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Scientific Report No. 4

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SUMMARY

An antenna which is a combination of the resonant V-antenna and the traveling-wave dipole antenna was studied both theoretically and experimentally. The universal curves which are useful for determining the values of the length of the antenna arms, apex angle, and loading resistors are presented. The theoretical curves for the radiation pattern agree in a general sense with those of experiments. The antenna has a pencil-beam radiation pattern. The two-wavelength long traveling-wave V-antenna has a gain 10.5 times as large as that of a half-wave dipole.

INTRODUCTION

A traveling-wave V-antenna is a V-antenna with a set of terminating resistors near its endsof such value that the current is substantially a traveling wave. The traveling-wave V-antenna is a combination of the resonant V-antenna described by P.S. Carter (ref. 1), et al., in 1931 and in recent books (ref. 2, 3, 4), and the traveling-wave dipole antenna studied by Hallén (refs.5), Altshuler (ref. 6), and quite recently by Wu and King (ref. 7).

A linear dipole antenna is characterized principally by a standing wave of current resulting from reflections at the ends. Its impedance is highly frequency sensitive. Hallén, and Wu and King suppressed the reflected current waves by gradually increasing the surface impedance of the wire as the ends were approached. It was achieved either by ohmic or purely reactive changes of the surface impedance of the antenna. Altshuler placed resistors at the current maximum nearest the ends of the antenna for the same purpose.

A traveling-wave V-antenna combines the advantages of a V-antenna and a traveling-wave dipole. It is characterized by simplicity in structure, a broad frequency band, and high directivity, but also by a loss of power in the terminating impedances. The absence of cylindrical symmetry leads to complications in the analysis. This report includes a discussion of theory and design procedures.

VALUES OF THE LOADING RESISTORS FOR TRAVELING-WAVE EXCITATION

Strictly speaking, a pure traveling wave of current can exist only on nonradiating transmission lines that are terminated in their characteristic impedances. However, the current in a properly loaded V-antenna with apex angle Δ, shown in Fig. 1, can be resolved into a large component which is a pure traveling wave, and a relatively small correction term. It follows that the traveling-wave part of the current must behave just as if it were maintained on a tapered transmission line with a spacing d that is proportional to the distance from the driving point. This line is terminated in a set of resistors at $r = h$ (or h_T away from the end).

In order to answer the question, what lumped resistance should be selected to minimize the wave reflected from the end, it is a satisfactory approximation to consider only the large traveling-wave component of the current which may be determined by transmission-line theory. For this purpose, it is convenient to introduce a characteristic impedance, Z_C, that varies with the distance from the feeding point. It is defined as the ratio of the scalar potential difference between the two arms of the V to the current in a wire – a quasi-stationary approximation. Alternatively, it is defined as the ratio $\sqrt{\frac{L}{C}}$ (refs. 3, 4, 18) where the distributed parameters L and C vary with distance and are calculated in a quasi-stationary manner. It is readily shown that in the special case $\frac{\Delta}{2} = 90^{o}$, this approximation is in general agreement with the more precise analysis by Hallén (ref. 5) (see Fig. 2). Moreover, measurements by Duff (ref. 9) show that the reflected current wave is always minimum when the loading resistance is near the values obtained with this approximation. (See Fig. 3 of ref. 9.)

The resistors are located where the current amplitude is a maximum in the absence of the resistors. The value of the current at this position is twice the incident wave, $2I_0$. When the resistors are inserted, the potential drop $I_h \cdot R$ is established across each. By the compensation theorem this drop may be replaced by a generator with a voltage $-I_h \cdot R$ where I_h is the total current in the resistor at $z = h$. The two generators — one in each arm of the V — are in series with the transmission-line impedances looking toward and away from the apex. If the length h_T toward the open end has the length $h_T \sim (2n+1)\,\lambda/4$ such that the input impedance is zero, the equivalent transmission-line circuit consists of a length of line h with a generator V_o at $z = 0$ and a generator $-2I_h \cdot R$ at $z = h$. The total current is

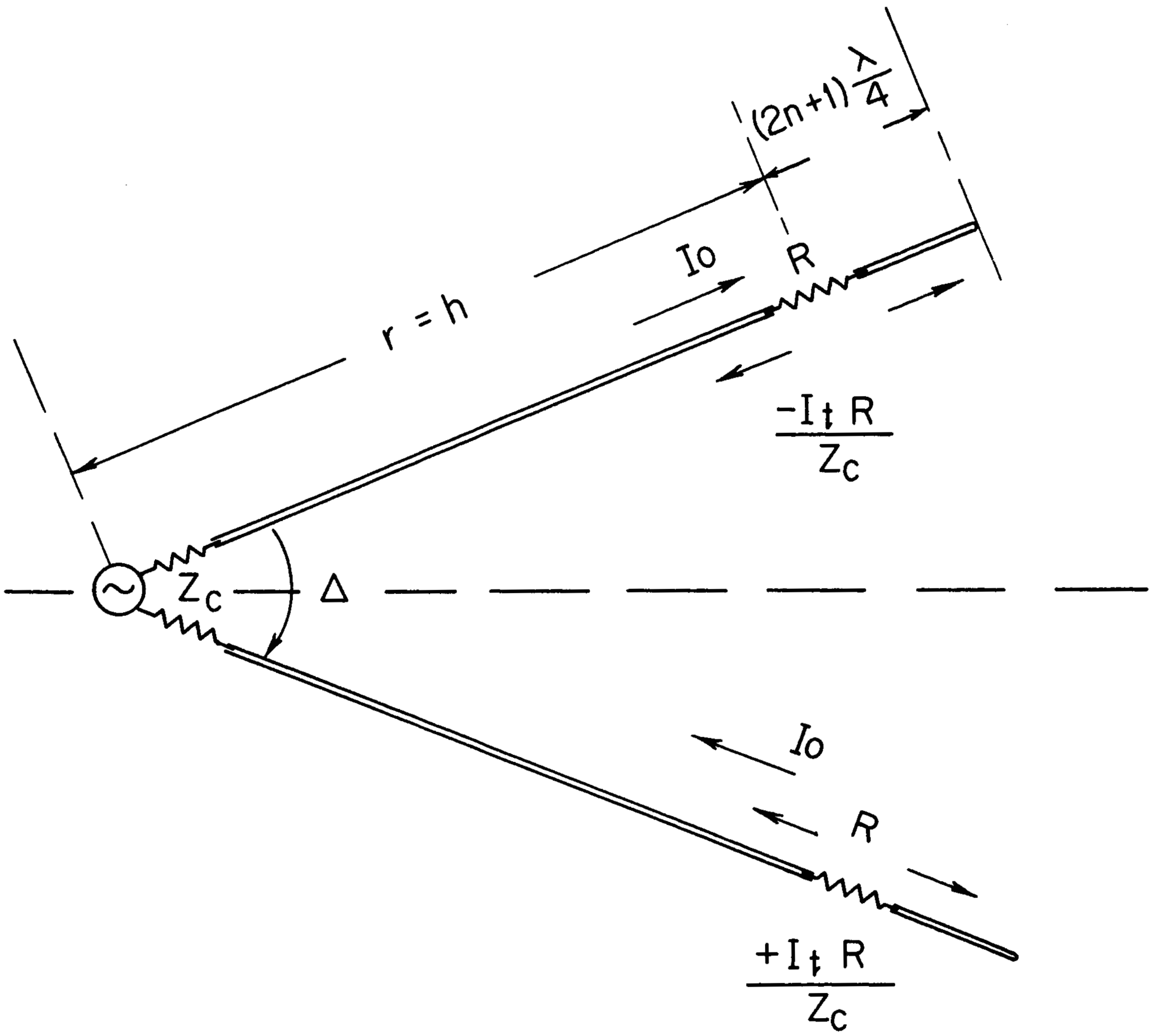

FIG. 1 GEOMETRIC CONFIGURATION OF A TRAVELING-WAVE V-ANTENNA

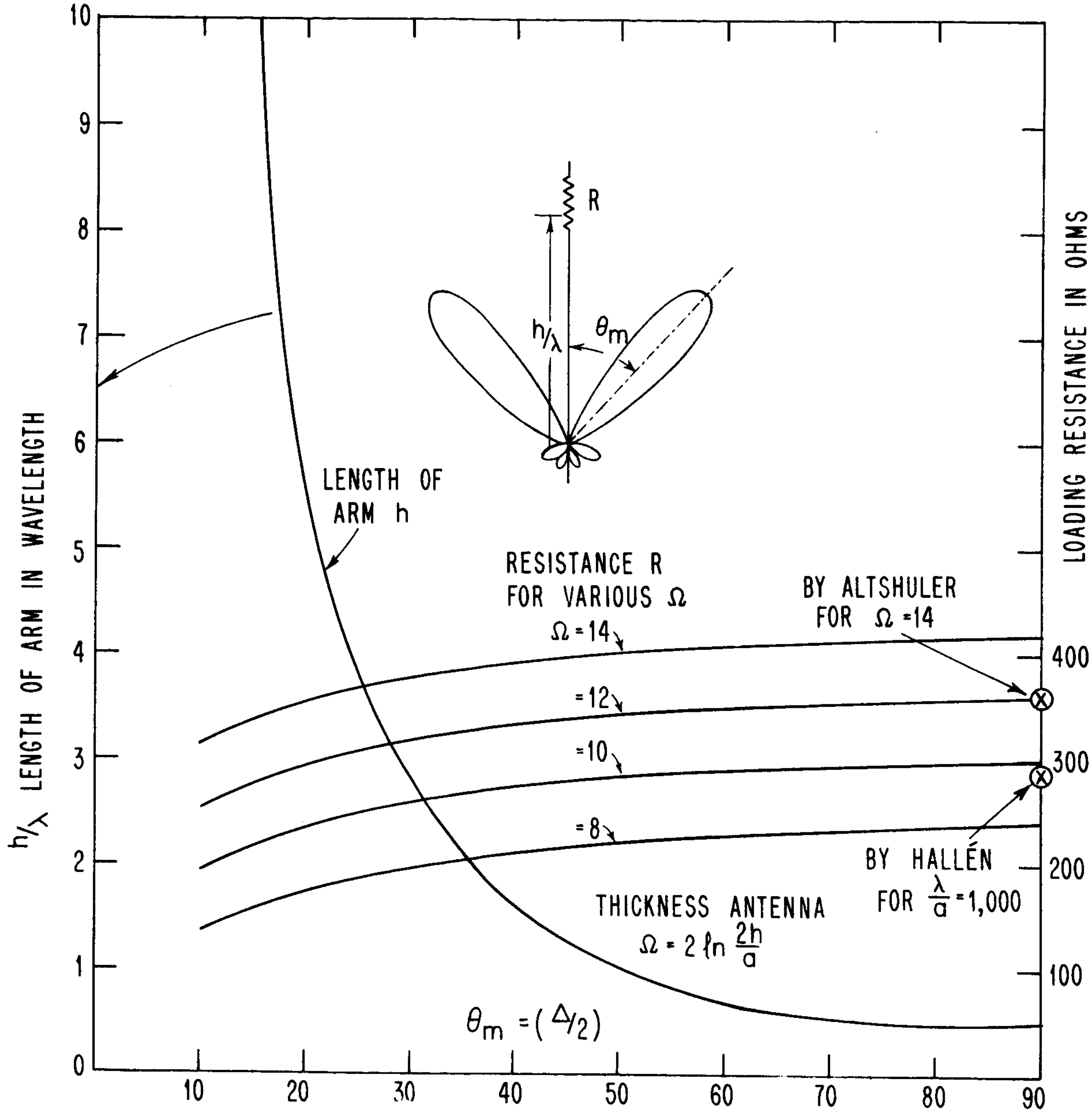

FIG. 2 DIRECTION OF THE MAJOR LOBE OF THE RADIATION PATTERN AND LOADING RESISTANCE

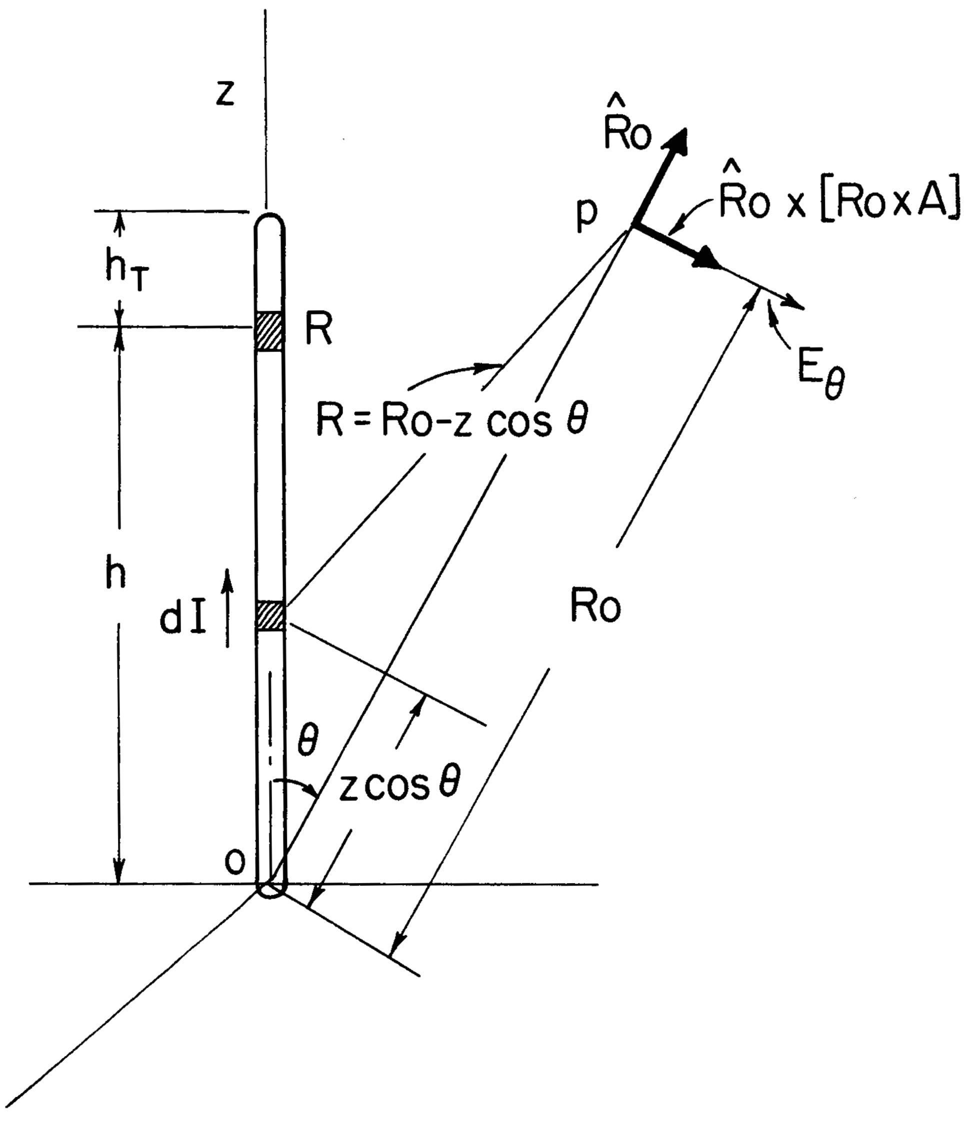

FIG. 3 GEOMETRIC CONFIGURATION OF A SINGLE WIRE EXCITED BY A TRAVELING CURRENT WAVE.

$$I(z) = \frac{1}{j Z_C \tan kh} \left[V \frac{\cos k(h-z)}{\cos kh} - 2 I_h R \frac{\cos kz}{\cos kh} \right] . \qquad (1)$$

If I_h is determined with $z = h$, the result is

$$I(z) = \frac{V}{Z_C} \left\{ \left(\frac{Z_C \cos kh + j\, 2R \sin kh}{2R \cos kh + j Z_C \sin kh} \right) \cos kz - j \sin kz \right\} . \qquad (2)$$

It follows at once that a traveling wave

$$I(z) = \frac{V}{Z_C} e^{-jkz} \qquad (3)$$

is obtained when

$$R = \frac{Z_C}{2} . \qquad (4)$$

An expression for the characteristic impedance of a tapered two-wire-line has been obtained by Schelkunoff (ref. 8).

$$Z_C = \frac{\zeta_o}{\pi} \log \frac{2h \sin \frac{\Delta}{2}}{a} \qquad (5)$$

where

$$\zeta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} .$$

The values of load resistance for a traveling wave were calculated from Eq. (4) and Eq. (5) and plotted in Fig. 2 as a function of the apex angle $\frac{\Delta}{2}$ for various values of the thickness parameter, $\Omega = 2\ln \frac{2h}{a}$ of the wire where a is the radius of the wire. R is rather sensitive to changes in $\frac{\Delta}{2}$ (a rapid increase of R with an increase in $\frac{\Delta}{2}$) when $\frac{\Delta}{2}$ is small, but quite insensitive to changes in $\frac{\Delta}{2}$ when $\frac{\Delta}{2}$ is larger. The range near $\frac{\Delta}{2} = 0$ was avoided because the two wires of finite radius either overlap or touch each other, and the meaning is lost.

When $\frac{\Delta}{2} = \frac{\pi}{2}$, the V becomes a dipole antenna, and for this particular case, detailed discussions made by Hallén (ref. 5), Altshuler (ref. 6), and Wu and King (ref. 7) are available.

Hallén defined a quantity, G_o, the characteristic wave conductance of an antenna of arbitrary length in the process of solving Hallén's integral equation for the cylindrical transmitting antenna. His expression is

$$\frac{1}{Z_C} = \frac{2\pi}{\zeta_o} \frac{2}{\pi^2} \int_0^1 \frac{du}{u\sqrt{1-u^2} \left| H_o^{(1)}(a\beta u) \right|^2} . \tag{6}$$

It is noteworthy that this expression is independent of the length of the wire.

Altshuler obtained an approximate expression for the characteristic impedance of the wire from a similarity of the expressions for the input impedance of an open-circuited transmission line and the zero-order impedance of a dipole antenna. The result is

$$Z_C = \frac{\zeta_o}{2\pi} \Psi \tag{7}$$

where Ψ is a parameter that depends on βh and βa. An approximate value for short antennas is

$$\Psi = \Omega - 2, \; \Omega = 2 \ln \frac{2h}{a} . \tag{8}$$

The values of the loading resistors for $\frac{\Delta}{2} = \frac{\pi}{2}$ obtained using Hallén's and Altshuler's results are also included in Fig. 2 for comparison. They are all in reasonable agreement.

The distribution of current measured by Duff (ref. 9) indicates that the traveling-wave V-antenna is predominantly excited by a traveling-wave when the relations among the apex angle, length of the antenna and the value of loading resistor are close to the ones specified by the graph in Fig. 2.

RADIATION FROM A SINGLE WIRE EXCITED BY A TRAVELING WAVE

A traveling-wave V-antenna consists of two identical arms meeting at the driving-point and, except in the standing-wave section between the resistor and the end of the antenna, both arms are excited by a traveling current wave of the same amplitude but opposite in sign. Due to the identity of the arms, the radiation pattern of the traveling-wave V-antenna can be obtained immediately by a proper superposition if the complex field factor (radiation pattern) of one of the two identical arms is known.

Consider a polar coordinate system with a straight wire stretched along the z-axis from z = 0 to z = h, as shown in Fig. 3. When the wire is excited solely by a traveling current wave

$$I^{s}(z) = I_o e^{-j\beta z}$$

where (9)

$$\beta = \frac{2\pi}{\lambda} \ .$$

From the general expression for the far-zone field (refs. 9 and 10), the field intensity at P is

$$E_\theta = \frac{j\omega\mu_o}{4\pi} \frac{e^{-j\beta R_o}}{R_o} \int_0^h I_o e^{-j\beta z'(1-\cos\theta)} \sin\theta \ dz' \qquad (10)$$

where $R = R_o - z\cos\theta$.

The integration can be performed immediately; the result is

$$E_\theta = I_o \zeta_o \frac{e^{-j\beta R_o}}{4\pi R_o} \cdot F(\theta) \qquad (11)$$

where

$$F(\theta) = \frac{1 - e^{-j\beta h(1-\cos\theta)}}{1-\cos\theta} \sin\theta \qquad (12)$$

$$\zeta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = 120\pi \ .$$

Figure 4a, b, and c, respectively, shows the real part, imaginary part, and the absolute value of the field factor (radiation pattern) calculated from Eq. (11) as a function of θ for a range of values of $\beta h = \pi$, 2π, 2.5π, and 3.5π. Note that the number of lobes increases by one with an increase in βh by π radians.

It should also be observed that with an increase in the length of the wire, a quantity of practical interest, the direction θ_m of the major lobe moves closer to the direction of the axis of the wire. The values of θ_m

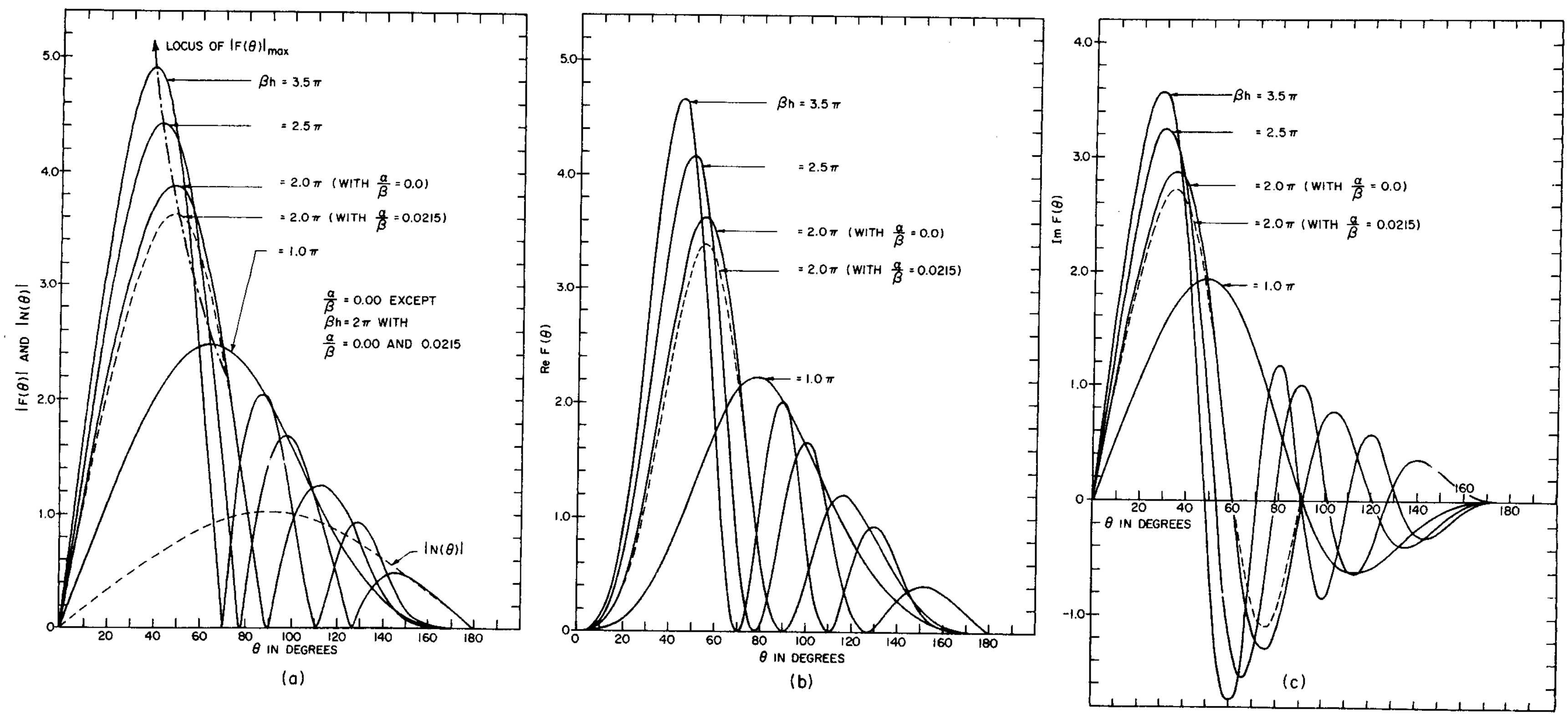

FIG. 4 RADIATION PATTERN FROM A SINGLE WIRE EXCITED BY A TRAVELING CURRENT WAVE.

can be found by equating the derivative of E_θ with respect to θ to zero. Calculated values of θ_m are shown in Fig. 2; θ_m is along the horizontal axis; the length of the wire is along the vertical axis. It is seen from the figure that the rate of decrease in θ_m with respect to an increase in βh becomes rapidly smaller as θ_m is reduced and θ_m can be zero only in the limit of infinite βh. The rate of decrease in θ_m is extremely small when βh is larger than 12π.

This information about θ_m for a single wire is important for determining the apex angle Δ of the traveling-wave V-antenna for which the maximum power is radiated in the direction of the y-axis (see Fig. 5). For the optimum value

$$\frac{\Delta_{opt}}{2} = \theta_m \quad , \tag{13}$$

since, with this apex angle, the direction of the major lobe of the one arm is superimposed on that of the other. It follows that the apex angle of the traveling-wave V-antenna should be decreased as the length of the arms is increased in order to maintain the maximum radiation along the y-axis.

Another interesting fact to be mentioned is that the narrowest beam width D (angle between the first minima of the azimuthal radiation pattern) of the traveling-wave V-antenna obtainable by adjusting the apex angle for a given length and resistor is equal to the apex angle of the antenna, i. e.,

$$D = \frac{\Delta_{opt}}{2} = \theta_m \quad . \tag{14}$$

The width of the major lobe of the antenna increases with either an increase or decrease in the apex angle from its optimum value.

In the above analysis, the attentuation of the traveling wave along the wire was not taken into consideration. This is not necessary unless the antenna is very long. When attenuation is included, the distribution of current along the wire to be used is

$$I = I_o e^{-(j\beta + \alpha)z} \quad . \tag{15}$$

The calculated results using Eq. (15) instead of Eq. (9) are compared with those based upon a loss-free wire in Fig. 4. The attenuation constant used in the calculations is that measured by Duff (ref. 9), viz., 0.6 dB per wavelength independent of the apex angle. This corresponds to $\alpha/\beta = 0.0215$. It is seen that the attenuation of the current has little effect on the radiation pattern.

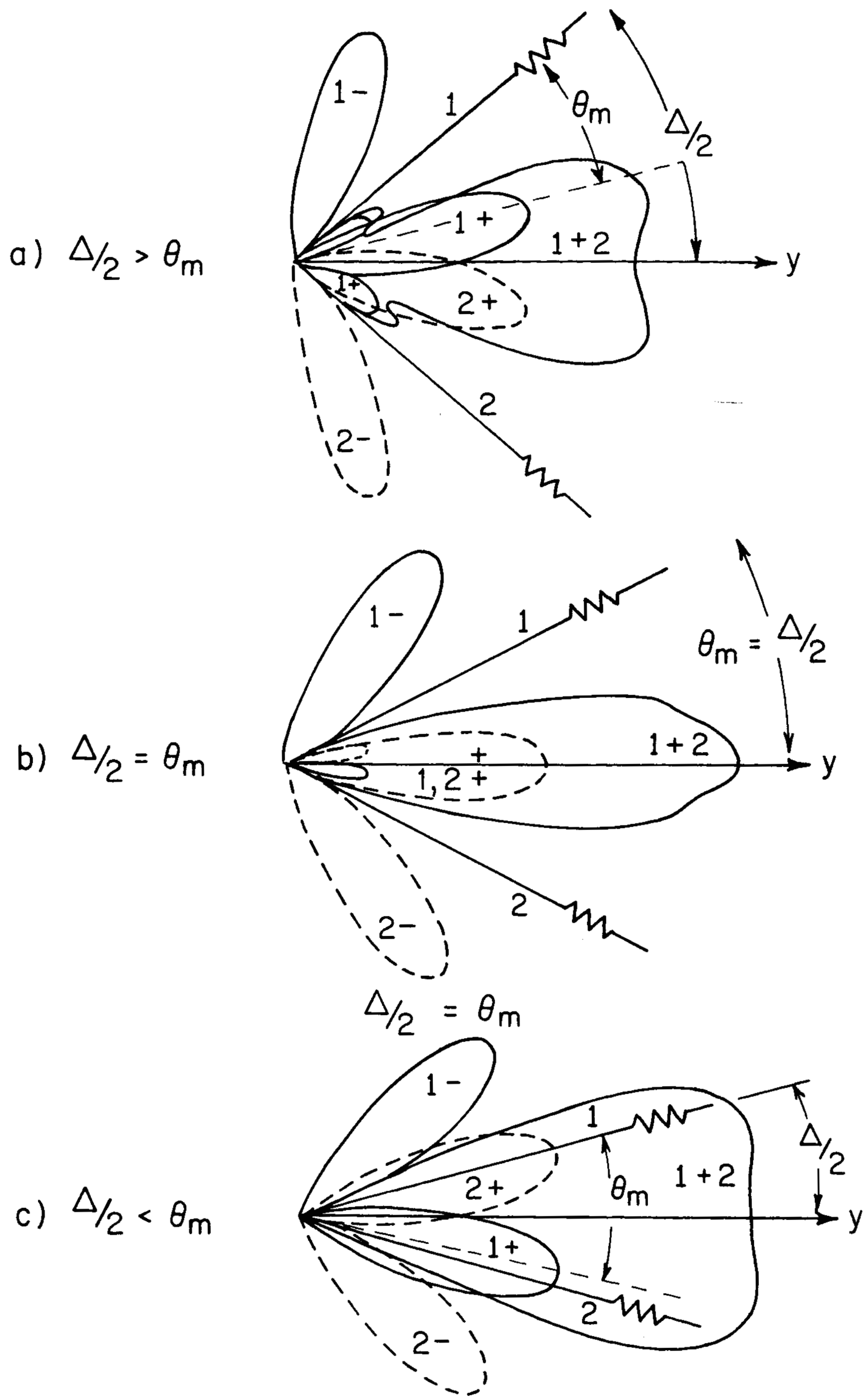

FIG. 5 QUALITATIVE DISCUSSION OF THE RADIATION PATTERN OF A TRAVELING-WAVE V-ANTENNA WITH RESPECT TO THE APEX ANGLE

CONTRIBUTION OF THE STANDING WAVE ALONG THE END SECTION

In the previous sections it has been assumed that the contribution of the standing wave along the end section (h_T in Fig. 3) to the radiation pattern can be ignored. In the following, the contribution from this section is examined. Consider the configuration of one of the two arms of the traveling-wave V-antenna shown in Fig. 3, again. The contribution of the section from the position of the loading resistor $z = h$ to the end of the antenna $h + h_T$, which is predominantly excited by a standing current wave, is

$$I_s = I_{so} \sin \beta \, (h + h_T - z). \tag{16}$$

The expression $N(\theta)$ for the far-zone field can be obtained in a similar form as Eq. (10) by replacing Eq. (9) by Eq. (16).

$$E_s = I_{so} \zeta_o \frac{e^{-j\beta R_o}}{4\pi R_o} N(\theta) \tag{17}$$

$$N(\theta) = \frac{e^{j\beta h \cos\theta}}{\sin\theta} \, j \left[\cos\left(\frac{\pi}{2}\cos\theta\right) + j \left\{ \sin\left(\frac{\pi}{2}\cos\theta\right) - \cos\theta \right\} \right]. \tag{18}$$

The calculated results of $|N(\theta)|$ are plotted in Fig. 4(a) for $\beta h_T = \frac{\pi}{2}$. The magnitude of $N(\theta)$ for the standing-wave section is seen to be much smaller than that for the traveling-wave section, $F(\theta)$ expressed by Eq. (12). The ratio of the main lobes of $F(\theta)$ to the maximum of the magnitude of $N(\theta)$ is 3.8, 5.7, 7.1, 8.3, 9.2, 10.3, or 11.0 for $h/\lambda = 1, 2, 3, 4, 5, 6,$ or 7 with constant $\beta h_T = \frac{\pi}{2}$. It may be concluded that in the calculations in Fig. 2, which were based only on the traveling-wave section, the error due to the exclusion of the contribution of the standing-wave section actually becomes insignificant at sufficiently great lengths of the antenna.

RADIATION PATTERN FROM A TRAVELING-WAVE V-ANTENNA

The radiation pattern of the single wire due to the contributions of both traveling wave and standing-wave sections is obtained as a sum of Eq. (11) and Eq. (17) with a consideration of the continuation of the traveling and standing wave currents at $z = h$;

$$E_{total} = I_o \zeta_o \frac{e^{-j\beta R_o}}{4\pi R_o} M(\theta) \tag{19}$$

$$M(\theta) = F(\theta) + \frac{e^{-j\beta h}}{\sin \beta h_T} N(\theta) \tag{20}$$

Since the field of each arm is known, the field of the V-antenna with an arbitrary apex angle Δ is simply the addition of the fields of the two arms. The E-plane pattern (in the plane of the antenna or ϕ dependence) is given by

$$F(\phi) = M(\phi - \pi + \frac{\Delta}{2}) - M(\phi - \pi - \frac{\Delta}{2}) . \tag{21}$$

The negative sign indicates that the currents in the two arms are equal and opposite in the sense that in the one it is away from the apex, in the other toward the apex. In Fig. 6, the calculated E-plane pattern is compared with experiment. The correlation is not perfect, but the expression of Eq. (21) with Eq. (20) is adequate for most uses in design.

POWER GAIN

The power gain in the direction of the maximum radiation of the traveling-wave V-antenna is defined as the ratio of the power required from an isotropic radiator to produce the given intensity in the desired direction to that required from the actual antenna (ref. 11)

$$g = \frac{4\pi R_o^2 P_r}{W} = \frac{4\pi R_o^2 P_r}{\frac{1}{2} R^e I_o^2} \tag{22}$$

where R^e is the external or radiation resistance of the traveling-wave V-antenna. An approximate expression for the field in the direction of the y-axis (in Fig. 5) is obtained from Eq. (20) and Eq. (21) with $\phi = \pi$, and the radiated power in this direction is

$$P_r = \frac{1}{2} E H^* = \frac{1}{2} \frac{1}{\zeta_o} E E^* . \tag{23}$$

Insertion of Eq. (23) into Eq. (22) leads to

$$g = \frac{120}{R^e} \sin\frac{\Delta}{2} \cdot \frac{\left\{1-\cos\beta h(1-\cos\frac{\Delta}{2})\right\}^2 + \left\{\sin\beta h(1-\cos\frac{\Delta}{2})\right\}^2}{(1-\cos\frac{\Delta}{2})^2} . \tag{24}$$

Note that the first factor in Eq. (24) is in the same form as that of the power gain of a half-wave dipole antenna

$$g_{dipole} = \frac{120}{R^e_{dipole}} . \tag{25}$$

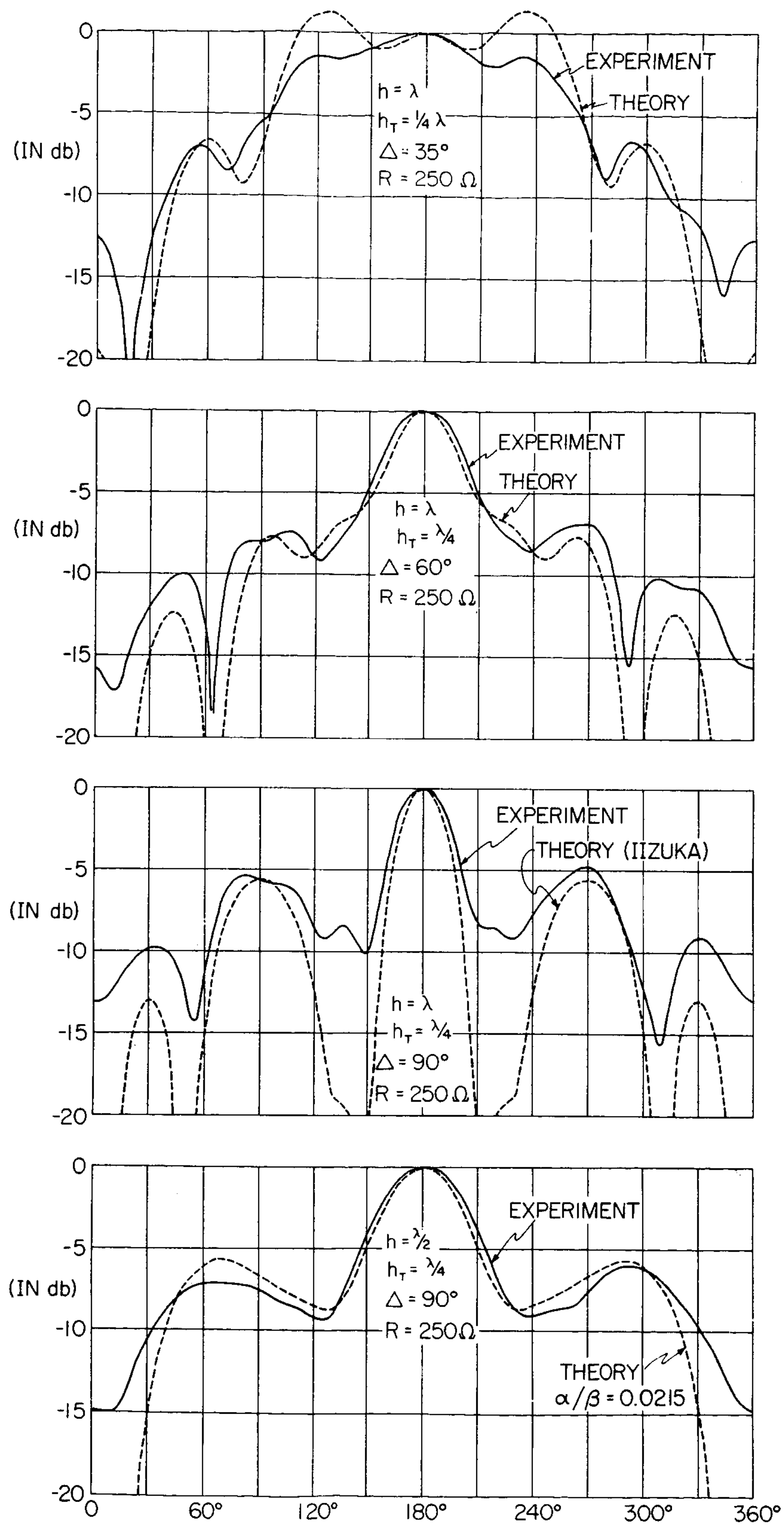

FIG. 6 THEORETICAL AND EXPERIMENTAL RADIATION PATTERNS OF TRAVELING WAVE V-ANTENNA. E-PLANE (IN THE PLANE OF V).

Hence, the second factor in Eq. (25) determines how much better the directivity of a traveling-wave V-antenna is than that of a half-wave dipole, when the radiation resistances are the same for both antennas. Taking the traveling-wave V-antenna with $h = 2\lambda$ (whose apex angle $\frac{\Delta}{2}$ is 36^{o} from Fig. 2) for example, the gain is

$$g = 10.5 \cdot g_{dipole} .$$

ACKNOWLEDGMENT

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