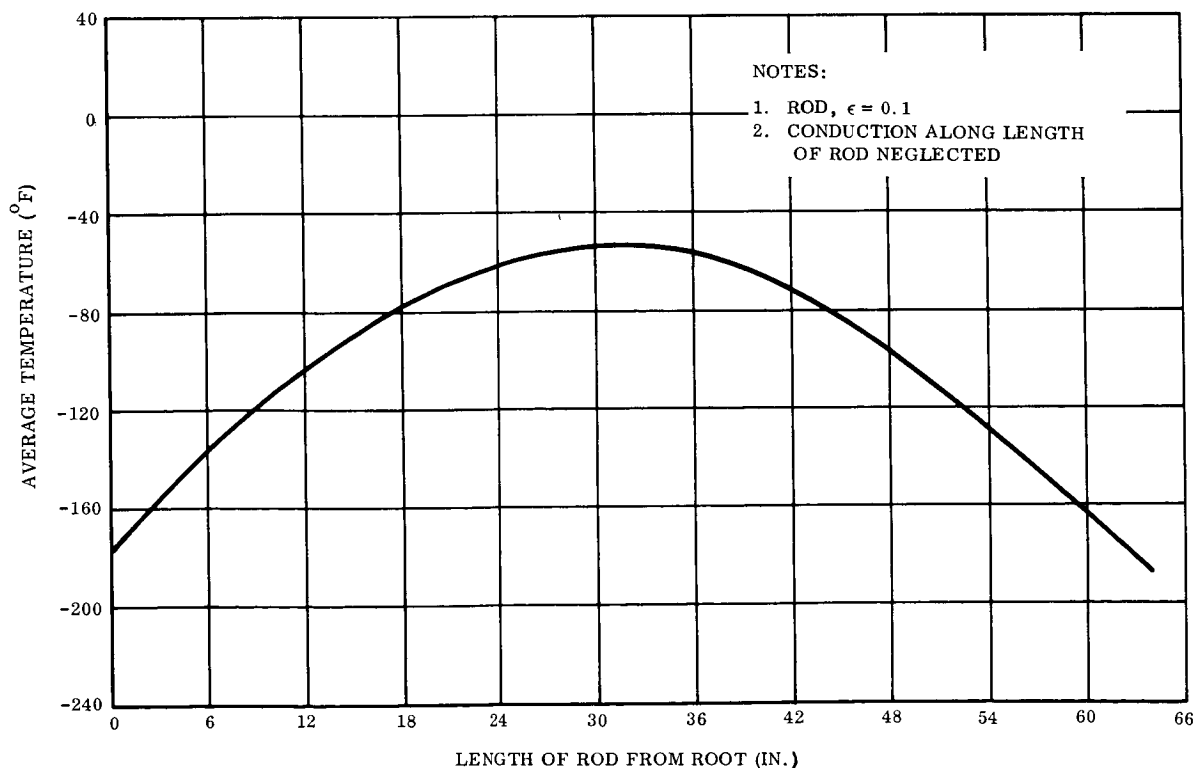


Figure 3-10. Distribution of Average Rod Temperature Induced by RTG Heating

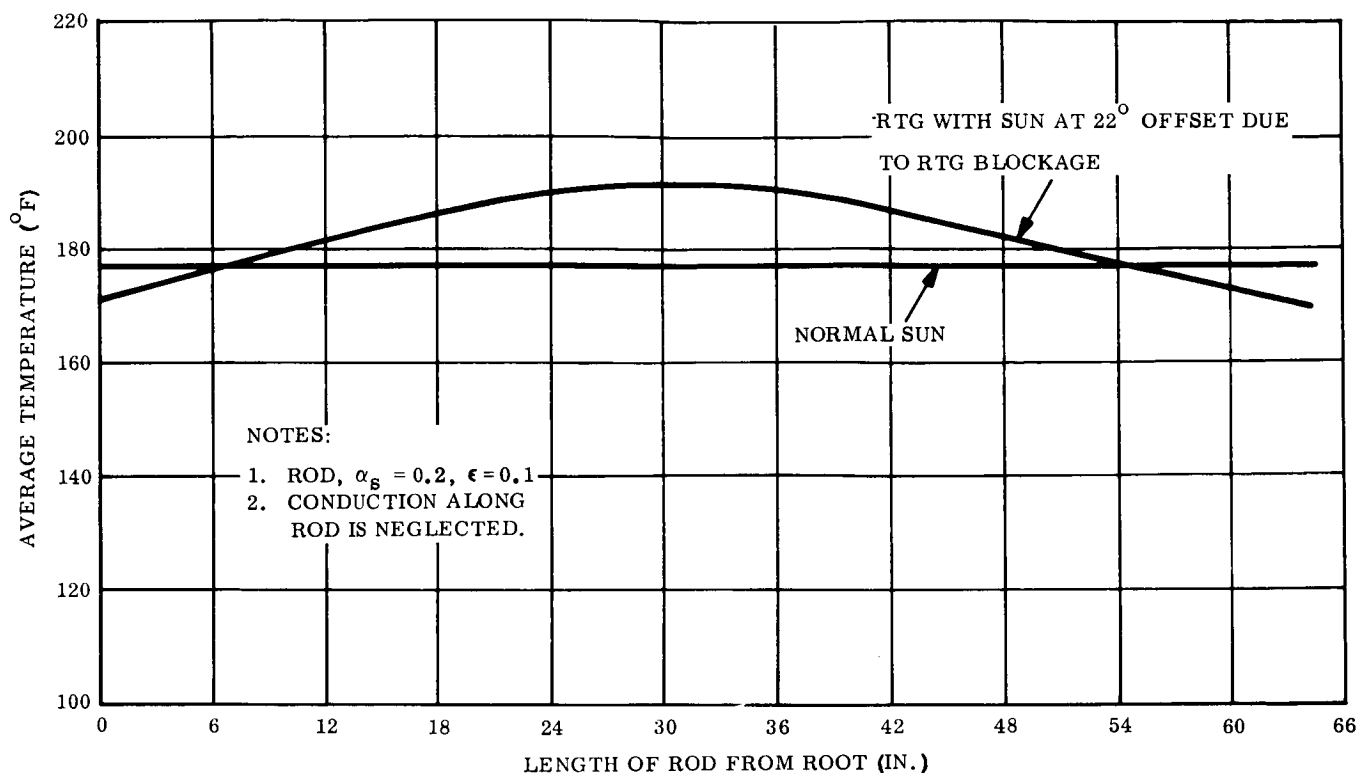


Figure 3-11. Distribution of Average Rod Temperature Induced by RTG and Solar Heating

Using the technique described in Reference 5, the maximum circumferential temperature distribution that was indicated by this evaluation of RTG heating is presented in Figure 3-12, as a function of circumferential position, for a worst-case rod seam orientation. These results are normalized by the average temperature at that location so that a graphic comparison can be made of the temperature distribution produced by a normal solar flux. Figure 3-12 illustrates that the maximum RTG-induced temperature difference is 1.9°F . This value was only experienced at rod positions (4), (5), and (6). At positions (0) and (9), the difference had decreased to 0.4°F . In comparison, normal solar flux results in a maximum temperature difference, for the same rods, of between 11.6°F and 17.4°F , for a solar absorptance of 0.2 and 0.3, respectively. At some satellite orbital position, the RTG-induced thermal difference will add to that induced by the sun. The rod circumferential temperature distribution for this worst case has been presented in Figure 3-12 for a rod solar absorptance of 0.2. The maximum temperature difference resulting from this combined heat flux will only be approximately 4 percent higher than that produced solely by the sun, and with a rise in average temperature of 14°F . From this it has been concluded that RTG heating will not produce significant thermal bending of the gravity-gradient rods in the selected spacecraft configuration.

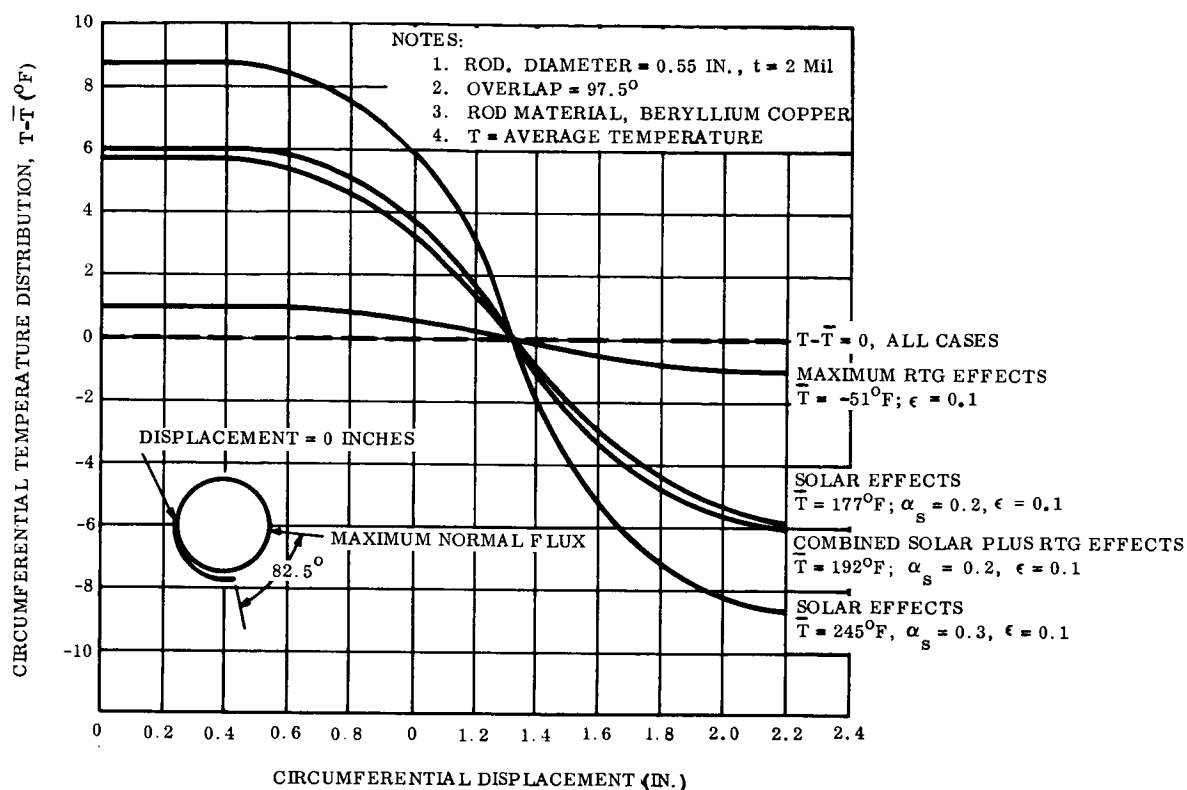


Figure 3-12. Comparison of Circumferential Temperature Distribution Induced by RTGs and Normal Sun

3.3.3 THERMAL ANALYSIS OF RTG AND SOLAR HEATING OF THE NIMBUS RING INSULATION

A detailed thermal analysis was performed to determine the effect of RTG radiant heating on the sensory ring insulation. The results of three cases investigated are shown in Figure 3-13.

In order to obtain temperatures of the outer layer of insulation on the sensory ring, geometric view factors were determined between the periphery and the base of the cylinder, simulating the RTGs and the entire top circular surface of the sensory ring insulation. For this purpose, the ring top surface was divided into six annular areas. Conduction between these areas was assumed to be negligible.

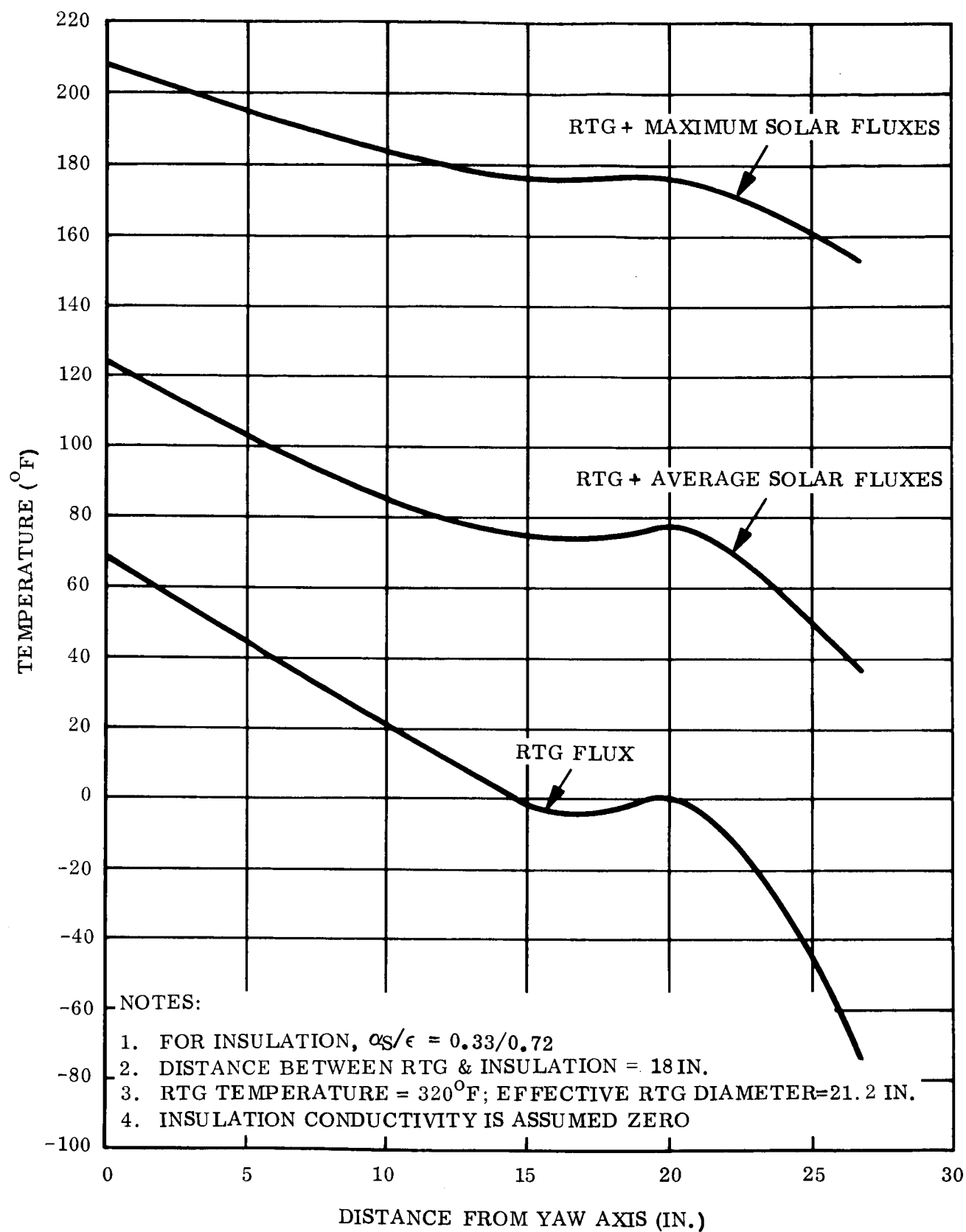


Figure 3-13. Temperature Distribution Along First Layer of Radiation Insulation On Top of NIMBUS Ring

The steady-state heat balance equation applicable to each area is given by:

$$\alpha S = \sigma T_1^4 (F_{e_{12}} F_{a_{12}} + F_{e_{13}} F_{a_{13}} + \epsilon_1 F_{a_{1-s}}) - \sigma T_2^4 (F_{a_{12}} F_{e_{12}} + F_{a_{13}} F_{e_{13}})$$

where α = solar absorptance
 S = solar constant
 F_e = radiant interchange factor
 F_a = geometric view factor
 ϵ = emittance (infrared)
 σ = Stefan Boltzmann constant
 T = temperature

subscripts

1 = insulation area
 2 = base of RTG simulated cylinder
 3 = periphery of RTG simulated cylinder
 s = black space

RTG heating resulted in temperatures along the first layer of the ring insulation ranging from +70°F to -74°F, with an approximate average of 0°F. These temperatures would be encountered during eclipse periods where solar heating would not take place. In order to assess the heat leak which would occur through the multilayer insulation, a more representative outer layer temperature distribution is that caused by the combination of the RTG load and an average orbital solar flux. This results in a range from 126°F to 36°F as illustrated in Figure 3-13. If an average sensory ring internal temperature of 70°F is assumed, the heat leak through the insulation will be small. For the optical properties chosen for the outer insulation layer, an orbit average solar flux yields -19°F for this layer, compared to an approximate 80°F when this average is combined with that caused by the RTGs.

When the solar flux is normal to the ring insulation, its temperature would rise to 126°F. This maximum solar flux, in combination with the RTG flux, causes the outer insulation layer to assume temperatures ranging from 208°F to 153°F. These will occur temporarily, due to orbital motion of the satellite.

From this analysis it has been concluded that the net effect of the RTGs is to decrease the heat leak from the internal sections of the sensory ring, since the insulation temperatures resulting from a combined RTG and orbit average solar flux are nearer the ring internal temperatures than those caused solely by the average solar flux.

3.3.4 SENSORY RING HEAT DISSIPATION CAPABILITY

The following tabulation illustrates the maximum orbit average heat rejection capacity for each of the 18 compartments of the sensory ring. A double value is given for each of the compartments: the first value listed is the net value for the shuttered surface, and the second value is the net rejection of the shuttered surface, plus the net rejection of the compartment lower surface with the insulation removed and a low α_s/ϵ coating applied.

Table 3-8. NIMBUS Sensory Ring Heat Rejection Capability

<u>Compartment No.</u>	<u>Net Radiation (Watts)</u>	<u>Net Radiation with Bottom Insulation Removed (Watts)</u>
1	13	17.5
2	11.5	16
3	12.5	17
4	17.5	21
5	18	22.5
6	17.5	21
7	12.5	17
8	11.5	16
9	13	17.5
10	13	17.5
11	12.5	17
12	12.5	17
13	17.5	21
14	18	22.5
15	17.5	21
16	12.5	17
17	11.5	16
18	13	17.5

In order to obtain the values listed in Table 3-8, a detailed analysis of each compartment was necessary. Two representative curves of heat rejection rate as a function of orbit time are given on Figure 3-14 and 3-15. These represent compartments No. 1 and No. 5, the leading edge and side compartments, respectively. It will be noted that the solar influence on compartment No. 1 is quite strong, but becomes negligible on No. 5. This influence is shown by the difference in orbit average capacity for the two compartments (see tabulation).

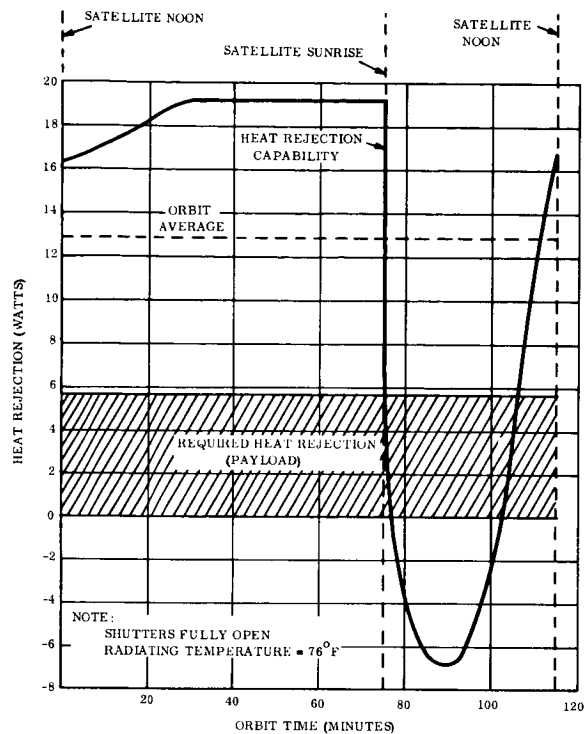


Figure 3-14. NIMBUS Sensory Subsystem Heat Rejection Characteristics, Compartment No.1

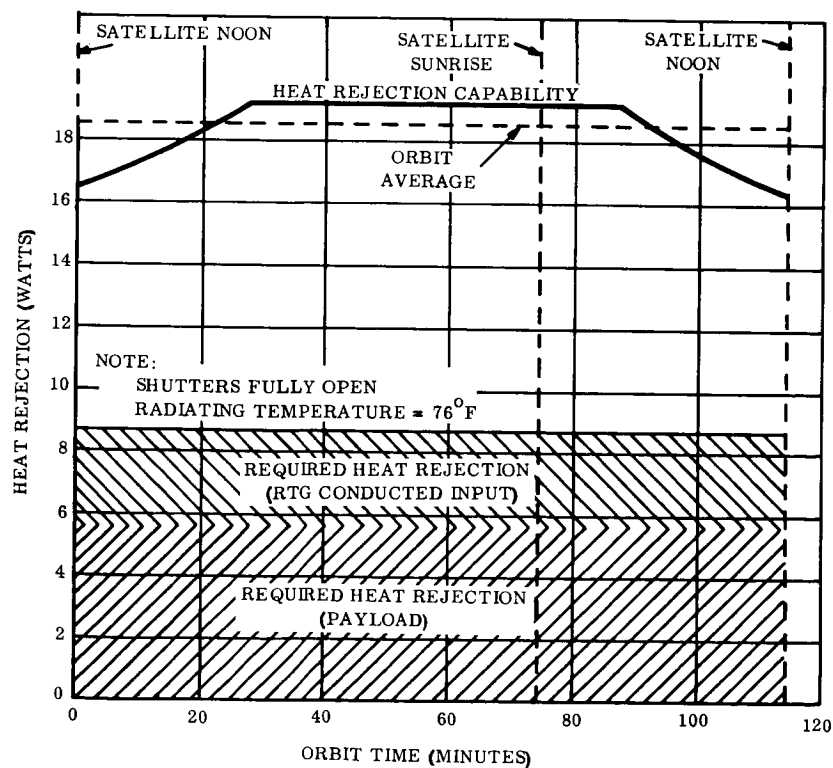


Figure 3-15. NIMBUS Sensory Subsystem Heat Rejection Characteristics, Compartment No.5

Contrary to what might be expected, the performance of compartment No. 1 will not significantly be effected by the dip in the curve as long as the heat dissipation of the equipment in the compartment is within the orbit average capability. The thermal mass of the components and local structure, as well as conduction to adjacent compartments, damps out the response effectively.

For the purpose of this analysis, a uniform distribution of the RTG electrical output was assumed. With a total power available of 100 watts, the heat load per compartment is 5.55 watts. At separators No. 4, No. 10, and No. 16, the structural supports of the RTG assembly are attached. The worst-case analysis of the conductance of these supports (see section 4.3) indicates that a heat leak value of 3.1 watts per strut, or 6.2 watts per attachment point, can be expected. Therefore compartments No. 3, No. 4, No. 10, No. 15, and No. 16 have an additional load of 3.1 watts each, as illustrated in Table 3-9.

Table 3-9. Sensory Ring Heat Rejection Requirements

<u>Compartment No.</u>	<u>Heat Load Watts</u>
1	5.55
2	5.55
3	8.65
4	8.65
5	5.55
6	5.55
7	5.55
8	5.55
9	8.65
10	8.65
11	5.55
12	5.55
13	5.55
14	5.55
15	8.65
16	8.65
17	5.55
18	5.55

The orbit average temperature and the orbital temperature variation of each of the sensory ring compartments have been calculated. This analysis included the effects of optimum shutter positions, conduction to adjacent compartments, and payload heat capacity. The results are presented in Table 3-10. Since the sensory ring payload has not been defined for this study, payload heat capacity was assumed to be the same as the NIMBUS A test re-

Table 3-10. Estimated Sensory Ring Thermal Performance of Selected Configuration

<u>Compartment</u>	<u>Average Temperature</u> (°F)	<u>Temperature Variation</u> (°F)
1	71	±3.6
2	72	±3.6
3	72	±1.8
4	71.8	±0.9
5	69	±3.6
6	68	±2.2
7	70	±9.0*
8	72	±1.8
9	73.5	--**
10	73.5	±0.5
11	72	±1.8
12	70.3	±2.7
13	69.2	±0.9
14	69	--**
15	71.8	±9.0*
16	72	±3.6
17	72	±0.9
18	71	±2.7

* Component hot spot temperature

** Either no data available or no component in this location.

sults. It is expected that the variation calculated using this data is greater than would be experienced by the actual equipment, since the flight data reflects a certain amount of cyclical component operation. The orbit average temperatures and calculated variations illustrated in Table 3-10 do not exactly follow the heat loads tabulated in Table 3-9, as might be expected. This apparent anomaly is the result of two different shutter characteristics of the NIMBUS spacecraft. PCM antenna, located between compartment No. 2 and No. 3,

No. 7 and No. 8, No. 11 and No. 2, and No. 16 and No. 17, require a different shutter design in order not to interfere with their ground planes. This shutter design takes the form of five shutter blades, each a different width, as compared with the nominal design of four blades of equal width. In addition, the ground plane provides a restriction in the "view angle" of the radiating surface. Figure 3-16 shows the variation of effective emissivity as a function of shutter angle for each of the configurations.

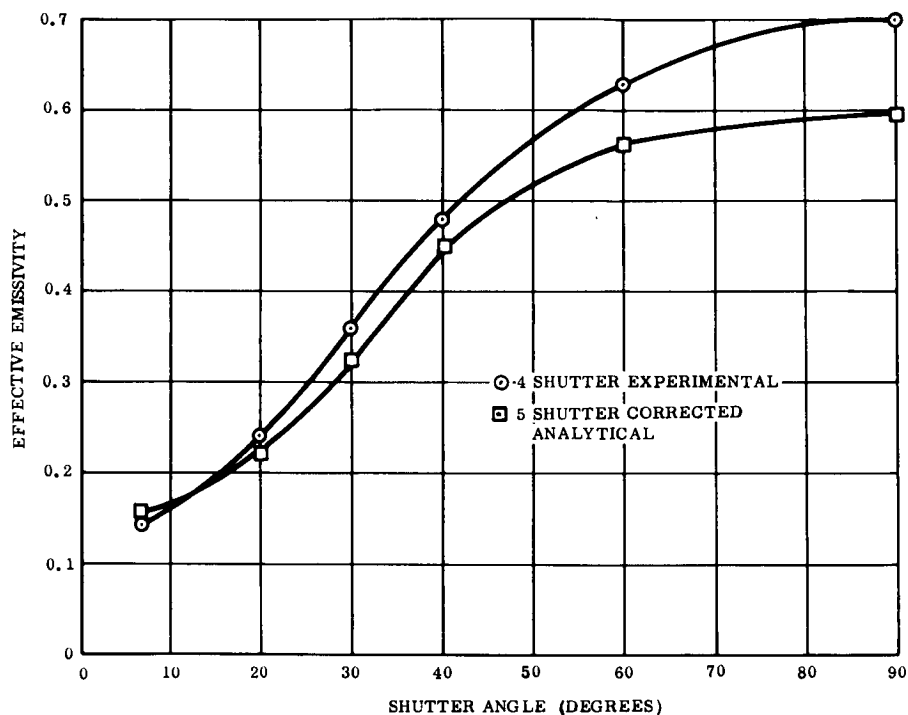


Figure 3-16. NIMBUS Sensory Subsystem Shutter Characteristics

3.4 PASSIVE ATTITUDE CONTROL SUBSYSTEM DESIGN

3.4.1 GENERAL

The Passive Attitude Control Subsystem selected in this study consists of four gravity-gradient erection mechanisms, two magnetically-anchored eddy-current dampers, Marmon clamp support and release mechanisms, and associated supporting structure, as shown in Figure 3-1.

In the launch configuration, the rods are fully retracted and the dampers are supported by the Marmon clamps. After injection into orbit and when the passive attitude control system is to be put into operation, the Marmon clamps are actuated and power is applied to the gravity-gradient rod erection mechanisms. Each of the two upper rods carries a damper. The two lower rods have no tip masses and do not require Marmon clamp release devices.

3.4.2 GRAVITY-GRADIENT ROD ERECTION MECHANISMS

The extendable gravity-gradient rods are the type manufactured by deHavilland Aircraft of Canada, Limited. This technique involves the formation of a tubular section from a flat metal BeCu strip which is formed and heat-treated in the tubular form and then flattened under stress and wound onto a storage drum. Erection in orbit is accomplished by playing out the stowed strip through a set of guides which allows the boom to form into its natural tubular shape. The edges of the metal strip overlap each other to render stiffness to the section. Two standard strip sizes have been developed; a 0.002-inch-thick, 2-inch-wide strip and a 0.005-inch-thick, 4-inch-wide strip. The smaller size has been selected because most of the flight experience is with this size. In addition, the smaller size results in a smaller lighter rod erection package. The precise diameter selected for all NIMPAC rods is 0.55 inches, which satisfies solar pressure requirements and provides a rod more than adequate to withstand the acceleration loads imposed by erection at about 1 foot/second with an initial vehicle tumble rate of 1 degree/second, while carrying the 12-pound damper. The rods will be plated with silver to a thickness of 0.002 inch and polished to a solar absorptivity of approximately 0.2 to minimize thermal bending.

The unit used to deploy the gravity-gradient rods is referred to as an erection mechanism. Deployment consists of guiding the rod into its natural tubular shape while being unwound from the storage drum. This process is somewhat critical, since the relatively thin rod is subject to crippling damage if not deployed properly. Two basic types of erection units have been developed by deHavilland Aircraft Company. The self-erecting unit, which uses the strain energy in the flattened stored rod to effect deployment, and the motorized unit,

which employs an electric motor to drive the rod storage drum for deployment and retraction. For this mission a motorized unit has been selected for the following reasons:

- a. Peak erection rates for self-erecting units are in the range of 4 feet/second, probably excessive from the standpoint of rod strength with 12-pound dampers.
- b. The motorized units effect a controlled erection and can be preselected, whereas the self-erecting units erect in a more uncontrolled fashion.
- c. The positive driving torque on the motorized units decreases the possibility of "hangup" during deployment.
- d. The majority of flight experience has been with motorized units.
- e. The motorized units permit boom retraction if required for turnover maneuver.

Figure 3-17 is a photograph of a rod erection unit very similar to the one for this application. This type of unit is flexible in its ability to meet specific requirements, such as erection rate, type of power consumed, and rod length. All erection units for this mission will be virtually identical, thus minimizing the design and development effort with no significant compromises. Each unit will require a 10-vdc power supply and will operate at less than 6.5 watts of power.

Alignment of each erection mechanism with respect to the satellite will be accomplished by shimming at the mounting lugs. An alignment mirror attached to the mechanism will be used as a reference during component acceptance tests and subsequently during installation of the mechanism to the satellite. Alignment is required to compensate for manufacturing tolerances. The deHavilland extendable rod principle is a qualified flight-proven system having flown on several spacecraft and rocket flights. The rod lengths proposed for this application are within present and planned flight experience.

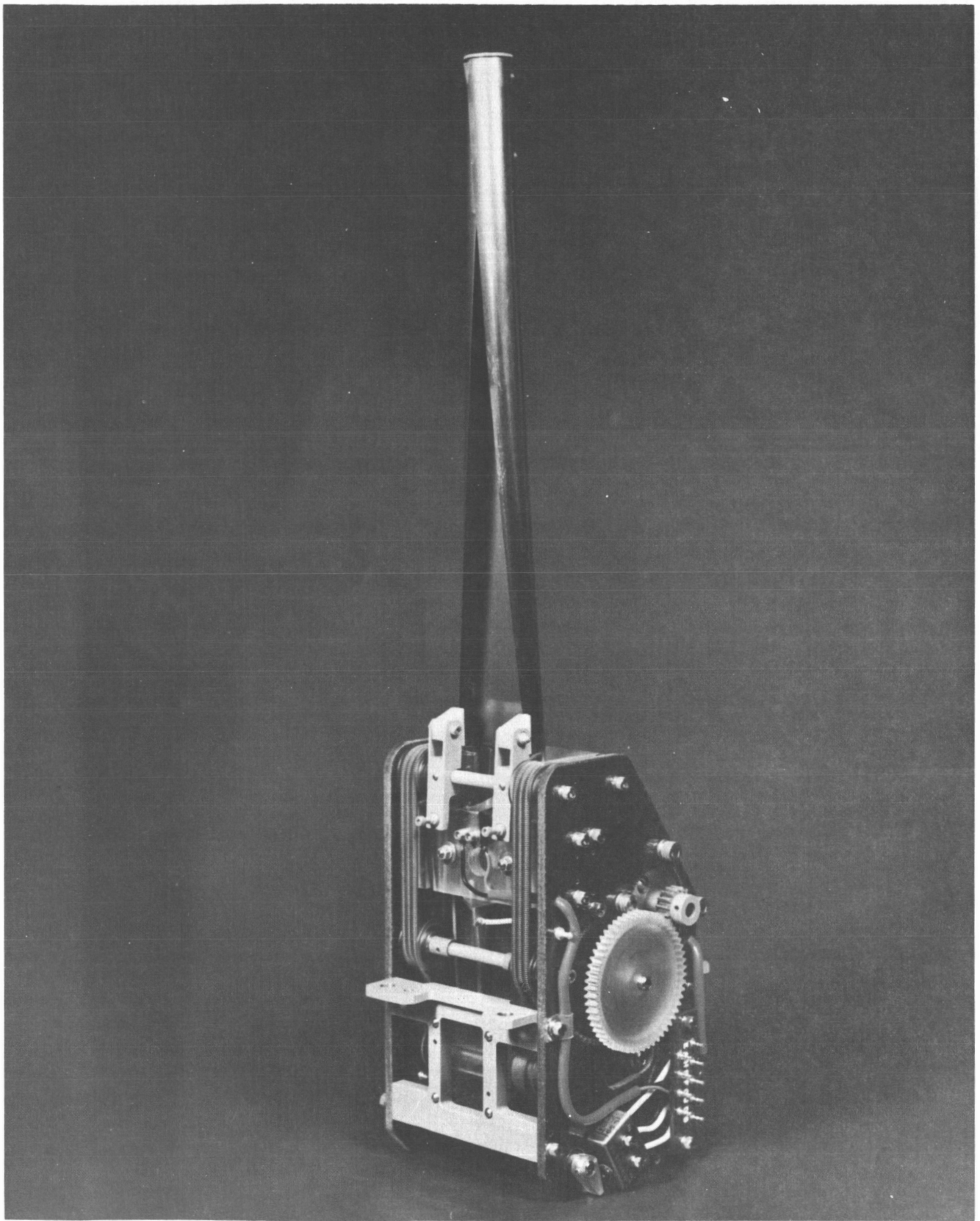


Figure 3-17. Motor-Driven Gravity-Gradient Rod Erection Unit

3.4.3 DAMPER DESIGN

The dampers selected for this application are magnetically-anchored, diamagnetically-suspended units utilizing eddy-current losses for energy dissipation. The damper performance characteristics include a magnetic-moment of 60,000 pole cm and a nominal damping coefficient of 172,000 dyne-cm/rad/sec. The weight of each unit is 12 pounds. The damper illustrated in Figure 3-3 consists of an uncaged magnet assembly within a sphere assembly. The magnet assembly is made up of six cast Alnico-5 bar magnets approximately 1 inch in diameter joined together by means of a spherical carbon steel fitting. The end of each bar magnet is inserted into a hole in the fitting with an interference fit providing the holding capability. The axes of the bar magnets, when assembled, form three mutually perpendicular axes. Plastic boots are bonded to the free end of each bar magnet to prevent metal-to-metal contact during handling, launching, and excessive accelerations during orbital capture. The outer spherical diameter of these boots is the same as the inner diameter of the copper shell which completely surrounds the magnet assembly, thereby avoiding concentrated loads.

The sphere assembly consists of three concentric shells: an exterior structural shell of aluminum alloy with a basic O.D. of 6.5 inches, an interior copper shell, and a pyrolytic graphite shell sandwiched between the aluminum and copper shells. Each shell is made of two hemispheres. Oxygen-free, high-conductivity (OFHC) copper is used in order to maximize the eddy-current losses. The rabbeted joint between the copper hemispheres is soldered in order to insure adequate electrical conductivity.

Pyrolytic graphite is used for the diamagnetic material because of its extremely high magnetic susceptibility to weight ratio. The selected thickness of graphite is sufficient for centering force capability but small enough to avoid delamination tendencies associated with higher thickness-to-diameter ratios. The copper shell previously described serves to protect the graphite from the loads associated with the response of the "loose" magnet assembly within the damper. A vent hole is provided so that air trapped within the damper will be vented immediately during the launching operation as the altitude increases. In

this manner an inadvertent disturbing torque due to escaping air during orbital operation is avoided.

For passive thermal control of the damper in orbit, D4D thermal coating is applied to the exterior surface. This coating is an aluminum paint with an alkyd base which provides an α/ϵ of about 1.0. A plastic fitting is used to attach the damper to the end of the rod thereby minimizing heat flow between these two items.

In orbital operation, the damper's function is as follows: Each sphere assembly is essentially tied to the satellite by means of a gravity-gradient rod and each magnet assembly, which is free to rotate within its sphere assembly, "locks on" to the Earth's magnetic field. As the satellite oscillates, there is a relative angular velocity between the magnet and sphere assemblies which produces eddy currents in the copper shell, causing a dissipation of energy.

The pyrolytic graphite shell serves as the suspension medium in conjunction with the magnet assembly, utilizing the principle based upon the repulsive interaction of a magnetic field upon a diamagnetic material. This interaction tends to center the magnet assembly within the sphere assembly during oscillatory motions so that no physical contact occurs (friction) between these two parts.

The proposed damper is an extremely simple, highly reliable, passive component with almost an unlimited operational lifetime.

The basic requirements for each damper, imposed by the overall system needs, are a damping coefficient of 172,000 dyne-cm/rad/sec, a centering force capability of at least 3 dynes per pound of magnet assembly at contact of the magnet and sphere assemblies, and a weight of 12 pounds. A damper sized to meet the above requirements results in a magnet assembly having a magnetic moment of 60,000 pole-cm, which is quite acceptable. No severe size restraint was imposed on the damper; however, the basic O.D. of 6.5 inches, necessary to meet the above requirements, is very compatible with the rod erection mechanism, resulting in a reasonable structural arrangement.

Eddy-current losses and centering force capabilities are a complex phenomenon. The proposed design is based heavily on a smaller, previously developed damper. Thus the problem becomes one of the scaling up known characteristics, which can be done with a fair degree of accuracy. In any event, ground tests can be readily conducted to verify the performance characteristics of the damper as well as its ability to withstand the applicable shock, vibration, acceleration, thermal, and vacuum environments.

3.5 SPACECRAFT ATTITUDE PERFORMANCE

3.5.1 GENERAL

The function of the gravity-gradient stabilization subsystem is to initially orient the spacecraft with a preferred axis, aligned to the local vertical, reduce the rates and error angles which exist after separation as quickly as possible, and maintain three-axis attitude control of the spacecraft to the orbit as accurately as possible. The degree to which the subsystem performs these functional requirements is discussed in this section. The steady-state pointing accuracy is evaluated for the reference-design orbit conditions. The damping time from tumble in pitch to steady state is also evaluated. These evaluations are made by means of a digital computer simulation of the motion of the spacecraft with all significant disturbance torques included. This simulation calculates torques on the spacecraft and integrates spacecraft angular accelerations and rates to determine spacecraft position as a function of time. A description of the simulation is given in Section 5.

The gravity-gradient stabilization subsystem selected is described in Section 3.4. The subsystem consists of four deHavilland type extendable rods which are deployed from the top surface of the NIMBUS sensory ring. Two rods extend upwards at an angle of $18\frac{1}{2}$ degrees to the yaw axis and carry 12-pound magnetically-anchored eddy-current dampers. These dampers give the large moments of inertia about the pitch and roll axis which are required for gravity-gradient stabilization. The dampers also remove energy from the spacecraft so as to damp large angular oscillations which occur after deployment of the rods. The other two rods extend $19\frac{1}{2}$ degrees below the horizontal and carry

no tip weight. These rods provide the correct relationship between the three moments of inertia about the principal axes for most accurate three-axis stabilization, as well as balance the solar pressure torques on the upper rods.

3.5.2 DESIGN-REFERENCE-ORBIT ATTITUDE PERFORMANCE

3.5.2.1 Steady-State Pointing Accuracy

The steady-state pointing accuracy of the four-rod gravity-gradient stabilization subsystem has been evaluated for the nominal 750-nautical-mile sun-synchronous orbit. The pointing accuracy is defined by the Euler angles between the spacecraft body fixed reference axes and the orbit reference axes. The steady-state time history of the Euler angles is plotted in Figure 3-18 for 100 hours (45 orbits). The peak values of the steady-state oscillation amplitudes are:

Pitch:	+ 1.6 - 1.0	degrees
Roll:	$\pm$ 0.3	degrees
Yaw:	+ 1.8 - 1.4	degrees

Figure 3-18 illustrates the effects of a small transient motion which is present because the initial conditions do not represent a steady-state condition. It can be seen that the amplitude of pitch motion decreases throughout the run. The steady-state conditions are reached after about 150 hours.

The disturbance effects which are included in this evaluation of steady-state pointing accuracy are:

- a. Nominal orbit eccentricity = 0.0089
 - Apogee = 787.5 nautical miles
 - Perigee = 712.5 nautical miles

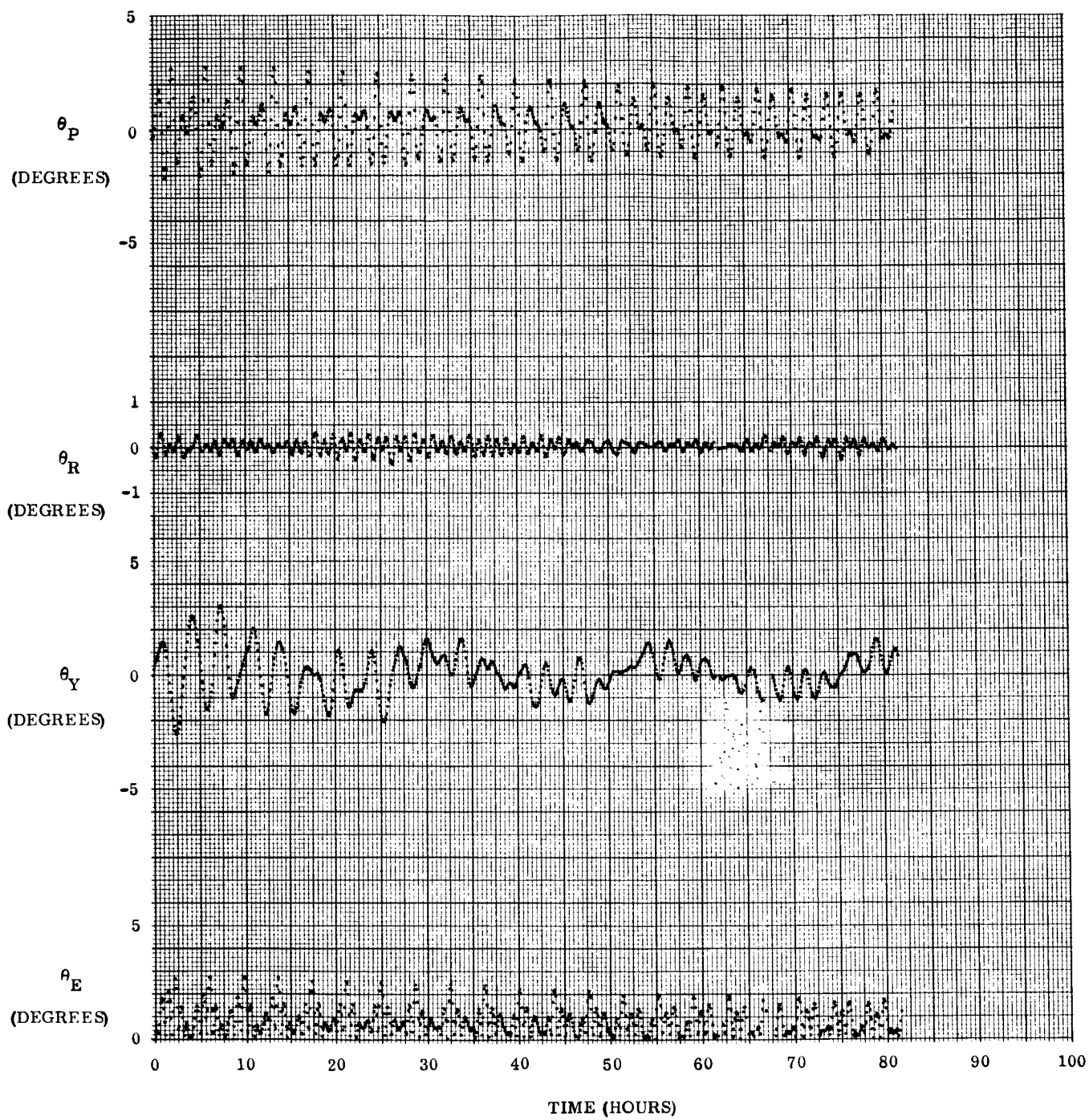


Figure 3-18. Steady-State Pointing Performance ($\epsilon = 0.0089$)

- b. Solar pressure torques in sun-synchronous orbit. The effects of solar pressure on the spacecraft and on rods curved from temperature distribution are included.
- c. Shift of the spacecraft principal axes due to thermal curvature of the rods.
- d. Motion of the damper magnet following the earth's magnetic field.
- e. Spacecraft residual magnetic dipole of 100 pole-cm in worst-case orientation.

Other effects which have been analyzed and found to have negligible effect on the steady-state accuracy are aerodynamic torques and the constant component of angular momentum due to internal moving parts.

An evaluation of the steady-state pointing accuracy over 200 hours (95 orbits) with the worst-case eccentricity of $\epsilon = 0.0179$ is shown in Figure 3-19. This evaluation continues well into steady state and shows the continuing decay of pitch amplitude from 100 hours to 150 hours. The steady-state values of error for $\epsilon = 0.0179$ are:

Pitch:	+ 2.5 degrees - 1.7
Roll:	+ 0.3 degrees
Yaw:	+ 1.8 degrees - 1.4

The error magnitude due to each of the various disturbance effects had been estimated by means of the linearized equations of motions in Section 4.5.3. These error magnitudes are presented in Table 3-11.

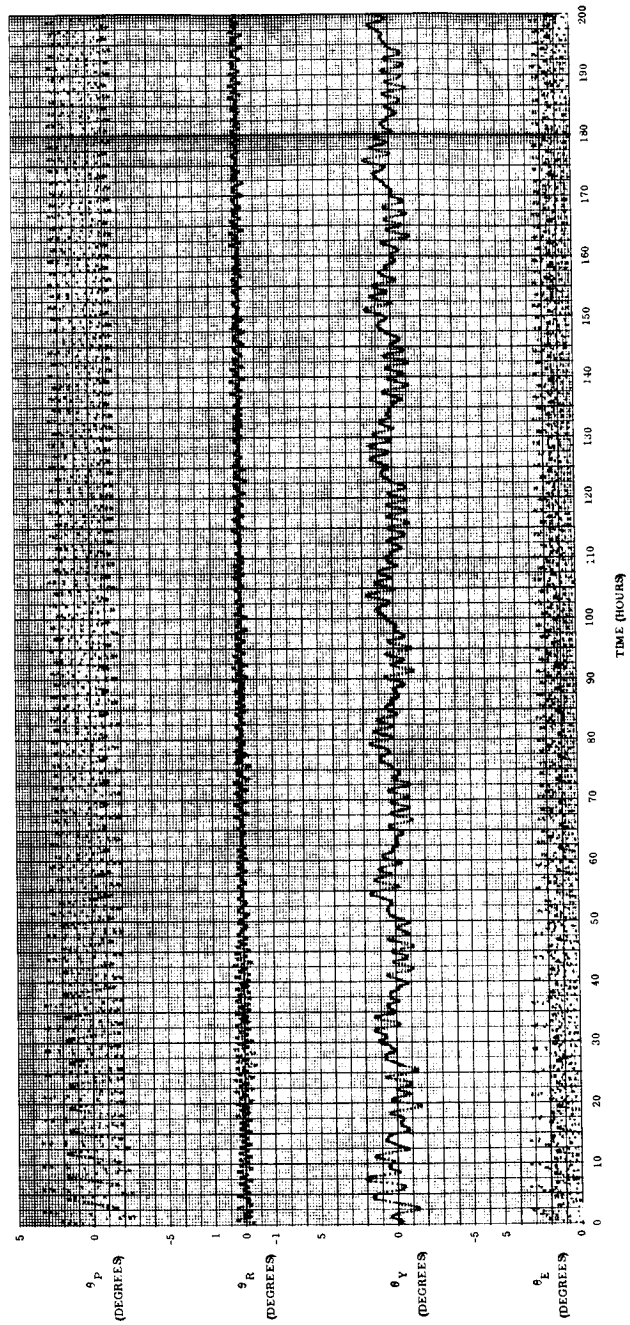


Figure 3-19. Steady-State Pointing Performance ($\epsilon = 0.0179$)

Table 3-11. Steady-State Error Sources

	Pitch	Roll	Yaw
Orbit Eccentricity ($\epsilon = 0.0089$) ($\epsilon = 0.0179$)	$\pm 0.85^\circ$ $\pm 1.7^\circ$	--	--
Damper Magnet Motion	$+ 0.26^\circ$ $\pm 0.33^\circ$	$\pm 0.08^\circ$	$- 0.85^\circ$
Vehicle Residual Dipole (100 pole cm on pitch and roll axis)	$\pm 0.05^\circ$	$\pm 0.02^\circ$	$- 0.17^\circ$ $\pm 0.21^\circ$
Solar Pressure Torques	$\pm 0.05^\circ$	--	--

It is clear that most of the pitch error is due to eccentricity, and most of the roll and yaw error is due to motion of the damper magnet. The pitch error due to orbit eccentricity is a sinusoidal oscillation at orbital frequency as can be seen in the time history of pitch error angle. The other significant pitch disturbance is due to motion of the damper magnet. This pitch error has a static bias component, which causes the positive values of pitch error to be larger than the negative values, and a sinusoidal component at twice orbital frequency.

The yaw error due to damper motion is primarily an orbital frequency oscillation. The rotation of the earth's magnetic field with respect to the orbit plane causes a 24-hour cyclic distortion of the yaw error, which can clearly be seen on the yaw time history. Similarly, the roll motion exhibits a 24-hour cycle due to rotation of the earth's magnetic field.

A time history of the spacecraft body rates (rates about the $\bar{X}$ (yaw), $\bar{Y}$ (roll), and $\bar{Z}$ (pitch) body fixed axes taken with respect to inertial space) is shown in Figures 3-20 and 3-21. These rates are from the same simulation run as the angles in Figures 3-18 and 3-19. The

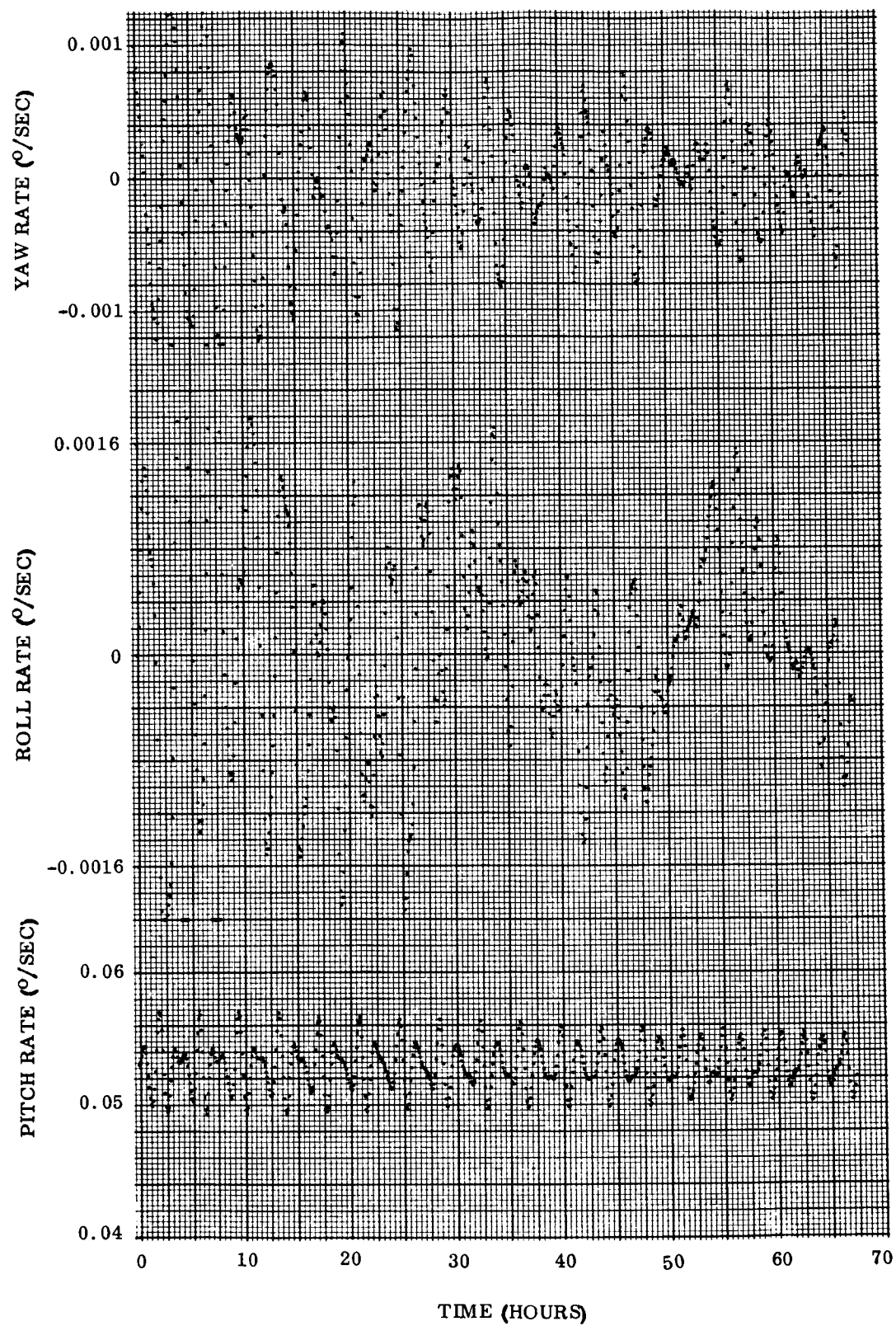


Figure 3-20. Steady-State Body Rates ($\epsilon = 0.0089$)

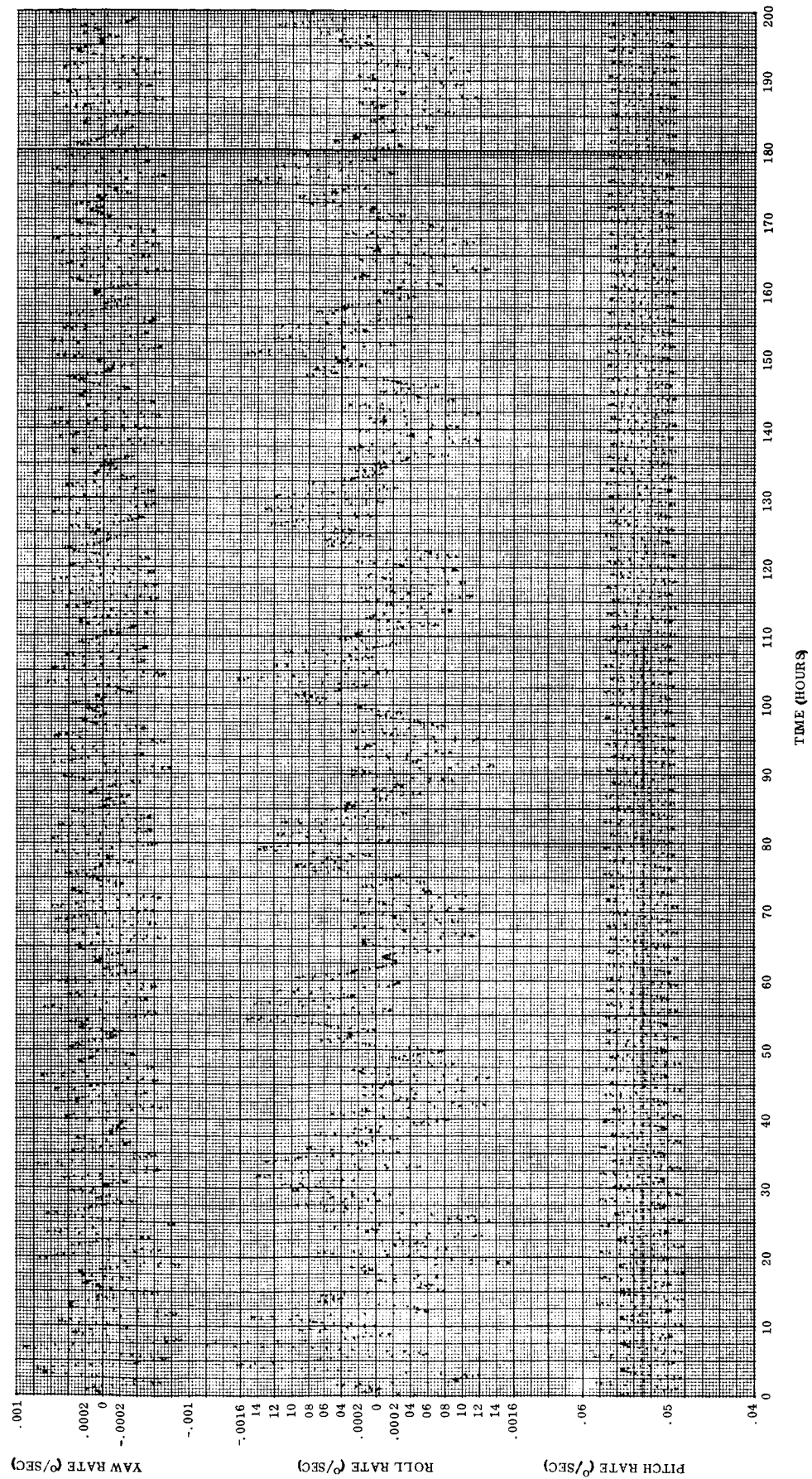


Figure 3-21. Steady-State Body Rates ($\epsilon = 0.0179$)

body rate about the yaw axis is less than 0.001 degree/second at all times during the oscillation. The roll rate is less than 0.0016 degree/second. The 24-hour fluctuation due to rotation of the earth's magnetic field can also be seen in these rates.

The pitch body rate with respect to inertial space is plotted about the average orbital rate of 0.053 degrees per second. The rates due to satellite oscillations in pitch are ± 0.004 degree per second with respect to the average orbital rate. This pitch rate, which is the highest of the body rates, is 25 times lower than the required minimum rate of 0.05 degree per second.

The only significant error source which has not been considered in the preceding discussion is the effect of the alignment tolerance of the gravity-gradient rods. The tolerance on the envelope of possible rod positions is $1/2$ degree half cone angle. The worst possible condition would occur if all of the rods were misaligned in a direction to cause the maximum shift in orientation of the principal axes. This maximum shift is $1/2$ degree. The effects of rod misalignment on the relation between the three moments of inertia, which determines the amplitude of response to disturbance torques, will cause less than $1/2$ degree additional error on any axis.

3.5.2.2 Damping Performance

The time for satellite oscillations to damp from the last pitch tumble to the steady-state pointing accuracy requirements of 5 degrees in pitch and roll and 10 degrees in yaw is 180 hours and has been evaluated by means of the nonlinear digital solution of the equations of motion. The time history of the Euler angles showing damping from initial conditions of tumble in pitch and yaw is shown in Figure 3-22. The pitch motion damps to ± 5 degrees with a $+1$ degree bias at about 180 hours. This represents 2.5 time constants based on the 66-hour exponential decay time obtained from the small angle equations of motion.

It can be seen that the spacecraft continues to yaw through many rotations while the large angle pitch motion is damping. This is because the energy of motion is coupled between

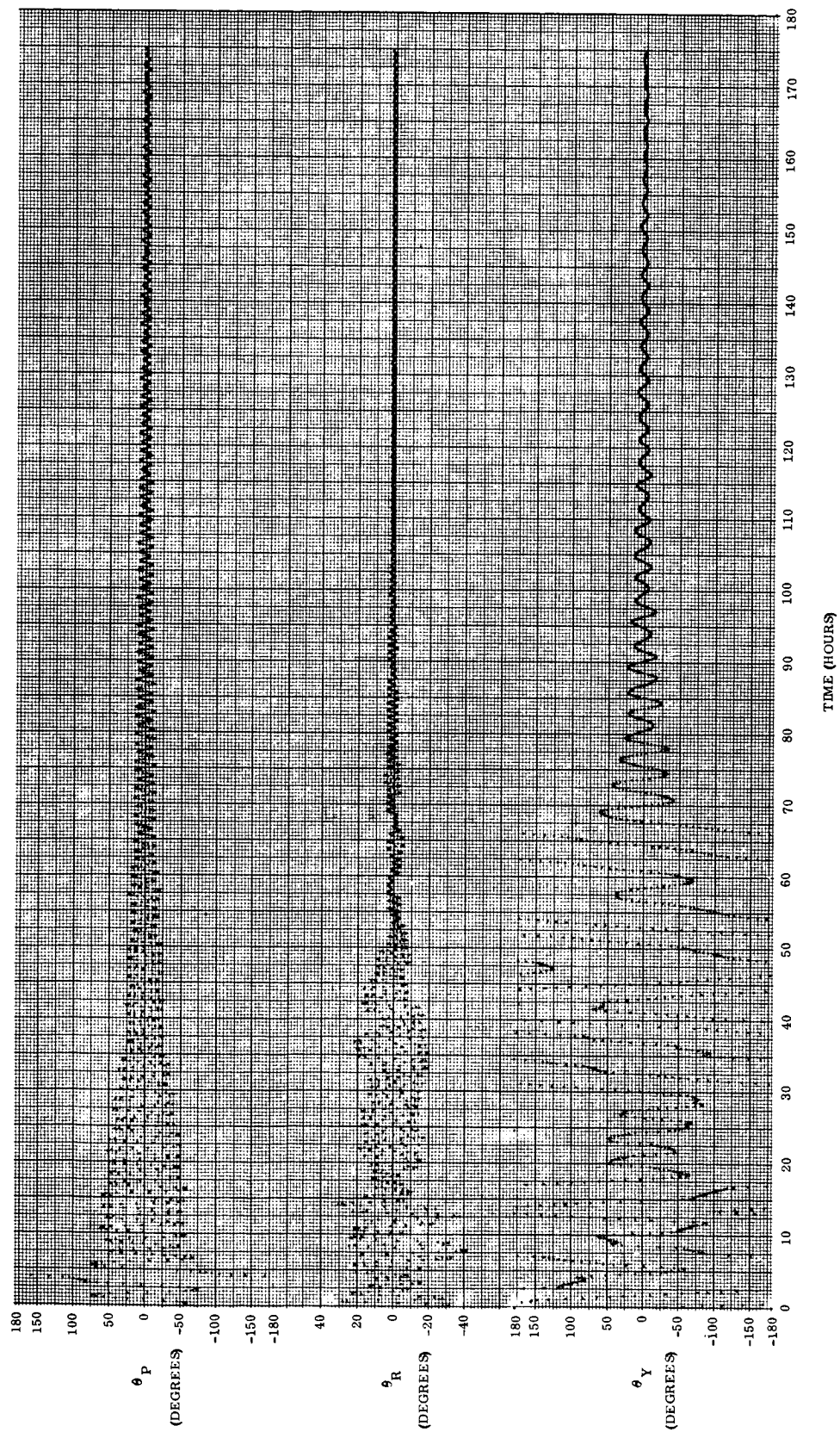


Figure 3-22. Damping Time to Steady State

the spacecraft body axes, and the energy needed to rotate 180 degrees in yaw is much less than that to tumble in pitch. As the damper removes energy, however, the yaw tumble stops at 70 hours after the last pitch tumble and damps to steady state at the same rate as pitch motion since it is disturbed by pitch motion coupling into the yaw axis.

3.5.2.3 Capture Performance

The measure of capture performance is the time from separation of the spacecraft from the final booster stage until the last tumble of the spacecraft with respect to the vertical direction. The spacecraft must ultimately stop tumbling and align itself with the axis of least moment of inertia in the vertical direction, since this is the only stable orientation with respect to vertical, and the damper continuously removes energy from the spacecraft.

The capture sequence begins immediately after separation of the spacecraft from the final booster stage. All four rods are deployed at a rate of 1 foot/second. The upper rods reach full length after 60 seconds and the lower rods after 80 seconds. The separation rates are reduced during rod extension due to conservation of momentum. The rates after rod retraction are approximately equal to the ratio of inertias before and after separation times the original separation rates. The inertia growth ratios for each of the three axes are:

$$\text{Pitch: } \frac{I_f}{I_o} = \frac{3000}{64} = 46.8$$

$$\text{Roll: } \frac{I_f}{I_o} = \frac{2800}{64} = 43.7$$

$$\text{Yaw: } \frac{I_f}{I_o} = \frac{575}{52} = 9.0$$

The roll and yaw rates and angles are -1 degree per second and -5 degrees, as shown in Figure 3-23. It can be seen that one complete revolution occurs in pitch followed by capture with a pitch oscillation of about ± 75 degrees. The separation conditions for Figure 3-24 are 1 degree per second roll and yaw rate and 5 degrees roll and yaw error. This figure also shows one tumble followed by capture.

An investigation has also been made of the range of separation conditions for which right-side-up capture without tumble can be attained. The investigation is based on calculations of kinetic and potential energy of the spacecraft as it rotates with respect to the orbit reference frame. The analytic approach is verified with the nonlinear digital simulation.

This investigation shows that right-side-up capture can be achieved for all separation conditions within the specified 1 degree per second rate and 5-degree error except for negative pitch rates greater than -0.8 degrees per second. For these pitch rates, roll and yaw rates must be below 0.8 degree per second.

3.5.3 OFF-DESIGN ORBIT ATTITUDE PERFORMANCE

3.5.3.1 Performance as a Function of Eccentricity and Mean Altitude

In determining the performance as a function of eccentricity, the variations between apogee and perigee considered are 0, 75, 150, and 300 nautical miles. These differences produce eccentricities of 0, 0.0089, 0.0179, and 0.0358, respectively. The results of the digital simulations for these eccentricities are shown in Figures 3-25 through 3-28, respectively. Not all of these simulations reach steady state (minimum pointing error amplitude); however, the steady-state amplitudes can be estimated accurately from the decay envelope of Figure 3-27. The total oscillatory part of the pitch oscillation amplitude taken from the digital simulation results is plotted in Figure 3-29. Comparison of Figures 3-25 through 3-29 shows that eccentricity has no effect on yaw or roll error amplitudes. The pitch amplitude is linear with eccentricity up to $\epsilon = 0.02$. The pitch amplitude for $\epsilon = 0.0358$ has been estimated as ± 4 degrees + 0.5 degree, although the run is not to steady state.

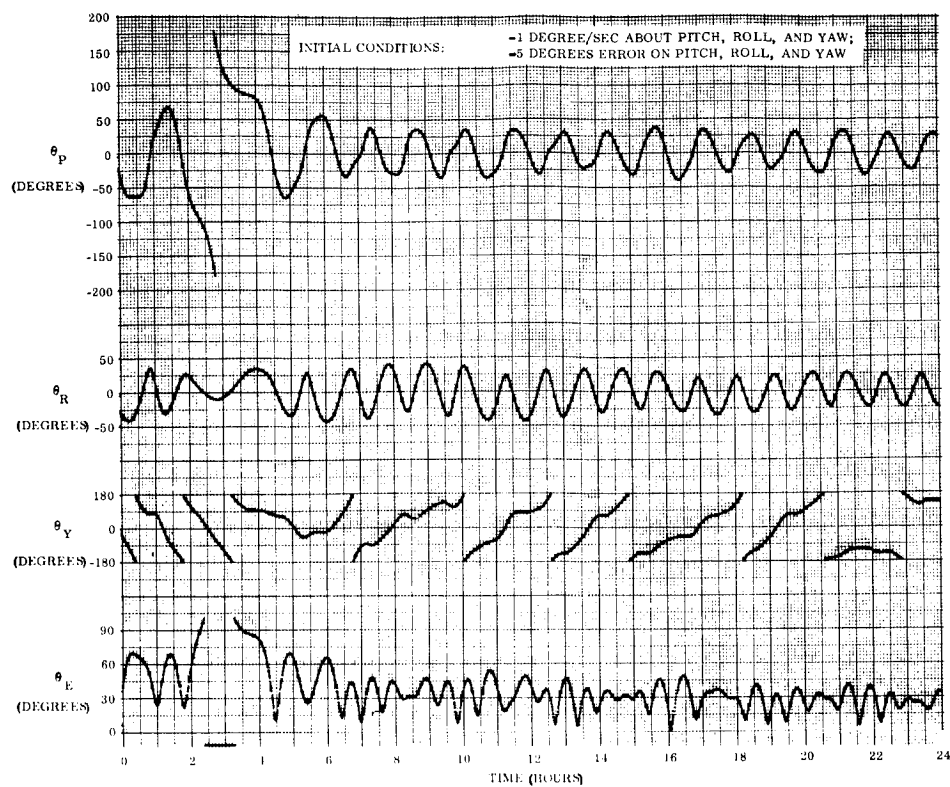


Figure 3-23. Capture Performance (Negative Roll and Yaw)

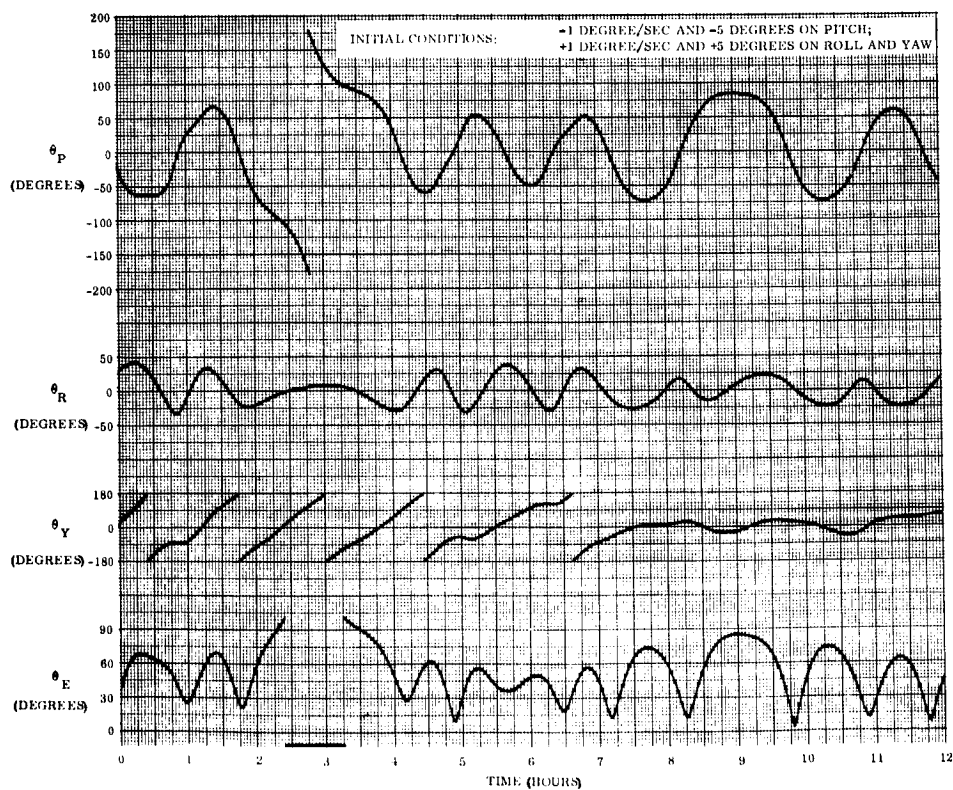


Figure 3-24. Capture Performance (Positive Roll and Yaw)

This estimation of the steady-state error shows a slightly greater than linear response amplitude.

An analytical expression for the pitch amplitude response to eccentricity disturbance is developed in Section 4.5. This expression is

$$\theta_p(t) = 93.5 \epsilon \sin(\omega_0 t + 21) \text{ (degrees)} \text{ and is also plotted in Figure 3-29. The}$$

analytic results are less than the digital simulation results by about 0.5 degree, since the digital results include the oscillatory response due to damper motion and vehicle residual magnetic dipole.

Typical values of body rates after rod deployment with separation rates of 1 degree per second are:

Pitch: 0.01-0.02 degrees per second

Roll: 0.01-0.02 degrees per second

Yaw: 0.08-0.1 degrees per second

This reduction in rate due to inertia growth enables the spacecraft to capture within 6 hours from any of the specified separation conditions of 1 degree per second and 5 degrees error.

The total time from separation to capture has been evaluated from initial conditions at separation of 1 degree per second rate about each body axis and 5 degrees error between the spacecraft and its nominal orientation. The total time to capture with the worst-case initial conditions of -1 degree per second pitch rate and -5 degrees pitch error is less than 6 hours. This capture time limit has been established by means of the nonlinear digital simulation of the equations of motion of the spacecraft with time varying moments of inertia to account for the extension of the four gravity-gradient rods. The time history of the Euler angles which define the vehicle position with respect to the orbit reference axes are shown in Figures 3-23 and 3-24. These plots show spacecraft motion during

the rod extension phase, which lasts for 80 seconds and the subsequent tumble and capture of the spacecraft. The separation conditions are -1 degree per second pitch rate and -5 degrees pitch angle for both figures.

The variation in mean altitude of plus or minus 25 nautical miles results in variations of eccentricity, gravity-gradient restoring torque, and disturbance torques from damper and vehicle dipole of fractions of a percent and causes no measurable change in vehicle steady-state performance.

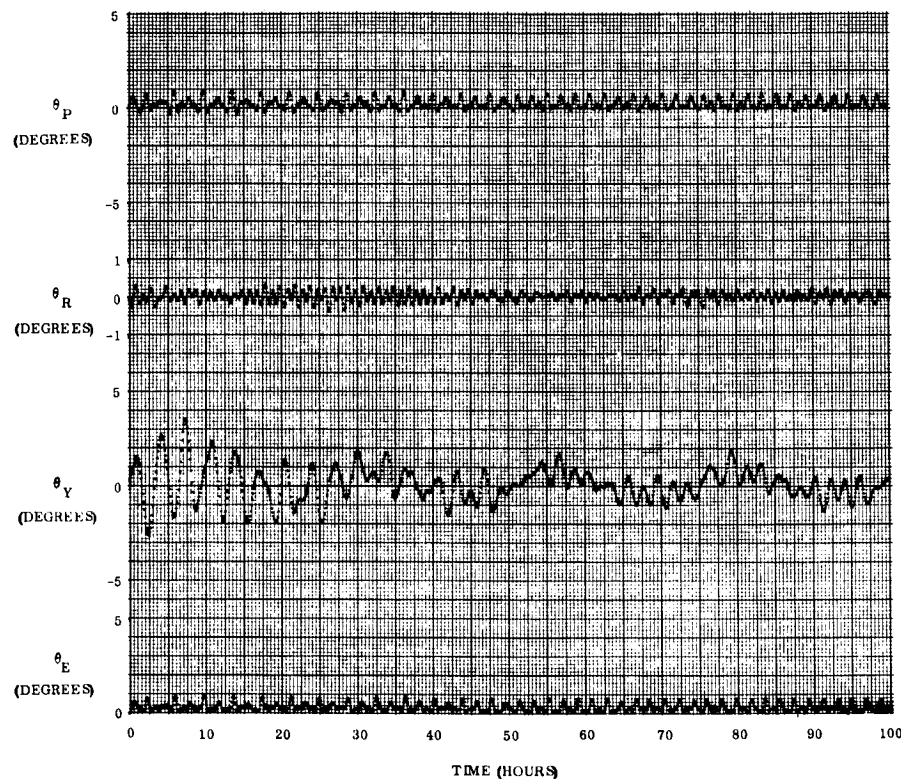


Figure 3-25. Steady-State Pointing Performance ($\epsilon = 0$)

3.5.3.2 Performance as a Function of Orbit Plane Drift

The effects on steady-state accuracy of orbit plane drift from the sun-synchronous position have been evaluated by means of the nonlinear digital simulation. Orbit plane drift affects the magnitude and time history of solar pressure torques on the spacecraft. Since the

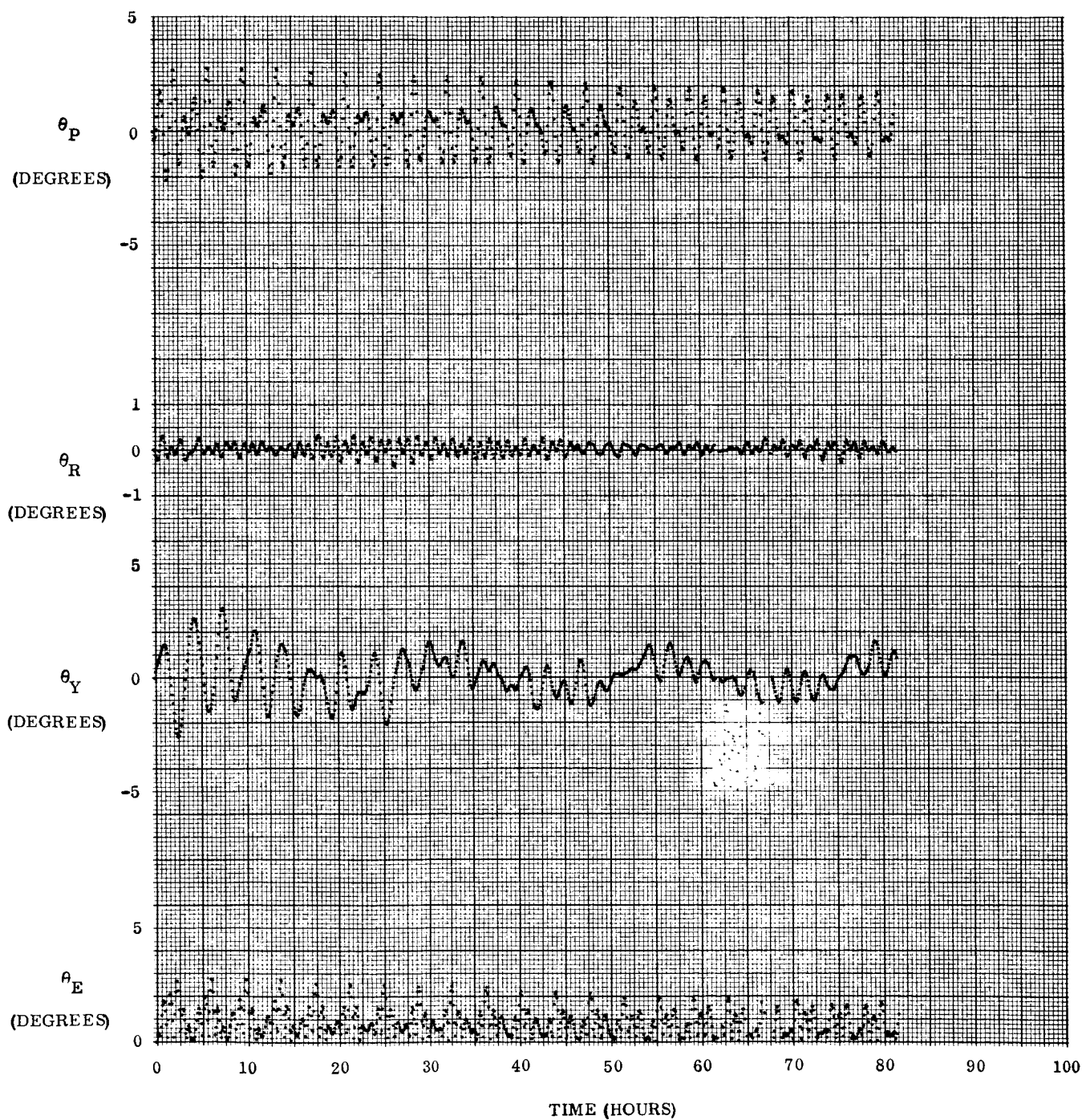


Figure 3-26. Steady-State Pointing Performance ($\epsilon = 0.0089$)

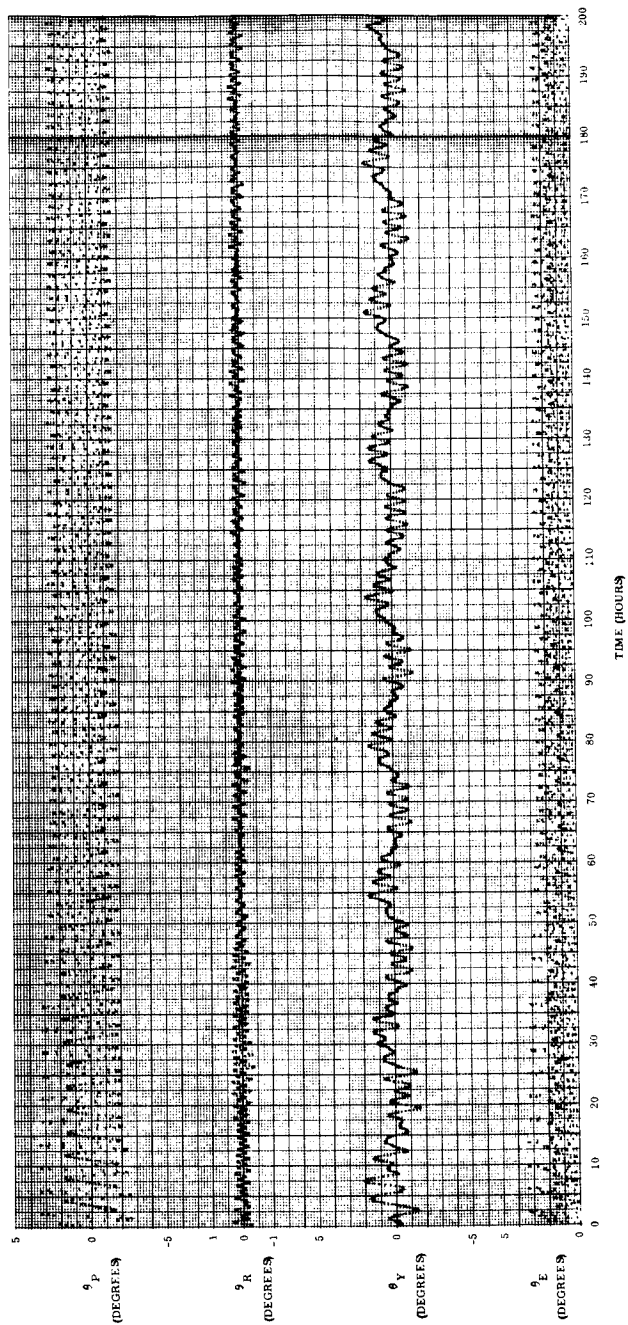


Figure 3-27. Steady-State Pointing Performance ($\epsilon = 0.0179$)

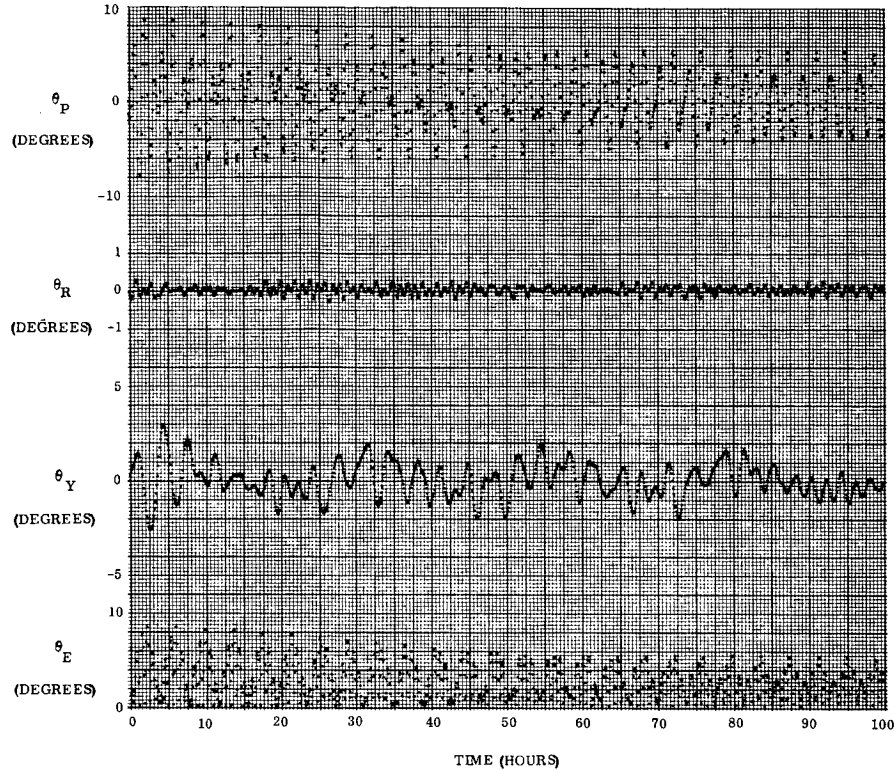


Figure 3-28. Steady-State Pointing Performance ($\epsilon = 0.0358$)

four-rod configuration is well balanced for solar pressure torques, the effects of orbit plane drift will be small. This result is confirmed by the digital simulation results.

Orbit plane drift positions of 30 degrees, 60 degrees, and 90 degrees from sun-synchronous were evaluated. The digital simulations are 100 hours long and are shown in Figure 3-30, 3-31, and 3-32. As in all of the steady-state runs, the error amplitudes are still decaying at 100 hours; however, the final amplitudes can be estimated accurately from the results of the 200-hour eccentricity ($\epsilon = 0.0178$) simulation of Figure 3-27. The orbit plane drift runs were made at the same eccentricity as Figure 3-27, which is the specified worst-case eccentricity of 150 nautical miles between apogee and perigee. Comparing the amplitudes of Figures 3-30, 3-31, and 3-32 with those of Figure 3-27 over the first 100 hours, it can be seen that the pitch and yaw errors are virtually identical. The only effect of orbit plane drift appears in roll error for a 60-degree

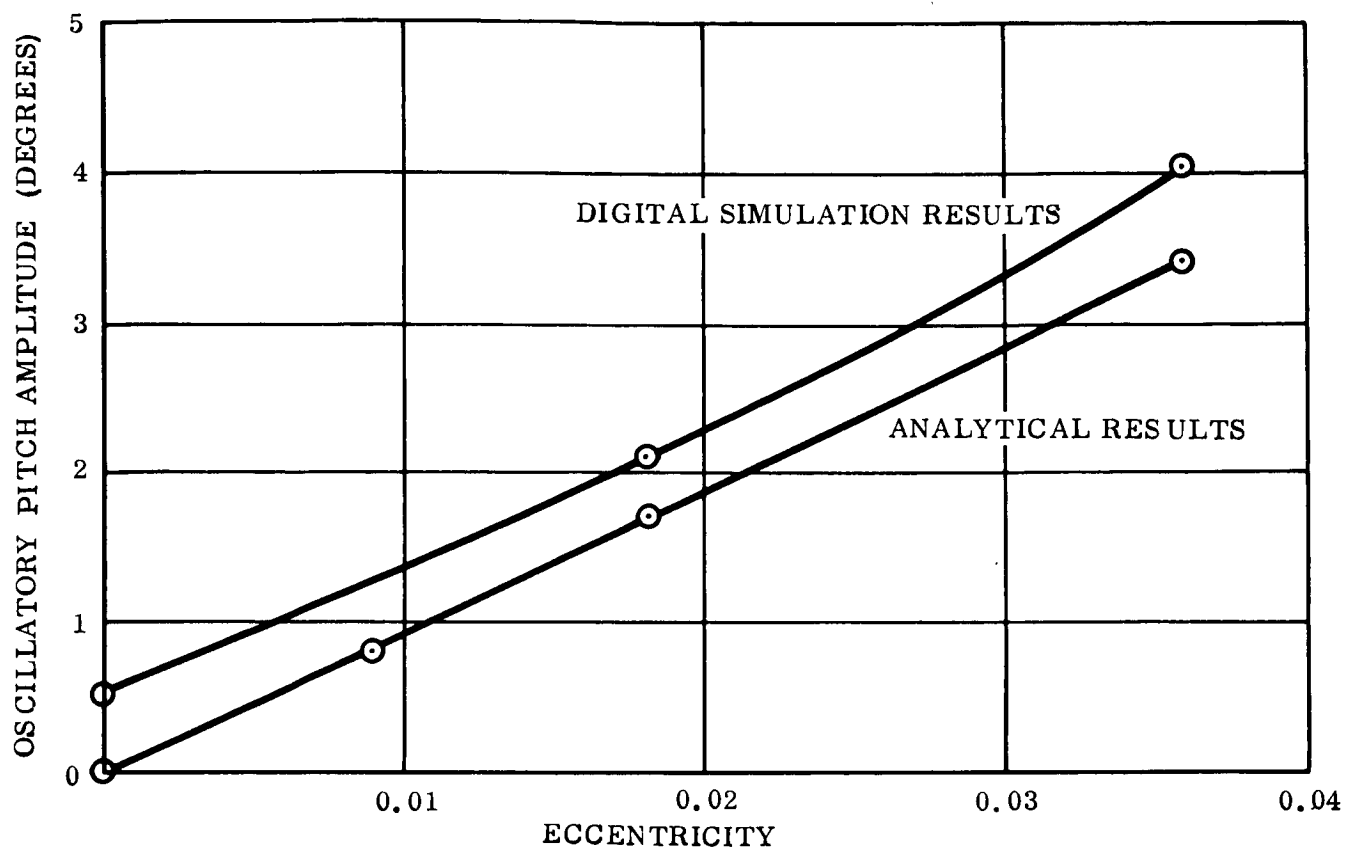


Figure 3-29. Pitch Amplitude vs Eccentricity

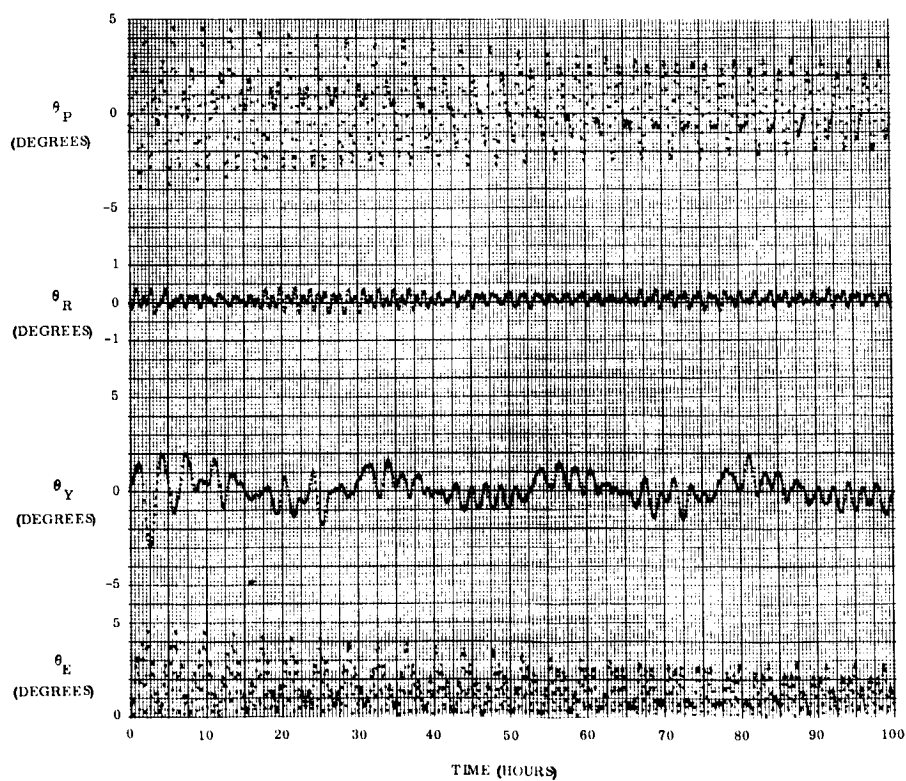


Figure 3-30. Steady-State Pointing Performance (30-Degree Orbit Plane Drift)

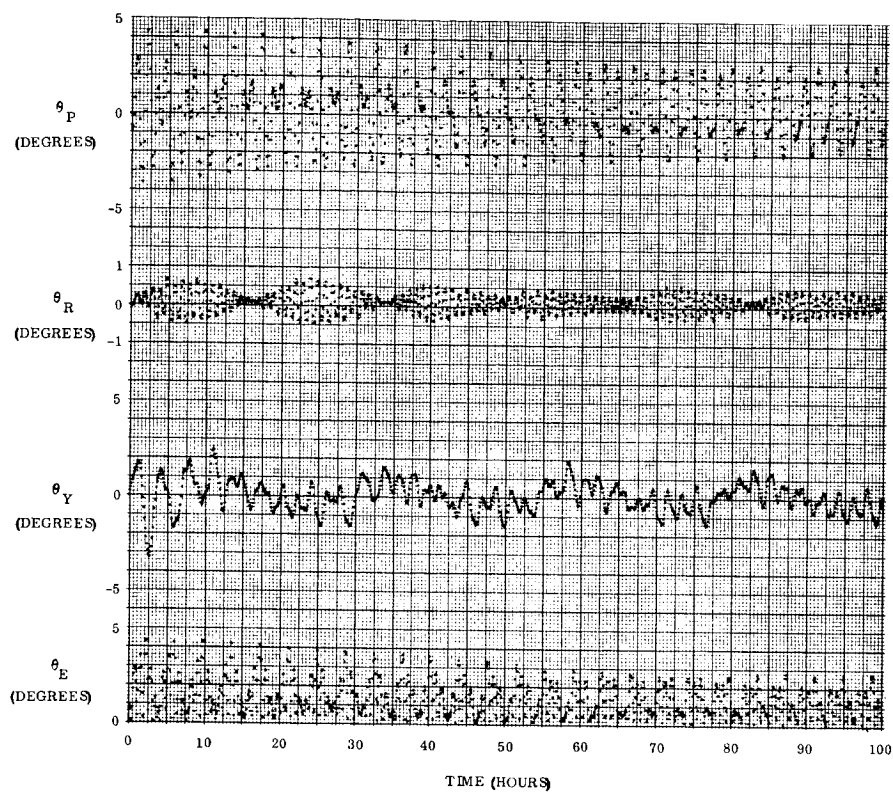


Figure 3-31. Steady-State Pointing Performance (60-Degree Orbit Plane Drift)

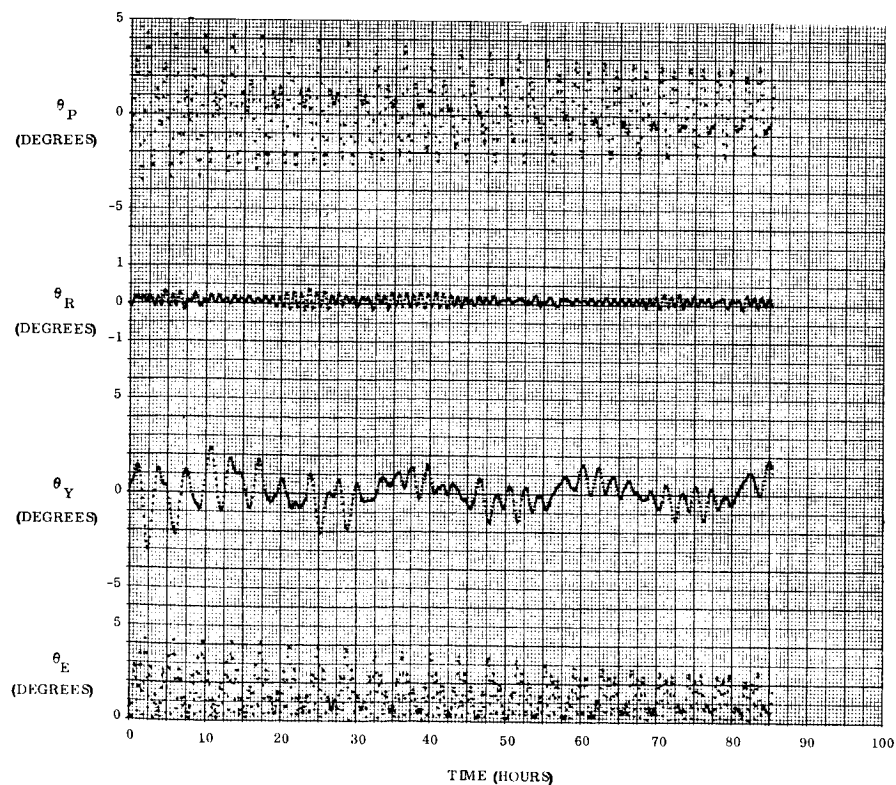


Figure 3-32. Steady-State Pointing Performance (90-Degree Orbit Plane Drift)

orbit plane drift from sun-synchronous position. A plot of roll error against orbit plane is shown in Figure 3-33. The peak steady-state roll error at 60-degree orbit plane drift is 0.6 degree.

3.5.3.3 Performance with Uncompensated Momentum.

The effects of uncompensated internal momentum on the steady-state pointing accuracy are discussed in this section. The momentum profiles analyzed are, 25×10^{-5} foot-pound-seconds constant momentum along the spacecraft pitch axis and 25×10^{-4} foot-pound-seconds momentum for 4 minutes per orbit with worst-case phasing. The conclusion from this investigation is that the uncompensated internal momentum has no appreciable effects on steady-state pointing accuracy.

The effects of the constant momentum along the spacecraft pitch axis have been evaluated by analytical techniques. This constant momentum produces torques only on the spacecraft roll and yaw axes, and only due to the effect of body rates about these axes. The torques are given by

$$T_{\text{pitch}} = 0$$

$$T_{\text{roll}} = H_{\text{pitch}} \omega_{\text{yaw}}$$

$$T_{\text{yaw}} = -H_{\text{pitch}} \omega_{\text{roll}}$$

where:

$$H_{\text{pitch}} = \text{uncompensated momentum}$$

$$\omega_{\text{roll}} = \text{body rate about roll axis taken with respect to inertial space}$$

$$\omega_{\text{yaw}} = \text{body rate about yaw axis.}$$

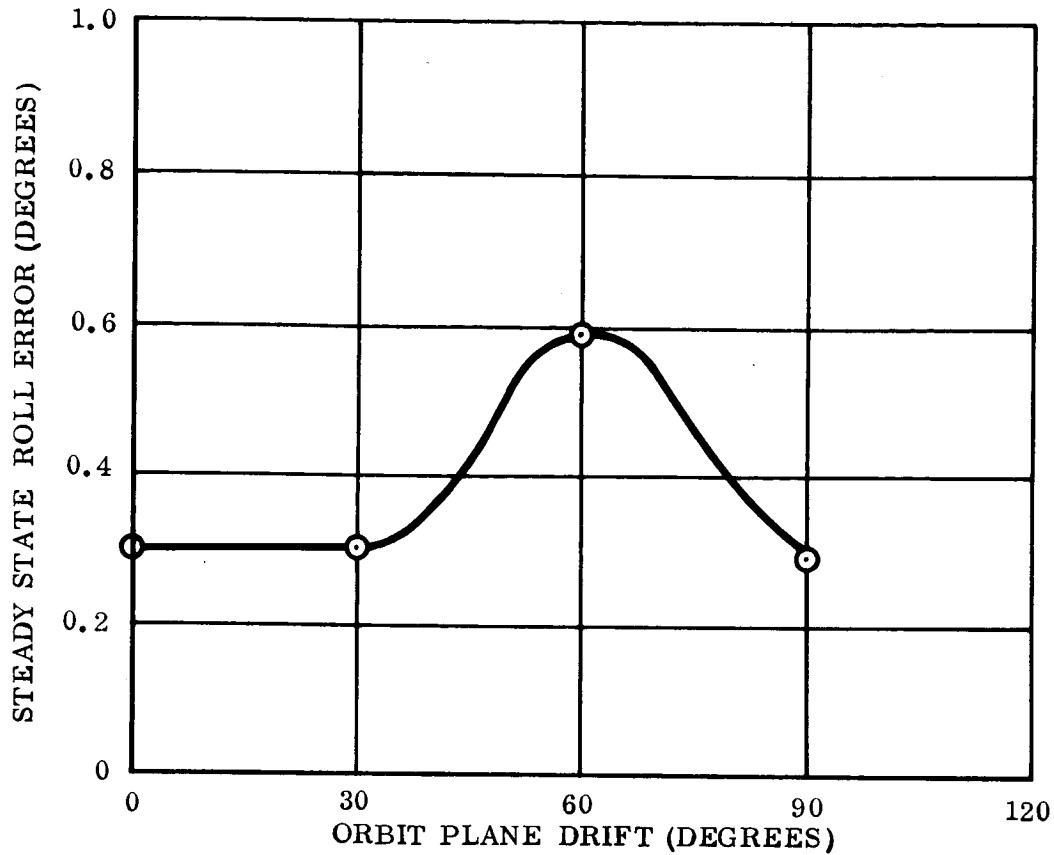


Figure 3-33. Steady-State Roll Error vs Orbit Plane Drift

The equations of motion for a rigid body with gravity-gradient and momentum torques are derived in Section 4.7. These equations are:

$$I_R \ddot{\theta}_R + \omega_o \dot{\theta}_Y (I_Y - I_P + I_R) + 4\omega_o^2 \theta_R (I_P - I_Y) = T_R = H_{\text{pitch}} \omega_{\text{yaw}}$$

$$I_Y \ddot{\theta}_Y + \omega_o \dot{\theta}_R (-I_Y + I_P - I_R) + \omega_o^2 \theta_Y (I_P - I_R) = T_Y = -H_{\text{pitch}} \omega_{\text{roll}}$$

A comparison of the magnitude of the momentum torque terms with the coupling torque terms on the left side of the equations shows that the momentum torque terms are much too small to cause any appreciable change in attitude performance. The steady-state body rates and Euler angles rates are less than 0.005 degree per second during steady-state conditions.

The magnitude of the momentum-caused torques will be less than:

$$T = H_{\text{pitch}} \omega_{\text{body}} \\ = (25 \times 10^{-5}) \left(\frac{0.005}{57.3} \right) = 2.18 \times 10^{-8} \text{ (ft-lb)}$$

The gravity-gradient and coupling torques present during steady-state conditions (approximately 1.6 degree oscillation amplitude) are:

$$T_{\text{gravity-gradient}} = \omega_o^2 (I_P - I_R) \\ = (0.923 \times 10^{-3})^2 (3000 - 2800) \\ = 1.7 \times 10^{-4} \text{ (ft-lb)}$$

$$T_{\text{coupling}} = \omega_o \dot{\theta} (I_Y - I_P + I_R) \\ = (0.923 \times 10^{-3}) \left(\frac{0.005}{57.3} \right) (575 - 3000 + 2800) \\ = 3 \times 10^{-5}$$

Since the restoring torques which are present during steady-state operation at amplitudes of 1.6 degrees are three orders of magnitude greater than the disturbance torques due to the constant component of uncompensated momentum, it has been concluded that this momentum will result in no appreciable disturbance effect.

The 4- minute pluse of uncompensated momentum will cause torques about all three spacecraft body axes. The torques about the spacecraft roll and yaw axes will be 10 times as large as the torques computed above, since the momentum is 10 times as large. This effect is still two orders of magnitude below the restoring torques and causes no appreciable effect.

The torques on the pitch axis due to changing momentum have been included in the nonlinear digital simulation in order to evaluate the resulting pitch motion. The torque profile which is repeated once each orbit is shown in Figure 3-34.

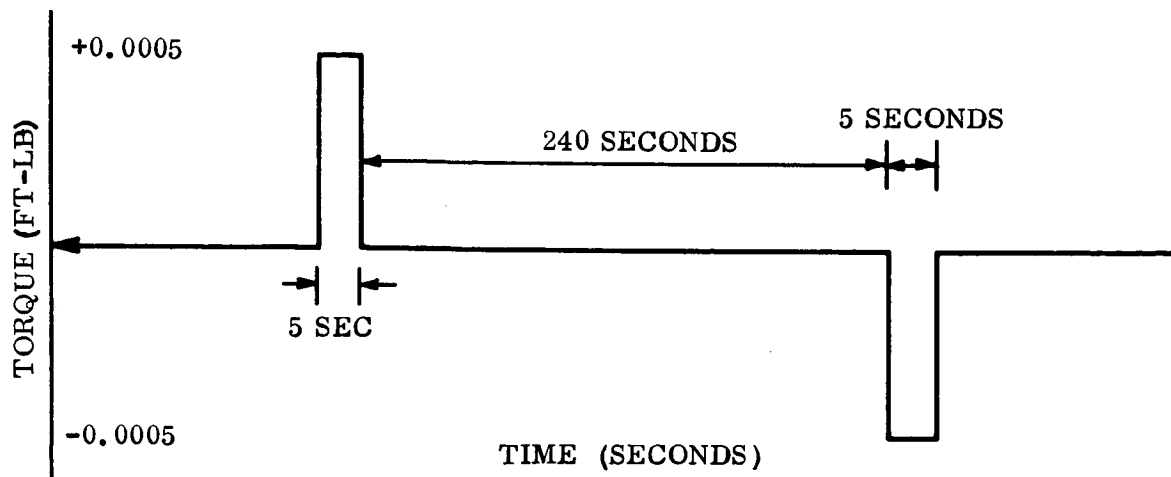


Figure 3-34. Torque Profile for Uncompensated Momentum

The nominal torque profile shown in Figure 3-34 was added to the other torques computed in the nonlinear digital simulation at three phase positions with respect to the basic pitch oscillation at orbital frequency. The midpoint of the torque profile (125 seconds after start of positive torque) was positioned at perigee, and at 45 degrees and 90 degrees of orbit angle after perigee. These simulations were made at the worst-case eccentricity of $\epsilon = 0.0179$. The results of these simulations are shown in Figures 3-35, 3-36, and 3-27. Comparison of these figures with each other and with the nominal performance without momentum disturbance shown in Figure 3-19 indicates that the pitch errors are all virtually the same at 80 to 90 hours.

The simulation with pulse torque at 45 degrees of orbit angle from perigee was repeated with pulse torque values of five times and 25 times the nominal specified value. The results of these simulations are shown in Figures 3-38 and 3-39, respectively. Figure 3-38, with five times the torque level, shows about 0.1 degree larger pitch error at 80 hours. Figure 3-39, with 25 times the torque level, covers 11 hours and shows pitch errors of 4.8 degrees compared to 4.8 degrees at the same time for the nominal torque case.

This investigation shows that uncompensated momentum along the pitch axis, both constant and pulsed, can be at least an order of magnitude larger than those specified without significantly degrading pointing accuracy.

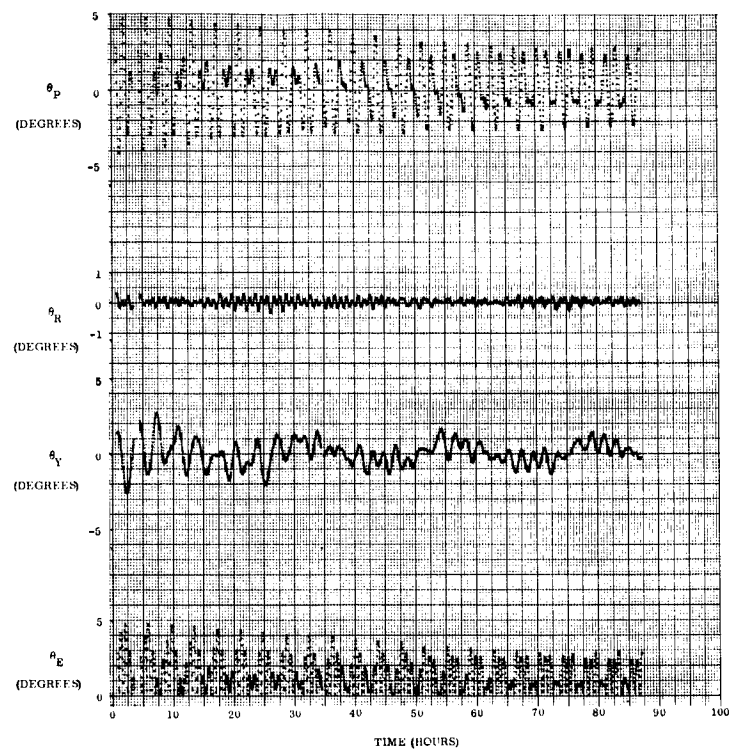


Figure 3-35. Momentum Pulse at Perigree

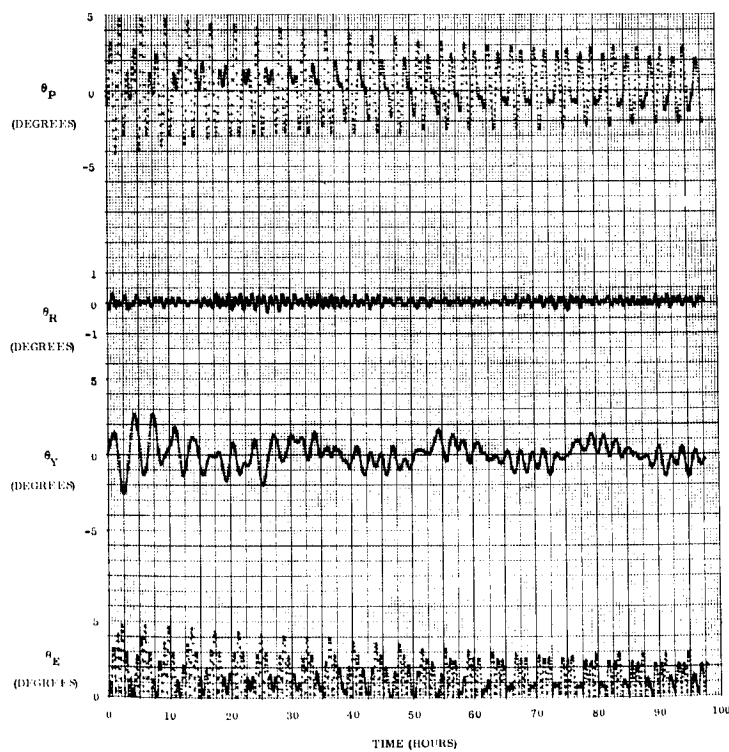


Figure 3-36. Momentum Pulse at 45 Degrees After Perigree

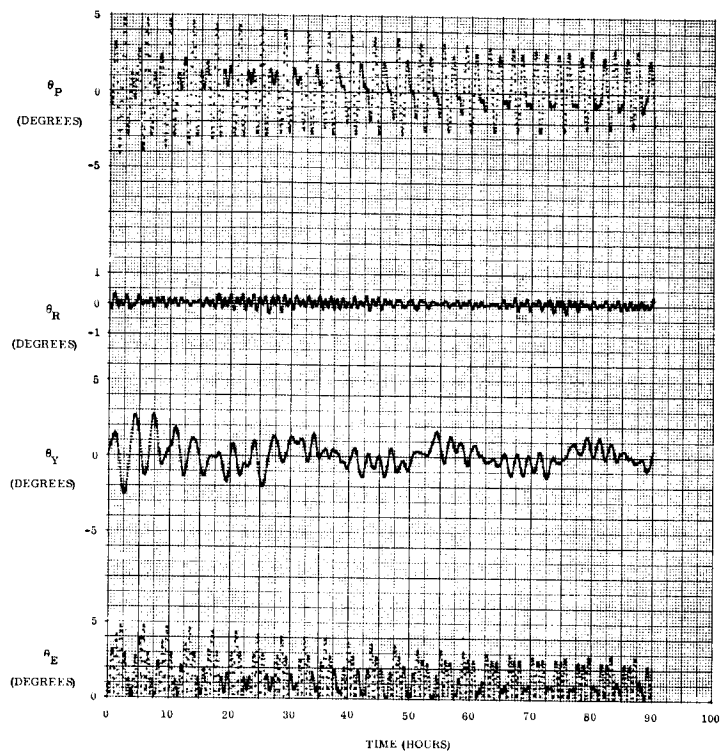


Figure 3-37. Momentum at Pulse 90 Degrees After Perigree

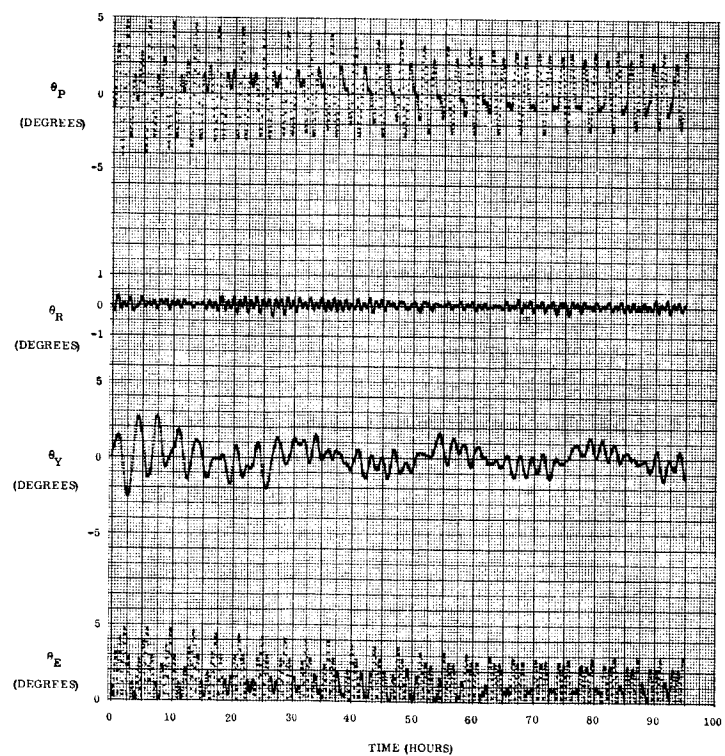


Figure 3-38. 5 Times Momentum Pulse at 45 Degrees After Perigree

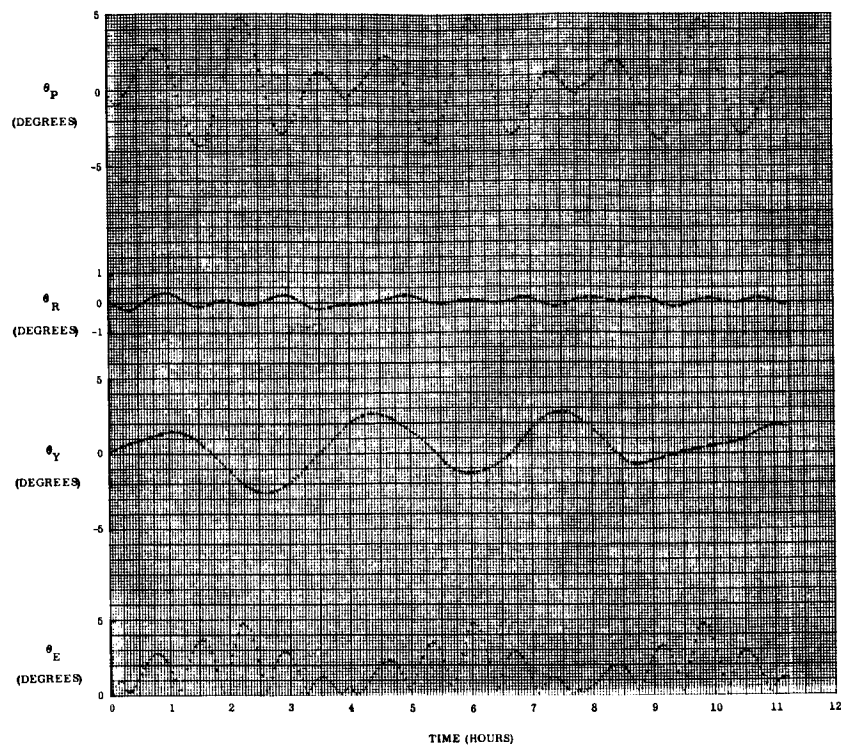


Figure 3-39. 25 Times Momentum Pulse at 45 Degrees After Perigree

SECTION 4

RATIONALE FOR SELECTED CONFIGURATION

4.1 GENERAL

The selected spacecraft and attitude control configurations presented in Section 3 were evolved as the results of tradeoff analyses conducted during the study. The results of these analyses and the selection rationale are summarized in this section.

The spacecraft configuration selection process, presented in Sections 4.2 and 4.3, consists of obtaining an arrangement of RTGs and the NIMBUS sensory ring which is compatible with the constraints of the gravity-gradient rod configuration, thermal design, and nuclear safety requirements.

The rationale for selection of the magnetically anchored eddy-current damper and the four rod gravity-gradient stabilization subsystem is presented in sections 4.4 and 4.5, respectively. All of the types of dampers which are presently under development for use with gravity-gradient systems are considered in Section 4.4. Gravity-gradient configurations are chosen for each type of damper and performance and weight estimates made. A selection criterion is developed based on specified objectives and the reasons for selecting the spherical eddy-current damper are detailed. The advantages of three-, four-, and six-rod configurations are compared in Section 4.5. It is concluded that a four-rod configuration offer the best combination of low solar pressure torques, minimum number of rods, and minimum weight. An optimization study to select the four-rod configuration which minimizes steady-state pointing errors is also presented in this section.

An analysis of all significant disturbance torques and the expected pointing errors which they cause is presented in Section 4.6.

An evaluation of the time from separation until capture for the specified separation conditions and an analysis of the range of separation conditions for which no pitch tumble will occur are presented in Section 4.7.

4.2 SPACECRAFT CONFIGURATION SELECTION

Several basic configurations were considered for combining the SNAP-19 RTGs with the NIMBUS sensory ring: (1) the RTGs can be separately mounted directly to the sensory ring, (2) they can be mounted as a single unit on a structure above the ring, or (3) they can be deployed from the ring after orbital insertion. Several variations of these techniques were considered, and nine of the most promising configurations are presented in Table 4-1 along with a summary of tradeoff considerations.

Configurations 1 and 2 are similar except in the mounting height of the RTGs above the sensory ring. These two configurations and configurations 6 and 9 are the most acceptable of the nine presented. Configuration 1 was selected as the basic configuration for this study. It offers the simplest interface between the RTGs and the sensory ring and eliminates mechanical and electrical complexities required of a deployable configuration. Furthermore, it offers flexibility in that variations in RTG size, number of units, or changes in nuclear safety requirements can be accommodated without invalidating the results of this study.

A particular concern in selecting configurations for this application was the aerospace nuclear safety requirement of re-entry burnup of the SNAP-19 radioisotope fuel. The configurations considered in Table 4-1 will meet this requirement, as each is provided with an ejection mechanism, or separation and deployment mechanism, which will assure that the RTGs will be free from the spacecraft and available for re-entry burnup in the case of an in-orbit abort or at the end of a useful mission.

Deployment was considered for two reasons: first, because deployed generators could add to the stabilization of the spacecraft and, second, to expose the RTGs to re-entry heating. However, separation and deployment introduces design complexity and extensive testing. For these reasons configurations 6, 7, 8 and 9 were eliminated from further consideration during the course of the tradeoff study.

1. <u>SPACECRAFT</u>					
a. Attitude Control	Two dampers, four rods, unsymmetrical solar pressure	Two dampers, four rods, S.P. symmetry worse than 1	One damper, four rods, S.P. symmetry good	Sa	
b. Thermal Effect Rods	Effect on upper rods may be significant.	Less influence than 1	Symmetrical heating of upper rod	Sa	
Sensory Ring	Will require conduction isolation & possible redesign of existing insulation	Long conductive path Minimal effect on sensory ring	More heat conducted to sensory ring than 1	Si bu on	
c. Structural	Short load path, relatively light support structure	Long load path, high CG; will be active in vibration	Lowest structure ring loads Restricts access to ring	Si bu in	
d. Moment of Inertia					
(Slug-Ft ²) Roll	40	124	39		
Pitch	40	124	39		
Yaw	56	56	65		
C. G. (from Base of Sensory S/S)	10"	20"	9"		
2. <u>RTG</u>					
a. Design	Thermal blockage, particularly of middle RTG RTG case in structure load path	Same as 1, except reduced thermal blockage Higher dynamic amplification than 1	Low thermal blockage Low dynamic amplification	S R lo In th	
b. Separation (Safety) and Handling	Single separation Three units can be handled as one.	Same as 1 Separation may not be required.	Multiple separation Individual RTG handling required	S	

Notes:

- (1) The orbit plane is in the plane of the paper.
- (2) The plan view of each configuration depicts the circumferential location of the gravity. The plan view of configuration (2) is the same as (1).

Same as 2	Same as 4	Two dampers, three rods, S. P. symmetry good	One damper, four rods, S. P. symmetry good	Two dampers, three rods, S. P. symmetry good	One damper, one rod, unsymmetrical solar pressure
Same as 2	Same as 4	Very small influence on rods	Nonsymmetrical heating of lower rods	Very small influence on rods	Very small influence on rods
Similar to 2, but more effect on sensory ring	Same as 4	Same as 1 prior to extension No thermal effects after extension	Restricts sensory ring heat rejection. May increase coating degradation.	Same as 3 prior to extension No thermal effects after extension	Similar to 7, but to a lesser degree
Similar to 2, but less active vibration	Same as 4	Same as 1, but will require deployment development	Same as 3, but will require deployment development	Same as 7	Similar to 2, but possibly more active and requires deployment development
		After 18' <u>Before</u> <u>Ext</u> 42 1030 40 1030 56 56 10" 169"	After 18' <u>Before</u> <u>Ext</u> 38 57 38 57 62 79 9" 11"	After 18' <u>Before</u> <u>Ext</u> 39 179 39 179 65 128 9" 198"	After 6-1/2' <u>Before</u> <u>Ext</u> 42 172 40 172 56 330 10" -4"
Same as 2 RTGs not in ad path Increased thermal blockage	Same as 4	Same as 1, plus long electrical conduction path	Low thermal blockage	Minimum thermal blockage Long electrical conduction path	Same as 8, plus high dynamic amplification
Same as 1	Same as 1 RTGs could be handled individually.	Same as 1 Extension rather than separation	Same as 3 Extension rather than separation	No separation - triple extension RTGs handled individually	No separation - single release Complexed RTG handling

rods (indicated by dashed arrows).

Table 4-1. Comparison of Alternative Configurations

The configurations illustrated in Table 4-1 were reviewed with The Martin Company's SNAP-19 Program Manager. From this review it was concluded that dynamic amplification to the generators would have to be minimized through proper structure design and vibration isolation, but that all configurations presented were feasible from the SNAP-19 design standpoint. However, if the generators are to be grouped, configurations 1, 2, and 6 were preferable to 4 and 5 because the latter two would result in higher thermal blockage to the generators (i.e., reduced view factor for heat dissipation).

Heat rejection for the series assemblies shown in configuration 1, 2, and 6 could be improved by the addition of spacers between the RTGs. The SNAP-19 Program Manager did not consider this to be a major problem.

4.3 OPTIMIZATION OF SPACECRAFT TO RTG SEPARATION DISTANCE

The selected configuration, configuration 1 of Table 4-1, features the series mounting of the three SNAP-19 units on a space structure above the NIMBUS sensory ring. An analysis was conducted to optimize the height of this structure on a thermal and structural basis. From a thermal standpoint, increasing the height of the structure reduces the thermal conduction to the sensory ring. From a structural standpoint, increasing the height will increase structure flexibility and hence will cause greater dynamic responses. Optimization of the support truss height, therefore, is a tradeoff between thermal and structural requirements. The approach taken was to determine the minimum truss height compatible with thermal requirements and evaluate it for structural adequacy.

Thermal optimization was based on an analysis of the heat conduction which would be acceptable to the sensory ring thermal control subsystem from which tube sizes were chosen. The RTG support structure will conduct heat into the sensory ring at the three truss tube fittings. An allowable conduction leak was established by considering that the NIMBUS thermal control system is designed to dissipate to space a maximum total of 280 watts at 25°C. The heat rejection capacity of any bay depends on the solar incidence angle to that bay, with a minimum total of 11.5 watts per bay. This minimum capacity is reduced

by 25%, to 8.6 watts, to account for end-of-life degradation. The study specification stipulates a uniformly distributed heat load of 100 watts, or 5.5 watts per bay. Therefore, the permitted conduction leak into the sensory ring at bays under truss tube fittings is the difference between the 8.6 watts per bay capability and the 5.5 watts per bay requirement, or 3.1 watts per bay. Since two truss tubes meet at each fitting, it is assumed that the heat conducted through them will be dissipated by the two bays on each side of the fitting. Hence, each tube must conduct no more than 3.1 watts.

Tube conduction is controlled by two basic features: (1) interface resistance between tube ends and fittings and (2) tube cross section, length, and material. Interface resistance is difficult to predict or measure and is therefore neglected. This results in a conservative estimate of heat leakage, since the additional thermal resistance of joining surfaces will restrict the heat flow and add to the margin-of safety. Several materials were selected for thermal evaluation, for possible use as truss tubes. These materials are:

- a. Titanium alloys.
- b. Steels (302, 303, 304, 310, 321, and 347 stainless).
- c. Nickel alloys (Inconel X, Hastelloy A).
- d. Aluminum alloy (2024).

Figure 4-1 indicates the heat conducted to the sensory ring for each of these materials as a function of cross-sectional area and length. Many tube sizes can be chosen to limit thermal conduction to the sensory ring to a maximum value of 3.1 watts. An aluminum tube of 0.2-square-inch cross-sectional area was selected, since similar tubes are used on NIMBUS A, B, and C to support the active control subsystem. Based on this tube material and cross-sectional area, tube lengths must be at least 23 inches. An 18-inch separation distance between the sensory ring and RTG was selected. This results in a tube length of approximately 25 inches.

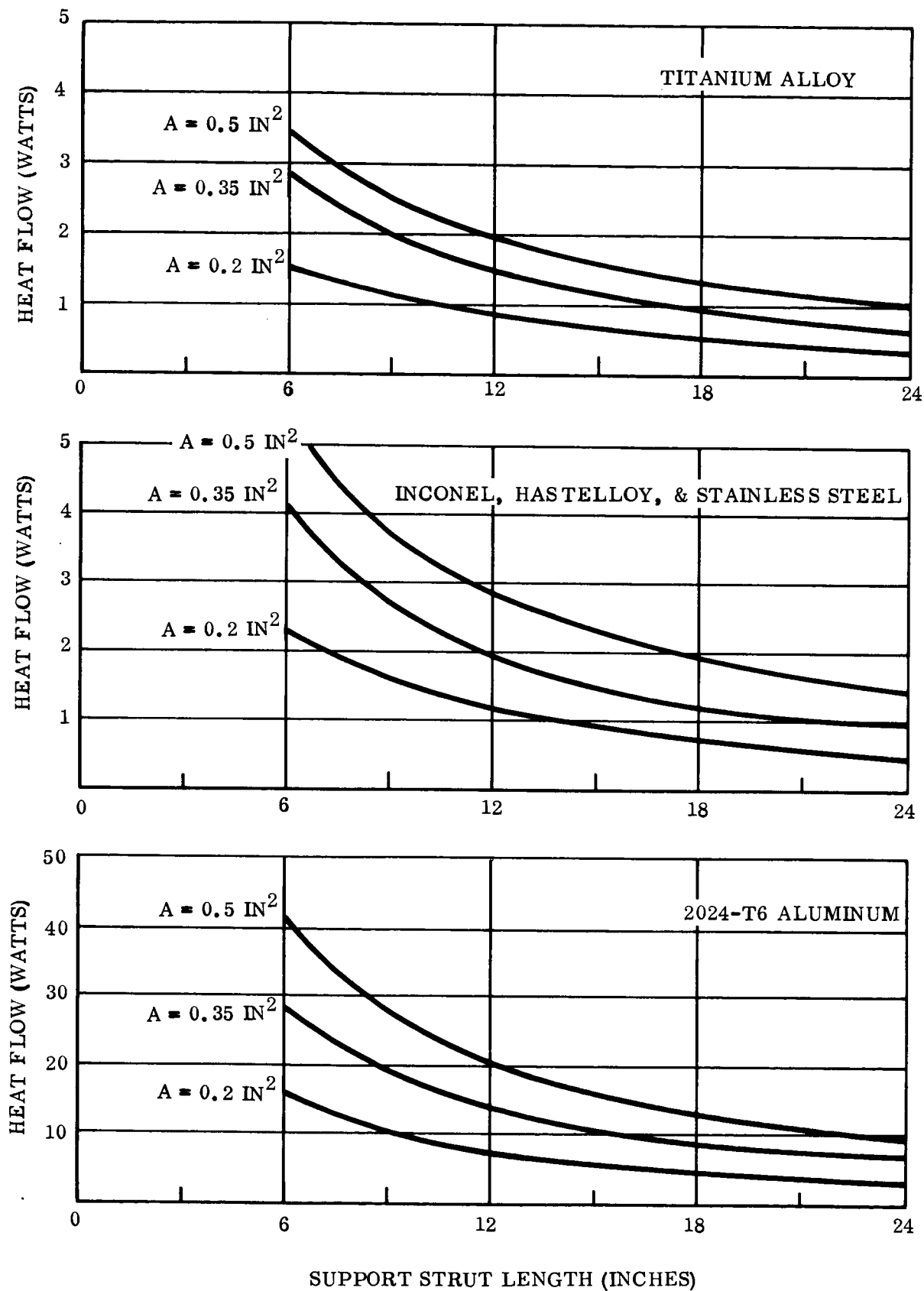


Figure 4-1. Heat Conduction in RTG Mount (Per Strut)

Structural analysis of the RTG height above the sensory ring was performed with truss tube weight and RTG dynamic response as parameters. Since the truss tube weight is a very small portion of the total vehicle weight, it is not a significant criterion for selecting an optimum separation distance. Therefore, only a preliminary analysis was made of truss tube weight variation under thrust load condition. Twenty inches was the optimum standoff distance, which compares well with the 18-inch thermal height and indicates that the tubes are used efficiently.

Dynamic response optimization of truss height cannot be analytically performed, because much depends on mode shapes and the all-important damping factors. These are experimentally determined parameters, which are used in analyses and are not directly proportional to any geometric dimension. The one major objective that is sought in a 2-degree-of-freedom system representing NIMPAC is to avoid coupling between the two masses. One truss height may be as good as another with respect to noncoupling, depending on the choice of parameters used in the analysis. Hence, the thermally selected 18-inch truss height configuration was studied for coupling. Figure 4-2 shows the response of the upper mass in a 2-degree-of-freedom system (corresponding to the RTGs of NIMPAC) expressed as a ratio of mass acceleration to input acceleration for a range of mode shape parameters expressed as functions of displacements of the upper and lower masses. Strong coupling of the upper mass is evident at a mode shape parameter of about 3. For the selected configuration, the first thrust mode shape parameter is about 104, which is well removed from the strongly coupled region. Hence, the truss height is acceptable, since it does not produce coupling between the RTGs and the sensory ring. The 104 factor is based on a first thrust mode g loading of 15.6 g for the RTGs and 0.15 g for the sensory ring, as illustrated in Table 3-3.

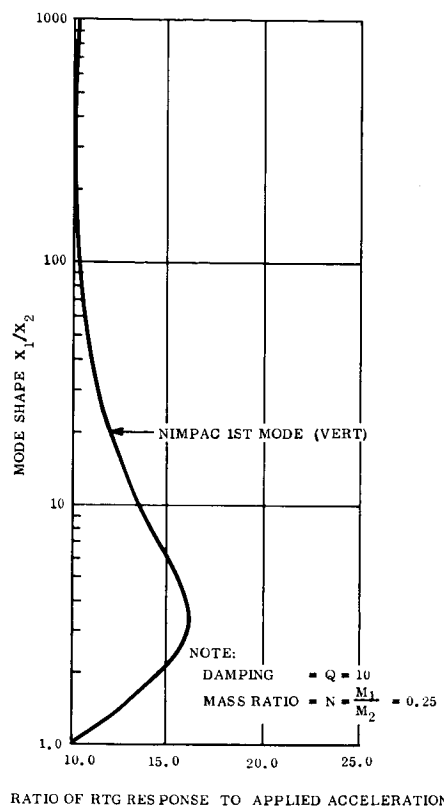


Figure 4-2. Variation in Peak Response of RTGs as a Function of Mode Shape

4.4 SELECTION OF PASSIVE ALTITUDE CONTROL DAMPING TECHNIQUE

A survey has been made of the various damping techniques which would be applicable to a passively stabilized NIMBUS. Four types of dampers were selected for consideration and a passive stabilization configuration selected for each damper type. The characteristics of these configurations were evaluated against mission requirements, and the magnetically-anchored eddy-current damper was selected for this application. The rationale for selection of the damping configuration is presented in the following discussion. Only dampers which have been designed and are under development have been considered for this study. This constraint leaves the following six types of dampers for consideration.

- a. Magnetically-anchored eddy-current damper.
- b. Magnetically-anchored viscous-fluid damper.
- c. Single-axis gravity-anchored eddy-current damper.

- d. Two-axis gravity-anchored eddy-current damper.
- e. Single-axis gravity-anchored magnetic-hysteresis damper.
- f. Two-axis gravity-anchored magnetic-hysteresis damper.

The magnetic-hysteresis dampers are eliminated from further consideration because of poor damping performance over a wide range of oscillation amplitudes. This occurs because the hysteresis energy loss during each cycle of operation is proportional to amplitude and not rate and gives damping which is too high at low amplitudes and too low at high amplitudes. For example, in comparison to an eddy-current damper, the damping times from large angles to steady state would be approximately 50% to 100% longer. The hysteresis damper is lighter than the eddy-current damper due to the small amount of hysteresis material required compared to the copper eddy-current conductor. This weight difference is of little importance compared to the difference in damping performance.

4.4.1 DAMPER DESCRIPTION

Typical design layouts of the remaining four types of dampers are shown in Figure 4-3, 4-4, and 4-5. The two-axis gravity-anchored eddy-current damper is basically a pair of single-axis units and is similar in layout to the illustrated in Figure 4-5. A discussion of the characteristics of each damper type is presented below.

4.4.1.1 Magnetically-Anchored Eddy-Current Damper

The spherical eddy-current damper is of the same design as that successfully flown in early 1965 on the gravity-gradient stabilization system of a Naval Research Laboratory satellite and on the NASA GEOS satellite launched in October 1965. The damper (Figure 4-3) consists of an outer spherical shell assembly and a loose magnet assembly free to rotate within the sphere. The magnet assembly consists of six individually cast Alnico-5 bar magnets joined to a common center fitting. Plastic tips are bonded to the ends of the magnets to prevent metal-to-metal contact during launch accelerations.

The sphere assembly consists of three concentric shells, a structural shell of aluminum on the outside, a copper shell on the inside, and a shell of pyrolytic graphite sandwiched between the other two. In orbital operation, the sphere assembly is rigidly fixed to the satellite, by means of the extended rod, and the magnet rotates within the sphere to maintain alignment to the earth's magnetic field. Oscillatory motion of the satellite causes relative motion between the magnet assembly and the sphere, resulting in the generation of eddy currents within the copper shell. The repulsive force between the magnet assembly and the diamagnetic shell produces a stable suspension for the magnet, which is strong enough to prevent contact between the magnet and sphere during orbital operation. Thus only rate proportional damping torques occur. The damping proportionality constant is 0.015 ft-lb-sec for a 12-pound flight-proven type of damper. Higher damping constants (as much as 0.03 ft-lb-sec) can be achieved by incorporating appropriate design changes.

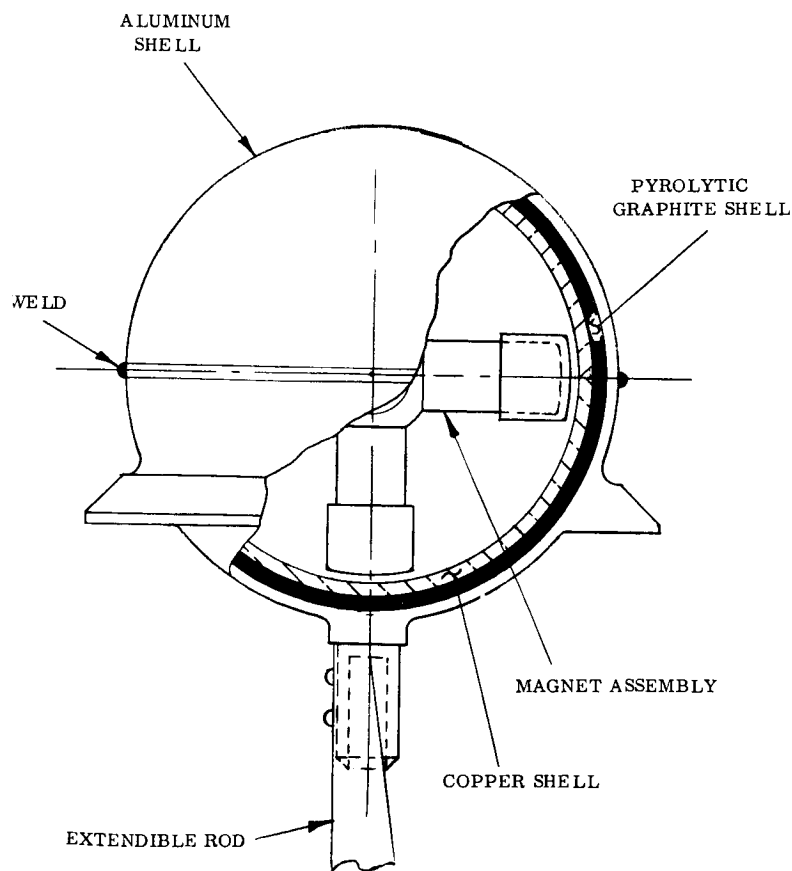


Figure 4-3. Magnetically-Anchored Eddy-Current Damper

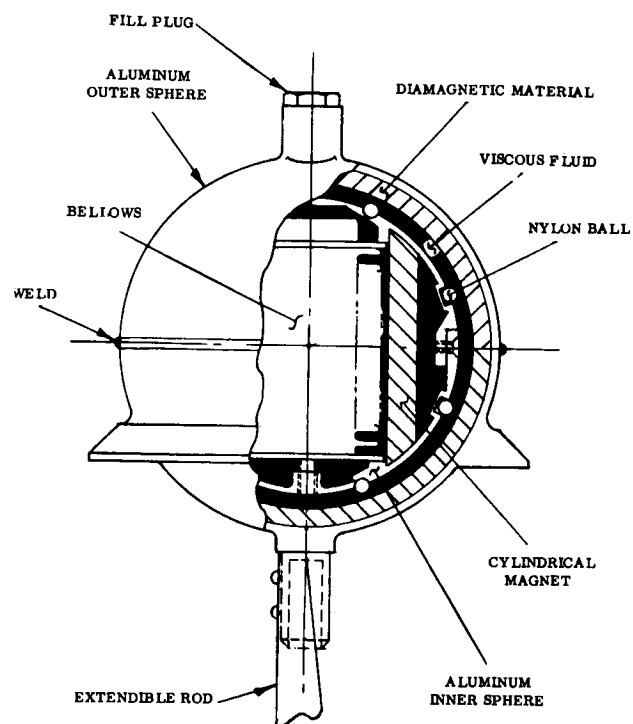


Figure 4-4. Magnetically-Anchored Viscous Damper

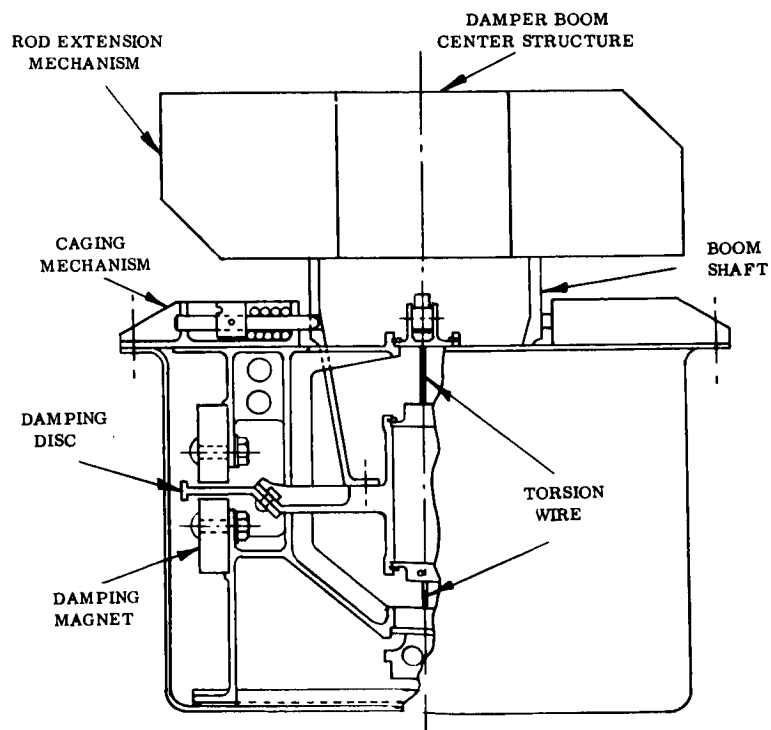


Figure 4-5. Single-Axis Eddy-Current Damper

4.4.1.2 Magnetically-Anchored Viscous Fluid Damper

The spherical viscous fluid damper is also of a design similar to that successfully flown in early 1964 on a Naval Research Laboratory satellite. The damper (Figure 4-4) consists of an inner sphere assembly and an outer sphere, separated by a spherical gap which is filled with viscous silicone fluid. The inner sphere assembly consists of a hollow cylindrical magnet and a bellows mounted in a spherical shell. The outer sphere is a structural aluminum housing with a pyrolytic graphite lining. The inner sphere magnet aligns the inner sphere to the earth's magnetic field and also provides a stable centering force suspension by means of the repulsive forces from the diamagnetic graphite lining. The bellows assembly maintains a nearly constant pressure of the trapped viscous fluid as volumetric changes occur with temperature. Oscillations of the satellite cause relative motion between the inner and outer spheres with resulting rate damping torques due to viscous shear in the silicone fluid. A damper of this type exhibits considerably higher maximum values of damping coefficient per pound of damper weight than the spherical eddy-current damper because of the high viscosity available from the silicone fluid. The weight limit is fixed by the magnet size needed to maintain alignment to the earth's magnetic field. A maximum damping coefficient of 1.0 ft-lb-sec can be obtained with a 12-pound damper.

4.4.1.3 Single-Axis Eddy-Current Damper

The single-axis eddy-current damper is used with one or two extendable rods to form a unit which provides rate proportional damping torques and a secondary body of high inertia. This secondary body inertia is selected as part of the system design to assure relative motion across the damper. The single-axis damper design must provide for freedom of rotation about one axis with rotational constraint about two axes and linear translation constraint in three axes. A restoring spring rate and damping torques about the rotation axis are also required. Suspension of the rotating member may be accomplished by torsion wire or by diamagnetic forces between a pyrolytic graphite shell on the rotating member and a magnet assembly on the fixed member. A typical design layout with torsion wire suspension is shown in Figure 4-5. The torsion wire provides rotational freedom with the desired spring rate and constraint of undesired motion.

The damping torques are obtained as reactions from eddy currents induced in the conducting disc by the relative rotation. The damper weight, excluding rod units, is 5 pounds. The maximum damping coefficient attainable is 0.03 ft-lb/rad/sec.

4.4.1.4 Two-Axis Eddy-Current Damper

The two-axis eddy-current damper is basically a pair of single-axis dampers and has identical design requirements for each axis. A damping level of 0.03 ft-lb/rad/sec about each axis can be obtained in a 10-pound unit, excluding rods.

4.4.2 CONFIGURATION FOR DAMPING EVALUATION

A passive attitude control configuration has been selected for each of the four types of dampers. Each of these configurations has been selected to best utilize the characteristics of the particular damper employed. Only one configuration has been selected for evaluation of the damping characteristics of the spherical eddy-current and the spherical viscous fluid damper. This has been done because of the similarity in damping characteristics of the two types of dampers. Although a three-rod configuration or a four-rod configuration can be used with either type of damper, as illustrated in Figures 4-6 and 4-7, only the four-rod configuration is considered in this evaluation to separate damper tradeoffs from rod configuration tradeoffs. The damping time from large angles to steady state and the errors due to damper motion are identical for the three- and four-rod configurations for a fixed value of damping coefficient with either type damper.

4.4.2.1 Magnetically Anchored Eddy-Current or Viscous-Fluid Configuration

This configuration employs four extendable rods with tip weights to achieve desired moments of inertia for gravity-gradient orientation, and two magnetically-anchored eddy-current or viscous-fluid dampers to damp the oscillatory motion of the spacecraft. The arrangement of the rods and dampers is shown in Figure 4-6. The two upper rods are 60-feet long, and the upper tip weights are 12 pounds each. The two lower rods are 80 feet long with no tip weights.

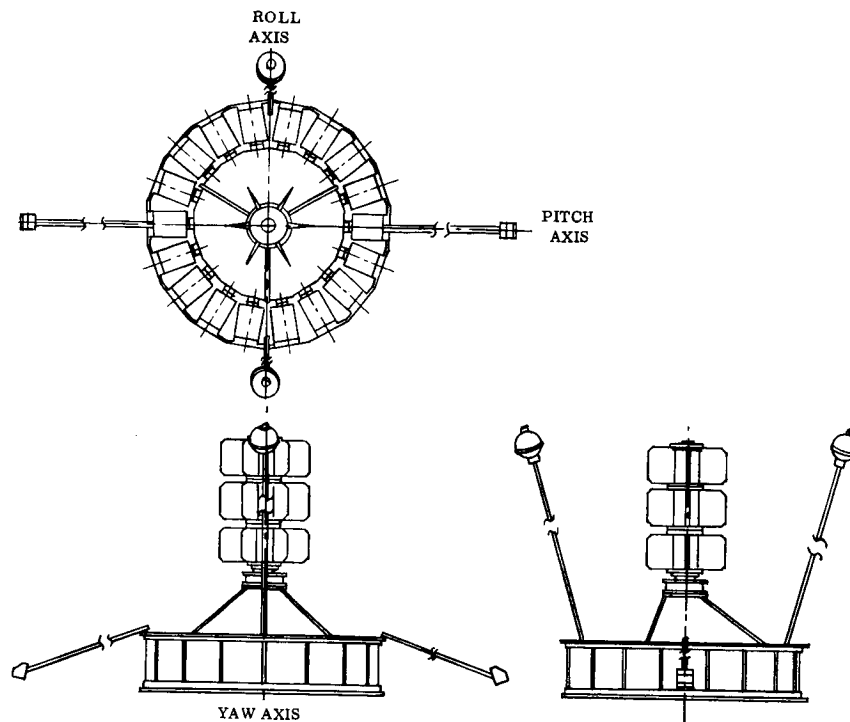


Figure 4-6. Magnetically-Anchored Eddy-Current or Viscous Damper (Four-Rod Configuration)

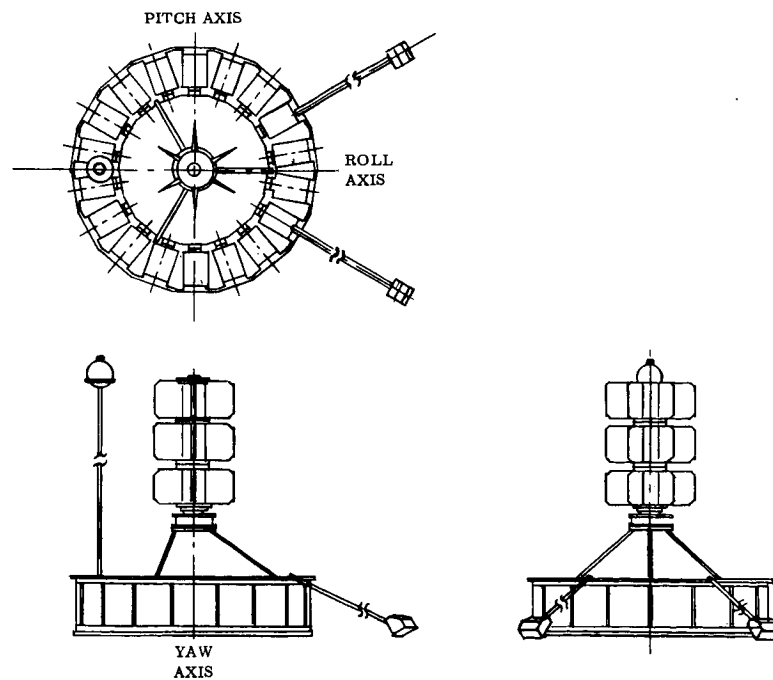


Figure 4-7. Magnetically-Anchored Eddy-Current or Viscous Damper (Three-Rod Configuration)

This combination of tip weights and rod lengths has been selected to give zero nominal solar pressure torque when the sun is on the roll or yaw axis. This is accomplished by balance of the areas of the rods and satellite above and below the total center of mass. A value of damping coefficient of 0.025 ft-lb-sec has been selected to provide low steady-state damper caused errors of 0.6 degree in pitch and 0.8 degree in yaw. The transient damping time for decay of large angle oscillations down to the steady-state pointing accuracy is 186 hours, (8 days), and the peak steady-state oscillation amplitudes are about 3 degrees in pitch, 1 degree in roll, and 3 degrees in yaw. Implementation of this gravity-gradient subsystem requires mounting four-rod extension units on the upper surface of the sensory ring, with clearance to extend in the directions shown. A command signal would initiate the deployment sequence. The estimated altitude weight control subsystem in this configuration is 40 pounds.

4.4.2.2 Gravity-Anchored Single-Axis Eddy-Current Damper Configuration

This configuration employs a total of five booms, three extended from the satellite body and two extended from the rotating side of the single degree-of-freedom rotating damper. A typical arrangement of the rods and damper is shown in Figure 4-8. The rod lengths, tip weights, and rod mounting angles are selected to provide moments of inertia of both the main body and damping boom, which results in low angular response to disturbance torques. It can be seen that the principal axes of the two bodies considered separately are at angles of 45 degrees and 20 degrees to the orbit reference axes. These angles are chosen to provide the greatest coupling of oscillations of main body about the orbit reference frame into oscillatory relative motion across the damper, thus minimizing decay time.

Implementation of this configuration requires the mounting of three rod units on the upper surface of the sensory ring with clearance to extend at the angles shown. The damper unit is mounted above the sensory ring and must be located to allow ± 45 degrees of rotation for the damper boom. The damper boom will penetrate the 70-degree cone during the initial time of decay from large angles but not during steady-state operation. Two opposing rod units are mounted on the rotor of the damper. A locking device to support the rotating

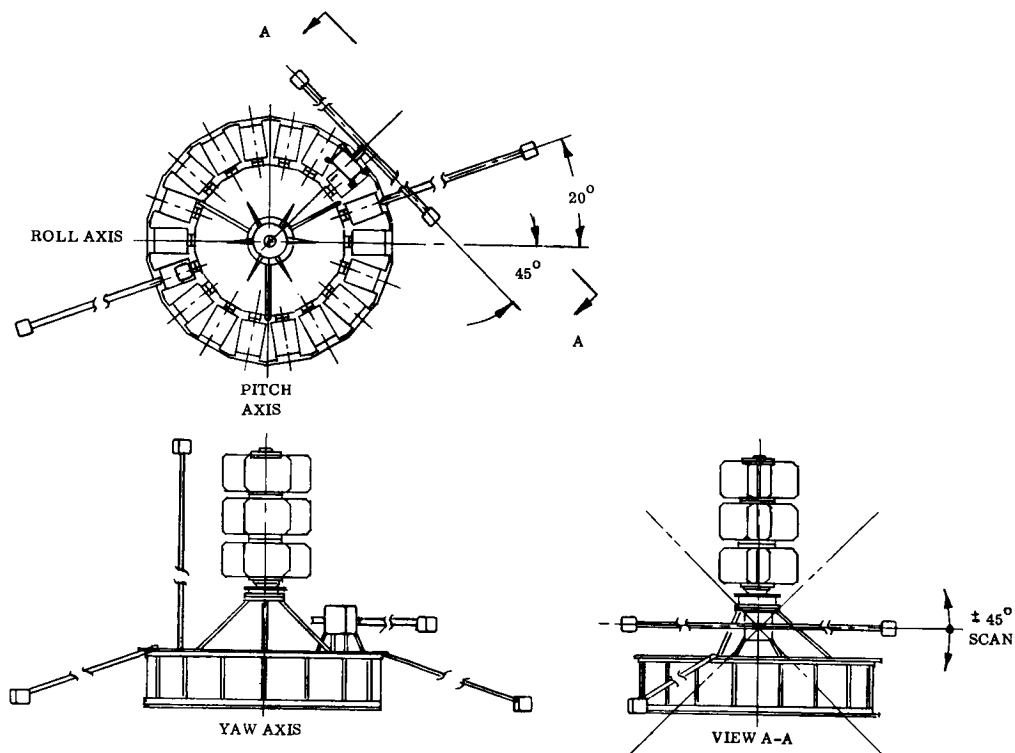


Figure 4-8. Gravity-Anchored Single-Axis Eddy-Current Damper Configuration

member during launch and a harness to carry rod extension signals to the rotating rod units are required. A separation system would free the harness and support from the rotor after rod deployment.

Damping time for decay from large-angle oscillations down to the steady-state oscillations is 8 hours. The steady-state oscillation amplitudes are 2.5 degrees in pitch, 1 degree in roll, and 4 degrees in yaw. The total system weight, including five-rod extension units and tip weights, the single-axis damper and mounting structure, is estimated as 41 pounds.

4.4.2.3 Gravity-Anchored Two-Axis Eddy-Current Damper

This configuration employs a single rod extended from the sensory ring and two rods extended from the rotating side of a two-axis damper. A typical arrangement of rods and

dampers is shown in Figure 4-9. The selection of rod lengths, tip weights and rod mounting angles is less restricted than the previously discussed configuration because the use of a two-axis damper eliminates the need for providing coupling of main body oscillations into relative motion of the damper boom about a single axis. Inertias of the two-part system with the damper at null position are selected to provide the highest damping fraction for the least-damped mode of oscillation. The two-axis damper is basically a pair of single-axis dampers, as described previously. The rotational axes are located along the spacecraft pitch and yaw axes. Implementation of this configuration requires mounting the damper and one rod extension unit on the sensory ring, damper mechanical caging, electrical harness to the rotating side, and a damper uncaging system. The damping time of this configuration is 8 hours. The steady-state oscillation amplitudes are somewhat higher than the single-axis configuration, since both the main body and the damper boom are nonsymmetrical. The hinge point, center of mass, and center of solar pressure are all at different locations. This results in varying solar pressure torques which drive oscillations in both bodies. Steady-state errors are 4 degrees in pitch, 1 degree in roll, and 4 degrees in yaw. The total systems weight is estimated as 34 pounds.

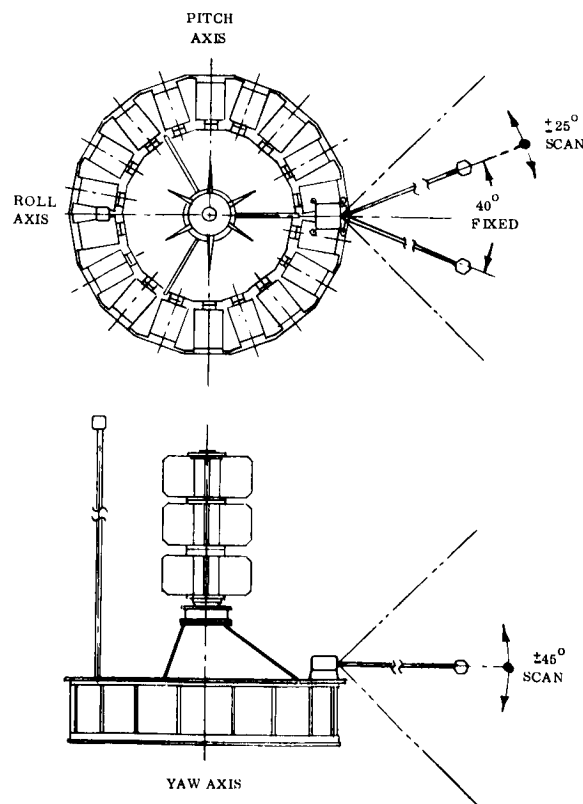


Figure 4-9. Gravity-Anchored Two-Axis Eddy-Current Damper Configuration

4.4.3 DAMPER SELECTION

Selection of the stabilization subsystems must be made in light of the mission objectives established in the study specification. Mission objectives which effect this selection are:

- a. Pointing accuracy requirements.
- b. Damping time to steady state.
- c. Weight.

The accuracy requirements for this study of 5-degree pitch and roll error and 10-degree yaw error can be met by any of the damping configurations. However, minimizing the steady-state pointing errors is the most important mission objective for selecting a damping technique.

For the meteorological satellite mission, high probability of successful initial deployment and mean time to failure of 2 to 3 years is highly desirable. All of the gravity-gradient configurations have these characteristics because of the small number of moving parts and complete absence of electrical control signals during operation.

The effects of the gravity-gradient subsystem on other subsystems can generally be minimized by making use of the great flexibility in choice of rod and damper location. The constraint of no rods within a 70-degree half-angle cone about the earth-pointing axis has been included in all damping configurations to eliminate effects on the sensory subsystems. No difficult interface problems are expected for any of the damping configurations. Damping time to steady state, as well as low weight, are desirable goals but must be obtained with minimum sacrifice in life and pointing accuracies. For all the systems discussed, the damping time is 8 days or less and the subsystem weight is less than 50 pounds; these are sufficiently small so that damping time and weight are not very influential factors in the selection of a configuration.

The characteristics of the four types of damping configurations previously described are evaluated below as they apply to the established mission objectives. A comparison of the steady-state pointing accuracy of the four damping configurations is made in the table 4-2.

Table 4-2. Pointing Accuracy of Damping Configurations

<u>Configuration</u>	<u>Errors</u>		
	<u>Pitch</u> (Degrees)	<u>Roll</u> (Degrees)	<u>Yaw</u> (Degrees)
1. Spherical eddy-current	3	1	3
2. Spherical viscous fluid	3.1	1	3.1
3. Single-axis eddy-current	2.5	1	4
4. Two-axis eddy-current	4	1	4

The nominal steady-state accuracy of configuration 1 and 2 are nearly identical, since the rod configurations are identical and the torques due to the motion of the damper magnet are nearly identical. The errors due to damper motion are 0.6 degree in pitch and 0.8 degree in yaw. The only significant difference in damping performance between the spherical eddy-current damper and the spherical viscous-fluid damper is in the variation in damping coefficient with temperature. A temperature variation from -5°F to $+15^{\circ}\text{F}$ can be expected during the 750-nautical-mile-altitude sun-synchronous orbit. This temperature difference will cause a change in damping coefficient of 0.002 ft-lb-sec (or 10%) in the viscous-fluid damper. The change in damping coefficient with temperature is negligibly small for the eddy-current damper. These changes in damping coefficient cause changes in steady-state error and damping time for the spherical viscous fluid damper and result in higher steady-state errors than would otherwise be indicated. The errors for configuration 3 are lower in pitch because of the absence of the damper motion disturbance but higher in yaw due to pitch motion of the main body, primarily due to orbit eccentricity coupling into yaw motion of the main body through the coupling action of the damper and secondary damping boom. The accuracy of configuration 4 is poor because of the torques on the nonsymmetrical damper boom due to solar pressure which will cause forces at the connection to the main body and result in steady-state errors particularly in the pitch axis.

The conclusion drawn from this evaluation is that the steady-state error of the first and third configurations are nearly the same and is better than either the second or fourth configurations.

An evaluation of the reliability of initial deployment for the four configurations can be made by comparing the number of parts which must be deployed. These parts include damper and tip weight supports for the spherical eddy-current and viscous fluid dampers, damper caging mechanism and electrical harness for the single-axis eddy-current damper configurations, and the rod extension units. The total number of deployments required are as follows:

<u>Configuration</u>	<u>Deployment</u>
1. Spherical eddy-current	8
2. Spherical viscous-fluid	8
3. Single-axis eddy-current	11
4. Two-axis eddy-current	7

It is apparent that the configuration with fewest rods will have the greatest reliability for initial deployment; however, many of these extendable rods have been deployed in orbit and reliability is extremely high.

All of the passively-damped gravity-gradient systems have excellent long-life characteristics. Each type of system has only a few moving parts while in operation, and no electronics or ground commands are required to control attitude. The only moving parts are found in the damper mechanism, so that a comparison of life potential must be made for the dampers. Comparing the damper layouts in the previous discussion on the basis of number of moving parts, size and strength of the parts, and number of close fits and alignments required, the dampers are rated:

- a. Spherical eddy-current
- b. Spherical viscous-fluid
- c. Single-axis eddy-current
- d. Two-axis eddy-current

The state of development and flight experience favors selection of one of the spherical types of dampers. Both the eddy-current and viscous types of dampers have already completed environmental qualification testing and successfully performed for many months in orbit. Several types of one-and two-axis eddy-current dampers are in development but none have flight experience. Damping times to steady state for the four damping techniques are listed below:

Single-axis eddy-current	8 hours
Two-axis eddy-current	8 hours
Spherical eddy-current	186 hours
Spherical viscous-fluid	186 hours

Subsystem weights are:

Two-axis eddy-current	34 lb
Spherical eddy-current	40 lb
Spherical viscous-fluid	40 lb
Single-axis eddy-current	41 lb

4.4.4 RATIONALE FOR SELECTION OF THE DAMPING CONFIGURATION

The rationale for selection of a damping configuration constitutes an evaluation of the characteristics of the damping techniques against the mission objectives and a justification of the logical steps used to select the configuration which best accomplishes the mission objectives. The objectives of primary importance are:

- a. Steady-state pointing accuracy
- b. Damping time to steady state
- c. System weight

These objectives are important and should be attained to the highest degree possible without significant sacrifice in steady-state pointing accuracy.

Rationale:

- a. The spherical eddy-current damper and single-axis eddy-current damper give the best steady-state pointing accuracy.
- b. The spherical eddy-current damper offers the highest potential for long life, has as much flight experience as any damper type, and has as simple a deployment sequence as any damping type.
- c. The spherical eddy-current and spherical viscous-fluid dampers have minimum effect on other subsystems.
- d. The single-axis and two-axis eddy-current dampers have the shortest damping time.
- e. The two-axis eddy-current damper configuration has minimum weight.

The conclusion from items a, b, c is clearly that the spherical eddy-current damper should be used for this application. The differences in damping time or system weight between the selected damper and other types do not warrant their choice.

4.4.5 SELECTION OF DAMPER CHARACTERISTICS

The selection of the linear damping coefficient of the spherical eddy-current damper involves a tradeoff between damper-caused steady-state errors and decay time for large initial oscillations. The linear proportionality factor between relative rate of the outer and inner sphere and reaction torque determines the damping torques, which act as the satellite oscillates.

Since the damper also induces steady-state pointing errors due to its motion while following the magnetic field, these errors are estimated from linearized equations of motion to form the basis for selection of the damping coefficient. The final evaluation of damping performance and steady-state pointing accuracy is made with the large-angle nonlinear digital simulation.

4.4.5.1 Linearized Equations for Damping Performance

The linearized equations of motion for the satellite which are used to estimate damping performance are based on the following assumptions:

- a. Superposition of the magnet motion due to damping torques upon magnet motion due to rotation of the earth's magnetic field with respect to the stabilized satellite.
- b. Linearization of the equations of motion of the satellite based upon:
 1. Small angle motion.
 2. Only gravity-gradient and damping torques.
 3. No cross products of inertia of the satellite.
- c. The cross coupling between roll and yaw axes is neglected in evaluating damping time, since pitch oscillation is the least damped mode, and the coupling increases the roll damping.

The linearized equations of motion for the satellite are shown below:

$$I_p \ddot{\theta}_p + B(\dot{\theta}_p + \dot{\theta}_d) + 3\omega_o^2 (I_r - I_y) \theta_p = 0$$

$$I_r \ddot{\theta}_r + B(\dot{\theta}_r - \dot{\theta}_d) + 4\omega_o^2 (I_p - I_y) \theta_r = 0$$

$$I_y \ddot{\theta}_y = B(\dot{\theta}_y - \dot{\theta}_d) + \omega_o^2 (I_p - I_y) \theta_r = 0$$

where:

I_p = satellite pitch moment of inertia

I_r = satellite roll moment of inertia

I_y = satellite yaw moment of inertia

θ_p Euler angles between pitch, roll,

θ_r and yaw body axes and the r, p, q

θ_y orbiting reference frame.

B = total linear damping coefficient for two dampers

ω = 0.0255 (ft-lb/ rad/sec)

ω_o = orbital rate = 0.923×10^{-3} (rad/sec)

$\dot{\theta}_d$ = damper magnet motion induced by oscillations of the satellite

It can be shown that $\dot{\theta}_d$ is small compared to any satellite rate for the damper design being considered. The equation of motion for the damper magnet driven by satellite oscillation of amplitude A is:

$$I_d \ddot{\theta}_d + B \dot{\theta}_d + K_m \theta_d = AB \omega_s \sin \omega_s t$$

where ω_s = frequency of satellite oscillation

I_d = damper magnet moment of inertia

K_m = damper magnet torque constant

The steady-state response is:

$$\frac{\theta_d(j\omega_s)}{A(j\omega_s)} = \frac{B\omega / Id}{(j\omega_s)^2 + \frac{B}{I_d}(j\omega_s) + \frac{K_m}{I_d}}$$

Substituting values for the nominal damper design

K_m = restoring spring constant of the damper magnet to the earth's magnetic field

$$= \frac{T_m}{\sin \theta} \text{ MH} \left(\frac{\text{ft-lbs}}{\text{oersted}} \right) (\text{oersted}) = 0.00882(0.1) = 8.82 \times 10^{-4} \left(\frac{\text{ft-lb}}{\text{rad}} \right)$$

M = magnet strength = 120,000 pole cm total

I_d = damper magnet inertia 0.001 (slug ft²)

B = 0.0255 $\left(\frac{\text{ft-lb}}{\text{rad/sec}} \right)$

$\omega_s = 2\omega_o$ (highest disturbance frequency) = $1.85 \times 10^{-3} \left(\frac{\text{rad}}{\text{sec}} \right)$

$$\frac{\theta_d}{A} = \frac{0.0474}{-3.5 \times 10^{-6} + .0474j + 0.882} \approx 0.054$$

Since the damper motion is at the same frequency as satellite motion the damper magnet rate induced by satellite oscillation is about 5% of the satellite rate.

The satellite equations of motion, without external disturbance torques, are then:

$$I_p \ddot{\theta}_p + B \dot{\theta}_p + 3\omega_o^2 (I_r - I_y) \theta_p = 0$$

$$I_r \ddot{\theta}_r + B \dot{\theta}_r + 4\omega_o^2 (I_p - I_y) \theta_r = 0$$

$$I_y \ddot{\theta}_y + B \dot{\theta}_y + \omega_o^2 (I_p - I_r) \theta_y = 0$$

The exponential decay time constant (τ) for transient oscillation is given by

$$\gamma = \frac{1}{\tau \omega_n} = \frac{2I}{B}$$

Evaluating for each axis:

$$\tau_p = 66 \text{ (hours)}$$

$$\tau_r = 61 \text{ (hours)}$$

$$\tau_y = 10.7 \text{ (hours)}$$

Thus, it can be seen that pitch damping will determine the time to reach steady state from the worst-case capture condition of ± 90 degrees pitch oscillation. The time required to reach steady state after rod deployment is 186 hours. The roll exponential decay will be faster than 61 hours because the roll-yaw coupling transfer roll energy into yaw, where it will be damped more rapidly. It can also be seen that the pitch-damping exponential decay time is inversely proportional to B for all cases where the magnet is large enough to maintain alignment to the earth's magnetic field. This variation is shown in Figure 4-10 for the nominal design and orbit altitude. A more general plot which includes variations in altitude, pitch moment of inertia, and magnet strength, as well as damping coefficient,

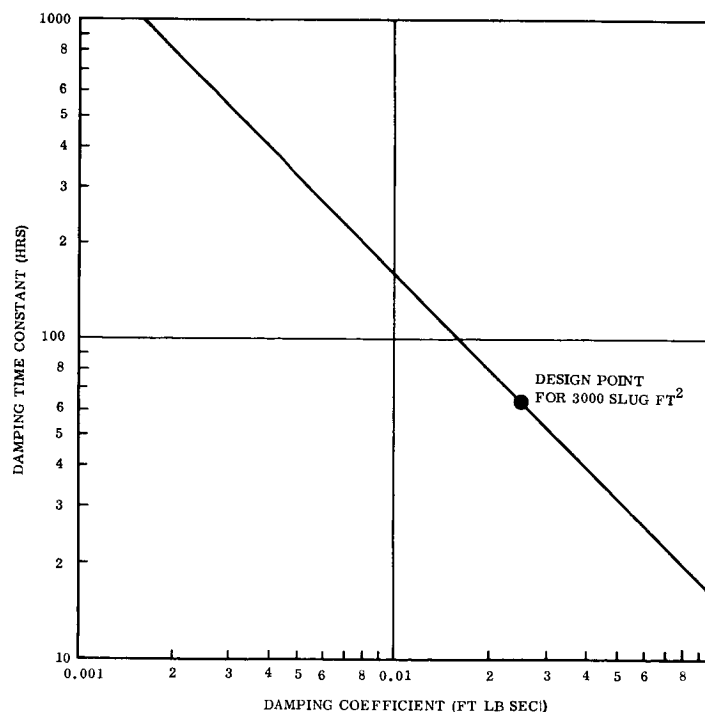


Figure 4-10. Pitch Damping Time vs Damping Coefficient

is shown in Figure 4-11. The exponential decay time constant for pitch oscillation, in number of orbits is given as a function of the damping parameter $\frac{B}{I_p \omega_o}$. This line labeled "Magnet Parameter Asymptote" gives damping for an infinitely large magnet. These results are based on solution of the two body equations of motion (satellite and damper) linearized for small-angle motion in pitch. The earth's magnetic field strength is included as a $\frac{1}{R^3}$ function of altitude. All altitude functions are expressed in terms of orbital rate. The effects of lower magnet strength are shown by the curved lines of constant magnet parameter (M/I_p). It can be seen that damping time decreases with increasing damping coefficient (B) for a fixed magnet strength, up to a point where the increasing damping torques pull the magnet off the earth's magnetic field and damping performance decreases. The nominal design point is shown in this figure and indicates that the magnet strength is very near "infinite." This large magnet strength is required to obtain the desired level of eddy-current damping, which is proportional to magnet strength.

The straight lines labeled $N = 1, 2, 3$ are lines for which a satellite rate of N times orbital rate will exert damping torques on the damper magnet greater than the peak magnetic torque possible with the magnet perpendicular to the earth's magnetic field. Under this condition, the magnet will tumble with respect to the earth's field, and damping will be sharply reduced. Satellite rates are proportional to amplitude and natural frequency of the satellite oscillation are less than 2.7 times orbital except when the satellite is tumbling. The satellite rate necessary to tumble the nominal damper design magnet is found by equating the peak magnetic torque and the damping torque.

$$\omega_t = \frac{MH}{B} \frac{(\text{ft-lb})}{\text{ft-lb}} \frac{\text{rad/sec}}{\text{rad/sec}}$$

$$\omega_t = \frac{8.82 \times 10^{-4}}{0.0255} = 0.0345 \text{ rad/sec}$$

$$\omega_t = 37.4 \omega_o$$

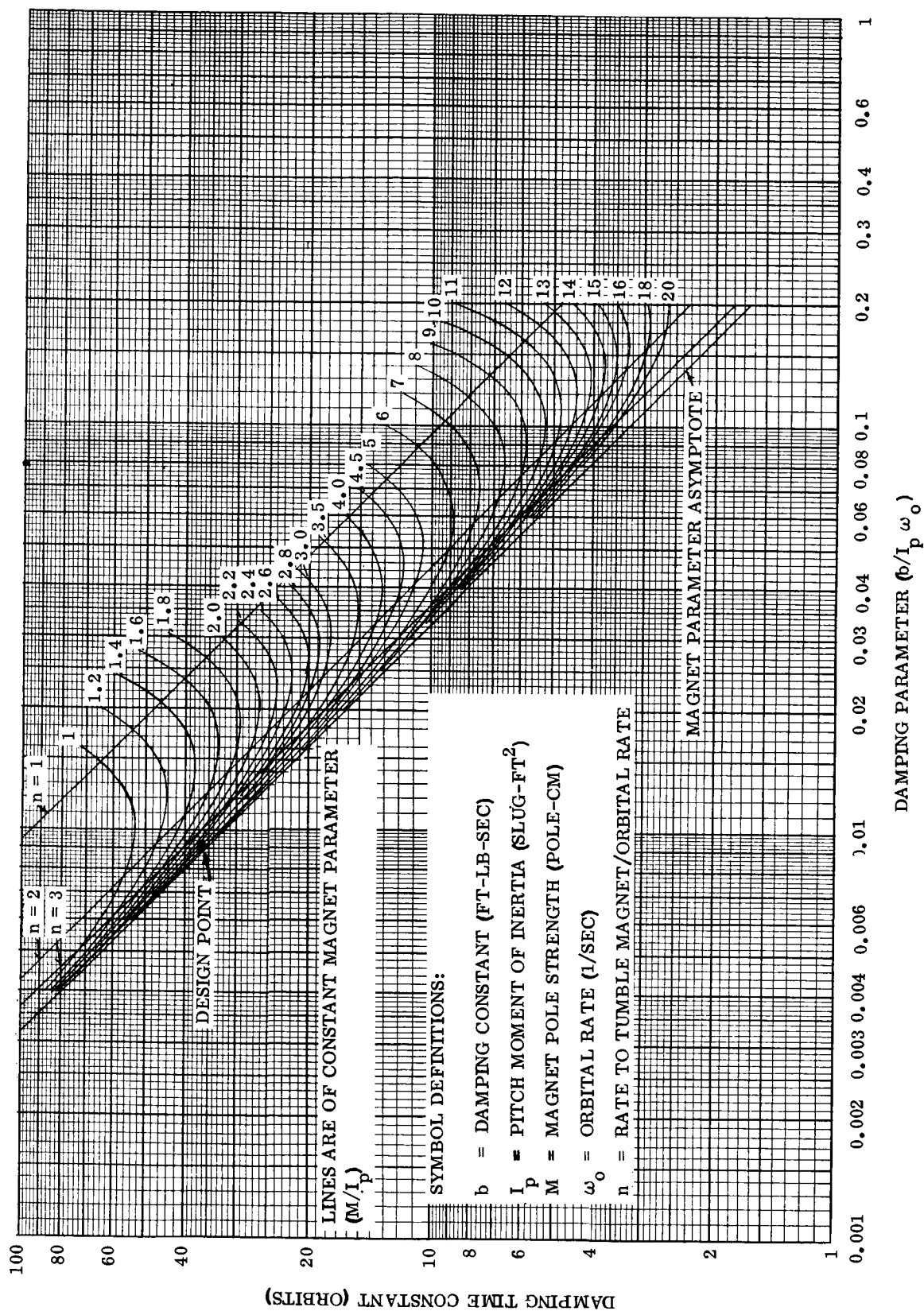


Figure 4-11. Effect of Damping Parameter and Magnet Parameter on Time Constant

The maximum rates which occur after rod deployment from a degree/second separation conditions are 0.003 rad/sec which occur about yaw. Thus there is ample margin in magnet strength to assure that the magnet will not tumble once the rods are deployed.

4.4.5.2 Damper-Induced Steady-State Errors

The damper torques on the satellite caused by motion of the magnet following the earth's magnetic field are derived in Section 4.6. The results of that derivation are:

$$T_{pitch} = B\omega_o(1 + \sin 2\omega_o t) = 2.6 \times 10^{-5}(1 + \sin 2\omega_o t)(\text{ft-lb})$$

$$T_{roll} = 8 \times 10^{-6} \sin \omega_o t (\text{ft-lb})$$

$$T_{yaw} = 5 \times 10^{-6} \sin \omega_o t (\text{ft-lb})$$

The steady-state pointing errors which result from these disturbance torques are calculated from the linearized amplitude response functions shown in Section 4.5.3. The linearized response functions are:

$$\begin{aligned} \theta_{pitch} &= \frac{1}{I_p \omega_o^2} \left[\frac{T_{static}}{3 \left(\frac{I_r}{I_p} - \frac{I_y}{I_p} \right)} + \frac{T_{2\omega_o}}{-4+3 \left(\frac{I_r}{I_p} - \frac{I_y}{I_p} \right)} \right] \\ \theta_{roll} &= \frac{1}{I_p \omega_o^2} \left[\frac{T_{roll}}{3(1 - I_y/I_p)} + \frac{j T_{yaw}}{3(1 - I_y/I_p)} \right] \\ \theta_{yaw} &= \frac{1}{I_p \omega_o^2} \left[\frac{-j T_{roll}}{3(1 - I_y/I_p)} + \frac{T_y \left(4-4 \frac{I_y}{I_p} - \frac{I_r}{I_p} \right)}{3 \left(1 - \frac{I_r}{I_p} - \frac{I_y}{I_p} \right) \left(1 - \frac{I_y}{I_p} \right)} \right] \end{aligned}$$

Substituting the nominal design values:

$$I_p = 3000 \text{ slug ft}^2$$

$$I_r = 2780 \text{ slug ft}^2$$

$$I_y = 490 \text{ slug ft}^2$$

$$B = \frac{0.0255 \text{ ft-lb}}{\text{rad/sec}}$$

$$\omega_o = 0.923 \times 10^{-3} \text{ rad/sec}$$

$$\theta_p = (0.26 \text{ degree} + 0.33 \text{ degree} \sin 2\omega_{ot}) \text{ degrees}$$

$$\theta_r = (0.08 \sin \omega_{ot}) \text{ degrees}$$

$$\theta_y = (0.85 \sin \omega_{ot}) \text{ degrees}$$

These steady-state errors are proportional to the damping coefficient B.

4.4.5.3 Tolerance Effects on Dampers

The effects of manufacturing and environmental tolerances on the damping performance are discussed below. These tolerances affect the actual value of torque rate proportionality (B) which is achieved by the damper in orbit. The affects of magnet strength and temperature are analyzed.

Variations in the dipole strength of the damper magnet can affect the value of damping coefficient achieved in two ways: change in torque rate proportionality factor and change in the anchoring of the magnet to the earth's magnetic field. The effect of tolerance changes in magnet strength on the amplitude of magnet motion with respect to the earth's magnetic field (anchoring) is negligible, since this magnet amplitude is only 5% of the satellite motion amplitude. The eddy-current damping coefficient is proportional to the magnet strength to the first power, and exponential decay time constant is inversely

proportional to damping coefficient. Thus the percent tolerance in damping equals the percent tolerance in magnet strength.

$$\tau = K_{\tau} \frac{1}{M}$$

$$\frac{d\tau}{dM} = \frac{K_{\tau}}{M^2} = \frac{\tau}{M}$$

$$\frac{d\tau}{\tau} = \frac{dM}{M} = \% \text{ variation}$$

Magnet strength is measured before and after environmental testing of the damper unit. The accuracy of the test result is 5% or better. Since the actual launch and flight environment may differ from the test environment, a total magnet strength tolerance of $\pm 10\%$ is estimated for in flight conditions. This $\pm 10\%$ variation in magnet strength causes a $\pm 10\%$ variation in pitch damping exponential decay time, or

$$= 66 \text{ hours} \pm 10\%$$

$$= 59 \text{ to } 73 \text{ hours}$$

Damping coefficient also depends upon temperature, since the thermal conductivity of the copper sphere within which the eddy-currents are generated changes with temperature. The change in conductivity of copper is approximately 30% per 100°F in the range from -50°F to +50°F as established by test and published literature. Thermal analysis for a damper coated with D4D in a 750-nautical-mile sun-synchronous orbit shows temperature variation of -5°F to +15°F at the copper sphere. The -5°F to +15°F range caused a 6% change in conductivity over a single orbit.

This represents a $\pm 3\%$ change in damping for a damper designed to $\pm 5^\circ\text{F}$ nominal temperature. Since the damping performance is determined by the average damping coefficient over many orbits, the $\pm 3\%$ variation in damping which occurs each orbit will have a

negligible effect on the overall damping performance. Thus a total damping time constant tolerance of

$$= 59 \text{ to } 73 \text{ (hours)}$$

is predicted.

Variations in damping coefficient (B) also cause proportional variations in steady-state pointing error due to damper magnet motion. The $\pm 10\%$ variation in B due to tolerance on magnet strength causes a $\pm 10\%$ tolerance on the damper caused errors for the nominal sun synchronous orbit. The steady errors due to damper magnet motion for $\pm 10\%$ tolerance on damping level are listed below:

$$\theta_p = (0.28 + 0.36 \sin 2\omega_{ot}) \text{ degrees}$$

$$\theta_r = (0.09 \sin \omega_{ot}) \text{ degrees}$$

$$\theta_y = (0.93 \sin \omega_{ot}) \text{ degrees}$$

4.5 SELECTION OF PASSIVE ATTITUDE CONTROL CONFIGURATION

4.5.1 GENERAL

Three passive attitude control configurations were considered for use with the NIMBUS sensory ring and RTG mounting arrangement which was selected as the final spacecraft design. All three configurations made use of the magnetically-anchored eddy-current damper, which was selected as the final damper design. The three configurations considered employed three, four, and six gravity-gradient rods, respectively. Each of the rod configurations will give moments of inertia about the principal axes in the range shown to be optimum for steady-state pointing accuracy in the linearized optimization study. The moments of inertia selected for preliminary studies of the three configurations are:

$$\begin{aligned} I_P &= 3000 \text{ (slug ft}^2\text{)} \\ I_R &= 2800 \text{ (slug ft}^2\text{)} \\ I_Y &= 650 \text{ (slug ft}^2\text{)} \end{aligned}$$

A discussion of the strengths and weaknesses of each configuration and a rationale for selection of the four rod configuration follow.

4.5.1.1 Three-Rod Configuration

The three-rod configuration provides the desired moments of inertia with near-minimum total system weight, but has high solar pressure torques.

The three-rod configuration, selected to give the moments of inertia listed above, has a single rod extending upward 4 degrees off the vertical (yaw) reference axis of the spacecraft body. The upper rod is 77 feet long and has a 12-pound damper mounted at the rod tip. Two rods extend 19 degrees below the horizontal plane at a 33-degree angle to either side of the satellite roll reference axis. The two lower rods are 64 feet long with 2-pound tip weights.

This configuration provides the three desired moments of inertia with the minimum number of rods. For the general arrangement shown, symmetry of the rod angles assures that the XZ and YZ cross products of inertia are zero. This leaves four conditions to be met by selection of the rod characteristics: three moments of inertia and $I_{XY} = 0$. Since there are seven variables allowed in selection of the rod characteristics, some additional design conditions can ideally be imposed. The seven rod variables are:

- a. Upper tip weight (damper).
- b. Lower tip weights.
- c. Upper rod length.
- d. Lower rod length.
- e. Upper rod angle from yaw axis.
- f. Lower rod angles (2) from roll axis.

The ground rule limiting rod locations to 20 degrees below the horizontal plane fixes one of the lower rod angles at 19 degrees, since having as much rod below the satellite as possible helps solar pressure balance. The upper tip weight is fixed at 12 pounds, since this is near the maximum weight which can be safely extended on the rod. The selection of a high damper weight (12 pounds), compared to the lower rod tip weights (2 pounds), gives a near-minimum overall system weight, since the large inertias (pitch and roll) are obtained from a single weight and rod very nearly perpendicular to the pitch and roll axes. Thus, for the three-rod design, four inertia values and minimum system weight are obtained by varying five-rod and tip-weight characteristics. The total system weight is 25.3 pounds.

The chief disadvantage of this design is that solar pressure torques are not balanced for the sun on the roll or yaw axis. The resulting solar pressure torque is a sinusoidal function at orbital frequency. The long upper rod, which gives minimum system weight, suffers from greater solar pressure torque due to thermal bending than would a shorter rod with heavier tip weight.

In summary, the three-rod configuration gives the desired moments of inertia for minimum steady-state pointing errors, but does not balance solar pressure torques.

4.5.1.2 Four-Rod Configuration

The four-rod configuration provides the three desired moments of inertia, as does the three-rod configuration, but in addition, the four-rod configuration provides zero nominal solar pressure torque when the sun is on the vehicle roll or yaw axis. The four-rod configuration is heavier than the three-rod configuration but is less affected by solar pressure torque on thermally bent rods.

The four-rod configuration has two rods extending upwards at an angle of $18\frac{1}{2}$ degrees to the satellite yaw axis. These rods are 60 feet long and each has a 12-pound damper at the tip. The upper rods lie in the roll-yaw plane of the satellite reference axes. Two rods extend down $19\frac{1}{2}$ degrees below the horizontal plane in the yaw-pitch satellite reference plane. These lower rods are 80 feet long and have no tip weights.

The rod and tip weight variables available for selection are:

- a. Upper tip weights.
- b. Lower tip weights.
- c. Upper rod length.
- d. Lower rod length.
- e. Upper rod angle.
- f. Lower rod angle.

While there are only six variables available, the symmetry of the rod configurations assures that all cross products of inertia about the reference axes are zero and that solar pressure torque is zero when the sun is on the yaw axis. Thus the requirements to be met are (1) three moments of inertia and (2) zero solar pressure torque with the sun on the roll axis. The digital program used to determine the four-rod design which meets

these requirements is described in Section 4.5.2. Two of the six variables were preselected before using the program to find values of the other four variables to meet moment of inertia and balance solar pressure. The upper tip weights (damper) were selected as 12 pounds, since this is near the maximum weight which can safely be extended on the rod. This maximum weight above the spacecraft assures that solar pressure will be balanced with the shortest possible rods below the spacecraft. A lower rod angle of $19\frac{1}{2}$ degrees (maximum allowable) was also selected to achieve minimum lower rod length. The total system weight of the four-rod configuration is 40 pounds. The pitch solar pressure torques for this configuration have been evaluated with the nonlinear digital simulation and is plotted in Figure 4-12.

4.5.1.3 Six-Rod Configuration

The six-rod configuration was conceived as a modification to the four-rod configuration to allow balancing the solar pressure with shorter lower rods than is possible with the four-rod configuration. The advantage of shorter rods for a balanced configuration is that the solar pressure torques due to thermal bending of the rods are reduced. This configuration was not pursued once the analysis of the four-rod configuration showed that solar pressure balance could be achieved with lower rods of 80 feet. The solar pressure torque due to thermal bending is less than 10^{-6} ft-lb for the four-rod configuration. The reduction in solar pressure torque from thermal bending by making the lower rods shorter than 80 feet is not considered worth the addition of two more rods and extension units.

4.5.1.4 Rationale for Selection of the Four-Rod Configuration

Since all of the configurations can provide the desired moments of inertia for minimum steady-state pointing error, evaluation of the configurations is made on the following characteristics.

- a. Reduction of solar pressure torques.
- b. Minimum number of rods.
- c. Minimum total system weight.

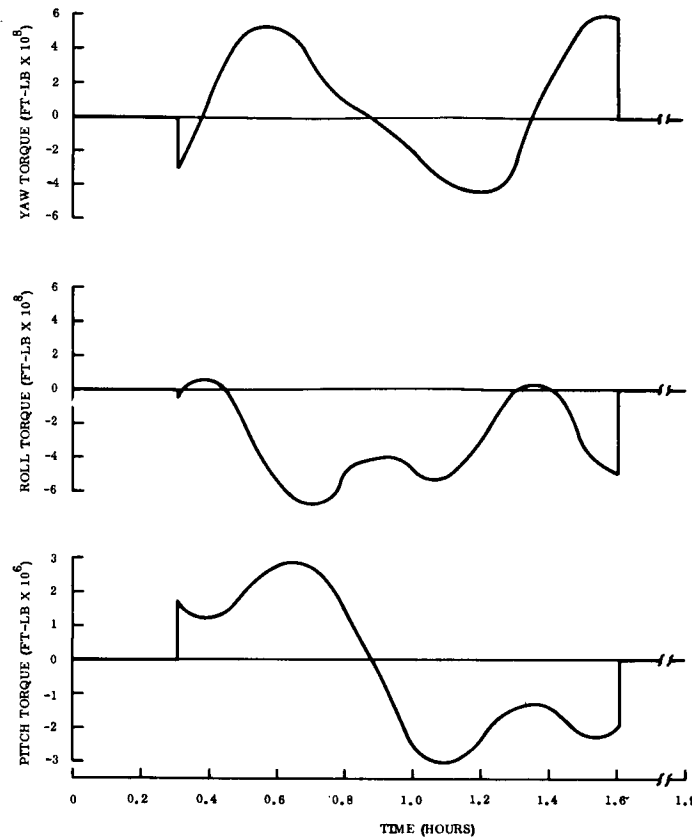


Figure 4-12. Solar Torque (Sun in Orbit Plane)

Since the emphasis of the design study is on achieving minimum steady-state errors, the reduction of solar pressure torques is considered most important. Minimum weight is considered less important than the number of rods so long as the total system weight budget of 50 pounds is not exceeded.

The four-rod configuration has been selected, since it achieves low solar pressure torques while using only one rod more than the minimum required and does not exceed the weight budget.

4.5.2 ROD CONFIGURATION FOR MINIMUM SOLAR TORQUE

A necessary consideration in the design of the rod configuration is the effect of solar radiation pressure. The torque acting on the vehicle due to this pressure is a function of the rod lengths, angles and body design. Not only must the rod configuration satisfy

the minimum solar torque criteria, it must also produce a desired set of moments of inertia. The optimum set of moments are selected from an analysis of the response to all disturbance torques as discussed in Section 4.5.3. Of the configurations discussed in Section 4.5.1, the four-rod configuration, which was selected as the final design, is analyzed to minimize solar pressure torque.

In the analysis of the solar torque, it is assumed that all the sun's rays at the satellite are parallel and that the satellite's distance from the sun is relatively constant so that the radiation pressure, P_o , is constant. It is also assumed for any surface on the configuration that a fraction (α) of the total solar energy is absorbed by the surface, a fraction (ρ_s) is specularly reflected, and a fraction (ρ_D) is diffusely reflected in accordance with the Lambert Law. Then

$$\alpha + \rho_s + \rho_D = 1$$

It is further assumed that these fractions are constant and do not vary with the angle of incidence of the rays. It is also assumed that the rods all extend from a common origin.

For the purpose of this study, it was necessary to consider the dissipation of energy on the sensory ring and RTGs separately. The best information available indicated the absorbance, specular reflectivity, and diffuse reflectivity for the sensory ring are 0.3, 0.0, and 0.7 respectively; for the RTGs they are 0.9, 0.0, and 0.1. For the rods, the absorbance and specular reflectivity are 0.2 and 0.8, respectively.

4.5.2.1 Solar Torque

Consider the expression for the force on a differential area dA . $\bar{S}$ is a unit vector along the direction of the sun's rays and $\bar{N}$ is a unit vector perpendicular to the surface dA and positive inward on the illuminated side. The force on dA due to the absorbed energy is

$$d^2 \bar{F}_\alpha = \bar{S} P_o \alpha (\bar{S} \cdot \bar{N})^2 dA$$

For the energy specularly reflected, the incident and reflected rays make equal contributions to the force on dA and the vector sum along $\bar{N}$ is

$$\overline{d^2 F_{\rho_s}} = 2\bar{N} P_o \rho_s (\bar{S} \cdot \bar{N})^2 dA$$

The force due to the diffusely reflected energy is

$$\overline{d^2 F_{\rho_d}} = P_o \rho_d (\bar{S} \cdot \bar{N}) dA (\bar{S} + 2/3\bar{N})$$

The total force on dA is the sum of the expressions given by the previous equation.

$$\overline{d^2 F} = P_o dA \left[\bar{S} (1-\rho_s) (\dot{S}N) + 2\bar{N} \rho_s (\dot{S}N)^2 + 2/3\bar{N} \rho_d (\dot{S}N) \right]$$

Defining R as the vector from the center of mass of the overall satellite to the surface dA , then the torque due to the illumination of dA is

$$\overline{T} = \iint \bar{R} \times \overline{d^2 F}$$

4.5.2.2 Torque on Rods

Define an orbit fixed coordinate system, $\bar{r} \bar{p} \bar{q}$, such that the $\bar{r}$ axis is along the vector from the earth's center to the satellite, $\bar{p}$ is along the velocity vector, and $\bar{q}$ forms the orthogonal set. The nominal NIMBUS orbit is sun-synchronous; plane therefore, a minimum type of solar torque rod configuration is obtained by symmetry about the yaw axis and balanced areas about the roll-pitch plane. The configuration selected has two upper rods in the yaw roll ($\bar{r}\bar{p}$) plane and the lower rods in the yaw pitch ($\bar{r}\bar{q}$) plane.

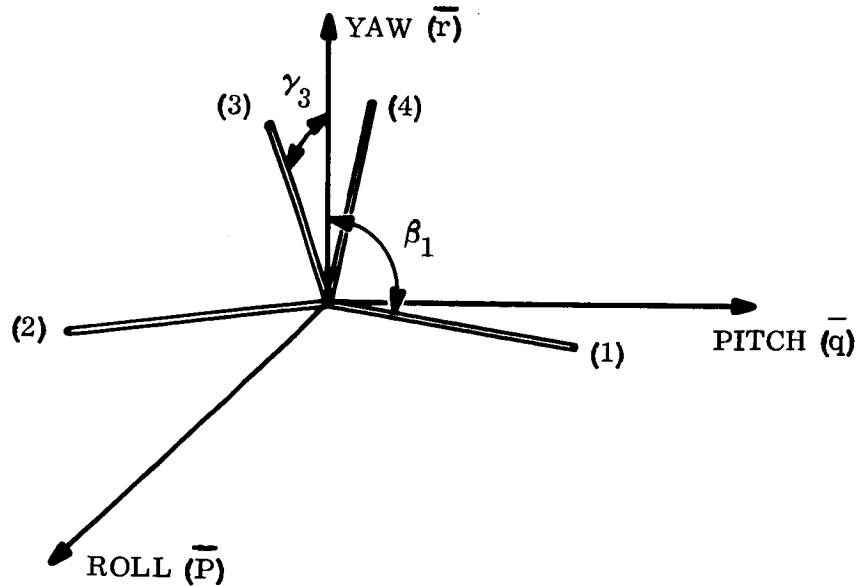


Figure 4-13. Rod Positions for the Four-Rod Configuration

This type of rod configuration will produce a torque which is nominally zero when the sun is on the $\bar{r}$ or $\bar{p}$ axis. For the lower rods, a body coordinate system $\bar{i}'_i, \bar{j}'_i, \bar{k}'_i$ is defined such that the transformation between this system and the orbit system is given by

$$\begin{bmatrix} \bar{r} \\ \bar{p} \\ \bar{q} \end{bmatrix} = \begin{bmatrix} -\sin \beta_i & 0 & \cos \beta_i \\ 0 & 1 & 0 \\ \cos \beta_i & 0 & \sin \beta_i \end{bmatrix} \begin{bmatrix} \bar{i}'_i \\ \bar{j}'_i \\ \bar{k}'_i \end{bmatrix}$$

where β is the angle in the yaw pitch plane measured from the positive yaw axis and $i = 1, 2$. A second coordinate system is then defined such that the sun lies in the $\bar{j}_i, \bar{k}_i$ plane making an angle σ with the $\bar{j}_i$ axis.

$$\bar{S} = \cos \sigma_i \bar{j}_i + \sin \sigma_i \bar{k}_i$$

The transformation from $\bar{i}_i' \bar{j}_i' \bar{k}_i'$ to $\bar{i}_i \bar{j}_i \bar{k}_i$ being defined as

$$\begin{bmatrix} \bar{i}_i' \\ \bar{j}_i' \\ \bar{k}_i' \end{bmatrix} = \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 \\ \sin \theta_i & \cos \theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{i}_i \\ \bar{j}_i \\ \bar{k}_i \end{bmatrix}$$

Since the NIMBUS orbit is sun-synchronous, the solar unit vector may also be expressed in the orbit frame as

$$\bar{S} = -\cos \mu \bar{r} - \sin \mu \bar{p}$$

From the following equations

$$\bar{S} = \cos \mu \sin \beta_i \bar{i}_i' - \sin \mu \bar{j}_i' - \cos \mu \cos \beta_i \bar{k}_i'$$

and using using the coordinate transformation

$$\bar{S} = -\cos \sigma_i \sin \theta_i \bar{i}_i' + \cos \sigma_i \cos \theta_i \bar{j}_i' + \sin \sigma_i \bar{k}_i'$$

From these equations

$$\begin{aligned} \sin \sigma_i &= -\cos \mu \cos \beta_i \\ \cos \sigma_i &= \left[\cos^2 \mu \sin^2 \beta_i + \sin^2 \mu \right]^{1/2} \\ \cos \theta_i &= -\sin \mu / \cos \sigma_i \\ \sin \theta_i &= -\cos \mu \sin \beta_i / \cos \sigma_i \end{aligned}$$

The normal to the rod and the vector from the origin to the elementary area dA are given by

$$\bar{N} = -\sin \psi_i \bar{i}_i + \cos \psi_i \bar{j}_i$$

$$\bar{R}_{o_\ell} = a_\ell \sin \psi_i \bar{i}_i - a_\ell \cos \psi_i \bar{j}_i + \ell \bar{k}_i$$

where a_ℓ is the radius of the lower rod and ℓ the length.

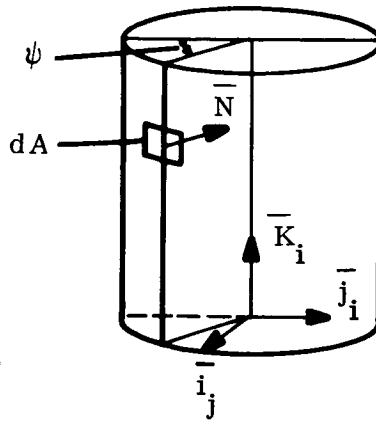


Figure 4-14. Rod Cross Section and Axes

The elementary area is given by

$$dA = a_\ell d\psi d\ell$$

Considering the configuration under study and a NIMBUS body configuration which has its center of mass on the yaw axis, the vector from the origin of the rods to the center of mass may be given as

$$\bar{R}_c = d \cdot \bar{r} = d \left[-\sin \beta_i \cos \phi_i \bar{i}_i + \sin \beta_i \sin \phi_i \bar{j}_i + \cos \beta_i \bar{k}_i \right]$$

where d is the distance from the common origin of the rods to the center of mass. Using these equations and integrating $d\ell$ from 0 to 1 and $d\psi$ from $-\pi/2$ to $\pi/2$, the torque about the $\bar{i}_i \bar{j}_i \bar{k}_i$ axis is

$$\bar{T} = T_{1i} \bar{i}_i + T_{2i} \bar{j}_i + T_{3i} \bar{k}_i$$

where :

$$T_{1i} = \rho_0 a_\ell \left[\cos^2 \sigma_i (2d \cos \beta_i - \ell) (1 + \rho_s/3) - \sin \sigma_i \cos \sigma_i (2 \sin \beta_i \sin \theta_i + \pi/2 a_\ell) (1 - \rho_s) \right]$$

$$T_{2i} = -2\rho_0 a_\ell \ell \sin \beta_i \cos \theta_i \sin \sigma_i (1 - \rho_s)$$

$$T_{3i} = \rho_0 a_\ell \ell \sin \beta_i \cos \theta_i \left[2 \cos^2 \sigma_i (1 + \rho_s/3) \right]$$

The torque about the pitch ($\bar{q}$) axis is

$$T_{q\ell} = a_{1i} T_{1i} + a_{2i} T_{2i} + a_{3i} T_{3i}$$

where :

$$a_{1i} = \cos \beta_i \cos \theta_i$$

$$a_{2i} = -\sin \beta_i \sin \theta_i$$

$$a_{3i} = \sin \beta_i$$

For the upper rods the body coordinate system $\bar{i}_j \bar{j}_j \bar{k}_j$, $j = 3, 4$ is defined such that $\bar{i}_j$ is in the direction of the pitch, $\bar{q}$, axis and

$$\begin{bmatrix} \bar{r} \\ \bar{p} \\ \bar{q} \end{bmatrix} = \begin{bmatrix} 0 & \sin \gamma_j & \cos \gamma_j \\ 0 & -\cos \gamma_j & \sin \gamma_j \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{i}_j \\ \bar{j}_j \\ \bar{k}_j \end{bmatrix}$$

where γ_j is the angle in the yaw roll plane measured from the positive yaw axis. It is therefore only necessary to consider the torque about the $\bar{i}_j$ axis. Since these rods are in the plane of the sun, it is necessary to allow for the rotation of this body frame such that the normal to the rod is always positive inward on the illuminated side. This is accomplished by employing a quantity, δ , which is positive or negative, depending on the position of the rod with respect to the sun. Thus

$$\bar{N} = -\delta_j \sin \psi_j \bar{i}_j + \delta_j \cos \psi_j \bar{i}_j$$

as in the case of the lower rods, the solar unit vector may be described in the body frame

$$\bar{S} = \cos \sigma_j \bar{j}_j + \sin \sigma_j \bar{k}_j$$

where

$$\begin{aligned} \cos \sigma_j &= -\cos \mu \sin \gamma_j + \sin \mu \cos \gamma_j \\ \sin \sigma_j &= -\cos \mu \cos \gamma_j - \sin \mu \sin \gamma_j \end{aligned}$$

and it can be shown that a suitable expression for δ_j is

$$\delta_j = \frac{\cos \sigma_j}{|\cos \sigma_j|}$$

The vector to the elementary area dA from the origin and the vector from the origin to the center of rotation are

$$\begin{aligned} \bar{R}_{ou} &= a_u \sin \psi_j \bar{i}_j - a_u \cos \psi_j \bar{j}_j + L \bar{k}_j \\ \bar{R}_{oc} &= d \bar{r} \end{aligned}$$

where a_u and L are the radius and length of the upper rods, respectively. The elementary area is given by

$$dA = a_u d\psi_j d_l$$

Using these equations in determining the torque about the pitch axis results in

$$T_{qu} = \rho_o a_u L \left[\delta_j \cos^2 \sigma_j (2 d \cos \gamma_j - L) (1 + \rho_{s/3}) - \sin \sigma_j \cos \sigma_j (2 d \sin \gamma_j + \pi / 2 a_u) (1 - \rho_s) \right]$$

The torque on the rods is then the sum of the equations which define T_{ql} and T_{qu}

4.5.2.3 Torque on NIMBUS Body

For the purpose of this study, the NIMBUS body has been approximated by an equivalent sphere. In general, the solar torque on a sphere is given by

$$\overline{T}_{SPH} = \pi r_s^2 P_o \left(1 + \frac{2}{3} \rho_D (\overline{R}_{cs} \times \overline{S}) \right)$$

where $\overline{R}_{cs}$ is the vector from the center of mass to the geometric center. Due to the symmetry involved

$$\overline{R}_{cs} = D \overline{r}$$

where D is the distance from the top of the sensory ring to the center of mass. Thus the torque about the pitch axis for the equivalent sphere is

$$T_{qs} = P_o \pi r_s^2 D \sin \mu \left(1 + \frac{2}{3} \rho_{DB} \right)$$

where r_s is the radius of the sphere

The position of the center of mass of the total system is dependent on the position of the center of mass of the equivalent sphere. Consider the following diagram.

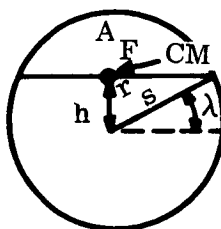


Figure 4-15. Equivalent Sphere Representation of NIMBUS Body

The distance of the center of mass from the geometric center is h .

$$h = r_s \sin \lambda$$

Assume that the area A_F will approximate the area of the RTGs and A_S will approximate the sensory ring. It can be shown that if A_S is the larger area

$$A_S = \frac{r_s^2 \pi}{2} + r_s^2 \sin \lambda \cos \lambda + r_s^2 \lambda$$

$$A_F = \frac{r_s^2 \pi}{2} - r_s^2 \sin \lambda \cos \lambda - r_s^2 \lambda$$

These areas are weighted, however, by the diffuse reflectivity, ρ_{DS} for the sensory ring and ρ_{DF} for the RTGs

$$A_S = \rho_{DS} A'_S$$

$$A_F = \rho_{DF} A'_F$$

Where A'_S and A'_F are the actual areas of the sensory ring and the RTGs respectively

$$A'_S/A'_F = 10.2$$

Therefore

$$10.2 = \frac{\pi/2 + \sin \lambda \cos \lambda + \lambda}{\pi/2 - \sin \lambda \cos \lambda - \lambda}$$

Solving 4-28 for λ it yields

$$\lambda \approx 19^\circ$$

and

$$h = 0.326 r_S$$

The diffuse reflectivity ρ_{DB} , to be used is determined as a weighted average.

$$\rho_{DS} A'_S + \rho_{DF} A'_F = \rho_{DB} \pi r_S^2$$

An analysis of the solar torque on the individual parts of the NIMBUS body produce the result that the torque on the NIMBUS body may be approximated by

$$T = P_o \left[0.25 A'_T \cos \mu \sin \mu + A'_S \left\{ 2.5 \sin \mu \cos \mu + 0.81 \sin \mu (\sin \mu + 0.467) \right\} + A'_F \left\{ 2.92 \sin \mu (\sin \mu + 0.467) \right\} \right]$$

where A'_T = area of the top of the sensory ring = 19.62 square feet

A'_S = area of the side of the sensory ring = 5.1 square feet

A'_F = equivalent area of the RTG's on which solar pressure is
 $= 3.38 + 0.795 \frac{\cos \mu}{\sin \mu}$

The equation for T is then related to the equation for T_{qs} and, along with the previous equation for the diffuse reflectivity, the radius of an equivalent sphere is determined for sun angles of 90 and 45 degrees. These two results of 1.6 and 45 degrees and 2.6 for 90 degrees are then averaged to produce the radius of the equivalent sphere. Thus

$$r_S = 2.1 \text{ feet}$$

With this radius the diffuse reflectivity ρ_{DB} , which will be used for the equivalent sphere is

$$\rho_{DB} = 0.28$$

4.5.2.4 Optimum Configuration

This analysis now takes the form of determining the rod lengths and angles which will satisfy the conditions of minimum solar torque and desired moments of inertia. From the equations for T_{qe} , T_{qu} and T_{qs}

$$\begin{aligned} T_q = & \sum_{i=1}^2 \left[a_{1i} T_{1i} + a_{2i} T_{2i} + a_{3i} T_{3i} \right] \\ & + P_o a_u L \sum_{j=3}^4 \left[\delta_j \cos^2 \sigma_j (2d \cos \gamma_j - L) (1 + \rho_S/3) \right. \\ & \quad \left. - \sin \sigma_j \cos \sigma_j (2d \sin \gamma_j + \pi/2 a_u) (1 - \rho_S) \right] \\ & + P_o \pi r_S^2 d \sin \mu (1 + 2/3 \rho_{DB}) \end{aligned}$$

Let M and m represent the tip weights of the upper and lower rods, respectively. Assuming it is permissible to neglect the mass of the upper and lower rods, the moments of inertia about the center of mass are

$$I_r = 2ML^2 \sin^2 \gamma + 2m\ell^2 \cos^2 (20^\circ)$$

$$I_p = 2M (L^2 \cos^2 \gamma + d^2 - 2Ld \cos \gamma) + M_B d^2 + 2m(\ell^2 + d^2 + 2\ell d \sin 20^\circ)$$

$$I_q = 2M (L^2 + d^2 - 2Ld \cos \gamma) + M_B d^2 + 2m(\ell^2 \sin^2 (20^\circ) + d^2 + 2\ell d \sin 20^\circ)$$

where M_B is the mass of the NIMBUS body. γ is the half angle between the upper rods and will be determined to minimize the torques

$$\gamma_3 = \gamma \quad \gamma_4 = 360^\circ - \gamma$$

and

$$\beta_1 = 90^\circ + 20^\circ \quad \beta_2 = 270^\circ - 20^\circ$$

The cross products of inertia must be zero because of the symmetry of the configuration. Choosing a specific upper tip mass (M) and initial distance (d) and an initial half angle (γ) the length and half angle for the upper rods is determined from the following equations.

$$x = m\ell^2 = (I_p + I_r - I_q)/3.532$$

Using equations for I_n and I_p

$$Y_o = I_p + I_r - 3.766 x - (M_B + 2M) d^2$$

$$\Delta Y_o = \frac{(M_B d + 2Md + 2md + 0.342 m \ell) \Delta d}{M}$$

$$\Delta Y = \Delta Y_o - \frac{(2Md^2 + 0.684 m \ell d)}{M}$$

$$Y = Y_0 + \Delta Y$$

$$Z = 2d \cos \gamma$$

$$L^2 - LZ - Y = 0$$

From the equation for I_r

$$\gamma = \arcsin \left[\frac{I_r - 1.766 m \ell^2}{2 M L^2} \right]$$

It is initially assumed that $\Delta Y \approx 0$ and the length (L) is determined from the equation. The half angle is then determined from the equation for γ . With these values, the equation for T_q is solved for the length ℓ by equating T_q identically equal to zero. The distance d is next determined by

$$d = \frac{2 M L \cos \gamma - 0.684 m \ell + 11.8}{M_B + 2M + 2m}$$

and Δd is taken as the difference between this value of d and the previous one. The tip mass for the lower rods is determined using equation 4-34. The procedure is repeated starting with equation 4-35. The error ΔY is determined,

$$\text{If } \Delta Y < 0.01 Y$$

it is assumed that the lengths and angles are correct under the assumptions which were made. These quantities are then further improved in the following manner.

Neglecting the offset (d) but including the mass of the rods, the moments of inertia are

$$I_r = 2 M_T L^2 \sin^2 \gamma + 2 m_T \ell^2 \cos^2 (20^\circ)$$

$$I_p = 2 M_T L^2 \cos^2 \gamma + 2 m_T \ell^2$$

$$I_q = 2 M_T L^2 + 2 m_T \ell^2 \sin^2 (20^\circ)$$

$$M_T = M + \frac{\lambda L}{3} \quad m_T = m + \frac{\rho \ell}{3}$$

where λ and ρ are the densities per unit length of the upper and lower rods, respectively. The tip mass of the lower rod is determined to satisfy equation for I_q . If it evolves that this mass is negative, this equation is again employed with the tip mass of the lower rods set equal to zero to determine a length L which will give the desired moment I_q . With the lengths and tip masses thus determined, equations for I_n and I_p are used in determining a suitable half angle, γ , which gives the appropriate moments and the desired minimum solar torques.

The results of this study for some of the moments of inertia considered are indicated in Table 4-3.

Table 4-3. Four-Rod Configurations for Zero Solar Pressure Torque

Desired Moment (slug-ft ²)			Upper Rod					Lower Rod			
			Length (ft)	Tip Mass (slugs)	Density (slug/feet)	Radius (ft)	Half Angle (deg)	Length (ft)	Tip Mass (slugs)	Density (slug/feet)	Radius (ft)
Pitch	Roll	Yaw									
3135	2920	790	64.2	0.374	0.485×10^{-5}	0.0229	23	61.5	0.016	1.1×10^{-5}	0.052
3000	2880	400	63	0.374	0.485×10^{-5}	0.0229	16	55	0	1.1×10^{-5}	0.052
3000	2800	580	63	0.374	0.485×10^{-5}	0.0229	20	83	0	0.485×10^{-5}	0.0229

4.5.3 LINEARIZED OPTIMIZATION OF INERTIAS

The steady-state pointing accuracy of a gravity-gradient system employing the magnetically-anchored damper is dependent on the disturbance torques and the response of the system to these disturbance torques. The significant disturbance torques which act on the system are solar pressure torques, orbit eccentricity torques, magnetic torques due to satellite residual dipole, and damper torques due to motion of the damper magnet following the earth's magnetic field. These torques occur at frequencies which are harmonics of orbital frequency. The system response to these torques may be calculated from the restoring torques which are functions of the moments of inertia.

This parametric study consisted of calculating the magnitudes of the disturbance torques for the four-rod configuration and evaluating the system response for many combinations of moments of inertia. A weighted sum of the pitch, roll, and yaw errors is plotted as a function of the inertias to show the combinations of inertias which give minimum error.

The effects of solar pressure torques and the geometric rotation of the principal axes due to thermal bending of the rods are not included in this optimization study. These are the only significant disturbances which are functions of the rod lengths, angles and tip weights. The approximate values of moments of inertia which give minimum error have been selected from this parametric study neglecting these solar effects. A final parametric study has been made for several values of moments of inertia in the optimum range and including all solar pressure effects. This parametric study was made by using the large angle non-linear digital simulation and is described in Section 4.5.4.

The orbit eccentricity response is a function of the moments of inertia and is calculated for each value of I_p , I_R , and I_y during the parametric evaluation. The damper motion torque is for a damper design which will damp initial oscillations of ± 90 degrees to steady state in 186 hours.

The disturbance torque magnitudes and frequencies which were used in this parametric study are listed in Table 4-4 and are calculated in Section 4.6.

The system response to the disturbance torques is calculated from Euler's dynamical equations for a rigid body with gravity-gradient restoring torques. These equations, when written in terms of Euler angles between the body reference axes and the orbit reference frame, and linearized with small angle approximations, are:

$$I_p \ddot{\theta}_p + 3 \omega_o^2 \theta_p (I_R - I_y) = T_P$$

$$I_R \ddot{\theta}_R + 4 \omega_o^2 \theta_R (I_p - I_y) + \omega_o \dot{\theta}_y (I_y - I_p + I_R) = T_R$$

$$I_y \ddot{\theta}_y + \omega_o^2 \theta_y (I_p - I_R) + \omega_o \dot{\theta}_R (I_p - I_R - I_y) = T_y$$

The variables used are:

ω_o = orbital rate (rad/sec)

I = moments of inertia (slug ft²)

$3 \omega_o^2 \theta_P (I_R - I_Y)$ = pitch gravity-gradient torque (ft-lb)

$3 \omega_o^2 \theta_R (I_P - I_Y)$ = roll gravity-gradient torque (ft-lb)

Table 4-4. Disturbance Torques for Linear Optimization of Inertias

	Pitch (ft-lb)	Roll (ft-lb)	Yaw (ft-lb)
<u>Zero Frequency (Bias)</u>			
Damper motion	24×10^{-6}	-	0.9×10^{-6}
Roll axis dipole	-	-	0.6×10^{-6}
Yaw axis dipole	-	0.6×10^{-6}	-
TOTAL	24×10^{-6}	0.6×10^{-6}	1.5×10^{-6}
<u>Orbit Frequency</u>			
Orbit eccentricity	$f(I_R, I_P, I_Y)$	-	-
Pitch axis dipole	-	2.5×10^{-6}	1×10^{-6}
Yaw axis dipole	1×10^{-6}	-	-
Roll axis dipole	2.2×10^{-6}	-	-
Damper motion	-	8×10^{-6}	5.0×10^{-6}
TOTAL	3.2×10^{-6} plus eccentricity	10.5×10^{-6}	6.3×10^{-6}
<u>Twice Orbital Frequency</u>			
Damper motion	23×10^{-6}	—	—
Solar pressure	1×10^{-6}		
TOTAL	24×10^{-6}		

$$\left. \begin{aligned}
& \omega_o^2 \theta_R (I_P - I_Y) \\
& \omega_o \dot{\theta}_Y (I_Y - I_P - I_R) \\
& \omega_o \dot{\theta}_R (I_P - I_R - I_Y) \\
& \omega^2 \theta_Y (I_P - I_R)
\end{aligned} \right\} = \text{coupling torques due to orbital rate}$$

$$\left. \begin{aligned}
& T_P \\
& T_R \\
& T_Y
\end{aligned} \right\} = \text{external torques}$$

These equations are linear differential equations with constant coefficients and are solved by standard Laplace Transform techniques.

The steady-state amplitudes for sinusoidal disturbance torques are evaluated by substitution of $j \omega_d$ (where subscript "d" refers to disturbance frequency) for the Laplace operator. Since ω_d has only values which are harmonics of orbital frequency, the response equations are functions only of inertia, orbital frequency, and torque magnitude. For static, sub-script(o), once orbital (1) and twice orbital (2), the roll (θ_R), yaw (θ_Y), and pitch (θ_P) amplitudes are:

$$\left[\theta_R \right]_o = \left(\frac{T_{R0}}{I_P \omega_o^2} \right) \frac{1}{4 (1 - I_Y/I_P)}$$

$$\left[\theta_R \right]_1 = \left(\frac{T_{R1}}{I_P \omega_o^2} \right) \frac{1}{3 (1 - I_Y/I_P)} + \left(\frac{T_{Y1}}{I_P \omega_o^2} \right) \frac{j}{3 (1 - I_Y/I_P)}$$

$$\left[\theta_R \right]_2 = \left(\frac{T_{R2}}{I_P \omega_o^2} \right) \frac{1 - \frac{I_R}{I_P} - 4 \frac{I_Y}{I_P}}{12 \left(\frac{I_Y}{I_P} + \frac{I_R}{I_P} - 1 \right) \left(\frac{I_Y}{I_P} \right)} - \left(\frac{T_{Y2}}{I_P \omega_o^2} \right) \frac{j}{6 \left(\frac{I_Y}{I_P} \right)}$$

$$\left[\theta_Y \right]_0 = \left(\frac{I_{Y0}}{I_P \omega_o^2} \right) \frac{1}{(1 - I_R/I_P)}$$

$$\left[\theta_Y \right]_1 = \left(\frac{T_{R1}}{I_P \omega_o^2} \right) \frac{j}{3 (1 - I_Y/I_P)} - \left(\frac{T_{Y1}}{I_P \omega_o^2} \right) \frac{(4 - 4 I_Y/I_P - I_R/I_P)}{3 (1 - I_R/I_P - I_Y/I_P) (1 - I_Y/I_P)}$$

$$\left[\theta_Y \right]_2 = \left(\frac{T_{R2}}{I_P \omega_o^2} \right) \frac{j}{6 (I_Y/I_P)} + \left(\frac{T_{Y2}}{I_P \omega_o^2} \right) \frac{1}{3 (I_Y/I_P)}$$

$$\left[\theta_P \right]_0 = \left(\frac{T_{P0}}{I_P \omega_o^2} \right) \frac{1}{3 (I_R/I_P - I_Y/I_P)}$$

$$\left[\theta_P \right]_1 = \left(\frac{T_{P1}}{I_P \omega_o^2} \right) \frac{1}{[-1 + 3 (I_R/I_P - I_Y/I_P)]}$$

$$\left[\theta_P \right]_2 = \left(\frac{T_{P2}}{I_P \omega_o^2} \right) \frac{1}{[-4 + 3 (I_R/I_P - I_Y/I_P)]}$$

The effect of damping on the steady-state amplitude and phase angle is neglected, since the oscillatory motion is lightly damped and the rigid body natural frequencies are not near the disturbance frequencies; thus $j = 1$. The amplitudes of the response functions have been added without regard to sign or phase, since the eccentricity phase is undetermined and the dipole phasing may have either polarity. This procedure gives a somewhat conservative result.

Pitch moments of inertia of 2000, 3000 and 4000 slug ft² have been evaluated in this study. These pitch inertias can be obtained with a total attitude control subsystem weight of 30 to 50 pounds. Pitch inertias lower than 2000 slug ft² would result in larger steady-state errors.

Roll axis moments of inertia were varied from 50% to 100% of pitch inertia for each value of pitch inertia. Yaw inertia was varied from 5% to 80% of pitch inertia. The damper design was changed proportionally with pitch moment of inertia to maintain constant damping time.

A weighted sum of pitch, roll, and yaw error has been established to evaluate overall system performance. Since the roll error is much smaller than pitch or yaw error and since the yaw accuracy requirement is one half that of pitch or roll, a weighted total error function of $2\theta_P + \theta_R + \theta_Y$ is used.

The results of linearized optimization study for pitch moments of inertia of 2000, 3000 and 4000 slug ft² are shown in Figures 4-16, 4-17, and 4-18, respectively. The general shapes of the curves show that a relative minimum total error is achieved in two areas. A total error function of about 8 degrees is achieved for inertias ratios near $\frac{I_R}{I_P} = 0.7$ and $\frac{I_Y}{I_P} = 0.6$. A much lower minimum error function is achieved for inertia ratios near $\frac{I_R}{I_P} = 0.93$ and $\frac{I_Y}{I_P} = 0.225$. Comparison of this minimum for different values of pitch moment of inertia shows that the error function decreases with increasing pitch moment of inertia. The minimum error functions are 4.7 degrees, 4.4 degrees, and 4.3 degrees for 2000, 3000 and 4000 slug ft², respectively. This indicates that increasing pitch moment of inertia above 3000 slug ft² is not effective in reducing the total error function.

Comparison of the linearized optimization results in Figure 4-17 with the natural frequency map shown in Figure 4-19 shows why the two areas of minimum error function are present. The natural frequency map is based on the small angle linearized equations of motion, given in section 4.5.2.1. It shows the natural frequency of oscillation about each axis as a function of the moment of inertia ratios $\frac{I_R}{I_P}$ and $\frac{I_Y}{I_P}$. The two areas of minimum error function are shown. It can be seen that these areas correspond to pitch natural frequencies above and below orbital frequency. That is, the moments of inertia for minimum error detune the resonant response to bias, orbital, and twice-orbital frequencies. The minimum error points both lie at a yaw natural frequency near 0.65 times orbital which provides minimum response to the bias and orbital frequency yaw disturbance torques. The minimum error function is lower when the pitch natural frequency is above orbital frequency because of the larger restoring torques.

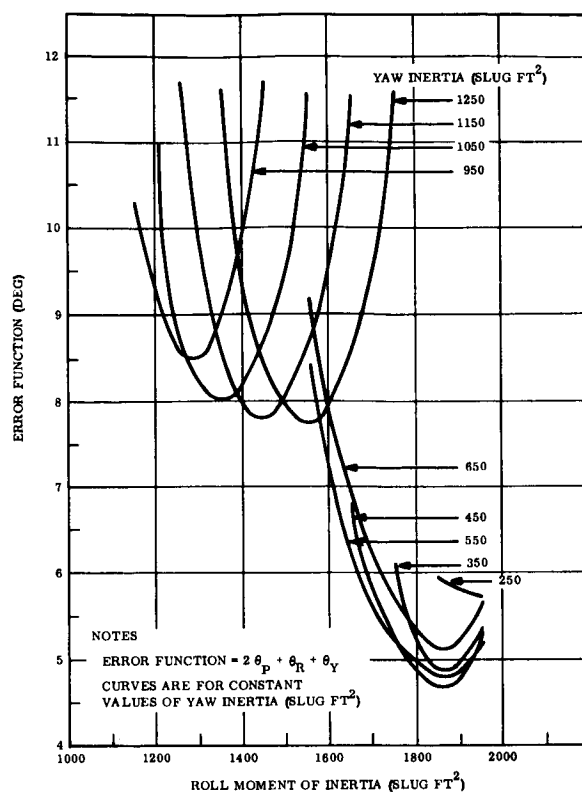


Figure 4-16. Parameter Study of Steady-State Pointing Accuracy ($I_{\text{Pitch}} = 2000 \text{ Slug Ft}^2$)

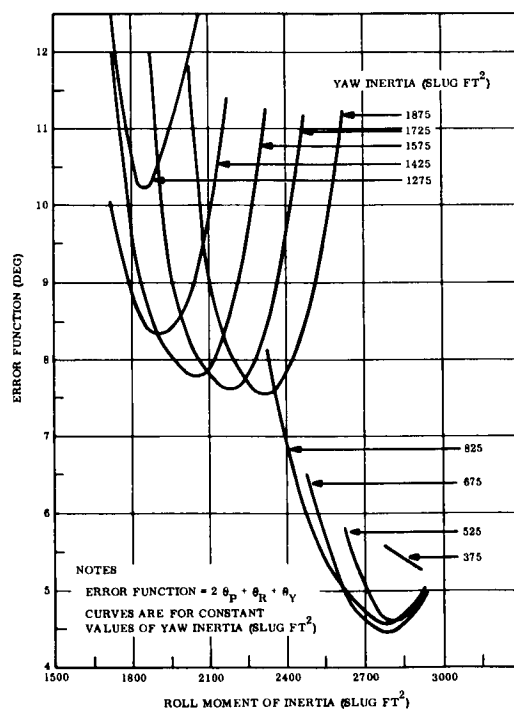


Figure 4-17. Parameter Study of Steady-State Pointing Accuracy ($I_{\text{Pitch}} = 3000 \text{ Slug Ft}^2$)

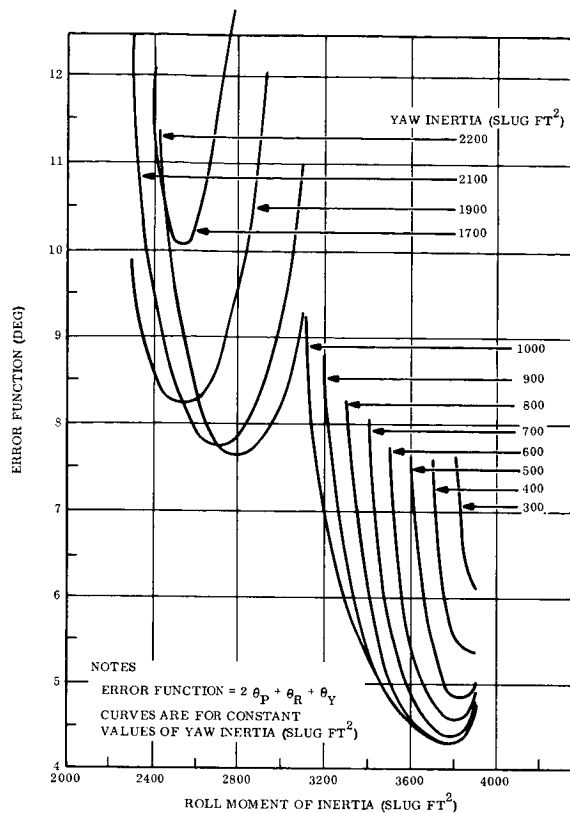


Figure 4-18. Parameter Study of Steady-State Pointing Accuracy ($I_{Pitch} = 4000 \text{ Slug Ft}^2$)

4.5.4 NONLINEAR OPTIMATION OF INERTIAS

This study consisted of selecting optimum values of the moment of inertias for which the satellite achieves the best steady-state pointing error. It was conducted by using the large-angle nonlinear digital simulation described in Section 5. The effect of solar pressure torques and the geometric rotation of the principal axis due to thermal bending of the rods are included in the program. The several values of the moment of inertias which were studied were selected from the optimum values determined from the linearized study. In selecting these values, the orbit eccentricity, which constitutes a position disturbance in pitch, and the gravity-gradient restoring torques were considered.

The eccentricity response is minimized by increasing the pitch natural frequency, which is dependent upon the quantity $\frac{I_R - I_Y}{I_P}$ and is maximized for $\frac{I_R}{I_P} = 1$ and $I_Y = 0$.

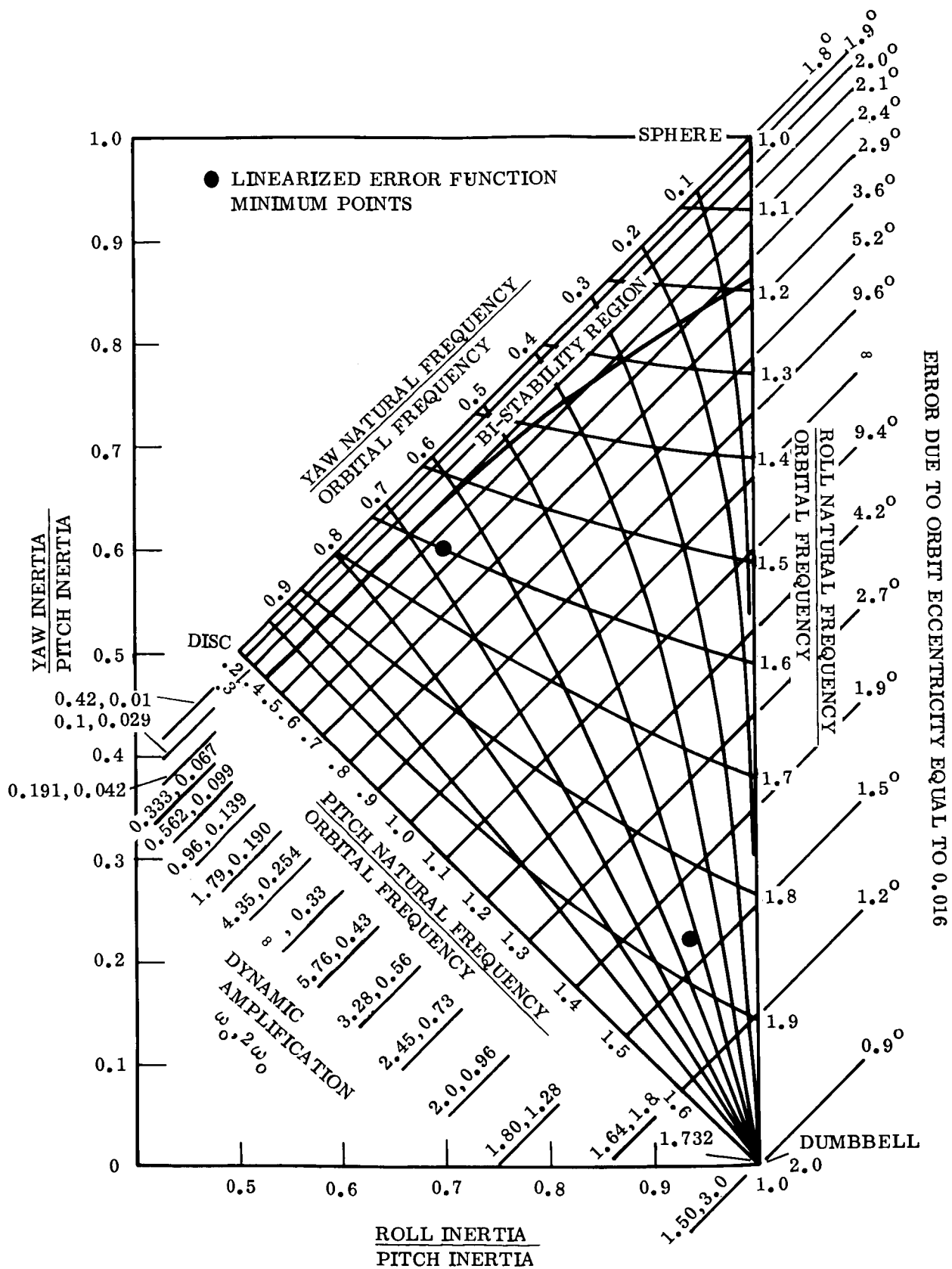


Figure 4-19. Natural Frequency Map

The gravity-gradient restoring torques on pitch and roll are also maximized by these conditions. It turns out, however, that these conditions minimize the gravity-gradient yaw restoring torque, which is dependent on $I_r - I_p$. It was observed in the previous section that the optimum linear case moments do not give $I_r/I_p = 1$ and $I_y = 0$. Tradeoff was possible for the linear case so that the yaw restoring torque is not nulled and the pitch natural frequency is in a maximum region. This type of tradeoff is desired also for the nonlinear case. Thus moments of inertia were selected bracketing the linear case moments.

As indicated in the previous section the linear optimum set of moments are

$$\begin{aligned} I_p &= 3000 \text{ slug ft} \\ I_r &= 2780 \text{ slug ft} \\ I_y &= 675 \text{ slug ft} \end{aligned}$$

The performance of the configurations with the bracketing moments was evaluated by the nonlinear digital simulation and are presented in Table 4-5 and Figures 4-20 and 4-21.

Table 4-5. Nonlinear Optimization Results

	I_p	I_r	I_y	θ_p	θ_r	θ_y	$(2\theta_p + \theta_r + \theta_y)$
Configuration No. 1	3000	2780	767	+3.0 -2.2	± 0.3	+1.6 -1.8	7.9
Configuration No. 2	3000	2770	396	+2.2 -1.5	± 0.5	+2.4 -2.5	7.3

A third computer run was then performed with the moments of inertia chosen between these two sets.

$$I_p = 3000; I_r = 2800; I_y = 575$$

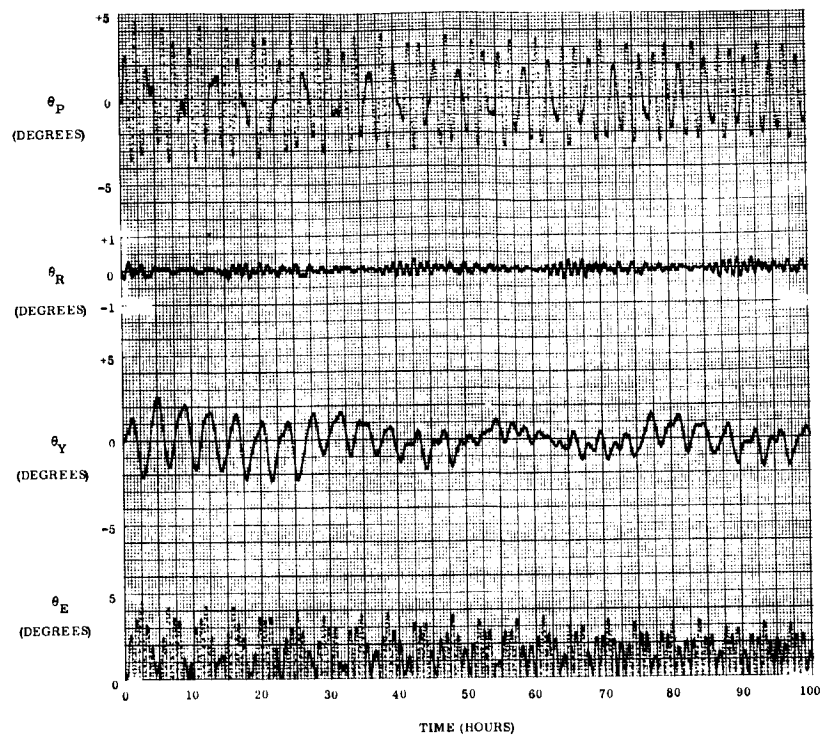


Figure 4-20. Steady-State Error (Configuration No. 1)

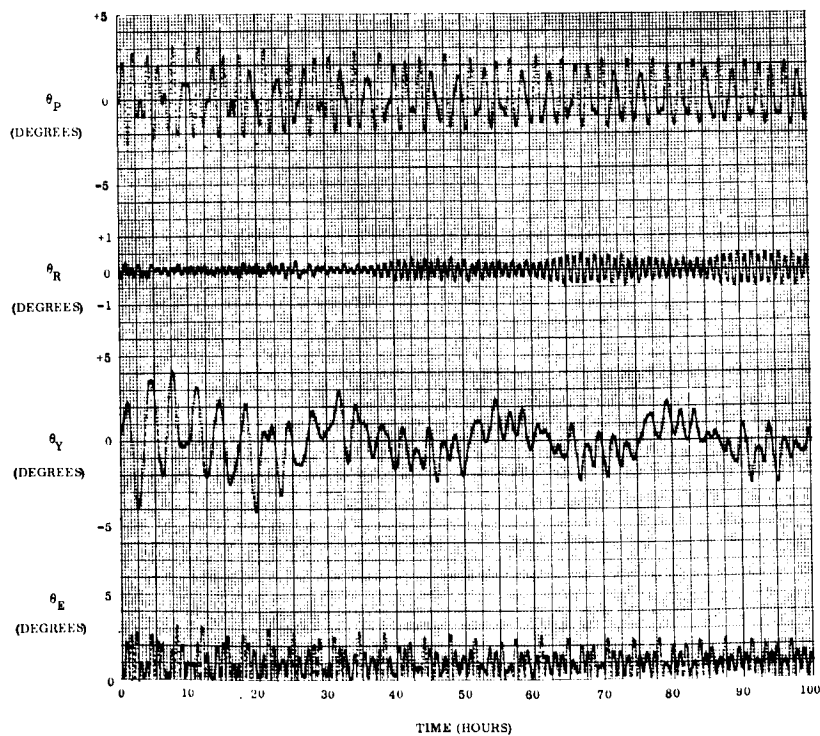


Figure 4-21. Steady-State Error (Configuration No. 2)

With these moments the maximum pitch, yaw and roll errors achieved are

$$\theta_p = \begin{matrix} +2.5 \\ -1.7 \end{matrix}$$

$$\theta_r = \pm 0.3$$

$$\theta_y = \begin{matrix} +1.8 \\ -1.4 \end{matrix}$$

$$2\theta_p + \theta_r + \theta_y = 7.1$$

All of these nonlinear digital simulation evaluations were made for the worst-case eccentricity of $\epsilon = 0.0179$. The linear optimization studies were made for the nominal eccentricity of $\epsilon = 0.0085$. The difference in eccentricity response is 0.85 degrees in pitch, which accounts for a 1.7 degree difference in error function between the linearized optimum and the nonlinear optimum. Thus the linearized optimum for worst eccentricity would have 6.1 degrees of error.

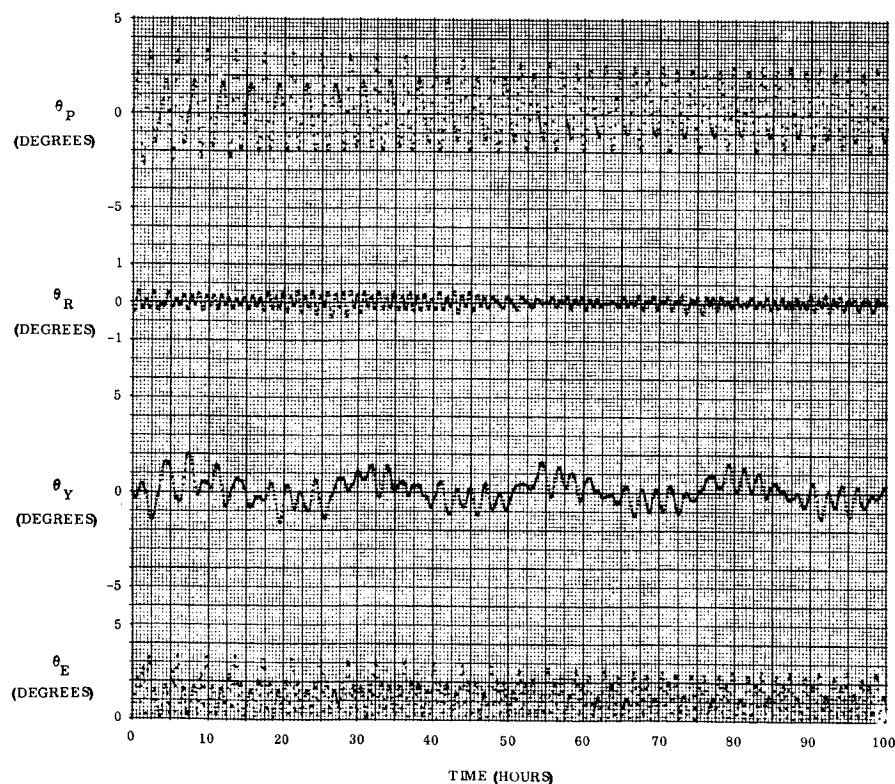


Figure 4-22. Steady-State Error (Configuration No. 3)

4.6 DISTURBANCE TORQUE ANALYSIS

The steady-state error angles between the satellite principal axes and the orbit reference axes are determined by the disturbance torques for a particular gravity-gradient stabilization configuration. The disturbance torques are calculated here and are used for prediction of the steady-state errors.

The significant disturbance torques which produce steady state errors are:

- a. Orbit eccentricity.
- b. Torques due to motion of the damper magnet.
- c. Torques from satellite residual magnetic dipole.
- d. Torques due to solar pressure.

Torques due to aerodynamic pressure are negligible for all altitudes within the worst-case eccentricity. The lowest mean altitude with ± 150 nautical mile eccentricity results in a minimum altitude of 625 nautical miles above the earth's surface. The most conservative estimates of atmospheric density at this altitude are about 10^{-16} slug/ft³. This density would occur if orbit perigee coincides with the peak density point of the atmospheric diurnal bulge during a maximum of the 11-year sunspot cycle. This density results in a dynamic pressure of 0.2×10^{-7} lb/ft², which is about 20% of solar pressure. For all nominal design cases (750 ± 75 nautical mile altitude) aerodynamic pressure is negligible.

Likewise, the change in nominal altitude of ± 25 nautical miles causes negligible variations in dynamic motion of the spacecraft. Twenty-five nautical miles represents about 0.6% of the total geocentric distance to the satellite. All important disturbance effects and the gravity-gradient torques are either nearly constant or proportional to the geocentric distance to a power so that the 0.6% change is negligible.

4.6.1 ORBIT ECCENTRICITY

Orbit eccentricity causes pitch errors in gravity-gradient oriented satellites because of the variation in the angular rate of the geocentric radius vector to the satellite. Consider the relationship between the true anomaly (η_0) and the eccentric anomaly (E) for an elliptical orbit, where ϵ is the eccentricity.

$$\text{TAN } \frac{\eta_0}{2} = \left[\frac{1 + \epsilon}{1 - \epsilon} \right]^{1/2} \text{TAN } \frac{E}{2}$$

The true anomaly is the angle measured from the perigee to the radius vector; i.e., it is the angular position of the radius vector with respect to an inertial coordinate frame.

$$\text{Let } \frac{1 + \beta}{1 - \beta} = \left[\frac{1 + \epsilon}{1 - \epsilon} \right]^{1/2}$$

For further applications the following expansions of powers of β in powers of ϵ are useful.

$$\beta = \frac{1}{2} \epsilon + \frac{1}{8} \epsilon^3 + \frac{1}{16} \epsilon^5 + \text{-----}$$

$$\beta^2 = \frac{1}{4} \epsilon^2 + \frac{1}{8} \epsilon^4 + \frac{5}{64} \epsilon^6 + \text{-----}$$

$$\beta^3 = \frac{1}{8} \epsilon + \frac{3}{32} \epsilon^5 + \frac{9}{128} \epsilon^7 + \text{-----}$$

Thus

$$\text{TAN } \frac{\eta_0}{2} = \frac{1 + \beta}{1 - \beta} \text{TAN } \frac{E}{2}$$

Let

$$A = e^{i\eta_0} \quad \text{and} \quad B = e^{iE}$$

Therefore

$$\frac{1}{i} \frac{A^{1/2} - A^{-1/2}}{A^{1/2} + A^{-1/2}} = \left(\frac{1}{i} \frac{1 + \beta}{1 - \beta} \right) \left(\frac{B^{1/2} - B^{-1/2}}{B^{1/2} + B^{-1/2}} \right)$$

From this

$$A = B \frac{1 - \beta B^{-1}}{1 - \beta B}$$

Taking the natural log of both sides

$$i \eta_0 = iE + \ln(1 - \beta B^{-1}) - \ln(1 - \beta B)$$

$$i \eta_0 = iE \left[\beta B^{-1} + \frac{1}{2} \beta^2 B^{-2} + \frac{1}{3} \beta^3 B^{-3} + \dots \right] \\ + \left[\beta B + \frac{1}{2} \beta^2 B^2 + \frac{1}{3} \beta^3 B^3 + \dots \right]$$

$$\eta_0 = E + 2 \left[\beta \sin(E) + \frac{1}{2} \beta^2 \sin(2E) + \frac{1}{3} \beta^3 \sin(3E) + \dots \right]$$

$$= E + 2 \sin(E) \left[\frac{1}{2} \epsilon + \frac{1}{8} \epsilon^3 + \dots \right]$$

$$+ \sin(2E) \left[\frac{1}{4} \epsilon^2 + \frac{1}{8} \epsilon^4 + \dots \right]$$

$$+ \frac{2}{3} \sin(3E) \left[\frac{1}{8} \epsilon^3 + \dots \right]$$

Neglecting terms beyond ϵ^3

$$\eta_o = E + \epsilon \sin(E) + 1/4 \epsilon^2 \sin(2E) + 1/4 \epsilon^3 [\sin(E) + 1/3 \sin(3E)]$$

The mean anomaly, M ,

$$M = E - \epsilon \sin(E)$$

is equivalent to the mean motion, ω_o , times time

$$M = \omega_{ot}$$

ω_o is the average orbital rate

Since $\epsilon < 1$

$$\sin(M) = \sin(E - \epsilon \sin(E))$$

$$\cong \sin(E) - \frac{\epsilon}{2} \sin(2E)$$

and

$$\sin(2M) \cong \sin(2E) - 2\epsilon \sin(E)$$

Thus

$$\eta_o \cong M + 2\epsilon \sin(E) + 1/4 \epsilon^2 \sin(2E) + 1/4 \epsilon^3 \sin(E)$$

$$= M + 2\epsilon (\sin(E) - \epsilon/2 \sin(2E)) + 5/4 \epsilon^2 \sin(2E) + 1/4 \epsilon^3 \sin(E)$$

$$\begin{aligned}\eta_o &= M + 2\epsilon \sin(M) + 5/4 \epsilon^2 (\sin(2E) - 2\epsilon \sin(E)) + 11/4 \epsilon^3 \sin(E) \\ &= M + 2\epsilon \sin(M) + 5/4 \epsilon^2 \sin(2M) + \dots\end{aligned}$$

Therefore for small values of ϵ , η_o can be expressed as

$$\eta_o = \omega_o t + 2\epsilon \sin(\omega_o t) + 5/4 \epsilon^2 \sin(2\omega_o t) + \dots$$

Since the worst-case eccentricity is $= 0.036$, the ϵ^2 and higher order terms may be neglected.

The gravity-gradient restoring torques which are present on the satellite are proportional to the angle between the satellite principal moment of inertia axis and the geocentric radius vector. The linearized equation for pitch motion, which is valid for small angles, can be written based on the equivalent dynamic system illustrated in Figure 4-23.

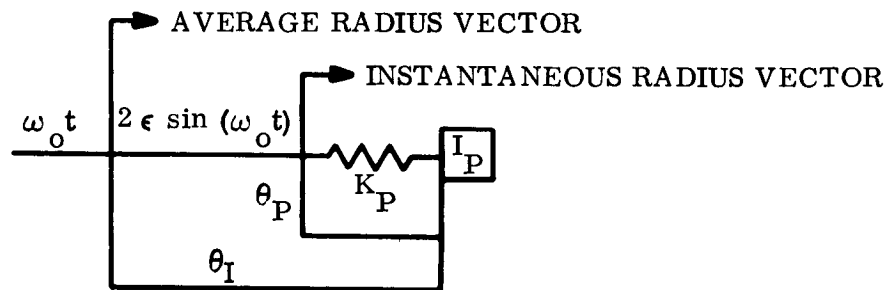


Figure 4-23. Equivalent Dynamic System for Pitch Motion

where θ_p = angle between satellite axis and instantaneous radius vector

θ_I = angle between satellite axis and average radius vector

The equation of motion is:

$$I_p \ddot{\theta}_I + K_p \theta_p = 0$$

However :

$$\theta_p = \theta_I - 2\epsilon \sin(\omega_o t)$$

Therefore:

$$I_p \ddot{\theta}_I + K_p \theta_I = K_p (2\epsilon \sin(\omega_o t))$$

The torques due to the eddy-current damper have a negligible effect on the steady-state response because the damping is far below critical and the natural frequency of the pitch motion is not near the disturbance frequency. The steady-state amplitude of θ_I at orbital frequency is given by:

$$\theta_I(t) = \frac{2\epsilon K_p}{K_p - I_p \omega_o^2} \sin(\omega_o t)$$

The gravity-gradient restoring torque can be written

$$K_p = \frac{T_p}{\theta_p} = 3\omega_o^2 (I_r - I_y)$$

Then

$$\theta_I(t) = \frac{2\epsilon \sin(\omega_o t)}{1 - \frac{I_p}{3(I_r - I_y)}}$$

But since

$$\theta_I = \theta_p + 2\epsilon \sin(\omega_o t)$$

$$\theta_p(t) = \left[\frac{1}{1 - \frac{I_p}{3(I_r - I_y)}} - 1 \right] 2\epsilon \sin(\omega_o t) = \left[\frac{1}{3\left(\frac{I_r}{I_p} - \frac{I_y}{I_p}\right) - 1} \right] 2\epsilon \sin(\omega_o t)$$

$(\theta_p(t))$ is the pitch error angle due to eccentricity between the satellite vertical reference axis and the instantaneous local vertical. It can be seen that this error depends only on the eccentricity and the inertia ratios. The quantity

$$\frac{I_r - I_y}{I_p}$$

which determines the pitch response is proportional to the pitch motion natural frequency squared as shown below:

$$\omega_n^2 = \frac{K_p}{I_p} = 3\omega_o^2 \left(\frac{I_r}{I_p} - \frac{I_y}{I_p} \right)$$

Thus, minimum response is obtained when $I_y = 0$ and $I_r = I_p$.

I_r cannot be greater than I_p or the vehicle will be unstable in yaw and rotate 90 degrees so that $I_p > I_r$ in the new position.

The moments of inertia selected for the NIMPAC final design very nearly optimize eccentricity response. Substituting the final inertia values of:

$$I_p = 3000$$

$$I_r = 2800$$

$$I_y = 575$$

gives an eccentricity response of $\theta_p(t) = 1.63 \sin \omega_o t$ (radians). For the nominal eccentricity of 0.0089 (75 nautical miles), the pitch error is ± 0.85 degrees at orbital frequency.

4.6.2 DAMPER DISTURBANCE TORQUES

Disturbance torques are produced on the satellite by the eddy-current damping effect of the spherical dampers as the magnets rotate to follow the earth's magnetic field. These disturbance torques are proportional to the relative rate between the satellite and the damper magnet. The proportionality constant is the damper linear damping coefficient B (ft-lb/rad/sec). The relative motion between damper magnet and spacecraft can be approximated by the relative angular motion between the earth's magnetic field vector and the orbit reference axes, since the magnet and spacecraft follow these vectors respectively within a few degrees.

The total magnetic field vector for a dipole approximation to the earth's magnetic field may be expressed as:

$$H = \frac{-Me}{r^3} \left[2 \cos \theta \bar{r} - \sin \theta \bar{n} \right]$$

where

M_e = earth's magnetic strength constant

$$= 8.06 \times 10^{25} \text{ gauss/cm}^3$$

r = satellite geocentric distance

$$= 7.76 \times 10^8 \text{ (cm) for 750 nautical-mile altitude}$$

θ = magnetic colatitude

$\bar{r}$ = radial unit vector

$\bar{n}$ = normal unit vector

The geomagnetic coordinate system is shown in Figure 4-24:

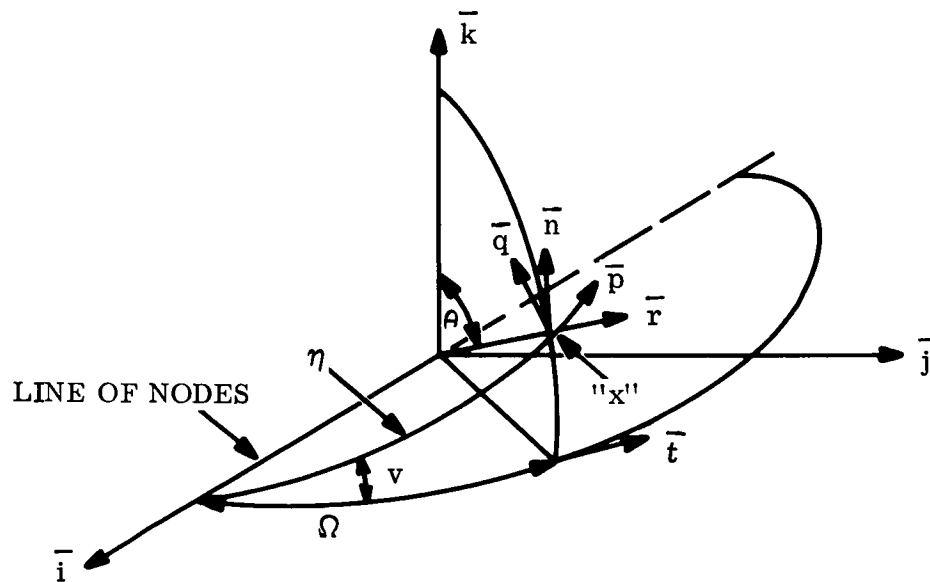


Figure 4-24. Geomagnetic Coordinate System

The location of the point "X" is also specified in terms of the angles of an orbit defined with respect to the magnetic equator.

- V = inclination of the orbit plane from the magnetic equator
- η = orbit angle from the line of nodes
- Ω = longitude from the line of nodes

The set of coordinate axes ($\bar{i}$, $\bar{j}$, $\bar{k}$) are defined to be inertially fixed.

- $\bar{i}$ = along the line of nodes, positive outward
- $\bar{j}$ = perpendicular to $\bar{i}$ in the equatorial plane, positive outward
- $\bar{k}$ = perpendicular to $\bar{i}$ and $\bar{j}$ such that $\bar{i} \times \bar{j} = \bar{k}$, positive north.

A set of coordinate axes ($\bar{r}$, $\bar{p}$, $\bar{q}$) are shown which move along the orbit path with point "X." These reference axes are defined as follows:

- $\bar{r}$ = along the geocentric line, positive outward
- $\bar{q}$ = perpendicular to the orbit plane, positive in the direction of orbit angular rate
- $\bar{p}$ = perpendicular to $\bar{r}$ and $\bar{q}$, positive along the velocity vector.

The matrix (A) defines the relationship between the orthogonal sets ($\bar{r}$, $\bar{p}$, $\bar{q}$) and ($\bar{i}$, $\bar{j}$, $\bar{k}$).

$$\begin{bmatrix} \bar{r} \\ \bar{p} \\ \bar{q} \end{bmatrix} = [A] \begin{bmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{bmatrix}$$

where

$$\begin{array}{llll}
 a_{11} & = & \cos \eta & \\
 a_{12} & = & \sin \eta \cos V & \\
 a_{13} & = & \sin \eta \sin V & a_{31} = 0 \\
 & & & a_{32} = -\sin V \\
 a_{21} & = & -\sin \eta & a_{33} = \cos V \\
 a_{22} & = & \cos \eta \cos V & \\
 a_{23} & = & \cos \eta \sin V &
 \end{array}$$

As shown in Figure 4-24, the vector $\bar{t}$ is perpendicular to the vectors $\bar{r}$ and $\bar{n}$ such that the vector product $\bar{r} \times \bar{n} = -\bar{t}$.

$$\begin{bmatrix} \bar{r} \\ \bar{n} \\ \bar{t} \end{bmatrix} = [B] \begin{bmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{bmatrix}$$

where

$$\begin{array}{llll}
 b_{11} & = & \sin \theta \cos \Omega & \\
 b_{12} & = & \sin \theta \sin \Omega & \\
 b_{13} & = & \cos \theta & b_{31} = -\sin \Omega \\
 & & & b_{32} = \cos \Omega \\
 b_{21} & = & -\cos \theta \cos \Omega & b_{33} = 0 \\
 b_{22} & = & -\cos \theta \sin \Omega & \\
 b_{23} & = & \sin \theta &
 \end{array}$$

The relationship between $(\bar{r}, \bar{p}, \bar{q})$ and $(\bar{r}, \bar{n}, \bar{t})$ is given by

$$\begin{bmatrix} \bar{r} \\ \bar{p} \\ \bar{q} \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} \bar{r} \\ \bar{n} \\ \bar{t} \end{bmatrix}$$

where

$$\begin{aligned} C_{11} &= 1 \\ C_{12} &= 0 \\ C_{13} &= 0 \\ C_{21} &= 0 \\ C_{22} &= \sin^2 \eta \sin V \cos \Omega - \cos \eta \cos V \sin \eta \sin V \sin \Omega + \cos \eta \sin V \sin \Omega \\ C_{23} &= \sin \eta \sin \Omega + \cos \eta \cos V \cos \Omega \\ C_{31} &= 0 \\ C_{32} &= \sin^2 V \sin \eta \sin \Omega + \cos V (1 - \sin^2 \eta \sin^2 V)^{1/2} \\ C_{33} &= -\sin V \cos \Omega \end{aligned}$$

The components of the magnetic field vector $\bar{H}$ along $\bar{r}, \bar{p}, \bar{q}$ are given by:

$$\bar{H} = -\frac{M_e}{r^3} \left[2 \sin \eta \sin V \bar{r} - \sin V \cos \eta \bar{p} - \cos V \bar{q} \right]$$

The angular rate vector of the magnetic field may be expressed by using components in the r, p, q system and by its components along and perpendicular to the H vector.

$$\begin{aligned} \omega &= \omega_r \bar{r} + \omega_p \bar{p} + \omega_q \bar{q} \\ &= \omega_{11} \bar{r} + \omega_\eta \bar{M} + \omega_t \bar{t} \end{aligned}$$

$$\begin{aligned}\omega = & \omega_{11} \bar{r} + \omega_{\eta} \left[C_{12} \bar{r} + C_{22} \bar{p} + C_{32} \bar{q} \right] \\ & + \omega_t \left[C_{13} \bar{r} + C_{23} \bar{p} + C_{33} \bar{q} \right]\end{aligned}$$

Thus

$$\begin{aligned}\omega_r &= \omega_{11} \\ \omega_p &= \omega_{11} C_{21} + \omega_{\eta} C_{22} + \omega_t C_{23} \\ \omega_q &= \omega_{11} C_{31} + \omega_{\eta} C_{32} + \omega_t C_{33}\end{aligned}$$

Since the damper magnet follows the orbital motion, it is necessary to consider the projection of the orbital rate on axes parallel and perpendicular to the magnet. In fact, it turns out that the parallel component of the magnet angular rate is the projection of the orbital rate and $\dot{\Omega}$ on the r axis.

$$\omega_{11} = C_{31} \omega_o + \dot{\Omega} \cos \theta$$

where ω_o is the orbital rate and $\eta = \omega_o t$

The perpendicular components are comprised of the projection of the orbital rate and the components of $(\dot{\theta}$ and $\dot{\Omega})$ with respect to time.

Thus

$$\omega_M = \dot{\Omega} \sin \theta + C_{32} \omega_0$$

$$\omega_t = -\dot{\theta} + C_{33} \omega_0$$

$$\omega_r = \dot{\Omega} \cos \theta$$

$$\omega_p = C_{22} (\dot{\Omega} \sin \theta + C_{32} \omega_0) + C_{23} (-\dot{\theta} + C_{33} \omega_0)$$

$$\omega_q = C_{32} (\dot{\Omega} \sin \theta + C_{32} \omega_0) + C_{33} (-\dot{\theta} + C_{33} \omega_0)$$

The rate of change of θ and Ω with respect to time is determined in the following manner:

$$\cos \theta = \frac{\bar{H} \cdot \bar{K}}{|\bar{H}|} = \frac{\sin^2 V (2 \sin^2 \eta - \cos^2 \eta) - \cos^2 V}{(4 \sin^2 V \sin^2 \eta + \sin^2 V \cos^2 \eta + \cos^2 V)^{1/2}}$$

$$\sin \theta = \frac{3 \sin V \sin \eta (1 - \sin^2 V \sin^2 \eta)^{1/2}}{(4 \sin^2 V \sin^2 \eta + \sin^2 V \cos^2 \eta + \cos^2 V)^{1/2}}$$

therefore

$$\dot{\theta} = \frac{-3 \omega_0 \sin V \cos \eta \left[1 + \sin^2 V \sin^2 \eta \right]}{\left[4 \sin^2 V \sin^2 \eta + \sin^2 V \cos^2 \eta + \cos^2 V \right] (1 - \sin^2 V \sin^2 \eta)^{1/2}}$$

The rate of change of $\dot{\Omega}$ is the projection of orbital rate on the $\bar{k}$ axis.

Thus

$$\dot{\Omega} = \omega_o \cos \nu$$

With this, all the quantities necessary in the determination of ω_r , ω_p and ω_q have been found in terms of the orbit angle, ξ , in the inclination ν and the orbital rate ω_o .

For an inclination of 90 degrees, polar orbit, the expression for the pitch angular rate is given by

$$\omega_q = \frac{-2 \omega_o}{1 + 3 \sin^2 \omega_o t}$$

and the pitch damping torque is

$$T_q = \beta \omega_q$$

expanding in a Fourier series

$$T_q = B \omega_o (1 + \cos 2 \omega_o t + \dots)$$

The damper disturbance torques which act on the spacecraft pitch, roll, and yaw axes are computed in the nonlinear digital simulation as a function of the body rates for the damper magnet and the spacecraft body. The time history of these torques is plotted in Figure 4-25. The pitch torque has a bias component and a twice-orbital-frequency component. The magnitude of the bias component is 2.4×10^{-5} ft-lb (which equals $B \omega_o$), since the magnet turns over about pitch with respect to the spacecraft once each orbit. The twice-orbital-frequency oscillations is about $\pm 1.8 \times 10^{-5}$ ft-lb. The magnet torque on the yaw axis of the spacecraft is not a simple orbital function, since it is distorted by the rotation of the earth's magnetic field. The largest component of torque occurs at orbital frequency and has a

magnitude of $\pm 5 \times 10^{-6}$ ft-lb. Likewise, the roll disturbance torque is distorted by earth's rotation but has a large component of orbital frequency with a magnitude of $\pm 8 \times 10^{-6}$ ft-lb.

The response of the spacecraft to these disturbances based on the linearized equations of motion are calculated below:

Pitch response:

$$\theta_p = \frac{1}{I_p \omega_o^2} \left[\frac{T_{BIAS}}{3 \left(\frac{I_R}{I_P} - \frac{I_Y}{I_P} \right)} + \frac{T_{2\omega_o}}{\left[-4 + 3 \left(\frac{I_R}{I_P} - \frac{I_Y}{I_P} \right) \right]} \right]$$

= 0.26 degrees bias

and 0.33 degrees at twice orbital frequency

Yaw response:

$$\theta_y = \frac{T \omega_o}{3 \omega_o^2} \left(\frac{4 - 4 \frac{I_Y}{I_P} - \frac{I_R}{I_P}}{\left(1 - \frac{I_R}{I_P} - \frac{I_Y}{I_P} \right) (I_P - I_Y)} \right)$$

= ± 0.85 degrees at orbital frequency

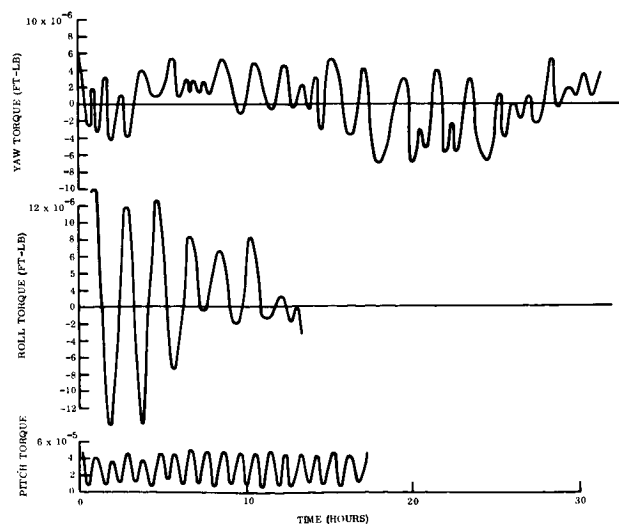


Figure 4-25. Damper Torque vs Time

Roll response:

$$\theta_R = \frac{T \omega_o}{I_P \omega_o^2} \left[\frac{1}{3 \left(1 - \frac{I_Y}{I_P} \right)} \right] = \pm 0.08 \text{ degrees at orbital frequency}$$

4.6.3 SPACECRAFT RESIDUAL MAGNETIC DIPOLE TORQUES

Disturbance torques are produced on the satellite by magnetic interactions between the earth's magnetic field and any residual magnetic dipole associated with the satellite. These torques can be calculated based on the dipole model of the earth's magnetic field assumed in the previous section. The spacecraft is assumed to be oriented along the $\bar{r}$, $\bar{p}$, $\bar{q}$ (yaw, roll, pitch) reference axis, and to have residual dipole magnitudes of M_r , M_p , and M_q along these axes. Using the expression for the components of earth's magnetic field along the $\bar{r}$, $\bar{p}$, $\bar{q}$ axes,

$$\bar{H} = -\frac{M_e}{r^3} \left[2 \sin \eta \sin v \bar{r} - \sin v \cos \eta \bar{p} - \cos v \bar{q} \right]$$

Components of torque about $\bar{r}$, $\bar{p}$, $\bar{q}$ are given by:

$$\begin{aligned} T_r &= -\frac{M_e}{r^3} \left[M_p \cos v + M_q \sin v \cos \omega_o t \right] \\ T_p &= -\frac{M_e}{r^3} \left[2 M_q \sin v \sin \omega_o t + M_r \cos v \right] \\ T_q &= -\frac{M_e}{r^3} \left[-M_r \sin v \cos \omega_o t - 2 M_p \sin v \sin \omega_o t \right] \end{aligned}$$

The worst-case pitch torque occurs when $v = 90$ degrees (magnetic polar orbit), which occurs twice each day due to rotation of the earth. The pitch torque is equal for a dipole along the roll (p) axis or the yaw axis (r). The pitch torque for a 100 pole cm dipole is:

$$T_q = 2.5 \times 10^{-6} \sin \omega_o t \text{ (ft-lb)}$$

The pitch error response is:

$$\theta_p = \frac{T_q}{I_p \omega_o^2} \left[\frac{1}{-1 + 3 \left(\frac{I_R}{I_P} - \frac{I_Y}{I_P} \right)} \right]$$

$$= 0.0457 \text{ degrees}$$

The worst-case orbit inclination and dipole location for yaw torque depends upon the amplitude response of the gravity-gradient system to constant and orbital frequency disturbances. It can be seen that a dipole along the roll axis causes a yaw bias and a dipole along the pitch axis causes a yaw oscillation at orbital frequency. The amplitude response functions based on small angle equations of motion (from linear optimization) are:

$$\text{Bias } \theta_y = \frac{T_r}{\omega_o^2 (I_P - I_R)}$$

Orbital frequency

$$\theta_y = \frac{T_r \left[4 - 4 \frac{I_Y}{I_P} - \frac{I_R}{I_P} \right]}{3 \omega_o^2 \left[1 - \frac{I_R}{I_P} - \frac{I_Y}{I_P} \right] (I_P - I_t)}$$

Substituting the moments of inertia of the final configuration

$$I_P = 3000$$

$$I_R = 2800$$

$$I_Y = 575$$

$$\text{Bias } \theta_y = 3.36 \times 10^{+5} T_r \text{ (degrees)}$$

Orbital frequency

$$\rho_y = \pm 1.67 \times 10^5 T_r \text{ (degrees)}$$

The maximum values of T_r are

Bias ($v = 67^\circ$)

$$T_r = -0.495 \times 10^{-6} \text{ (ft-lb)}$$

Orbital frequency ($v = 90^\circ$)

$$T_r = -1.27 \times 10^{-6} \text{ (ft-lb)}$$

The peak yaw errors are:

$$\text{Bias } \theta_y = -0.17 \text{ degrees}$$

Orbital frequency

$$\theta_y = \pm 0.21 \text{ degrees}$$

The worst-case orbit inclination and dipole position for roll torque are $v = 90$ degrees, and a dipole along $\bar{q}$ (pitch). The peak roll torque is:

$$T_p = 2.5 \times 10^{-6} \sin \omega_o t$$

The roll response at orbital frequency is:

$$\theta_R = \frac{T_P}{I_p \omega_o^2} \left[\frac{1}{3 \left(1 - \frac{I_Y}{I_P} \right)} \right] = \pm 0.023 \text{ degree}$$

4.6.4 SOLAR PRESSURE TORQUES

Solar pressure torques occur on the spacecraft because of the asymmetry of the rod configuration and the thermal bending of the gravity-gradient rods due to uneven temperature distribution. The solar pressure torques have been held to a low value by careful selection of the rod lengths and rod tip weights so that the center of solar pressure coincides with the center of mass when the sun is on the vehicle roll or yaw axis. Solar torques are not zero, however, even for straight gravity-gradient rods when the sun is in other positions relative to the spacecraft.

Additional solar pressure torques are introduced by the effects of thermal bending of the gravity-gradient rods. The rods tend to bend away from the sun due to the greater expansion of the heated side of the rod. The shape of the rod is very close to a circular arc for rod lengths below 200 feet. This curvature of the rod causes a shift in the overall spacecraft center of mass position, which unbalances the solar pressure forces and results in torque. In addition, the solar pressure on the curved rods results in a shift in the center of solar pressure due to rod curvature. All of these contributions to solar pressure torque are evaluated in the nonlinear digital simulation as a function of instantaneous vehicle position with respect to the sun. The time history of these torques on the pitch, roll, and yaw body axes are plotted in Figure 4-26. The torques are for the nominal case of the sun in the orbit plane. As expected, the roll and yaw torques are extremely small, on the order of 5×10^{-8} ft-lb. The solar torque about the pitch axis is also very small compared to other disturbance torques. The pitch solar torque has a relatively large orbital frequency component as well as components at higher frequencies. Estimating the orbital frequency component as 3×10^{-6} ft-lb, the pitch angular response at orbital frequency is:

$$\theta_p = \frac{T\omega_o}{I_p \omega_o^2} \left[\frac{1}{-1 + 3\left(\frac{I_R}{I_P} - \frac{I_Y}{I_P}\right)} \right]$$

= 0.055 degrees at orbital frequency

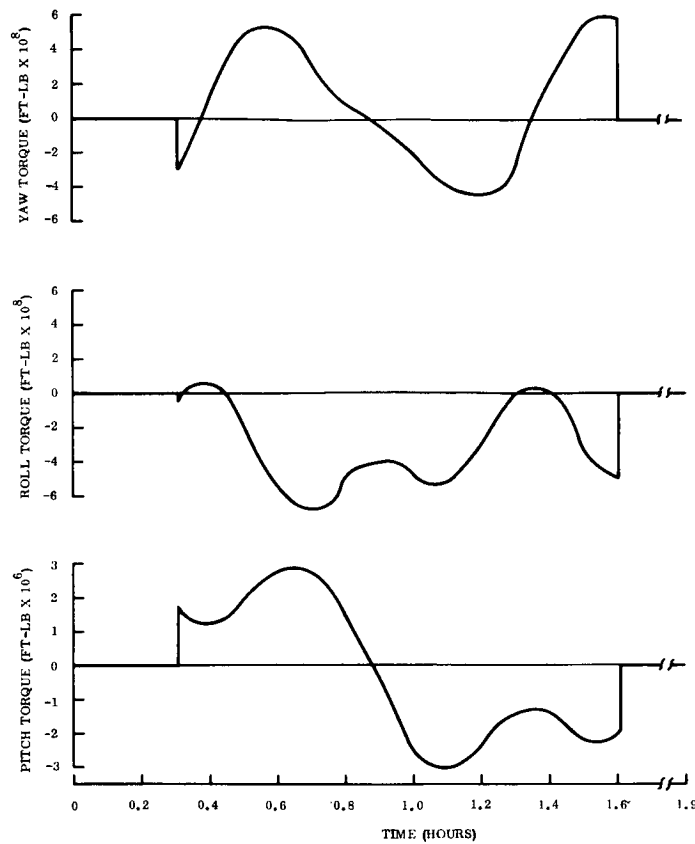


Figure 4-26. Solar Torque (Sun in Orbit Plane)

The response to other frequency components in pitch and all solar torques in roll and yaw will be smaller than this and are considered negligible.

In addition to solar pressure torques, the curvature of the rods introduces a shift in the angular position of the spacecraft principal axes with respect to the spacecraft central body. This effect is calculated in the nonlinear digital simulation by summing the moment of inertia contributions of rod and tip masses in the curved position about reference axes fixed to the central body.

4.6.5 ANALYSIS OF UNCOMPENSATED MOMENTUM

The uncompensated momentum specified is along the pitch axis of the spacecraft and has a constant magnitude component of 25×10^{-5} ft-lb-sec and an additional 25×10^{-4} ft-lb-sec momentum for 4 minutes per orbit with worst-case phasing.

A linearized analysis has been made to show that the constant component of momentum along the pitch axis results in negligible torques on the spacecraft.

The torques on the spacecraft due to a constant momentum along the pitch (Z) axis are:

$$T_x = -H \omega_y \quad (\text{yaw})$$

$$T_y = -H \omega_x \quad (\text{roll})$$

$$T_z = 0 \quad (\text{pitch})$$

where

$$\left. \begin{array}{l} \omega_x \\ \omega_y \end{array} \right\} \text{ rates with respect to inertial space about the spacecraft body fixed axes.}$$

The linearized equations of motion for a rigid body in orbit assuming small angle motions are given below:

$$I_R \ddot{\theta}_R + \omega_o \dot{\theta}_y (I_Y - I_P + I_R) + 4 \omega_o^2 \theta_R (I_P - I_Y) = T_y \quad (\text{roll})$$

$$I_Y \ddot{\theta}_y + \omega_o \dot{\theta}_R (-I_Y + I_P - I_R) + \omega_o^2 \theta_y (I_P - I_R) = T_x \quad (\text{yaw})$$

$$I_P \ddot{\theta}_p + 3 \omega_o^2 \theta_p (I_R - I_Y) = T_z = 0$$

where:

R, Y, P refers to roll, yaw, and pitch

X, Y, Z refers to yaw, roll, and pitch

I_r , I_y , I_p are the roll, yaw and pitch moments of inertia

θ_p , θ_r , θ_y are the Euler rotations, in order, which specify the spacecraft position with respect to the orbit reference frame.

The torques on the spacecraft body may be expressed in terms of the Euler angle rates by means of the following equations which relate body rates to Euler angle rates:

$$\omega_x = \dot{\theta}_y - (\dot{\theta}_p + \omega_o) \sin \theta_R$$

$$\omega_y = \dot{\theta}_R \cos \theta_y + (\dot{\theta}_p + \omega_o) \sin \theta_y \cos \theta_R$$

$$\omega_z = -\dot{\theta}_R \sin \theta_y + (\dot{\theta}_p + \omega_o) \cos \theta_y \cos \theta_R$$

Assuming small angles and dropping second order terms, the torques due to the constant uncompensated momentum are:

$$T_x = -H (\dot{\theta}_R + \omega_o \theta_y)$$

$$T_y = H (\dot{\theta}_y - \omega_o \theta_R)$$

A comparison of the magnitudes of T_x and T_y with similar terms in the spacecraft body equations of motion shows these torques to be negligible.

$$T_x = -25 \times 10^{-5} (\dot{\theta}_R + \omega_o \theta_y) \quad (\text{ft-lb})$$

$$T_y = 25 \times 10^{-5} (\dot{\theta}_y - \omega_o \theta_R) \quad (\text{ft-lb})$$

$$\omega_o (I_y - I_p + I_R) \dot{\theta}_y = .346 \theta_y \quad (\text{ft-lb})$$

$$4\omega_o (I_p - I_y) \omega_o \theta_R = 9 \omega_o \theta_R \quad (\text{ft-lb})$$

$$\omega_o (-I_y + I_p - I_R) \dot{\theta}_R = -.346 \theta_R \quad (\text{ft-lb})$$

$$\omega_o (I_p - I_R) \omega_o \theta_y = .185 \omega_o \theta_y \quad (\text{ft-lb})$$

Thus the torques due to constant uncompensated momentum are three orders of magnitude below the gravity-gradient restoring torques.

It can also be concluded that the time varying pitch momentum will effect only the pitch axis and can be evaluated by applying a pitch body torque which will produce the specified variation in momentum. Such a torque has been added to the nonlinear digital simulation in order to evaluate its effect on spacecraft steady-state pointing accuracy. The torque used in the evaluation is plotted in Figure 4-27.

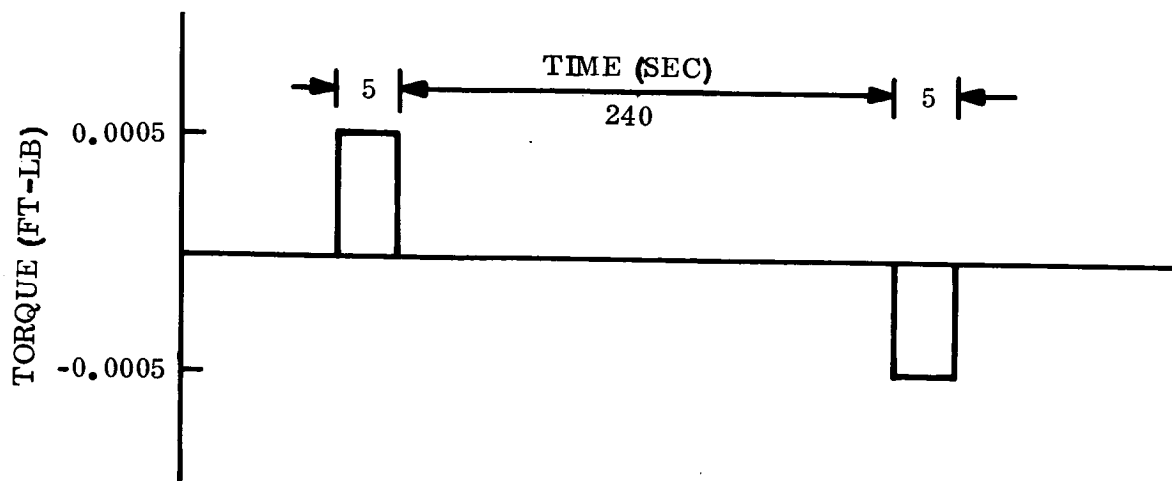


Figure 4-27. Torque Profile for Uncompensated Momentum Pulse

4.7 CAPTURE STUDIES

4.7.1 GENERAL

A satellite is captured when it is oscillating about the local vertical. Gravity-gradient torque acting on the satellite tends to align the axis of least moment of inertia along the local vertical, the axis of greatest moment of inertia along the normal to the orbit plane, and the third axis along the velocity vector.

Insight into the capture phenomena can be gained by the simple analogy with a pendulum in the earth's gravitational field. If the pendulum has a very large amount of energy it will "go over the top" and continue to rotate, slowing down and speeding up as energy is transferred between kinetic and potential energy. There is a critical energy, dependent on initial velocity and position, where the pendulum will just make it to the top. This critical energy provides the capture bound. Inside the capture bound the pendulum will oscillate, or "be captured" while outside the capture bound the pendulum will rotate, or "tumble."

This analogy provides the basic definition of capture relative to an orbiting satellite, in that a "critical" level of total system energy exists below which the satellite will be incapable of tumbling.

After separation from the last booster stage, the satellite may have an initial rate of rotation about any or all three of the principal axes. The capture phase is initiated by extending the gravity-gradient rods. Since angular momentum will be conserved (light damping) these initial rates will be reduced by the reciprocal of the ratio of system inertia growth, that is, by the ratio of initial to final inertias. The total energy of the system, kinetic plus potential, at the point of full rod extension will therefore determine whether the satellite will capture without tumbling or whether time must be allowed for energy to be dissipated (by way of the eddy-current dampers) until the level is less than "critical."

The primary task relative to capture evaluation to be performed under this study is to determine the maximum time which may elapse between initial rod extension and final capture (to the local vertical), for the "worst case" initial conditions within the specified range. During this evaluation, the actual limits of initial conditions for capture occurring without tumbling were also determined.

The maximum initial rates and position angles to be considered in this analysis are 1 degree/second and 5 degree/seconds, respectively, on all three principal axes.

The final results of the Capture studies are as follows:

- a. The "worst case" initial conditions were determined to be a negative pitch rate and angle coupled with any combination of positive and/or negative roll and yaw conditions.
- b. For the worst-case initial conditions, the maximum expected time from rod extension to capture does not exceed 6 hours.
- c. If the separation rates do not exceed -0.8 degree/second about each of the three axes, the satellite will capture immediately without tumbling. If the pitch rate should be positive, a rate significantly greater than 1 degree/second may be tolerated without tumbling.

4.7.2 TOTAL ENERGY CRITERIA FOR CAPTURE WITHOUT TUMBLE

As stated previously, the total system energy must be established at the end of rod extension in order to predict capture capability. The total energy consists of the potential energy associated with a body being acted upon by an external force field (earth's gravity) and the kinetic energy due to the motion of the body.

The potential energy of the spacecraft taken with respect to the orbit reference frame (i. e. , potential energy is minimum at nominal orientation) is given by:

$$\begin{aligned}
 (P. E.) = & \frac{1}{2} \omega_o^2 I_z \left[1 - 3 \cos^2 \theta_p \cos^2 \theta_R + 3 \sin^2 \theta_p \sin^2 \theta_y \sin^2 \theta_R \right. \\
 & \left. - 2 \cos^2 \theta_R - 3 \cos^2 \theta_y (\cos^2 \theta_p \cos^2 \theta_y - \cos^2 \theta_p) \right] \\
 & + \frac{1}{4} \omega_o^2 I_y \left[1 - 3 \cos^2 \theta_p \cos^2 \theta_R - 3 \sin^2 \theta_p \sin^2 \theta_y \sin^2 \theta_R \right. \\
 & \left. - 2 \sin^2 \theta_y \cos^2 \theta_R + 3 \cos^2 \theta_y (\cos^2 \theta_p \cos^2 \theta_R - \right. \\
 & \left. 2 \cos^2 \theta_p) \right] \\
 & + \frac{1}{2} \omega_o^2 I_x \left[3 \cos^2 \theta_R \cos^2 \theta_p - \sin^2 \theta_R - 1 \right]
 \end{aligned}$$

Where θ_p , θ_r , θ_y are the Euler angles with respect to the orbiting reference frame; I_x , I_y , I_z are the system moments of inertia about the body principal axes; and ω_o is the orbital rate of the spacecraft.

Once the orbit is defined and the system moments of inertia established, the potential energy is a function of the Euler angles which specify the vehicle position with respect to the orbit reference frame. Figure 4-28 represents one quadrant of the surface for positive values of pitch and roll angles and for zero yaw rotation.

Figure 4-29 is an identical plot except for a yaw position of 90 degrees. (Boundary lines for the zero yaw case of Figure 4-28 are superimposed in phantom.)

This potential energy map serves two purposes in this analysis. First, it establishes the critical energy level which the systems may possess without subsequent tumbling. This level is represented by the difference in energy between a pitch angle of 0 and + 90 degrees.

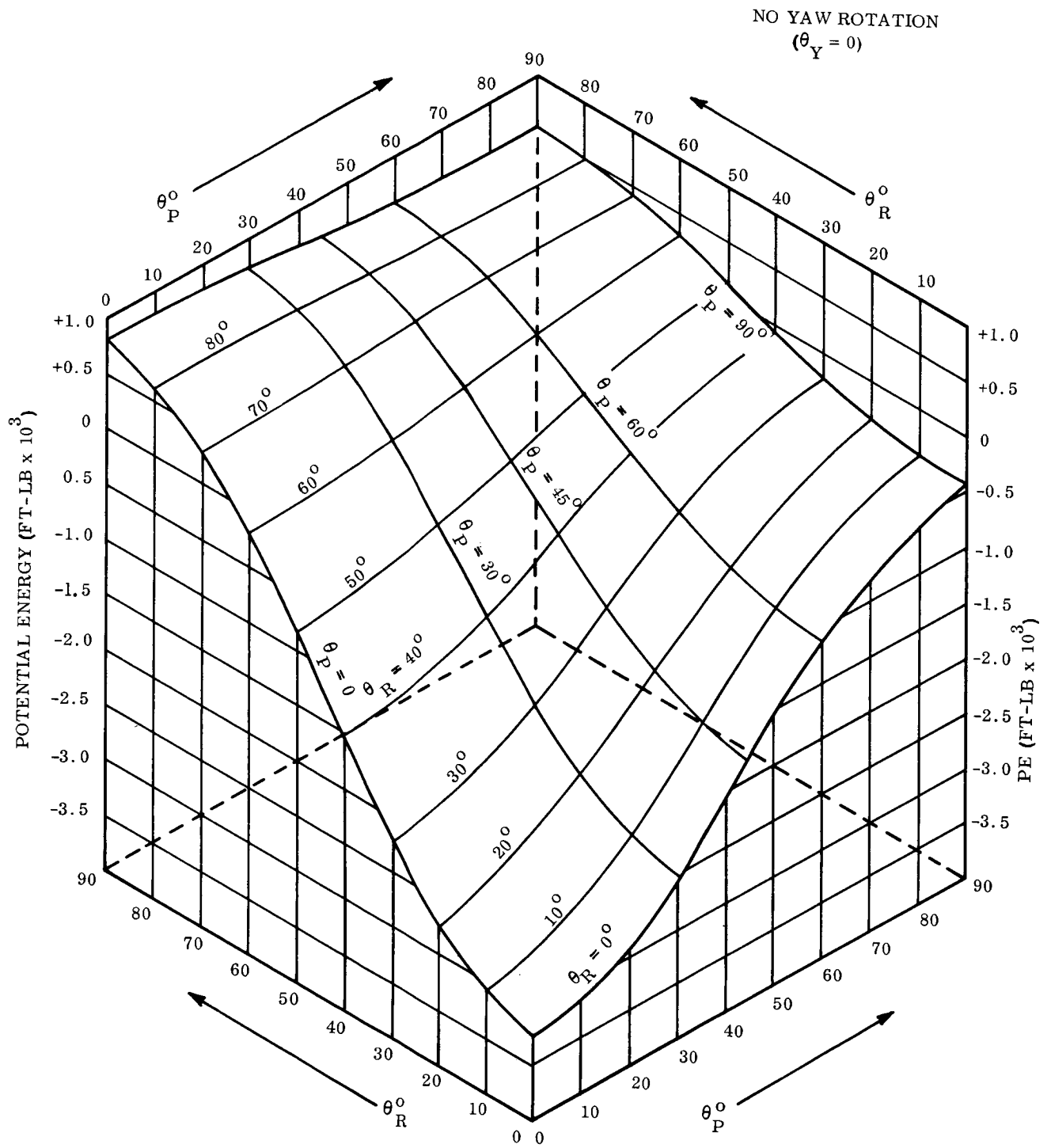


Figure 4-28. NIMPAC Potential Energy Map With Yaw Rotation

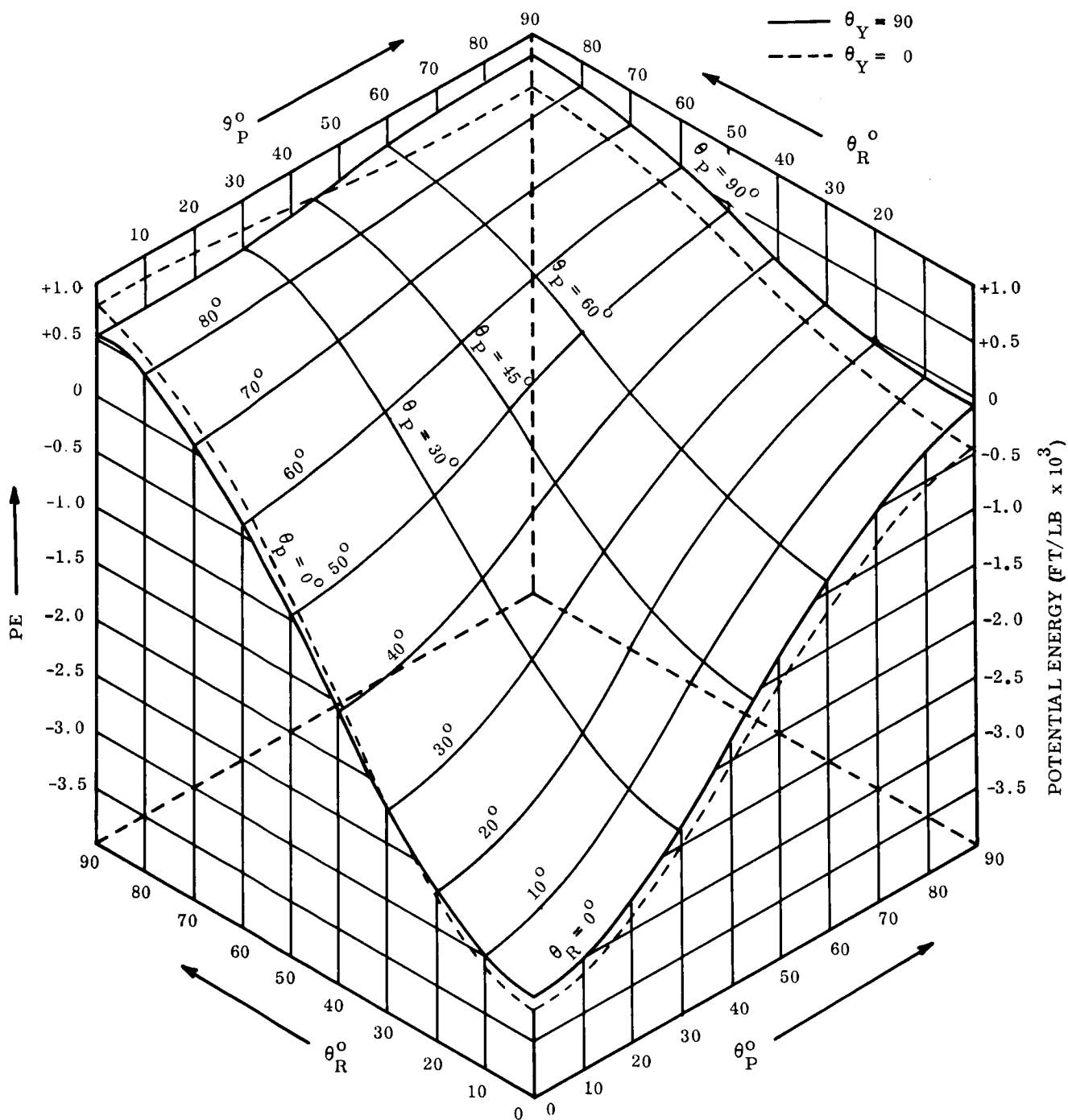


Figure 4-29. NIMPAC Potential Energy Map With 90-Degree Yaw Rotation

Secondly, it enables a quick determination of the system potential energy for any values of body orientation relative to the orbiting reference frame. The kinetic energy is added, and this sum compared to the critical level.

For the purposes of defining the maximum energy which the system may possess without tumbling, a slightly conservative approach was taken. The maximum permissible energy (kinetic plus potential) was set equal to the "minimum" difference in potential energy between the condition of zero error relative to the local vertical and minimum at the horizontal point where either pitch, roll, or combination thereof are 90 degrees. In Figure 4-28, the minimum point occurs where $\theta_p = \theta_r = \theta_y = 0$, and the potential energy at this point is equal to 3.1×10^{-3} ft-lb whereas the minimum "max" point occurs at $\theta_p = 90$ degrees and $\theta_r = \theta_y = 0$ degrees where $PE = -0.42 \times 10^{-3}$ ft-lb.

This by definition then results in a maximum permissible energy level of $(-0.42 + 3.1) \times 10^{-3} = \underline{2.68 \times 10^{-3} \text{ ft-lb}}$ beyond which the satellite will tumble.

For this purpose, the "nonlinear digital simulation" computer program was modified to account for changing moments of inertia, simulating the extension of four rods at individual rates of travel.

The maximum body rates to be considered in this study were specified at 1 degree/second about all axes and initial errors of 5 degrees from the principal body axes. Significant combinations of the maximum values of rates and angles were evaluated, as well as -0.8 degree/second pitch rate which represents the limit of negative pitch rate for which capture occurs without tumble. Short computer runs were made for each of the cases in Table 4-6.

Table 4-6. Values of Separation Conditions Considered For Capture Study

	θ_p	θ_r	θ_y	$\dot{\theta}_p$	$\dot{\theta}_r$	$\dot{\theta}_y$
	(Degrees)			(Degree/Seconds)		
a.	-5	-5	-5	-1	-1	-1
b.	+5	+5	+5	+1	+1	+1
c.	-5	+5	+5	-1	+1	+1
d.	+5	-5	-5	+1	-1	-1
e.	+5	+5	-5	+1	+1	-1
f.	+5	-5	+5	+1	-1	+1
g.	-5	-5	+5	-1	-1	+1
h.	-5	+5	-5	-1	+1	-1
i.	-5	-5	-5	-0.8	-0.8	-0.8

The satellite orientation and rates, relative to the orbiting reference frame at the point of full extension of the last rod, were used to compute the total energy of the system. The potential was computed from the expression given previously. The kinetic energy was computed from the following expression:

$$\begin{aligned}
 (\text{K. E.}) = & \frac{1}{2} I_z (\omega_z - \omega_o \cos \theta_R \cos \theta_Y)^2 \\
 & + \frac{1}{2} I_Y (\omega_Y - \omega_o \cos \theta_R \sin \theta_Y)^2 \\
 & + \frac{1}{2} I_X (\omega_X - \omega_o \sin \theta_R)^2
 \end{aligned}$$

where I_z , I_y , I_x are again the moments of inertia about pitch, roll, and yaw axes; ω_z , ω_y , and ω_x are body rates with respect to inertial space, and θ_r , θ_y are Euler angles, relative to the orbiting reference frame.

Table 4-7 summarizes the results and compares the total energies to the "critical" value for tumbling.

Table 4-7. Comparison of Total Energy for Capture

<u>Case</u>	<u>Potential Energy</u> (ft-lb x 10 ³)	<u>Kinetic Energy</u> (ft-lb x 10 ³)	<u>Total Energy</u> (ft-lb x 10 ³)	<u>Capture Without Tumble</u>	
				<u>Theoretical</u>	<u>Actual*</u>
a.	1.51	2.99	4.5	No	No
b.	1.15	1.24	2.4	Yes	Yes
c.	1.50	3.0	4.5	No	No
d.	1.23	1.21	2.4	Yes	Yes
e.	1.30	1.13	2.4	Yes	Yes
f.	1.23	1.10	2.3	Yes	Yes
g.	1.67	2.90	4.6	No	No
h.	1.67	2.8	4.5	No	No
i.	1.15	2.48	3.6	No	Yes

"Critical" total energy = 2.7×10^3 ft-lb

These results demonstrate that the total energy concept is a good criterion for predicting whether tumble will occur. Of the nine cases evaluated, the results of eight were verified by computer runs. The ninth case (i) for -0.8 degree/second about all axes indicated an apparent disagreement. However, since this is a known marginal case (determination of a threshold level) the results are not surprising and a simple explanation exists. Although the computed total system energy at the end of rod extension exceeds the critical level, there is not enough energy in pitch and/or roll to tumble the vehicle immediately, and, before sufficient additional energy is transferred from yaw (to pitch and/or roll), the damper has dissipated enough to reduce to total energy below critical.

4.7.3 FINAL EVALUATION OF CAPTURE CAPABILITY FOR "WORST CASE" INITIAL CONDITIONS

Figures 4-30 through 4-38 represent the results of the nine "initial condition" cases evaluated by means of the large-angle, nonlinear, digital simulation. Of the nine cases run, the first eight make use of the maximum rates and angles as specified by requirements, i.e., 1 degree/second and 5 degree/second error on all three axes. These, therefore, represent all possible combinations of positive and negative conditions on all three axes.

The following conclusions may be drawn from these results:

- a. Of the four cases which tumbled, two captured and remained captured within the first orbit and two within the third orbit. In no case was any tumbling observed after 3 hours (See Figures 4-30 through 4-38). It may, therefore, be conservatively concluded that if the initial rates do not exceed 1 degree/second and initial angles are within 5 degrees, final capture will be realized within 6 hours after rod extension.
- b. Within the rates and angles investigated, the only cases which tumble before capturing are those for which the initial pitch rate is in the negative direction; regardless of the direction of roll and yaw rates.
- c. The limiting case for a negative pitch rate and no tumbling appears to be approximately -0.8 degree/second (Figure 4-38).
- d. If the initial pitch rate should be in the positive direction, a rate significantly larger than 1 degree/second may be tolerated without tumbling.

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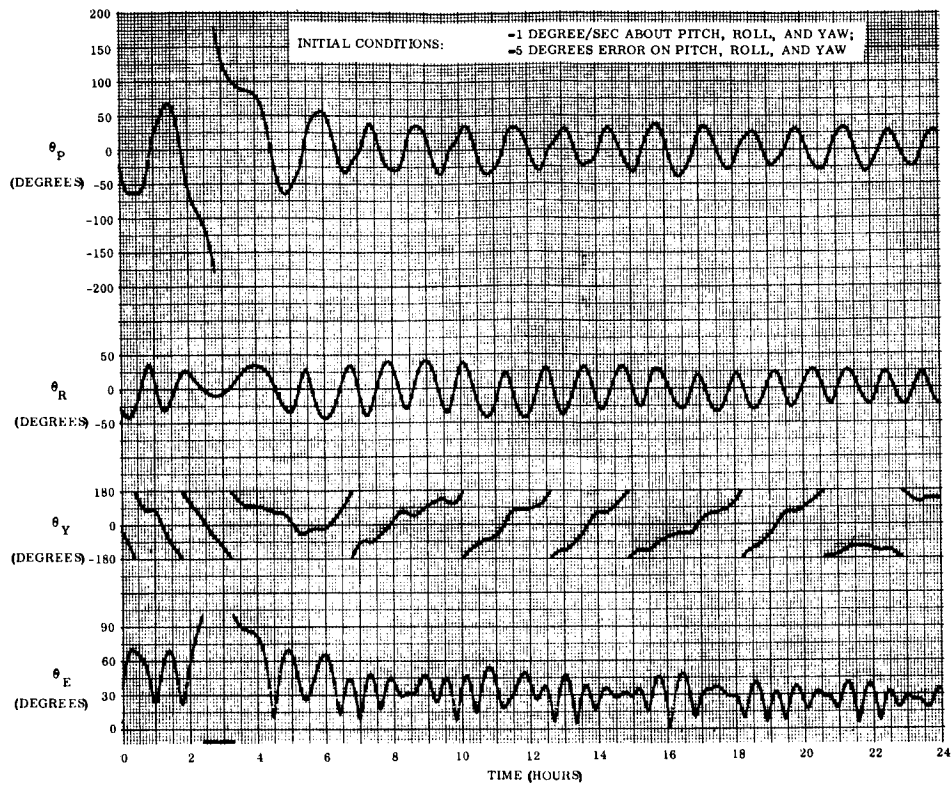


Figure 4-30. Capture Performance No. 1

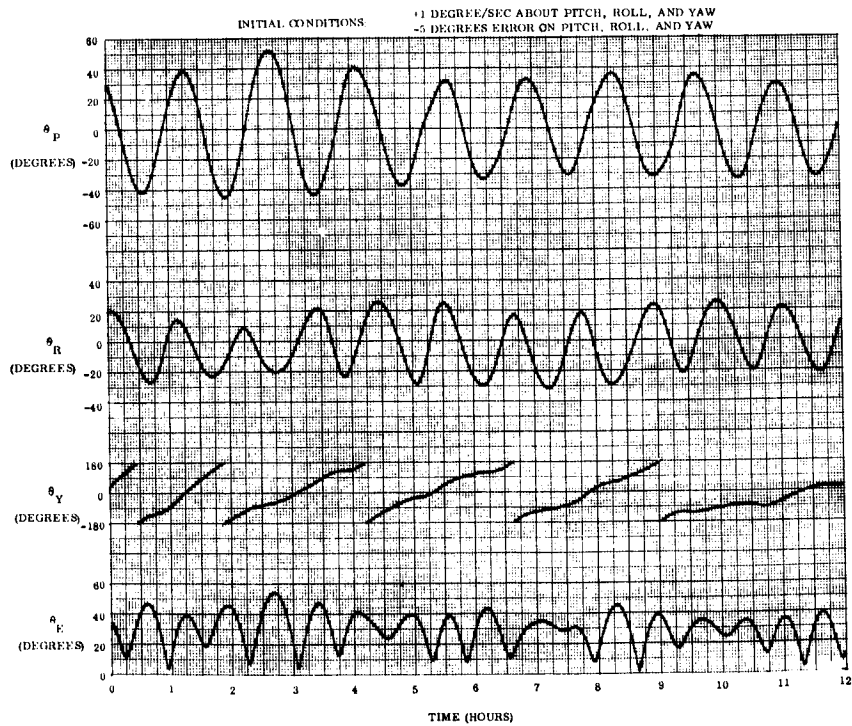


Figure 4-31. Capture Performance No. 2

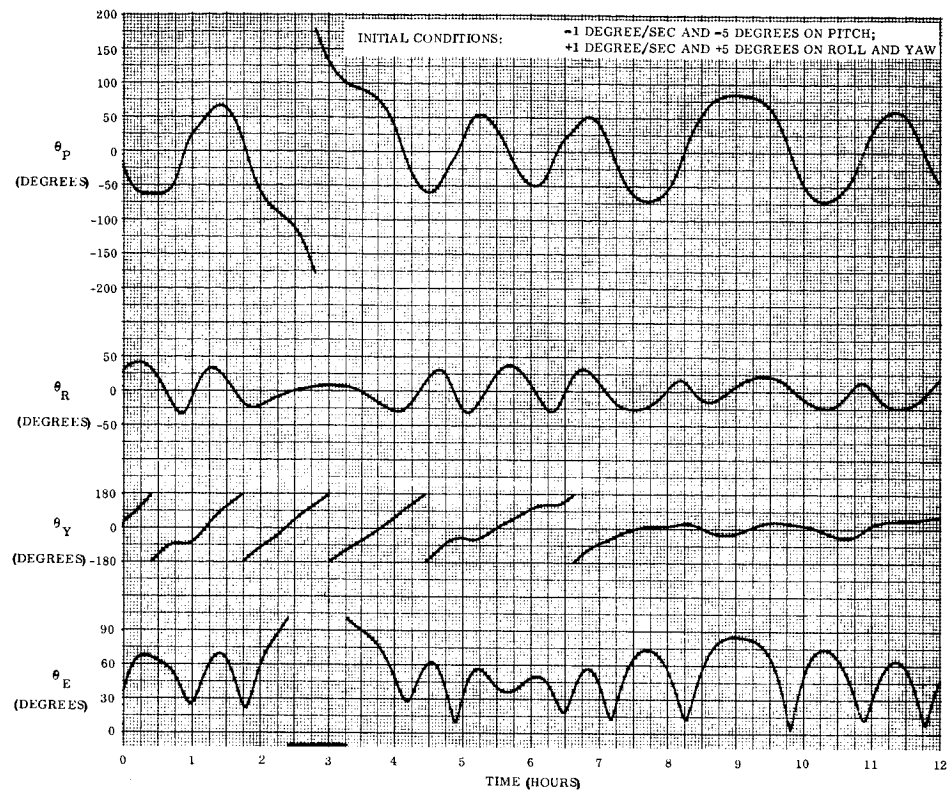


Figure 4-32. Capture Performance No. 3

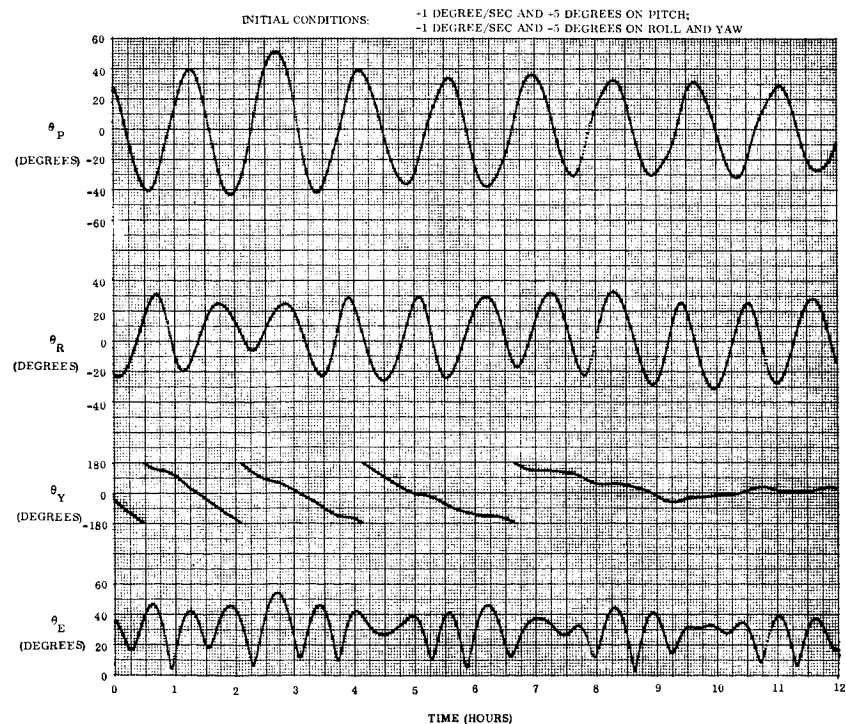


Figure 4-33. Capture Performance No. 4

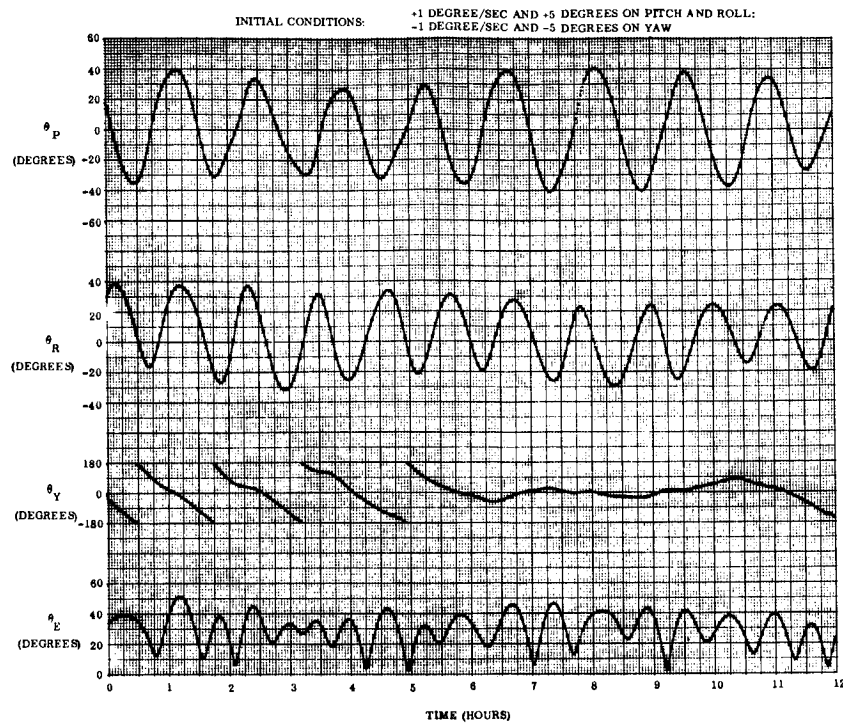


Figure 4-34. Capture Performance No. 5

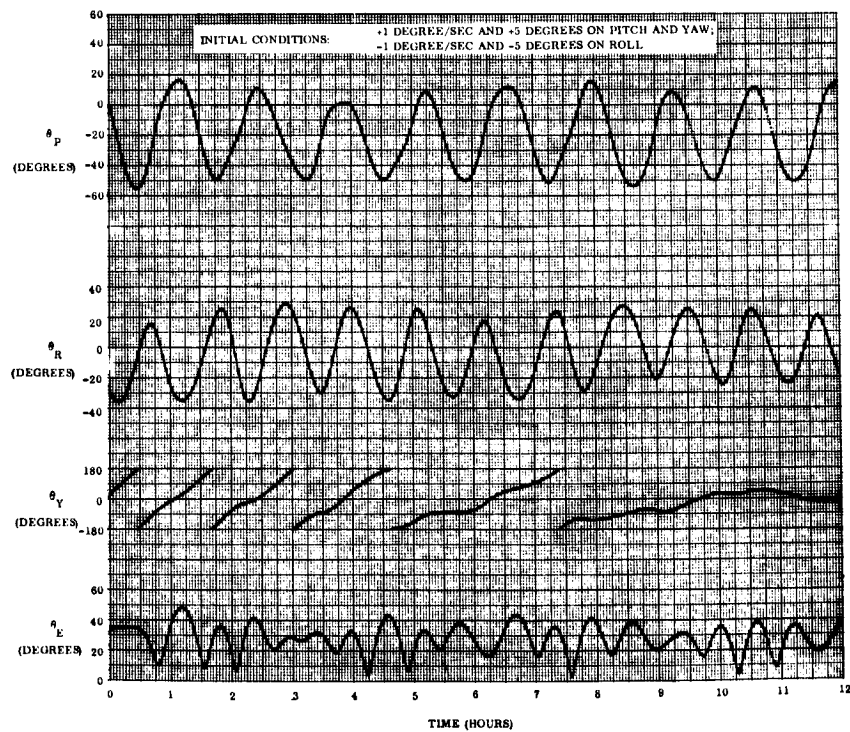


Figure 4-35. Capture Performance No. 6

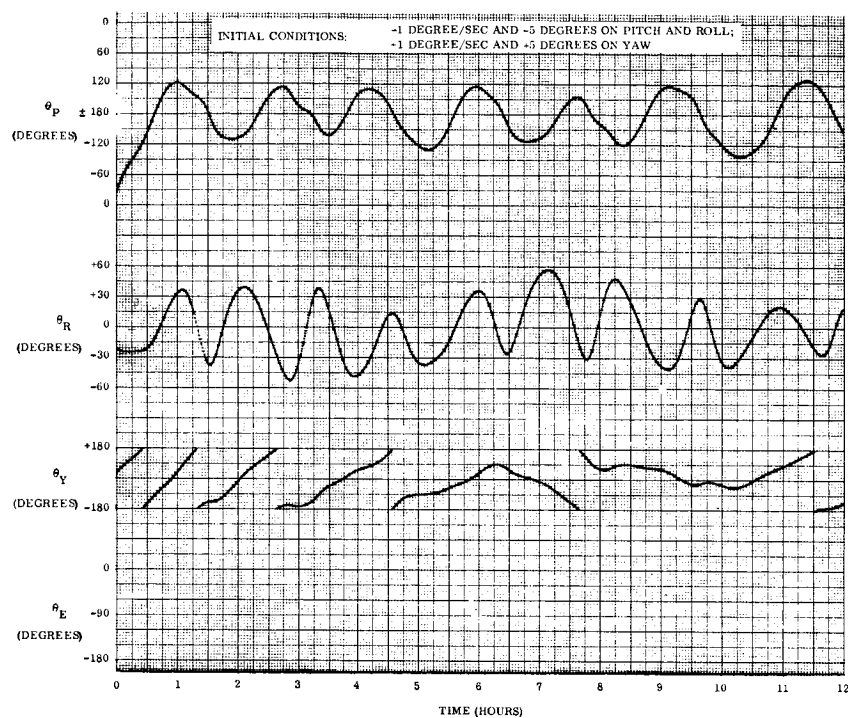


Figure 4-36. Capture Performance No. 7

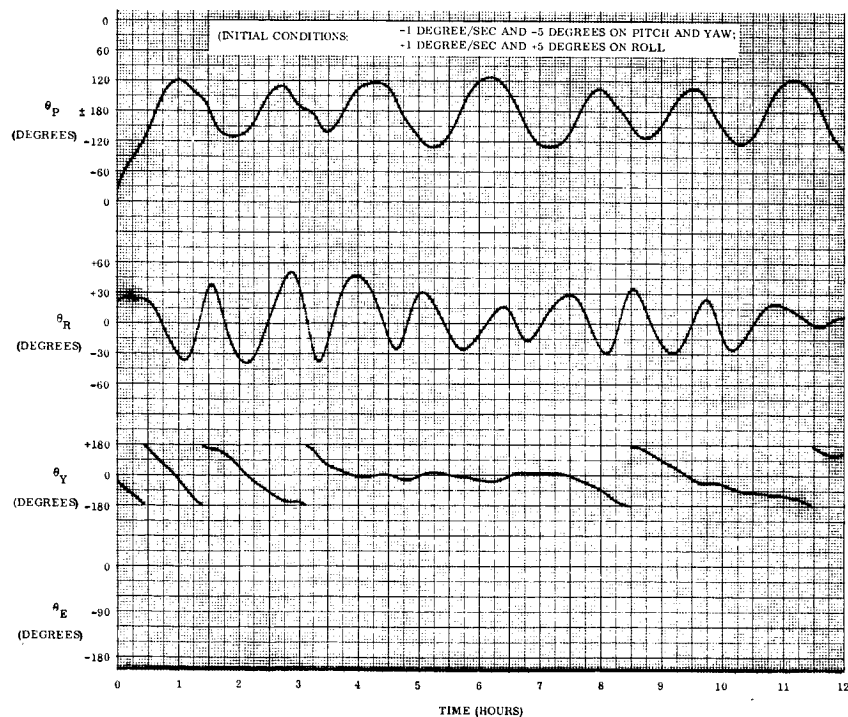


Figure 4-37. Capture Performance No. 8

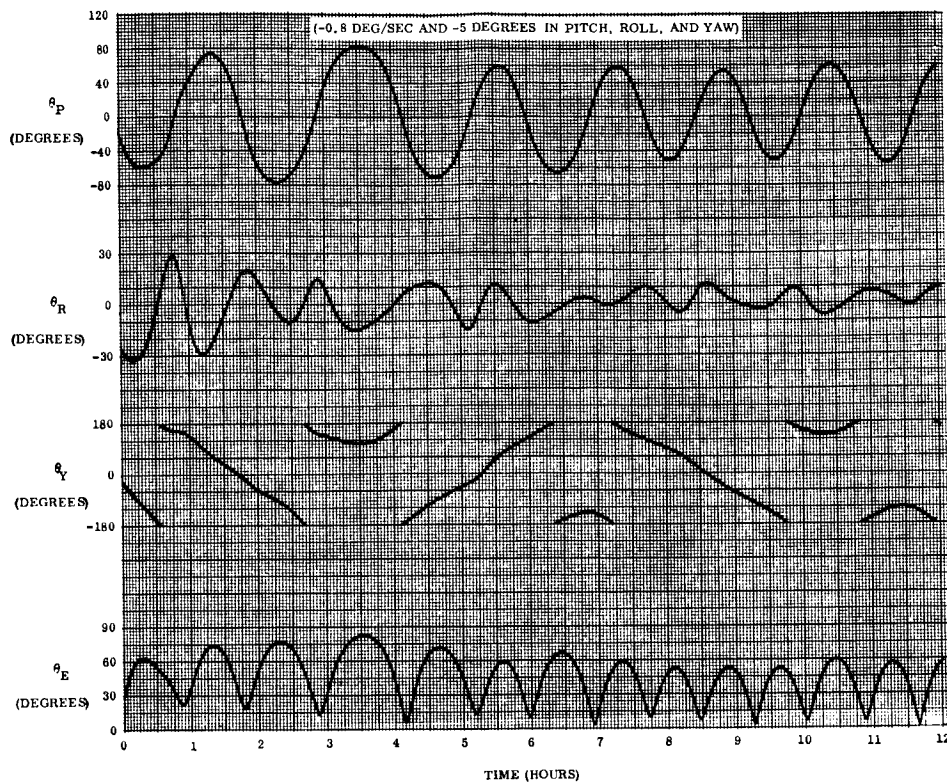


Figure 4-38. Capture Performance No. 9

SECTION 5

DESCRIPTION OF DIGITAL COMPUTER SIMULATIONS

5.1 GENERAL

Existing digital computer simulations were used during the course of the study to support the analytical effort. Approximately 40 hours of 7094 digital computer operational time was required for this study. The majority of this effort was expended in the simulation of the passive attitude control subsystem. This section presents a descriptive summary of the major computer programs used. The mathematical models used in many of these programs have been substantiated by testing or flight experience. The passive attitude control simulation has been substantiated by two-axis control flight experience, and three-axis flight programs are in process.

5.2 PASSIVE ATTITUDE CONTROL SUBSYSTEM SIMULATION

An existing digital computer simulation was used for the steady-state, transient damping, and capture performance analyses of the selected configuration.

The digital computer simulation is based on solution of Euler's dynamical equations for the main body and the inner spheres of the dampers, using step-by-step integration of the time variables. Up to six separate dampers may be used. The satellite model can accommodate up to six long, extensible rods to provide large moments of inertia, which orient the satellite by means of the gravity-gradient effect. All attitude motions are by magnetically-anchored viscous and eddy-current dampers.

The following torques are included in the program:

- a. Gravity-gradient.
- b. Solar radiation pressure.
- c. Magnetic dipole moment.

- d. Viscous or eddy-current damping.
- e. Aerodynamic pressure (optional).
- f. Magnetic hysteresis (optional).
- g. Thruster (optional).
- h. Miscellaneous disturbance (optional).

The magnetic torques are those on the bar magnets in the dampers and those due to any residual magnetic dipole of the satellite. The miscellaneous disturbance torques have direct components, sinusoidal components of any amplitude, frequency, and phase and pulsed components. They may be used to simulate the effects of tape recorders or other periodic disturbances. Whenever the rods are exposed to the sun, the uneven heating of the illuminated and shaded sides causes a differential thermal expansion which bends the rods. Because of this thermal bending of the rods, the satellite center of mass and its moments and products of inertia are continually changing. These effects are accounted for in Euler's dynamical equations. All torques are computed about body geometric axes translated without rotation to the instantaneous center of mass. The effects of a spherical satellite body shadow on the rods are included. When the satellite is in the earth's shadow, the rods are straight, and all solar torques are zero. The instantaneous center of mass of the satellite is assumed to follow a circular or elliptical path about the geocenter. The orbital plane is inclined at any constant angle to the equatorial plane. The precession rate of the orbital plane is treated as a constant, equal to the time-average rate precalculated from the harmonics of the earth's gravitational field. Coupling between the orbital motion and the attitude motion has been neglected, because studies have shown the effects are insignificant for practical applications. The effects of orbital eccentricity and regression have been instantaneous position of the satellite and the consequent effects on the control and disturbance torques.

The derivatives of the absolute angular rates of the satellite are computed from Euler's dynamical equations. These are integrated numerically to obtain the absolute angular rates about the body geometric axes. From these, the Euler angular rates are computed. The latter are numerically integrated to obtain the Euler angles. The numerical integration method used provides for a variable integration interval. The length of the interval is controlled by the program in accordance with specified tolerable errors, which are compared with the estimated numerical integration errors.

This provides a balance between accuracy and economy. The maximum degree of flexibility in the program is achieved by the use of flexible subroutines for input, initialization, orbital computations, bent rod geometry, applied torques, derivatives, numerical integration, and output.

Various simplifying assumptions have been made in areas where the effects are small, in order to keep the simulation program size practical and to keep machine running costs as economical as possible. The assumptions are listed and discussed below:

- a. The instantaneous center of mass of the vehicle with rods is assumed to follow the circular or elliptic orbit path.
- b. Coupling between satellite motion and orbit motion is small and is neglected.
- c. The gravity field of the earth is assumed to be spherical and vary as $1/R^3$ from the geocenter.
- d. The solar pressure flux is assumed to be on parallel rays and of constant magnitude.
- e. The thermally bent shape of the rods is assumed to be circular in the plane of the rod and sun.
- f. Small dynamic terms in Euler's equations of motion for the flexible satellite with bending rods are ignored.
- g. Step-by-step integration of the time variables is controlled by means of several error criteria and step size varied to allow maximum step size within the allowable errors. The error criteria are placed on the absolute and relative values of the integrated variables and are chosen so that reduction of the error criteria does not change the results of long computer runs.

5.3 SIMULATION OF TEMPERATURE DISTRIBUTIONS IN GRAVITY-GRADIENT RODS

The thermal analysis of the axial and circumferential temperature distribution in the gravity-gradient rods resulting from RTG heat dissipation was conducted during the study by using a digital computer simulation. The simulation is the closed-form analytical solution to the steady-state thermal balance of temperature distributions in a discontinuous cylindrical tube. The discontinuity results from a uniform cleavage or separation in the lateral surface. Computer solutions are available as a function of gravity-gradient rod geometries and physical properties and as a function of the normal component of the heat flux relative to the rod seam. Analytical results from the program indicate that the temperature distributions are a strong function of the location of the heat flux relative to the rod seam.

The following assumptions have been made in the analytical formulation:

- a. If the rod is sufficiently long such that the surface area is very much larger than the cross-sectional area, the end effects may be considered negligible. Hence, the temperature distribution will not vary along the length of the rod.
- b. For a given rod of moderate length and thin wall, the variation due to rod bending of incident solar energy along the length of the rod has an insignificant effect on the circumferential temperature distribution.
- c. A thin-walled rod develops negligible gradients along its radius.
- d. Internal radiation exchange within the rod may be regarded as having a relatively insignificant effect on the overall temperature gradient.
- e. Thermal contact conductance at the rod overlap is not considered.

5.4 STRUCTURAL CONFIGURATION SIMULATION

Flexibility coefficients and unit load solutions of the selected configuration were determined during the study by using a structural computer simulation developed on the NIMBUS Program. This simulation is used to compute the internal loads and deflections within the three-dimensional sensory ring frame work and in the truss support structure. Deflection information is then used in forming a flexibility matrix for subsequent use in the dynamic analysis. This routine computed member stiffnesses from basic geometric and physical parameters. Using a "deflection" technique, a total stiffness matrix for the entire space structure is obtained. The routine sets up the force-deflection relationships, which are then solved for internal loads and deflections. Thermal effects may also be incorporated, if desired. There is also an option to this routine which will yield a matrix of stiffness influence coefficients for the entire structure. This routine is capable of handling structures of up to 150 joints but is limited to structures idealized as straight or curved bars, beams, or tubes.

Geometrical and physical properties of structural members, known loadings, deflections, thermal environments, and boundary conditions are used as input data. Input (including the feature of having data for each structural member listed separately), deflections at each joint (three displacements and three rotations), member internal loads, and stiffness influence coefficients are listed as output data.

5.5 DYNAMIC STRUCTURAL SIMULATION

The structural dynamic analysis conducted during the study used the appropriate mass matrix, the flexibility matrix determined from the structural configuration simulation (Section 5.4), and a damping matrix from NIMBUS test results to calculate the resonant frequencies, dynamic response, and mode shapes of the combined structure. The computer program used for this analysis is basically a matrix algebra system having the capability to compute eigen values and eigen vectors. Effects of the predicted response on the internal stress distribution were obtained by using the unit solutions of the structural simulation.

SECTION 6
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