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PLASTIC BUCKLING OF AXIALLY COMPRESSED ECCENTRICALLY STIFFENED CYLINDERS

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SUMMARY

A solution for the plastic buckling of axially compressed eccentrically stiffened cylinders is derived by use of the J_2 deformation theory of plasticity (maximum energy of distortion theory) for a set of simply supported edge boundary conditions. Many effects which have not been studied previously are included, and a closed-form stability criterion is obtained. The present results are more realistic for practical stiffened cylinders than an existing solution. A numerical example is given to illustrate the effect of plastic action on the buckling load of a typical aerospace structure.

INTRODUCTION

The ring-and-stringer-stiffened cylinder is a commonly used aerospace structure for supporting axially compressive loads. Although considerable information is available for determination of the elastic buckling load of such structures, information is scarce on buckling at stresses above the proportional limit of the shell material. Prior work on this problem is apparently confined to reference 1 wherein conventional orthotropic shell theory is employed to represent a stiffened cylinder. Two deficiencies occur in the approach of reference 1; the essentially uniaxial character of the stiffeners and the eccentricity of the stiffeners are not considered.

In the present report, plastic buckling of axially compressed, simply supported, eccentrically stiffened cylinders such as shown in figure 1 is treated. The uniaxial character of the stiffeners and the stiffener eccentricity (asymmetry about the cylinder middle surface) are explicitly accounted for by following the approach of references 2 and 3. Simple J_2 deformation theory of plasticity is utilized (also called maximum octahedral shear stress theory, maximum energy of distortion theory, etc.). Experimental results for buckling are in reasonably good agreement with J_2 deformation theory.

The present solution is an extension of Stowell's work (ref. 4) in which it is assumed that unloading does not occur during buckling. In contrast to reference 4, however, a variable Poisson's ratio is included in the present theory as was done by Bijlaard (ref. 5).

Donnell-type stability differential equations are employed in the study, and it is assumed that buckling is a bifurcation from a membrane prebuckled shape. In addition, the circumferential stiffeners are considered not loaded, that is, ring constraint is neglected, prior to buckling. An equation is developed which gives the buckling stress in terms of the geometry of the cylinder and the properties of the cylinder material. A numerical example is given to illustrate the use of the present solution and to show some typical results.

SYMBOLS

The units for the physical quantities used in this paper are given both in U.S. Customary Units and in the International System of Units (SI) (ref. 6). Factors relating these two systems are given in the appendix.

a	ring spacing (see fig. 1)
a_{ij}	defined by equations (34)
A_{ij}	plasticity coefficients defined by equations (19)
A_r	cross-sectional area of a ring
A_s	cross-sectional area of a stringer
b	stringer spacing (see fig. 1)
B	extensional stiffness, $\frac{E_{sec} t}{1 - \mu^2}$
D	bending stiffness, $\frac{E_{sec} t^3}{12(1 - \mu^2)}$
E	Young's modulus
E_{sec}	secant modulus
E_{tan}	tangent modulus

e_i	strain intensity (see eq. (2))
G	shearing modulus, $\frac{E}{2(1 + \mu_e)}$
G_{sec}	secant shearing modulus, $\frac{E_{sec}}{2(1 + \mu)}$
I_{os}	moment of inertia of stringer about middle surface of cylinder
I_r	moment of inertia of ring about ring centroid
I_s	moment of inertia of stringer about stringer centroid
J_r	torsional constant of a ring
J_s	torsional constant of a stringer
K	material stress-strain curve parameter
K_μ	function defined by equation (14)
l	cylinder length
m	number of longitudinal buckle half-waves
M_x, M_y, M_{xy}, M_{yx}	moments per unit length
n	number of circumferential buckle waves
N_x, N_y, N_{xy}	in-plane forces per unit length
$\overline{N}_x$	applied axial force per unit length at buckling
R	radius of cylinder middle surface

t	cylinder thickness
u,v,w	displacements from a membrane prebuckled shape in longitudinal, circumferential, and radial directions, respectively
x,y,z	longitudinal, circumferential, and radial coordinates, respectively, on cylinder middle surface
$\bar{z}_r$	distance from ring centroid to cylinder middle surface (see fig. 1), positive when ring on outside
$\bar{z}_s$	distance from stringer centroid to cylinder middle surface (see fig. 1), positive when stringer on outside
ϵ	uniaxial strain
$\epsilon_1,\epsilon_2,\epsilon_3$	variations in middle surface strain
$\epsilon_x,\epsilon_y,\gamma_{xy}$	strains
$\lambda = \dfrac{1 - \dfrac{E_{tan}}{E_{sec}}}{4K_{\mu}(1 - \mu^2)}$	
μ	Poisson's ratio, $\frac{1}{2} - \left(\frac{1}{2} - \mu_e\right)\frac{E_{sec}}{E}$
μ_e	Poisson's ratio in elastic state
σ	uniaxial stress
$\bar{\sigma}$	axial buckling stress
σ_i	stress intensity defined by equation (1)
σ_{pl}	stress at proportional limit
σ_{oy}	0.2-percent offset yield stress

$\sigma_x, \sigma_y, \tau_{xy}$ stresses

χ_1, χ_2, χ_3 variations in middle surface curvature

The prefix δ denotes variation of the principal symbol during buckling. When the subscripts x or y follow a comma, they denote partial differentiation of the principal symbol with respect to x or y. Otherwise, the subscripts x and y denote the direction with which the principal symbol is associated.

DERIVATION OF THEORY

By use of simple J_2 deformation theory of plasticity, expressions are obtained for the variations of stresses during buckling in terms of the variations of strains and material properties during buckling. The reference surface is taken as the cylinder middle surface. The results are specialized for axially compressive loading. Subsequently, the variations in stresses are integrated over the shell and stiffeners in order to obtain expressions for the variations in forces and moments during buckling. Finally, the variations in forces and moments are substituted in Donnell-type stability differential equations which are then solved to yield a closed-form stability criterion in terms of the geometric and material properties of the stiffened cylinder.

Fundamental Relations in J_2 Deformation Theory

The fundamental relations in the J_2 deformation theory of plasticity (see ref. 7) are the stress intensity $\sigma_i\left(\sigma_i = 3\tau_o/\sqrt{2}\right.$, where τ_o is the octahedral shear stress$\left.\right)$:

$$\sigma_i = \left(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2\right)^{1/2} \tag{1}$$

the strain intensity $e_i\left(e_i = 3\gamma_o/\left[2\sqrt{2}(1 + \mu)\right]\right.$, where γ_o is the octahedral shear strain$\left.\right)$:

$$e_i = \frac{1}{1 - \mu^2}\left[(1 - \mu + \mu^2)\left(\epsilon_x^2 + \epsilon_y^2\right) - (1 - 4\mu + \mu^2)\epsilon_x\epsilon_y + \frac{3}{4}(1 - \mu)^2\gamma_{xy}^2\right]^{1/2} \tag{2}$$

and the stress-strain relations which are compatible with the stress and strain intensities:

$$\left.\begin{aligned}\sigma_x &= \frac{E_{sec}}{1 - \mu^2}\left(\epsilon_x + \mu\epsilon_y\right)\\ \sigma_y &= \frac{E_{sec}}{1 - \mu^2}\left(\epsilon_y + \mu\epsilon_x\right)\\ \tau_{xy} &= \frac{E_{sec}}{2(1 + \mu)}\gamma_{xy}\end{aligned}\right\} \tag{3}$$

In addition, certain material properties are defined in terms of the stress and strain intensities: first, the secant modulus E_{sec}

$$E_{sec} = \frac{\sigma_i}{e_i} \tag{4}$$

and second, the tangent modulus E_{tan}

$$E_{tan} = \frac{d\sigma_i}{de_i} \tag{5}$$

Poisson's ratio is a variable which is defined on page 387 of reference 7 as

$$\mu = \frac{1}{2} - \left(\frac{1}{2} - \mu_e\right)\frac{E_{sec}}{E} \tag{6}$$

where μ_e is the elastic value of μ.

Variations of Strains and Stresses During Buckling

During buckling, the strains vary from their prebuckling values. Let the variation be denoted by δ; then,

$$\left.\begin{aligned}\delta\epsilon_x &= \epsilon_1 - z\chi_1\\ \delta\epsilon_y &= \epsilon_2 - z\chi_2\\ \delta\gamma_{xy} &= 2\epsilon_3 - 2z\chi_3\end{aligned}\right\} \tag{7}$$

where ϵ_1, ϵ_2, and ϵ_3 are the variations in middle surface strain and χ_1, χ_2, and χ_3 are the variations in middle surface curvature.

Corresponding variations in stresses can be obtained from equations (3); for example,

$$\delta\sigma_x = \frac{E_{sec}}{1 - \mu^2}(\delta\epsilon_x + \mu\delta\epsilon_y) + (\epsilon_x + \mu\epsilon_y)\frac{\delta E_{sec}}{1 - \mu^2} + \epsilon_x E_{sec}\,\delta\left(\frac{1}{1 - \mu^2}\right) + \epsilon_y E_{sec}\,\delta\left(\frac{\mu}{1 - \mu^2}\right) \qquad (8)$$

Accordingly, the following variations are required:

$$\delta E_{sec} = -\frac{E_{sec}^2}{\sigma_i}\left(1 - \frac{E_{tan}}{E_{sec}}\right)\delta e_i \qquad (9)$$

$$\delta\left(\frac{1}{1 - \mu^2}\right) = \left[\frac{2\mu}{(1 - \mu^2)^2}\right]\delta\mu \qquad (10)$$

$$\delta\left(\frac{\mu}{1 - \mu^2}\right) = \left[\frac{1 + \mu^2}{(1 - \mu^2)^2}\right]\delta\mu \qquad (11)$$

From equations (6) and (9),

$$\delta\mu = \left(\frac{1}{2} - \mu_e\right)\frac{E_{sec}^2}{E\sigma_i}\left(1 - \frac{E_{tan}}{E_{sec}}\right)\delta e_i \qquad (12)$$

so that all variations of the material parameters are expressed in terms of δe_i which, in turn, can be obtained from the expression for e_i (eq. (2)):

$$\delta e_i = \frac{1}{2K_\mu e_i(1 - \mu^2)^2}\Big[2(1 - \mu + \mu^2)(\epsilon_x\delta\epsilon_x + \epsilon_y\delta\epsilon_y) - (1 - 4\mu + \mu^2)(\epsilon_x\delta\epsilon_y + \epsilon_y\delta\epsilon_x) + \frac{3}{2}(1 - \mu)^2\gamma_{xy}\delta\gamma_{xy}\Big] \qquad (13)$$

where

$$K_\mu = 1 - \frac{\frac{1}{2} - \mu_e}{1 - \mu^2} \frac{E_{sec}}{E} \left(1 - \frac{E_{tan}}{E_{sec}}\right) \left\{ 2\mu + \frac{1}{2\sigma_i^2} \left[-(1 + 2\mu)(\sigma_x^2 + \sigma_y^2) + 2(2 + \mu)\sigma_x\sigma_y - 6(1 + \mu)\tau_{xy}^2 \right] \right\} \tag{14}$$

Upon substitution of the stress-strain relations (eqs. (3)) and the variations of strains (eqs. (7)), equation (13) becomes

$$\delta e_i = \frac{1}{2K_\mu \sigma_i (1 - \mu^2)} \Bigg(\left[(2 - \mu)\sigma_x - (1 - 2\mu)\sigma_y\right]\epsilon_1 + \left[(2 - \mu)\sigma_y - (1 - 2\mu)\sigma_x\right]\epsilon_2 + 6(1 - \mu)\tau_{xy}\epsilon_3 - z\left\{ \left[(2 - \mu)\sigma_x - (1 - 2\mu)\sigma_y\right]\chi_1 + \left[(2 - \mu)\sigma_y - (1 - 2\mu)\sigma_x\right]\chi_2 + 6(1 - \mu)\tau_{xy}\chi_3 \right\} \Bigg) \tag{15}$$

Finally, the relation for $\delta\sigma_x$ (eq. (8)) becomes

$$\delta\sigma_x = \frac{E_{sec}}{1 - \mu^2} \Bigg[\left[\epsilon_1 + \mu\epsilon_2 - z(\chi_1 + \mu\chi_2)\right] + \left(1 - \frac{E_{tan}}{E_{sec}}\right) \frac{1}{2K_\mu \sigma_i^2} \left[-\sigma_x + \frac{\frac{1}{2} - \mu_e}{1 - \mu^2}(\sigma_y + \mu\sigma_x)\frac{E_{sec}}{E} \right] \times \Bigg(\left[(2 - \mu)\sigma_x - (1 - 2\mu)\sigma_y\right]\epsilon_1 + \left[(2 - \mu)\sigma_y - (1 - 2\mu)\sigma_x\right]\epsilon_2 + 6(1 - \mu)\tau_{xy}\epsilon_3 - z\left\{ \left[(2 - \mu)\sigma_x - (1 - 2\mu)\sigma_y\right]\chi_1 + \left[(2 - \mu)\sigma_y - (1 - 2\mu)\sigma_x\right]\chi_2 + 6(1 - \mu)\tau_{xy}\chi_3 \right\} \Bigg) \Bigg] \tag{16}$$

The expression for $\delta\sigma_y$ is obtained by permutation of the x and y subscripts in equation (16) whereas the expression for $\delta\tau_{xy}$ is

$$\delta\tau_{xy} = \frac{E_{sec}}{2(1 - \mu^2)}\left[2(1 - \mu)\left(\epsilon_3 - z\chi_3\right) - \left(1 - \frac{E_{tan}}{E_{sec}}\right)\frac{\tau_{xy}}{K_\mu \sigma_i^2}\left(1 + \frac{\frac{1}{2} - \mu_e}{1 + \mu}\frac{E_{sec}}{E}\right)\right.$$

$$\times\left(\left[(2 - \mu)\sigma_x - (1 - 2\mu)\sigma_y\right]\epsilon_1 + \left[(2 - \mu)\sigma_y - (1 - 2\mu)\sigma_x\right]\epsilon_2 + 6(1 - \mu)\tau_{xy}\epsilon_3\right.$$

$$\left.\left. - z\left\{\left[(2 - \mu)\sigma_x - (1 - 2\mu)\sigma_y\right]\chi_1 + \left[(2 - \mu)\sigma_y - (1 - 2\mu)\sigma_x\right]\chi_2 + 6(1 - \mu)\tau_{xy}\chi_3\right\}\right)\right] \tag{17}$$

Equations (16) and (17) give the variations in stresses in terms of variations in cylinder middle surface strains and curvatures, membrane prebuckling stresses, and material properties in the prebuckled state.

Hereafter, the equations are restricted to the case of axially compressive loading wherein $\sigma_i = \sigma_x$ because $\sigma_y = \tau_{xy} = 0$. The variations in stresses can now be written as

$$\left.\begin{aligned}
\delta\sigma_x &= \frac{E_{sec}}{1 - \mu^2}\left[A_{11}\left(\epsilon_1 - z\chi_1\right) + \mu A_{12}\left(\epsilon_2 - z\chi_2\right)\right] \\
\delta\sigma_y &= \frac{E_{sec}}{1 - \mu^2}\left[\mu A_{12}\left(\epsilon_1 - z\chi_1\right) + A_{22}\left(\epsilon_2 - z\chi_2\right)\right] \\
\delta\tau_{xy} &= \frac{E_{sec}}{2(1 - \mu^2)} A_{33}\left(\epsilon_3 - z\chi_3\right)
\end{aligned}\right\} \tag{18}$$

where

$$\left.\begin{aligned}
A_{11} &= 1 - \lambda(2 - \mu)^2 \\
A_{12} &= 1 + \frac{\lambda}{\mu}(1 - 2\mu)(2 - \mu) \\
A_{22} &= 1 - \lambda(1 - 2\mu)^2 \\
A_{33} &= 2(1 - \mu)
\end{aligned}\right\} \tag{19}$$

in which

$$\lambda = \frac{1 - \frac{E_{tan}}{E_{sec}}}{4K_{\mu}(1 - \mu^2)} \tag{20}$$

For axial compression, K_{μ} reduces to

$$K_{\mu} = 1 + \frac{\left(1 - \frac{E_{tan}}{E_{sec}}\right)(1 - 2\mu)^2}{4(1 - \mu^2)} \tag{21}$$

Variations of Forces During Buckling

The variation of the forces during buckling is obtained by integration of the variation of stresses over the appropriate area, for example,

$$\delta N_x = \int_{-t/2}^{t/2} \delta\sigma_x \, dz + \frac{1}{b} \int_{A_s} \delta\sigma_x \, dA_s \tag{22}$$

which, upon substitution for $\delta\sigma_x$ and integration, becomes

$$\delta N_x = \frac{E_{sec}\, t}{1 - \mu^2}\left(A_{11}\epsilon_1 + \mu A_{12}\epsilon_2\right) + E_{tan} \frac{A_s}{b}\left(\epsilon_1 - \bar{z}_s \chi_1\right) \tag{23}$$

The last term on the right-hand side of equation (23) is a result of the assumption that the stringers carry only axial stress (hence, the tangent modulus is appropriate). The expressions for δN_y and δN_{xy} are obtained in a similar manner; however, because the rings are assumed to carry no load (that is, ring constraint is neglected) prior to buckling, all ring terms have elastic moduli. In summary,

$$\left.\begin{aligned}
\delta N_x &= \left(BA_{11} + E_{tan}\frac{A_s}{b}\right)\epsilon_1 + \mu BA_{12}\epsilon_2 - \bar{z}_s \frac{E_{tan} A_s}{b} \chi_1 \\
\delta N_y &= \mu BA_{12}\epsilon_1 + \left(BA_{22} + \frac{EA_r}{a}\right)\epsilon_2 - \bar{z}_r \frac{EA_r}{a} \chi_2 \\
\delta N_{xy} &= (1 - \mu)B\epsilon_3
\end{aligned}\right\} \tag{24}$$

where

$$B = \frac{E_{sec}\, t}{1 - \mu^2} \tag{25}$$

Variations of Moments During Buckling

In a manner similar to that used to obtain the variation of the forces during buckling, the variation of the moments is obtained as

$$
\left.
\begin{aligned}
\delta M_x &= -\left[\left(DA_{11} + \bar{z}_s{}^2 E_{tan}\frac{A_s}{b} + E_{tan}\frac{I_s}{b}\right)\chi_1 + \mu DA_{12}\chi_2 - \bar{z}_s E_{tan}\frac{A_s}{b}\epsilon_1\right] \\
\delta M_y &= -\left[\mu DA_{12}\chi_1 + \left(DA_{22} + \bar{z}_r{}^2\frac{EA_r}{a} + \frac{EI_r}{a}\right)\chi_2 - \bar{z}_r\frac{EA_r}{a}\epsilon_2\right] \\
\delta M_{xy} &= \left[(1 - \mu)D + G_{sec}\frac{J_s}{b}\right]\chi_3 \\
\delta M_{yx} &= -\left[(1 - \mu)D + \frac{GJ_r}{a}\right]\chi_3
\end{aligned}
\right\} \qquad (26)
$$

where

$$D = \frac{E_{sec}\, t^3}{12(1 - \mu^2)} \qquad (27)$$

Equations (24) and (26) for the variations in forces and moments during buckling reduce for the elastic eccentrically stiffened cylinder to equation (27) of reference 3. At this stage in the derivation, it is apparent that equations (11) and (12) of reference 1 apply only to orthotropic shells and, if applied to stiffened cylinders, imply that stiffeners have a two-dimensional stress state. The present theory, as does that of reference 3, treats the stiffeners as one-dimensional elements, a representation which is more realistic than an orthotropic shell model for practical stiffened cylinders.

Stability Differential Equations

The Donnell-type stability differential equations for axially compressed cylinders are

$$
\left.
\begin{aligned}
&\delta N_{x,x} + \delta N_{xy,y} = 0 \\
&\delta N_{xy,x} + \delta N_{y,y} = 0 \\
&-\delta M_{x,xx} + \delta M_{xy,xy} - \delta M_{yx,xy} - \delta M_{y,yy} + \frac{\delta N_y}{R} + \bar{N}_x w_{,xx} = 0
\end{aligned}
\right\} \qquad (28)
$$

and the variations of middle surface strains and curvatures are

$$\left.\begin{aligned} \epsilon_1 &= u_{,x} & \chi_1 &= w_{,xx} \\ \epsilon_2 &= v_{,y} + \frac{w}{R} & \chi_2 &= w_{,yy} \\ \epsilon_3 &= \frac{u_{,y} + v_{,x}}{2} & \chi_3 &= w_{,xy} \end{aligned}\right\} \tag{29}$$

Upon substitution of the expressions for the variations in forces and moments during buckling from equations (24) and (26), respectively, and the variations of the middle surface strains and curvatures from equations (29), the stability differential equations (eqs. (28)) become

$$\left.\begin{aligned}
&\left(BA11 + E_{tan}\frac{A_s}{b}\right)u_{,xx} + \mu BA12\left(v_{,yx} + \frac{w_{,x}}{R}\right) - \bar{z}_s\left(E_{tan}\frac{A_s}{b}\right)w_{,xxx} + \frac{1-\mu}{2}B\left(u_{,yy} + v_{,xy}\right) = 0 \\
&\frac{1-\mu}{2}B\left(u_{,yx} + v_{,xx}\right) + \mu BA_{12}u_{,xy} + \left(BA22 + \frac{EA_r}{a}\right)\left(v_{,yy} + \frac{w_{,y}}{R}\right) - \bar{z}_r\frac{EA_r}{a}w_{,yyy} = 0 \\
&\left(DA11 + \bar{z}_s^{\,2}E_{tan}\frac{A_s}{b} + E_{tan}\frac{I_s}{b}\right)w_{,xxxx} + \left[2D\left(1 - \mu + \mu A12\right) + G_{sec}\frac{J_s}{b} + \frac{GJ_r}{a}\right]w_{,xxyy} \\
&\quad + \left(DA22 + \frac{\bar{z}_r^{\,2}EA_r}{a} + \frac{EI_r}{a}\right)w_{,yyyy} - \bar{z}_s E_{tan}\frac{A_s}{b}u_{,xxx} - \bar{z}_r\frac{EA_r}{a}\left(v_{,yyy} + \frac{w_{,yy}}{R}\right) \\
&\quad + \frac{\mu BA_{12}u_{,x}}{R} + \left(BA22 + \frac{EA_r}{a}\right)\left(v_{,y} + \frac{w}{R}\right)\frac{1}{R} - \bar{z}_r\frac{EA_r}{a}w_{,yy} + \bar{N}_x w_{,xx} = 0
\end{aligned}\right\} \tag{30}$$

Stability Criterion

It is desired to find the solution to the stability differential equations for the simply supported edge boundary conditions

$$\delta N_x = v = w = \delta M_x = 0 \tag{31}$$

The following buckling displacements satisfy the boundary conditions (eq. (31)):

$$\left.\begin{aligned} u &= \bar{u} \cos \frac{m\pi x}{l} \cos \frac{ny}{R} \\ v &= \bar{v} \sin \frac{m\pi x}{l} \sin \frac{ny}{R} \\ w &= \bar{w} \sin \frac{m\pi x}{l} \cos \frac{ny}{R} \end{aligned}\right\} \tag{32}$$

(where $\bar{u}$, $\bar{v}$, and $\bar{w}$ are the amplitudes of the buckling displacements) and are substituted in the stability differential equations (eqs. (30)). In order to obtain a nontrivial solution to the resulting equations, the determinant of the coefficients of $\bar{u}$, $\bar{v}$, and $\bar{w}$ must be zero, and the following stability criterion results:

$$\bar{N}_x = \left(\frac{l}{m\pi}\right)^2\left[a_{33} + \left(\frac{a_{13}a_{12} - a_{11}a_{23}}{a_{11}a_{22} - a_{12}^2}\right)a_{23} + \left(\frac{a_{12}a_{23} - a_{13}a_{22}}{a_{11}a_{22} - a_{12}^2}\right)a_{13}\right] \tag{33}$$

where

$$\left.\begin{aligned} a_{11} &= \left(BA_{11} + E_{tan}\frac{A_s}{b}\right)\left(\frac{m\pi}{l}\right)^2 + \left(\frac{1 - \mu}{2}\right)B\left(\frac{n}{R}\right)^2 \\ a_{12} &= \left[\mu BA_{12} + \left(\frac{1 - \mu}{2}\right)B\right]\frac{m\pi}{l}\frac{n}{R} \\ a_{13} &= \frac{\mu BA_{12}}{R}\frac{m\pi}{l} + \bar{z}_s E_{tan}\frac{A_s}{b}\left(\frac{m\pi}{l}\right)^3 \\ a_{22} &= \left(\frac{1 - \mu}{2}\right)B\left(\frac{m\pi}{l}\right)^2 + \left(BA_{22} + \frac{EA_r}{a}\right)\left(\frac{n}{R}\right)^2 \\ a_{23} &= \left(BA_{22} + \frac{EA_r}{a}\right)\frac{\frac{n}{R}}{R} + \bar{z}_r\frac{EA_r}{a}\left(\frac{n}{R}\right)^3 \\ a_{33} &= \left(DA_{11} + \bar{z}_s^2 E_{tan}\frac{A_s}{b} + E_{tan}\frac{I_s}{b}\right)\left(\frac{m\pi}{l}\right)^4 + \left[2D(1 - \mu + \mu A_{12}) + G_{sec}\frac{J_s}{b} + \frac{GJ_r}{a}\right]\left(\frac{m\pi}{l}\right)^2\left(\frac{n}{R}\right)^2 \\ &\quad + \left(DA_{22} + \bar{z}_r^2\frac{EA_r}{a} + \frac{EI_r}{a}\right)\left(\frac{n}{R}\right)^4 + \frac{1}{R^2}\left(BA_{22} + \frac{EA_r}{a}\right) + \frac{2}{R}\bar{z}_r\frac{EA_r}{a}\left(\frac{n}{R}\right)^2 \end{aligned}\right\} \tag{34}$$

The buckling stress is

$$\bar{\sigma} = \frac{\bar{N}_x}{t + \frac{A_s}{b}} \tag{35}$$

Equation (33) is solved by trial and error for the plastic buckling load. Because of the numerous parameters in equation (33) and the need to investigate a large range of buckling modes (values of m and n), it is necessary from a practical standpoint to use a digital computer for numerical work.

The solution represented by equation (33) reduces to: the solution of reference 8 for unstiffened plastic cylinders, the classical Euler load for unstiffened elastic cylinders, and the solution of reference 2 for stiffened elastic cylinders. For a plastic symmetrically stiffened plate column $\left(\bar{z}_s = 0\right)$ with $\mu = \mu_e = \frac{1}{2}$, equation (33) reduces to

$$\bar{N}_x = \left(\frac{\pi}{l}\right)^2 \left[\frac{E_{tan} I_{os}}{b} + \frac{E_{sec} t^3}{9}\left(\frac{1}{4} + \frac{3}{4}\frac{E_{tan}}{E_{sec}}\right)\right] \tag{36}$$

whereas the results of reference 1 imply

$$\bar{N}_x = \left(\frac{\pi}{l}\right)^2 \left[\frac{1}{4}\frac{E_{sec} I_{os}}{b} + \frac{3}{4}\frac{E_{tan} I_{os}}{b} + \frac{E_{sec} t^3}{9}\left(\frac{1}{4} + \frac{3}{4}\frac{E_{tan}}{E_{sec}}\right)\right] \tag{37}$$

(See eqs. (53) and (54) of ref. 1.) Equation (37) contains a term associated with the stiffeners which involves the secant modulus whereas intuition suggests that terms associated with stiffeners should involve only the tangent modulus in agreement with equation (36). Accordingly, equation (37) always gives higher plastic buckling stresses than does equation (36).

NUMERICAL EXAMPLE

Because of the many geometric and material parameters in the theory, general results cannot be presented, even for a specific stress-strain relation. However, a numerical example is presented which is representative of current large-diameter booster interstage structures.

The stress-strain curve for 7075-T6 aluminum alloy is shown in figure 2. This stress-strain curve was represented numerically by the Nádai stress-strain curve for a 0.2-percent offset yield stress:

$$\left.\begin{aligned} \epsilon &= \frac{\sigma}{E} & & (\sigma \leqq \sigma_{pl}) \\ \epsilon &= \frac{\sigma}{E} + 0.002\left(\frac{\sigma - \sigma_{pl}}{\sigma_{oy} - \sigma_{pl}}\right)^K & & (\sigma > \sigma_{pl}) \end{aligned}\right\} \tag{38}$$

where for 7075-T6 aluminum alloy

$\sigma_{pl} = 55$ ksi (0.38 GN/m^2)

$\sigma_{oy} = 72$ ksi (0.50 GN/m^2)

$E = 10.5 \times 10^3$ ksi (72 GN/m^2)

$K = 2.2$

The cylinder has a radius of 135 in. (345 cm), a length of 135 in. (345 cm), and a thickness of 0.10 in. (0.25 cm). The stiffener dimensions are shown in figure 3; the stringer spacing is held constant at 2.00 in. (5.08 cm).

For general instability (buckling in which both rings and stringers participate), buckling stresses are shown in figure 4 for a range of ring spacings. The increase in stress obtained by placing stringers externally as opposed to internally is seen to be considerably lessened for small ring spacings in this example.

For panel instability (buckling in which only stringers participate, that is, buckling between rings), buckling stresses are shown in figure 5 for a range of ring spacings. Large effects of plastic action are seen in figure 5 for small ring spacings. However, only for large ring spacings are the panel instability stresses lower than the general instability stresses. The effects of eccentricity are seen to decrease with increasing plastic action.

CONCLUDING REMARKS

A solution for the plastic buckling of axially compressed eccentrically stiffened cylinders is derived by use of a deformation theory of plasticity for a set of simply supported edge boundary conditions. Stiffeners are treated as one-dimensional elements, a representation which is more realistic than the orthotropic shell model employed in a previous plastic buckling analysis. A numerical example is given to illustrate the effect of plastic action on the buckling load of typical stiffened cylinders in aerospace applications. For this example, the effects of eccentricity are seen to decrease with increasing plastic action.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., August 1, 1966,
124-11-06-04-23.

APPENDIX

CONVERSION OF U.S. CUSTOMARY UNITS TO SI UNITS

The International System of Units (SI) was adopted by the Eleventh General Conference on Weights and Measures, Paris, October 1960, in Resolution No. 12 (ref. 6). Conversion factors for the units used herein are given in the following table:

Physical quantity	U.S. Customary Unit	Conversion factor (*)	SI Unit
Length	inch	0.0254	meters (m)
Stress	ksi = kipsf/in^2	6.895×10^6	newtons per square meter (N/m^2)

*Multiply value given in U.S. Customary Unit by conversion factor to obtain equivalent value in SI Unit.

Prefixes to indicate multiple of units are as follows:

Prefix	Multiple
centi (c)	10^{-2}
giga (G)	10^9

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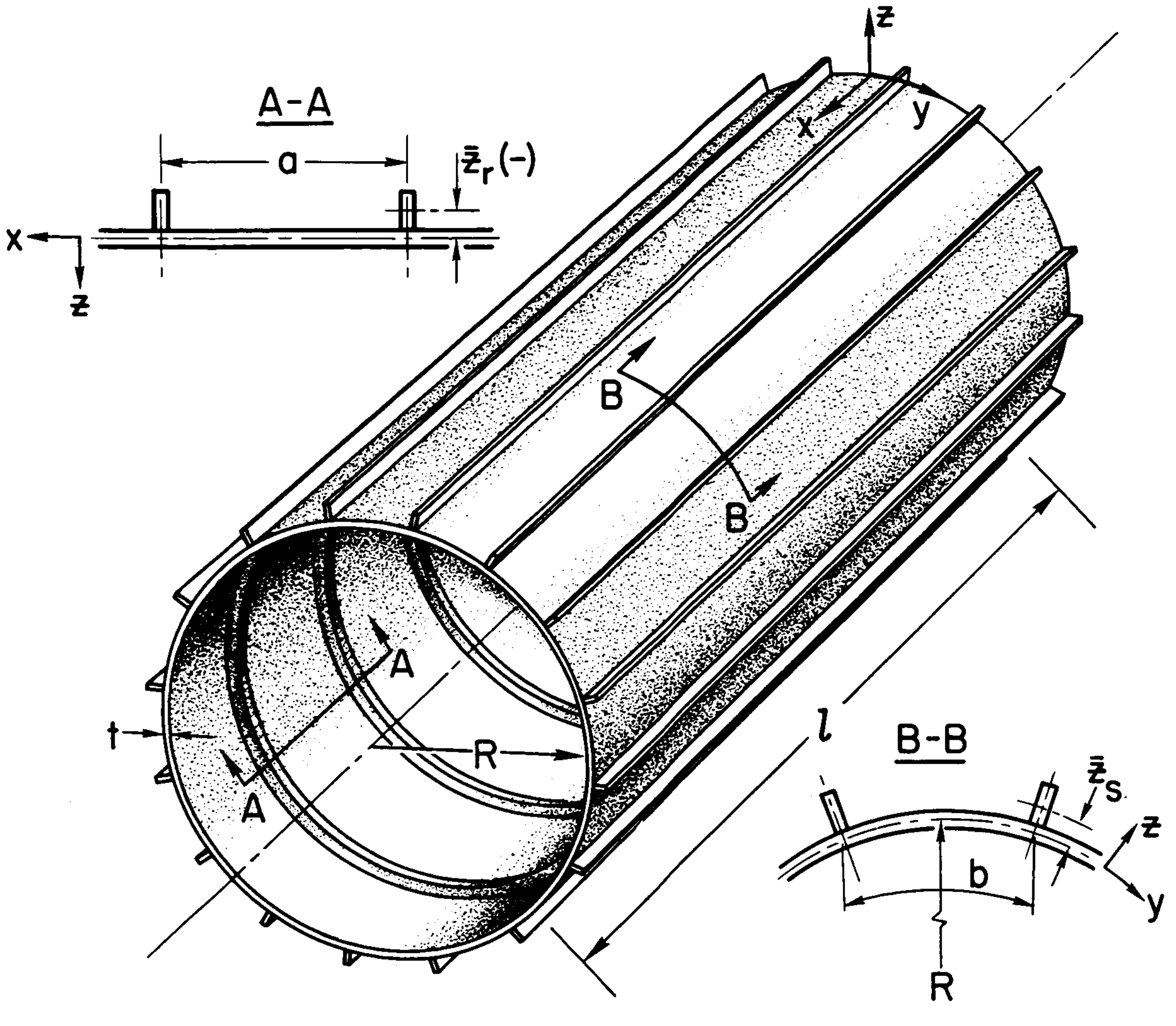


Figure 1.- Stiffened cylinder configuration.

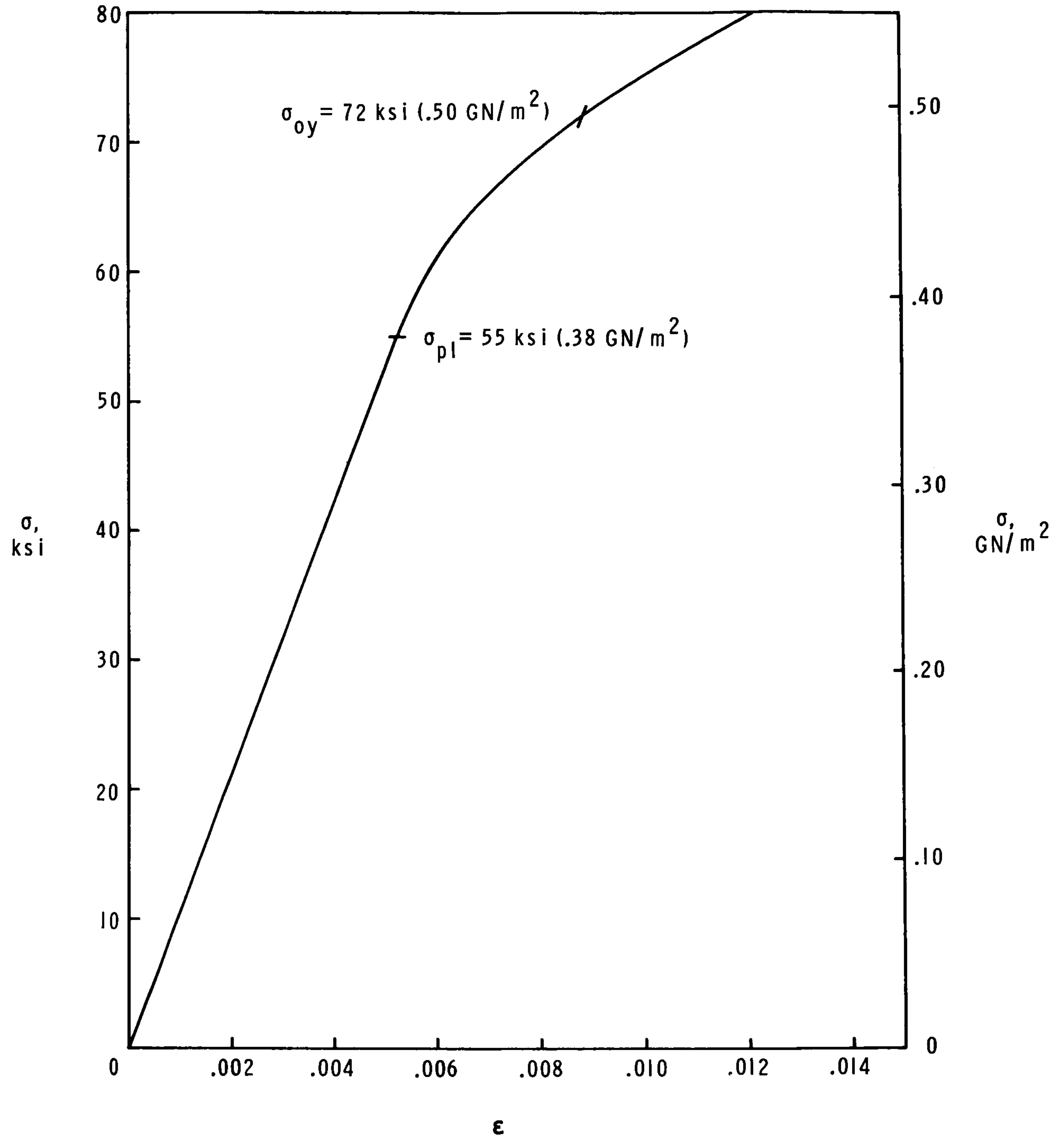
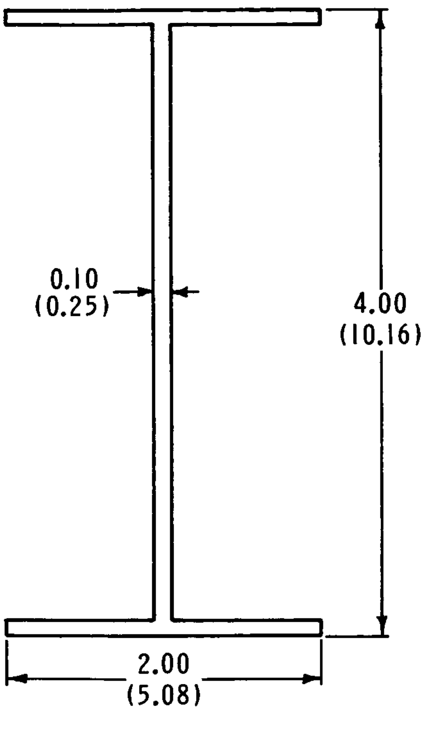
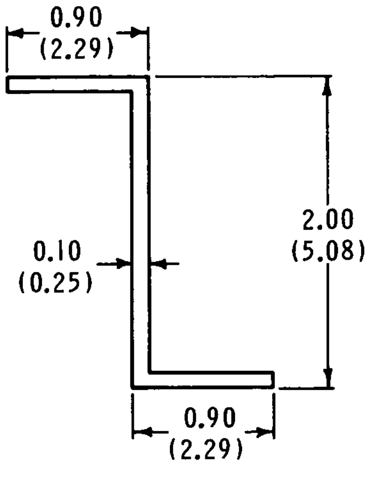


Figure 2.- Stress-strain curve for 7075-T6 aluminum alloy.



(a) Ring.



(b) Stringer. Spacing = 2.00 in. (5.08 cm).

Figure 3.- Stiffener dimensions in inches (cm).

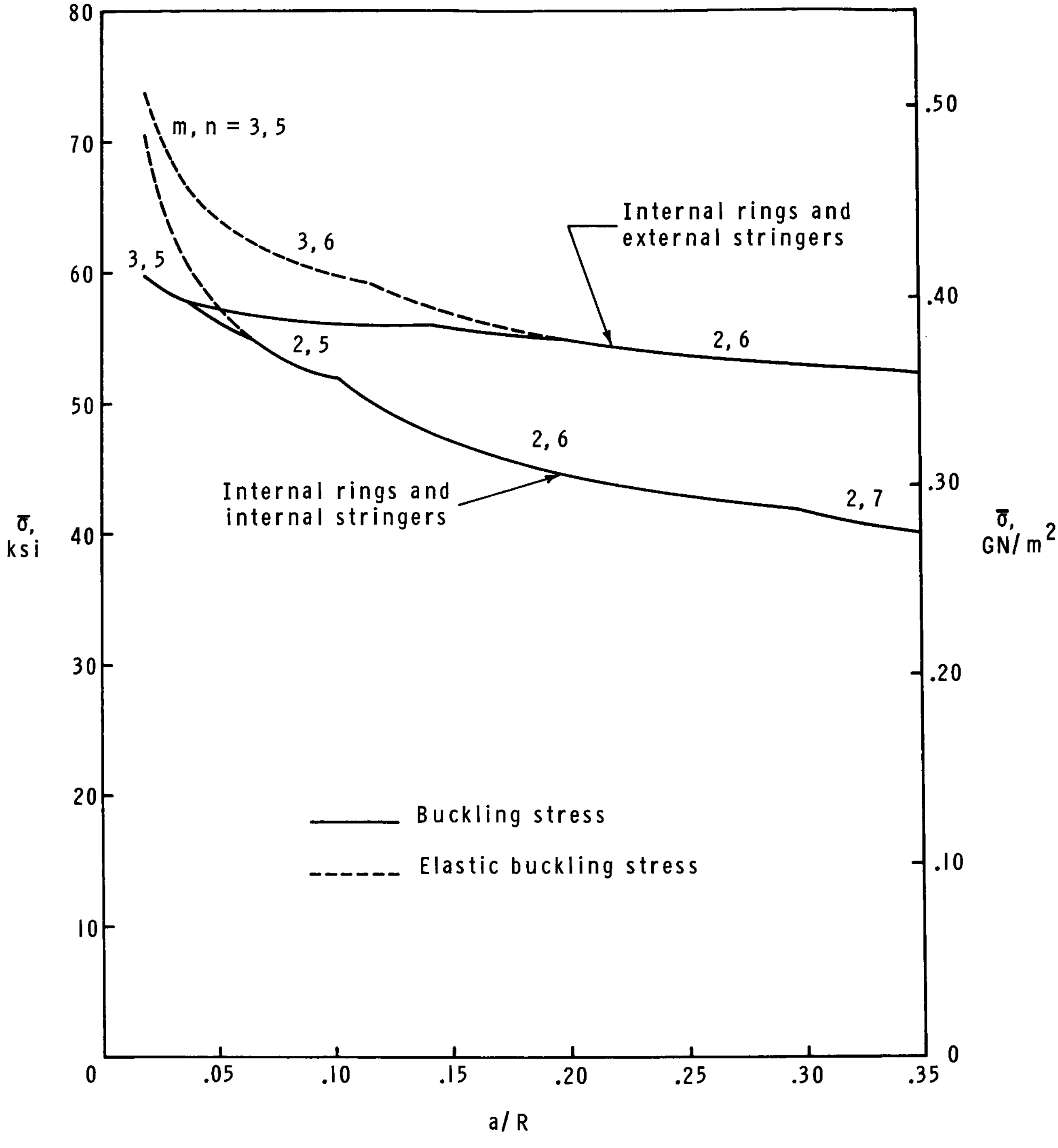


Figure 4.- General instability stresses as a function of a/R for axially compressed ring-and-stringer-stiffened cylinders. L = R = 135 inches (345 cm); t = 0.10 inch (0.25 cm); b = 2.00 inches (5.08 cm); 7075-T6 aluminum alloy.

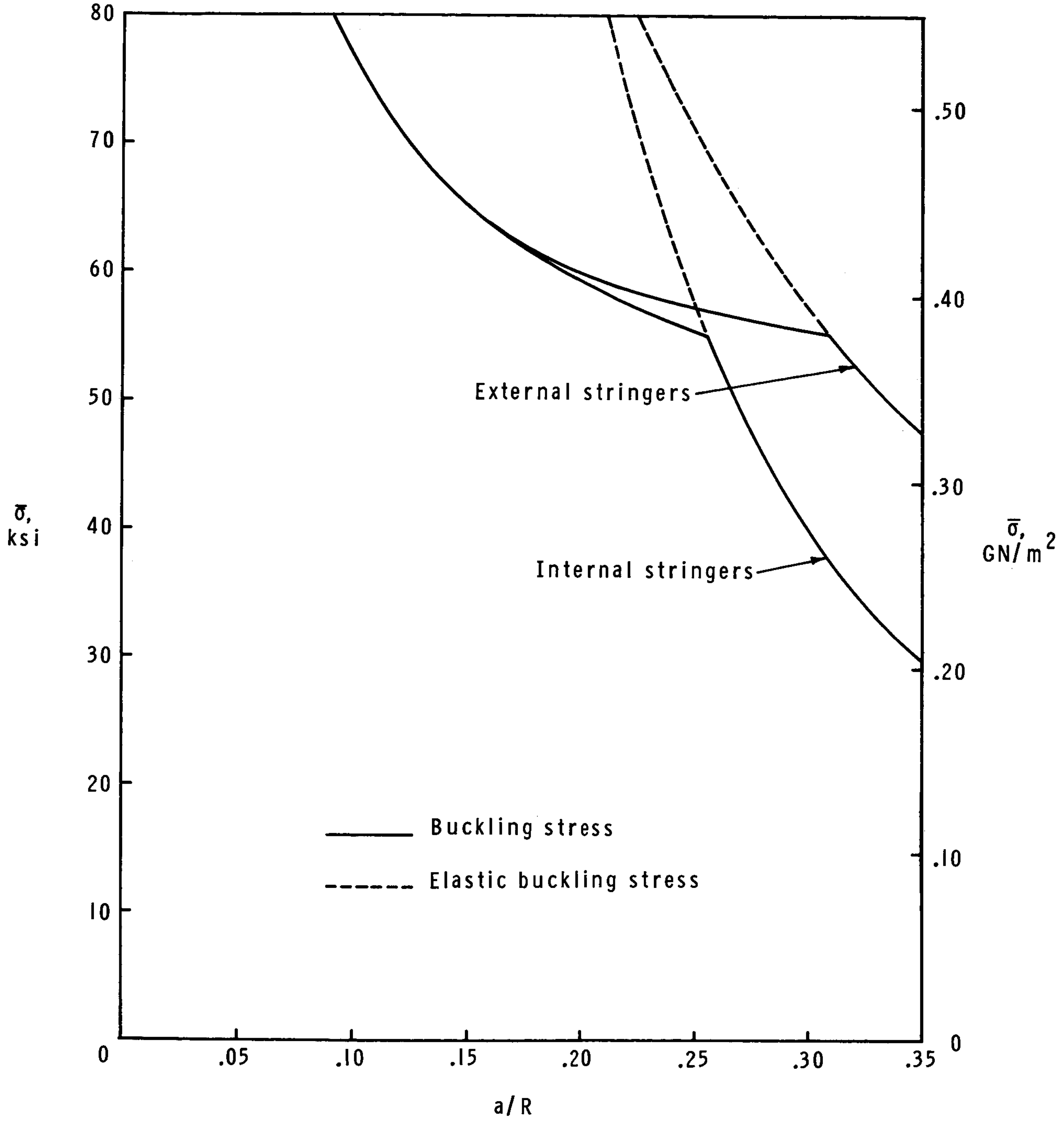


Figure 5.- Panel instability stresses as a function of a/R for axially compressed ring-and-stringer-stiffened cylinders. R = 135 inches (345 cm); t = 0.10 inch (0.25 cm); b = 2.00 inches (5.08 cm); 7075-T6 aluminum alloy.

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