STRUCTURAL MECHANICS OF DEFORMATION AND FRACTURE

QUARTERLY PROGRESS REPORT

ON

CONTRACT NASW 1190

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- November 1966 -

<u>STRUCTURAL MECHANICS OF DEFORMATION AND FRACTURE</u>

The general objective of this study is the development of an approach to mechanical design and material development which takes account of some of the contemporary understanding of the physics of solids. Particular attention is given to the temperature dependence of the irreversible part of the deformation which is related to the atomic scale structure of the solid through an energy of activation. Part of the work involves the examination of the behavior of model materials in selected test situations. The responses of the models are then compared with features of the behavior of real materials for which physical rationalizations are being sought.

During the current quarter, a hammer impact problem was considered from this point of view. The results of this study have been presented at a meeting of the Society of Rheology and submitted for publication in their Transactions. That presentation constitutes the body of this report.

This work augments the earlier studies of the responses of similar models to simpler stress and strain program which are cited in the bibliography. In addition to illuminating the relationships between material response and material structure, the new results are relevant to material forming processes, laboratory testing of materials and the design of devices for the attenuation of impact loads.

The material in this report is part of a proposed thesis to be submitted to Prof. W. F. Ames of the University of Delaware in connection with the candidacy of Mr. Mueller for the Ph. D. degree.

ABSTRACT

As part of a continuing exploration of the mechanical behavior of materials which have temperature dependent properties, the effect of a hammer blow on a small sample of model viscoelastic material is considered. In this situation, the duration of the experiment is long compared with the time required for stress equilibrium to develop and short in comparison with the thermal relaxation time of the sample. The analysis shows how a material which responds in an almost perfectly elastic manner to a light blow can respond in a viscous manner to a heavy blow. It also suggests that a homogeneous continuum with properties that would make it acceptable as a structural material must be brittle. This result may contribute to the understanding of the important relationship between microstructure and ductility.

The hammer experiment involves relatively high strain rates which vary continuously during the deformation. In earlier work, the constant deformation rate case was studied. The latter situation is difficult to produce in the laboratory in the range of rates of interest here and is less likely to arise in practice.

RESPONSES OF MODEL VISCOELASTIC MATERIALS TO IMPACT

Introduction

This report is one of a series describing exploratory studies of the mechanical behavior of materials which have temperature dependent properties. With such materials, the heat produced by the irreversible part of the work of deformation can have a strong influence on the outcome of an experiment (1-8). The present study relates to the effect of impact loads on viscoelastic materials. As before, the approach that is taken involves the examination of the responses of selected models of materials to simple test situations. The relevance of the models is then estimated by comparing these responses with features of the behavior of real materials for which physical rationalizations are being sought. The results supplement the earlier studies and are a further step in the development of an understanding of the effects of energy conservation on the behavior of materials.

The problem that is considered here is similar to that which arises when a short cylinder of material, standing on an anvil, is hit with a hammer. Real experiments of this type are easy to perform in the laboratory and they simulate a condition which is of practical interest. These experiments involve relatively high strain rates which vary continuously during the process. The present study is thus distinguished from one described earlier in which the deformation of model viscous cylinders at various constant rates was considered (6). That constant rate case, in the range of interest here, is difficult to produce in the laboratory and is less likely to occur in nature.

In the absence of thermal effects and changes of the shape of the test piece, the solutions of this problem are familiar. For example, with perfectly elastic materials the hammer will bounce away from the sample with a velocity equal and opposite to its initial velocity. With viscoelastic materials the bounce velocity will be lower and there will be no bounce when the conditions for critical damping are met.

With heating, the effective viscosity is reduced and the damping increases during the experiment. As a consequence, the amount of bounce will depend on the severity of the blow as well as the properties of the material. In particular, it is shown that a material which responds in a generally elastic manner to a light blow that does not produce much heat may respond inelastically to a heavy blow. This is, of course, precisely what is observed with real materials. The model material can also show a strain rate dependent yield strength similar to that observed in real experiments. Under some conditions very high temperatures are produced in the model material which might lead to catastrophic failures comparable with brittle fracture in real materials.

Discussion of the Model Experiment

The model experiment which is considered has the following features. The duration of the experiment is long in comparison with the time required for stress equilibrium to develop in the system and short in comparison with the thermal relaxation time of the test piece. In other words, the deformation is treated as quasi-static and adiabatic. These conditions are quite appropriate in many real hammer experiments.

The instantaneous deformation is considered to be divided into two independent parts. One is reversible and proportional to the current value of the force. The other is irreversible and depends on the time integral of the force. In other words the system is treated as a Maxwell Model composed of an ideal elastic element in series with an ideal viscous element. This also implies that the model material is homogeneous and isotropic which is not generally true of real materials. The results suggest an important role for heterogeneity in determining the behavior of real materials.

The temperature effect is considered to arise from the work done on the viscous element. On the basis of the contemporary view of the physics of materials, it may be expected that the temperature dependence of the viscosity is very much stronger than the temperature dependence of the elasticity. For these computations the latter is neglected. The pressure dependence of the viscosity is also neglected. Similar idealizations were made in the earlier studies which gave results in general agreement with experience.

The changing shape of the test piece is also considered, subject to the assumption of constant volume. In compression experiments, the increase in cross-sectional area tends to offset the force reduction caused by heating. In tension experiments the change in section area augments the effect of heating. In practice, when the sample becomes very short or very long, heat losses to the environment can be expected so that the adiabatic assumption loses its validity. Also, in real tensile experiments, non-uniform strain distribution are likely to arise.

Details of the Computation

The instantaneous velocity of the hammer after impact is the sum of the rate of displacement of the elastic element $\dot{\epsilon}_E$ and that of the viscous element $\dot{\epsilon}_V$.

$$\dot{\epsilon} = \dot{\epsilon}_E + \dot{\epsilon}_V \tag{1}$$

The force producing the deceleration of the hammer is the restoring force of the strained elastic element which is equal to the force on the deforming viscous element.

$$-M\ddot{\epsilon} = E\int_0^t \dot{\epsilon}_E \, dt = H\dot{\epsilon}_V \tag{2}$$

in which M is the mass of the hammer, **E** is the effective spring constant and H is the effective viscosity.

The substitutions

$$\omega = \dot{\epsilon}/\dot{\epsilon}_0 \tag{3}$$

$$\tau = \sqrt{E/M}\; t \tag{4}$$

in which $\dot{\epsilon}_0$ is the hammer velocity at impact and t is time, define the non-dimensional velocities

$$\omega_E = \dot{\epsilon}_E/\dot{\epsilon}_0 = -\frac{d^2\omega}{d\tau^2} \tag{5}$$

$$\omega_V = \dot{\epsilon}_V/\dot{\epsilon}_0 = -\frac{\sqrt{ME}}{H}\frac{d\omega}{d\tau} \tag{6}$$

so that equation 1 becomes

$$\frac{d^2\omega}{d\tau^2} + \frac{1}{Q}\frac{d\omega}{d\tau} + \omega = 0 \tag{7}$$

in which

$$Q = H/\sqrt{ME} \tag{8}$$

For small deformations Q is essentially a constant, Q_0, and equation (7) leads to the linear damped oscillations referred to in the introductory remarks. As would be expected the value of Q_0 is an important property of the system. It is similar to the quantity Γ cited in reference 8 and its designation is intended to suggest the quality factor used by electrical engineers in the discussion of the behavior of circuits and components. The value of Q_0 determines the damping in the system. When Q_0 is high, bounce will occur, when it is low, there will be no bounce. Furthermore, for systems that bounce, the value of $1/Q_0$ indicates the fraction of the total energy that is lost per bounce.

In general, however, Q is not constant. The effective viscosity, for example, can be factored to give

$$H = \eta A / L_V \tag{9}$$

in which η, A, and L_V are respectively the coefficient of viscosity, cross-sectional area and length of the viscous element, which vary in the course of the deformation.

Having in mind a hammer made of an elastic material and much larger than the test piece, the elastic deformation of the sample will be negligible in comparison with that of the hammer. Changes of the effective spring constant and mass in the experiment are, therefore, neglected.

Equation 9 can also be written

$$H = H_0 \frac{L_0}{L_V} \frac{A}{A_0} \frac{\eta}{\eta_0} \tag{10}$$

in which the zero subscripts refer to the initial values of the parameters. Assuming incompressibility this becomes

$$H = H_0 \left(\frac{L_0}{L_V}\right)^2 \frac{\eta}{\eta_0} \tag{11}$$

The current length of the sample is then given by

$$L_V = L_0 \pm \int_0^t \dot{\epsilon}_V \, dt \tag{12}$$

which leads to

$$\mathcal{L} = \frac{L_V}{L_0} = 1 \pm \sqrt{M/E} \; \frac{\dot{\epsilon}_0}{L_0} \int_0^\tau \left(\omega + \frac{d^2\omega}{d\tau^2}\right) d\tau \tag{13}$$

The sign option allows the computation to be applied to both compression and elongation of the sample.

The viscosity ratio in equations 10 and 11 is now assumed to depend only on the temperature, T, according to

$$\eta/\eta_0 = e^{-a(T-T_0)} = e^{-\phi} \tag{14}$$

in which "a" is a constant of the material and ϕ is defined by equation 14. The current temperature is determined by the time integral of the power input to the viscous element divided by its heat capacity so that

$$\phi = -\int_0^t \frac{aM}{cA_0L_0} \dot{\epsilon}\, \dot{\epsilon}_v \, dt = -\int_0^\tau \frac{aM\dot{\epsilon}_0^2}{cA_0L_0}\left(\omega + \frac{d^2\omega}{d\tau^2}\right)\frac{d\omega}{d\tau}d\tau \tag{15}$$

in which c is the specific heat of the material.

Equation 7 can now be rewritten

$$\frac{d^2\omega}{d\tau^2} - \frac{1}{Q_0}\left\{\left[1 \pm V_0\int_0^\tau \left(\omega + \frac{d^2\omega}{d\tau^2}\right)d\tau\right]^2 \exp -2V_0^2\delta\int_0^\tau\left(\omega + \frac{d^2\omega}{d\tau^2}\right)\frac{d\omega}{d\tau}d\tau\right\}\frac{d\omega}{d\tau} + \omega = 0 \tag{16}$$

in which

$$\delta = \frac{aL_0E}{2cA_0} \tag{17}$$

$$V_0 = \sqrt{M/E}\;\frac{\dot{\epsilon}_0}{L_0} \tag{18}$$

and the negative sign in front of V_0 indicates that the impact compresses the sample and the positive sign indicates that the impact stretches the sample. Equations 7 and 16 describe experiments in which the hammer sticks to the sample. If, when $d\omega/d\tau$ changes sign the hammer separates from the sample, the solution is complete when this sign change occurs.

Results of Computation

The responses of nine different representative model materials are shown in Figures 1-12. A range of initial hammer velocities is applied to each material and both compression and tension are considered. The experiments are characterized by the values of the non-dimensional groups defined by Q_o, δ, and V_o. The first of these, Q_o, completely characterizes the small deformation, isothermal response of the model material which is linearly related to the initial hammer velocity. When Q_o is large, the response is generally elastic. When Q_o is small, it is generally viscous. Critical damping occurs for $Q_o = 0.5$.

The group, δ, contains the temperature coefficient of the viscosity and the heat capacity of the sample. This quantity together with the reduced, initial hammer velocity, V_o, determine the magnitude of the thermal effect. The value of δ can be qualitatively correlated with the nature of a real material on the atomic scale through the energy of activation for the deformation process.

The significance of these groups is further indicated by the fact that the adiabatic temperature developed in the test piece when all of the kinetic energy of the hammer is converted to heat is given by

$$\phi_{max} = V_o^2 \delta \tag{19}$$

This is the actual temperature when the damping is supercritical and no bounce occurs. For high Q_o systems the actual temperature generated by the blow before the hammer bounces depends on the new group

$$B = \frac{V_o^2 \delta}{Q_o} \tag{20}$$

The results are presented just as they are plotted by the computer and are discussed as if they were real experimental results. The figures show the history of the reduced temperatures, ϕ, the reduced forces, $d\omega/d\tau$ (which are proportional to the deformation of the elastic element) and the reduced lengths of the viscous element (which in this case corresponds to the sample, $\lambda = L_V/L_0$) in terms of the reduced time, τ. The curves are dotted after the force changes sign. The dotted portions would apply if the hammer adhered to the sample after impact. They sometimes indicate the residual kinetic energy in the bounce.

The responses are quite diverse. Figures 1-3 show the behavior of a fairly elastic material ($Q_0 = 10$) in compression. In Figure 1, the heating effect is the lowest ($\delta = 2.0$) the range of B is 0.05 to 0.450, and the range of ϕ_{max} is 0.5-4.5. Gross non-linearity appears only in the dotted portion of the curves; that is, after the bounce. Notice how at the highest velocity the bouncing hammer might stretch the heated sample if it adhered.

Figure 2 shows the results when $\delta = 10$ the range of B is 0.01-1.0, and the range of ϕ_{max} is 0.1-10. The most significant feature of this figure and also of Figure 3, for which $\delta = 20$ is the abrupt onset of the effect of the heat. That is, at the highest hammer velocity the temperature is a small fraction of ϕ_{max} until the damping builds up to the critical value. It then rises rapidly to the full value. In Figure 3, the range of values of B is 0.02-0.50 and the range of ϕ_{max} is 0.2 to 5.0.

Figures 4-6 show the behavior in compression of a material which has more initial damping ($Q_o = 1.0$). Figure 4, for which $\delta = 2$ shows the responses in the range $B = 0.02$-2.0 and ϕ_{max} 0.02-2.0. This material shows a more gradual onset of damping because of the lower value of Q_o. Figure 5, for $\delta = 10$, $B = 0.1$-10 and $\phi_{max} = 0.1$-10, shows a new effect. At the highest hammer velocity, the sample collapses before the kinetic energy of the hammer is spent. As a result of the geometric effect the force rises again. In effect the hammer bounces off the anvil after using some of its energy to produce the deformation. Figure 6, for $\delta = 20$, $B = 0.2$-20 and $\phi_{max} = 0.2$-20 shows generally similar behavior.

Figures 7-9 show the behavior, in compression, of a material for which the initial damping is supercritical ($Q_o = 0.1$). Here the value of B is irrelevant and only ϕ_{max} must be considered. In Figure 7 the range of ϕ_{max} is 0.02 to 4.5. Oscillations are shown in the figures at the higher initial velocities which are due to the bounce of the hammer after the sample is flattened. Stretching of the sample, if it remains attached to the hammer is conspicuous in this figure.

Figures 8 and 9, similarly, show no oscillations at the lowest hammer velocities and saturation or bottoming at the higher velocities.

Figures 10-12 are derived from tensile experiments. In these the bottoming does not occur. However, the question of whether the flight of the hammer is arrested by the impact arises. Figure 10 shows the response of the generally elastic sample (Q_{θ}= 10). The abrupt yield of this material appears here as in the compression experiment. The hammer is arrested at each velocity but it almost gets away at the highest velocity. Figure 11 shows the response of the material with Q_{o}= 1. Here a damped oscillation is shown at the lowest velocity. At the highest velocity the hammer does not stop. Figure 12 shows almost typical viscous behavior (Q_{o}= .1). It appears that the hammer is arrested at the lower velocity.

Discussion of Results

In the problem considered above the force and deformation programs are determined by the properties of the material and certain features of the test situation. This is in contrast to the earlier studies in which these programs were inputs. The discussion here is, therefore, somewhat more complicated.

In any case, it is desirable to establish criteria for the significance of heating effects in any particular experiment. The numbers B and ϕ_{max} are appropriate.

The results conform generally with experience with fairly soft materials and with some types of harder materials which are brittle. The application of the first law of thermodynamics provides an explanation for certain mechanical responses which is certainly incomplete. However, whatever additional rationalization is applied would be enriched by these energy considerations.

One of the somewhat unexpected consequences of this study is that it suggests that any homogeneous, viscoelastic material which has properties which would make it acceptable for structural applications would necessarily be brittle. This follows from the abrupt development of criticality in the damped system even when the effective viscosity changes slowly. It would follow then that the ductility of such materials must depend on their heterogeneity or microstructure. This inference is in agreement with some of the prevailing views of students of material behavior. A mechanism for this effect was proposed in an earlier report (5).

It may be noticed that if a purely viscous model were hit by the hammer, the initial value of the force would be infinite. It would, however, rapidly decay to essentially the same form as shown in the low Q_0 cases described above. In any real system some elasticity will exist and the force will be zero at the instant of impact.

Acknowledgements

The work described above has been supported in part by the Office of Advanced Research and Technology of the Material Aeronautics and Space Administration under Contract NASW-1190 monitored by Messrs. Howard Wolko and Melvin Rosche. The authors are also indebted to Dr. J. D. Stewart of the General Electric Company and Professor W. F. Ames of the University of Delaware for encouragement and guidance.

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CAPTIONS FOR FIGURES

Figure 1 - Reduced temperature, reduced force and reduced length for a system characterized by $Q_o = 10.0$ and $\delta = 2.0$ are plotted as a function of reduced time. Solution curves for values of reduced initial compressive hammer velocity, $V_o = 0.5$, 1.0, and 1.5 are superimposed.

Figures 2 through ~~13~~ 9 are the same as Figure 1 except for differences as noted below.

Figure 2 -	$Q_o = 10.0$, $\delta = 10.0$	$V_o = 0.1$, 0.5, 1.0
Figure 3 -	$Q_o = 10.0$, $\delta = 20.0$	$V_o = 0.1$, 0.5, 0.6
Figure 4 -	$Q_o = 1.0$, $\delta = 2.0$	$V_o = 0.1$, 0.5, 1.0
Figure 5 -	$Q_o = 1.0$, $\delta = 10.0$	$V_o = 0.1$, 0.5, 1.0
Figure 6 -	$Q_o = 1.0$, $\delta = 20.0$	$V_o = 0.1$, 0.5, 1.0
Figure 7 -	$Q_o = 0.1$, $\delta = 2.0$	$V_o = 0.1$, 0.5, 1.5
Figure 8 -	$Q_o = 0.1$ $\delta = 10.0$	$V_o = 0.1$, 0.5, 1.0
Figure 9 -	$Q_o = 0.1$, $\delta = 20.0$	$V_o = 0.1$, 0.5, 1.0

Figure 10 - Reduced temperature, reduced force and reduced length for a system characterized by $Q_o = 10.0$ and $\delta = 10.0$ are plotted as a function of reduced time. Solution curves for values of reduced initial elongative hammer velocity, $V_o = 0.1$, 0.6, 0.7 are superimposed.

Figure 11 - $Q_o = 1.0$, $\delta = 2.0$

(Same as Figure 10 except $V_o = 0.1$, 0.5, 1.0)

Figure 12 - $Q_o = 0.1$ $\delta = 20.0$ (V_o 0.05, 0.5 are......)

(Same as Figure 10)

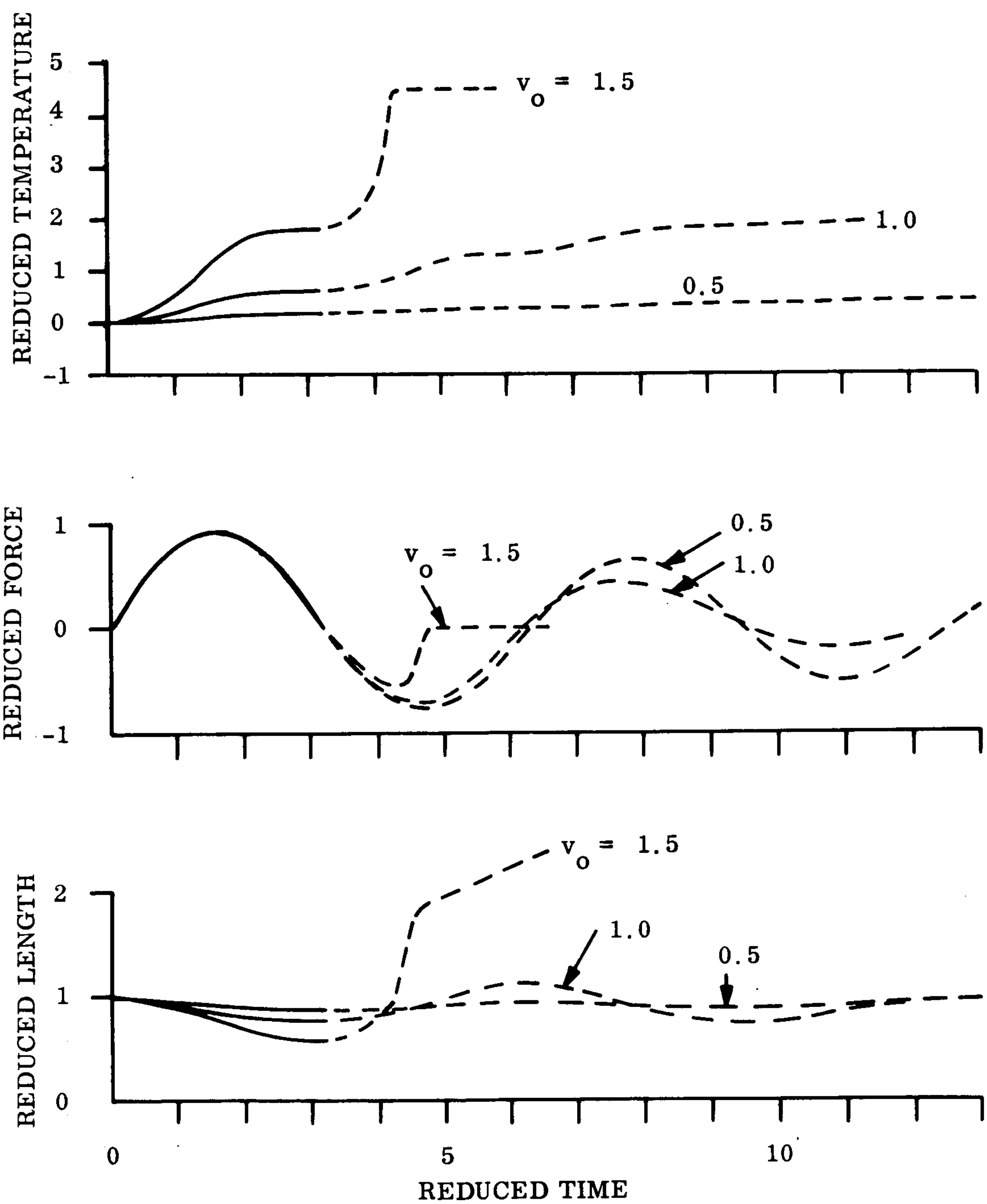
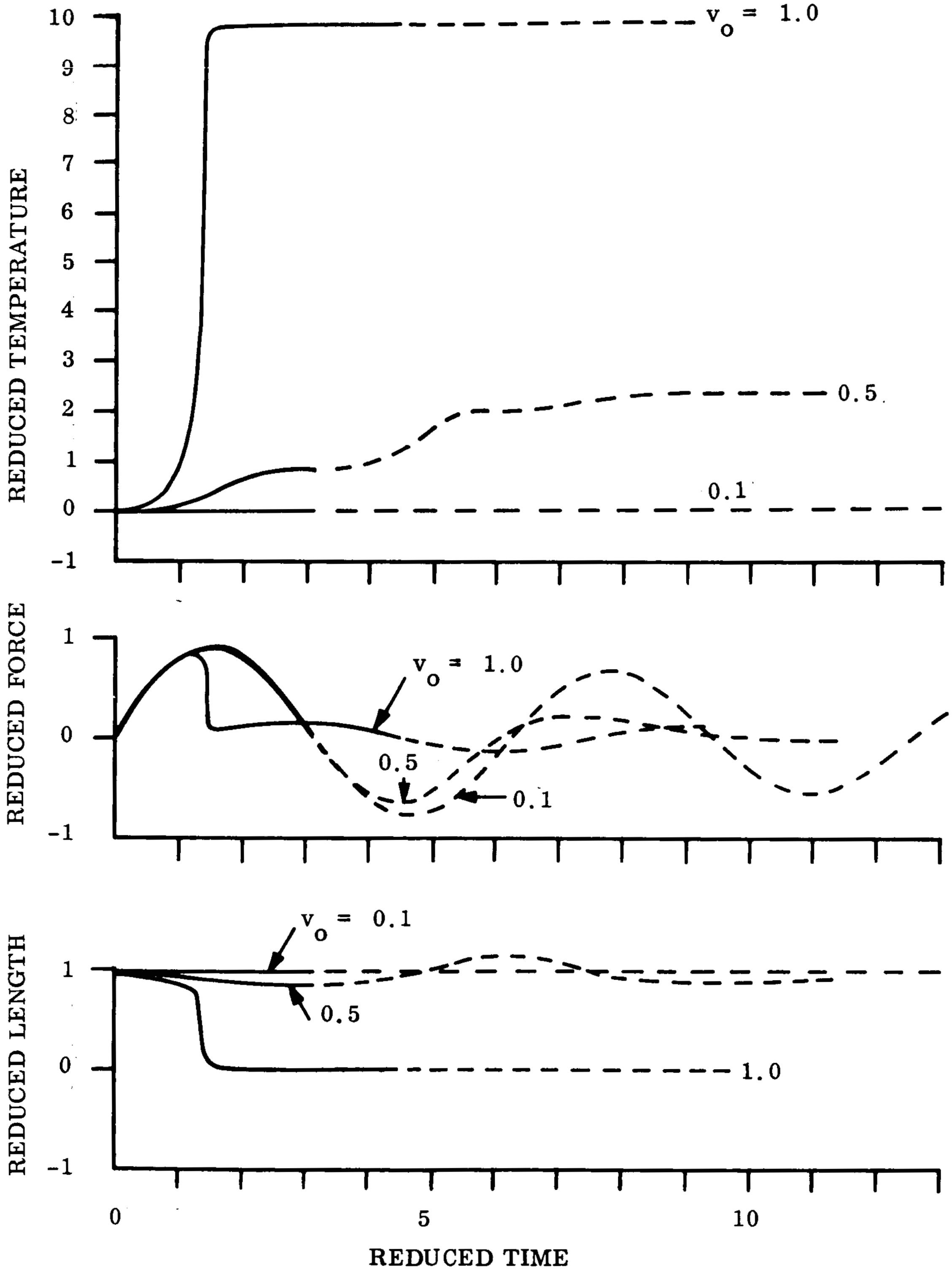


Figure One. Compression

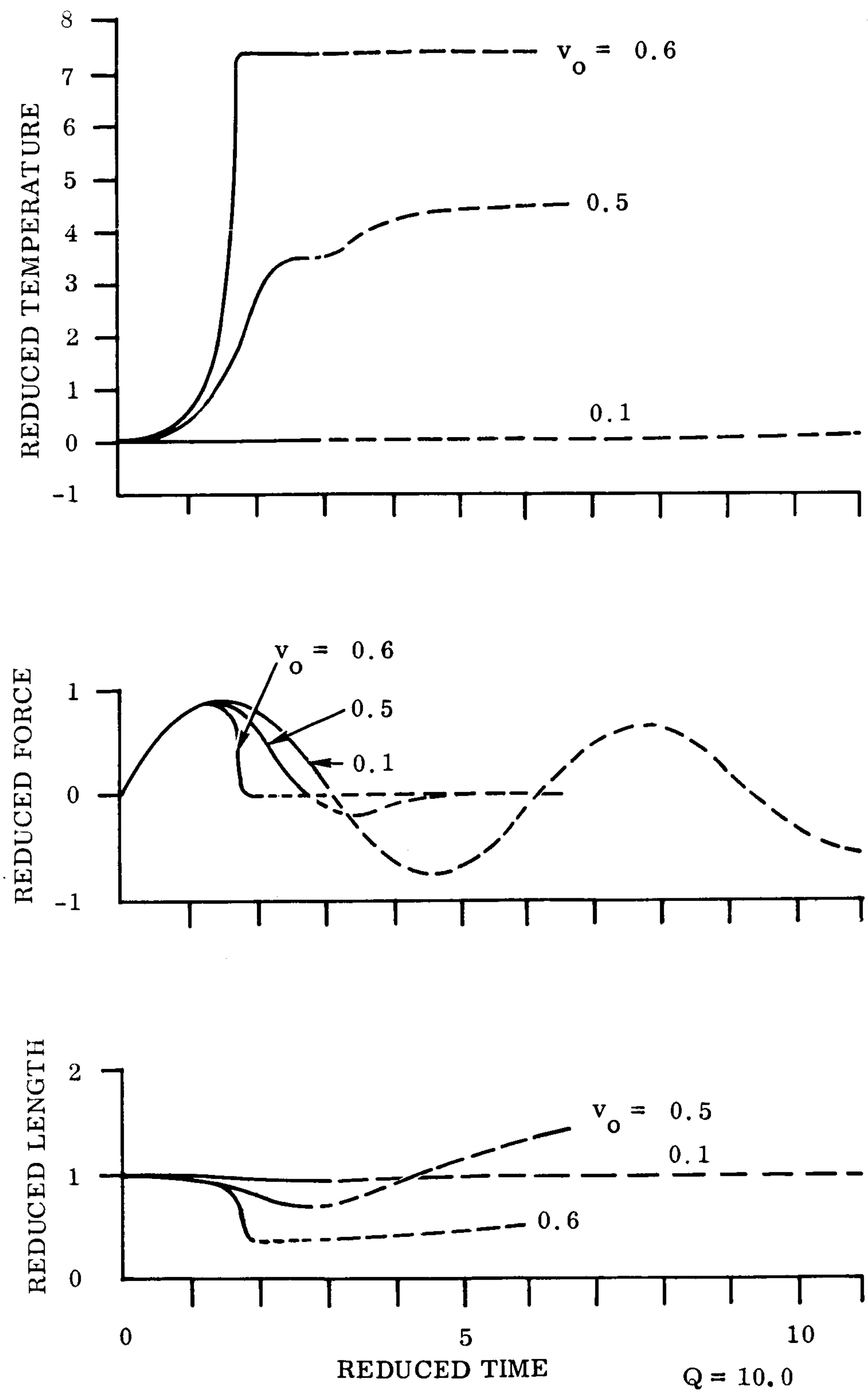
$Q = 10.0$
$\delta = 2.0$



Q = 10.0

Figure Two. Compression

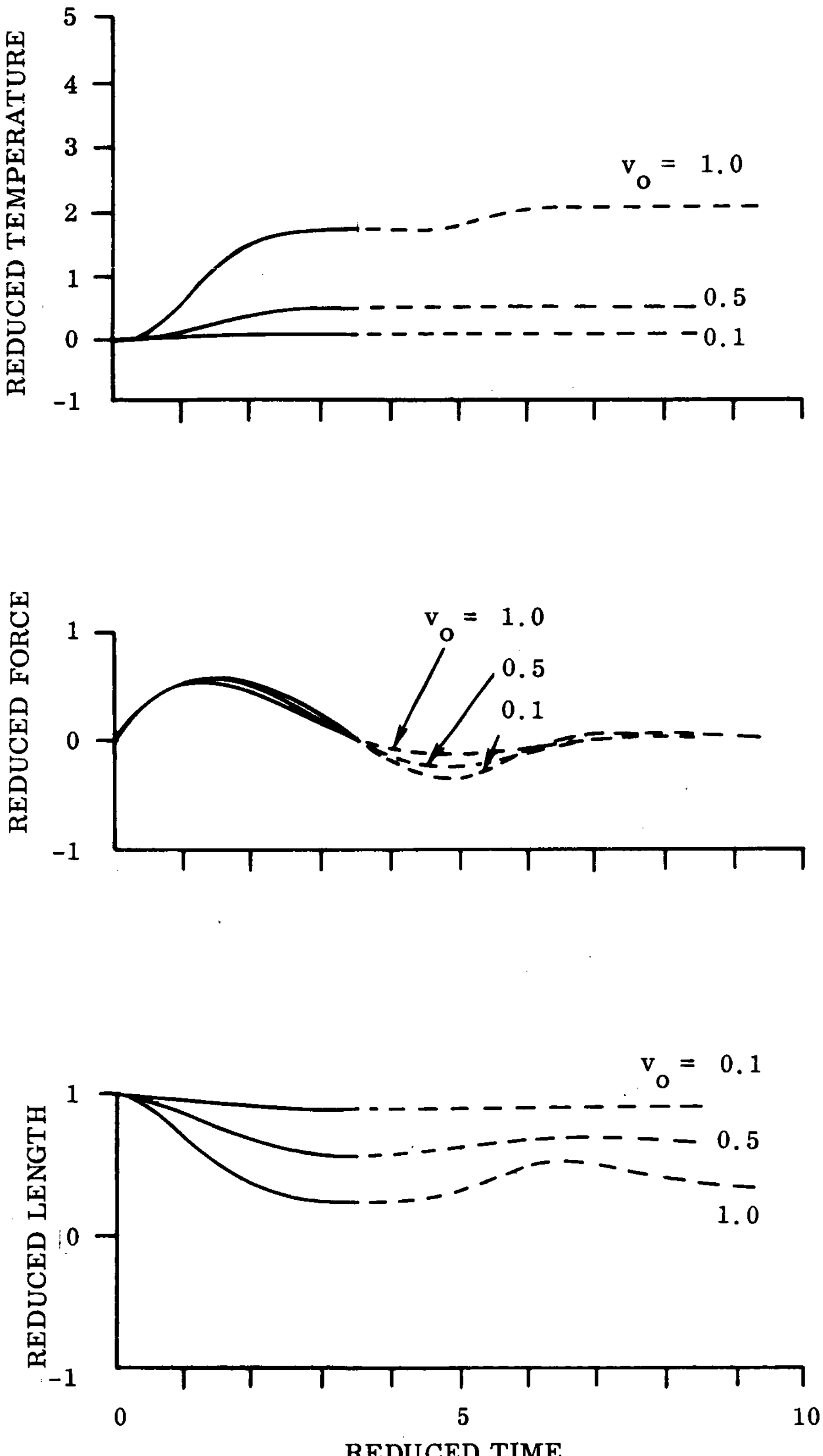
δ = 10.0



$Q = 10.0$

$\delta = 20.0$

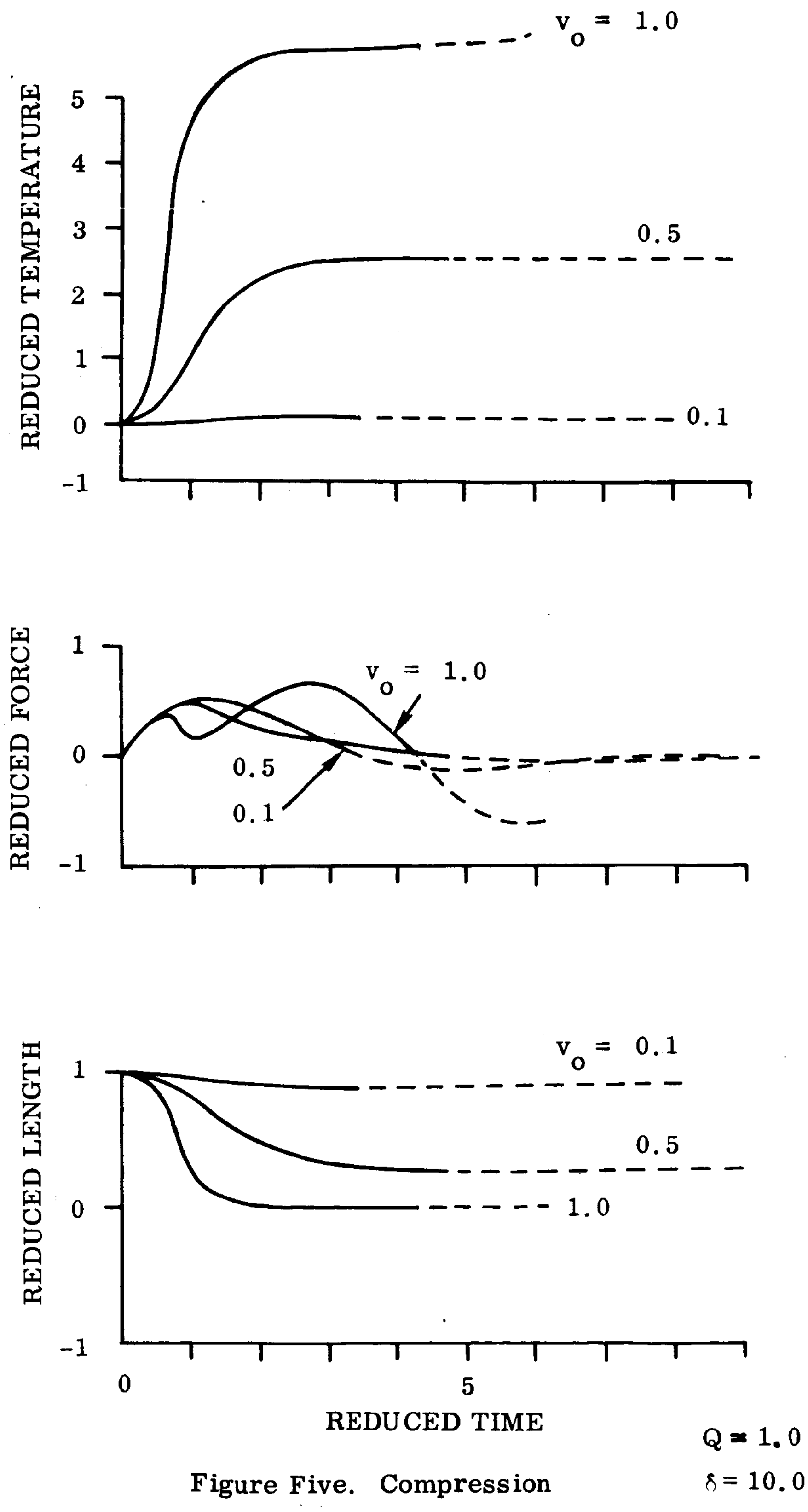
Figure Three. Compression



Q = 1.0

Figure Four. Compression

δ = 2.0



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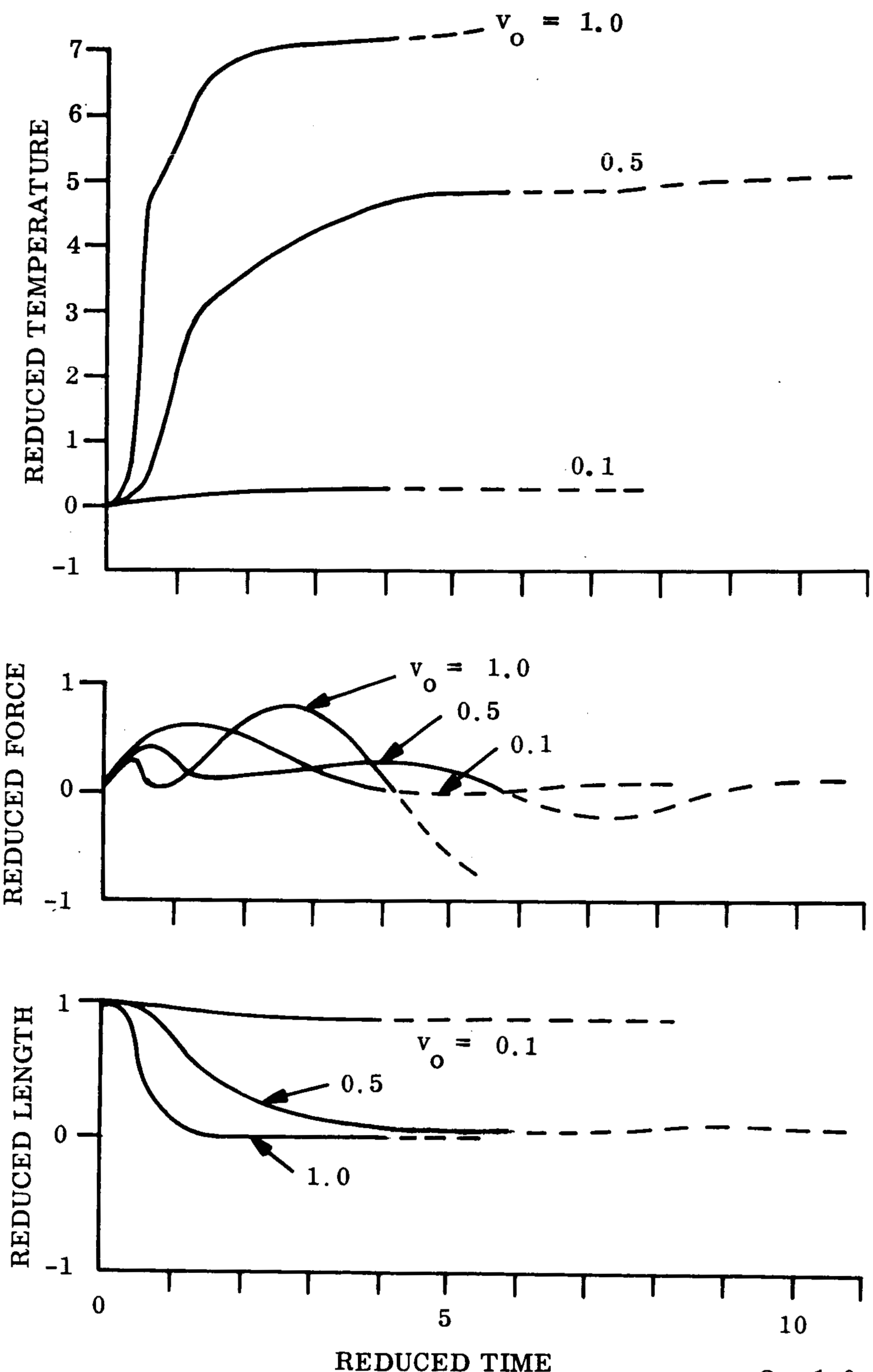

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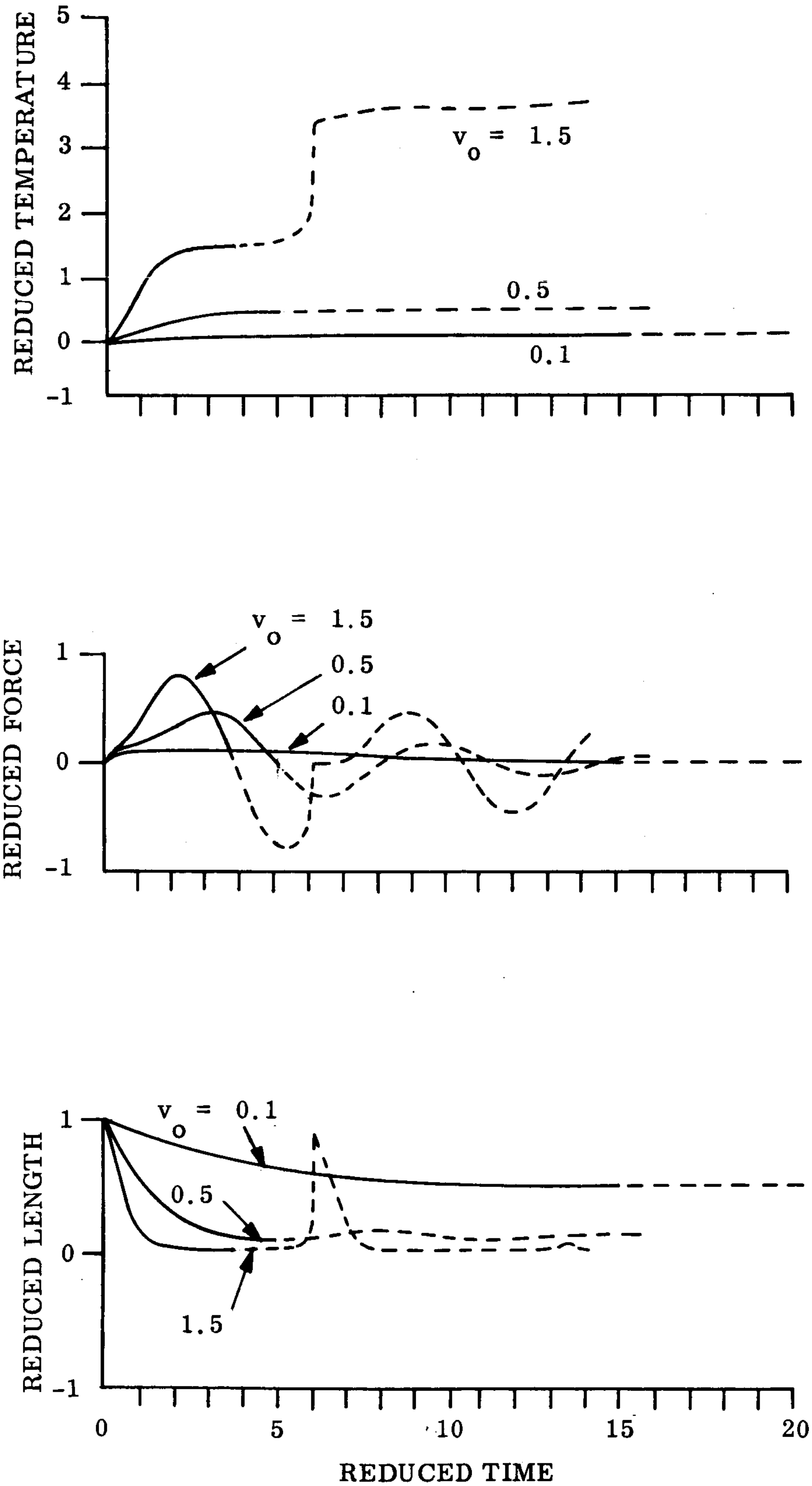
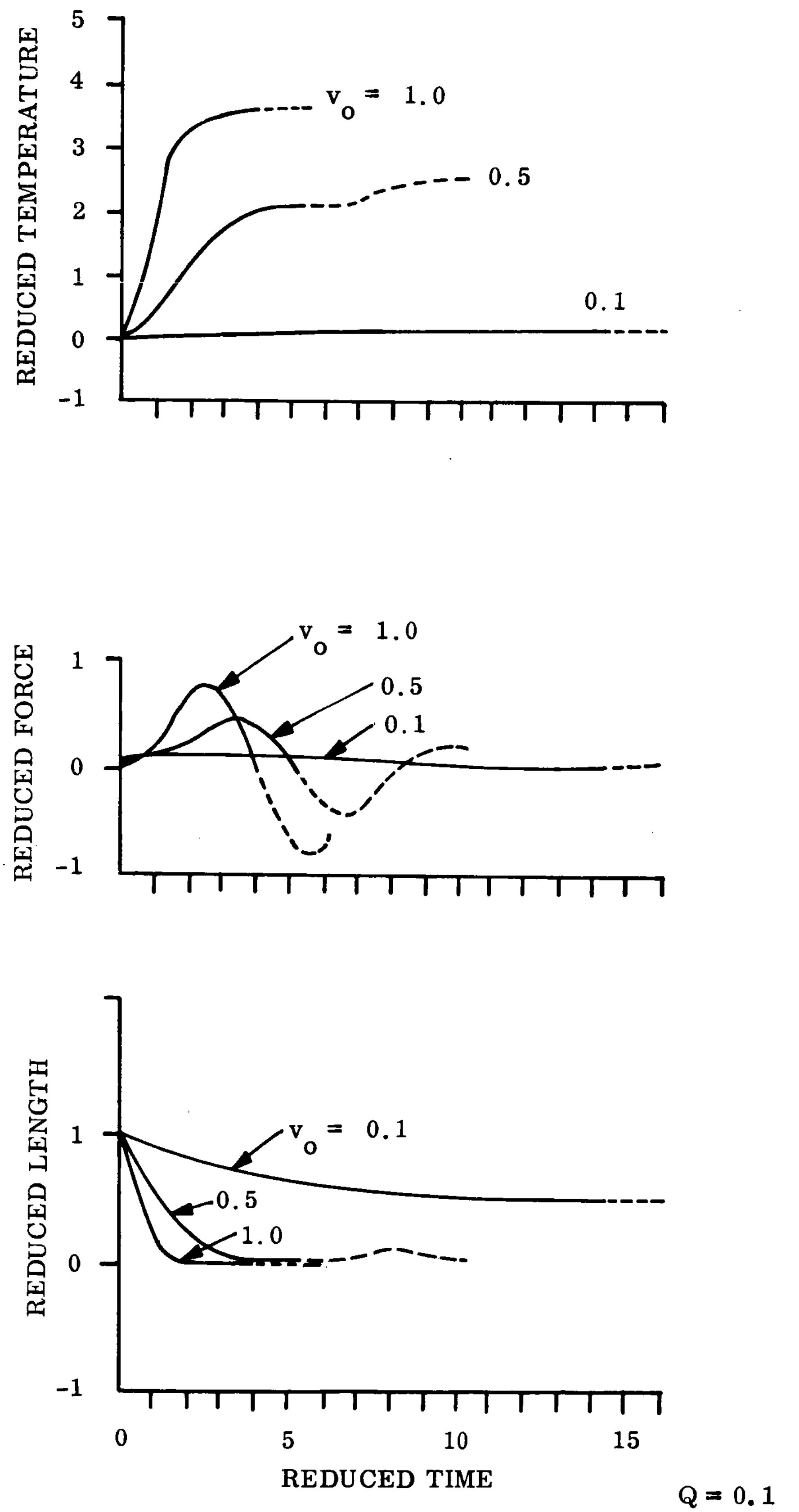


Q = 0.1

Figure Seven. Compression δ = 2.0



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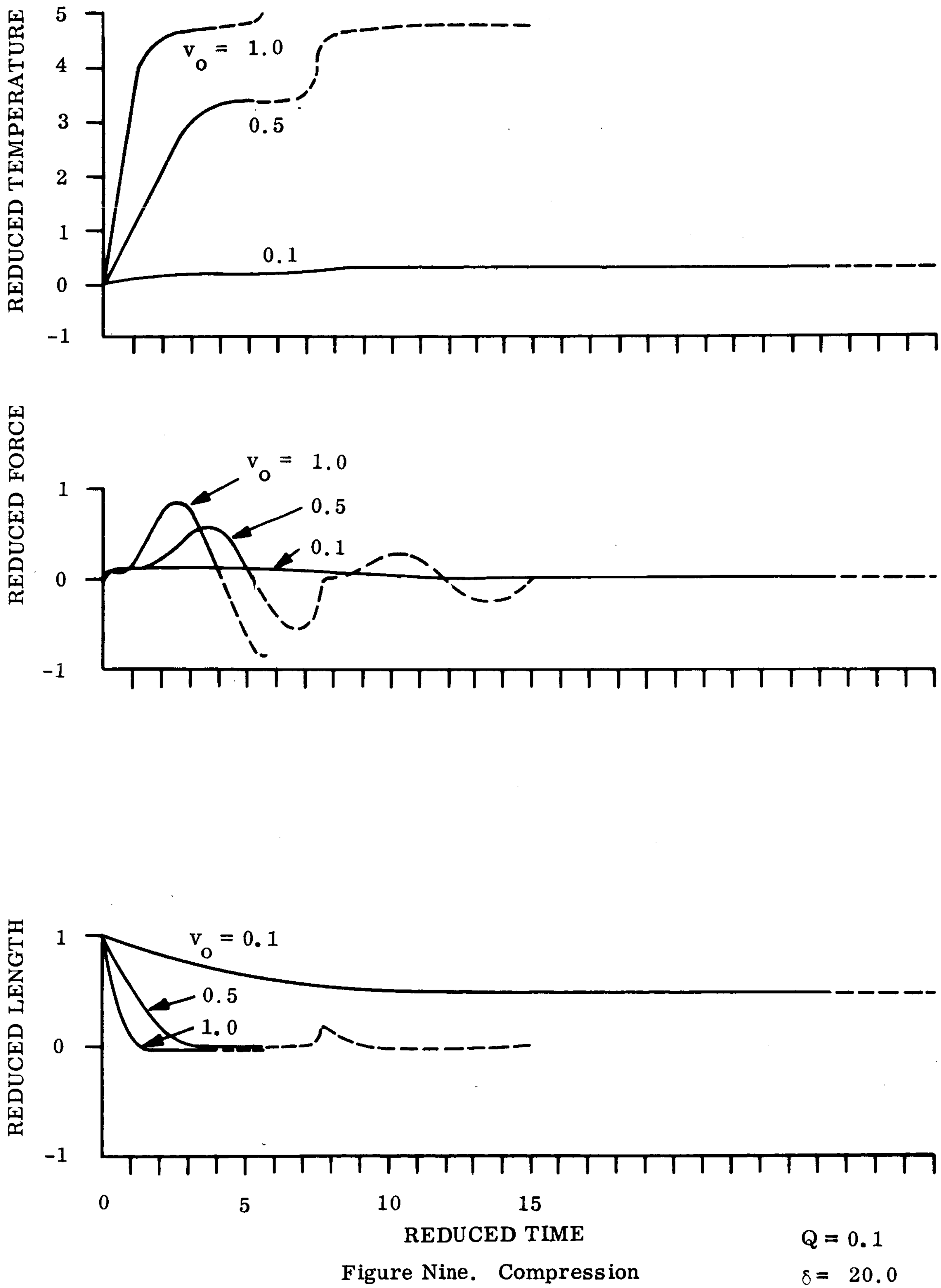

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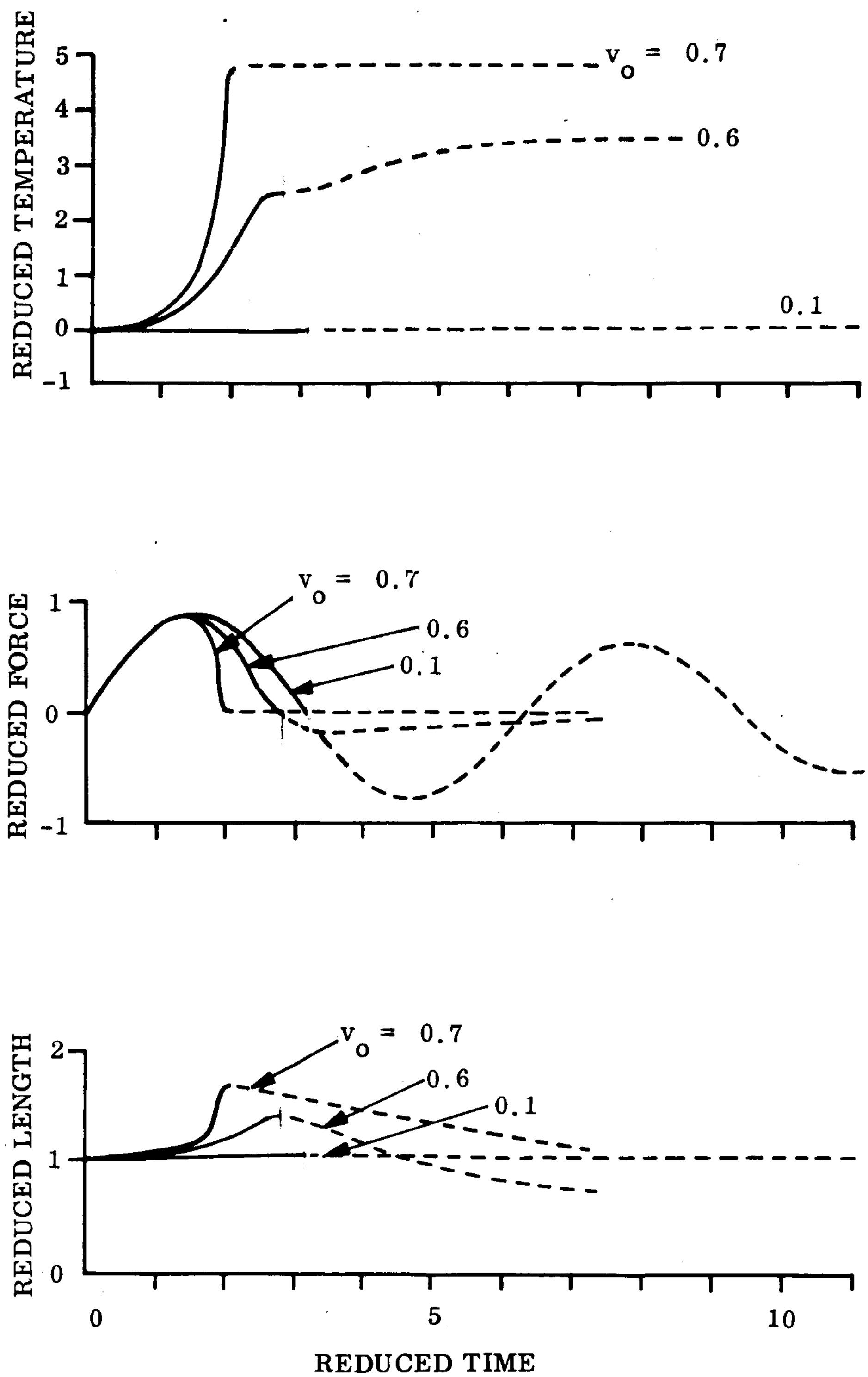


$Q = 10.0$

Figure Ten. Elongation

$\delta = 10.0$

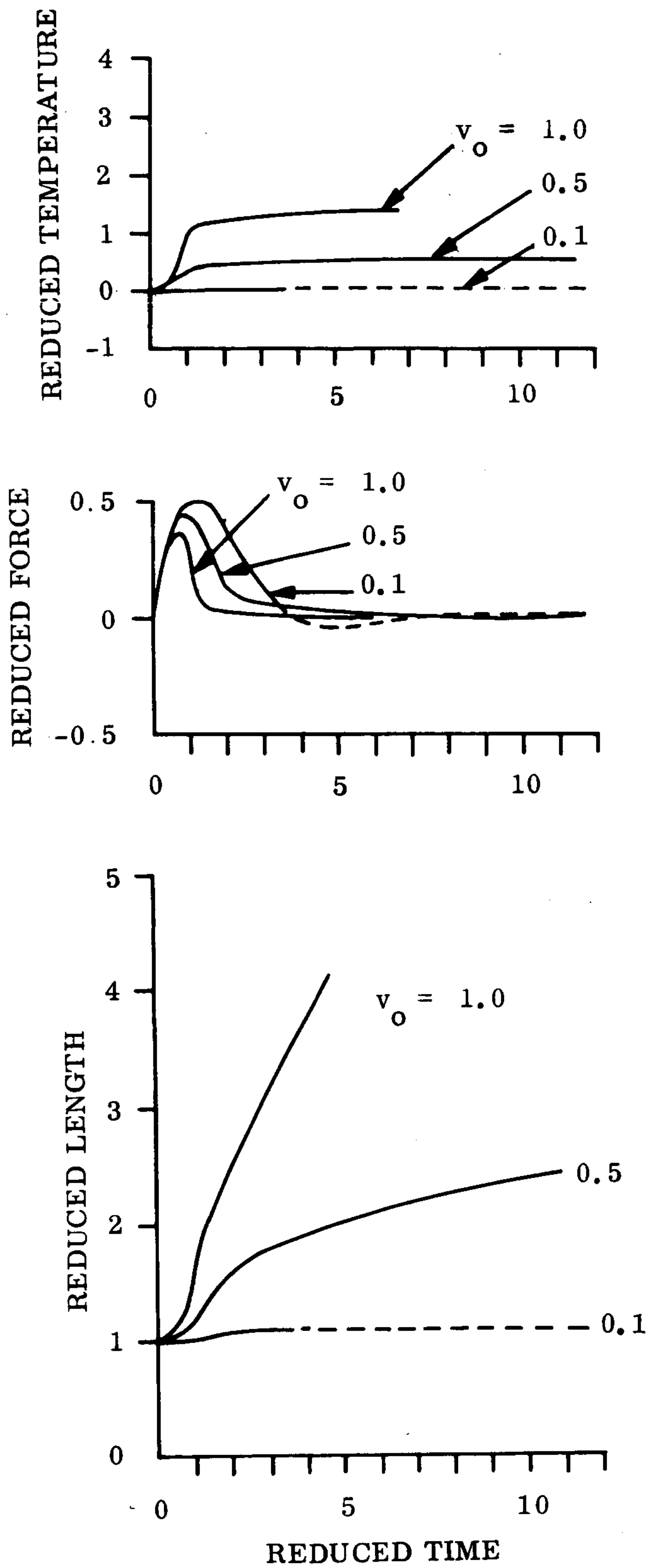
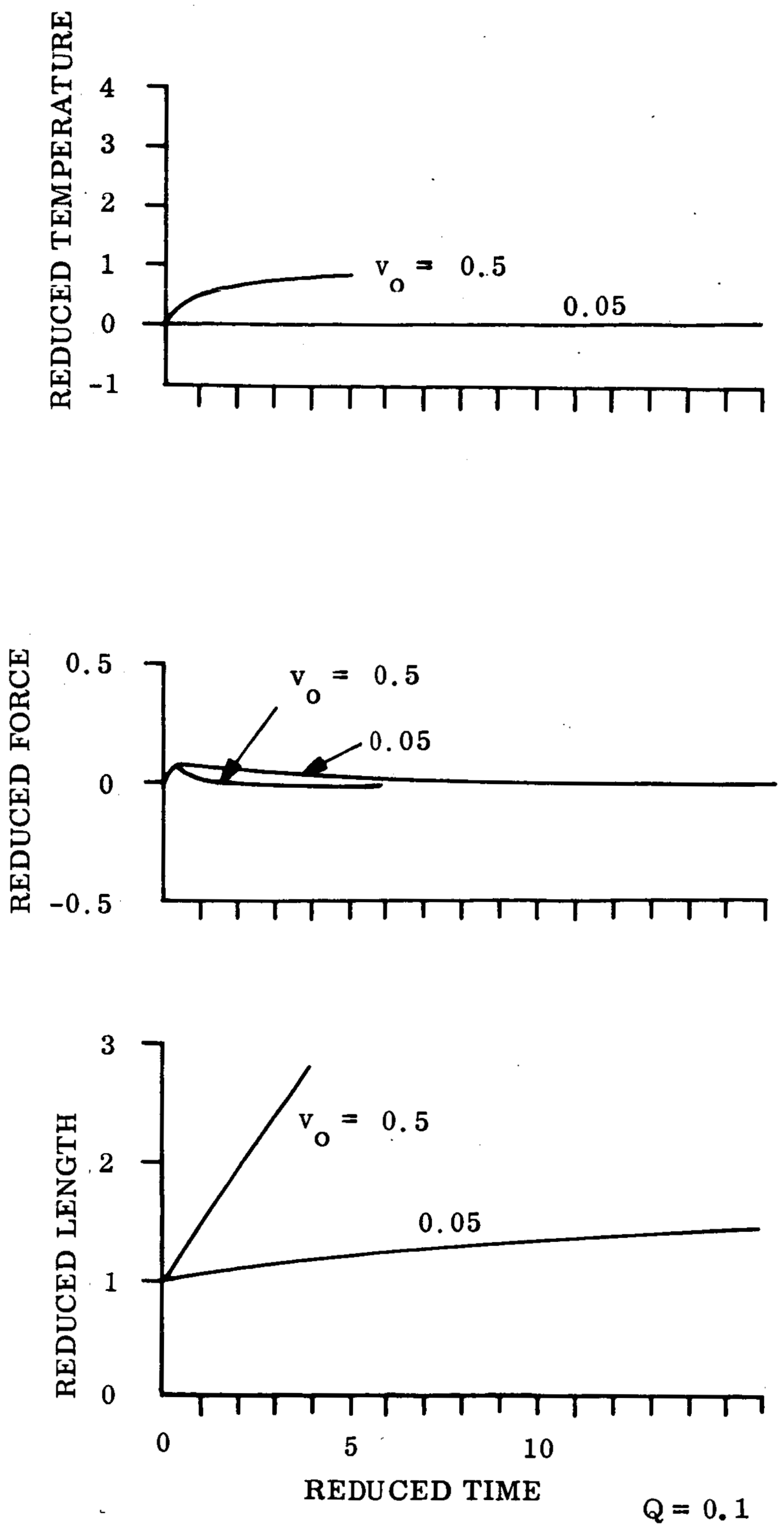


Figure Eleven. Elongation

Q = 1.0

δ = 2.0



Q = 0.1

Figure Twelve. Elongation δ = 20.0