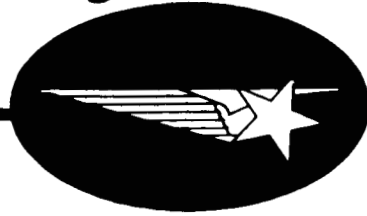


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CONVECTIVE AND RADIATIVE HEAT TRANSFER TO BLUNT BODIES AT ANGLE OF ATTACK

by

H. HOSHIZAKI

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July 1966

Final Report, Part II,
Prepared Under Contract NAS 7-386

FOREWORD

The work described in this report was completed for the National Aeronautics and Space Administration Headquarters, under the terms and specifications of Contract NAS 7-386, issued through NASA Resident Office - JPL, 4800 Oak Grove Drive, Pasadena, California.

The work was performed in the Aerospace Sciences Laboratory, R. Capiiaux, Manager, of the Lockheed Palo Alto Research Laboratory.

ABSTRACT

The convective and radiative heat transfer to blunt bodies at large angles of attack is investigated. In the analysis the shock shape is assumed to be a known quantity. Approximate solutions to the integrated tangential and azimuthal momentum equations are used to determine the shock layer velocity profiles. A finite difference type method is used to solve the energy equation. A crucial assumption made in the analysis is that the crossflow is locally similar. This assumption reduces the general three-dimensional problem to a two-dimensional one where the azimuthal angle appears as a parameter. The solution yields the convective and radiative heat transfer distribution as well as the detailed structure of the shock layer. The radiative heat transfer is evaluated spectrally taking into account the coupling between the flow field and the radiation transport. Typical results for one flight condition are presented.

ACKNOWLEDGMENTS

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NOMENCLATURE

A	Avogadro's number
a_i	tangential velocity profile coefficients
b_i	crossflow velocity profile coefficients
B_ν	spectral Planckian intensity
C_p	total specific heat at constant pressure
c	light velocity
D_{ij}	diffusion coefficient for a multicomponent system
D_i^T	thermal diffusion coefficient
E	radiant emission per unit time per unit volume
ϵ_n	exponential function
f	tangential velocity ratio, u/u_δ
h	static enthalpy, also Planck constant
h_i	static enthalpy of i^{th} species, including enthalpy of formation
$\bar{h}$	altitude in 10^5 ft.
I_i	integrals defined by Eqs. (22), (29) or (33)
j	crossflow velocity ratio, w/w_δ
k	total thermal conductivity, also Boltzmann constant
K	defined by Eq. 45
$\bar{k}$	frozen thermal conductivity
L	reference length
M_i	molecular weight of species i

M_o	molecular weight of undissociated gas
N	number density, also defined by Eq. (46)
n_i	moles of species i per unit volume
n_t	total number of moles per unit volume
P	static pressure
Pr	total Prandtl number, $Pr = C_p \mu / k$
$\dot{q}_c$	convective energy flux
$\dot{q}_r$	radiative energy flux
R	body radius of curvature
Re	Reynolds number, $\rho_{\delta,o} U_{\infty} R / \mu_{\delta,o}$
r	body radius measured from body centerline
T	temperature
U_{∞}	free-stream velocity
$\bar{U}$	velocity in 10^4 fps
u	velocity component parallel to body in x -direction
v	velocity component normal to surface
w	velocity component parallel to body in ϕ -direction
X_j	mole fraction of species j , $X_j = N_j / N_t$
$\tilde{x}$	distance from geometric center, (Fig. 1)
x, y, ϕ	body-oriented coordinate system
δ	shock detachment distance
$\tilde{\delta}$	transformed shock detachment distance
η	Dorodnitzyn variable
θ	body angle, also kT

κ	body curvature
$\tilde{\kappa}$	$1 + \kappa y$
κ_{ν}	absorption coefficient, cm^{-1}
μ	dynamic viscosity
ν	kinematic viscosity, also frequency
ξ	nondimensional surface distance x/R
ρ	density
$\bar{\rho}$	density ratio across shock, $\rho_{\infty} / \rho_{\delta}$
σ	Stefan-Boltzmann constant
σ^*	effective cross-section, $\sigma^* = \kappa_{\nu}/N$
τ_{ν}	monochromatic optical thickness
φ	azimuthal angle
χ	h_{ν}/kT

Subscripts

a	atom
e	electron
i	i^{th} chemical species, also ion
in	inviscid
m	molecule, also maximum
N_2	nitrogen molecule
N	nitrogen atom
O_2	oxygen molecule
O	oxygen atom
s	shock

t	total
w	wall quantities
δ	quantities immediately behind shock
∞	free-stream condition
0	stagnation point

barred quantities are non-dimensional

Section 1

INTRODUCTION

In this paper the problem of determining the convective and radiative heat transfer to blunt bodies at large angles of attack is considered. Flow fields about blunt bodies at angles of attack are of interest since space vehicles entering planetary atmospheres will perform lifting maneuvers to limit the deceleration and to minimize aerodynamic heating. Previous investigations of flow fields about bodies at angles of attack have been restricted to non-radiating, inviscid flows (Ref. 1-4) or inviscid, radiating flows about pointed cones (Ref. 5).

Webb (Ref. 1) used the inverse method while Waldman (Ref. 2) employed the method of integral relations to solve the angle-of-attack blunt body problem. Bohachevsky (Ref. 3) on the other hand, has examined the unsteady, three-dimensional flow about blunt bodies where the asymptotic solution (in time) gives a good description of the steady-state solution. Kaattari (Ref. 4) has developed a semi-empirical method for predicting the shock shape of blunt bodies at angle-of-attack.

In the present investigation the radiating, viscous shock layer on blunt bodies at large angles-of-attack is examined. The objective of the analysis is to simultaneously determine the convective and radiative heat transfer. Energy loss by radiation, self-absorption and the coupling between the inviscid and viscous regions due to radiative energy transfer are taken into account. Mass injection effects are currently neglected.

Section 2

ANALYSIS

2.1 METHOD OF SOLUTION

The general problem considered is the radiating, viscous blunt body problem at angle-of-attack. This is a direct problem in that the shock shape and the flow structure in the shock layer are to be determined for a specified body shape and angle-of-attack. In this paper we do not attempt to solve the general problem. Instead we investigate the flow structure of the three-dimensional shock layer when both the body and shock shape are prescribed. The thin shock layer approximations are used to reduce the elliptic conservation equations to parabolic equations. The upstream influence, namely, the influence of the corner region (see Fig. 1) on the flow near the stagnation point, is accounted for by the prescribed shock shape. In this analysis Kaattari's method will be used to determine the shock shape. Since this method does not include the effect of radiation transfer on the shock shape, the present analysis is limited to conditions where such effects are small.

The thin shock layer approximation limits the present analysis to the front face of blunt bodies. This is not a serious limitation since the convective and radiative heating are most severe in this region. In this analysis the shock layer is divided into inviscid and viscous regions. The extent of the viscous region is unknown and is to be determined as part of the solution. The shock layer gas is assumed to be in thermodynamic equilibrium. The emission and absorption of radiant energy is evaluated spectrally with the one-dimensional, parallel slab approximation.

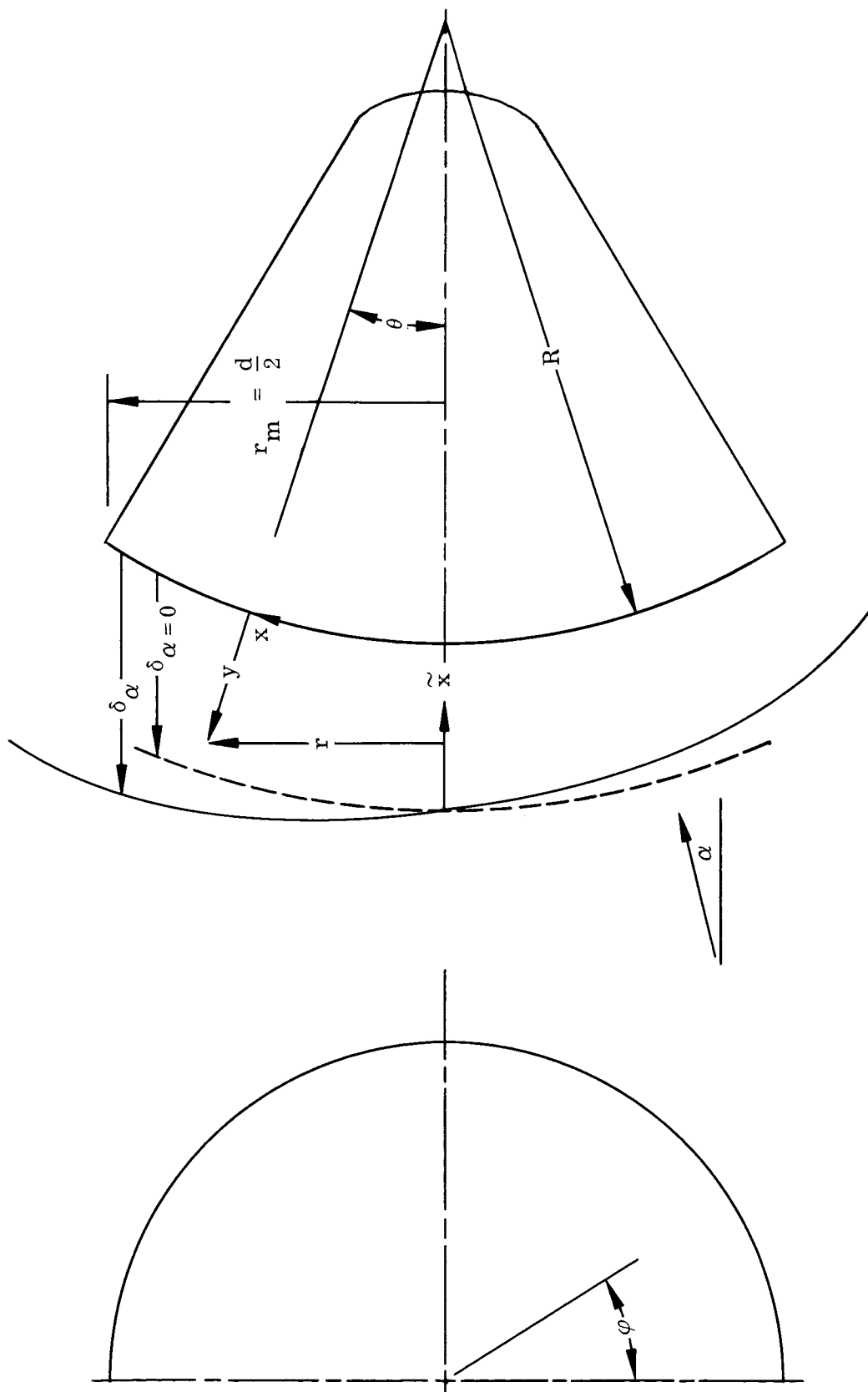


Fig. 1 Body-Oriented Coordinate System

A crucial part of the analysis is the effective decoupling of the crossflow from the axial flow by the assumption that the shock detachment distance in the transformed coordinate system and the enthalpy and crossflow velocity profiles are slowly varying functions of the azimuthal angle. This approximation enables solutions to be obtained along geodesics, independently of the flow along neighboring geodesics. The solution along a geodesic is, however, dependent on the local crossflow. This approximation is similar to the "locally similar" approximation employed in two-dimensional boundary-layer theory. In this analysis we are essentially assuming that the flow is locally similar in the azimuthal direction.

In order to obtain satisfactory tangential velocity profiles, the tangential velocity is assumed to be linear in the inviscid region and cubic in the viscous region. The velocity is matched at the inviscid-viscous interface to obtain a continuous profile. The inviscid tangential velocity gradient is obtained from the integrated (across the shock layer) tangential momentum equation. The cross-flow velocity is also determined in a similar manner with the tangential momentum equation replaced by the integrated azimuthal momentum equation. The edge of viscous layer is determined from the integrated continuity equation. The justification for constructing the velocity field in this manner rests on the knowledge that the inviscid velocity profiles are nearly linear (Ref. 6-8). A further justification is that approximate velocity profiles will result in accurate total enthalpy profiles if the energy equation is solved by finite differences or similar numerical methods.

2.2 THIN SHOCK-LAYER EQUATIONS

The conservation equations which are valid for thin shock layers when the viscous region thickness is the same order-of-magnitude as the shock-detachment distance will be presented. The additional simplification which results when the viscous region is assumed to be thin with respect to the shock-layer thickness will then be indicated. Since the shock-detachment distance is $O(\bar{\rho})$ and the thickness of the viscous region is $O(\sqrt{Re})$, we have for a

viscous shock layer, $\bar{\rho} \sim 1/\sqrt{\text{Re}}$. The approximate equations can be obtained from the conservation equations (Ref. 9) by neglecting all terms of $O(\bar{\rho}^2)$ and higher. The body-oriented coordinate system used in this analysis is shown in Fig. 1. Carrying out the order-of-magnitude analysis results in the following equations valid to $O(\bar{\rho})$.

x-momentum

$$\begin{aligned} \rho u \frac{\partial u}{\partial x} + \tilde{\kappa} \rho v \frac{\partial u}{\partial y} + \kappa \rho uv + \frac{\tilde{\kappa} \rho w}{r} \frac{\partial u}{\partial \varphi} - \frac{\rho w^2}{r} \frac{\partial r}{\partial x} = - \frac{\partial P}{\partial x} \\ - \frac{1}{\tilde{\kappa} r} \frac{\partial}{\partial y} \left[r \tilde{\kappa}^2 u \frac{\partial \mu}{\partial y} \right] + \frac{\tilde{\kappa}}{r} \frac{\partial}{\partial y} \left[\frac{r}{\tilde{\kappa}} \frac{\partial}{\partial y} (\tilde{\kappa} \mu u) \right] \end{aligned} \quad (1)$$

y-momentum

$$\begin{aligned} \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} - \frac{\kappa \rho u^2}{\tilde{\kappa}} + \frac{\rho w}{r} \frac{\partial v}{\partial \varphi} - \frac{\rho w^2}{r} \frac{\partial r}{\partial y} = - \frac{\partial P}{\partial y} + \frac{4}{3} \frac{\partial}{\partial y} \left[\frac{\mu}{r} \frac{\partial (ru)}{\partial x} \right. \\ \left. + \mu \frac{\partial v}{\partial y} + \frac{\mu}{r} \frac{\partial w}{\partial \varphi} \right] + \frac{\partial}{\partial y} \left(u \frac{\partial \mu}{\partial x} \right) + \frac{\partial u}{\partial y} \frac{\partial \mu}{\partial x} - \frac{1}{r} \frac{\partial \mu}{\partial y} \frac{\partial (ru)}{\partial x} \\ - \frac{1}{r} \frac{\partial}{\partial x} \left[r \frac{\partial (\mu u)}{\partial y} \right] + \frac{\partial}{\partial y} \left(\frac{w}{r} \frac{\partial \mu}{\partial \varphi} \right) - \frac{1}{r} \frac{\partial \mu}{\partial y} \frac{\partial w}{\partial \varphi} - \frac{1}{r} \frac{\partial}{\partial \varphi} \left[\frac{\partial}{\partial \varphi} (\mu w) \right] \end{aligned} \quad (2)$$

φ -momentum

$$\begin{aligned} \frac{\rho u}{r} \frac{\partial (rw)}{\partial x} + \frac{\tilde{\kappa} v}{r} \frac{\partial (rw)}{\partial y} + \frac{\tilde{\kappa} w}{r} \frac{\partial w}{\partial \varphi} = - \frac{1}{r} \frac{\partial P}{\partial \varphi} \\ - \frac{1}{r^2} \frac{\partial}{\partial y} \left(r^2 \tilde{\kappa} w \frac{\partial \mu}{\partial y} \right) + \frac{\partial}{\partial y} \left[\frac{\tilde{\kappa}}{r} \frac{\partial}{\partial y} (r \mu w) \right] \end{aligned} \quad (3)$$

continuity

$$\frac{\partial}{\partial x} (r \rho u) + \frac{\partial}{\partial y} (\tilde{\kappa} r \rho v) + \frac{\partial}{\partial \varphi} (\tilde{\kappa} r \rho w) = 0 \quad (4)$$

energy

$$\begin{aligned} \frac{\rho u}{\tilde{\kappa}} \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} + \frac{\rho w}{r} \frac{\partial h}{\partial \varphi} &= \frac{u}{\tilde{\kappa}} \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} + \frac{w}{r} \frac{\partial P}{\partial \varphi} + \frac{1}{r \tilde{\kappa}} \frac{\partial}{\partial y} \left(\frac{1}{r \tilde{\kappa}} \frac{\partial T}{\partial y} \right) \\ &+ \mu \left[\left\{ \tilde{\kappa} \frac{\partial}{\partial y} \left(\frac{u}{\tilde{\kappa}} \right) \right\}^2 + \left\{ r \frac{\partial}{\partial y} \left(\frac{w}{r} \right) \right\}^2 \right] - E \end{aligned} \quad (5)$$

For terms of $O(\bar{p})$ in the above equations we have set $\tilde{\kappa} = 1$ and $r = r_w$ to be consistent with our order-of-magnitude approximation. These equations when reduced to the two-dimensional case, differ slightly from the thin shock layer equations presented by Hayes and Probstein (Ref. 10) since our analysis is not restricted to stagnation-point flows.

The thermal conductivity which appears in the energy equation is the total thermal conductivity defined as the sum of the translational and reaction conductivities. This very useful concept of combining the transport of energy by ordinary conduction and diffusion of reacting species has been discussed by Hirschfelder (Ref. 11) and employed by several authors (Ref. 12-14) in obtaining solutions to the boundary-layer equations. The total thermal conductivity is defined by the following equation.

$$k = \bar{k} - \sum_i h_i \left[\frac{n^2}{\rho} \sum_{j \neq i} M_i M_j D_{ij} \frac{dX_j}{dT} - \frac{D_i^T}{T} \right] \quad (6)$$

The radiation term E in the energy equation specifies the net emission or absorption of radiant energy per unit volume per unit time. It is calculated

assuming the shock layer is locally one-dimensional. That is, for any value of x and ϕ , temperature and density gradients in the x and ϕ directions at all points across the shock layer are neglected insofar as radiation transport is concerned. In addition, the geometric approximation is made that the shock layer geometry is that of an infinite plane slab.

It is clear that without the one-dimensional approximation the radiation transport calculation is much more difficult to perform. More significantly, the parabolic character of the thin shock-layer equations is destroyed as influence can now be propagated upstream via the radiation field. The approximation of replacing the shock-layer geometry by an infinite plane slab requires, for optically thin shock layers, that the shock-layer thickness be small relative to the local radius of curve, i.e., $\delta/R \ll 1$. For optically thick shock layers the requirement for the plane slab approximation is that the mean free path for radiation λ_R be small compared to the local radius of curvature, i.e., $\lambda_R/R \ll 1$.

Hence, the plane slab approximation becomes more accurate in the important spectral ranges when the shock layer is optically thick. Indeed, Kennet, and Strack (Ref. 15) show that for optical depth unity, the plane slab approximation is in error by only 5% for an isothermal, spherical shock layer. Concerning the approximation of one-dimensional temperature and density fields, Koh (Ref. 16) has investigated the effects of the non-isothermal temperature on the radiation flux at the stagnation point of a sphere and found only a 3% difference from an isothermal solution when the shock layer has an optical depth of unity. Although these results are for axially symmetric flows, the conclusion that the one-dimensional plane-parallel slab model can be used to obtain quantitatively valid results will hold for asymmetric flows as long as the azimuthal and axial variations are comparable.

Except for the locally one-dimensional treatment, the radiation term is evaluated in an exact manner. No approximations restricting the analysis to opti-

cally thin or optically thick shock layers are made. Instead a full integral treatment of the emission and absorption processes is made. Moreover, the radiation transport is performed in terms of a detailed spectral analysis using realistic absorption coefficients. The spectral transport solution shows that grey gas approximations using a Planck mean opacity produce results which are in error by as much as an order of magnitude (Ref. 17).

The radiation flux divergence can be expressed as

$$E = \nabla \cdot \mathbf{q}_r = - \int_0^\infty \int_{\omega=4\pi} \kappa_\nu (I_\nu - B_\nu) d\omega d\nu \quad (7)$$

In Eq. (7), κ_ν is the volumetric (spectral) absorption coefficient; I_ν is the local radiation intensity; B_ν is the equilibrium (i.e., Planckian) radiation intensity.

Solving for the intensity under the one-dimensional approximation and performing the integration over solid angle indicated by Eq. (7) leads to the following expression for the flux divergence (Ref. 18).

$$\begin{aligned} -E = 2\pi \int_0^\infty K_\nu(t) \left[\int_0^\tau B_\nu [T(\tau_\nu)] \epsilon_1 (\tau_\nu - t_\nu) dt_\nu \right. \\ \left. + \int_{\tau_\nu}^\tau B_\nu |T(t_\nu)| \epsilon_1 (t_\nu - \tau_\nu) dt_\nu - 2B_\nu |T(\tau_\nu)| \right] d\nu \end{aligned} \quad (8)$$

where τ_ν is the optical depth at frequency ν

$$\tau_\nu = \int_0^{y'} \kappa_\nu(y'') dy'' \quad (9)$$

B_v is the spectral Planckian intensity,

$$B_v = \frac{2hy^3}{c^2} \frac{1}{\exp(hv/kT)-1} \quad (10)$$

$\epsilon_n(t)$ is the exponential-integral of order n ,

$$\epsilon_n(t) = \int_0^\infty \frac{\exp(-tz)}{z^n} dz \quad (11)$$

In arriving at Eq. (8) it has been assumed that the medium outside the shock wave neither emits, absorbs, or reflects radiant energy and the body surface acts as a perfect absorber and does not reradiate. The quadratures in Eq. (8), for a given shock layer temperature distribution, are evaluated numerically using the spectral absorption coefficients described in Subsection 3.3.

The thin shock layer equations (Eqs. 1-5) were derived on the basis that the thickness of the viscous region is comparable to the shock layer thickness. Significant entry heating usually occurs when the Reynolds number is between 10^4 and 10^6 so that the viscous layer is much thinner than the shock layer. For these conditions we can further simplify the conservation-equations by setting $\tilde{u} = 1$ and $r = r_w$ in all of the viscous terms.

The energy equation can be more conveniently dealt with when written in terms of total enthalpy. This is accomplished in the usual manner by combining the momentum equations with the energy equation written in terms of the static enthalpy (Eq. 5). The appropriate form of the x and φ -momentum equations are given by Eqs. (1) and (3) with $\tilde{u} = 1$ and $r = r_w$ in the viscous terms only.

The appropriate y -momentum is obtained by neglecting all terms which are of higher order than $\partial P/\partial y$ since the pressure gradient term in the energy equa-

tion is $O(\bar{p})$. Neglecting the higher order term reduces the y-momentum equation to

$$\rho u^2 + \frac{\rho w^2}{r} \frac{\partial r}{\partial y} = \frac{\partial P}{\partial y} \quad (12)$$

The justification for the additional simplification can perhaps be better explained by pointing out that the original form of the y-momentum equation (Eq. 2) is correct to $O(\bar{p})$ with respect to itself. When the y-momentum equation is used in conjunction with the energy equation the higher order terms in the y-momentum become of $O(\bar{p}^2)$ and can be neglected since the pressure gradient terms in the energy equation are of $O(\bar{p})$.

Combining the momentum equations with the energy equation (Eq. 5) results in

$$\frac{\rho u}{\tilde{\kappa}} \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} + \frac{\rho w}{r} \frac{\partial H}{\partial \phi} = \frac{\partial}{\partial y} \left[\mu \frac{\partial H}{\partial y} + \mu \frac{\partial h}{\partial y} \left(\frac{1}{P_r} - 1 \right) \right] - E \quad (13)$$

where the total enthalpy is $H = h + \frac{u^2}{2} + \frac{w^2}{2}$ and P_r is the total Prandtl number.

2.3 INTEGRATION OF MOMENTUM EQUATIONS

The velocity flow field will be determined according to the method outlined in Subsection 2.1. In accordance with this method the viscous x-momentum equation (Eq. 1) is integrated across the shock layer and is used to determine the inviscid tangential velocity gradient. The viscous x-momentum equation is thus satisfied in the large. The azimuthal momentum equation is also integrated across the shock layer and is used to determine the inviscid, crossflow velocity gradient. With the aid of the continuity equation, the x and ϕ -momentum equations can be manipulated into forms convenient for integration. Carrying

out the manipulations results in the following equations.

x-momentum

$$\begin{aligned} \frac{\partial}{\partial x} (\rho u^2) + \frac{1}{r_w} \frac{\partial}{\partial \varphi} (\rho w u) - \frac{\rho_w^2}{r_w} \frac{\partial r_w}{\partial x} + \frac{\partial}{\partial y} (\rho v u) + \frac{\rho u^2}{r_w} \frac{dr}{dx} \\ = - \frac{\partial P}{\partial x} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \end{aligned} \quad (14)$$

φ -momentum

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{r}{r_w} \rho u w \right) + \frac{r}{r_w} \frac{\partial}{\partial y} \left[\left(\frac{r}{r_w} \right)^2 \rho v w \right] + \frac{r}{r_w} \rho u w \left(\frac{1}{r_w} \frac{\partial r_w}{\partial x} + \frac{1}{r} \frac{\partial r}{\partial x} \right) \\ + \frac{\tilde{\kappa}}{r_w} \frac{\partial}{\partial \varphi} (\rho w^2) = \frac{1}{r_w} \frac{\partial P}{\partial \varphi} + \frac{\partial}{\partial y} \left(\mu \frac{\partial w}{\partial y} \right) \end{aligned} \quad (15)$$

Integrating the x and φ -momentum equations across the shock layer yields

x-momentum

$$\begin{aligned} u_\delta^2 \frac{\partial}{\partial x} \int_0^\delta \frac{\rho}{\rho_\delta} f^2 dy + 2 u_\delta \frac{\partial u_\delta}{\partial x} \int_0^\delta \frac{\rho}{\rho_\delta} f^2 dy - u_\delta^2 \frac{\partial \delta}{\partial x} + (v u)_\delta \\ + \frac{1}{r_w} \frac{dr_w}{dx} u_\delta^2 \int_0^\delta \frac{\rho}{\rho_\delta} f^2 dy + \frac{w_\delta u_\delta}{r_w} \frac{\partial}{\partial \varphi} \int_0^\delta \frac{\rho}{\rho_\delta} f dy \end{aligned} \quad (16)$$

$$\begin{aligned}
& + \frac{1}{r_w} \frac{\partial}{\partial \varphi} (w_\delta u_\delta) \int_0^\delta \frac{\rho}{\rho_\delta} f j \, dy - (wu)_\delta \frac{1}{r_w} \frac{\partial \delta}{\partial \varphi} - \frac{1}{r_w} \frac{\partial r_w}{\partial x} \int_0^\delta \frac{\rho}{\rho_\delta} j^2 \, dy \\
& = \frac{\delta}{\rho_\delta} \left(\frac{\partial P}{\partial x} \right)_\delta + \frac{1}{\rho_\delta} \left(\mu \frac{\partial u}{\partial y} \right)_w
\end{aligned}$$

φ -momentum

$$\begin{aligned}
& (uw)_\delta \frac{\partial}{\partial x} \int_0^\delta \frac{r}{r_w} \frac{\rho}{\rho_\delta} f j \, dy + \frac{\partial}{\partial x} (uw)_\delta \int_0^\delta \frac{r}{r_w} \frac{\rho}{\rho_\delta} f j \, dy - \frac{r_\delta}{r_w} (uw)_\delta \frac{\partial \delta}{\partial x} \\
& + \left(\frac{r_\delta}{r_w} \right)^2 (\tilde{\kappa} v w)_\delta \left(1 - \frac{\delta \sin \theta}{r_w} \right) + (wv)_\delta \frac{\sin \theta}{r_w} \int_0^\delta \left(\frac{r}{r_w} \right)^2 \tilde{\kappa} \frac{\rho}{\rho_\delta} \frac{v}{v_\delta} j \, dy \\
& \quad \quad \quad (17)
\end{aligned}$$

$$\begin{aligned}
& + (uw)_\delta \int_0^\delta \frac{r}{r_w} \frac{\rho}{\rho_\delta} f j \left(\frac{1}{r_w} \frac{\partial r_w}{\partial x} + \frac{1}{r} \frac{\partial r}{\partial x} \right) dy + \frac{1}{r_w} \frac{\partial}{\partial \varphi} (w^2)_\delta \int_0^\delta \tilde{\kappa} \frac{\rho}{\rho_\delta} j^2 \, dy \\
& + \frac{w_\delta^2}{r_w} \frac{\partial}{\partial \varphi} \int_0^\delta \tilde{\kappa} \frac{\rho}{\rho_\delta} j^2 \, dy - \tilde{\kappa}_\delta w_\delta^2 \frac{\partial \delta}{r_w \partial \varphi} = - \frac{\delta}{\rho_\delta r_w} \left(\frac{\partial P}{\partial \varphi} \right)_\delta - \frac{1}{\rho_\delta} \left(\mu \frac{\partial w}{\partial y} \right)_w
\end{aligned}$$

where

$$f = \frac{u}{u_\delta} \quad \text{and} \quad j = \frac{w}{w_\delta}$$

The density ratio across the shock wave has been assumed to be a constant along the shock surface. This is a good approximation as long as the free-stream normal velocity component remains hypersonic (Ref. 19). In addition it has been assumed that the pressure gradient in the azimuthal direction is independent of y . This approximation is justified on the basis that the

pressure gradient is a second order term in the momentum equation.

The compressibility effects can be reduced considerably by introducing the Dorodnitsin transformation.

$$\eta = \frac{\int_0^y \left(\frac{r}{r_w} \right) \frac{\rho}{\rho_\delta} dy}{\int_0^\delta \left(\frac{r}{r_w} \right) \frac{\rho}{\rho_\delta} dy} = \frac{\int_0^y \frac{r}{r_w} \frac{\rho}{\rho_\delta} dy}{\delta} \quad (18)$$

The shock detachment distance is related to the transformed shock detachment distance $\tilde{\delta}$ by

$$\delta = \tilde{\delta} \int_0^1 \frac{\rho_\delta}{\rho} d\eta - n\tilde{\delta}^2 \frac{\sin\theta}{2r_w} \left[\int_0^1 \frac{\rho_\delta}{\rho} d\eta \right]^2 \quad (19)$$

Transforming the momentum equations (Eqs. 16-17) from x, y, φ to ξ, η, φ results in

x-momentum

$$\begin{aligned} \frac{\partial I_1}{\partial \xi} + \left[\frac{2}{\bar{u}_\delta} \frac{\partial \bar{u}_\delta}{\partial \xi} + \frac{\tilde{\kappa}_\delta}{\bar{r}_w} \frac{d\bar{r}_w}{d\xi} \right] I_1 - \left(1 + \tilde{\delta} \frac{\sin\theta}{\bar{r}_w} \right) \frac{\partial \tilde{\delta}}{\partial \xi} + \tilde{\kappa}_\delta^2 \frac{\bar{r}_\delta}{\bar{r}_w} \left(\frac{\bar{v}}{\bar{u}} \right)_\delta \\ + \frac{1}{\bar{r}_w} \left(\frac{\bar{w}}{\bar{u}} \right)_\delta \frac{\partial I_2}{\partial \varphi} + \frac{\tilde{\kappa}_\delta I_2}{\bar{u}_\delta \bar{r}_w} \frac{\partial}{\partial \varphi} (\bar{u}_\delta \bar{w}_\delta) - \frac{\bar{w}_\delta}{\bar{u}_\delta \bar{r}_w} \frac{\partial (\tilde{\delta})}{\partial \varphi} \\ - \left(\frac{\bar{w}}{\bar{u}} \right)_\delta^2 \frac{I_4}{\bar{r}_w} \left(\frac{3}{4} \frac{\partial \bar{r}_\delta}{\partial \xi} + \frac{1}{4} \frac{\partial \bar{r}_w}{\partial \xi} \right) = - \frac{\tilde{\delta} \bar{\rho}}{\bar{u}_\delta^2} \left(\frac{\partial \bar{\rho}}{\partial \xi} \right)_\delta \\ - \frac{1}{\bar{u}_\delta \text{Re}} \left[\left(\frac{\rho \mu}{\rho \mu} \right)_{w,0} \frac{I}{\tilde{\delta}} f'(0) \right] \end{aligned} \quad (20)$$

φ -momentum

$$\begin{aligned}
 & \frac{\partial I_2}{\partial \xi} + \frac{\partial}{\partial \xi} (u_\delta w_\delta) \frac{I_2}{u_\delta w_\delta} - \frac{r_\delta}{r_w} \frac{\partial \bar{\delta}}{\partial \xi} + \left(\frac{v}{u}\right)_\delta \left[\left(\frac{r_\delta}{r_w}\right)^2 \tilde{\kappa}_\delta \left(1 - \frac{\bar{\delta} \sin \theta}{r_w}\right) \right. \\
 & \quad \left. + \sin \theta \frac{\bar{\delta}/L}{r_w} \int_0^1 \frac{\bar{r}}{\bar{r}_w} \tilde{\kappa} \frac{v}{v_\delta} j \, d\eta \right] \\
 & + \frac{\bar{\delta}}{L} \int_0^1 f j \left(\frac{1}{\bar{r}_w} \frac{\partial \bar{r}_w}{\partial \xi} + \frac{1}{\bar{r}} \frac{\partial \bar{r}}{\partial \xi} \right) d\eta + \frac{1}{u_\delta \bar{w}_\delta \bar{r}_w} \left[\bar{w}_\delta^2 \frac{\partial I_4}{\partial \varphi} + \frac{\partial (\bar{w}_\delta^2)}{\partial \varphi} I_4 \right. \\
 & \quad \left. - \hat{\kappa}_\delta \bar{w}_\delta \frac{\partial \bar{\delta}}{\partial \varphi} \right] = - \frac{\bar{\delta} \bar{p}}{u_\delta \bar{w}_\delta \bar{r}_w} \left(\frac{\partial \bar{P}}{\partial \varphi} \right)_\delta - \frac{(\rho \mu)_w}{(\rho \mu)_{\delta,0}} \frac{L}{\delta} \frac{j'(0)}{\bar{u}_\delta \text{Re}}
 \end{aligned} \tag{21}$$

where primes denote differentiation with respect to η and

$$\begin{aligned}
 I_0 &= \frac{\bar{\delta}}{L} \int_0^1 f d\eta & I_1 &= \frac{\bar{\delta}}{L} \int_0^1 f^2 d\eta
 \end{aligned} \tag{22}$$

$$\frac{\bar{\delta}}{L} \int_0^1 \tilde{\kappa}^2 \frac{r_w}{r} f j \, d\eta \approx \tilde{\kappa}_\delta \frac{\bar{\delta}}{L} \int_0^1 f j d\eta = \tilde{\kappa}_\delta I_2$$

$$\bar{w}_\delta = w_\delta / u_\infty$$

$$\bar{r}_w = r_w / L$$

$$\xi = x / L$$

$$\bar{u}_\delta = u_\delta / u_\infty$$

$$\bar{\delta} = \delta / L$$

$$\bar{P} = P / \rho_\infty u_\infty^2$$

$$\bar{v}_\delta = v_\delta / u_\infty$$

In the x-momentum equation we have made the approximation that

$$\frac{\tilde{\delta}}{L} \int_0^1 \frac{\tilde{\kappa}}{\tilde{r}} \frac{\partial \tilde{r}}{\partial \xi} j^2 d\eta \approx \frac{1}{\tilde{r}_w} \left(\frac{1}{4} \frac{\partial \tilde{r}_w}{\partial \xi} + \frac{3}{4} \frac{\partial \tilde{r}_\delta}{\partial \xi} \right) I_4$$

The coupling between the crossflow and the axial flow in the momentum equations is represented by the underlined terms in Eqs. 20 and 21. Neglecting these terms is equivalent to assuming that $\tilde{\delta}$ and both the crossflow and axial velocity profiles are slowly varying functions of the azimuthal angle. Note that the momentum equations are still dependent on the local crossflow. By neglecting the underlined terms, the momentum equations can be solved along geodesics by treating the azimuthal angle as a parameter. One can then construct the entire asymmetric flow field by obtaining a sufficient number of solutions along geodesics.

Additional approximations can be made to the integrated φ -momentum equation (Eq. 21). The fourth term on the L.H.S. can be approximated by

$$\left(\frac{\tilde{v}}{\tilde{u}} \right)_\delta \left(\frac{\tilde{r}_\delta}{\tilde{r}_w} \right)^2 \tilde{\kappa}_\delta \left[1 - \frac{\sin\theta}{\tilde{r}_w} \left(\tilde{\delta} - \frac{\tilde{\delta}/L}{\tilde{\kappa}_\delta} \left(\frac{\tilde{r}_w}{\tilde{r}_\delta} \right)^2 \int_0^1 \frac{\tilde{r}}{\tilde{r}_w} \tilde{\kappa} \frac{\tilde{v}}{\tilde{u}_\delta} j d\eta \right) \right] \approx \left(\frac{\tilde{v}}{\tilde{u}_\delta} \right) \left(\frac{\tilde{r}_\delta}{\tilde{r}_w} \right)^2 \tilde{\kappa}_\delta$$

By considering linear and constant velocity profiles, the value of the integral is estimated to be between 1/3 and 1. Since $\tilde{\delta}$ is slightly greater than δ for a cold wall (see Eq. 18), the two terms in parentheses tend to cancel each other, and hence, can be neglected. In the sixth term the following approximation is made

$$\frac{1}{\tilde{r}_w} \frac{d\tilde{r}_w}{d\xi} + \frac{1}{\tilde{r}} \frac{\partial \tilde{r}}{\partial \xi} = \frac{\partial}{\partial \xi} \frac{\partial \tilde{r}_w}{\partial \xi} - \frac{\tilde{v}}{\tilde{r}_w} \left(\frac{1}{\tilde{r}_w} \frac{\partial \tilde{r}_w}{\partial \xi} \sin\theta - \cos\theta \frac{d\theta}{d\xi} \right) \approx \frac{2}{\tilde{r}_w} \frac{d\tilde{r}_w}{d\xi}$$

since the term in parentheses is identically zero for spherical bodies and small for near spherical bodies. In the sixth term the approximation $\tilde{\kappa}/(r/r_w) \approx 1$ was employed.

With these approximations, the φ -momentum equation reduces to

$$\begin{aligned} \bar{u}_\delta \bar{w}_\delta \frac{\partial I_2}{\partial \xi} + I_2 \left[\frac{\partial}{\partial \xi} (\bar{u}_\delta \bar{w}_\delta) + \frac{2}{\bar{r}_w} \frac{d\bar{r}_w}{d\xi} \bar{u}_\delta \bar{w}_\delta \right] - \bar{u}_\delta \bar{w}_\delta \frac{\bar{r}_\delta}{\bar{r}_w} \frac{\partial \delta}{\partial \xi} + \left(\frac{\bar{r}_\delta}{\bar{r}_w} \right)^2 (\tilde{\kappa} \bar{w} \bar{v})_\delta \\ + I_4 \frac{\partial (\bar{w}_\delta^2)}{\bar{r}_w \partial \varphi} + \frac{\bar{w}_\delta^2}{\bar{r}_w} \frac{\partial I_4}{\partial \varphi} - \tilde{\kappa}_\delta \bar{w}_\delta^2 \frac{\partial \delta}{\bar{r}_w \partial \varphi} = - \frac{\delta \bar{p}}{\bar{r}_w} \left(\frac{\partial \bar{p}}{\partial \varphi} \right)_\delta \end{aligned} \quad (23)$$

$$- \frac{(\rho \mu)_w}{(\rho \mu)_{\delta,0}} \frac{L}{\tilde{\delta}} \frac{j'(0)}{\bar{u}_\delta \text{Re}}$$

The solution to the integrated momentum equations requires the evaluation of the integrals and the boundary conditions at the shock. These two items are discussed in subsequent sections.

2.4 EVALUATION OF INTEGRALS

The integrals in the integrated momentum equations (Eqs. 20, 23) can be evaluated by specifying the form of the velocity profiles. In the viscous region, the velocity profiles are assumed to be cubics.

$$0 \leq \eta \leq \eta_v$$

$$f_v = \frac{u}{u_\delta} = a_0(x) + a_1(x)\eta + a_2(x)\eta^2 + a_3(x)\eta^3 \quad (24)$$

$$j_v = \frac{w}{w_\delta} = b_0(x) + b_1(x)\eta + b_2(x)\eta^2 + b_3(x)\eta^3 \quad (25)$$

In the inviscid flow, the profiles are assumed to be linear.

$$n_v \leq \eta \leq 1$$

$$f_{in} = 1 - f'_{in}(1)(1-\eta) \quad (26)$$

$$j_{in} = 1 - j'_{in}(1)(1-\eta) \quad (27)$$

The following boundary conditions are used to determine the viscous velocity profile coefficients.

$$(1) \quad \eta = 0 \quad ; \quad u = w = 0$$

$$(2) \quad \eta = 0 \quad ; \quad \text{momentum equations evaluated at the wall}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right)$$

$$\frac{\partial P}{r_w \partial \varphi} = \frac{\partial}{\partial y} \left(\mu \frac{\partial w}{\partial y} \right)$$

$$(3) \quad \eta = \eta_v \quad ; \quad u_v = u_{in}$$

$$w_v = w_{in}$$

$$(4) \quad \eta = \eta_v \quad ; \quad \left(\frac{\partial u}{\partial y} \right)_v = \left(\frac{\partial u}{\partial y} \right)_{in}$$

$$\left(\frac{\partial w}{\partial y} \right)_v = \left(\frac{\partial w}{\partial y} \right)_{in}$$

By means of the above boundary conditions, the velocity profile coefficients and the integrals can be expressed in terms of three unknowns, namely, the inviscid axial and azimuthal velocity gradients and the thickness of the viscous layer, η_v . These unknowns are determined by means of the integrated tangential and azimuthal momentum equations (Eqs. 20 and 23) and the continuity equation integrated across the shock layer. Satisfying the integrated continuity equation insures that mass is conserved along a geodesic. The integrated continuity equation can be written as

$$\begin{aligned} \bar{r}_w \bar{u}_\delta \frac{\partial I_0}{\partial \xi} + I_0 \frac{\partial}{\partial \xi} (\bar{r}_w \bar{u}_\delta) + \bar{w}_\delta \frac{\partial I_3}{\partial \varphi} + I_3 \frac{\partial \bar{w}_\delta}{\partial \varphi} \\ = \bar{r}_\delta \bar{u}_\delta \frac{\partial \bar{\delta}}{\partial \xi} + \bar{\kappa}_\delta \bar{w}_\delta \frac{\partial \bar{\delta}}{\partial \varphi} - \bar{\kappa}_\delta \bar{r}_\delta \bar{v}_\delta \end{aligned} \quad (28)$$

$$I_3 = \frac{\bar{\delta}}{L} \int_0^1 j \, d\eta \quad (29)$$

The underlined term in Eq. (28) will be neglected to be consistent with the assumptions made previously.

2.5 ENERGY EQUATION

In the energy equation (Eq. 13) only one crossflow term appears explicitly. Convincing arguments can be put forth which justify the neglect of this term. We note, first of all, that the total enthalpy behind the shock wave is a constant. Furthermore, for a constant wall temperature the total enthalpy is also a constant at the wall. We therefore suspect that the variation of the total enthalpy in the azimuthal direction will be rather small. Additional reasons for suspecting that this is the case arise by inspection of the expression for the convective heat transfer. In the transformed coordinate

system the convective heat transfer can be written as

$$\frac{\dot{q}_c}{\rho_\infty U_\infty^3} = - \frac{(\rho\mu)_w}{(\rho\mu)_{\delta,o}} \frac{L}{2\bar{p}(Pr)_w Re_\delta} g'(o) \quad (30)$$

When normalized to some reference point such as the stagnation point, the heat transfer distribution point in either the axial or azimuthal direction takes on the following form:

$$\frac{\dot{q}_c}{(\dot{q}_c)_o} = \frac{(\rho\mu)_w}{(\rho\mu)_{o,w}} \left[\frac{g'(o)}{g'(o)_o} \right] \left[\frac{(Pr_w \tilde{\delta})_o}{Pr_w \tilde{\delta}} \right]$$

Again for a constant wall temperature the $\rho\mu$ ratio is proportional to the pressure ratio so that the heat transfer distribution can be written as

$$\frac{\dot{q}_c}{(\dot{q}_c)_o} = \frac{P}{P_o} \left[\frac{g'(o)}{g'(o)_o} \right]$$

since the term in the bracket is nearly unity.

Now if the total enthalpy gradient at the wall is a weak function of the azimuthal direction, as suspected, then the heat transfer and pressure distributions should be quite similar. In order to check this hypothesis the pressure and heat transfer distribution around elliptic cones at angle-of-attack were examined. In Figs. 2 and 3, the normalized heat transfer and surface pressure distributions are compared. It is seen that there is a close similarity up to the point where the flow separates ($\varphi = .6\pi$). The heat transfer and pressure data were obtained from two separate tests carried out at AEDC (20, 21). Although the pressure model is slightly different from the heat transfer model the differences are insignificant at large angles-of-attack.

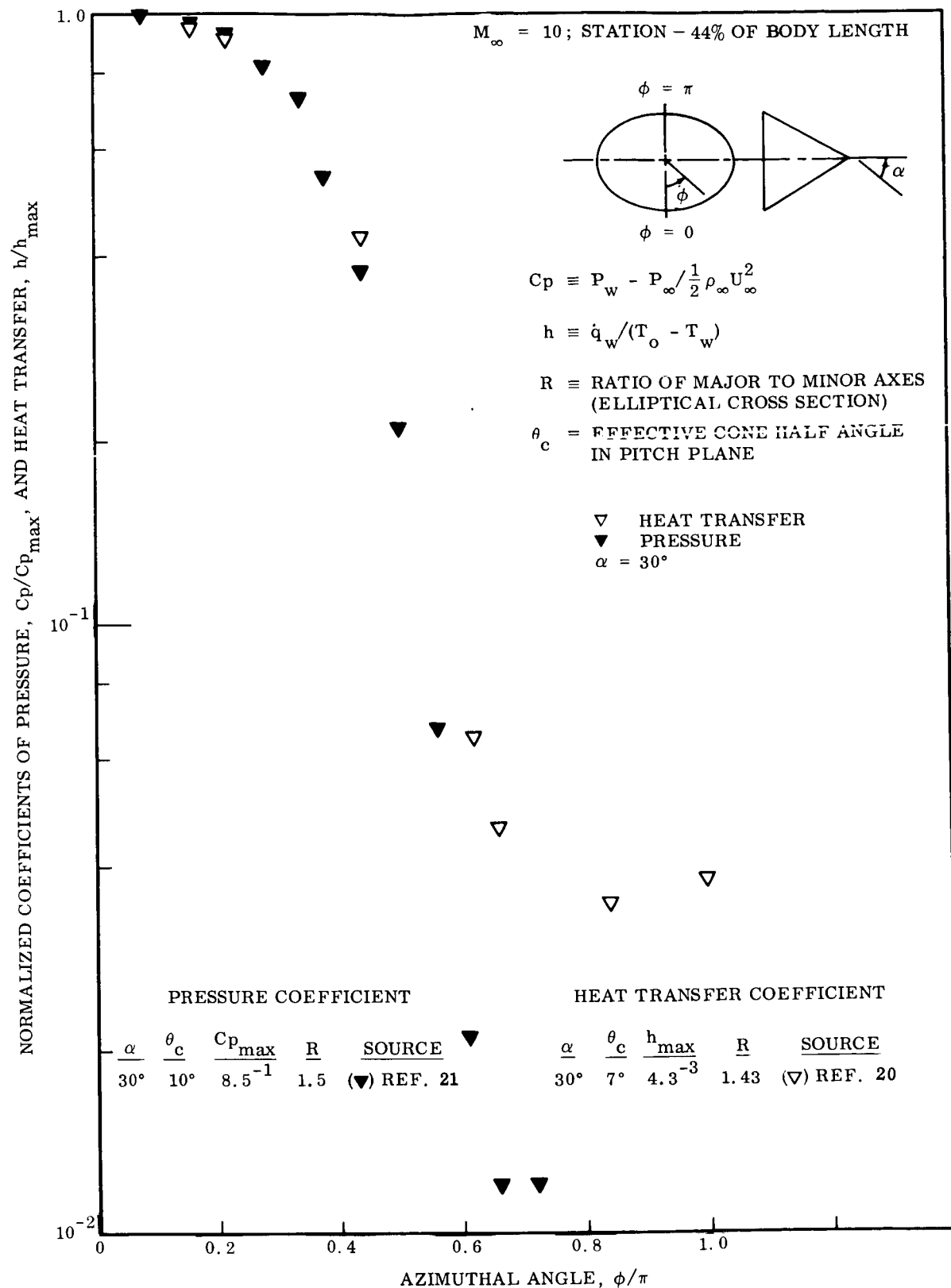


Fig. 2 Azimuthal Distribution of Surface and Heat Transfer Over an Elliptical Cone at Angle of Attack

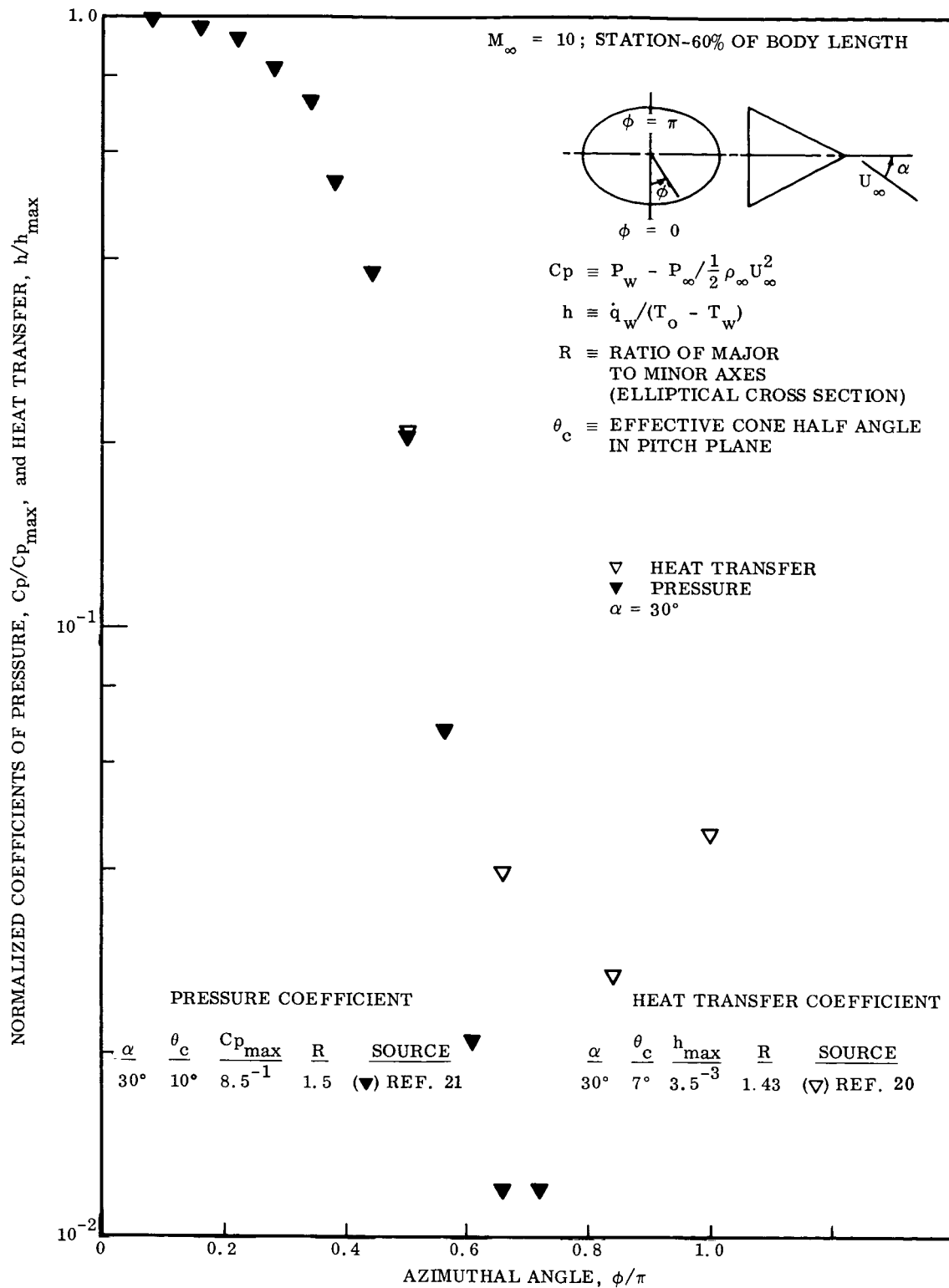


Fig. 3 Azimuthal Distribution of Surface Pressure and Heat Transfer Over an Elliptical Cone at Angle of Attack

The results for the pressure and heat transfer distribution in the azimuthal direction indicate that the total enthalpy gradient at the wall is a weak function of the crossflow velocity as suspected. In other words, the crossflow velocity has a small effect on the convective heating. For this reason the crossflow term in the energy equation will be neglected. However, the crossflow velocity can play an important role in determining the radiative heat transfer and the thermodynamic and transport properties through its influence on the static enthalpy profiles.

In transformed variables, the energy equation becomes

$$\begin{aligned} \frac{\partial}{\partial \eta} \left(\frac{1}{Pr} \frac{\rho \mu}{(\rho \mu)_{\delta,0}} \frac{\partial g}{\partial \eta} \right) + \frac{\tilde{\delta}}{L} Re \bar{u}_{\delta} \frac{\partial g}{\partial \eta} \left[\frac{\partial I_{0,\eta}}{\partial \xi} + \frac{I_{0,\eta}}{\bar{u}_{\delta} \bar{r}_w} \frac{\partial}{\partial \xi} (\bar{u}_{\delta} \bar{r}_w) + \frac{\bar{w}_{\delta}}{\bar{r}_w \bar{u}_{\delta}} \frac{\partial I_{3,\eta}}{\partial \varphi} \right. \\ \left. + \frac{1}{\bar{r}_w \bar{u}_{\delta}} \frac{\partial \bar{w}_{\delta}}{\partial \varphi} I_{3,\eta} \right] \\ = Re \left(\frac{\tilde{\delta}}{L} \right)^2 \left[\bar{u}_{\delta} f \frac{\partial g}{\partial \xi} + \frac{\tilde{\eta} L}{\rho H_{\delta} u_{\infty}} E \right] + \frac{\partial}{\partial \eta} \left[\frac{2 \rho \mu}{(\rho \mu)_{\delta,0}} \bar{u}_{\delta}^2 \left(\frac{1}{Pr} - 1 \right) \left(f \frac{\partial f}{\partial \eta} + \frac{w_{\delta}^2}{u_{\delta}^2} j \frac{\partial j}{\partial \eta} \right) \right] \end{aligned} \quad (31)$$

where the mass flux normal to the surface was replaced by the following expression obtained by integrating the global continuity equation.

$$\begin{aligned} f \frac{\partial \eta}{\partial \xi} \frac{\tilde{\delta}}{L} + \tilde{\eta} \frac{\bar{r}}{\bar{r}_w} \frac{\rho \bar{v}}{(\rho \bar{u})_{\delta}} = - \frac{\partial I_{0,\eta}}{\partial \xi} - I_{0,\eta} \frac{\frac{\partial}{\partial \xi} (\bar{u}_{\delta} \bar{r}_w)}{\bar{u}_{\delta} \bar{r}_w} - \frac{\bar{w}_{\delta}}{\bar{r}_w \bar{u}_{\delta}} \frac{\partial I_{3,\eta}}{\partial \varphi} \\ - \frac{1}{\bar{r}_w \bar{u}_{\delta}} \frac{\partial \bar{w}_{\delta}}{\partial \varphi} I_{3,\eta} \end{aligned} \quad (32)$$

In deriving Eq. (32), it was assumed that $\partial \eta / \partial \varphi = 0$. This assumption is consistent with the assumption that $\tilde{\delta}$ and the total enthalpy and velocity

profiles are slowly varying functions of the azimuthal angle.

The quantities $I_{0,n}$ and $I_{3,n}$ are defined by

$$I_{0,\eta} = \frac{\tilde{\delta}}{L} \int_0^\eta f d\eta \quad , \quad I_{3,\eta} = \frac{\tilde{\delta}}{L} \int_0^\eta j d\eta \quad (33)$$

Following the general procedure of Smith and Jaffe (Ref. 22) for obtaining nonsimilar solutions, the streamwise gradients are replaced by backward differences.

$$\left[\frac{\partial(\quad)}{\partial \xi} \right]_i = \frac{(\quad)_i - (\quad)_{i-1}}{\xi} \quad (34)$$

It is again assumed that the crossflow velocity profiles are slowly varying functions of φ so that the underlined term in Eq. 32 can be neglected.

In order to determine the gas properties and radiative flux divergence, the pressure variation across the shock layer must be known. The required pressure variation is obtained from an approximate solution to the inviscid y-momentum equation. Integration of the inviscid y-momentum equation yields

$$\begin{aligned} P = P_\delta + \int_y^\delta \rho u \frac{\partial v}{\partial x} dy + \int_y^\delta \rho u \frac{\partial v}{\partial y} dy - \int_y^\delta \frac{\kappa \rho u^2}{\tilde{\kappa}} dy \\ + \int_y^\delta \frac{\rho w}{r} \frac{\partial v}{\partial \varphi} dy - \int_\delta^y \frac{\rho w^2}{r} \frac{\partial r}{\partial y} dy \end{aligned} \quad (35)$$

It is now assumed for purposes of evaluating the pressure profile only that the v-velocity profiles are linear and only a function of η , i.e., $v/v_\delta = \eta$. Transforming to ξ, η results in

$$\begin{aligned} \bar{p} = \bar{p}_\delta + \frac{\bar{u}_\delta}{\bar{\rho}} \frac{\partial \bar{v}_\delta}{\partial \xi} \frac{\tilde{\delta}}{\bar{L}} \int_0^1 \eta f d\eta + \frac{\bar{v}_\delta^2}{\bar{\rho}} \int_0^1 \eta \frac{\rho}{\rho_\delta} d\eta \\ - \frac{\bar{u}_\delta^2}{\bar{\rho}} \frac{\tilde{\delta}}{\bar{L}} \int_0^1 \eta^2 d\eta + \frac{\bar{w}_\delta}{\bar{\rho}} \frac{\partial \bar{v}_\delta}{\bar{r}_w \partial \varphi} \frac{\tilde{\delta}}{\bar{L}} \int_0^1 \eta j d\eta - \frac{\bar{w}_\delta^2}{\bar{\rho}} \frac{\sin \theta}{\bar{r}_w} \frac{\tilde{\delta}}{\bar{L}} \int_0^1 j^2 d\eta \end{aligned} \quad (36)$$

The density profile in the third term on the R.H.S. of Eq. (36) is approximated by

$$\frac{\rho}{\rho_\delta} \approx \frac{h_\delta}{h}$$

which in effect means that we are ignoring the effect of pressure variation on the density profile. In all other parts of this analysis, the effect of pressure on the density profile is taken into account.

The energy equation (Eq. 31) is seen to be of the form

$$\frac{\partial}{\partial \eta} \left(\frac{1}{y_1} \frac{\partial g}{\partial \eta} \right) + \left(\frac{1}{y_1} \frac{\partial g}{\partial \eta} \right) P_1 = Q_1 + \frac{\partial Q_2}{\partial \eta} \quad (37)$$

The solution for g is

$$g = \int_0^\eta \left\{ y_1 \int_0^\eta \left(Q_1 + \frac{\partial Q_2}{\partial \eta} \right) e^{-\int_0^\eta P_1 d\eta'} d\eta \right\} d\eta + \int_0^\eta y_1 C_1 e^{-\int_0^\eta P_1 d\eta} d\eta + C_2 \quad (38)$$

where the constants of integration of C_1 and C_2 are determined from the known values of enthalpy at the wall and shock.

2.6 SHOCK SHAPE AND SHOCK BOUNDARY CONDITIONS

The shock shape can be obtained from either theory or experiment. In this analysis the semi-empirical method of Kaattari (Ref. 4) is used to determine

the shock shape. According to Kaattari, the shock detachment distance for blunt bodies (see Fig. 1) at angle-of-attack can be expressed as

$$\left(\frac{\delta}{d}\right)_\alpha = \left(\frac{\delta}{d}\right)_{\alpha=0} - \cos\varphi \sin\alpha \left[C_1 \frac{r\delta}{r_m} + C_3 \left(\frac{r\delta}{r_m}\right)^3 \right] \quad (39)$$

where C_1 and C_3 are known constants. The shock shape can be related conveniently to the body geometry by the radiation

$$r_\delta = r_w + \delta_\alpha \sin\theta \quad (40)$$

where θ is the body surface angle.

The boundary conditions on the shock will be expressed in terms of the shock slope in the r and φ direction. The displacement of the shock front from the shock front location at the geometric center, $\tilde{x}$, (see Fig. 1) is given by

$$\tilde{x} = R_s \left[1 - \cos\left(\sin^{-1} \frac{r\delta}{R_s}\right) \right]_{\alpha=0} + d \left[C_1 \frac{r\delta}{r_m} + C_3 \left(\frac{r\delta}{r_m}\right)^3 \right] \cos\varphi \sin\alpha \quad (41)$$

from which the required slopes can be determined.

In our body-oriented coordinate system the required velocity components are the normal and tangential components in the x and φ direction. These components can be readily obtained from the analysis presented in Ref. (1).

$$\bar{u}_\delta = \sin\theta \left[\cos\alpha - \frac{K}{N^2} (1-\bar{p}) \right] - \cos\theta \left[\sin\alpha \cos\varphi - \frac{K}{N^2} \frac{\partial \tilde{x}}{\partial r} (1-\bar{p}) \right] \quad (42)$$

$$\bar{v}_\delta = -\sin\theta \left[\sin\alpha \cos\varphi - \frac{K}{N^2} \frac{\partial \tilde{x}}{\partial r} (1-\bar{p}) \right] - \cos\theta \left[\cos\alpha - \frac{K}{N^2} (1-\bar{p}) \right] \quad (43)$$

$$\bar{w}_\delta = \sin\alpha \sin\varphi + \frac{K}{N^2} \frac{\partial \tilde{x}}{r \partial \varphi} \quad (44)$$

$$K = \cos\alpha + \frac{\partial \tilde{x}}{\partial r} \cos\varphi \sin\alpha - \frac{\partial \tilde{x}}{r \partial \varphi} \sin\varphi \sin\alpha \quad (45)$$

$$N^2 = 1 + \left(\frac{\partial \tilde{x}}{\partial r} \right)^2 + \left(\frac{\partial \tilde{x}}{r \partial \varphi} \right)^2 \quad (46)$$

The pressure at the shock is given by

$$\frac{P_\delta}{\rho_\infty u_\infty^2} = \bar{P} = \left(\frac{K}{N} \right)^2 (1-\bar{p}) \quad (47)$$

The solutions to the momentum equations (Eqs. 20, 23) must proceed from the point where $u_\delta = 0$ to eliminate the streamwise derivative of the integrals I_1 and I_2 . The curve on the shock surface where $u_\delta = 0$ can be obtained from Eq. (42). Note that since we have assumed that the velocity profiles can be expressed as polynomials of the normal (to the surface) variable η , the stagnation point on the body is on the normal which intercepts the shock at the point where $u_\delta = 0$. In other words, the stagnation streamline is assumed to be a straight line. Although it is realized that this is not correct (Ref. 1, 23), the inaccuracy introduced is not serious since both the convective and radiative fluxes vary slowly in the immediate vicinity of the stagnation point.

Section 3

GAS PROPERTIES

3.1 THERMODYNAMIC AND TRANSPORT PROPERTIES

The analysis up to this point is valid for any multicomponent gas in thermodynamic equilibrium. Numerical solutions can be obtained if the thermodynamic, transport, and radiative properties are known. The thermodynamic and transport properties, as computed by Hansen (Ref. 24) and correlated by Viegas and Howe (Ref. 25), are used in the numerical solutions. The correlation formulas are valid for temperatures between 1000° and $15,000^{\circ}\text{K}$ and for pressures between 0.1 and 100 atmospheres.

3.2 NUMBER DENSITIES

The absorption coefficients which are discussed in Subsection 3.3, are most conveniently expressed in terms of the gas-particle number densities. The number densities can be easily obtained from the air equation of state if an idealized air chemistry model is adopted. In this model, oxygen dissociation is assumed to begin first as the air temperature is increased. After all the oxygen molecules are dissociated, the nitrogen molecules commence dissociating. When all of the molecules are dissociated, ionization of the atoms begins and no distinction is made between the ionization of the oxygen and nitrogen atoms.

The number density of the molecules, atoms, and ions can be expressed in terms of the total number density and the compressibility Z , by means of the definitions of the total mass and total number density.

$$\rho_t = \rho_m + \rho_a + \rho_i + \rho_e \quad (48)$$

$$N_t = N_m + N_a + N_i + N_e \quad (49)$$

With the equation of state

$$Z = \frac{M_o}{M_t} = \frac{M_o}{RT} \frac{\rho}{\rho_t} \quad (50)$$

Equation 48 can be rewritten as

$$N_t = N_m \frac{M_m}{M_o} Z + N_a \frac{M_a}{M_o} Z + N_i \frac{M_i}{M_o} Z \quad (51)$$

By considering particular intervals of the compressibility factor Z in which the gas chemistry is specified, the number densities can be obtained in terms of the total number density and the compressibility factor by means of Eqs. 49-51. The number density equations for the specified compressibility intervals have been previously presented in Ref. (26) and are given in Appendix A for convenience.

3.3 ABSORPTION COEFFICIENTS

The radiative energy transfer within the shock layer depends critically on the absorption coefficient of the gas particles. The absorption coefficient is dependent on the nature of the gas particle, its number density, the gas temperature, and the radiation frequency. The absorption coefficients utilized in the present analysis are summarized in Fig. 4. An examination of the molecular absorption coefficients for air given in Refs. 27, 28, and 29 shows that the only important air molecular absorption coefficients are the O_2 Schumann-Runge continuum and the N_2 Birge-Hopfield band.

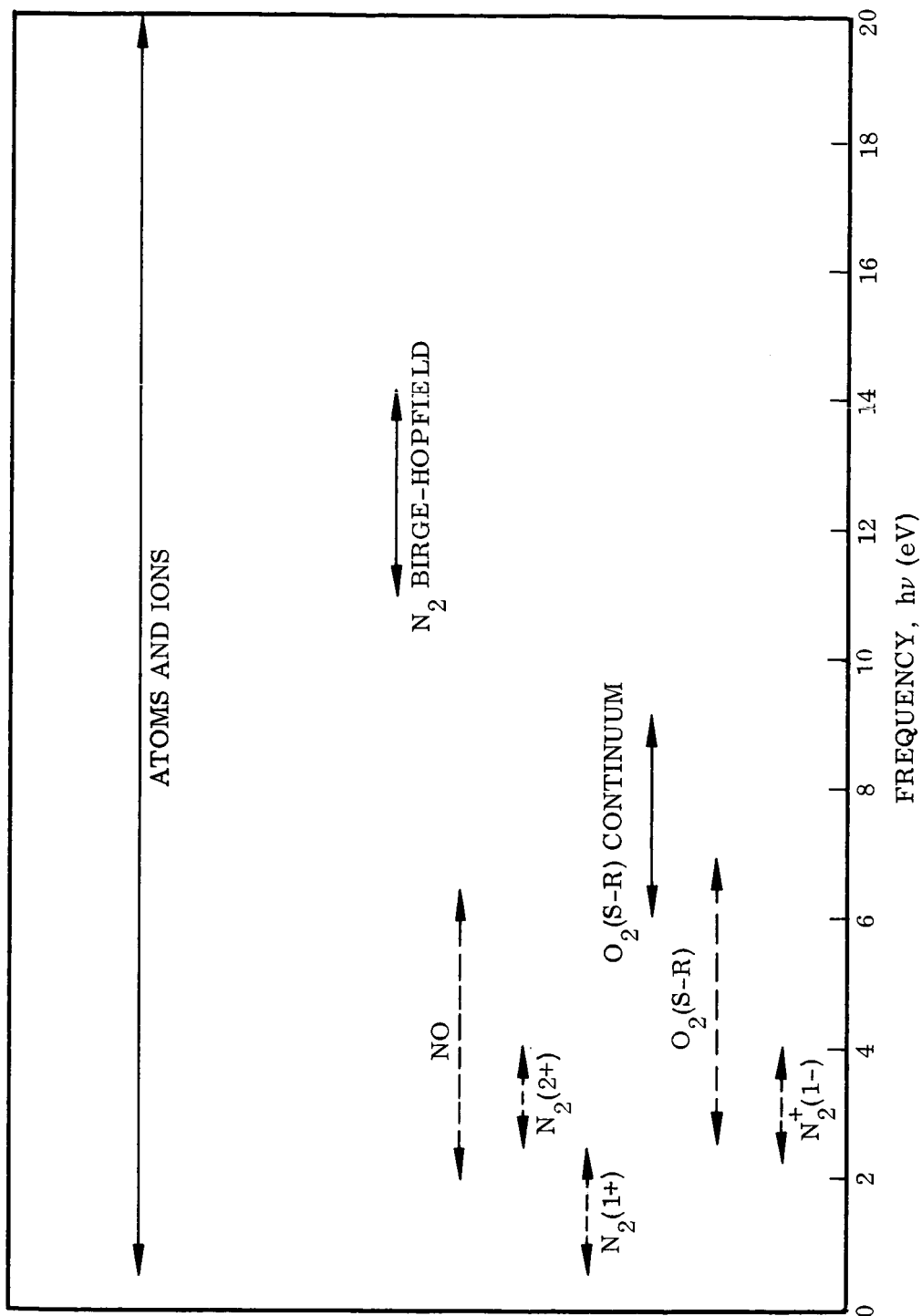


Fig. 4 Summary of Absorption Coefficients

All of the important absorption coefficients summarized in Fig. 4 are given, by simple analytical expressions in terms of the particle number density, temperature, and the radiation frequency. The equations are used to evaluate the radiation flux gradient term in the energy equation to obtain numerical solutions.

The continuum absorption coefficients due to bound-free and free-free transitions for the neutral and singly ionized atoms of nitrogen and oxygen were obtained from the theory of Biberman (Ref. 30) and Armstrong et al (Ref. 31). Biberman's results are used for the free-free and bound-free transitions from the higher excited states while Armstrong's detailed quantum-mechanical calculations are used to provide cross-sections for photon absorption due to transitions from the ground and low lying excited states.

The absorption coefficient for air is written as the sum of the individual species absorption coefficients

$$\kappa_v(x) = [1 - e^{-x}] \left[(\kappa_v)_n + (\kappa_v)_o + (\kappa_v)_i \right]$$

where the term in the first bracket accounts for induced emission. The absorption coefficient of the oxygen ion is assumed to be equal to that of the nitrogen ion. This approximation is acceptable since the ions contribute little to the total absorption coefficient except at the higher frequencies where the radiation from the shock layer gas is likely to be black body.

The absorption coefficients for the various species are presented in Ref. (26) and in Appendix B.

Section 4

NUMERICAL METHOD

The integrated momentum and energy equations are solved by an iterative method. A solution is first obtained at the point where $u_\delta = 0$. The point on the body where $u_\delta = 0$ depends on the value of azimuthal angle which is treated as a parameter in the solution. At this point a total enthalpy profile and a value for the transformed shock detachment distance, $\tilde{\delta}$, are assumed. With these two quantities known, Eqs. 20, 23, and 28 are solved simultaneously for the inviscid axial and azimuthal velocity gradients and the viscous layer thickness, η_v .

With these three quantities known, the velocity profiles can be evaluated. The pressure variation across the shock layer can now be determined from Eq. (36). Equation (19) is now used to check the assumed value of $\tilde{\delta}$. We now iterate on $\tilde{\delta}$ until we have a converged solution for the assumed enthalpy profile. With this solution for the velocity field, the energy equation (Eq. 38) is solved to obtain a new enthalpy profile. The procedure is repeated until a converged solution for the enthalpy profile is obtained.

To obtain solutions around the body the equations are integrated both in the windward and leeward direction. The procedure for solving the equations is identical to that used at the starting point.

Section 5

DISCUSSION OF THE RESULTS

Numerical results have been obtained for one flight condition ($U_\infty = 34,500$ fps, $h = 170,000$ ft) for the body shape shown in Fig. 1 with $R = 2.64$ ft and $r_m = 1.03$ ft. These results are for an angle attack of 10° . Velocity and enthalpy profiles at the stagnation point are shown in Fig. 5. The tangential and azimuthal velocity profiles should be identical at the stagnation point since the body being considered is a spherical segment. The slight difference in the velocity profiles is due to the approximations made in the solutions to the x and φ -momentum equations. The total enthalpy near the edge of the viscous layer is seen to be slightly less than the value behind the shock. This decrease in the inviscid layer is due to radiation cooling. The large gradients near the wall are, of course, caused by viscous effects. Similar results are presented in Fig. 6 for $\varphi = 30^\circ$ and $\xi = 0.18$. The velocity profiles are again seen to be quite similar although in this case the cross flow velocity of w_δ is approximately twice the tangential velocity u_δ . The total enthalpy profile at this body station is also quite similar to the stagnation-point profile. These results indicate that the flow over the face of the blunt body considered is not too different from the flow at the stagnation-point.

Convective heat transfer distributions are shown in Fig. 7. The stagnation point is located at $\xi = 0.155$. The convective heat transfer increases on the windward side and decreases on the leeward side of the stagnation point. The increase in the convective heat on the windward side is due to the decrease in the shock-detachment distance which results in an increase in the wall enthalpy gradient. This increase in the wall enthalpy gradient more than compensates for the decrease in density which results in an

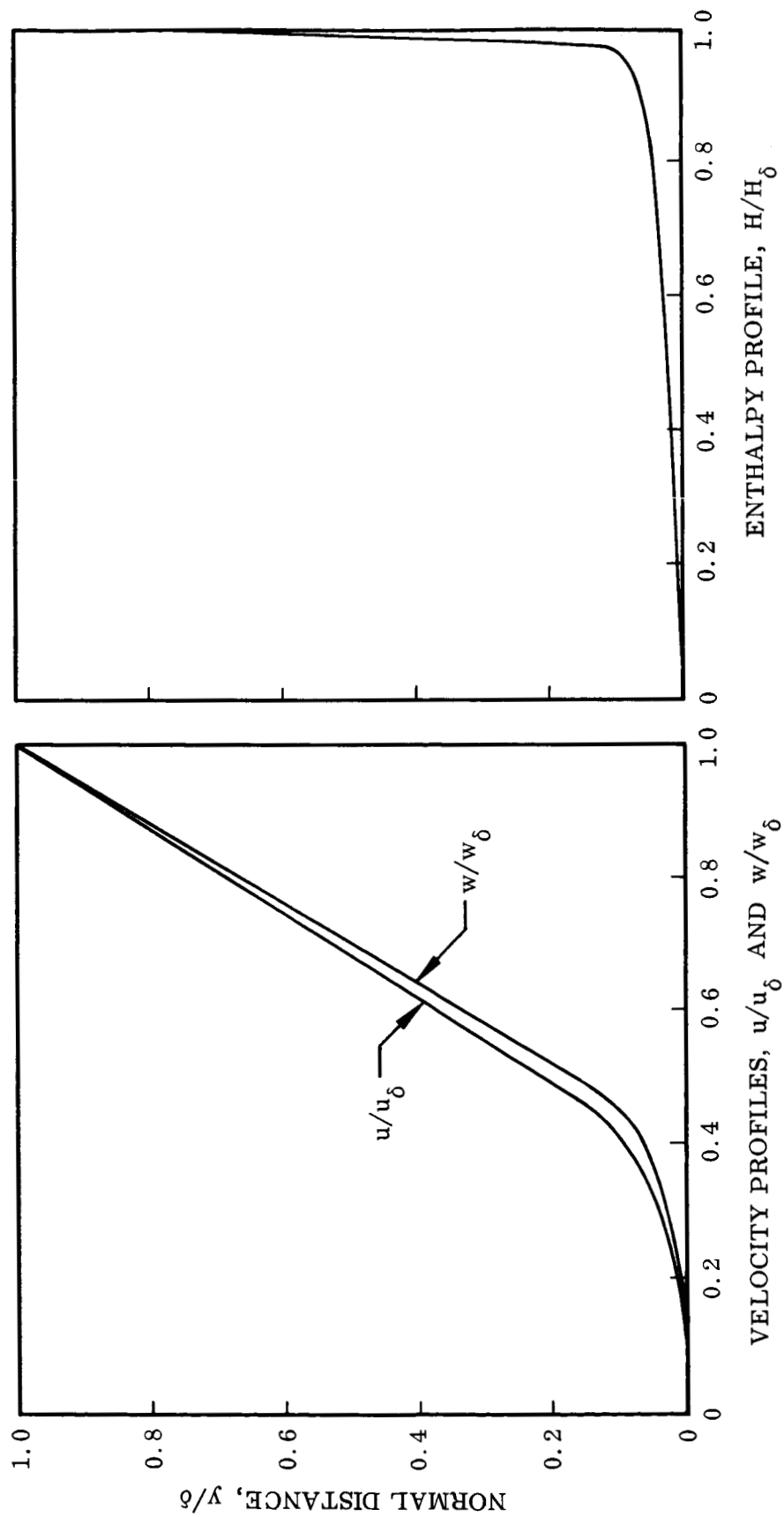


Fig. 5 Velocity and Enthalpy Profiles for $\alpha = 10^\circ$, $\varphi = 0$, $\xi = 0.155$ (Stagnation Point), $\bar{U} = 3.45$, $\bar{h} = 1.7$, $R = 2.64$ ft, and $r_m = 1.03$ ft

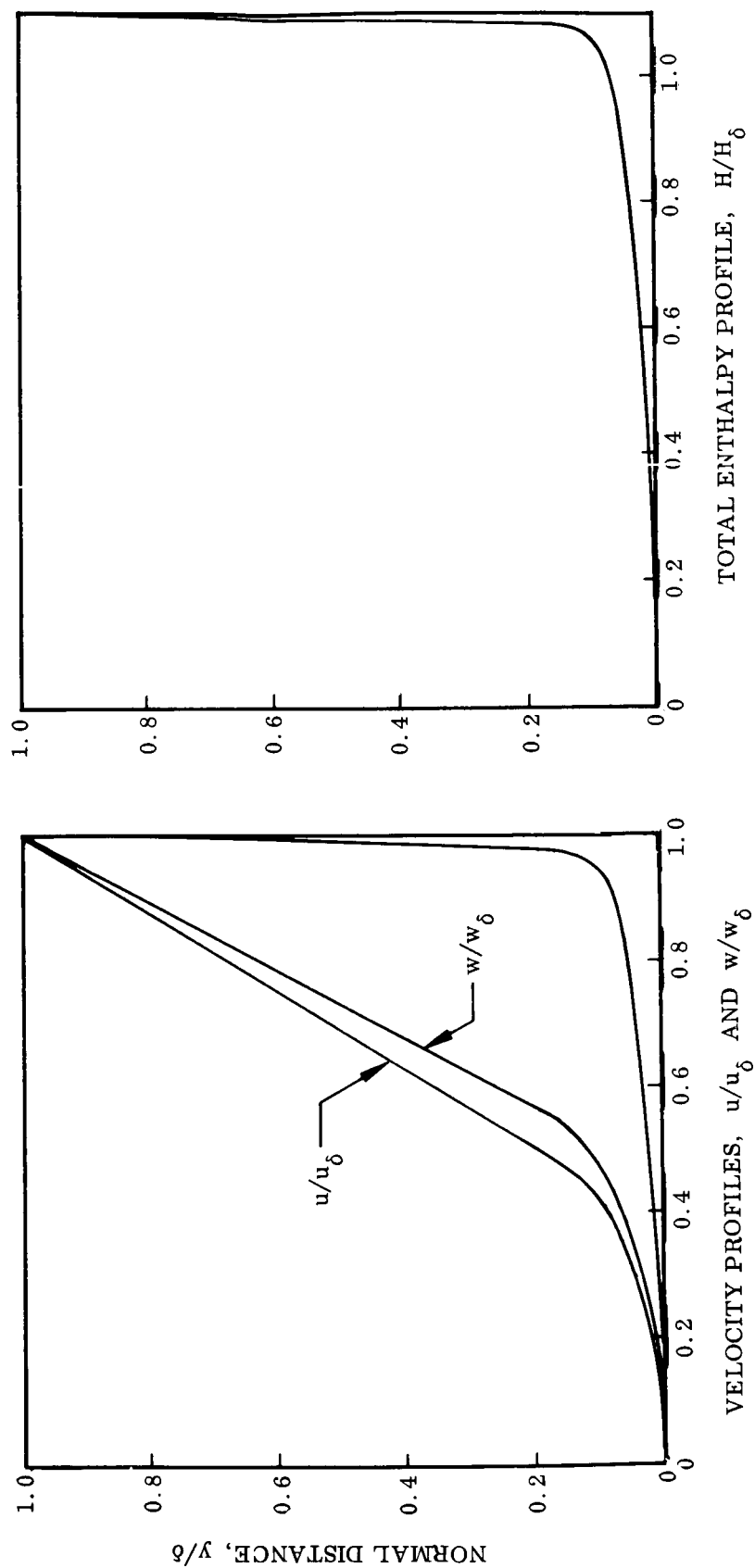


Fig. 6 Velocity and Total Enthalpy Profiles for $\alpha = 10^\circ$, $\varphi = 30^\circ$, $\xi = 0.18$, $\bar{U} = 3.45$, $\bar{h} = 1.7$, $R = 2.64$ ft and $r_m = 1.03$ ft

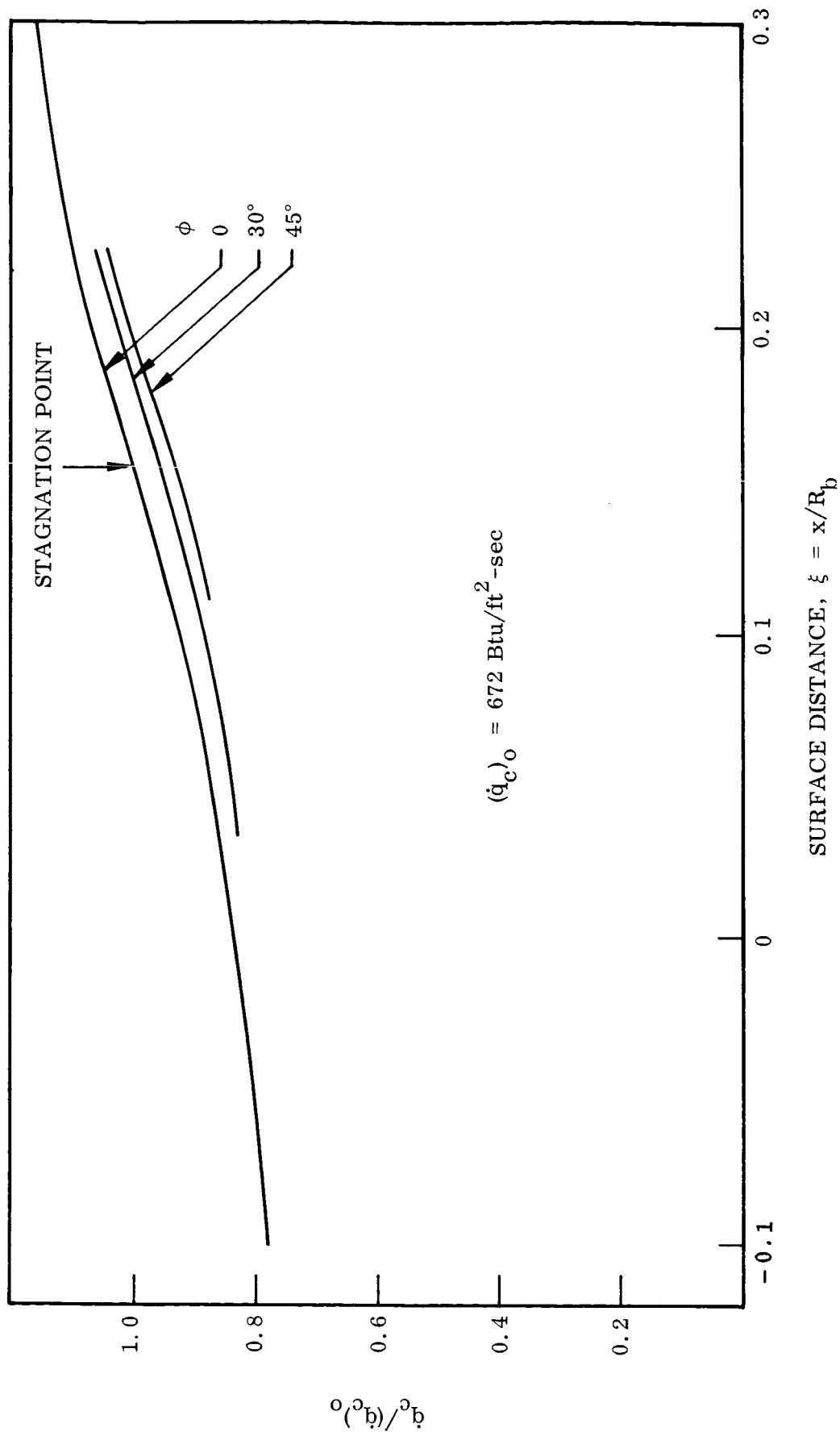


Fig. 7 Convective Heat Transfer Distribution, $\alpha = 10^\circ$, $\bar{U} = 3.45$, $\bar{h} = 1.7$, $R_b = 2.64 \text{ ft}$, and $r_m = 1.03 \text{ ft}$

increase in convective heating. On the leeward side both the wall enthalpy gradient and density decrease which causes convective heating to decrease. The convective heat transfer is seen to be a weak function of the azimuthal angle. This is not surprising since the angle attack was taken to be 10° . As the angle attack approaches zero, the distributions should become independent of φ . If the angle attack is increased, the effect of the azimuthal angle on the distributions will become more pronounced.

Radiative heat transfer distributions are shown in Fig. 8. These distributions are seen to decrease in both the windward and leeward directions. The reason for this behavior is that the radiative heat transfer is dependent on the entire temperature profiles in the shock layer rather than on local quantities. The temperature profiles are quite insensitive to body location on the face of a blunt body. The temperature behind the shock, however, falls off slightly from the stagnation point value in both the leeward and windward directions. This decrease in the temperature behind the shock is responsible for the decrease in the radiative heat transfer away from the stagnation point. The radiative heat transfer distribution is also a weak function of the azimuthal angle for the angle of attack considered. For larger angles of attack we would again expect a pronounced effect of azimuthal angle.

The monochromatic radiative flux for two body stations is shown in Fig. 9. These results show that approximately half the radiant energy is in the ultraviolet region with the remaining half in the visible and infrared region. The difference between $\varphi = 0$ and $\varphi = 45^\circ$ is seen to be quite small which is consistent with the results shown in Fig. 8. These results and the distributions shown in Fig. 8 do not include line radiation.

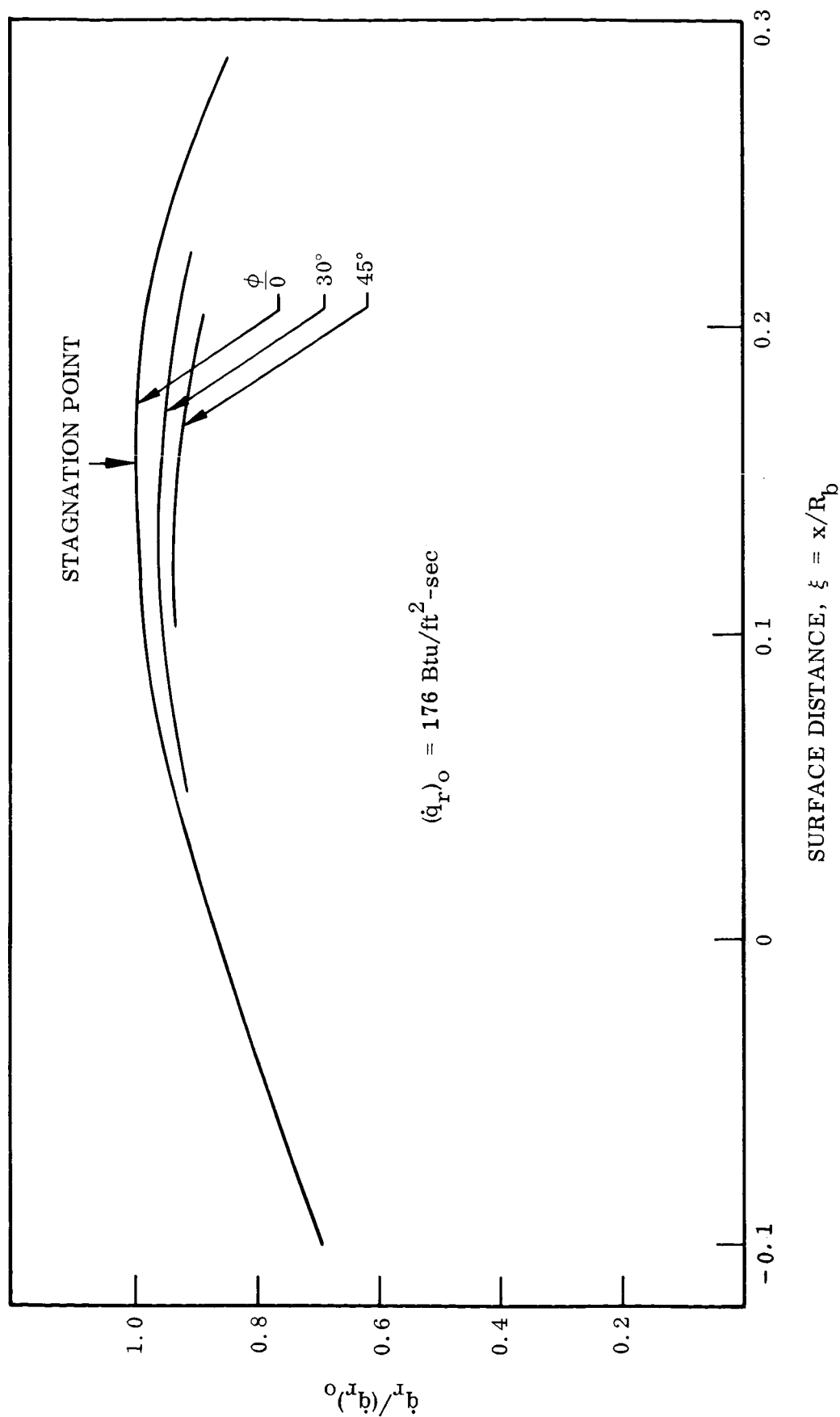


Fig. 8 Radiative Heat Transfer Distribution, $\alpha = 10^\circ$, $\bar{U} = 3.45$, $\bar{h} = 1.7$, $R = 2.64 \text{ ft}$, and $r_m = 1.03 \text{ ft}$

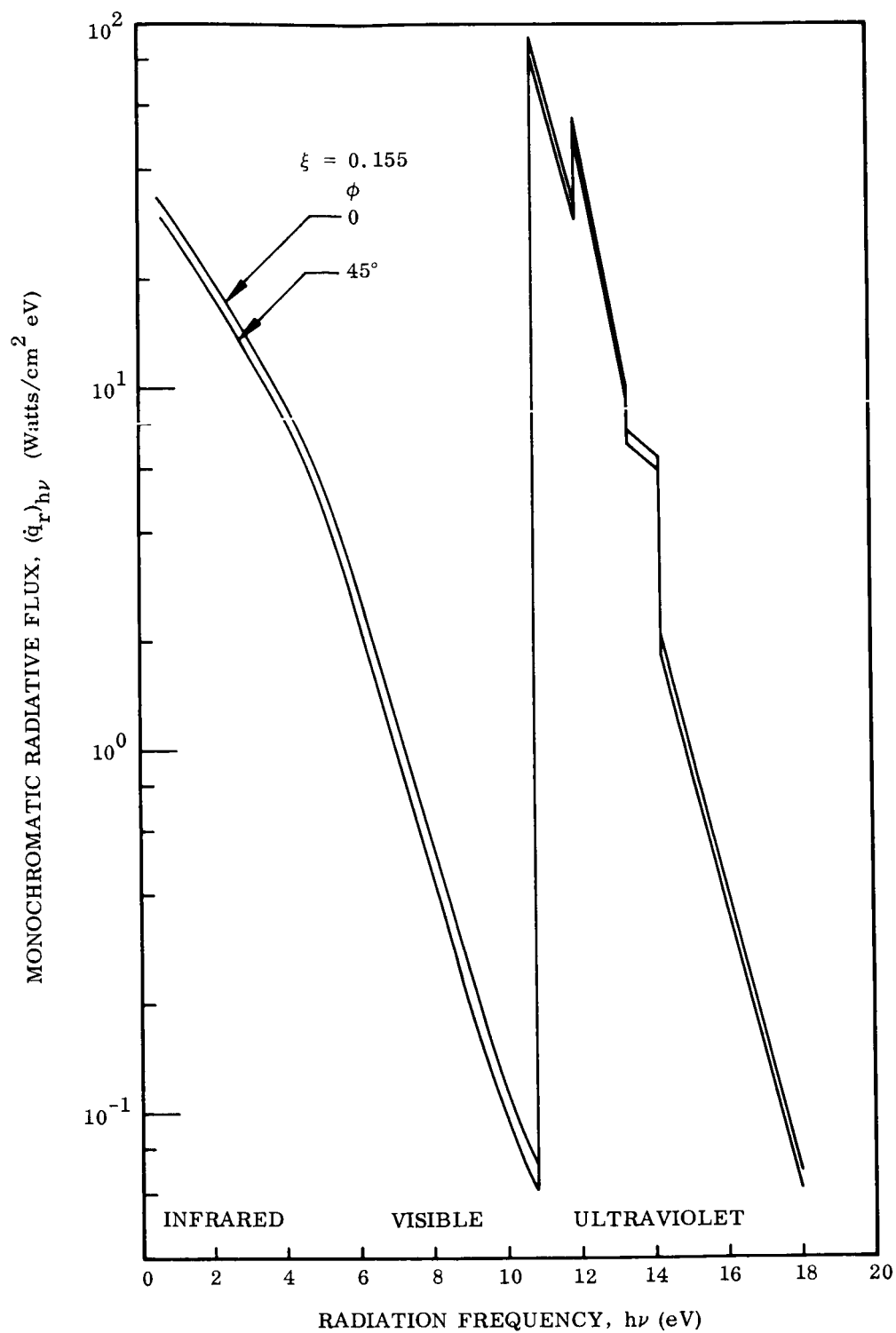


Fig. 9 Monochromatic Radiative Flux, $\alpha = 10^\circ$, $\bar{U} = 3.45$, $\bar{h} = 1.7$, $R = 2.64$ ft, and $r_m = 1.03$ ft

Section 6

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Appendix A

NUMBER DENSITY

The expressions which give the number density as a function of the total number density and the compressibility factor are easily derived from Eqs. 49-51 and are listed below:

Dissociation of Oxygen: $1.0 < Z \leq 1.2$

$$N_{O_2} = N_t \left(\frac{1.2 - Z}{Z} \right) \quad (A-1)$$

$$N_O = N_t \left(\frac{2Z - 2}{Z} \right) \quad (A-2)$$

$$N_{N_2} = N_t \frac{0.8}{Z} \quad (A-3)$$

$$N_t = \frac{Z \rho_t A}{M_o} \quad (A-4)$$

Equation A-3 states that the number density of N_2 is equal to 0.8 times the total number density of air calculated, assuming no dissociation.

Dissociation of Nitrogen: $1.2 < Z \leq 2.0$

$$N_O = N_t \frac{0.4}{Z} \quad (A-5)$$

$$N_{N_2} = N_t \left(\frac{2 - Z}{Z} \right) \quad (A-6)$$

$$N_N = N_t \left(\frac{2Z - 2.4}{Z} \right) \quad (A-7)$$

Single Ionization of Atoms: $2 < Z \leq 4$

$$N_O + N_N = N_t \left(\frac{4-Z}{Z} \right) \quad (A-8)$$

$$N_i = N_t \left(\frac{Z-2}{Z} \right) \quad (A-9)$$

Double Ionization of Atoms: $4 < Z \leq 6$

$$N_i = N_t \left(\frac{6}{Z} - 1 \right) \quad (A-10)$$

$$N_{ii} = N_t \left(1 - \frac{4}{Z} \right) \quad (A-11)$$

Appendix B
ABSORPTION COEFFICIENTS

The absorption coefficients for the various species are listed below for the indicated intervals.

$$0 \leq h\nu \leq 4.22$$

$$(\kappa_v)_N = 4.5 \tilde{a} N_N \theta e^{-(x_N - x)} \frac{\xi_N}{(h\nu)^3} \quad (B-1)$$

$$(\kappa_v)_O = \frac{8}{9} \tilde{a} N_O \theta e^{-(x_O - x)} \frac{\xi_O}{(h\nu)^3} \quad (B-2)$$

$$(\kappa_v)_i = 5.32 \tilde{a} N_i \theta e^{-(x_i - x)} \frac{\xi_i}{(h\nu)^3} \quad (B-3)$$

where

$$\tilde{a} = 7.25 \times 10^{16} \text{ cm}^2\text{-ev}^2$$

$$x_N = 14.3/\theta, x_O = 13.4/\theta, x_i = 25.5/\theta$$

$$x = h\nu/kT = h\nu/\theta$$

ξ_N, ξ_O, ξ_i = quantum-mechanical correction factor (Fig. 10)

$$4.22 \leq h\nu \leq 10.8$$

$$(\kappa_v)_N = (\text{Eq. (B-1)}) e^{(4.22/\theta - x)} \quad (B-4)$$

$$(\kappa_v)_O = (\text{Eq. (B-2)}) e^{(4.22/\theta - x)} \quad (B-5)$$

$$(\kappa_v)_i = (\text{Eq. (B-3)})$$

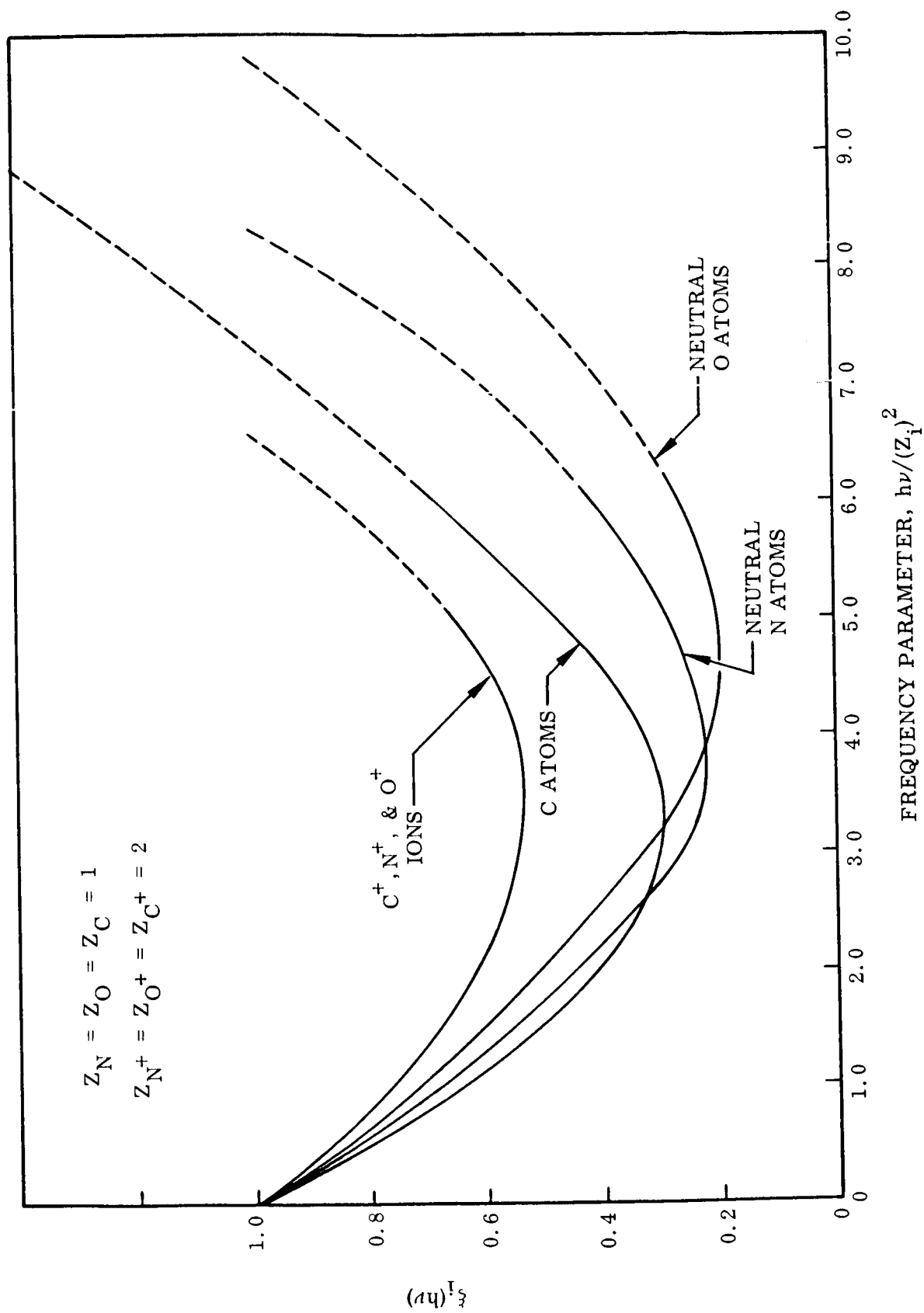


Fig. 10 Quantum-Mechanical Correction Factor $\xi(h\nu)$

$$10.8 \leq h\nu \leq 12.0$$

$$(n_v)_N = (\text{Eq. (B-4)} + N_N \cdot \varphi_{N,1} \quad (\text{B-6})$$

$$(n_v)_O = (\text{Eq. (B-5)})$$

$$(n_v)_i = (\text{Eq. (B-3)}) e^{(11.2/\theta - X)} \quad (\text{B-7})$$

$$12 \leq h\nu \leq 13.4$$

$$(n_v)_N = (\text{Eq. (B-4)} + N_N (\varphi_{N,1} + \varphi_{N,2}) \quad (\text{B-8})$$

$$(n_v)_O = (\text{Eq. (B-5)})$$

$$(n_v)_i = (\text{Eq. (B-7)})$$

$$13.4 \leq h\nu \leq 14.3$$

$$(n_v)_N = (\text{Eq. (B-8)})$$

$$(n_v)_O = (\text{Eq. (B-5)} + N_O \cdot \varphi_{O,1} \quad (\text{B-9})$$

$$(n_v)_i = (\text{Eq. (B-7)})$$

$$14.3 \leq h\nu \leq 20$$

$$(n_v)_N = (\text{Eq. (B-4)} + N_N (\varphi_{N,1} + \varphi_{N,2} + \varphi_{N,3}) \quad (\text{B-10})$$

$$(n_v)_O = (\text{Eq. (B-9)})$$

$$(n_v)_i = (\text{Eq. (B-7)})$$

where

$$\varphi_{N,1} = 5.16 \times 10^{-17} \frac{e^{-(X_N - 10.8/\theta)}}{[4 + 10e^{-(2.38/\theta)} + 6e^{-(3.57/\theta)}]}$$

$$\varphi_{N,2} = 6.4 \times 10^{-17} \frac{e^{-(X_N - 12/\theta)}}{[4 + 10e^{-(2.38/\theta)} + 6e^{-(3.57/\theta)}]}$$

$$\varphi_{N,3} = 3.16 \times 10^{-17} \frac{1}{[4 + 10e^{-(2.38/\theta)} + 6e^{-(3.57/\theta)}]}$$

$$\varphi_{O,1} = 3.6 \times 10^{-17} \frac{1}{[9 + 5e^{-(1.98/\theta)} + e^{-(4.18/\theta)}]}$$

The absorption coefficients for molecular oxygen Schumann-Runge continuum were obtained from the results of Evans and Schexnayder (Ref. 28). In Ref. 28 the authors discuss the approximate formula of Sulzer-Wieland and point out its deficiency at elevated temperatures ($T \approx 10,000^\circ\text{K}$). At lower temperatures where the molecular oxygen is present in a gas mixture in chemical equilibrium, the approximate results of Sulzer-Wieland given below are adequate for present purposes.

$$\kappa_v = 1.49 \times 10^{-17} N_{O_2} \left[\tanh \left(\frac{0.0975}{\theta} \right) \right]^{1/2} \exp \left[-\tanh \left(\frac{0.0975}{\theta} \right) \right. \\ \left. \times \left(\frac{h\nu - 8.56}{0.805} \right)^2 \right] \quad (B-11)$$

The absorption coefficient for the nitrogen Birge-Hopfield band was obtained from results of R. Allen (Ref. 29). The cross section for $T = 7000^\circ\text{K}$ was approximated and used to obtain the following expression for the absorption coefficient.

$$\kappa_v = 1.2 \times 10^{-17} N_{N_2} \exp \left[- \left| hv - 13.6 + \left(1 - \frac{0.603}{\theta} \right) \right|^{1.3} \right] \quad (\text{B-12})$$

The effective cross sections, defined by

$$\sigma^* = \frac{\kappa_v}{N} \quad (\text{B-13})$$

for the two molecules discussed above are presented in Fig. 11.

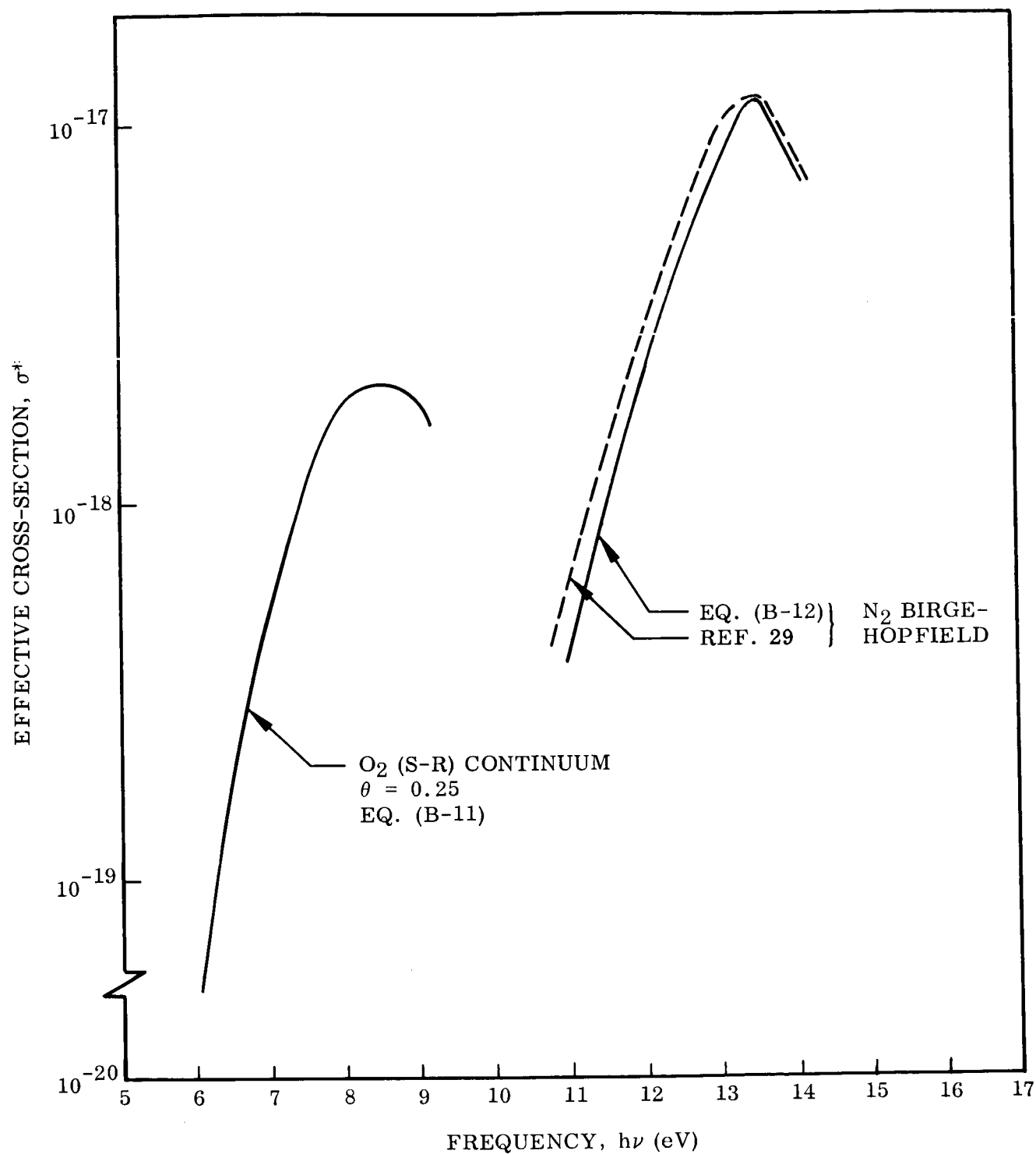


Fig. 11 Effective Cross Section of Molecules