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The $d_{3/2}$ Neutron Scattering Resonance in the Shell Model of O^{16}

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The shell model approach to reaction theory (SMART) [1] provides an exact expression for nucleon reaction amplitudes in terms of matrix elements between A-nucleon eigenstates of a model Hamiltonian H_o. This theory rests on an expression for the reaction matrix element [2] derived from a formulation of reaction theory in which the scattering states are not asymptotic solutions of the exact Schrodinger equation. In this respect SMART differs from superficially similar approaches [3] which are based on approximating nuclear states of shell model wave functions in some adaptation of the theory of Feshbach.

The model Hamiltonian H_o of this paper is that which leads to the shell model of light nuclei

$$H_o = -\frac{\hbar^2}{2M}\sum_{i=1}^{A} \nabla_i^2 + \sum_{i=1}^{A} U(i) \qquad (1)$$

The central potential U is chosen to be real, finite, and spherically symmetric, although this last is not necessary. The H_o therefore has both discrete and continuum A-particle states

$$H_o \chi_a^{\pm} = E_a \chi_a^{\pm} \qquad H_o|n\rangle = \varepsilon_n|n\rangle \qquad (2)$$

which are simply constructed from the bound and continuum states of U. With the zero of energy defined for the state with A - 1 nucleons in the lowest bound states of U and one particle in the continuum state of zero energy, the discrete bound states of H_o will occur at both negative and positive energy. With a suitable choice of H_o a correspondence is established between the states

of H_o and those of H by an adiabatic transition from H_o to H. The negative energy (discrete) states of H_o generally evolve into the bound states of H, and the discrete positive energy states of H_o go into the resonant continuum states of H.

The S-matrix element

$$S_{ba} = \langle \chi_b^- | \chi_a^+ \rangle - 2\pi i \delta(E_a - E_b) \langle \chi_b^- | \hat{T} | \chi_a^+ \rangle \qquad (3)$$

appears to come from an approximation in which wave functions for the initial and final states are approximated by eigenstates of H_o. However, the correlations introduced into both the target and scattering wave function are included through the reduced transition operator $\hat{T}$, which satisfies

$$\hat{T} = V + V(E^+ - H_o)^{-1} \hat{T} \qquad V \equiv H - H_o + (\text{Energy Shift}) \qquad (4)$$

For a proper choice of H_o, the observed resonances in S_{ba} will arise from single particle continuum resonances in the χ_a^+ or from virtual excitation of the discrete states of H_o. These latter resonances are exhibited explicitly by

$$\langle \chi_b^- | \hat{T} | \chi_a^+ \rangle = \langle \chi_b^- | \hat{\hat{T}} | \chi_a^+ \rangle + \sum_n \frac{\langle \chi_b^- | \hat{\hat{T}} | \tilde{\chi}_n \rangle \langle \chi_n | \hat{\hat{T}} | \chi_a^+ \rangle}{E - E_\mu + i \frac{\Gamma_\mu}{2}} \qquad (5)$$

where $\hat{\hat{T}}$ is a non-resonant effective interaction. The operator $\hat{\hat{T}}$ does not contain the resonances arising from discrete state of H_o, but it does contain the short range modifications of the two-nucleon interaction due to virtual transitions to the high momentum continuum states of H_o. This can be seen from the equation for $\hat{\hat{T}}$

$$\hat{\hat{T}} = V + V(E^+ - H_o)^{-1} P_c \hat{\hat{T}} \qquad (6)$$

where P_c projects on the continuum states of H_o.

The intermediate states X_n are simply linear superpositions of the finite set of discrete states $|n\rangle$ of H_o found by diagonalizing $(H_o + \hat{\hat{T}})$. Since $\hat{\hat{T}}$ is complex the eigenvalues $E_n + i\Gamma_n/2$ are complex and the adjoint states $\tilde{X}_n$ are not complex conjugates of the X_n. If only the $\mathrm{Re}\hat{\hat{T}}$ is used in the diagonalization to determine the X_n and $\mathrm{Im}\hat{\hat{T}}$ is treated as a perturbation, then the intermediate states are exactly those of the shell model. The effective interaction is associated with a particular choice of H_o and is calculable from the two-nucleon interaction. In the shell model $\mathrm{Re}\hat{\hat{T}}$ is treated as a phenomenological interaction which is the sum of two-body, energy independent, local potentials whose form is to be determined from fitting the resonance energies. From Eq. (8) it is clear that $\hat{\hat{T}}$ has these properties only in some approximation. Nevertheless, insight may be gained from this approach if only a more complete calculation of the reaction amplitude could be carried out with a minimum of additional approximations.

A program for this calculation can be developed by using the dispersion relations satisfied by $\hat{\mathcal{T}}$. These are summarized in the single equation

$$\hat{\mathcal{T}} = V + \hat{\mathcal{T}} P_c (E^+ - H_o)^{-1} P_c \hat{\mathcal{T}} \qquad (7)$$

In the approximation that $\mathrm{Im}\hat{\mathcal{T}} \ll \mathrm{Re}\hat{\mathcal{T}}$ an equation for the widths follows immediately from an optical theorem for $\mathrm{Im}\hat{\mathcal{T}}$

$$\Gamma_n(E) = 2\pi \sum_c |\langle X_n | \mathrm{Re}\hat{\mathcal{T}} | X_c^+(E) \rangle|^2 \qquad (8)$$

In addition to the imaginary part of the energy contributed by $\mathrm{Im}\hat{\mathcal{T}}$, a small energy shift is also produced.

$$\Delta E_n = -\Sigma_m |\langle X_m | \mathrm{Im}\hat{\mathcal{T}} | X_n \rangle|^2 / (E_n - E_m) \qquad (9)$$

This shift can be calculated from $\mathrm{Re}\hat{\mathcal{T}}$ by using the same optical theorem to evaluate the off-diagonal matrix elements of $\mathrm{Im}\hat{\mathcal{T}}$. Finally, the assumption of an energy-independent $\mathrm{Re}\hat{\mathcal{T}}$ can be checked for consistency by using the equation

$$\langle X_n | \mathrm{Re}\hat{\mathcal{T}} | X_n \rangle = \langle X_n | V | X_n \rangle + \pi^{-1} \int_o^\infty dE' \, \Gamma_n(E')/(E - E') \qquad (10)$$

The principal value integral can be evaluated by using the expression for the width given in Eq. (8).

A preliminary calculation based on this program has been carried out by choosing a square well for the model potential U. The depth of this well is different in each angular momentum state. The depths were adjusted to yield the single particle energies for the bound shell model levels in O^{16}. The crucial part of this whole analysis, however, is the treatment of the $d_{3/2}$ state, which is actually observed as a resonance in neutron and proton scattering on O^{16}. Although it is usually convenient for U to be the Hartree potential, this choice is neither necessary nor convenient for the $d_{3/2}$ state [4]. Instead U is chosen to have this state bound near zero energy. The difference between U and the Hartree potential then generates a one-body effective interaction $\hat{\mathcal{T}}_p$ which shifts the resonance energy to the observed value and produces the observed single particle width. The resonances in O^{17} and F^{17} are completely described by the virtual excitation of this state.

The shell model states of O^{16} are to be found by diagonalizing $H_o + \mathrm{Re}\hat{\mathcal{T}}_{ph} + \mathrm{Re}\hat{\mathcal{T}}_p$ on the bound states of H_o. These now include states formed by coupling a hole to the bound $d_{3/2}$ state. In the diagonalization $\hat{\mathcal{T}}_p$ merely shifts this state to the energy usually chosen for it. The particle-hole interaction $\mathrm{Re}\hat{\mathcal{T}}$ will therefore be that usually found, except for differences introduced by the use of the more realistic single particle wave functions for a finite potential instead of harmonic oscillator wave functions. These preliminary calculations have been simplified therefore by the use of the particle-hole interaction and the expansion coefficients found by G. E. Brown and collaborators [5].

The widths are calculated with $\mathrm{Re}\,\hat{\Sigma} \approx \mathrm{Re}\,\hat{\Sigma}_{ph} + \mathrm{Re}\,\hat{\Sigma}_{p}$ by using these wave functions in Eq. (8). The particle-hole interaction enters because it permits the exchange of energy which enables the particle to leave. The $\hat{\Sigma}_{p}$ enters because the $d_{3/2}$ state is itself unstable. The two amplitudes add coherently for two reasons. First, the d-state has been shifted upward and consequently broadened by $\hat{\Sigma}_{ph}$. Second, the particle-hole width is affected by different shapes of the d-state wave function, reflecting different choices of U, and the term in $\hat{\Sigma}_{p}$ compensates to first order.

The widths and energy shifts due to damping are shown in Table I for the $J = 1^-$ particle-hole states of O^{16}. From the widths the energy dependence of the interaction matrix element as given by Eq. (10) has been calculated for several levels and is shown in the figure. The small energy dependence of the widths and resonance energies of all these states shows that this is a satisfactory shell model description of all the resonances including that at 24.3 MeV which is principally $d_{3/2}$ coupled to $p_{3/2}$ and $p_{1/2}$ holes. Nevertheless, the model correctly includes the very substantial configuration interaction with the $d_{3/2}$ state which is found in any consistent calculation.

References

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TABLE I

ENERGY	Γ_n MeV	Γ_p MeV	$\Gamma_n + \Gamma_p$ MeV	ΔE_n LEVEL SHIFT keV
24.3	.65	.75	1.40	−42
22.2	.30	.56	.86	+18
20.0	.01	.09	.10	0
17.6	.13	.23	.36	+12
13.7	0	.03	.03	0

NEUTRON AND PROTON WIDTHS, TOTAL WIDTH, AND LEVEL SHIFT DUE TO DAMPING FOR $J=1^-$ PARTICLE-HOLE STATES OF O^{16}.

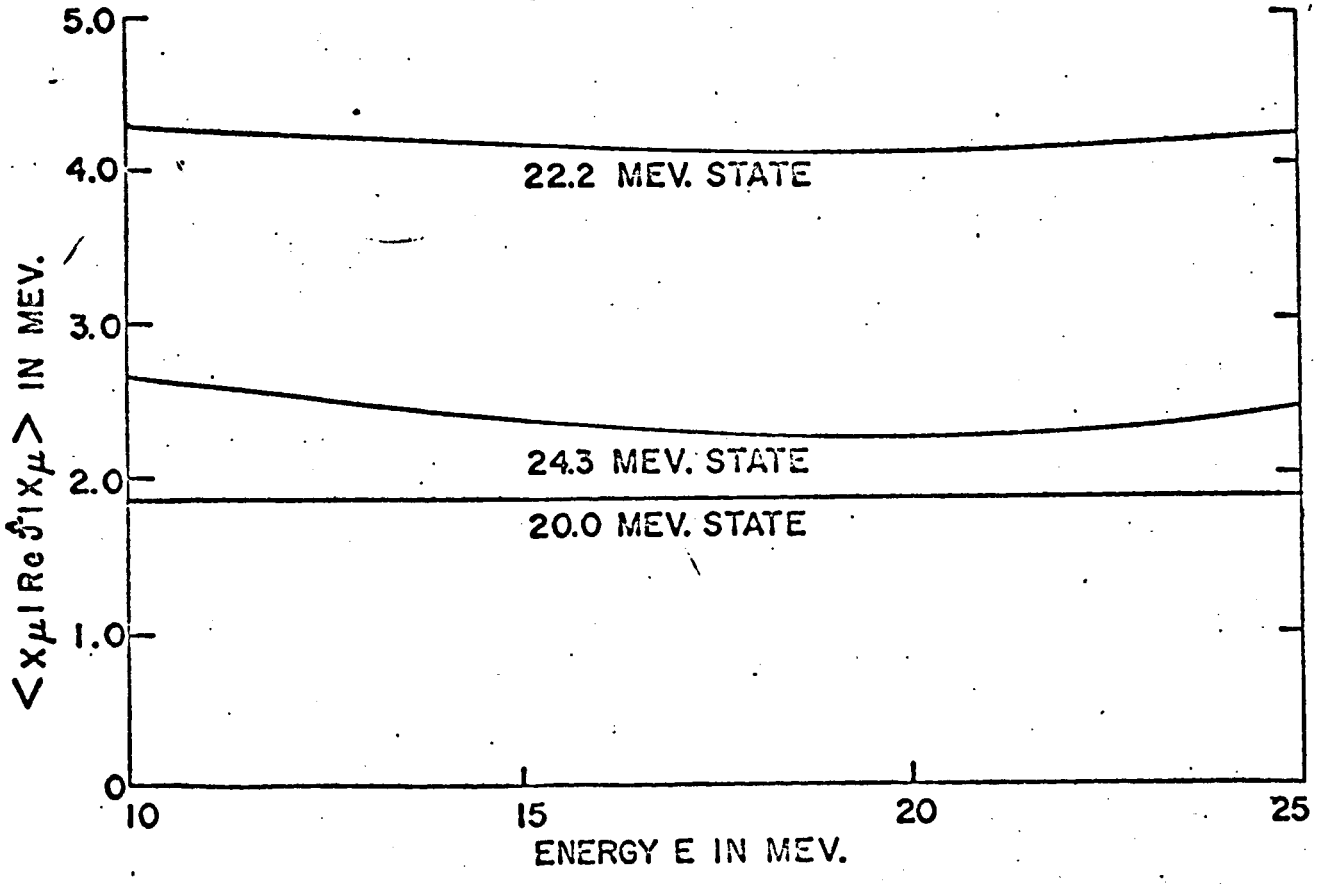

FIGURE I. THE ENERGY DEPENDENCE OF THE DIAGONAL MATRIY ELEMENT OF THE REAL PART OF THE EFFECTIVE MTERACTION FOR THREE $J=1^-$ PARTICLE-HOLE STATES.