NASA CONTRACTOR REPORT

NASA CR-664

MULTIVARIABLE ANALYSIS OF THE MECHANICS OF PENETRATION OF HIGH SPEED PARTICLES

by D. D. Bouma and W. C. Burkitt

Prepared by
MARTIN MARIETTA CORPORATION
Denver, Colo.
for Western Operations Office

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • DECEMBER 1966

NASA CR-664

MULTIVARIABLE ANALYSIS OF THE MECHANICS OF PENETRATION OF HIGH SPEED PARTICLES

By D. D. Bouma and W. C. Burkitt

Distribution of this report is provided in the interest of information exchange. Responsibility for the contents resides in the author or organization that prepared it.

Prepared under Contract No. NAS 7-219 by
MARTIN MARIETTA CORPORATION
Denver, Colo.

for Western Operations Office

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – Price $3.25

CONTENTS

Table

SUMMARY

The original purpose of the investigation was to deduce from a statistical analysis of the crater depth data for spherical projectiles striking massive targets, what properties of the materials involved were most closely related to crater depth. It was realized that probably none of the static, low temperature and pressure properties which are conveniently available to describe materials, played a direct role in controlling the cratering process. However, it was hoped that by surveying a large number of physical and mechanical properties of the target and projectiles, a number of such properties would be found to be statistically related to the data. A formulation of the relationship could then be used to predict crater depth results, or at least correlate the available results.

The most difficult aspect of the problem was that of correctly accounting for the effects of impact velocity, so that the variation in the data due to material properties could be isolated for analysis. This problem has been solved, at least for the available data, by fitting all data to the functional form

$$Z \left(\frac{h}{d}\right) = \ell n \left[1 + (Xv)^2\right]$$

where $Z \left(\frac{h}{d}\right)$ is a dimensionless crater depth, Z being the product of a number of materials properties, each raised to some exponent. The term (Xv) is a dimensionless velocity, X also containing the product of different powers of several material properties.

Both X and Z can be calculated from material properties with a coefficient of multiple correlation* of 0.84 for the available data. Practically, this amounts to an accuracy of ±25% in the prediction of crater depth at any velocity for any of the 34 combinations of projectile and target materials in the available data.

The steps by which this has been accomplished have been formalized into a generally applicable procedure for problems of this type. This procedure consists of:

1) Data collection and reduction to common terms;

2) Isolation of the effects of physical and geometrical variables that can be controlled so that their trends can be observed;

*Explained in Appendix D.

3) Extraction of these effects from the data;

4) Assignment of the remaining variation in the data to changes in the properties of the materials involved, in the form of a power law;

5) Preliminary statistical analysis to eliminate from an arbitrarily chosen large group of candidate properties, those which show only negligibly slight correlation to the data;

6) Arrangement of the remaining candidate properties into dimensionless groups, using as guides the exponents from the preliminary analysis;

7) Final statistical analysis in terms of these dimensionless groups.

The method described above was applied to the problem of crater volume in massive targets. This was related to the crater depth solution in the form

$$\frac{V}{d^3} = A \left(\frac{h}{d}\right)^n$$

where A is a shape parameter, and n an exponent reflecting the change of volume with depth of crater. Both A and n can be calculated from material properties with coefficient of multiple correlation greater than 0.95.

The method has been extended to the problem of bumpered targets without filler materials. In this problem, trends are observed in the data reflecting the effects of standoff distance, thickness of bumper, thickness of second wall, and impact velocity. Each of these trends has been approximated by a suitable functional form. Analysis of a group of data for one combination of projectile and target materials shows satisfactory agreement between the model and the reduced data. Time did not permit analysis of data for other combinations, or subsequent analysis of materials properties effects.

I. INTRODUCTION

This final report presents the results of a three-year study beginning in March 1963 and extending through March 1966. The initial objective of this study was to determine, by statistical analysis of the data available from the literature,* those properties of projectile and target materials that influence the penetration of very high velocity projectiles. Experimental results from the high-velocity impact of spherical projectiles on massive targets were chosen as the basis for this analysis, since this type of data was most plentiful.

Only the usual physical and mechanical properties of metallic materials, and derivatives of these, were to be considered, since to be useful any formulation resulting from such an analysis must be expressible in terms of commonly available information. No attempt was made to consider the actual properties of the materials under conditions of very high pressure, temperature, and strain rate. Since in the final analysis all differences between metallic materials must reflect basic differences in crystalline structure (or perhaps even in atomic structure), the underlying assumption was that consideration of enough common properties could provide an adequate description of each material so that its behavior under conditions of hypervelocity impact could be predicted by correlating crater depth data to as many such material properties as might prove statistically significant. Preliminary work along this line had produced encouraging results (Ref 1).

Work during the study justified this assumption both in the correlation of crater depth data and of crater volume data, but disclosed the necessity of removing from the data all variation that was not directly attributable to materials properties. The objective of the study remained essentially unchanged, even though it was extended to include crater volume and damage to thin and multiwall targets. However, the bulk of the analysis effort had to be expended on devising means of extracting from the data the effects of variables other than projectile and target material properties. These variables, principally impact velocity and geometric parameters, affect the data in ways which cannot be included in a linear regression analysis with the effects of materials properties.

*Summarized in Appendix A.

A general procedure was evolved that produced successful analyses of crater depth and crater volume data and appears to be promising for the analysis of bumpered target data. It consists of the following steps:

1) Data collection and reduction to common terms;

2) Isolation of the effects of physical and geometrical variables that can be controlled so their effects can be observed;

3) Extraction of these effects from the data;

4) Assignment of the remaining variation in the data to changes in the properties of the materials involved, in the form of a power law;

5) Preliminary statistical analysis to eliminate from an arbitrarily chosen large group of candidate properties (Table 1), those that show only negligibly slight correlation to the data;

6) Arrangement of the remaining candidate properties into dimensionless groups, using as guides the exponents from the preliminary analysis;

7) Final statistical analysis in terms of these dimensionless groups.

To permit accurate appraisal of the effects of impact velocity over a wide range, a small experimental program was planned and executed at the Martin Company during the third year in order to fill specific gaps in the available data, and to provide a check on the correlative equations derived. These data are presented in Appendix B and shown in Fig. 1.

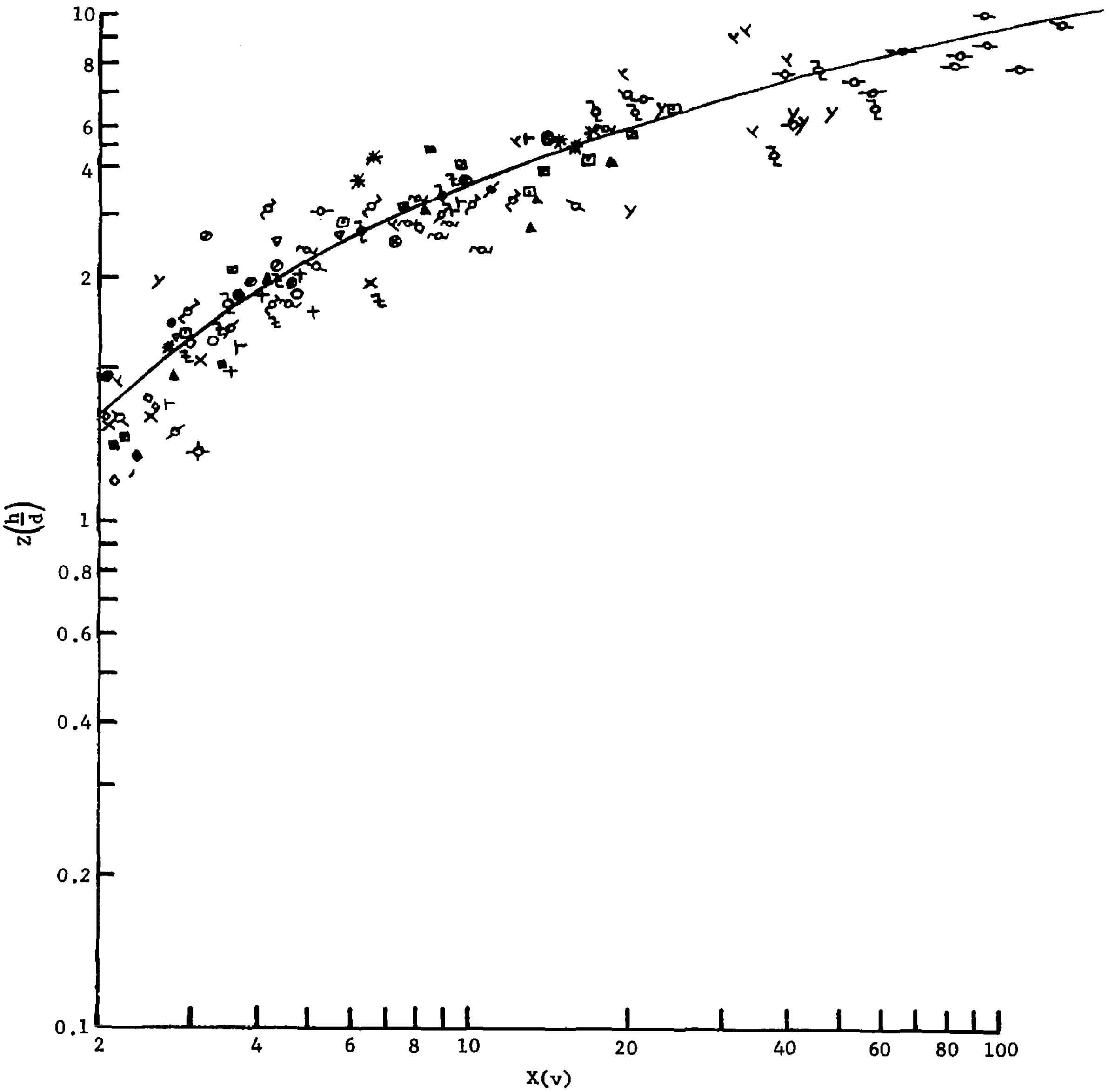

Fig. 1 Composite Variation of Crater Depth with Impact Velocity
(Shows Only Data Obtained at Martin-Denver)

II. CRATER DEPTH IN SEMI-INFINITE TARGETS

To determine the effect of material properties, a multivariable analysis was first performed on all the published data available up to 1965. They are tabulated in Appendix A and plotted in Fig. 2. (The data symbols are defined in Appendix A. Other symbols are defined in Table 2.) If the effect of material properties is not accounted for, it can be seen from Figure 2 that a considerable scatter will occur in the plotted data points.

The functional form that was found to bring all the penetration data from different materials into the closest agreement with each other was

$$Z\left(\frac{h}{d}\right) = \ell n\left[1 + (Xv)^2\right] \qquad [1]$$

where h is the crater depth, d the diameter of the spherical projectile that produced it, v the impact velocity in km/sec and ℓn is the natural logarithm. Z and X are functions of material properties. The degree of data correlation that can be achieved by this means is shown in Fig. 3. The measured values of Z and X required to bring about this correlation are given in Table 2.

Since the effects of impact velocity changes are contained in the form of Equation [1], the values of the parameters X and Z reflect only the effects of the properties of the different materials used as projectiles and targets in the different data sets. The measured values of X and Z were, therefore, subjected to separate regression analyses in terms of power laws in the materials properties variables of Table 1.

Table 1 Values of Parameters X and Z

Symbol	Projec-tile	Target	Measured Values		Calculated Values	
			X	Z	Eq [3] X	Eq [2] Z
+	2024T3	2024T4	.80	1.85	.67	1.63
×	Cu	2024T4	1.50	1.20	1.66	1.34
○	Pb	2024T4	2.00	1.60	1.68	1.19
✱	Stl	2024T4	2.00	1.70	1.70	1.42
◘	HPAL	HPAL	1.35	1.65	1.19	1.37
λ	HPCu	HPAL	2.60	1.00	3.50	1.62
△	2024T3	BeCu	.40	2.05	.42	2.93
⊥	2024T3	BeCu	.28	1.40	.39	1.88
⊢	2024T3	Cu	1.30	3.00	1.27	2.86
⊣	HPAL	HPCu	1.10	2.40	1.19	1.91
•	HPCu	HPCu	1.70	1.50	1.96	1.59
⊤	Cu(B65)	Cu(B36)	.65	2.01	3.28	2.27
⋈	Cu(B65)	Cu(B65)	2.10	1.90	1.78	1.67
■	Pb	Cu(B65)	2.00	1.30	1.80	1.56
▲	Stl	Cu(B65)	3.10	2.20	3.25	2.38
≺	2024T3	Pb	4.60	3.10	4.84	2.99
⋎	Cu(B65)	Pb	12.50	2.40	12.05	2.47
⌽	Pb	Pb	5.00	1.50	6.84	1.62
⊖	Stl	Pb	14.00	2.40	12.37	2.63
▽	2024T3	Stl	.78	3.00	.75	2.61
▼	2024T3	Stl (1030)	.46	2.00	.46	2.03
◇	Cu	Stl(1015)	1.10	1.50	1.05	1.51
◆	Pb	Stl	1.35	1.80	1.05	1.43
⌀	Stl	Stl	1.20	1.80	1.07	1.58
◐	2017T4	2024T4	.74	1.74	.72	1.60
‡	Zn	2024T4	11.50	2.90	12.02	3.21
∾	Zn	Cu(B65)	2.00	1.85	1.77	2.20
≀	Zn	Pb	11.50	2.90	12.02	3.21
♂	Zn	Stl(1015)	1.10	2.00	1.04	1.98
●	Zn	Zn	1.55	2.00	1.24	1.65
⊝	2024T3	Zn	.92	2.80	.89	2.47
▣	Cu(B65)	Zn	2.50	2.20	2.22	1.70
⌖	Pb	Zn	.90	.45	1.26	.76
▪	Stl	Zn	3.00	2.40	2.28	1.80

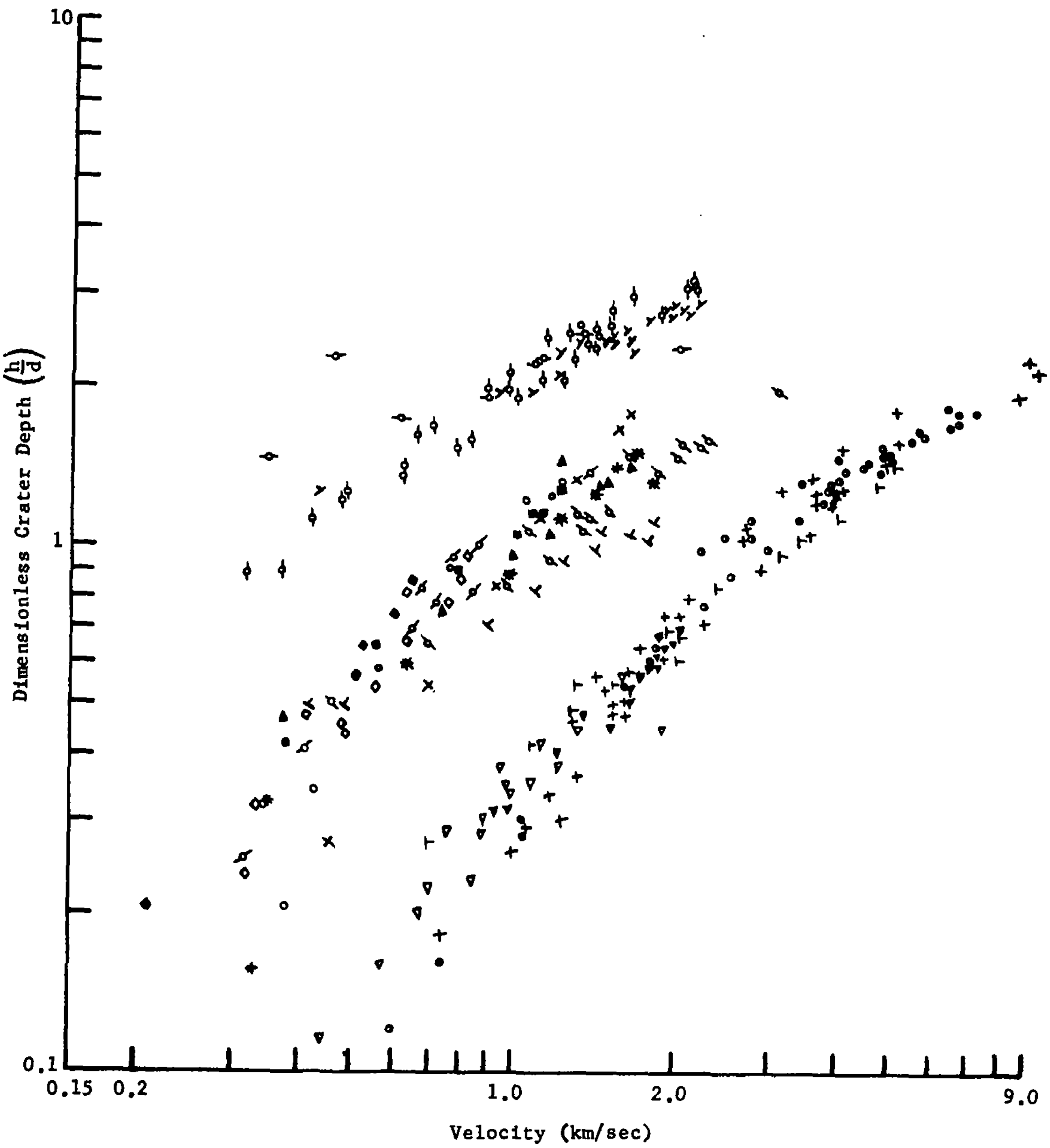

Fig. 2 Dependence of Crater Depth on Projectile Impact Velocity (Without Consideration of the Material Properties)

Table 2 List of Symbols

Experimental Variables

B	10^{-2} x Brinell hardness on 500-kg scale $(\mathrm{kg/mm^2})$
c	Mean specific heat of solid, 300°K to melting point (cal/gm°C)
C	Dilational velocity of sound (km/sec)
	$C = 9.90\sqrt{\frac{E}{\rho}\ t\ \frac{(1-\nu)}{(1+\nu)\ (1-2\nu)}}$
d	Projectile diameter (cm)
D	Crater diameter (cm)
E	10^{-6} x Young's modulus $(\mathrm{kg/cm^2})$
f	Shape factor { For a spherical projectile harder than the target, f = 2/3; For a spherical projectile not harder than the target, f = 1.0
F	10^{-2} x latent heat of fusion (cal/gm)
h	Crater depth (cm)
H	10^{-3} x heat to vaporize - sensible heats above 300°K plus latent heats (cal/gm)
H'	10^{-3} x latent heat of vaporization (cal/gm)
k	Thermal conductivity at 20°C (cal/cm sec°C)
K	10^{-6} x bulk modulus $(\mathrm{kg/cm^2}) = \frac{E}{3(1-2\nu)}$
K'	10^{-6} x dilational modulus $(\mathrm{kg/cm^2}) = \frac{E(1-\nu)}{(1+\nu)\ (1-2\nu)}$
m	Projectile mass (gm)
Q	10^{-2} x heat to melt - sensible heat above 300°K plus latent heat of fusion (cal/gm)
U	10^{-3} x ultimate strength $(\mathrm{kg/cm^2})$
v	Projectile velocity at impact (km/sec)

V	Crater volume (cc)
X	Velocity correction factor (sec/km), see Equation [3]
Y	10^{-3} x yield strength at 0.2% offset $\left(kg/cm^2\right)$
Z	Crater depth correction factor, see Equation [2]
BL	Ballistic limit - velocity at which the target is just punctured (km/sec)
Δ	Acoustical impedance ratio $\left(\sqrt{E_t \rho_t} + \sqrt{E_p \rho_p}\right) / \sqrt{E_t \rho_t}$ (Ref 3)
ϵ	Ductility - % elongation in 2-in. gage length at fracture
ρ	Density (gm/cc)
ν	Poisson's ratio
π_1	Dimensionless crater depth (h/d)

<u>Subscripts</u>

t	Target
p	Projectile, punctured

<u>Abbreviations</u>

$\longrightarrow$	Impacting
Proj	Projectile
Targ	Target
CP	Commercial purity
CR	Cold rolled
Stl	Steel
AISI	American Institute of Steel Industries
HP	High purity
VHN	Vickers hardness number

Standard Chemical Symbols and Alloy Designations

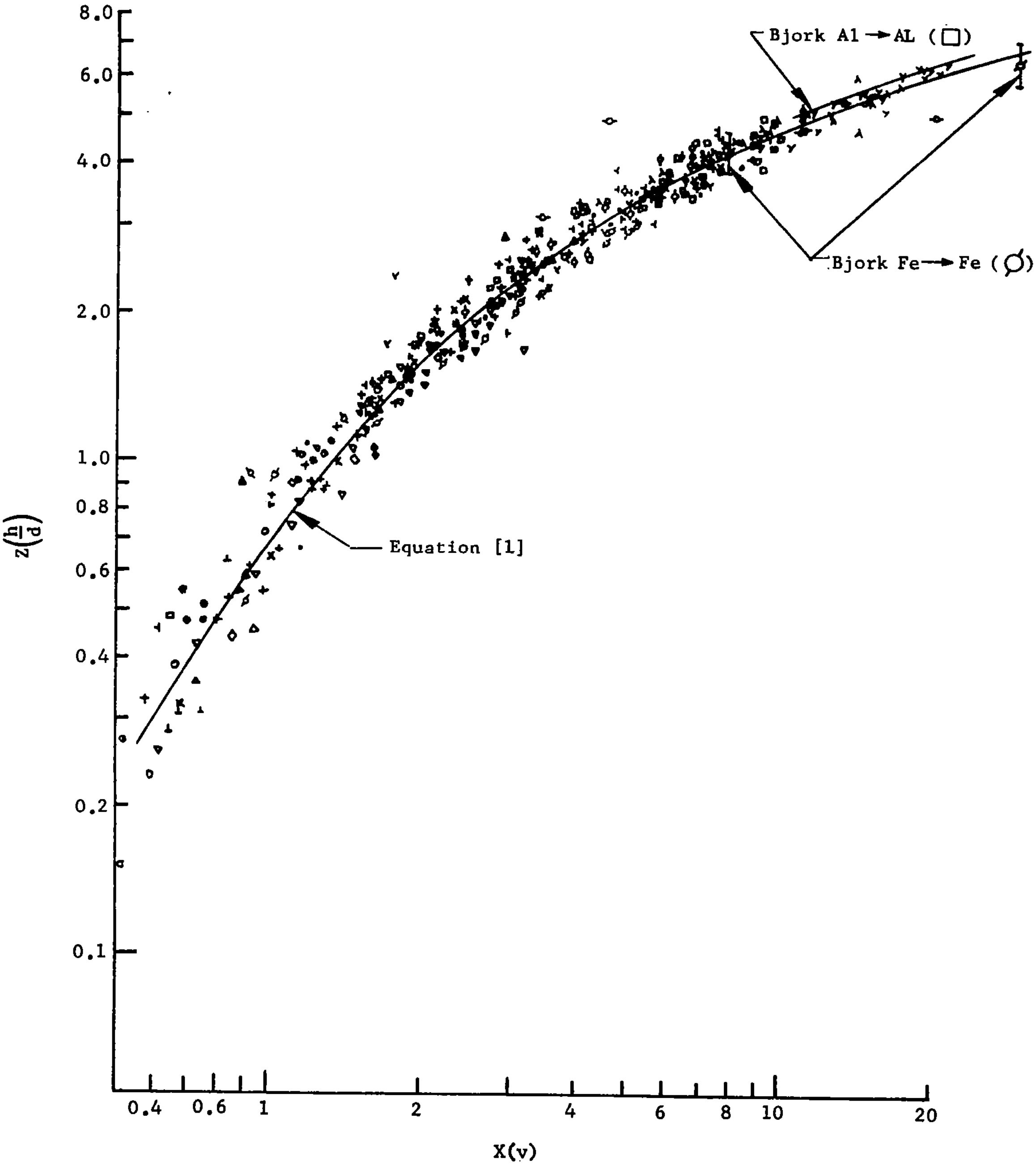

Fig. 3 Composite Variation of Crater Depth with Velocity

A preliminary analysis was made to eliminate many of the material properties which either were statistically unrelated to the values of X or Z, or were strongly related to other materials properties. The surviving candidate properties were grouped into dimensionless ratios for convenience in use, and a final statistical analysis was made. In this process, care was taken to ensure that X acquired the inverse dimensions of velocity so that the product (Xv) should be a dimensionless velocity term. $\frac{h}{d}$ was already dimensionless so that Z had only to be also dimensionless. The resulting equations were:

Coefficient of
Multiple Correlation

$$Z = 1.131\left(\frac{Q_p}{Q_t}\right)^{-0.185}\left(\frac{\rho_p}{\rho_t}\right)^{-0.36}\left(\frac{E_t}{B_t}\right)^{0.073}\left(\frac{E_p}{E_t}\right)^{0.149}\left(\frac{Y_t}{B_t}\right)^{0.132}\epsilon_p^{-0.053}\, f^{-0.755} \quad [2]$$

Coefficient of multiple correlation = 0.842

$$X = 0.479\left(\frac{\rho_t}{E_t}\right)^{0.50}\left(\frac{\rho_p}{\rho_t}\right)^{0.472}\left(\frac{E_\rho}{B_\rho}\right)^{0.406}\left(\frac{B_p}{B_t}\right)^{0.130}\left(\frac{Y_t}{B_t}\right)^{0.253}\epsilon_t^{0.216}\, f^{-1.44} \quad [3]$$

Coefficient of multiple correlation = 0.956

The symbols are defined in Table 2. The values of X and Z calculated from these equations are also given in Table 1. Note that the coefficients of multiple correlation are 0.842 and 0.956. A value of 1.0 would represent perfect correlation.

Figures 1 and 3 from which the measured values of X and Z were taken represent ideal correlations in the sense that no better correlations were found to be possible.

Because of the paucity of many of the data sets, and the short ranges of impact velocity in some, a small experimental program was planned and executed to fill out these sets and to provide a check on the predictive ability of the preceding analytical results. Figure 1 shows the results of this program. Values of X and Z were measured for these data also and are given in Table 2. Figures 4 and 4a represent the correlations actually achieved for the same groups of data using the calculated values of X and Z, except that projectile and target combinations involving zinc are omitted from Figure 4a and shown separately in Figure 4b.

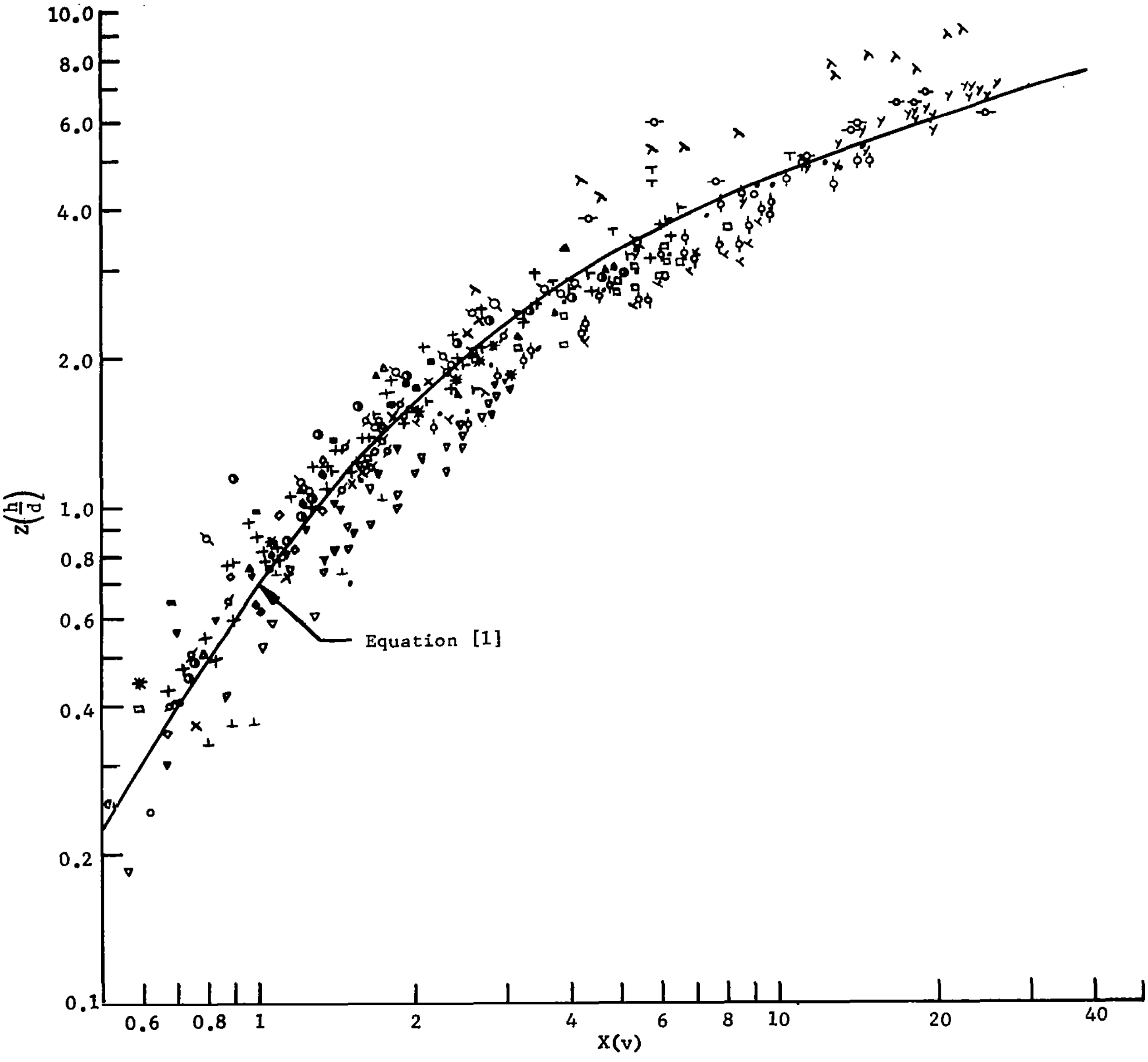

Fig. 4 Comparison of Predicted Crater Depth with Experimental Results Using Equations [1], [2], and [3]

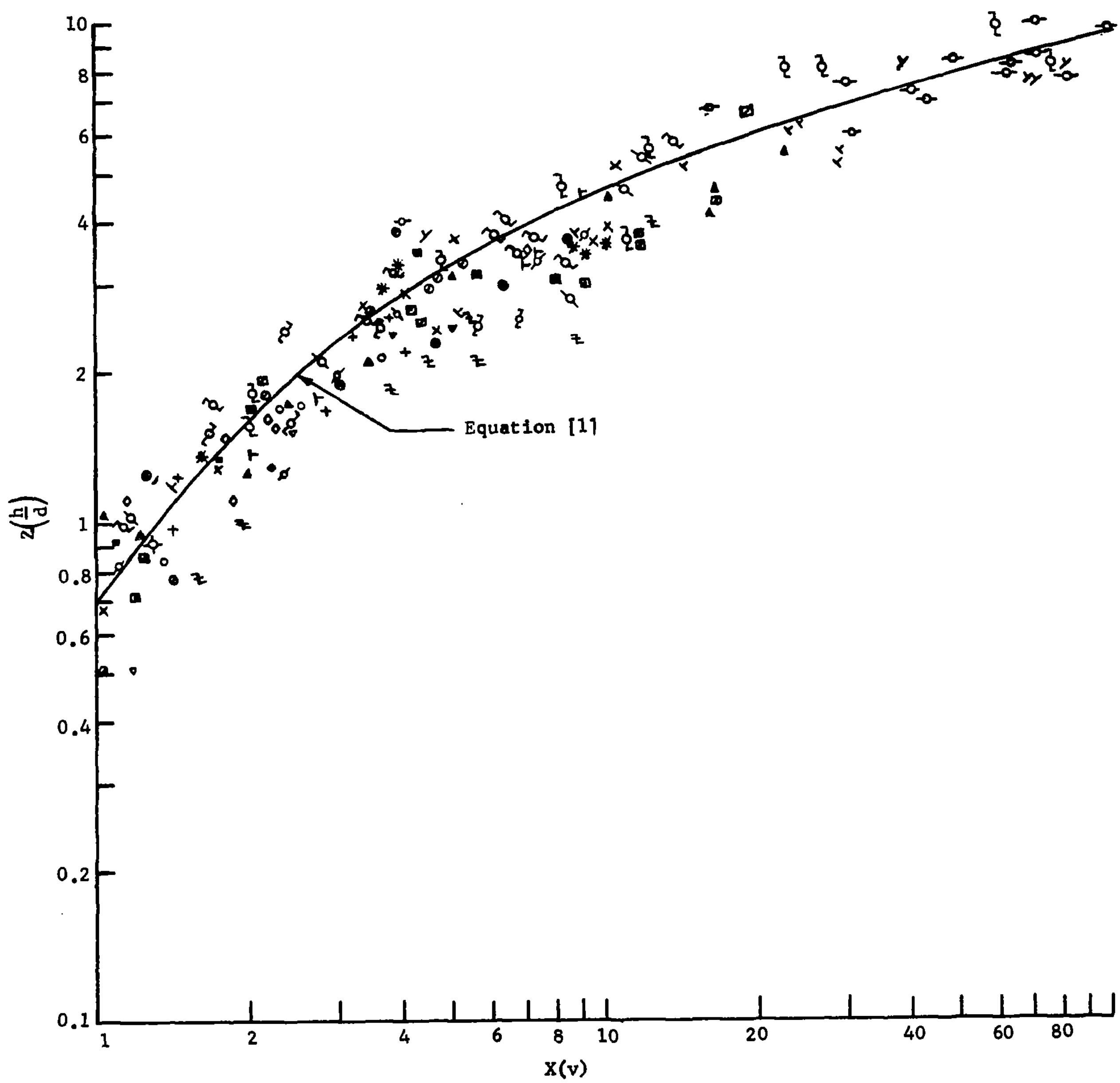

Fig. 4a Comparison of Predicted Crater Depth with Experimental Results Using Equations [1], [2], and [3] (Shows only Data Obtained at Martin-Denver)

As shown in Appendix B, all projectile and target material combinations in the experimental program included in this study were chosen so as to fill in or extend already existing data sets, with the exception of combinations involving zinc. This material was chosen to resolve a question that arose during the analyses of X and Z.

In these analyses, the Poisson's ratios of the projectile and target materials appeared as rather strong variables. This was believed to be spurious because Poisson's ratio was strongly cross-correlated with other properties, and because out of a range of 0 to 0.50 in nature, only the range 0.30 to 0.45 was represented in the available data, with a vast preponderance of values of 0.33. All of these facts tend to degrade the statistical validity of any conclusions regarding Poisson's ratio. Accordingly, Poisson's ratio was artificially suppressed in favor of other variables strongly related to it, in arriving at Equations [2] and [3].

For this reason, it was desirable to include in the experimental program some material with high or low Poisson's ratio. Beryllium with a Poisson's ratio of about 0.03 was originally considered but was ruled out on the basis of cost and toxicity. Zinc with a Poisson's ratio of about 0.43 was used instead.

Experimental data obtained in this program with zinc as either target or projectile material, or both, are well represented by Equations (1), (2), and (3), (as shown in Figure 4a) without reference to Poisson's ratio. No improvement in the multiple correlation coefficient could be obtained by including Poisson's ratio in Equations (2) and (3), so it was concluded that the early indications were in fact spurious, as originally assumed.

At the time of the inception of this study it was widely believed, or at least hoped, that crater depth due to impact by spherical projectiles was proportional to impact velocity raised to some power. However, several values for this exponent, based on more or less rational grounds, were suggested by various investigators, and supported by data. A value of 4/3 has long been used by ordnance researchers to predict penetration of steel armor. Summers (Ref 4) found a value of 2/3 for his data and suggested a direct proportionality between crater volume and projectile kinetic energy on this basis. Bjork (Ref 3), on the basis of his hydrodynamic calculations, suggested a value of 1/3, as have others, primarily those who used lead targets. It was suggested that this indicated a dependence of crater volume on projectile momentum.

It was not until the excellent experimental work of Goodman and Liles (Ref 5) was published, near the end of the first year of this study, that it became possible to unify these divergent viewpoints and to begin to approach a general correlation of all the then available data on the basis of differences in material properties. Figure 5 shows the data from Ref 5 for aluminum projectiles (◘) and copper projectiles (λ) on aluminum targets, as well as several other selected sets of data including steel-on-steel (ø).

Figure 5 has been extracted for clarity from the much more general Fig. 3, which contains all the data collected from the literature through 1965. These data are tabulated in Appendix A. By virtue of the logarithmic coordinates in which the figures are plotted, a power law in velocity is represented by a straight line with a slope equal to the velocity exponent. In Fig. 5, lines have been constructed through the data, with slopes corresponding to velocity exponents of 4/3, 2/3, and 1/3, as marked. Each of these lines fits the data over a fairly wide range of velocity and it is easy to see how different investigators arrived at different exponents, depending on their choices of projectile and target materials, and the impact velocities they could obtain. In particular, it is of interest to note how well the steel-on-steel data (ø) fits the n = 4/3 line, and in Fig. 3 how well Bjork's hydrodynamic prediction fits the data for pure alumimum projectiles and targets (◘) to which it pertains.

However, the main feature of interest in the two figures is the fact that while the data over a very wide range of impact velocity cannot adequately be represented by one exponent (straight line), it can be closely fitted by a single curve, the functional form of which is given by Equation [1]. It is another feature of the logarithmic coordinate system used in these figures that simply by multiplying the abscissas and the ordinates of a group of data by X and Z, the whole group can be translated horizontally

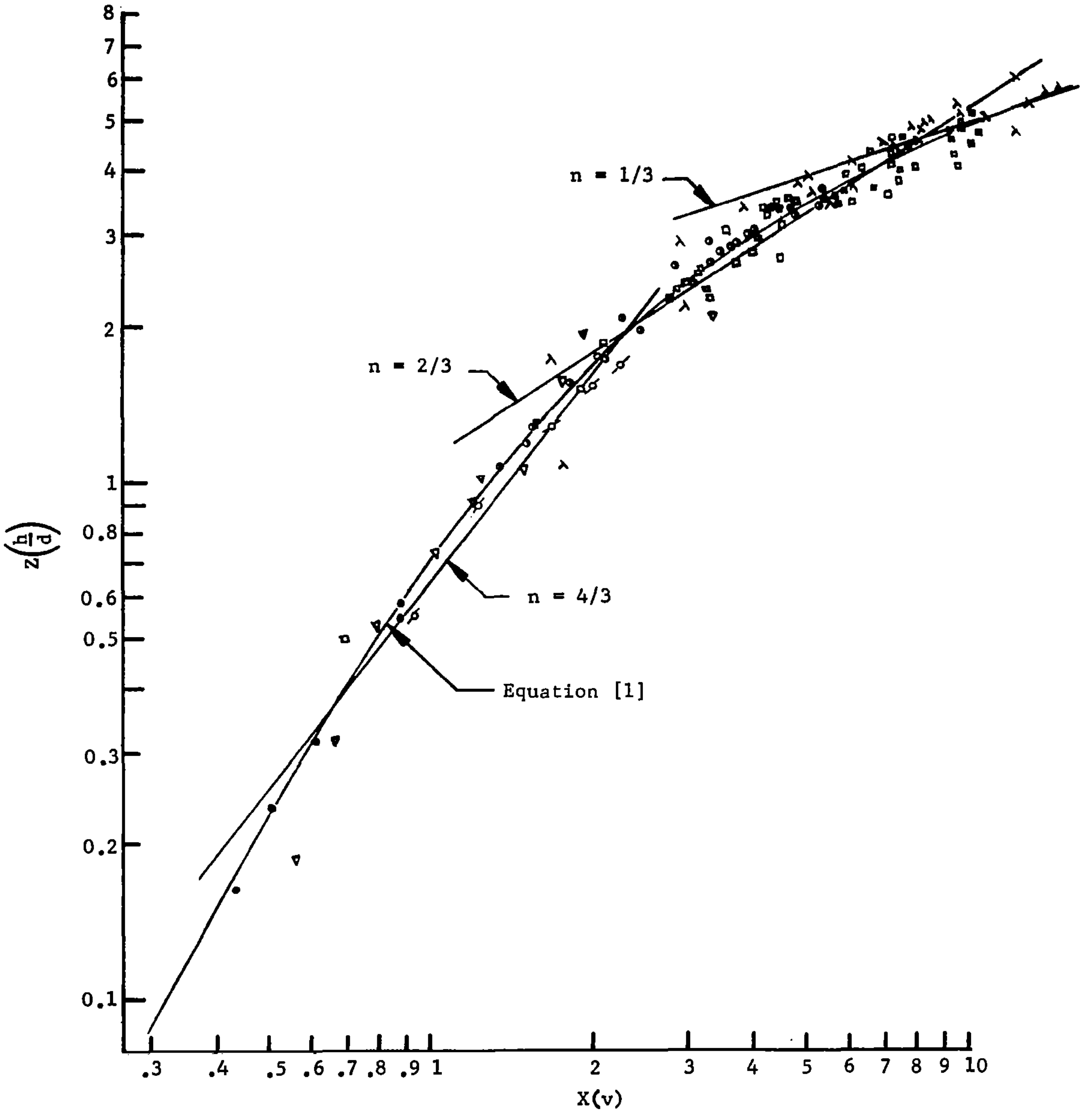

Fig. 5 Composite Variation of Crater Depth with Velocity

and vertically without distorting the pattern of the group, to any desired position. The line marked "Equation [1]" was calculated for X = Z = 1.0. The correlations of Fig. 3 and 5 were obtained by plotting individual data sets on transparent overlays, adjusting them to the best visual fit to this curve and measuring the values of X and Z that produced this fit. These measured values are tabulated in Table 2, and are analyzed to determine their dependence on material properties.

It is of considerable historical interest to note that when Equation [3] is written in the form of a dimensionless velocity it becomes

$$Xv = 0.479 \left(\frac{\rho_t v^2}{E_t}\right)^{0.500} \left(\frac{Y_t}{B_t}\right)^{0.253} \left(\frac{\rho_p}{B_t}\right)^{0.472} \left(\frac{E_p}{B_p}\right)^{0.406} \left(\frac{B_p}{B_t}\right)^{0.130} \epsilon_t^{0.216} f^{-1.44} \quad [3$$

which is a compromise between accuracy and complexity. Longer forms of this equation containing the ratios $\left(\frac{E_t}{B_t}\right)$ and $\left(\frac{\rho_t Q_t}{E_t}\right)$ were discarded because the additional complexity was not warranted by the small increase in goodness of fit.

It is apparent that in an equation similar to Equation [3] which contains the extra terms given above, the first term after the constant could, by algebraic rearrangement, be made to contain the "Best number"

$$\left(\frac{\rho_t v^2}{B_t}\right) \text{ (References 2, 7, 8),}$$

an unnamed ratio $\left(\frac{\rho_t v^2}{Y_t}\right)$ (Reference 9), or Whipple's thermal parameter $\left(\frac{v^2}{Q_t}\right)$ (Reference 7) instead of the "Mach number" $\left(\frac{\rho_t v^2}{E_t}\right)$ (References 4, 6).

Because these quantities are interrelated in the metallic materials considered, each of them has been used with some success in correlating selected data sets, in the references noted. However, no one of them has produced the high degree of general correlation obtained with Equation [3] which contains additional material parameters.

The above discussion appears to provide a sufficient explanation for the confusion regarding dimensionless velocities which prevades the literature.

III. CRATER VOLUME IN SEMI-INFINITE TARGETS

The purpose of this investigation was to define the dependence of crater volume on material properties, using a similar multivariable analysis on experimental data for the hypervelocity impact of spherical projectiles on semi-infinite targets. (see Appendix D).

The problem of crater volume is beset with many of the same difficulties as that of crater depth, plus a few of its own. The problem of scarcity and spottiness of available data is even more severe since the well-documented data sets are the same and the volume data are not reported for all of these. The special problems center around the considerable experimental error in the data that may effectively disguise any natural trends in sensitive parameters and limits analysis to those parameters whose trends are not completely obscured.

The volume of a crater is obviously dependent upon two factors; a typical dimension, and its shape. These are essentially independent since craters of a given depth may be either wide or narrow at the surface, and vary from conical to cylindrical in cross-section. Ideally then, three parameters are required to describe a given crater; one to specify a typical dimension, one to describe its relative depth, and one to describe its cross-sectional shape.

Consider the shape parameter first. For a crater of volume V having depth h, and surface diameter D, caused by a spherical projectile of diameter d, the parameter $\frac{V}{D^2h}$ is an indicator of the kind of cross-sectional shape involved. The shapes of interest are:

Shape	Cone	Paraboloid	Ellipsoid	Cylinder	Square Box
$\frac{V}{D^2h}$	$\frac{\pi}{12} = 0.262$	$\frac{\pi}{8} = 0.393$	$\frac{\pi}{6} = 0.524$	$\frac{\pi}{4} = 0.785$	1.00

Relative depth is well described by the ratio $\left(\frac{h}{D}\right)$. Given a shape factor $\frac{V}{D^2h}$ and a relative depth $\left(\frac{h}{D}\right)$, the volume of a crater is uniquely determined by either depth, h, or dimensionless depth, $\frac{h}{d}$.

Two useful parametric relationships that may be verified simply by reduction to simplest terms (leading to the identity $V \equiv V$) are:

$$\frac{V}{h^3} = \frac{V}{D^2h} \Big/ \left(\frac{h}{D}\right)^2 \qquad [4]$$

$$\frac{V}{d^3} = \frac{V}{h^3} \quad \left(\frac{h}{d}\right)^3 \qquad [5]$$

It is readily apparent from Equation [4] that the dimensionless volume $\frac{V}{h^3}$ is a sort of composite shape parameter since it can be expressed in terms of the shape parameter $\frac{V}{D^2h}$ and the relative depth $\frac{h}{D}$. Obviously the analysis would be more significant and perhaps more revealing if $\frac{V}{D^2h}$ and $\frac{h}{D}$ could be independently formulated in terms of the impact velocity and the properties of the projectile and target materials. This might mean, for instance, that $\frac{h}{D}$ changed while $\frac{V}{D^2h}$ remained constant, defining perhaps a family of ellipsoids varying from prolate to oblate. If $\frac{V}{D^2h}$ were found to vary while $\frac{h}{D}$ remained constant, a variety of shapes from cones to cylinders would be indicated, all within the same envelope defined by h and D.

However, one might intuitively tend to suspect that the shape parameter and depth parameter are not entirely independent of each other, and that shallow craters might tend to approach a conical shape, while deep ones might approach a cylindrical shape. If such a dependence could be shown to exist, one or the other of these parameters could be eliminated from Equation [4].

From the literature on hypervelocity cratering one would be led strongly to expect to find that all craters were special ellipsoids

$\frac{V}{D^2h} = \frac{\pi}{6} = 0.524$ such that $\frac{h}{D} = 0.50$, that is, hemispherical. If this were, in fact, the case, the volume problem would be solved automatically as soon as the crater depth problem were solved.

Unfortunately, the data seem to indicate that each of these three ideal situations occurs for some materials some of the time. Figure 6 shows a plot of $\frac{V}{D^2h}$ vs $\frac{h}{D}$. If these were truly independent of each other, a random scatter of data over the figure would be expected. The data are certainly scattered enough, but are visibly not random. A definite clustering around the point indicating hemisphericity is shown, particularly in the data set for high purity aluminum impacting the same material, symbolized by □. This is of interest with reference to Bjork's calculations for pure aluminum impacting the same material. His calculations have been shown to fit the crater depth data for this material quite well (see Fig. 3) and Fig. 6 shows that the hemisphericity predicted by Bjork (Ref 8) is borne out by the same data.

If shallow craters tended to be conical, and deep ones cylindrical, there would be a distinct trend in all the data to run from lower left to upper right in Fig. 6. Such a trend is clearly apparent in the symbols λ and ⊣ (Line A).

Figure 6 permits the following observations, based on the available data:

1) The shape and deepness parameters are not generally independent of each other;

2) An explicit dependence between them is not observable;

3) A tendency to hemisphericity is noticeable, but not general.

For these reasons, an analysis based on Equation [4] could not be accomplished. An attempt was made based on Equation [5], which was found to be less sensitive to experimental scatter and hidden relationships. Figure 7 shows the relative sensitivity of the parameter $\frac{V}{h^3}$ compared to $\frac{V}{D^2h}$ in Fig. 6. The four lines representing the data in Fig. 6, either as boundaries or means, are

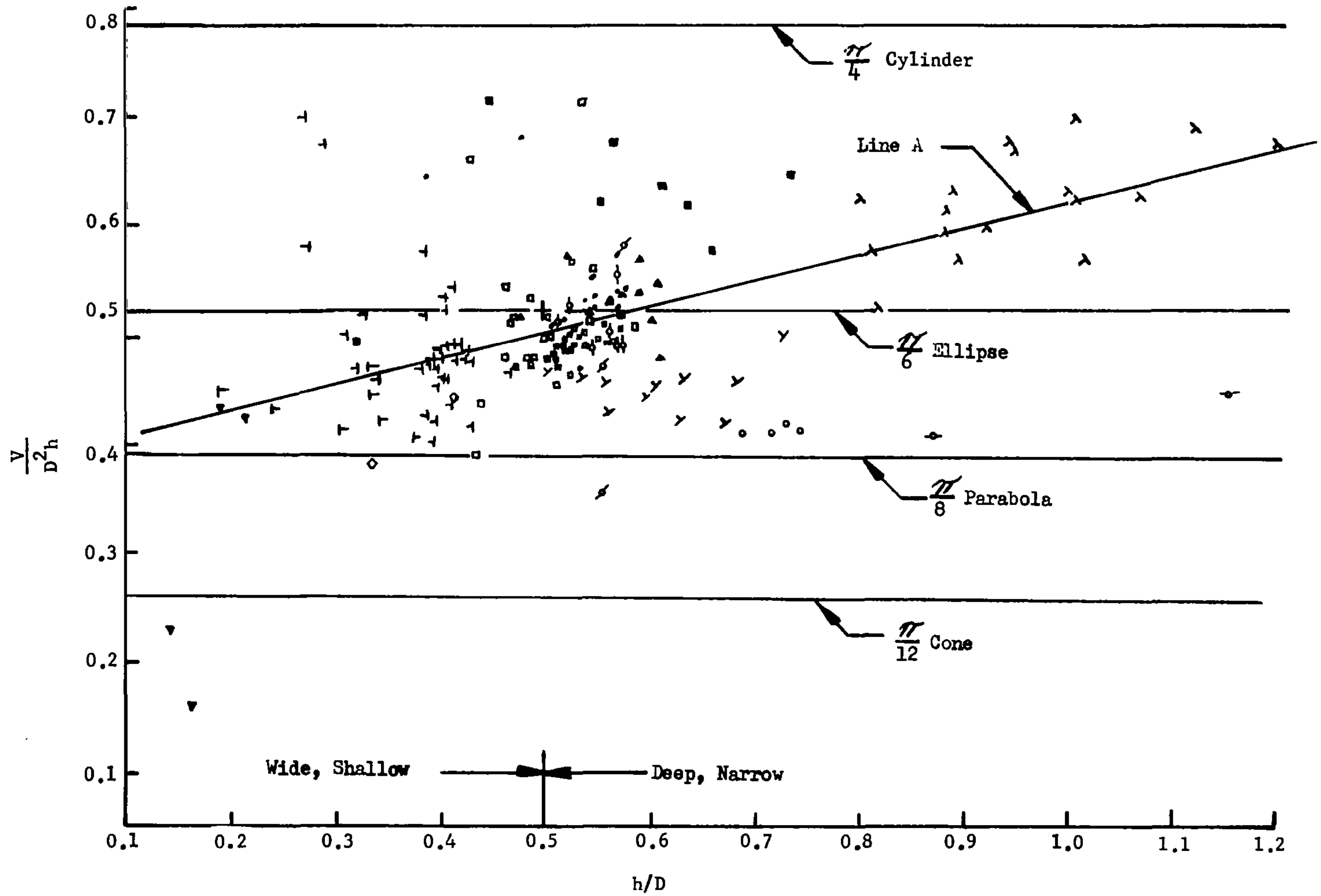

Fig. 6 Shape Parameter vs Depth Parameter

shown in Fig. 7. Obviously, the variation of $\frac{V}{h^3}$ compared to its range is much less than the variation of $\frac{V}{D^2h}$ compared to its range, so that experimental scatter does not obscure the trends of the data.

It was found that by plotting the volume parameter $\frac{V}{d^3}$ versus the size parameter, $\frac{h}{d}$, the composite shape parameter $\frac{V}{h^3}$ could be observed. Writing Equation [5] in the general form

$$\frac{V}{d^3} = A\left(\frac{h}{d}\right)^n \qquad [5]$$

The justification of the above choice for a regression surface is shown in Fig. 8 where all the data fall on straight lines on a log-log plot. For hemispherical craters, Equation [5] takes the form shown in Equation [5a].

$$\frac{V}{d^3} = \frac{2\pi}{3}\left(\frac{h}{d}\right)^3 = 2.094\left(\frac{h}{d}\right)^3 \qquad [5a]$$

The general shape of crater for a given data set can be inferred by seeing whether the data fall above or below the line for a hemisphere. Whether or not the shape remains the same as crater depth changes can be inferred from the exponent n, as will be explained presently.

In Fig. 8, all the available crater volume data are shown. In the first part of this figure, only the symbol ▼ and the left hemisphere line pertain to the left-hand vertical scale. Only the symbols ⊣ and ♦ and the right hemisphere line pertain to the right-hand scale. All other data points and reference lines pertain to the middle scale.

In particular, Figure 8 shows data falling above the hemisphere line, indicating more volume for a given depth than a hemisphere, hence implying a larger diameter, which describes a relatively shallow crater. This usually implies a tendency to approach a conical shape. Data points falling below the hemisphere line then tend to be deep and narrow and to approach cylindrical shape.

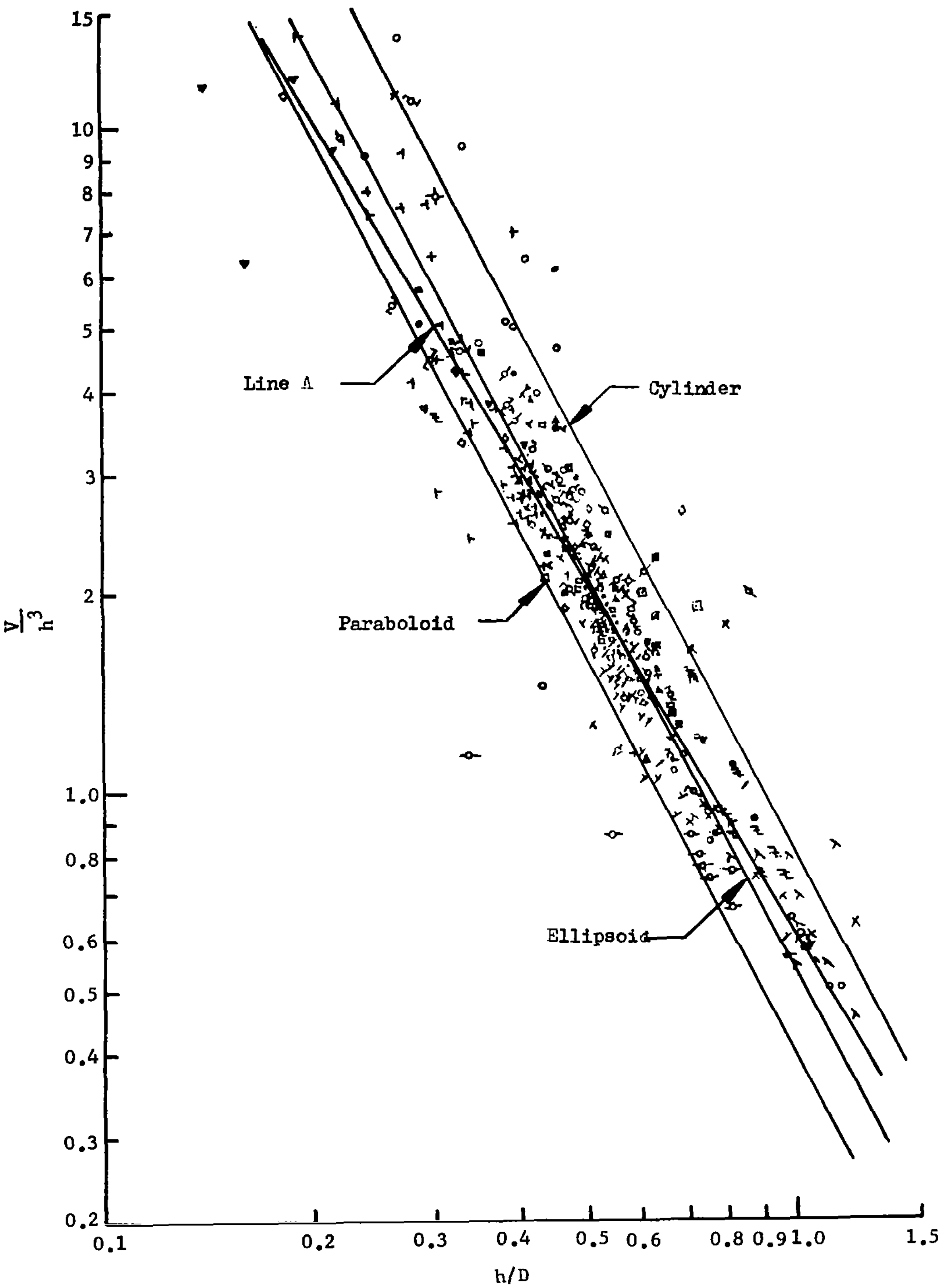

Fig. 7 Composite Shape Parameter vs Depth Parameter

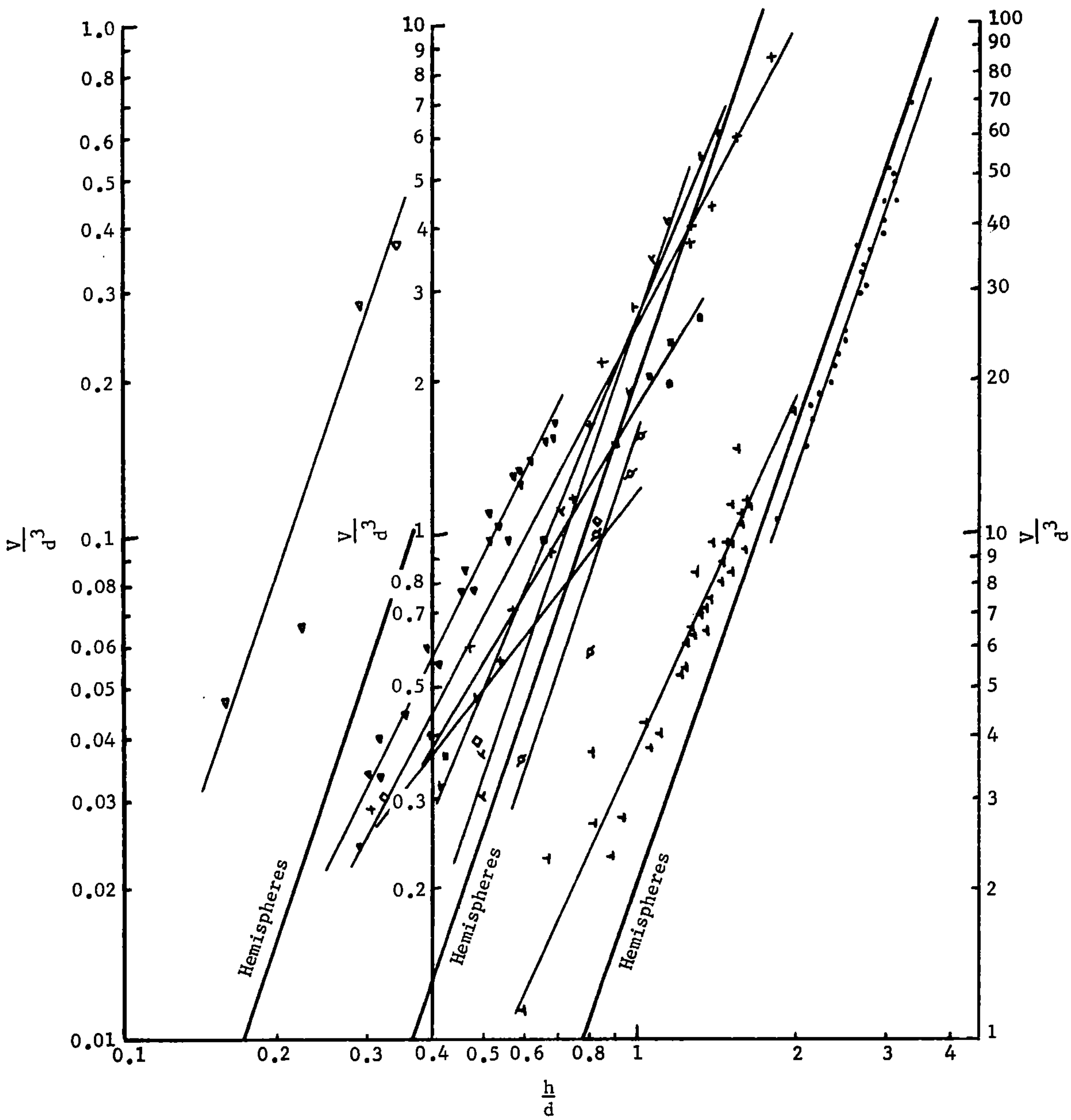

Fig. 8 Crater Volume vs Crater Depth

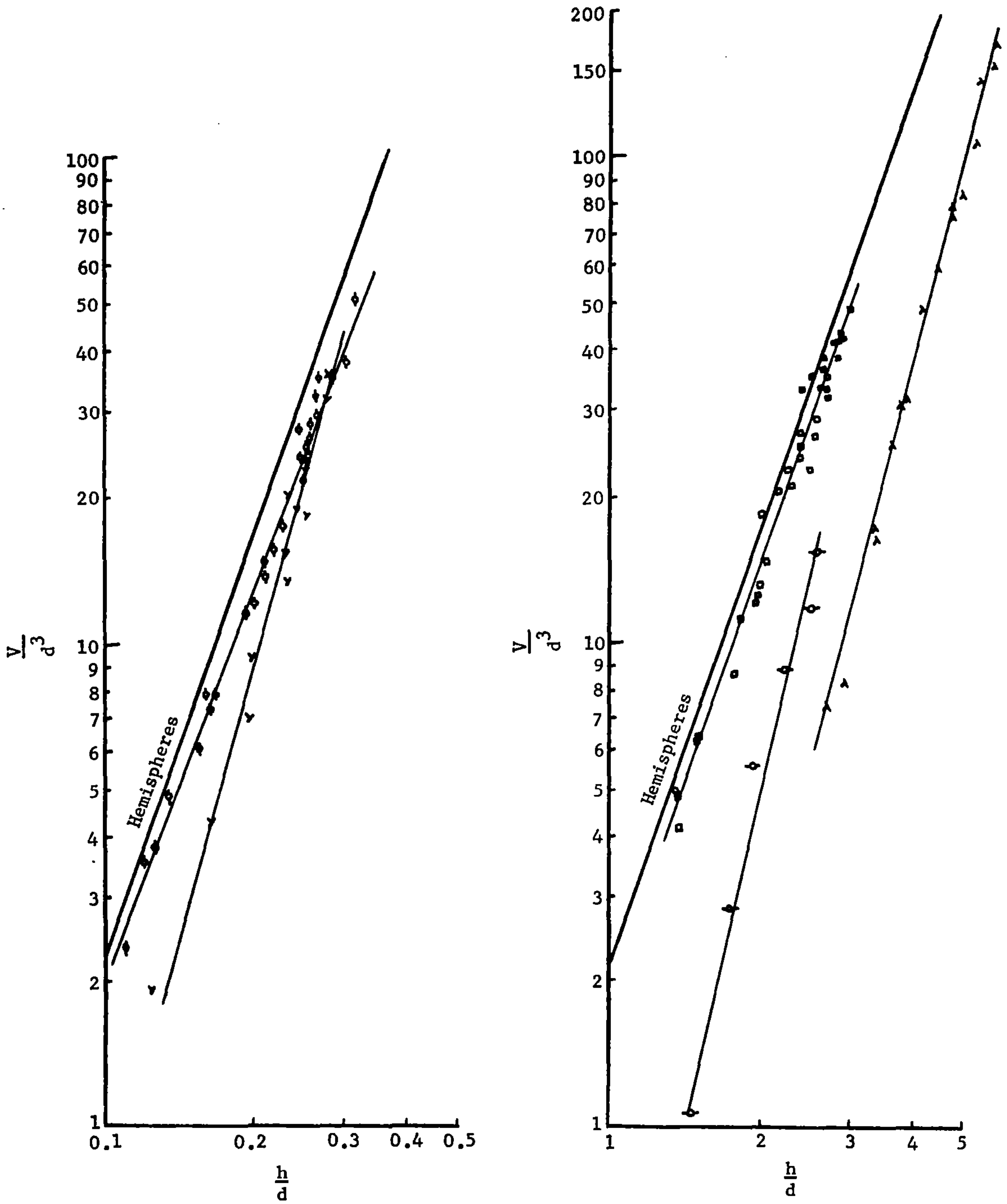
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0.1
0.2
0.3
0.4
0.5
V/d³
h/d
Hemispheres
200
150
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
V/d³
h/d
Hemispheres

Fig. 8 (concl)

Data that follow an exponent greater than 3.0 tend to become wider and shallower with increasing size, while for an exponent less than 3.0, they tend to become narrower and deeper. An exponent of 3.0 indicates a constant crater shape with increasing size, regardless of what the shape is.

The values of A and n in Equation [5] were measured graphically from Fig. 7, 8 and 9. The measured values are shown in Table 3.

The values of A and n were analyzed independently to determine their dependence on material properties. The following regression equations developed:

$$A = 2.14 \left(\frac{\rho_p}{\rho_t}\right)^{-0.70} \left(\frac{E_p}{B_p}\right)^{0.159} \left(\frac{E_t}{B_t}\right)^{-0.105} \epsilon_t^{-0.426} f^{0.729} \Delta^{-0.952} \qquad [6]$$

Coefficient of multiple correlation = 0.95

$$n = 2.00 \left(\frac{\rho_Q}{E}\right)_t^{0.13} \left(\frac{\rho_p}{\rho_t}\right)^{0.181} \left(\frac{E_t}{B_t}\right)^{0.116} \left(\frac{E_p}{E_t}\right)^{0.148} f^{-0.137} \Delta^{-0.161} \qquad [7]$$

Coefficient of multiple correlation = 0.95

Calculated values of A and n are also given in Table 3.

The conclusions which may be drawn from the existing data are reflected in Table 3. If all craters were, in fact, hemispherical, all values of A would be 2.094, and all values of n would be 3.00. There is a general tendency to approach hemispheres as the impact velocity (and h/d) increases, as shown by low values of A accompanied by high values of n and vice versa, but this trend is by no means universal. A number of low values of A are accompanied by values of n = 3.00, indicating a tendency for craters to be narrow and deep regardless of impact velocity. It is, however, not necessarily true that these conclusions will be applicable for larger projectiles.

As a general conclusion, then, it may be stated that the applicability of any approximate theory that assumes hemispherical crater shape is sharply limited.

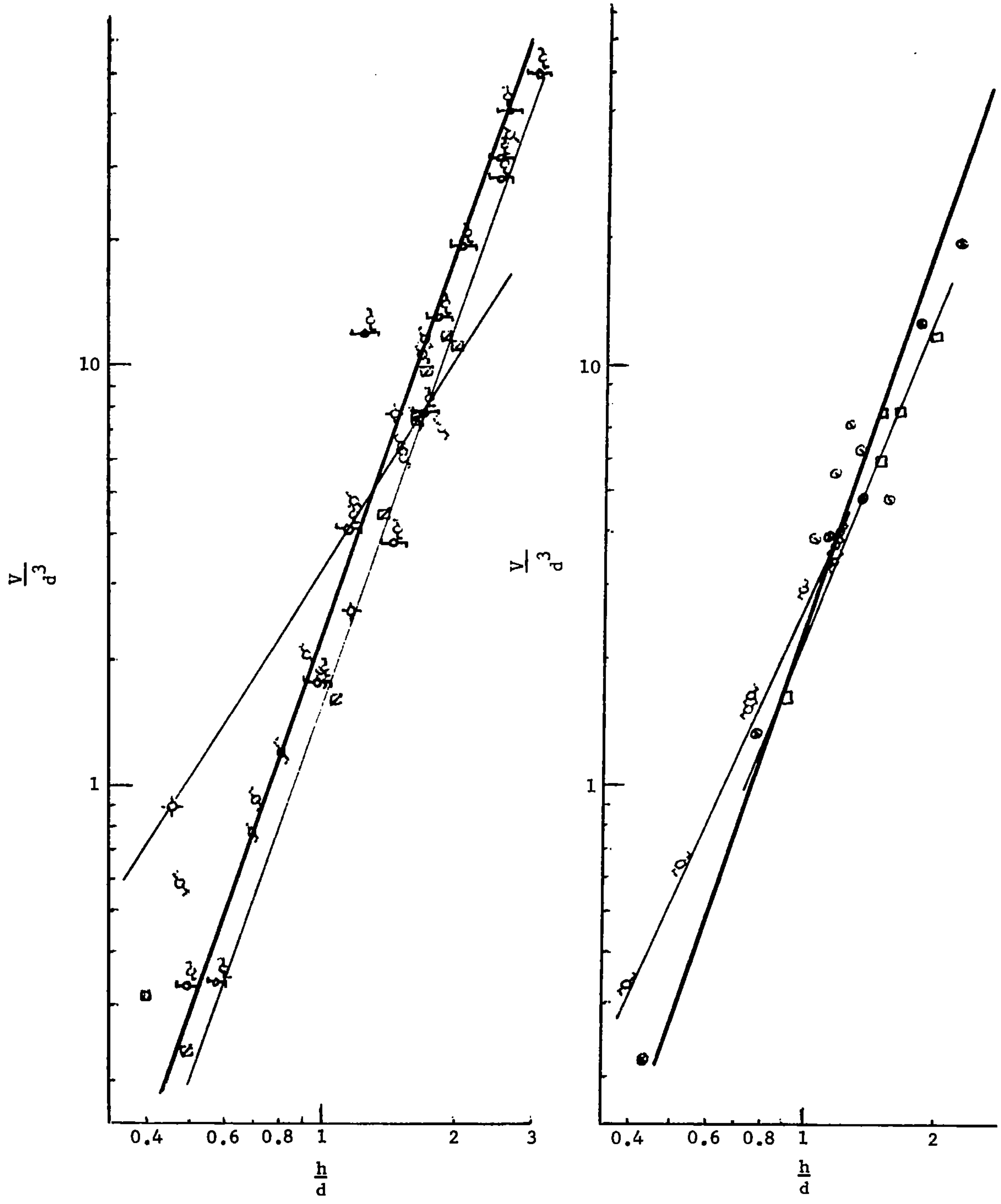

Fig. 9 Zinc Combinations Only

Table 3 Values of Parameters A and n

Symbol	Materials Projectile	Target	Measured Values		Calculated Values	
			A	n	Eq [17] A	Eq [18] n
+	2024T3	2024T4	2.50	2.20	2.35	2.26
✳	StL	2024T4	.84	2.57	.73	2.87
×	CPCu	2024T4	.64	3.10	.77	2.77
○	Pb	2024T4	1.26	2.30	1.24	2.30
⊢	2024T3	Cu	3.00	2.51	3.28	2.22
	Zn	Cu	2.25	2.50	2.32	2.51
	Cu	Cu	2.30	2.63	1.69	2.63
■	Pb	Cu	1.85	1.84	2.26	2.13
	2024T3	Pb	2.20	2.74	2.30	2.98
	StL	Pb	.41	4.11	.69	3.76
	Cu	Pb	.80	3.58	.74	3.63
	Pb	Pb	2.05	2.72	1.63	2.87
▽	2024T3	StL	3.50	2.16	3.45	2.10
	Zn	StL	2.60	2.28	2.51	2.39
◇	Cu	StL	2.40	2.56	1.84	2.52
	StL	StL	2.00	2.60	1.84	2.64
▲	StL	Cu	1.70	2.90	1.24	2.91
	Zn	2024T4	1.60	2.17	1.90	2.15
◆	Pb	StL	2.50	2.10	2.36	2.02
⊘	2024T3	Zn	3.00	1.98	2.53	1.98
	Zn	Pb	1.50	3.20	1.06	3.47
⊗	Zn	Zn	2.40	2.40	3.12	2.46
⧄	StL	Zn	1.70	2.80	1.63	2.85
⊡	Cu	Zn	1.70	2.69	1.68	2.73
⌖	Pb	Zn	3.25	2.14	3.14	2.10
⊣	HPAL	HPCu	3.90	2.24	5.05	2.13
•	HPCu	HPCu	1.70	2.91	1.46	2.72
□	HPAL	HPAL	1.55	3.00	1.57	2.83
	HPCu	HPAL	.21	3.80	.28	3.71
▼	2024T3	StL	3.75	1.97	3.65	2.06
	CPAL	CPAL	2.30	2.77	2.29	2.63

IV. THIN AND BUMPERED TARGETS

The analysis of the available data for finite and multiple wall targets required an extension of the techniques employed in the crater depth and crater volume problems, in the sense that a great deal more work had to be done on the data before the data could be subjected to regression analysis to determine the effects of material properties.

As in the crater depth problem, the depth of damage was found to depend on impact velocity in such a way that its functional representation did not lend itself to linear regression analysis, so that this effect had to be removed from the data.

Furthermore, as in the crater volume problem, geometric variables influenced the data in ways that also could not be handled by linear regression analysis so that these effects also had to be removed.

Figure 10 shows all the geometric variables of the case of normal impact of a spherical projectile upon a bumpered target of arbitrary thickness. When it is considered that a general case might involve three different materials in the projectile, bumper, and second sheet or target, and that depth of damage includes a hole in the bumper and a crater in the target, it becomes apparent that some refinement is necessary to permit a sufficiently concise statement of the problem so that an analysis can be made.

The first step in reducing the complexity of the problem is the elimination of one of the three materials which may be involved. This was accomplished by means of the well-attested "mass concept" which simply states that, at least to a first approximation, the effectiveness of a bumper depends only upon its density-thickness product (Ref 11 and 12). Accordingly, all the data used herein (Ref 11 and 13) have had the actual bumper thickness, of whatever material, converted to equivalent thickness of 2024-T3 by the ratio of the density of the actual material to that of 2024-T3 which is 2.79 gm/cc, since all the targets were made of this material or 7075-T6 aluminum. Any effects due to the minor differences between these two aircraft structural aluminum alloys, primarily hardness and strength, have been neglected. This leaves only the projectile and target materials to be considered.

The second step is the selection of a measure of damage to be considered as the dependent variable. The total penetration, consisting of the equivalent bumper thickness plus the crater depth

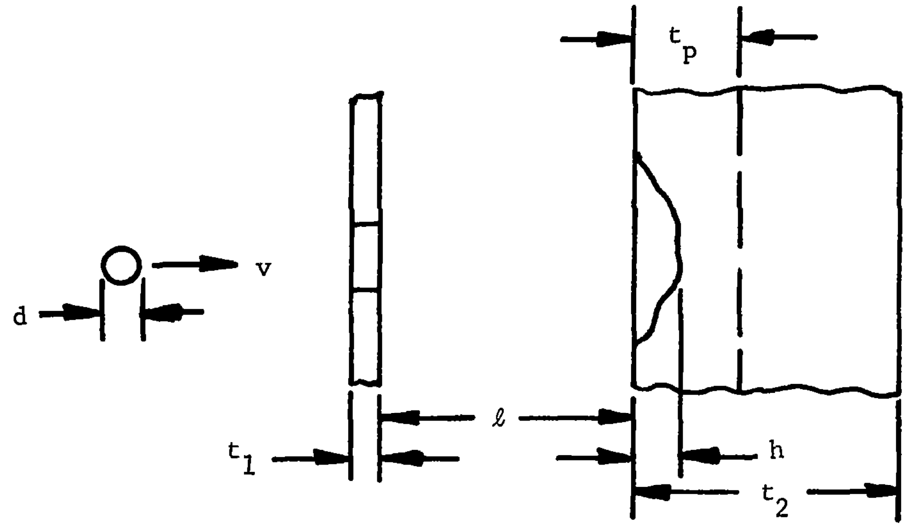

Fig. 10 Bumpered Target Geometry

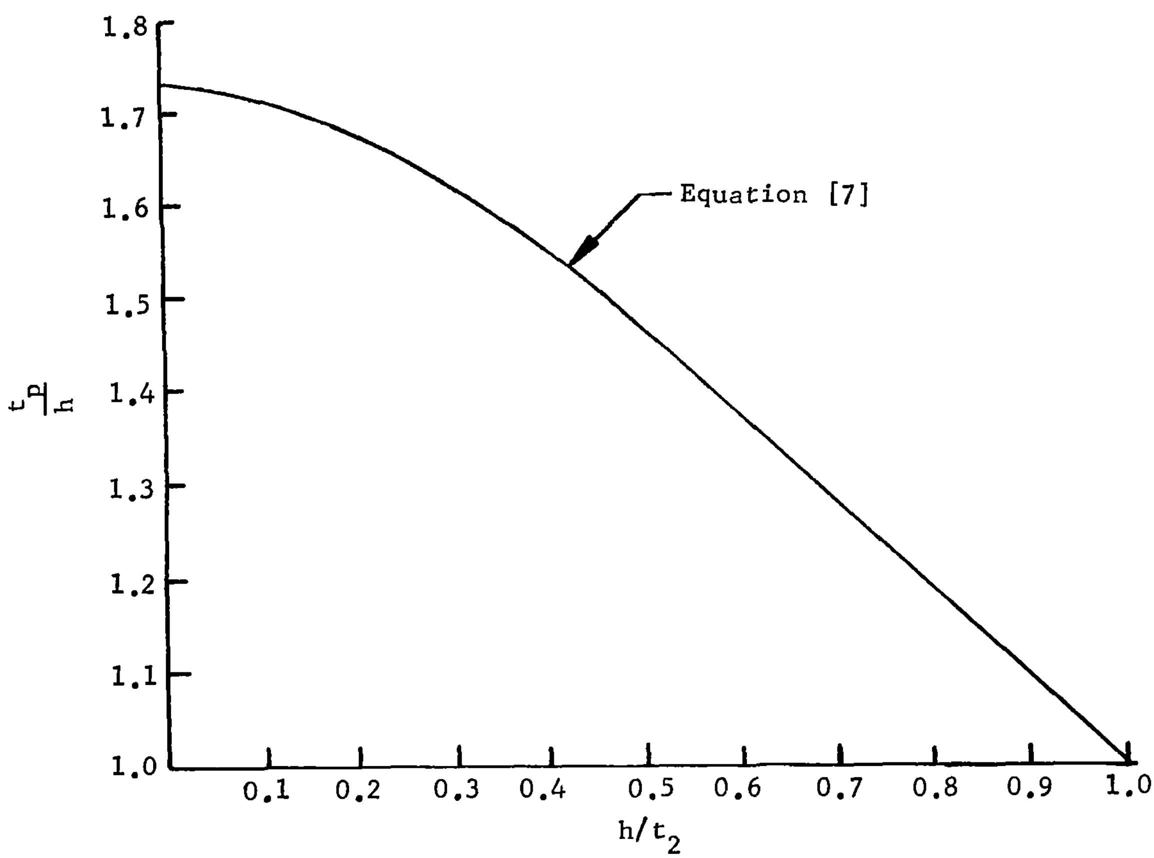

Fig. 11 Correction from Crater Depth to Thickness Just Penetrated

in the second sheet, is obviously the most convenient dependent variable since it is directly related to the weight of the configuration required to defeat a given projectile at a given velocity. However, the problem was found to become more tractable with only a minor loss in convenience by using only second sheet damage as the dependent variable.

In this connection it was found possible to remove the effect of the second sheet thickness from the data by introducing the concept of the thickness, t_p, which would have just been perforated by the same impact which produced the crater depth h in the second sheet thickness, t_2.

It was found that a correction developed for the unbumpered case worked satisfactorily for the bumpered case as well. Reference 14 gives the following relationship in the nomenclature of Fig. 10

$$\frac{h}{t_2} = A\sqrt{\frac{h - h_\infty}{h_\infty}} \text{ or } \left(\frac{h}{t_2}\right)^2 = A^2\left[\frac{h}{h_\infty} - 1\right] \qquad [6]$$

where A is simply a constant and h_∞ is the crater depth which would have been produced had t_2 been infinitely thick.

Consider the case where perforation just occurs. Both h and t_2 are then equal to t_p. Equation [6] becomes

$$1 = A^2\left[\frac{t_p}{h_\infty} - 1\right] = A^2[R - 1] \qquad [6a]$$

where R is a familiar concept. It is simply the ratio of the thickness which would have just been perforated to the crater depth in a semi-infinite target. It has been variously estimated as falling between 1.5 and 2.0. Reference 11 gives R = 1.74 for aluminum projectiles striking 2024-T3 targets at 7.4 km/sec.

Rewriting Equations [6a] and [6] as

$$A^2 + 1 = A^2\left(\frac{t_p}{h_\infty}\right)$$

$$A^2 + \left(\frac{h}{t_2}\right)^2 = A^2\left(\frac{h}{h_\infty}\right)$$

and dividing one by the other

$$\frac{A^2 + 1}{A^2 + \left(\frac{h}{t_2}\right)^2} = \frac{t_p}{h} = \frac{\frac{1}{R-1} + 1}{\frac{1}{R-1} + \left(\frac{h}{t_2}\right)^2} = \frac{R}{1 + (R - 1)\left(\frac{h}{t_2}\right)^2}$$

$$\frac{t_p}{h} = \frac{1.74}{1 + 0.74\left(\frac{h}{t_2}\right)^2} \qquad [7]$$

$$\frac{t_p}{d} = \frac{1.74}{1 + 0.74\left(\frac{h}{t_2}\right)^2}\left(\frac{h}{d}\right) \qquad [7a]$$

Equation [7] is plotted in Fig. 11 and gives the factor that corrects measured second sheet damage to the thickness of a sheet that would just have been punctured under the same impact conditions.

This device has the property of describing all bumpered target damage, from a thin second sheet (provided that not more than a marginal puncture occurs) to a semi-infinite second sheet, in terms of equivalent structures which would sustain marginal punctures under the same conditions. This makes nearly all the data of Ref 11 and 13 available for analysis on a common basis. Furthermore, since every equivalent structure will just be punctured, the most useful and meaningful kind of damage is described.

This device also has the property of converting the impact velocity of each shot to the so-called "ballistic limit" for the equivalent structure. The ballistic limit is defined as that velocity at which a structural configuration will just defeat the projectile. It is not a particularly meaningful concept for bumpered targets since, as shown in Fig. 15a and 16a, a critical velocity exists for each such configuration at which perforation will occur, but above and below which it will not. This is the "velocity barrier" found in Ref 15.

It may be explained on the basis that at low velocities the projectile passes through the bumper more or less intact. At the critical velocity the shock generated in the projectile is sufficiently strong to disrupt it, and at higher velocities the projectile is more finely divided and spread over a larger target area, with decreased depth of damage. As the velocity becomes very great the value of t_p should decrease asymptotically toward zero.

A functional form which has the necessary characteristics to describe this behavior is

$$\frac{t_p}{d} = a\, v^n\, e^{-b\, v^n} \qquad [8]$$

Differentiating once with respect to v and setting the derivative equal to zero gives the three values of v which locate the extrema.

$$0 = anv^{n-1}\, e^{-bv^n}\left[1 - bv^n\right]$$

whence

$$v^{n-1} = 0 \text{ or } v = 0$$

$$e^{-bv^n} = 0 \text{ or } v = \infty$$

$$1 - bv^n = 0 \text{ or } \bar{v} = \left(\frac{1}{b}\right)^{\frac{1}{n}} \text{ (critical velocity)}$$

Substituting the critical velocity in the function gives the maximum value

$$\left(\frac{t_p}{d}\right)_{max} = \frac{a}{eb}$$ where e is the base of natural logarithms.

As an aid in evaluating a and b the function may be written

$$\ell n\left[\left(\frac{t_p}{d}\right) v^{-n}\right] = \ell n\ a - bv^n$$

In semilogarithmic coordinates, this is a straight line of intercept a and slope (-b). A value of the exponent n = 2.0 has been found to provide good correlation. The technique is simply to compute $\left[(t_p / d)v^{-2}\right]$ for a series of shots in which only the impact velocity and $\left(t_p / d\right)$ change and to plot these values against v^2 on semilog paper as in Fig. 12 thru 18.

The data, so adjusted, were found to be reasonably well approximated by straight lines, as shown in Fig. 12 thru 18, where the intercepts, a , and slopes, b, were measured. These values, for aluminum projectiles only, are given in Table 4. Note that the angles made by the lines are such that

$$\tan \theta = 14.5b$$

where the constant 14.5 is a scale factor resulting from the fact that on the vertical scale the length of a decade, representing $\ell n\ 10 = 2.3026$, is the same as the length representing 33.4 units on the horizontal scale. The analysis needs to be extended to the data for other projectile materials that are given in Appendix C.

The intercepts and slopes measured above became new dependent variables, dependent only on the dimensionless adjusted bumper thickness (t_1/d) and the dimensionless standoff (ℓ/d) since only one projectile material and one target material were involved, and the effects of bumper material, second sheet thickness, impact velocity, and projectile size were already accounted for. The bumper thickness was adjusted on the basis of the mass concept explained previously by equating products of density and thickness (Ref 11 and 12).

The slopes, b, were found to be, at least to a first approximation, dependent only on the dimensionless adjusted bumper thickness (t_1/d), and negligibly affected by scale standoff distance variations. The approximate relationship was found to be

$$b = 0.0293 + 0.127\left(\frac{t_1}{d}\right)^{0.547} \qquad [9]$$

The intercepts, a, were found to depend linearly upon the dimensionless adjusted bumper thickness (t_1/d) with a constant intercept at $(t_1/d) = 0$ and slopes which were exponentially dependent upon the scale standoff distance (ℓ/d). This dependence is approximated by

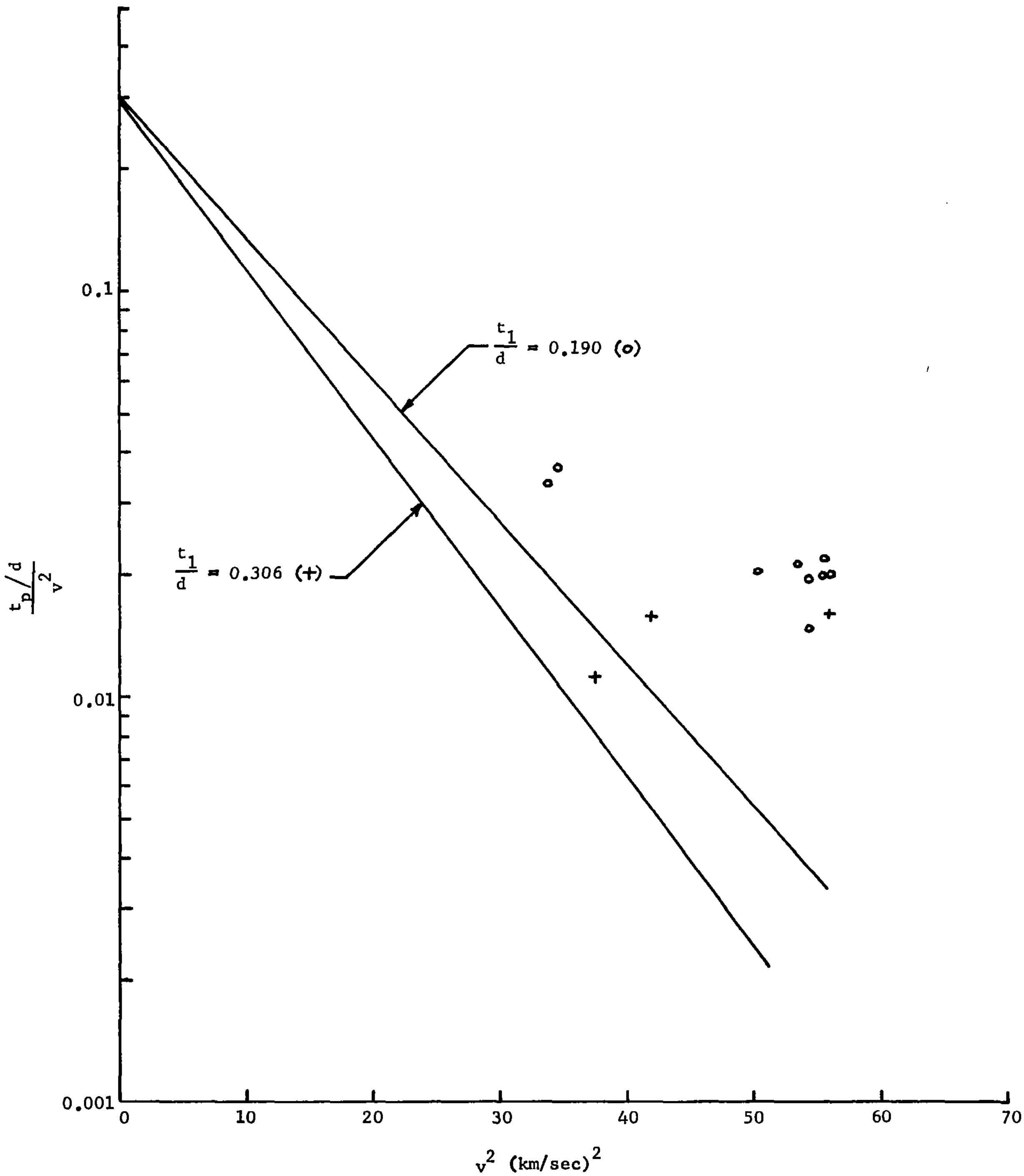

Fig. 12 Bumpered Target Reduced Data, Scale Standoff = 4.0

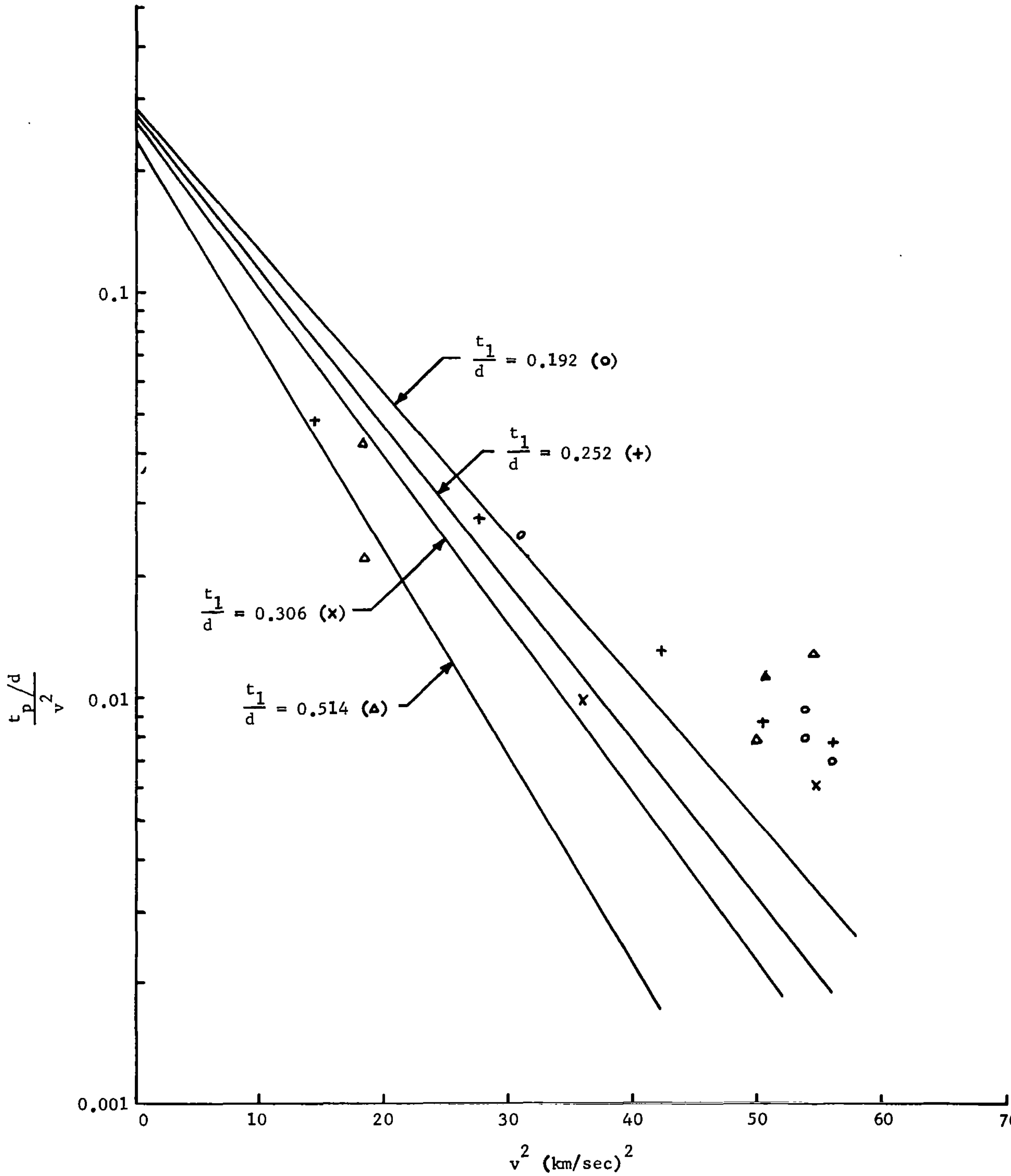

Fig. 13 Bumpered Target Reduced Data, Scale Standoff $\ell/d = 8.0$

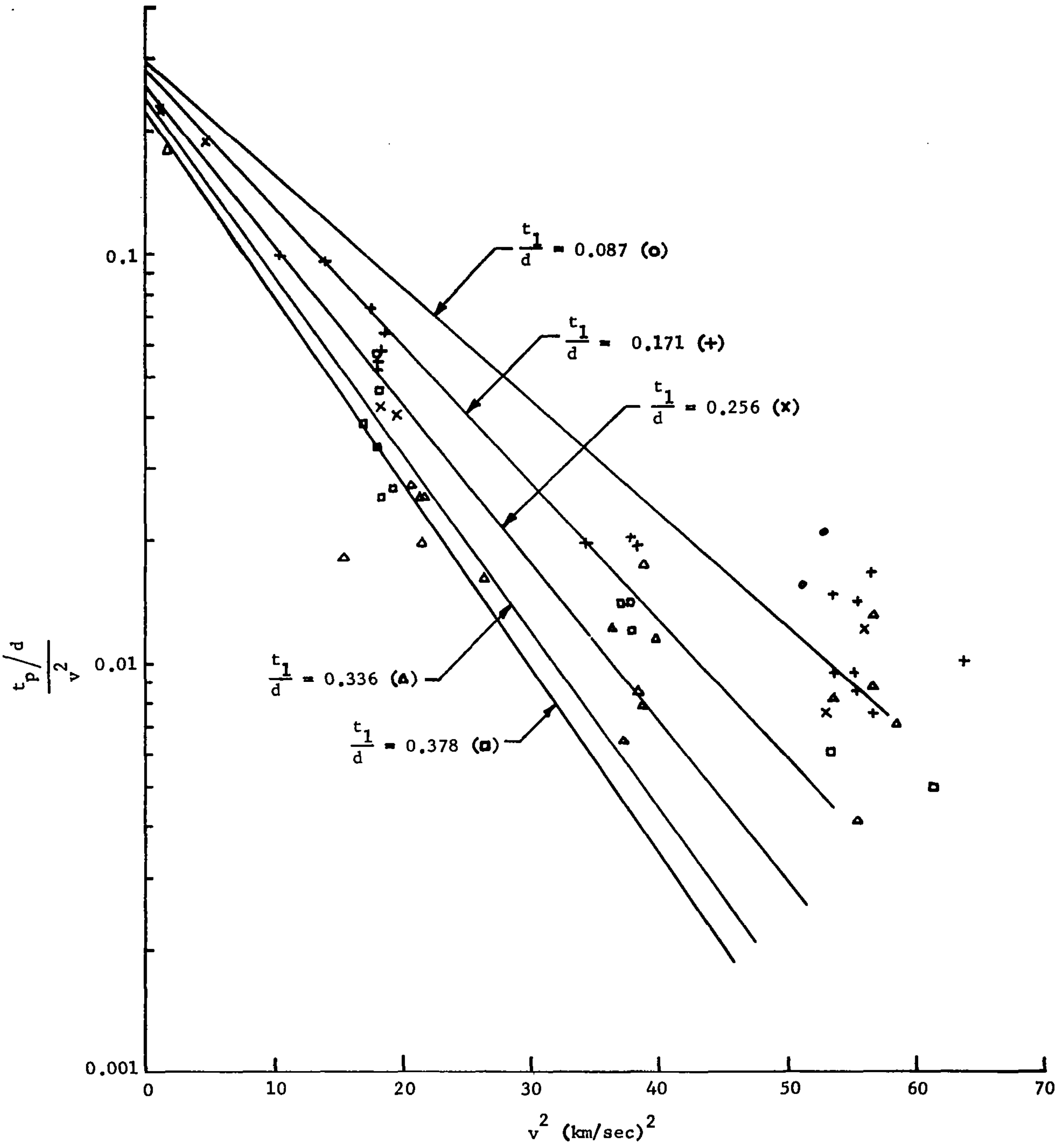

Fig. 14 Bumpered Target Reduced Data, Scale Standoff ℓ/d = 10.7

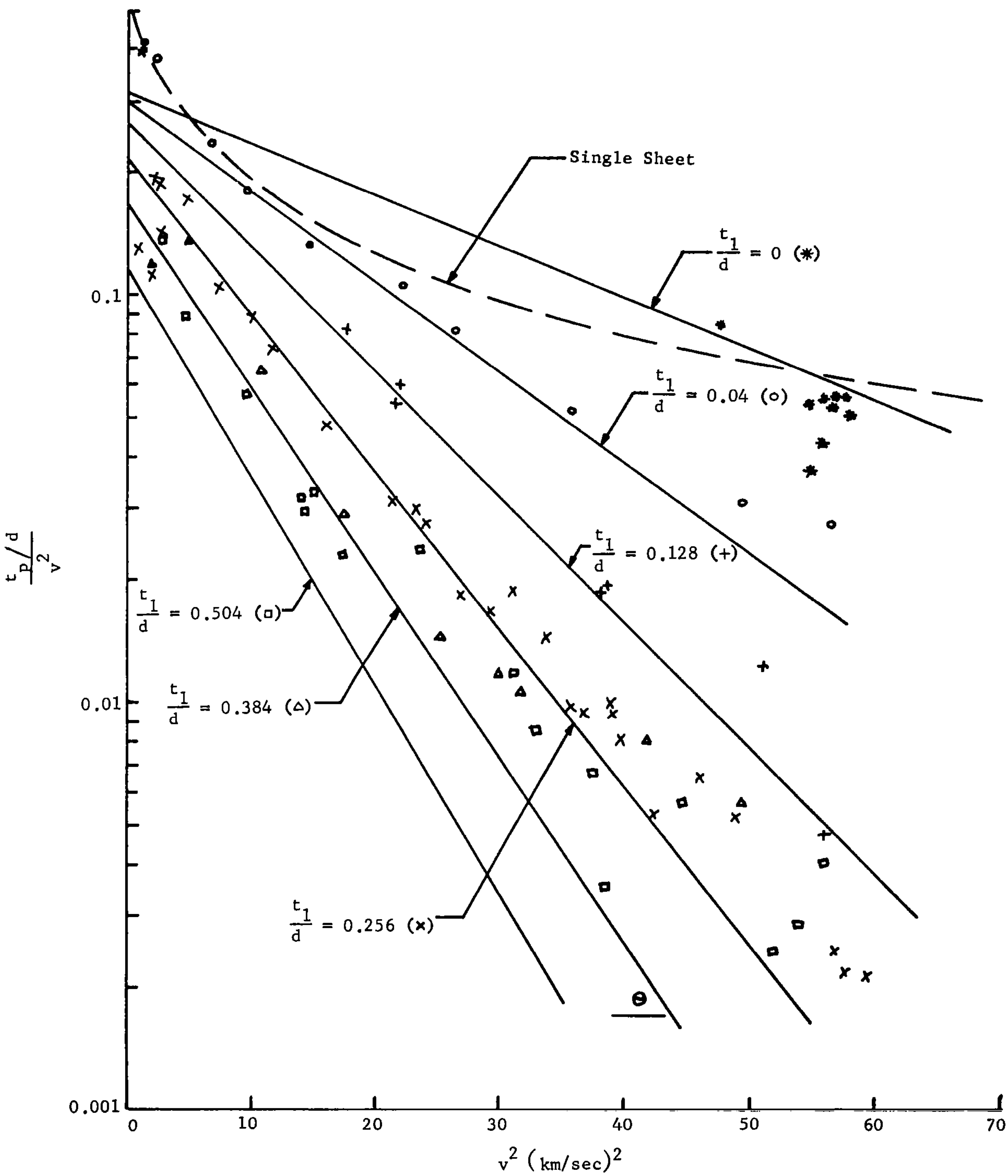

Fig. 15 Bumpered Target Reduced Data, Scale Standoff ℓ/d = 16.0

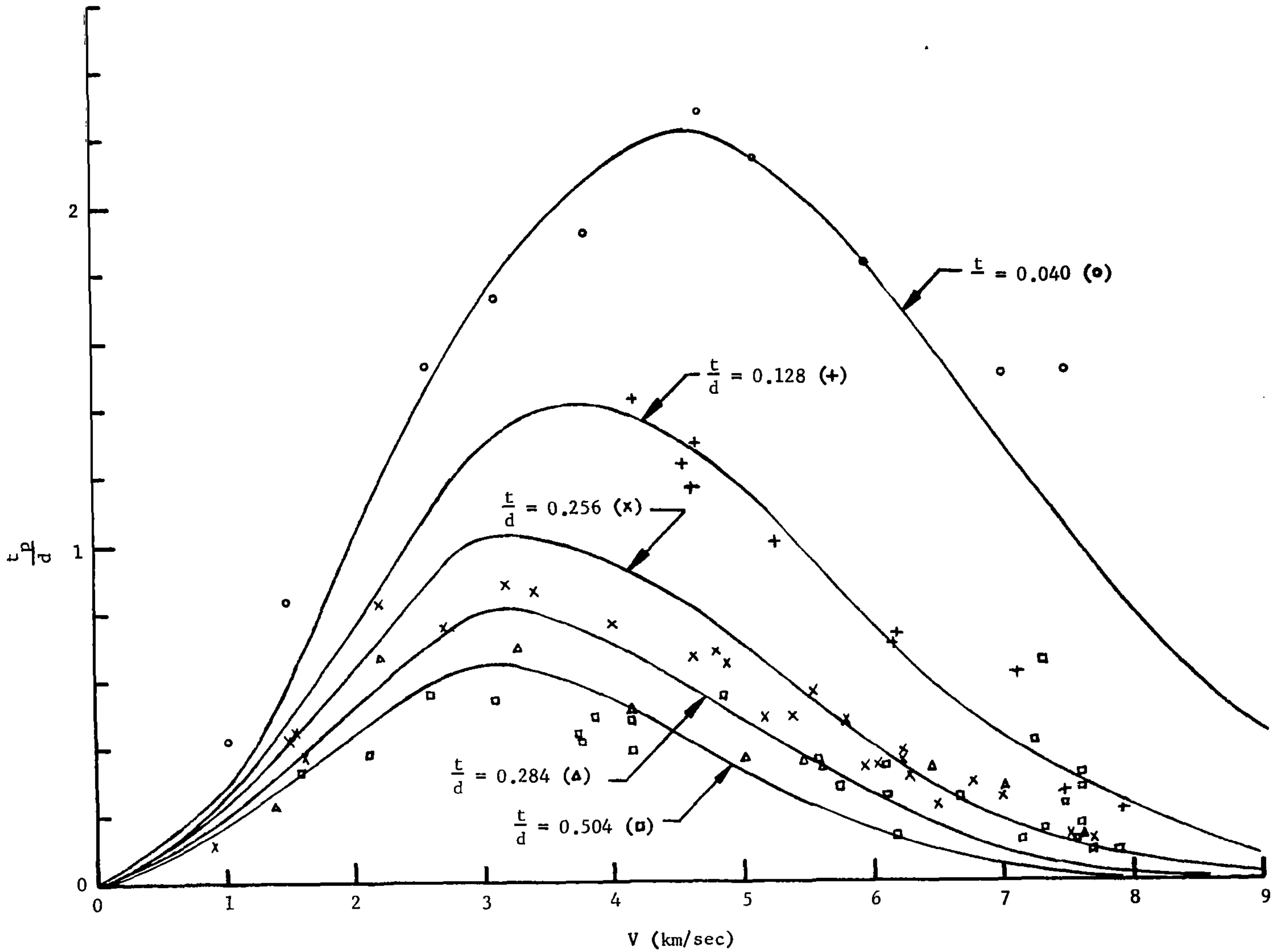

Fig. 15a Bumpered Target Data, Scale Standoff ℓ/d = 16.0

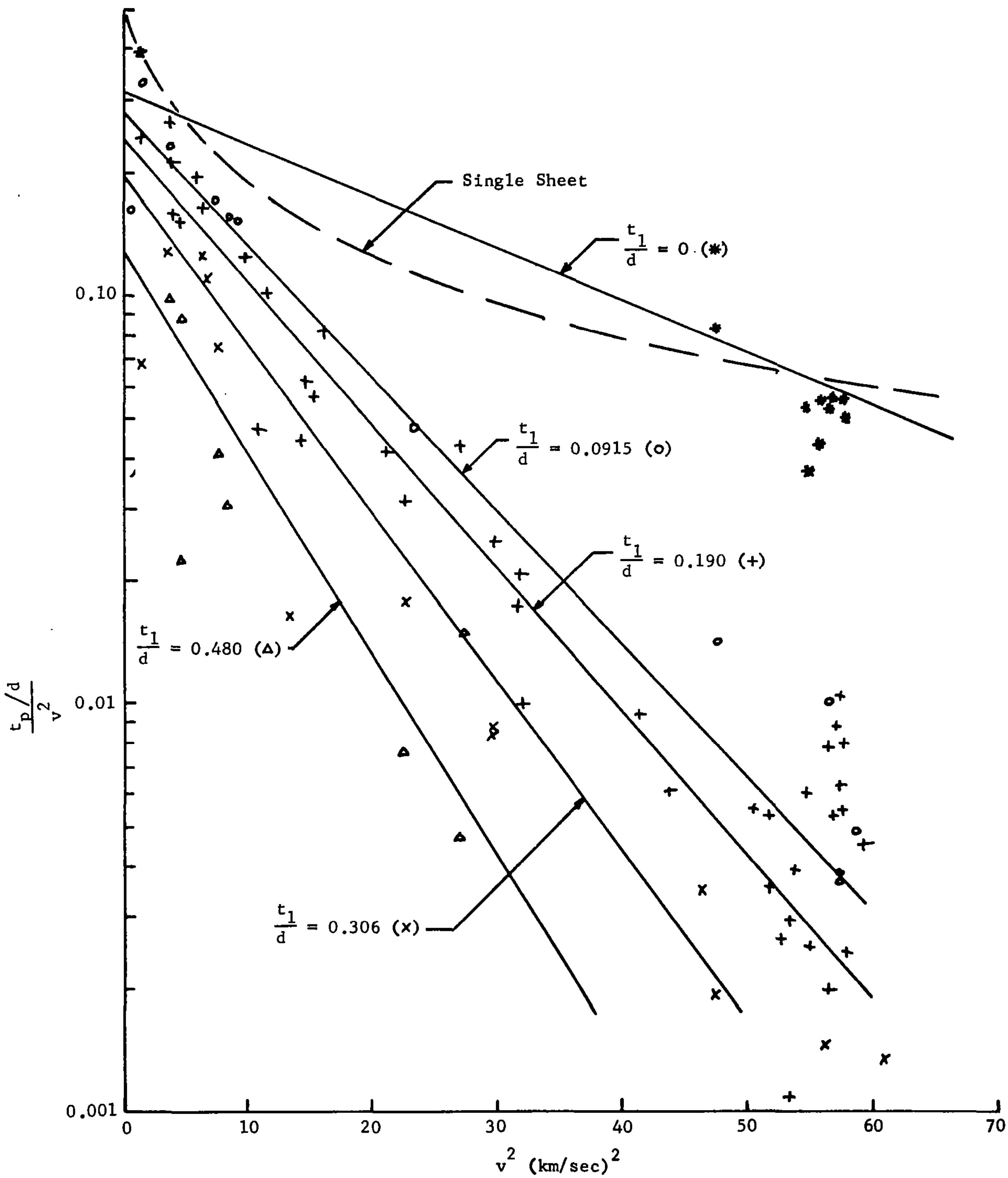

Fig. 16 Reduced Data for Bumpered Targets, Scale Standoff $\ell/d = 16.0$

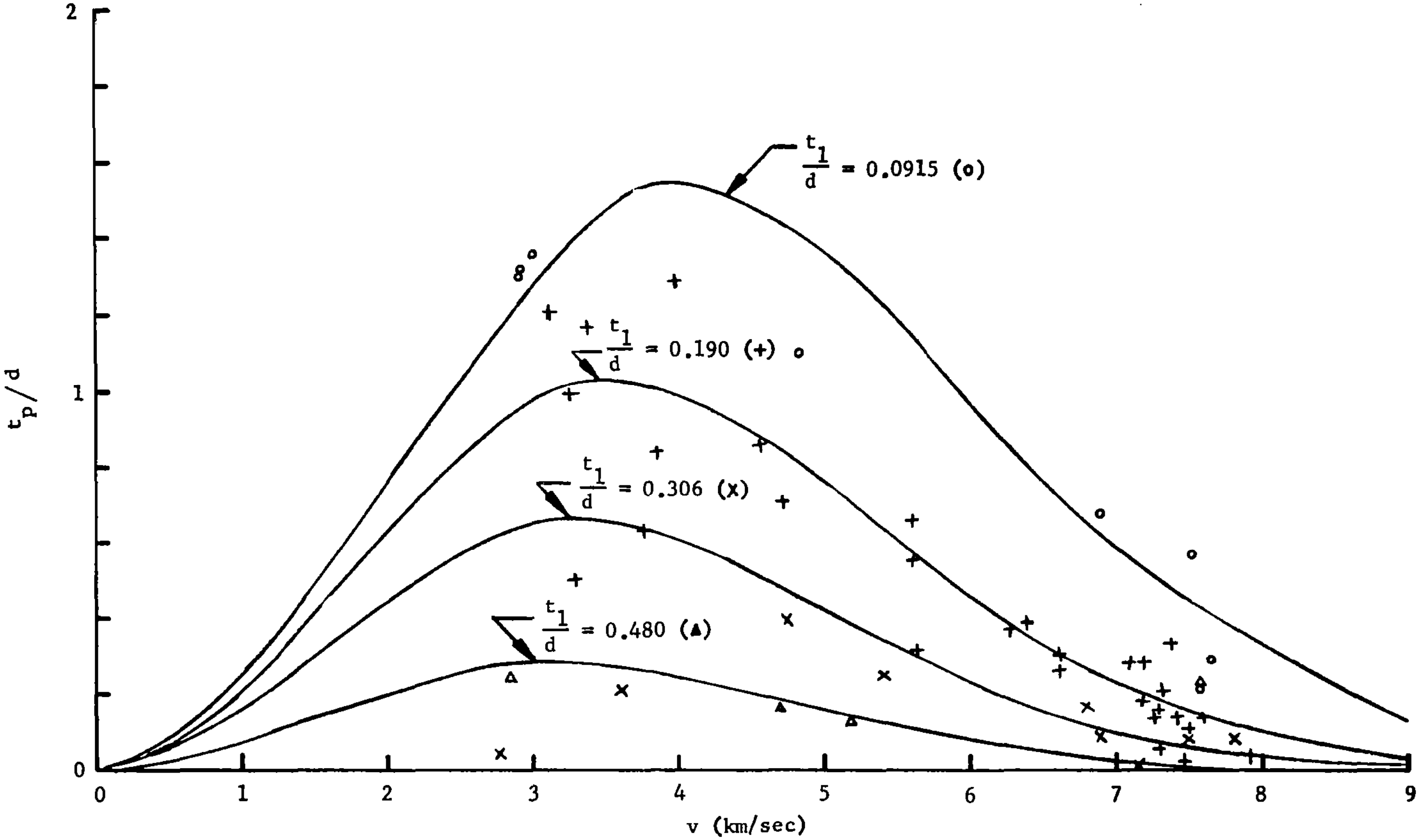

Fig. 16a Bumpered Target Data, Scale Standoff ℓ/d = 16.0

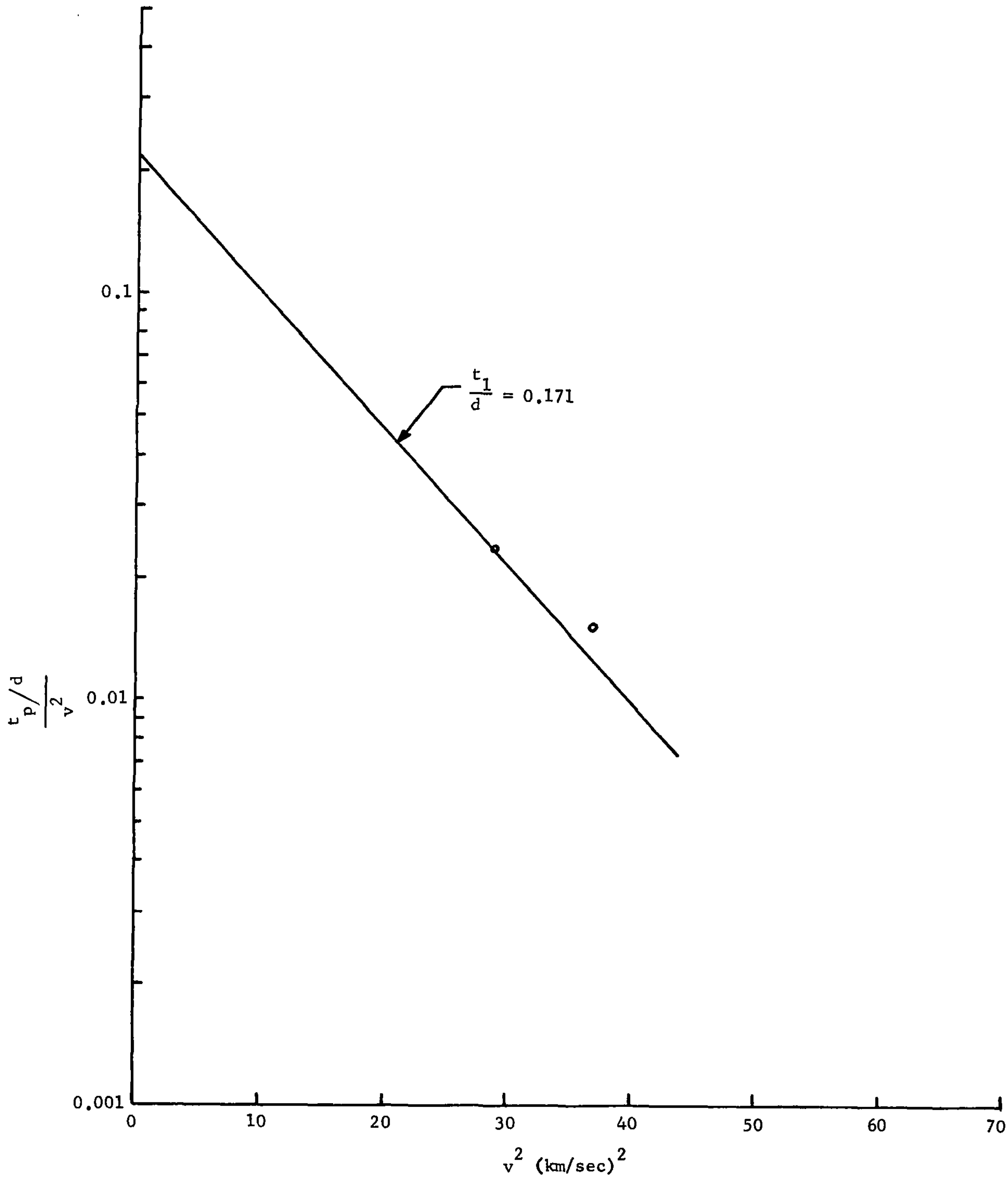

Fig. 17 Bumpered Target Reduced Data, Scale Standoff $\ell/d = 21.3$

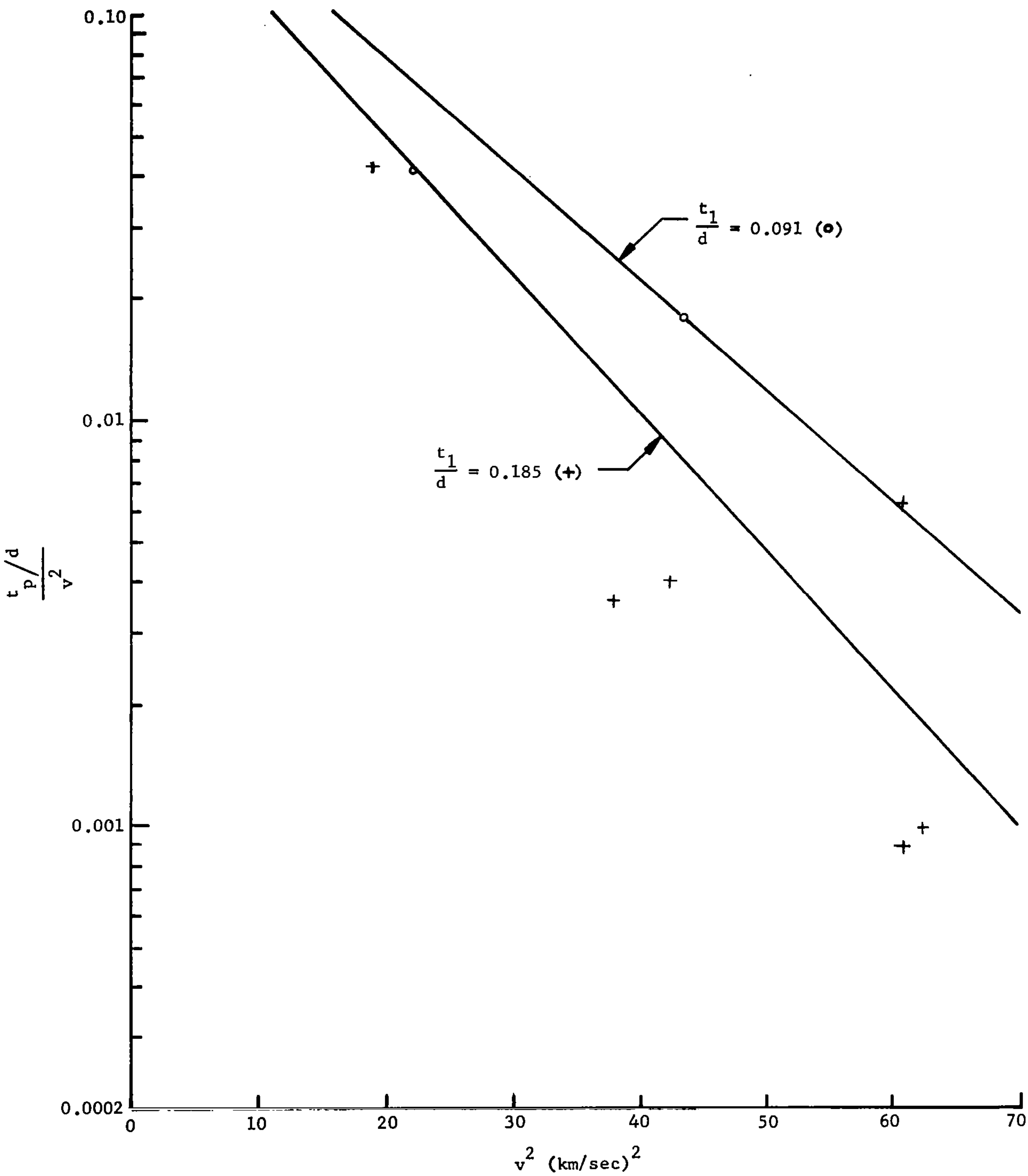

Fig. 18 Bumpered Target Reduced Data, Scale Standoff ℓ/d = 32.0

Table 4 Calculated Intercepts and Slopes for Bumpered Target Data

ℓ/d	4.0		5.3		8.0		10.7		16.0		21.3		32.0	
$t_{1/d}$	a	b	a	b	a	b	a	b	a	b	a	b	a	b
.190	.300	.0807												
.306	.292	.0958												
.171			.295	.0777										
.192					.282	.0807								
.252					.272	.0894								
.306					.263	.0958								
.514					.231	.1171								
.087							.291	.0631						
.171							.272	.0777						
.256							.253	.0894						
.336							.234	.0993						
.378							.224	.1043						
0									.312	.0293				
.0915									.278	.0631				
.190									.238	.0807				
.306									.194	.0958				
.480									.126	.1142				
.040									.297	.0511				
.128									.262	.0706				
.256									.214	.0894				
.384									.165	.1043				
.504									.115	.1171				
.171											.216	.0777		
.091													.228	.0631
.183													.139	.0792

$$a = 0.312 - 0.011\left(\frac{t_1}{d}\right)\left(\frac{\ell}{d}\right)^{1.284} \qquad [10]$$

In order to obtain a picture of the degree of correlation effected by the adjustments made in the data, analogous to Fig. 1 and 3, Equation [8] may be rewritten as

$$\ell n \left[\left(t_p/d\right) a^{-1} v^{-2}\right] = -bv^2 \qquad [8a]$$

In Fig. 19, all the data of Fig. 12 thru 18 have been adjusted by the values of a and b from Table 4 according to Equation [8a]. The equation itself plots as a straight line of intercept 1.0 and slope -1.0 ($\tan \theta = 1.45$ since the horizontal scale is ten times as large as in previous figures). The majority of the data is seen to be well represented by the line. The greatest errors occur at high velocities and consequent low second sheet damage where scatter may be expected to be relatively large, especially in view of the approximate adjustments made in accounting for the effects of bumper material, second sheet thickness, and projectile size.

Analyses similar to the above should be made for other projectile materials, and equations similar to Equations [9] and [10] generated. The constants in these equations would reflect only the changes in the properties of the projectile material and regression analyses of these constants could be performed in terms of these properties.

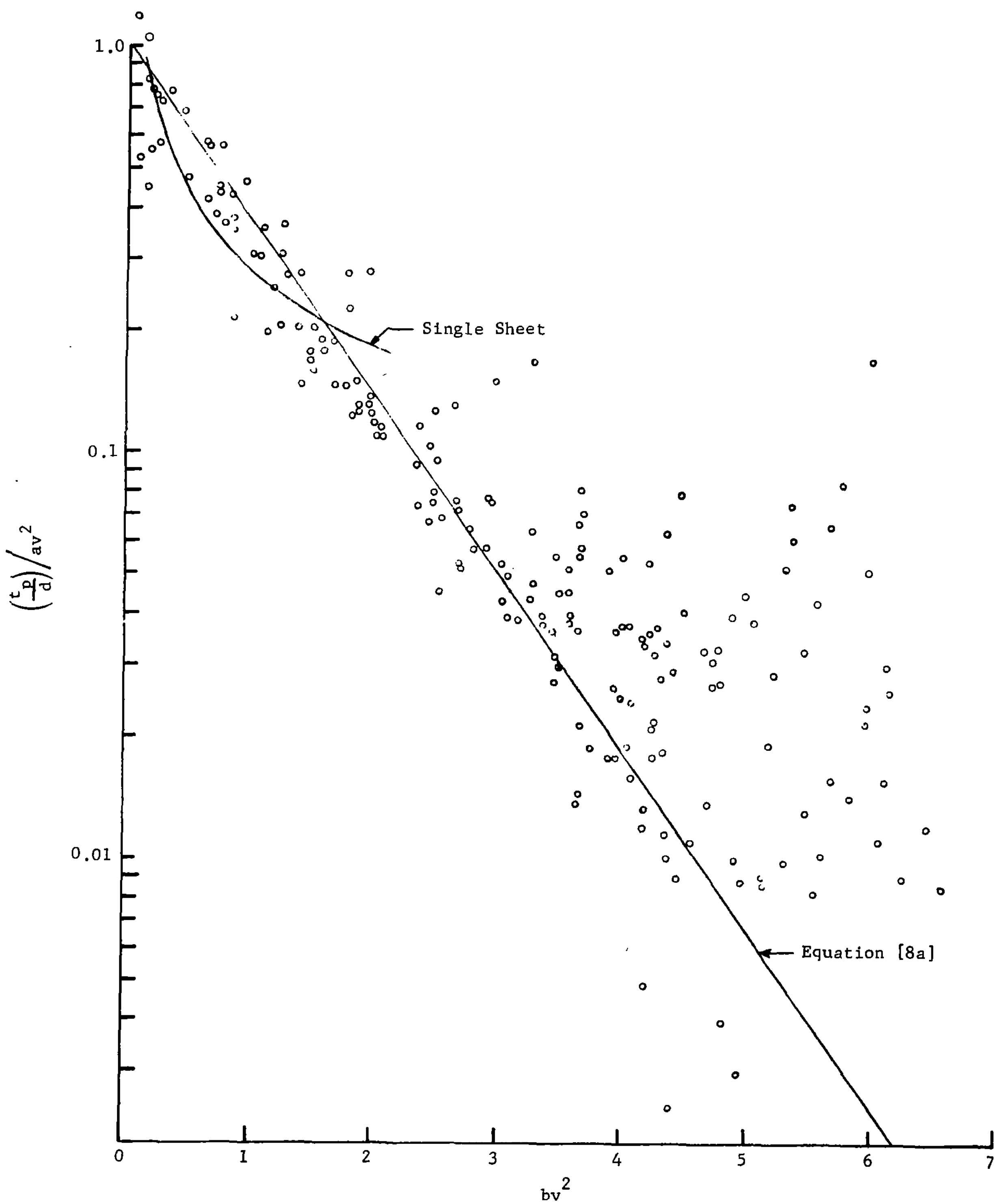

Fig. 19 Composite Correlation, Data for Scale Standoffs 4.0, 8.0, 10.7, 16, 21.3, and 32.0

V. FINITE SINGLE SHEET TARGETS

Data on the ballistic limit of single sheet, finite thickness targets are difficult to obtain and, consequently, rare. However, the data in Appendix C contain several shots in which the first or bumper sheet was not completely punctured, or in which the second sheet received essentially no damage. These have been treated as zero-bumper thickness data, and converted to marginal punctures in the same way as all the other data.

Furthermore, Ref 11 gives a curve of crater depth for semi-infinite targets for aluminum projectiles and 2024-T3 targets. This curve has been multiplied by R = 1.74 and reproduced in Fig. 15 and 16, and labeled "single sheet."

With this information as a guide, a straight line has been superimposed upon Fig. 15 and 16, representing zero bumper thickness, and Equations [9] and [10] give the intercept and slope of this line where $(t_1/d) = 0$.

The physical possibility of this refinement, that is that no discontinuity exists between the unbumpered case and those of very thin bumpers, is borne out by the routine use of metallic foil velocity sensors in many hypervelocity launching facilities. These sensors consist of pieces of metal foil placed known distances apart ahead of the target. The breaking of these pieces of foil by the passage of the projectile provides timing signals so that the projectile velocity can be calculated. The integrity of the projectile is not affected.

Conclusions and Recommendations - The multivariable analysis techniques used in this study have provided a great deal of insight into the nature of the material properties influencing crater depth and crater volume and have served to bring into relationship several empirical formulas developed by previous investigations. Prediction equations allowing quantitative estimates of the effect of material properties on crater depth have been given in Equations [2] and [3], and on crater volume in Equations [6] and [7].

For bumpered targets the work of others on the effect of material properties has been used as a basis for analyzing the relationship between bumper thickness, bumper spacing, and corresponding thickness of bumpered targets that will just be penetrated.

A multivariable analysis of the ballistic limit of single sheets was not possible because of the scarcity of useful experimental data. However, the possibility has been disclosed in this study that no discontinuity exists between the unbumpered case and that of very thin bumpers. For bumpered targets analyses need to be made of data for other projectile materials given in Appendix C.

REFERENCES

1. A. A. Ezra, J. D. Donahue, and A. A. Holston, Jr.: Effect of Material Properties on the Penetration of Target by Projectiles. AIAA Preprint 63-194.

2. O. P. Fuchs: "Impact Phenomena." AIAA Journal, Vol 1, No. 9, September 1963.

3. Olive G. Engel: "Hypervelocity Cratering Data and a Crater-Depth Model for the Regime of Partial Fluidity." Proceedings of the Sixth Symposium on Hypervelocity Impact, Vol II, Part 2, August 1963.

4. J. L. Summers: Investigations of High Speed Impact: Regions of Impact and Impact at Oblique Angles. NASA TND-94. July 1959.

5. E. H. Goodman and C. D. Liles: "Particle-Solid Impact Phenomena." Proceedings of the Sixth Symposium on Hypervelocity Impact, Vol II, Part 2, 1 August 1963. (Also AEDC-TDR-62-202, November 1962, ASTIA AD287808).

6. S. W. Yuan and C. N. Scully: "A New Approach to Hypervelocity Impact Theory." Advances in the Astronautical Sciences, Vol 13.

7. Walter Herrmann and Arfan H. Jones: Survey of Hypervelocity Impact Information. ASRL Report 99-1. September 1961, plus Addendum, October 1961, MIT Aeroelastic and Structures Research Laboratory.

8. L. V. Belyakov, F. F. Vitman, and N. A. Zlatin: "Collision of Deformable Bodies and its Modelling." Soviet Physics - Technical Physics, Vol 8, No. 8, February 1964.

9. J. L. Luttrell: "A Hypervelocity Impact Model for Completely Deforming Projectiles." Proceedings of the Sixth Symposium on Hypervelocity Impact, Vol II, Part 1, August 1963.

10. R. C. Bjork: "Review of Physical Process in Hypervelocity Impact and Penetration." Proceedings of the Sixth Symposium on Hypervelocity Impact, Vol II, Part 1, August 1963.

11. C. J. Maiden, J. W. Gehring, and A. R. McMillan: Final Report on Investigation of Fundamental Mechanism of Damage to Thin Targets by Hypervelocity Projectiles. TR-63-225. General Motors Defense Research Laboratories, September 1963.

12. R. F. Rosten, H. H. Hunt, and J. L. Wellnitz: Study of Principles of Meteoroid Protection. AE62-0413. Marshall Space Flight Center, NASA, Huntsville, Alabama.

13. C. J. Maiden et al.: Experimental Investigations of Simulated Meteoroid Damage to Various Spacecraft Structures. Progress Reports 1 thru 10 (Contract NAS9-3081). NASA/Manned Spacecraft Center, Houston, Texas, October 1964 thru March 1966.

14. W. H. Kinard et al.: Effect of Target Thickness on Cratering and Penetration of Projectiles Impacting at Velocities to 13,000 fps. NASA Memo 10-18-58. December 1958.

15. F. C. Posever and C. N. Scully: Investigation of Meteoroid Impact on Two-Sheet Configurations. AFFDL-TR-65-196. November 1965.

16. R. B. Mortensen et al.: "Effects of 3 to 12 Km/sec Impacts on Finite Targets." Proceedings of the Sixth Symposium on Hypervelocity Impact, Vol III, August 1963.

17. J. H. Perry (ed): Chemical Engineer's Handbook, Third Edition, McGraw-Hill Book Company, Inc, New York, New York, 1950.

18. NASA TM X-54,046

19. S. M. Halperson: "Some Phenomena Associated with Impacts into Aluminum." Proceedings of the Sixth Symposium on Hypervelocity Impact, Vol II, Part 2, August 1963.

20. F. C. Posever and C. N. Scully: Investigation of Structural Implications of Meteoroid Impact. FDL TDR 64-96. Air Force Flight Dynamics Laboratory, Research and Technology Division. AFSC, Wright-Patterson AFB, Ohio.

21. R. W. Watson: "The Perforation of Thin Plates by High Velocity Fragments." Proceedings of the Fifth Symposium on Hypervelocity Impact, Vol I, Part 2, April 1962.

22. W. S. Partridge, C. R. Morris, and M. D. Fuller: "Perforation and Penetration Effects of Thin Targets." Proceedings of the Third Symposium on Hypervelocity Impact, Vol I, February 1959.

23. R. W. Watson, K. R. Becker, and F. C. Gibson: "Thin Plate Perforation Studies with Projectiles in the Velocity Range from 2 to 5 Km/sec." Proceedings of the Sixth Symposium on Hypervelocity Impact, Vol III, August 1963.

24. C. R. Nysmith and J. L. Summers: Preliminary Investigation of Impact of Multiple-Sheet Structures and an Evaluation of the Meteoroid Hazard to Space Vehicles. NASA TN D-1039. September 1961.

<u>BIBLIOGRAPHY</u>

Binder, R. C.: <u>Fluid Mechanics</u>. Prentice-Hall, Inc, New York, New York.

Eichelberger, R. J. and Gehring, J. W.: "Effects of Meteoroid Impacts on Space Vehicles." ARS Paper 2030-61 (also <u>ARS Journal</u>, Vol 32, No. 10, October 1962, p 1583).

Halperson, S. M.: <u>Observations of Hypervelocity Impact</u>. (report number unknown), U. S. Naval Research Laboratory, Washington, D. C.

Hastings, E. C., Jr.: <u>The Explorer XVI Micrometeoroid Satellite</u>. NASA TMX-810. February 1963.

Humes, D. H.: <u>An Experimental Investigation of the Effectiveness of Single Aluminum Meteoroid Bumpers</u>. NASA TND-1784. May 1963.

Moore, H. J., MacCormack, R. W., and Gault, D. E.: "Fluid Impact Craters and Hypervelocity -- High-Velocity Impact Experiments in Metals and Rocks." <u>Proceedings of the Sixth Symposium on Hypervelocity Impact</u>, Vol II, Part 2, August 1963.

<u>Study of Target Penetration Prediction by High Speed and Ultra High Speed Ballistic Impact - Final Report</u>. APGC-TDR-63-22 (also 62-11, 35, 51). Air Proving Ground Center, Eglin Air Force Base, Florida.

APPENDIX A

TABULATED DATA FROM LITERATURE

All the crater depth and volume data collected from the technical literature to the end of 1965 is given to this appendix. At the top of each data set is the identification of the materials used, as given by the experimenters where the original reference was available, or by Herrmann and Jones (Ref 9) where it was not. In many cases, because of inadequate identification, it was necessary to make reasonable assumptions as to what was actually used, and the assumptions are also given.

The symbol given for each data set is the symbol used to plot the data in Fig. 1 thru 6 and is keyed to Table A-1. The thermal properties used are given in Table A-2, except for thermal conductivities, for which enough data were available to distinguish between alloys of the same parent metal.

It is important to note that a number of properties have been arbitrarily reduced in order of magnitude to the order of 1.0 by factors involving powers of ten as defined in Table 1. This was done because of the power law forms of Equations [2] and [3]. Regression analysis was performed after transformation to logarithms, and the accuracy of such an analysis is improved if all the logarithms are approximately the same size.

In the data pages, the symbol Π_1, has been used to designate the dimensionless crater depth (h/d).

Table A-1 Data Summary

Targ / Proj	2024T4	CPAL	HPAL	BeCu B=2.17	BeCu B=2.97	HPCu	Cu B=.36	Cu B=.65	Pb	Stl B=1.10	Stl B=1.30	Zn
2024T3	A-8,9 B-2 +	Pages Symbol		A-16 Δ	A-17 ⊥			A-18,19 B-7 ⊢	A-25 B-12	A-29 B-16 ∇	A-30 ▼	B-21 ⊘
CPAL		A-6										
HPAL			A-4,5 □			A-14,15 ⊣						
HPCu			A-10 λ			A-20 •						
Cu B=.65	A-11 B-3 ×						A-21 ⊤	A-22 B-8	A-26 B-13 Y	A-31 B-17 ◇		B-22 ⊡
Pb	A-12 B-4 ○							A-23 B-9 ■	A-27	A-32 B-18 ◆		B-23
Stl B=1.10	A-13 B-5 ✳							A-24 B-10 ▲	A-28 B-14	A-33 B-19		B-24
2017T4	A-7 ◑											
Zn	B-6							B-11	B-15	B-20		B-25 ⊗

Table A-2 Thermal Properties

		Al	Cu	Pb	Fe	Zn
Room Temperature	°C	27	27	27	27	27
Melting Point	°C	660	1083	327	1539	419
Mean Specific Heat of Solid	Cal/gm°C	.252	.1045	.0322	.155	.0977
Latent Heat of Fusion	Cal/gm	93	49	5.9	65	24.4
Heat to Melt	Cal/gm	252	159	15.6	299	62.7
Boiling Point	°C	2057	2595	1744	2735	907
Mean Specific Heat of Liquid	Cal/gm °C	.259	.118	.0328	.146	.120
Latent Heat of Vaporization	Cal/gm	2260	1145	203	1515	420
Heat to Vaporize	Cal/gm	2874	1482	265	1989	541

Much of this information is available in Ref 17

Reference 3
Identification: HPA1 → HPA1
Assumed Materials:

Item	Proj	Targ
B	.17	.17
E	.633	.633
F	.93	.93
ρ	2.67	2.67
ν	.33	.33
Y	.077	.077
U	.553	.553
ϵ	.52	.52
H	2874	2874
c	.252	.252
k	.54	.54
Q	252	252
C	5.86	5.86
K	.621	.621
K'	.938	.938
H'	2260	2260

f = 1.0
Δ = 2.0
Symbol □

d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
.159	.489	.296				
	1.289	.930				
	2.629	1.556				
	3.200	1.575	13.19	.661	.428	3.604
	3.216	1.820				
	4.076	1.997	18.41	.712	.535	2.484
	4.128	2.142				
	4.345	2.041				
	4.436	2.255				
	4.875	2.192				
	4.900	2.142				
	5.128	2.488				
	5.322	2.312				
	5.322	2.230	22.64	.516	.501	2.051
	6.654	2.683	35.33	.512	.466	2.357
	6.793	2.488	35.33	.481	.462	2.252
	6.924	2.381	33.09	.546	.462	2.563
	7.292	2.639	38.56	.477	.486	2.019
.159	7.507	2.759	41.79	.493	.538	1.704
.318	1.498	1.087				
	2.048	1.370	4.167	.393	.431	2.110
	2.106	1.420				
	2.254	1.477	6.250	.536	.485	2.281
	2.263	1.487	6.437	.518	.469	2.349
	2.317	1.367	4.820	.439	.439	2.282
	2.348	1.351	4.975	.494	.564	1.551
	2.548	1.808	11.35	.501	.504	1.971
	3.315	2.028				
	3.987	2.054				
	4.026	2.079				
	4.549	2.362	24.10	.481	.491	1.992
	4.566	2.277				
	4.820	2.151	20.56	.473	.472	3.121
	5.060	2.094				
	5.148	2.384	27.02	.499	.500	1.995
	5.289	2.491	22.67	.511	.587	1.484
	5.612	2.605	33.40	.486	.508	1.885
	5.694	2.564	28.89	.490	.510	1.884
	5.751	2.378	25.47	.478	.484	2.046
	5.880	2.624	36.69	.569	.523	2.079
.318	6.920	3.005	44.47	.456	.511	1.744

Reference 3

Identification: HPA1 → HP A1

Assumed Materials:

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.17	.17	.476	2.548	1.772	8.614	.493	.521	1.821
E	.633	.633		2.977	1.955	12.17	.501	.523	1.831
F	.93	.93		3.024	1.940	12.13	.472	.511	1.806
ρ	2.67	2.67		3.049	1.966	12.54	.507	.527	1.824
ν	.33	.33		3.107	1.955	12.86	.477	.502	1.890
Y	.077	.077		3.121	2.008	13.04	.516	.540	1.771
U	.553	.553		3.146	1.966	13.07	.490	.514	1.853
ϵ	.52	.52		3.447	2.039	14.68	.478	.510	1.841
H	2874	2874		4.223	2.283	21.07	.489	.526	1.764
c	.252	.252		4.727	2.533	26.58	.465	.525	1.689
k	.54	.54		5.166	2.698	32.23	.496	.559	1.589
Q	.252	.252		5.404	2.705	33.28	.497	.549	1.649
C	5.86	5.86		6.632	2.795	41.90	.563	.545	1.891
K	.621	.621		6.884	2.852	43.31	.497	.519	1.845
K'	.938	.938		6.876	2.787	38.34	.504	.538	1.742
H'	2260	2260		6.925	2.906	44.40	.504	.533	1.776
			.476	7.306	2.992	48.23	.491	.525	1.783

f = 1.0

Δ = 2.0

Symbol □

Reference 5

Identification: CPA1 → CPA1

Assumed Materials: 1100H14 → 1100H14

Item	Proj	Targ.	d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.32	.32	.159	6.173	2.526	38.31	.574	.490	2.388
E	.703	.703		6.429	2.992	44.03	.561	.583	1.652
F	.93	.93		6.633	3.042	45.28	.567	.593	1.615
ρ	2.72	2.72		6.684	2.961	41.55	.500	.558	1.609
ν	.33	.33		6.786	2.898	51.25	.584	.525	2.116
Y	1.196	1.196		6.837	2.658	50.00	.597	.473	2.675
U	1.267	1.267		6.939	2.753	46.52	.583	.510	2.241
ϵ	.20	.20		7.449	2.803	49.76	.624	.524	2.270
H	2.874	2.874		7.806	2.847	46.02	.641	.566	2.003
c	.252	.252	.159	7.806	2.721	45.03	.632	.531	2.245
k	.53	.53	.318	4.031	2.010	13.10	.461	.533	1.621
Q	2.52	2.52		4.745	2.230	19.25	.506	.538	1.749
C	6.12	6.12		5.918	2.397	23.23	.454	.518	1.695
K	.689	.689		6.275	2.552	20.90	.324	.506	1.264
K'	1.042	1.042	.318	6.582	2.574	31.66	.495	.515	1.867
H'	2.260	2.260							

f = 1.0

Δ = 2.0

Symbol $\prec$

Reference: 18
Identification: 2017T4 → 2024T4
Assumed Materials: 2017T4 → 2024T4

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	105	120	1.27	3.353	1.336			.352	
E	.745	.745		3.883	1.480			.379	
F	.93	.93		1.312	.720			.379	
ρ	2.79	2.77		2.185	1.012			.375	
ν	.33	.33		1.960	1.094			.405	
Y	2820	3300		.722	.162			.125	
U	4360	4780		2.403	1.046			.361	
ϵ	.22	.185		1.030	.300			.188	
H	2874	2874		.507	.086			.102	
c	.252	.252		.451	.072			.090	
k	.29	.29		1.849	.920			.368	
Q	252	252		2.687	1.142			.357	
C		6.25	1.27	1.809	.646			.281	
K	.730	.730	.635	4.695	1.540			.356	
K'				3.944	1.356			.361	
H'				4.842	1.500			.344	
				2.412	.888			.342	
			.635	2.696	1.060			.390	
			.318	5.497	1.680			.389	
				4.753	1.504			.376	
				4.787	1.512			.378	
				3.711	1.280			.364	
				4.392	1.440			.375	
				4.008	1.416			.393	
				3.755	1.296			.368	
				5.594	1.656			.390	
				6.480	1.824			.386	
				4.331	1.432			.373	
				3.278	1.152			.351	
				6.269	1.850			.402	
				2.886	1.000			.342	
				2.470	.888			.342	
				2.222	.776			.323	
				1.573	.544			.283	
				.591	.120			.115	
				1.762	.608			.292	
			.318	1.023	.280			.175	
			.159	3.569	1.232			.385	
				4.651	1.528			.424	
				4.724	1.520			.413	
				5.369	1.600			.398	
				6.440	1.744			.376	
				6.239	1.712			.369	
				4.688	1.392			.348	
				3.673	1.232			.416	
				3.819	1.280			.381	
			.159	6.998	1.840			.383	

$f = 1.0$

$\Delta = 2.004$

Symbol ●

Reference 7, Page 7

Identification: Al → Al 24ST

Assumed Materials: 2024T3 → 2024T4

Item	Proj.	Targ.	d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	120	.508	5.05	1.820	8.574			1.420
E	.745	.745	1.016	1.22	.300	.290			10.714
F	.93	.93		1.28	.470	.601			5.768
ρ	2.77	2.77		1.43	.570	.706			3.812
ν	.33	.33		1.48	.535				
Y	3510	3300		2.03	.740	1.138			2.904
U	4,920	4,780		2.09	.800	1.640			3.201
ϵ	.16	.16		2.35	.850	2.171			3.530
H	2,874	2,874		2.63	1.050				
c	.252	.252		2.65	1.110				
k	.29	.29		3.06	1.270				
Q	252	252		3.50	1.370	4.390			1.707
C	6.25	6.25		3.98	1.530	5.968			1.668
K	.730	.730							
K'	1.104	1.104		2.98		3.312			
H'	2260	2260		3.12		3.719			
			1.016	3.12		4.000			

Reference 7, Page 8

Identification: 2024T3 ⟶ 2024T3

Assumed Materials: 2024T3 ⟶ 2024T4

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	120	.157	4.02	1.387				
E	.745	.745	.559	.381	.045				
F	.93	.93		.725	.182				
ρ	2.77	2.77		1.05	.291				
ν	.33	.33		1.31	.364				
Y	3510	3300		1.53	.500				
U	4920	4780		1.87	.614				
ϵ	.16	.16	.559	1.99	.736				
H	2874	2874							
c	.252	.252							
k	.29	.29							
Q	252	252							
C	6.25	6.25							
K	.730	.730							
K'	1.104	1.104							
H'	2260	2260							

f = 1.0

Δ = 2.0

Symbol +

Reference 3

Identification: HPCu → HPAl

Assumed Materials:

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.50	.17	.159	.772	1.077	2.488	1.092	1.145	.833
E	1.132	.633		1.312	2.683	7.463	.691	1.124	.547
F	.49	.93		1.912	3.364	17.17	.570	1.016	.552
ρ	8.77	2.67		2.276	3.597	25.62	.635	1.002	.633
ν	.33	.33		2.419	3.490				
Y	.992	.077		3.627	4.838				
U	2.203	.553		4.284	5.096				
6	.58	.52		4.770	4.964				
H	1.482	2.874		5.380	4.699				
c	.1045	.252		5.687	5.354	140.6	.627	.801	.977
k	.934	.54		6.090	5.644	151.8	.528	.817	.791
Q	159	252	.159	6.436	5.745	168.9	.580	.812	.881
C	4.32	5.86	.318	.741	1.723				
K	1.110	.621		1.278	2.926	8.396	.677	1.204	.467
K'	1.677	.938		1.678	3.377	16.51	.629	1.069	.551
H'	1.145	2.760		2.133	3.758	31.19	.679	.946	.760
				2.240	3.887	31.81	.626	1.009	.615
				2.707	3.701				
				3.200	4.486				
				3.449	4.457				
				3.580	4.577				
				3.604	4.712	79.61	.632	.891	.796
				4.206	5.301	105.8	.616	.886	.784
			.318	5.340	5.925	150.9	.571	.896	.711
			.476	2.690	4.141	48.33	.701	1.011	.686
				3.086	4.450	59.00	.670	.950	.742
				3.496	4.759	75.50	.601	.922	.707
			.476	3.740	4.958	83.84	.599	.884	.767

f = .667
Δ = 3.42
Symbol λ

Reference 7, Page 9

Identification: CPCu → 2024T3

Assumed Materials: CPCu → 2024T4

Item	Proj	Targ.	d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	1.20	.559	.457	.273			.231	
E	1.195	.745		.693	.545			.522	
F	.49	.93		.936	.845			.641	
ρ	8.88	2.77		1.109	1.127			.775	
ν	.33	.33		1.289	1.341			.922	
Y	1800	3300		1.539	1.705			1.014	
U	2300	4780	.559	1.618	1.805			1.045	
ϵ	.40	.16							
H	1482	2874							
c	.1045	.252							
k	.934	.29							
Q	159	252							
C	4.42	6.25							
K	1.172	.730							
K'	1.771	1.104							
H'	1145	2260							

$f = 1.0$

$\Delta = 3.27$

Symbol ×

Reference 7, Page 10

Identification: CP Pb → 2024T3

Assumed Materials: CP Pb → 2024T4

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.04	120	.559	.375	.205				
E	.141	.745		.420	.341				
F	.059	.93		.632	.636				
ρ	11.20	2.77		.757	.909				
ν	.45	.33		1.04	1.227				
Y	.065	3300		1.16	1.250				
U	.166	4780	.559	1.22	1.318				
ϵ	.47	.16							
H	.265	2874							
c	.0322	.252							
k	.083	.29							
Q	.156	252							
C	2.16	6.25							
K	.470	.730							
K'	.535	1.104							
H'	.203	2260							

f = 1.0

Δ = 1.875

Symbol O

Reference 7, Page 11

Identification: CR Stl ⟶ 2024T3

Assumed Material: AISI 1015 ⟶ 2024T4

Item	Proj.	Targ.	d	v	l	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	110	120	.157	1.79	1.339				
E	2.11	.745	.559	3.44	.323				
F	.65	.93		.625	.595				
ρ	7.85	2.77		.958	.886				
ν	.30	.33		1.18	1.136				
Y	1800	3300		1.37	1.255				
U	3950	4780		1.53	1.436				
ϵ	.33	.16	.559	1.59	1.518				
H	1989	2874							
c	.155	.252							
k	.107	.29							
Q	299	252							
C	6.25	6.25							
K	1.758	.730							
K'	2.840	1.104							
H'	1515	2260							

f = 1.0

Δ = 3.83

Symbol ✳

Reference 3
Identification: HPAl → HPCu
Assumed Materials:

Item	Proj	Targ
B	.17	.50
E	.633	1.132
F	.93	.49
ρ	2.67	8.77
ν	.33	.33
Y	.077	.992
U	.553	2203
σ	.52	.58
H	2874	1482
c	.252	.1045
k	.54	.934
Q	252	159
C	5.86	4.32
K	.621	1.110
K'	.938	1.677
H'	2260	1145

f = 1.0

Δ = 1.412

Symbol ⊣

d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
.159	1.408	.498				
	1.417	.391				
	1.748	.598	1.741	.582	.276	7.667
	2.373	.668				
	2.777	.756				
	2.955	.838				
	3.134	.819	3.732	.676	.293	7.866
	3.318	.964				
	3.513	.957				
	3.603	1.033	4.229	.462	.343	3.925
	4.525	1.121				
	4.566	1.027				
	5.106	1.304	8.458	.521	.388	3.469
	6.020	1.524	8.458	.401	.396	2.554
	6.106	1.398	9.702	.539	.407	3.248
	6.539	1.568	10.946	.547	.415	3.183
.159	7.205	1.512	11.444	.464	.406	2.807
.318	.562	.161				
	1.465	.466				
	1.833	.611	1.151	.555	.223	11.185
	2.160	.737				
	2.324	.684	2.301	.702	.273	9.390
	2.990	.939	2.768	.470	.321	4.572
	3.828	1.040				
	3.912	1.074				
	4.357	1.241	5.504	.430	.389	2.850
	4.566	1.099				
	4.616	1.203				
	4.710	1.184				
	5.201	1.058				
	5.514	1.348	7.183	.473	.398	2.988
	6.141	1.446	8.116	.425	.399	2.664
	6.400	1.493	9.640	.455	.400	2.838
.318	6.944	1.629	11.630	.438	.412	2.585
.159	7.574	1.566	14.68	.578	.389	3.822
.159	8.819	2.000	17.17	.468	.467	2.146

Reference 3

Identification:

Assumed Materials: HPAl → HPCu

Item	Proj	Targ	d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.17	.50	.476	2.697	.901	2.327	.522	.330	4.798
E	.633	1.132		2.973	.827	2.689	.502	.313	5.122
F	.93	49		3.729	1.063	3.802	.470	.382	3.222
ρ	2.67	8.77		3.914	1.109	4.061	.484	.403	2.976
ν	.33	.33		4.396	1.357	6.472	.481	.426	2.650
Y	.077	.992		4.645	1.220	5.331	.489	.401	3.040
U	.552	2203		4.969	1.247	6.082	.525	.406	3.178
ϵ	.52	.58		5.309	1.384	7.501	.494	.421	2.780
H	2874	1482		5.328	1.327	6.963	.492	.410	2.933
c	.252	.1045		5.377	1.281	6.342	.480	.398	3.028
k	.54	.934		6.022	1.460	8.818	.492	.415	2.857
Q	252	159		6.106	1.438	8.159	.464	.409	2.774
C	6.21	4.32		6.262	1.283	6.583	.475	.393	3.082
K	.621	1.110		6.386	1.546				
K'	.938	1.677		6.563	1.514	9.559	.479	.418	2.739
H'	2260	1145		6.869	1.613	9.365	.419	.432	2.247
				6.896	1.653	11.303	.478	.435	2.530
			.476	6.950	1.588	10.579	.488	.429	2.651

f = 1.0

Δ = 1.412

Symbol ⊣

Reference 7, Page 23

Identification: AL ⟶ BeCu

Assumed Materials: 2024T3 ⟶ BeCu

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	217	.159	1.05	.078				
E	.745	1.195		1.85	.176				
F	.93	.49		2.26	.269				
ρ	2.77	8.23		2.32	.288				
ν	.33	.33		2.41	.221				
Y	3510	7000		4.00	.640				
U	4920	9420		4.14	.621				
ϵ	.18	.06	.159	4.18	.691				
H	2874	1482							
c	.252	.1045							
k	.29	.19							
Q	252	159							
C	6.25	4.58							

f = 1.0

$\triangle$ = 1.46

Symbol $\triangle$

Reference 7, Page 24

Identification: AL ⟶ BeCu

Assumed Materials: 2024T3 ⟶ BeCu

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	297	.159	1.15	.043				
E	.745	1.195		1.65	.125				
F	.93	49		2.21	.173				
ρ	2.77	8.23		2.34	.189				
ν	.33	.33		2.56	.189				
Y	3510	9490		2.90	.387				
U	4920	12300		3.48	.386				
ϵ	.18	.05	.159	4.55	.544				
H	2874	1482							
c	.252	.1045							
k	.29	.25							
Q	252	159							
C	6.25	4.42							

f = 1.0

Δ = 1.485

Symbol $\perp$

Reference 7, Page 27

Identification:

Assumed Materials: 2024T3 → Cu

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.20	.65	.318	4.66	1.312	5.504			2.441
E	.745	1.195	.318	5.00	1.408	6.188			2.228
F	.93	.49	1.016	1.29	.540	.566			3.585
ρ	2.77	8.90		1.98	.607	1.245			5.560
ν	.33	.33		3.09	.975	2.780			2.996
Y	3510	1800	1.016	3.34	1.030	3.000			1.144
U	4920	2300							
ϵ	.16	.40							
H	2874	1482							
c	.252	.1045							
k	.29	.934							
Q	252	159							
C	6.25	4.42							
K	.730	1.172							
K'	1.104	1.771							
H'	2260	1145							

f = .667

Δ = 1.44

Symbol ⊢

Reference 7, Page 29

Identification:

Assumed Materials: 2024T3 → Cu

Item	Proj	Targ	d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	.65	.157	3.962	1.129				
E	.745	1.195	.559	.329	.155	.047	.451	.189	13.851
F	.93	.49		.683	.273	.150	.434	.240	7.460
ρ	2.77	8.88		1.058	.414	.319	.417	.303	4.519
ν	.33	.33		1.274	.482	.479	.474	.331	4.295
Y	3510	1800		1.518	.545	.647	.448	.333	3.986
U	4920	2300		1.611	.577	.694	.425	.343	3.600
ϵ	.16	.40	.559	1.908	.682	.925	.409	.375	2.933
H	2874	14 82							
c	.252	.1045							
k	.29	.934							
Q	252	159							
C	6.25	4.42							
K	.730	1.172							
K'	1.104	1.771							
H'	2260	1145							

f = .667

Δ = 1.44

Symbol ⊢

Reference 3
Identification: HPCu → HPCu
Assumed Materials:

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.50	.50	.159	.806	.724	1.493	1.751	.257	26.43
E	1.132	1.132		1.146	.976	2.239	.646	.388	4.292
F	.49	.49		1.289	.970	1.990	1.243	.449	6.178
ρ	8.77	8.77		1.468	1.209				
ν	.33	.33		1.798	1.298	4.478	.681	.478	2.986
Y	.992	.992		2.373	1.682				
U	2203	2203		2.766	1.858				
ϵ	.58	.58		3.156	2.009				
H	1482	1482		4.030	2.375				
c	.1045	.1045		4.752	2.809				
k	.934	.934		5.038	2.772	35.57	.554	.546	1.863
Q	159	159		5.716	2.614	36.32	.511	.507	1.985
C	4.32	4.32	.159	6.775	3.024	51.75	.534	.540	1.835
K	1.110	1.110	.318	.782	.444				
K'	1.677	1.677		1.408	1.232				
H'	1145	1145		1.425	1.266				
				2.006	1.632				
				2.448	1.849	10.70	.503	.525	1.828
				3.287	2.151	16.70	.490	.545	1.649
				3.518	2.218	19.00	.472	.533	1.658
				3.583	2.136	17.79	.479	.503	1.893
				3.833	2.388	21.39	.496	.550	1.637
				3.978	2.416	22.48	.489	.543	1.660
				4.088	2.489	24.01	.489	.546	1.639
				4.718	2.312				
				4.908	2.700	33.40	.538	.548	1.793
			.318	6.304	3.078	48.82	.512	.558	1.648
			.476	2.926	2.102	14.91	.544	.578	1.627
				3.520	2.342	19.69	.527	.569	1.627
				4.038	2.486	25.04	.430	.549	1.627
				4.538	2.656	29.59	.524	.570	1.610
				4.605	2.730	30.88	.508	.573	1.544
				4.736	2.669	32.46	.528	.549	1.750
				5.270	2.919	38.58	.539	.591	1.546
				5.586	2.946	41.00	.538	.570	1.657
				5.967	2.959	44.71	.537	.550	1.776
			.318	6.146	3.104	44.78	.549	.606	1.498
			.318	6.440	3.057	50.69	.574	.569	1.775
			.159	7.574	3.308	69.66	.516	.518	1.924

f = 1.0

Δ = 2.0

Symbol •

Reference 7, Page 26

Identification: Cu $\longrightarrow$ Cu

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	.36	.318	1.17	1.232				
E	1.195	1.195		1.59	1.480				
F	.49	.49		2.09	1.728				
ρ	8.90	8.90		2.17	1.800				
ν	.33	.33	.318	3.33	2.288				
Y	2110	.703							
U	2680	2250							
ϵ	.35	.50							
H	1482	1482							
c	.1045	.1045							
k	.934	.934							
Q	159	159							
C	4.42	4.42							
K	1.172	1.172							
K'	1.771	1.771							
H'	1145	1145							

f = .67

Δ = 2.0

Symbol $\top$

Reference 7, Page 30,31

Identification: Cu → Cu

Assumed Materials:

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	.65	.559	.451	.500			.407	
E	1.195	1.195		.680	.645			.490	
F	.49	.49		.954	.836			.497	
ρ	8.90	8.90		1.149	.945			.495	
ν	.33	.33		1.317	1.068			.511	
Y	1800	1800		1.481	1.173			.506	
U	2300	2300	.559	1.582	1.236			.523	
ϵ	.40	.40	1.270	1.061	1.080			.614	
H	1482	1482		1.305	1.150			.558	
c	.1045	.1045		1.518	1.420			.664	
k	.934	.934	1.270	1.631	1.466				
Q	159	159	.302	2.03	1.563				
C	4.42	4.42	.317	1.35	1.120				
K	1.172	1.172		1.81	1.384				
K'	1.771	1.771		1.99	1.480				
H'	1145	1145		2.20	1.552				
				2.28	1.600				
			.317	3.06	1.952				

f = 1.0

Δ = 2.0

Symbol

Reference 7, Page 33

Identification: Pb → Cu

Assumed Materials:

Item	Proj	Targ	d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.04	.65	.559	.371	.423	.366	.494	.321	4.862
E	.141	1.195		.549	.650	.976	.715	.447	3.562
F	.059	.49		.768	.905	1.492	.623	.553	2.023
ρ	11.20	8.90		1.010	1.059	2.027	.636	.613	1.706
ν	.45	.33		1.085	1.159	2.392	.620	.637	1.536
Y	.065	1800		1.120	1.136	1.980	.579	.658	1.351
U	.166	2300	.559	1.213	1.295	2.627	.647	.731	1.209
ϵ	.47	.40							
H	.265	1482							
c	.0322	.1045							
k	.083	.934							
Q	.156	1.59							
C	2.16	4.42							
K	.470	1.172							
K'	.535	1.771							
H'	.203	1145							

f = 1.0

Δ = 1.385

Symbol ■

Reference 7, Page 36

Identification: Stl → Cu

Assumed Materials:

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.10	.65	.157	1.204	1.452				
E	2.11	1.195	.559	.372	.464	.216		.486	
F	.65	.49		.733	.745			.586	
ρ	7.85	8.90		.991	.955			.636	
ν	.30	.33		1.158	1.064			.632	
Y	1800	1800		1.426	1.282			.671	
U	3660	2300		1.481	1.323			.693	
ϵ	.33	.40	.559	1.640	1.432			.685	
H	1989	1482							
c	.107	.934							
Q	299	159							
C	6.25	4.42							
K	1.758	1.172							
K'	2.840	1.771							
H'	1515	1145							

f = .667

Δ = 2.248

Symbol ▲

Reference: 7, Page 43

Identification:

Assumed Materials: 2024T3 → Pb

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.20	.04	.157	.430	.669			.593	
E	.745	.141		1.103	.823			.425	
F	.93	.059		1.451	1.081			.515	
ρ	2.77	11.20	.157	1.762	1.032			.400	
ν	.33	.45	.559	.479	.495	.300	.508	.454	2.465
Y	3510	.065		.413	.491	.366	.535	.415	3.106
U	4920	.166		.890	.700	1.097	.500	.395	3.207
ϵ	.16	.47		1.219	.945	1.876	.438	.443	2.226
$\dot{H}$	2874	.265		1.411	.995	2.674	.482	.421	2.717
c	.252	.0322		1.640	1.059	3.452	.542	.431	2.906
k	.29	.083	.559	1.838	1.118	4.100	.568	.439	2.933
Q	252	.156							
C	6.25	2.16							
K	.730	.470							
K'	1.104	.535							
H'	2260	.203							

f = .667

Δ = 2.142

Symbol ✕

Reference: 7, Page 49,50

Identification: $Cu \rightarrow Pb$

Assumed Materials:

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	.04	.559	.430	1.259	1.895	.500	.729	.949
E	1.195	.141		.707	1.650	4.419	.459	.685	.985
F	.49	.059		.945	1.959	6.989	.420	.673	.930
ρ	8.90	11.20		1.079	1.964	9.288	.456	.608	1.229
ν	.33	.45		1.219	2.114	14.429	.470	.554	1.530
Y	1800	.065		1.487	2.527	18.200	.461	.639	1.129
U	2300	.166	.559	1.649	2.318	19.983	.464	.537	1.607
ϵ	.40	.47	.476	1.19	2.309	15.345	.444	.598	1.052
H	1482	.265		1.29	2.331	13.342	.423	.634	1.052
c	.1045	.0322		1.31	2.416	18.989	.473	.593	1.347
k	.934	.083		1.31	2.416	18.841	.412	.556	1.336
Q	159	.156		1.52	2.539	23.097	.454	.567	1.406
C	4.42	2.16		1.53	2.501	23.857	.455	.547	1.526
K	1.172	.470		1.60	2.560	22.939	.431	.562	1.362
K'	1.771	.535		1.60	2.560	24.914	.463	.559	1.480
H'	1145	.203		1.64	2.459	27.352	.468	.505	1.842
				1.79	2.752	31.451	.481	.565	1.509
				1.92	2.853	34.186	.437	.546	1.466
				1.97	2.853	34.186	.444	.550	1.466
			.476	1.99	2.832	35.095	.431	.529	1.538
			.302	2.03	2.798				
f = .667				2.11	2.782				
Δ = 3.585				2.19	2.882				
Symbol γ			.302	1.97	2.728				

Reference 7, Page 53, 54, 55

Identification: Pb → Pb

Assumed Materials:

Item	Proj	Targ
B	.04	.04
E	..141	.141
F	.059	.059
ρ	11.20	11.20
ν	.45	.45
Y	.065	.065
U	.166	.166
ϵ	.47	.47
H	.265	.265
c	.0322	.0322
k	.083	.083
Q	.156	.156
C	2.16	2.16
K	.470	.470
K'	.535	.535
H'	.203	.203

d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
.559	.417	1.118	2.364	.556	.572	1.692
	.613	1.350	4.803	.515	.512	1.957
	.830	1.595	7.824	.530	.524	1.926
	.991	1.914	11.49	.489	.547	1.643
	1.119	2.091	13.65	.495	.575	1.496
	1.213	2.095	14.64	.505	.562	1.592
.559	1.274	2.264	17.36	.491	.572	1.498
.277	2.147	3.046				
.317	.312	.880				
	.361	.896				
	.614	1.440				
	.959	2.000				
	1.387	2.504				
	1.841	2.808				
	2.086	3.112				
.317	2.099	3.200				
.476	.468	1.205	3.523	.518	.508	2.009
	.475	1.264	3.802	.480	.505	1.879
	.648	1.632	7.306	.482	.536	1.680
	.690	1.675	7.853	.519	.558	1.667
	.767	1.552	6.036	.484	.548	1.613
	.879	2.000	12.137	.504	.577	1.517
	.960	2.165	15.49	.469	.555	1.525
	1.120	2.501	21.72	.464	.578	1.387
	1.241	2.560	26.43	.510	.569	1.574
	1.269	2.416	27.50	.523	.518	1.945
	1.289	2.517	24.16	.479	.563	1.512
	1.300	2.517	25.38	.481	.550	1.588
	1.311	2.624	29.33	.478	.543	1.619
	1.340	2.437	24.16	.494	.545	1.665
	1.449	2.560	27.96	.484	.539	1.664
	1.480	2.624	32.52	.495	.525	1.796
	1.480	2.667	34.80	.504	.525	1.832
	1.500	2.832	35.25	.477	.555	1.549
	1.640	2.981	37.53	.442	.559	1.414
.476	2.140	3.131	50.74	.446	.519	1.651

f = 1.0

Δ = 2.0

Symbol ϕ

Reference: 7, Page 62

Identification: Stl → Pb

Assumed Materials: AISI 1015 → Pb

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	110	.04	.157	.460	2.274			1.410	
E	2.11	.141		1.103	2.194			.716	
F	.65	.059		1.451	2.500			.705	
ρ	7.85	11.20	.157	2.018	2.355			.503	
ν	.30	.45	.559	.346	1.468	1.060	.448	1.154	.336
Y	1800	.065		.607	1.741	2.842	.407	.870	.539
U	3660	.166		.887	1.927	5.582	.419	.731	.780
ϵ	.33	.47		1.109	2.218	8.819	.415	.718	.808
H	1989	.265		1.332	2.505	11.73	.414	.745	.747
c	.155	.0322	.559	1.524	2.600	15.29	.413	.689	.871
k	.107	.083							
Q	299	.156							
C	6.25	2.16							
K	1758	.470							
K'	2.840	.535							
H'	1515	.203							

$f = .667$

$\Delta = 4.24$

Symbol -o-

Reference 7, Page 89

Identification: 2024T3 → Stl

Assumed Materials: 2024T3 → AISI 1015

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	110	.157	3.810	.452				
E	.745	2.11	.559	.135	0				
F	.93	.65		.625	.041			.060	
ρ	2.77	7.85		.878	.114			.132	
ν	.33	.30		1.140	.159	.047	.232	.140	11.63
Y	3510	1800		1.405	.223	.0658	.160	.163	6.030
U	4920	3660		1.774	.286	.2817	.436	.191	12.01
ϵ	.16	.33	.559	1.969	.341	.3658	.425	.214	9.316
H	2874	1989	1.270	.741	.070			.081	
c	.252	.155		1.344	.200			.149	
k	.29	.107		1.686	.230			.158	
Q	252	299		1.899	.380			.244	
C	6.25	6.25	1.270	2.192	.418			.255	
K	.730	1.758							
K'	1.104	2.840							
H'	2260	1515							

$f = .667$
$\Delta = 1.353$

Symbol ∇

Reference 7, Page 94
Identification: 2024T3 → Stl
Assumed Materials: 2024T3 → AISI 1030

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	130	1.106	1.50	.285	.243			10.46
E	.745	2.11		1.78	.297	.334			12.71
F	.93	.65		1.87	.312	.326			10.74
ρ	2.77	7.85		1.94	.350	.439			10.20
ν	.33	.30		1.95	.315	.398			12.73
Y	3510	3660		2.18	.355				
U	4920	5410		2.44	.385	.592			10.39
ϵ	.18	.30		2.44	.405	.543			8.210
H	2874	1989		2.64	.450	.764			8.392
c	.252	.155		2.64	.462	.848			8.563
k	.29	.107		2.71	.475	.769			7.153
Q	252	299		3.04	.455	.857			9.117
C	6.25	6.25		3.04	.512	.973			7.213
K	.730	1.758		3.23	.555	.969			5.663
K'	1.104	2.840		3.25	.508	1.104			8.429
H'	2260	1515		3.29	.532	1.020			6.758
				3.44	.570	1.287			6.955
				3.53	.585	1.334			6.675
				3.70	.585	1.241			6.208
				3.70	.615	1.397			6.001
				3.77	.640				
				3.77	.680	1.544			4.907
				3.86	.692	1.640			4.930
			1.016	4.00	.657	1.518			5.341

f = 1.0
Δ = 1.353
Symbol ▼

Reference 7, Page 90

Identification: Cu $\rightarrow$ Stl

Assumed Materials: Cu $\rightarrow$ AISI 1015

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	1,10	.559	.222	.041			.069	
E	1.195	2.11		.640	.236	.150	.394	.186	11.39
F	.49	.65		.847	.486	.394	.386	.334	3.419
ρ	8.90	7.85		1.106	.536			.358	
ν	.33	.30		1.271	.659			.403	
Y	1,800	1,800		1.457	.768			.445	
U	2,670	3,660	.559	1.600	.864			.452	
ϵ	.40	.33	1.270	.686	.320	.304	.543	.242	9.305
H	1,482	1,989		1.045	.648	.704	.447	.415	2.587
c	.1045	.155		1.289	.826	1.064	.406	.464	1.888
k	.934	.107	1.270	1.637	.950			.490	
Q	1,59	2,99							
C	4.42	6.25							
K	1.172	1.758							
K'	1.771	2.840							
H'	1,145	1,515							

f = 1.0

Δ = 1.801

Symbol ◇

Reference 7, Page 91

Identification: Pb → Stl

Assumed Materials: Pb → AISI 1015

Item	Proj	Targ	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.04	110	.559	.261	.055			.075	
E	.141	2.11		.419	.205			.155	
F	.059	.65		.963	.441			.313	
ρ	11.20	7.85		.957	.459			.289	
ν	.45	.30		1.024	.564			.413	
Y	.065	1800		1.207	.736			.491	
U	.166	3660	.559	1.286	.850			.505	
ϵ	.47	.33							
H	.265	1989							
c	.0322	.155							
k	.083	.107							
Q	.156	299							
C	2.16	6.25							
K	.470	1.758							
K'	.535	2.840							
H'	.203	1515							

f = 1.0

Δ = 1.309

Symbol ◆

Reference: 7, Page 92

Identification: Stl → Stl

Assumed Materials: AISI 1015 → AISI 1015

Item	Proj	Targ	d	v	Π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	110	110	.157	2.707	1.371			.125	
E	2.11	2.11	.559	.315	.091				
F	.65	.65		.631	.250			.212	
ρ	7.85	7.85		.817	.409			.310	
ν	.30	.30		1.125	.573			.382	
Y	1800	1800		1.353	.691			.447	
U	3660	3660		1.515	.750			.458	
ϵ	.33	.33	.559	1.682	.818			.474	
H	1989	1989	1.270	.732	.592	.362	.584	.580	1.746
c	.155	.155		1.096	.798	.592	.357	.554	1.167
k	.107	.107		1.353	.832	1.008	.498	.533	1.748
Q	299	299		1.551	.950	1.303	.475	.559	1.523
C	6.25	6.25	1.270	1.710	1.000	1.551	.490	.562	1.552
K	1.758	1.758							
K'	2.840	2.840							
H'	1515	1515							

$f = 1.0$

$\Delta = 2.0$

Symbol $\varnothing$

APPENDIX B

TABULATED DATA OBTAINED DURING THIS PROGRAM

The crater depth and volume data obtained during the experimental part of this program, using the hypervelocity launching facilities to the Martin Company and the Denver Research Institute, at Denver, Colorado, are tabulated in this appendix.

The data involving aluminum alloys, copper, lead, and steel were obtained specifically to fill out gaps in the data taken from the literature, or to extend the velocity ranges of such data. Data including zinc were obtained specifically to resolve the question of the statistical significance of the Poisson's ratios of the projectile and target materials, as described in Chapter II.

All symbols, except the data symbols themselves, are the same as in Appendix A.

The accuracy of a few velocity measurements is doubtful. These velocities are noted by an asterisk. However, the data are useful to investigate the dependence of crater volume on material properties.

Identification: 2024T3 → 2024T4

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	120	.345	1.13	.331	.293	.465	.24	305
E	.745	.745	.317	2.10	.600	1.41	.576	.297	6.50
F	.93	.93	.317	2.19	.770	1.74	.530	.373	3.79
ρ	2.77	2.77	.317	4.20	1.02	7.55	1.11	.395	7.15
ν	.33	.33	.328	4.80	1.44	5.12	.530	.557	1.16
Y	3510	3300	.338	5.58	1.61	6.45	.609	.626	1.55
U	4920	4780	.323	5.95	1.37	4.13	.532	.570	1.65
ϵ	.16	.16							
H	2874	2874							
c	.252	.252							
k	.29	.29							
Q	252	252							
C	6.25	6.25							
K	.730	.730							
K'	1.104	1.104							
H'	2260	2260							

$f = 1.0$

$\triangle = 3.27$

Symbol +

Identification: CPCu→2024T4

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	120	.328	.295	.194	.284	1.25	.179	39.1
E	1.195	.745	.325	.335	.344	.416	.845	.271	11.4
F	.49	.93	.323	.634	.511	.536	.650	.404	4.00
ρ	8.90	2.77	.333	1.09	.994	1.62	.858	.718	1.66
ν	.33	.33	.338	2.09	1.13	2.60	1.13	.790	1.81
Y	1800	3300	.326	2.13	2.07	5.50	.675	1.05	.624
U	2300	4780	.328	2.51	2.22	7.67	.628	.950	.696
ϵ	.40	.16	.325	2.97	1.83	4.63	.570	.872	.752
H	1482	2874	.325	6.00	2.81	14.1	.921	1.20	.640
c	.1045	.252	.318	6.37	3.00	14.6	1.11	1.27	.966
k	.934	.29	.242	6.70	4.00	55.9	.513	.76	.906
Q	159	252							
C	4.42	6.25							
K	1.172	.730							
K'	1.771	1.104							
H'	1145	2260							

$f = 1.0$

$\Delta = 3.27$

Symbol ×

Identification: Pb $\longrightarrow$ 2024T4

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.04	1.20	.322	.564	.536	.750	.584	.347	4.86
E	.141	.745	.320	.610	.357	.639	1.02	.270	14.0
F	.059	.93	.320	.823	.698	2.21	1.12	.415	6.52
ρ	11.20	2.77	.319	1.38	1.39	2.46	.593	.806	.910
ν	.45	.33	.319	1.52	1.43	3.39	.550	.693	1.15
Y	.065	3.300	.322	2.22	1.77	4.80	.472	.750	.855
U	.166	4.780							
ϵ	.47	.16							
H	.265	2.874							
c	.0322	.252							
k	.083	.29							
Q	.156	2.52							
C	2.16	6.25							
K	.470	.730							
K'	.535	1.104							
H'	.203	2.260							

f = 1.0

Δ = 1.875

Symbol O

Identification: Stl ⟶ 2024T4

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	110	120	.356	.533	.428	.400	.793	.395	5.07
E	2.11	.745	.353	.975	.979	1.14	.633	.724	1.21
F	.65	.93	.356	2.19	2.11	4.88	.647	1.11	.521
ρ	7.85	2.77	.346	2.44	2.35	6.75	.680	1.14	.520
ν	.30	.33	.343	5.28	2.52	11.1	.625	.980	.650
Y	1800	3300	.346	5.60	2.48	8.68	.624	1.01	.604
U	3950	4780	.348	5.78	2.55	10.2	.611	1.00	.611
ϵ	.33	.16							
H	1989	2874							
c	.155	.252							
k	.107	.29							
Q	299	252							
C	6.25	6.25							
K	1.758	.730							
K'	2.840	1.104							
H'	1515	2260							

f = 1.0

Δ = 3.83

Symbol ✳

Identification: Zn → 2024T4

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.34	120	.340	.362	.187	.204	1.35	.208	31.1
E	.985	.745	.338	.635	.413	.258	.433	.344	3.68
F	.244	.93	.339	.983	.786	1.21	.743	.547	2.47
ρ	7.15	2.77	.341	1.22	.990	1.51	.760	.703	1.54
ν	.43	.33	.318	2.34	1.86	5.28	.712	.933	.821
Y	.379	3300	.318	2.77	2.08	6.84	.594	.882	.762
U	.632	4780	.325	3.36	2.50	9.64	.654	1.03	.614
ε	.01	.16	.313	3.75*	1.72	4.90	.562	.800	.882
H	.541	2874	.318	3.46	2.10	8.08	.664	.867	.882
c	.0977	.252	.333	4.35*	1.62	4.60	.735	.823	1.09
k	.0957	.29	.330	5.40	2.27	8.90	.689	.953	.760
Q	.627	252	.231	7.64	4.00	60.5	.555	.770	.940
C	3.87	6.25							
K	2.34	.730							
K'	2.81	1.104							
H'	.420	2260							

f = 1.0

$\triangle$ = 1.61

Symbol Ƶ

Identification: 2024T3 → CPCu

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	120	.65	.312	.353	.122	.164	2.25	.158	90.5
E	.745	1.195	.318	.625	.176	.156	.760	.163	28.7
F	.93	.49	.323	1.09	.268	.295	.613	.199	15.5
ρ	2.77	8.90	.318	1.54	.480	.311	.248	.297	2.83
ν	.33	.33	.320	2.10	.627	.912	.042	.336	2.43
Y	3510	1800	.333	5.36	1.17	4.60	.532	.422	2.90
U	4920	2300	.323	5.45	1.19	5.03	.524	.417	3.01
ϵ	.16	.40	.325	7.02	1.60	11.1	.557	.455	3.69
H	2874	1482							
c	.252	.1045							
k	.29	.934							
Q	252	159							
C	6.25	4.42							
K	.730	1.172							
K'	1.104	1.771							
H'	2260	1145							

f = .67

Δ = 1.44

Symbol ⊢

Identification: CPCu $\rightarrow$ CPCu

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	.65	.323	.381	.275	.208	.507	.226	9.84
E	1.195	1.195	.326	.661	.608	.662	.482	.404	2.96
F	.49	.49	.315	1.61	1.22	4.80	.778	.536	2.71
ρ	8.90	8.90	.323	2.82*	1.02	2.95	.557	.448	2.78
ν	.33	.33	.333	3.60*	1.19	2.78	.583	.510	2.25
Y	1,800	1,800	.328	4.61*	1.08	2.27	4.64	.505	1.82
U	2,300	2,300	.318	4.95	1.62	8.08	.516	.517	1.93
ϵ	.40	.40	.330	5.43	1.44	6.12	.585	.535	2.05
H	1,482	1,482	.338	9.00*	.970	2.84	.585	.433	3.13
c	.1045	.1045	.269	6.20	2.64	32.3	.595	.583	1.75
k	.934	.934	.264	6.80	3.13	46.7	.553	.602	1.53
Q	159	159							
C	4.42	4.42							
K	1.172	1.172							
K'	1.771	1.771							
H'	1,145	1,145							

f = 1.0

Δ = 2.0

Symbol

Identification: Pb → CPCu

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.04	.65	.322	.350	.426	.452	.469	.284	5.81
E	.141	1.195	.333	.608	.587	.945	.594	.357	4.67
F	.059	.49	.330	.985	.845	2.22	.630	.415	3.67
ρ	11.20	8.90	.323	2.38	2.29	6.18	3.49	.820	.520
ν	.45	.33							
Y	.065	1.800							
U	.166	2.300							
ϵ	.47	.40							
H	.265	1.482							
c	.0322	.1045							
k	.083	.934							
Q	.156	1.59							
C	2.16	4.42							
K	.470	1.172							
K'	.535	1.771							
H'	.203	1.145							

$f = 1.0$

$\Delta = 1.385$

Symbol ■

Identification: Stl ⟶ CPCu

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.10	.65	.308	.311	.445	.505	.666	.415	3.88
E	2.11	1.195	.348	.380	.401	.119	.328	.424	1.83
F	.65	.49	.305	.603	.541	.808	.794	.394	5.34
ρ	7.85	8.90	.353	1.05	894	1.37	.558	.539	1.92
ν	.30	.33	.353	1.52	1.30	2.41	.398	.600	1.11
Y	1.800	1.800	.343	2.14*	.630	.495	.419	.460	1.98
U	3.950	2.300	.353	4.44*	.734	1.59	.664	.406	4.02
ϵ	.33	.40	.356	3.02	1.89	10.9	.538	.578	1.61
H	1.989	1.482	.340	4.88	1.76	10.0	.525	.614	1.45
c	.155	.1045	.348	5.04	1.89	11.8	.564	.568	1.75
k	.107	.934	.343	6.82*	1.11	2.47	.526	.540	1.81
Q	2.99	1.59	.270	7.02	2.83	46.7	.137	.546	2.08
C	6.25	4.42							
K	1.758	1.172							
K'	2.840	1.771							
H'	1.515	1.145							

f = .67

Δ = 4.24

Symbol ▲

Identification: Zn → CPCu

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.34	.65	.338	.311	.286	.260	.878	.281	11.1
E	.985	1.195	.328	.634	.480	.568	.723	.376	5.13
F	.244	.49	.328	2.51*	.697	.855	.574	.445	2.51
ρ	7.15	8.90	.350	2.72*	.710	.910	.482	.432	2.59
ν	.43	.33	.327	.945	.816	1.14	.471	.473	2.10
Y	.379	1.800	.322	4.10*	.975	1.80	.596	.538	2.07
U	.632	2.300	.328	3.47*	.930	1.99	.616	.500	2.46
ε	.10	.40	.318	4.55*	.920	2.18	.620	.470	2.80
H	.541	1.482	.318	1.93	1.20	4.67	.612	.477	2.71
c	.0977	.1045	.323	2.20	1.54	5.94	.630	.620	1.64
k	.0957	.934	.322	4.68*	1.54	6.31	.585	.582	1.73
Q	.627	1.59	.315	3.12	1.92	7.10	.482	.697	.995
C	3.87	4.42	.320	3.91	1.65	7.62	.490	.523	1.80
K	2.34	1.172	.323	4.06	1.73	11.6	.594	.516	2.23
K'	2.81	1.771	.282	7.50	2.70	35.6	.557	.556	1.80
H'	.420	1.145							

f = 1.0

Δ = 1.815

Symbol

Identification: 2024T3 → Pb

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.20	4.0	.320	.305	.405	.304	.486	.325	4.80
E	.745	.141	.330	.412	.770	.555	.510	.658	1.22
F	.93	.059	.318	1.10	.850	2.18	.736	.453	3.59
ρ	2.77	11.20	.320	5.32*	1.31	3.95	.490	.527	1.77
ν	.33	.45	.310	1.88	1.23	6.66	.615	.413	3.62
Y	3.510	.065	.318	6.05*	1.68	9.98	.491	.483	2.10
U	4.920	.166	.323	3.06	1.69	11.0	.478	.455	2.30
ϵ	.16	.47	.323	6.04*	1.81	12.8	.484	.474	2.15
H	2.874	.265	.318	4.90	2.02	14.4	.411	.487	1.80
c	.252	.0322	.323	5.12	2.13	19.2	.485	.491	2.01
k	.29	.083							
Q	2.52	.156							
C	6.25	2.16							
K	.730	.470							
K'	1.104	.535							
H'	2.260	.203							

f = .67

Δ = 2.142

Symbol

Identification: CPCu $\longrightarrow$ Pb

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	.04	.327	.360	1.55	2.29	.573	.97	.610
E	1.195	.141	.318	5.02*	1.88	7.50	.395	.592	1.13
F	.49	.059	.307	2.83*	2.11	9.70	.718	.835	1.03
ρ	8.90	11.20	.323	4.03*	2.22	15.3	.438	.562	1.39
ν	.33	.45	.325	5.73	3.09	49.3	.443	.513	1.68
Y	1800	.065	.333	6.72	3.23	59.6	.482	.520	1.78
U	2300	.166	.325	3.25	3.36	61.4	.498	.555	1.58
ϵ	.40	.47	.330	5.80	3.00	65.4	.482	.446	2.42
H	1482	.265							
c	.1045	.0322							
k	.934	.083							
Q	1.59	.156							
C	4.42	2.16							
K	1.172	.470							
K'	1.771	.535							
H'	1145	.203							

f = .67

Δ = 3.585

Symbol γ

Identification: Stl ⟶ Pb

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.10	.04	.361	.314	1.55	2.13	.900	.945	.572
E	2.11	.141	.338	2.48*	2.28	10.3	.524	.774	.878
F	.65	.059	.315	4.27*	2.58	11.3	.422	.794	.670
ρ	7.85	11.20	.277	3.48	2.66	14.2	.506	.809	.772
ν	.30	.45	.383	1.28	2.59	18.8	.470	.66	1.08
Y	1.800	.065	.330	4.79*	2.40	19.5	.496	.595	1.41
U	3.950	.166	.322	6.37*	2.53	23.4	.496	.590	1.43
ε	.33	.47	.343	3.14	2.81	30.9	.483	.590	1.39
H	1.989	.265	.318	5.00	3.00	43.8	.519	.564	1.63
c	.155	.0322	.274	2.40	2.87	45.0	.553	.539	1.90
k	.107	.083	.285	6.82	2.96	51.7	.562	.520	2.08
Q	2.99	.156	.275	5.02	3.15	52.2	.523	.557	1.69
C	6.25	2.16	.338	3.94	3.21	61.0	.504	.524	1.84
K	1.758	.470	.248	5.75	3.34	65.0	.490	.522	1.78
K'	2.840	.535	.274	5.68	3.78			.537	
H'	1.515	.203	.298	7.80	3.63	84.2	.502	.532	1.77

f = .67

Δ = 4.24

Symbol -o-

Identification: Zn $\longrightarrow$ Pb

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.34	.04	.328	.169	.500	.341	.591	.450	2.93
E	.985	.141	.323	.169	.590	.345	.595	.586	1.74
F	.244	.059	.320	.396	1.09	1.82	.618	.658	1.43
ρ	7.15	11.20	.338	.692	1.50	3.93	.480	.645	1.15
ν	.43	.45	.254	.932*	1.18	4.24	.510	.444	2.59
Y	.379	.065	.338	.986	1.77	8.00	.479	.574	1.45
U	.632	.166	.333	3.79*	1.94	13.5	.510	.524	1.86
ϵ	.10	.47	.320	4.13*	2.14	20.0	.562	.524	2.04
H	.541	.265	.338	1.91	2.60	29.1	.526	.563	1.66
c	.0977	.0322	.328	2.26	2.59	32.6	.509	.522	1.87
k	.0957	.083	.325	6.35	2.73	42.4	.515	.500	2.06
Q	.627	.156	.320	4.95	3.18	51.5	.449	.526	1.62
C	3.87	2.16							
K	2.34	.470							
K'	2.81	.535							
H'	.420	.203							

$f = .67$

$\Delta = 3.12$

Symbol ~~? ~~

Identification: 2024T3 ⟶ Stl

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.20	1.10	.325	1.58	.195			.143	
E	.745	2.11	.320	3.25	.586	.770	.335	.298	3.76
F	.93	.65	.323	5.19	.920	2.97	.493	.360	3.80
ρ	2.77	7.85	.328	6.74	.915	2.56	.537	.401	3.35
ν	.33	.30							
Y	3.510	1.800							
U	4.920	3.950							
ϵ	.16	.33							
H	2.874	1.989							
c	.252	.155							
k	.29	.107							
Q	2.52	2.99							
C	6.25	6.25							
K	.730	1.758							
K'	1.104	2.840							
H'	2.260	1.515							

f = .67

Δ = 1.353

Symbol ∇

Identification: CPCu ➡ Stl

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	1.10	.330	1.09	.770	2.36	.424	.286	5.19
E	1.195	2.11	.330	1.80	.760	1.53	.504	.375	3.46
F	.49	.65	.325	1.74	.993	2.63	.665	.500	2.68
ρ	8.90	7.85	.325	2.18	1.09	3.51	.532	.442	2.73
ν	.33	.30	.328	2.24	1.06	2.86	.533	.469	2.43
Y	1.800	1.800	.330	4.45*	1.00	2.50	.588	.495	2.50
U	2.300	3.950	.341	5.04*	1.20	4.66	1.28	.690	2.70
ϵ	.40	.33	.336	5.09*	1.27	4.73	.557	.473	2.36
H	1.482	1.989	.263	6.78	2.55	22.5	.494	.555	1.60
c	.1045	.155							
k	.934	.107							
Q	1.59	2.99							
C	4.42	6.25							
K	1.172	1.758							
K'	1.771	2.840							
H'	1.145	1.515							

f = 1.0

Δ = 1.80

Symbol ◊

Identification: Pb → Stl

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.04	110	.331	.469	.153	.305	1.49	.133	84.0
E	.141	2.11	.325	.803	.508	.585	.469	.325	4.45
F	.059	.65	.325	2.10	.914	2.50	.488	.389	3.23
ρ	11.20	7.85							
ν	.45	.30							
Y	.065	1800							
U	.166	3950							
ϵ	.47	.33							
H	.265	1989							
c	.0322	.155							
k	.083	.107							
Q	.156	299							
C	2.16	6.25							
K	.470	1.758							
K'	.535	2.840							
H'	.203	1515							

f = 1.0

Δ = 1.31

Symbol ◆

Identification: Stl ➡ Stl

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.10	1.10	.350	1.01	.508	.558	.616	.380	4.27
E	2.11	2.11	.282	4.27*	.657	1.33	.980	.456	4.72
F	.65	.65	.348	2.17	.790	1.91	.552	.378	3.86
ρ	7.85	7.85	.356	3.84*	.800	1.89	.563	.390	3.70
ν	.30	.30	.330	4.05*	1.16	3.34	.538	.504	2.13
Y	1800	1800	.353	2.87	1.37	3.98	.486	.560	1.56
U	3950	3950	.348	5.61*	1.13	4.05	.595	.463	2.78
ϵ	.33	.33	.345	6.10*	1.41	6.32	2.30	1.00	2.30
H	1989	1989	.254	7.10	2.20	22.4	.640	.550	2.11
c	.155	.155	.157	8.57	2.42	30.8	.780	.600	2.17
k	.107	.107							
Q	299	299							
C	6.25	6.25							
K	1.758	1.758							
K'	2.840	2.840							
H'	1515	1515							

f = 1.0

Δ = 2.0

Symbol $\varnothing$

Identification: Zn → Stl

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.34	110	.341	.762	.253	.325	.855	.206	20.2
E	.985	2.11	.342	.762	.297	.350	.696	.228	13.3
F	.244	.65	.326	.896	.389	.314	.384	.267	5.36
ρ	7.15	7.85	.320	2.97*	.516	.606	.395	.298	4.45
ν	.43	.30	.302	1.60	.755	1.45	.597	.423	3.36
Y	.379	1800	.317	2.29	.785	1.56	.518	.400	3.23
U	.632	3950	.318	2.28	1.02	2.81	.550	.459	2.65
ε	.10	.33	.313	3.52	1.20	3.28	.500	.516	1.87
H	.541	1989	.333	5.40	1.22	3.52	.627	.570	2.38
c	.0977	.155	.308	6.50	1.25	3.93	.562	.530	2.00
k	.0957	.107							
Q	.627	299							
C	3.87	6.25							
K	2.34	1.758							
K'	2.81	2.840							
H'	.420	1515							

f = 1.0

Δ = 1.65

Symbol

Identification: 2024T3 ⟶ Zn

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	1.20	.34	.328	.396	.023			.025	
E	.745	.985	.318	.545	.080			.077	
F	.93	.200	.325	.693	.117			.094	
ρ	2.77	7.15	.318	1.09	.200	.311	.985	.152	39.1
ν	.33	.43	.320	1.60	.318	.606	.753	.199	19.1
Y	3.510	.379	.318		.935	2.66	.556	.415	3.23
U	4.920	.632	.328	3.75	1.09	3.72	.528	.428	3.11
ϵ	.16	.10	.333	4.35	1.62	4.60	.734	.823	1.09
H	2.874	.541	.333	5.04	1.22	5.15	.665	.485	2.83
c	.252	.0977	.320	5.83	1.41	6.06	.592	.524	2.16
k	.29	.0957	.318	5.18	1.32	7.00	.645	.460	3.06
Q	2.52	.627							
C	6.25	3.87							
K	.730	2.34							
K'	1.104	2.81							
H'	2.260	.420							

$f = .67$

$\Delta = 2.85$

Symbol ⊗

Identification: CPCu ⟶ Zn

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.65	.34	.307	.367	.331	.345	1.06	.333	9.52
E	1.195	.985	.324	.545	.414			.327	
F	.49	.244	.329	.914	.942	1.61	.652	.578	2.02
ρ	8.90	7.15	.339	1.85	1.55	5.90	.540	.580	1.60
ν	.33	.43	.338	4.12	1.74	7.74	.574	.627	1.46
Y	1.800	.379	.330	5.32	2.08	11.7	.657	.711	1.52
U	2.300	.632	.263	7.25	2.58	7.70	.61	1.15	.458
e	.40	.10							
H	1.482	.541							
c	.1045	.0977							
k	.934	.0957							
Q	1.59	.627							
C	4.42	3.87							
K	1.172	2.34							
K'	1.771	2.81							
H'	1.145	.420							

f = .67

Δ = 2.23

Symbol ⊡

Identification: Pb → Zn

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.04	.34	.324	.346	.204	.294	.740	.146	34.7
E	.141	.985	.324	.564	.470	.883	.763	.300	8.50
F	.059	.244	.316	1.01	1.22	2.60	.439	.550	1.45
ρ	11.20	7.15	.325	2.29	1.52	7.55	10.8	.557	2.13
ν	.45	.43							
Y	.065	.379							
U	166	632							
ϵ	.47	.10							
H	.265	.541							
c	.0322	.0977							
k	.083	.0957							
Q	.156	.627							
C	2.16	3.87							
K	.470	2.34							
K'	.535	2.81							
H'	.203	.420							

f = 1.0

Δ = 1.475

Symbol

Identification: Stl ⟶ Zn

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2 h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	110	.34	.317	.335	.400	.313	.970	.446	34.7
E	2.11	.985	.443	.565	.490	.230	.710	.419	4.05
F	.65	.244	.316	.914	1.09	1.62	.575	.675	1.27
ρ	7.85	7.15	.336	1.89	1.40	4.47	.634	.617	1.66
ν	.30	.43	.351	2.45	1.74	9.78	.720	.624	1.85
Y	1800	.379	.356	3.53	1.69	7.34	.605	.630	1.52
U	3950	.632	.337	5.10	2.03	11.3	.572	.655	1.33
ϵ	.33	.10	.340	7.96	1.89	11.6	.600	.632	1.51
H	1989	.541	.157	8.30	3.71	115.	.870	.622	2.25
c	.155	.0977							
k	.107	.0957							
Q	299	.627							
C	6.25	3.87							
K	1.758	2.34							
K'	2.840	2.81							
H'	1515	.420							

f = .67

$\triangle$ = 2.54

Symbol ⊠

Identification: Zn ⟶ Zn

Item	Proj.	Targ.	d	v	π_1	$\frac{V}{d^3}$	$\frac{V}{D^2h}$	$\frac{h}{D}$	$\frac{V}{h^3}$
B	.34	.34	.325	.355	.172			.157	
E	.985	.985	.312	.545	.278	.328	6.45	.674	14.2
F	.244	.244	.335	1.01	.796	1.32	.585	.470	2.63
ρ	7.15	7.15	.323	2.39	1.18	3.83	.650	.527	2.76
ν	.43	.43	.310	3.76	1.46	4.34	.530	.615	1.41
Y	.379	.379	.313	5.10	1.93	11.3	.593	.607	1.61
U	.632	.632	.282	7.15	2.34	18.7	.266	.428	1.45
ϵ	.10	.10							
H	.541	.541							
c	.0977	.0977							
k	.0957	.0957							
Q	.627	.627							
C	3.87	3.87							
K	2.34	2.34							
K'	2.81	2.81							
H'	.420	.420							

f = 1.0

Δ = 2.00

Symbol ⊗

APPENDIX C

TABULATED DATA FOR THIN AND BUMPERED TARGETS

The data for the ballistic limit (B.L.) of thin sheets, and the crater depth in bumpered targets are presented in this appendix. The ballistic limit is defined as that velocity at which a given configuration just defeats a given projectile, that is, a marginal puncture occurs. Data for complete punctures were useless for the purposes of this study, and were not collected.

These data were taken from the technical literature, with the exception of few shots performed at Denver Research Institute under an unrelated Martin contract.

The bumpered target data were collected in punched card form from Ref 11 and 13 and are presented as a tab run. Because of the limitations of the symbols available, the headings do not agree exactly with the symbols of Table 1 and are defined below.

RHO P = ρ_p, projectile density;

VEL = v, impact velocity;

T1EQ = (t_1/d), actual bumper thickness multiplied by the ratio of bumper material density to 2.79, divided by projectile diameter;

L/D = (ℓ/d), scale standoff distance;

H/D = (h/d), scale crater depth in second sheet;

H/T2 = (h/t_2), ratio of depth of damage in second sheet to thickness of second sheet;

FACT = factor from Fig. 8 which corrects second sheet damage depth to thickness which would just be punctured under the same conditions. This factor is always greater than 1.0 but the 1 was omitted on some cards;

TP/D = (t_p/d) = FACT x H/D, scale thickness of second sheet which would have been punctured under the same conditions;

T/D = $(t_1/d) + (t_p/d)$, total thickness of equivalent structure which would just be punctured under the same conditions;

RHO T = ρ_t, second sheet density;

No. = identifying shot number from Ref 11 and 13.

The data have been segregated by bumper thickness and standoff distance, and arranged in order of increasing impact velocity.

Only data for aluminum projectiles were analyzed in Chapter IV due to time limitations.

Reference 11

Identification: Al → 2024T3

Assumed Materials: 2017T4 → 2024T3

Item	Proj	Targ	B.L.	t	d	$\frac{t}{d}$
B	1,05	1,20	7.4	1.16	.32	3.625
E	.745	.745				
F	.93	.93				
ρ	2.79	2.77				
ν	.33	.33				
Y	2,820	3,300				
U	4,360	4,780				
ϵ	.22	.185				
H	2,874	2,874				
c	.252	.252				
k	.29	.29				
Q	2,52	2,52				
C		6.25				

Reference 19

Identification: 2024 → 2024

Assumed Materials: 2024T3 → 2024T3

Item	Proj	Targ	B.L.	t	d	$\frac{t}{d}$
B	120	120	5.74	1.90	.476	3.99
E	.745	.745				
F	.93	.93				
ρ	2.77	2.77				
ν	.33	.33				
Y	3300	3300				
U	4780	4780				
ϵ	.185	.185				
H	2874	2874				
c	.252	.252				
k	.29	.29				
Q	252	252				
C	6.25	6.25				

Reference: 20

Identification: BOROSILICATE GLASS → CRS 302

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B		164	8.79	.00660	.00483	$.131x15^{6}$	1.368
E		1.968	6.45	.00660			1.368
F			5.74	.00660			1.368
ρ	2.23	7.92	5.53	.00521			1.079
ν			7.99	.00521			1.079
Y		4140	7.85	.00521			1.079
U		7170	7.41	.00521			1.079
ϵ		.44	8.24	.00521			1.079
H			6.98	.00521			1.079
c			10.00	.00521			1.079
k			5.18	.00254			.526
Q			4.63	.00254			.526
C			5.18	.00254			.526
			3.43	.00254			.526
			4.19	.00254			.526
			3.12	.00254			.526
			3.52	.00254			.526
			2.67	.00254			.526
			4.33	.00254			.526
			3.98	.00254			.526
			3.08	.00254			.526
			2.41	.00127			.263
			8.31	.00673			1.395
			9.51	.00673			1.395
			13.14	.00762			1.579
			8.70	.00737			1.526
			13.73	.00749			1.553
			10.27	.00749			1.553
			2.59	.00178			.368
			2.29	.00190			.395
			2.16	.00216			.447

Reference 20

Identification: **BOROSILICATE GLASS → Mg AZ31B-O**

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B		.58	7.91	.0104	.00483	$.131 \times 10^{-6}$	2.158
E		.457	4.10	.00864			1.789
F			6.51	.00648			1.392
ρ	2.23	1.770	6.25	.00622			1.290
ν			5.12	.00572			1.184
Y		1620	2.94	.00508			1.053
U		2530	3.72	.00826			1.702
ϵ		.229	2.35	.00622			1.290
H			11.34	.01676			3.474
c			8.40	.01575			3.263
k			7.85	.01549			3.210
Q			9.40	.01626			3.368
C			9.66	.01422			2.947
			12.30	.01473			3.053
			5.50	.01397			2.895
			6.00	.01016			2.105
			7.30	.01372			2.842
			4.05	.01054			2.184
			7.74	.01118			2.316
			2.09	.00470			.974

Reference 20

Identification: BOROSILICATE GLASS → QMV BERYLLIUM Hot Rolled

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B		.70	6.55	.00508	.00513	158×10^{-9}	.990
E			4.71	.00762			1.485
F			4.60	.00762			1.485
ρ	2.23	1.828	5.81	.00762			1.485
ν			5.14	.00927			1.807
Y		2080	8.12	.00927			1.807
U		3060	5.07	.00914			1.782
ϵ		.01	8.61	.01156			2.252
H			12.69	.01206			2.351
c			7.47	.01206			2.351
k			4.71	.00521			1.015
Q			2.58	.00495			.965
C			7.20	.00952			1.856

Reference 20

Identification: BOROSILICATE GLASS → 5052-0

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B		.45	6.16	.00851	.00513	158×10^{-6}	1.658
E		.717	6.77	.00851	.00513		1.658
F		.93	5.90	.00711	.00513		1.386
ρ	2.23	2.687	5.42	.00686	.00513		1.337
ν		.33	8.91	.01092	.00513		2.129
Y		.900	5.93	.00864	.00513		1.683
U		1970	7.42	.00864	.00513		1.683
ϵ		.247	9.63	.0127	.00513		2.475
H		2874					
c		.252					
k		.33					
Q		252					
C							

Reference 21

Identification: Ketos Stl → Mag AZ 51X,B90-46T

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B	169		3.17	.70	.78	.023	3.93
E							
F							
ρ	7.85						
ν							
Y							
U							
ϵ							
H							
c							
k							
Q							
C							

Reference 21

Identification: Ketos Stl $\rightarrow$ 2024T3

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B	169		3.17	.49	.178	.023	2.75
E							
F							
ρ	7.85						
ν							
Y							
U							
ϵ							
H							
c							
k							
Q							
C							

Reference 21

Identification: Ketos Stl → 1780

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B	1.69	.45	3.17	.58	.178	.023	3.26
E		.745					
F		.93					
ρ	7.85	2.79					
ν		.33					
Y		.703					
U		1.830					
σ		.22					
H		2.874					
c		.252					
k		.41					
Q		2.52					
C							

Reference 21

Identification: Ketos (Stl) → 2S0

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	$\frac{t}{d}$
B	1.69	.23	3.17	.70	.178	3.93
E		.745				
F		.93				
ρ	7.85	2.71				
ν		.33				
Y		.352				
U		.913				
ϵ		.45				
H		2.874				
c		.252				
k		.53				
Q		2.52				
C						

Reference 22

Identification: 2024

Assumed Materials: Stl → 2024T3

Item	Proj	Targ	B.L.	t	d	$\frac{t}{d}$
B	.98	1.20	.43	.159	.238	.668
E		.745	.83	.318		1.336
F		.93	1.20	.635		2.67
ρ		2.77	1.87	.794	.238	3.335
ν		.33				
Y		3300				
U		4780				
ϵ		.185				
H		2874				
c		.252				
k		.29				
Q		252				
C		6.25				

Reference 22

Identification: **glass**

Assumed Materials: Stl → Glass

Item	Proj	Targ	B.L.	t	d	$\frac{t}{d}$
B			.25	.127	.238	.533
E			.63	.318		1.336
F			1.27	.635		2.67
ρ			2.46	1.27	.238	5.33
ν						
Y						
U						
ϵ						
H						
c						
k						
Q						
C						

Reference 23

Identification: Piano Wire → 2024T3

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B		1.20	4.0	.695	.178	.023	3.90
E		.745					
F		.93					
ρ		2.77					
ν		.33					
Y		3.300					
U		4.780					
ϵ		.185					
H		2.874					
c		.252					
k		.29					
Q		2.52					
C		6.25					

Reference: 23

Identification: Ketos Stl → 2024T3

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B	169	120	2.0	.319	.178	.023	1.791
E		.745	3.2	.628	.178	.023	3.53
F		.93	3.2	-	.356	.187	
ρ	7.85	2.77	5.0	2.14	.352	.180	6.08
ν		.33					
Y		3300					
U		4780					
ϵ		.185					
H		2879					
c		.252					
k		.29					
Q		252					
C		6.25					

Reference: 16

Identification: AL → 2024T4

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B		1.20	10.0	1.27	.380	.08	3.34
E		.745	9.0	2.54	.591	.3	4.30
F		.93					
ρ	2.77	2.77					
ν		.33					
Y		3300					
U		4780					
ϵ		.185					
H		2874					
c		.252					
k		.29					
Q		252					
C		6.25					

Reference: 24

Identification: PYREX GLASS → 2024 T3 CLAD

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B		120	.762	.1575	.3175		.496
E		.745					
F		.93					
ρ	2.23	2.77					
ν		.33					
Y		3300					
U		4780					
ϵ		.185					
H		2874					
c		.252					
k		.29					
Q		252					
C		6.25					

Reference: **Martin Denver Eighth Shot**

Identification: GLASS → 2024T3

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m	$\frac{t}{d}$
B		1.20	5.37	.2565		.001	
E		.745				.0006	
F		.93					
ρ		2.77					
ν		.33					
Y		3.300					
U		4.780					
ϵ		.185					
H		2.874					
c		.252					
k		.29					
Q		2.52					
C		6.25					

Reference: Martin Denver Sixth Shot

Identification: GLASS BALL → 2014T6

Assumed Materials:

Item	Proj	Targ	B.L.	t	d	m
B		133	5.58	.475		.0062
E		.745				
F		.93				
ρ		2.77				
ν		.33				
Y		4220				
U		2780				
ϵ		.11				
H		2874				
c		.252				
k		.37				
Q		252				
C		6.25				

BUMPERED TARGET DATA

References 11 and 13

ALUMINUM 2017 PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#3.99

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.79	5.86	.192	3.994	.7421	.186	1.69	1.254	1.446	2.80	1560
2.79	7.07	.190	3.99	.5991	.150	1.70	1.018	1.208	2.80	1352
2.79	7.29	.190	3.994	.6619	.166	1.70	1.125	1.315	2.80	1336
2.79	7.35	.192	3.994	.6132	.154	1.70	1.042	1.234	2.80	1557
2.79	7.44	.192	3.994	.6541	.164	1.70	1.112	1.304	2.80	1544
2.79	7.44	.192	3.994	.7799	.391	1.56	1.217	1.409	2.80	1545
2.79	7.47	.190	3.994	.6557	.164	1.70	1.115	1.305	2.80	1333
2.79	6.10	.321	3.99	.2390	.060	1.730	.413	.734	2.80	1226
2.79	6.46	.306	3.99	.3899	.195	1.69	.6589	.965	2.80	996
2.79	7.47	.306	3.99	.5189	.130	1.71	.8873	1.193	2.80	1351

ALUMINUM 2017 PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#5.3

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.77	5.79	.171	5.33	.827	.620	.355	1.12	1.29	2.77	008
2.79	7.35	.204	5.336	.4685	.117	1.72	.8058	1.010	2.80	1767

ALUMINUM 2017 PROJECTILES @ ALUMINUM 2024T3 TARGETS L/D#8.0

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.79	5.55	.192	7.987	.4623	.231	1.67	.7720	.964	2.80	1559
2.79	7.32	.192	7.987	.2453	.061	1.73	.4244	.616	2.80	1556
2.79	7.32	.190	7.987	.2877	.072	1.73	.4977	.688	2.80	1335
2.79	7.47	.190	7.987	.2248	.056	1.73	.3889	.579	2.80	1334
2.77	3.78	.252	8.00	.392	.049	.735	.680	.932	2.77	263
2.77	4.74	.252	8.00	.392	.049	.735	.680	.932	2.77	264
2.77	5.23	.252	8.00	.432	.054	.730	.747	.999	2.77	262
2.77	6.49	.252	8.00	.316	.040	.735	.548	.800	2.77	261
2.77	7.07	.252	8.00	.252	.032	.720	.433	.685	2.77	166
2.77	7.47	.252	8.00	.248	.031	.720	.426	.678	2.77	169

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.79	5.88	.306	7.99	.1981	.050	1.730	.3427	.649	2.80	1225
2.79	7.38	.306	7.99	.1918	.048	1.73	.3318	.638	2.80	1350
2.77	4.25	.514	8.00	.536	.536	.435	.769	1.28	2.77	271
2.77	4.28	.501	8.00	.244	.244	.660	.405	.906	2.77	273
2.77	7.06	.501	8.00	.240	.240	.670	.401	.902	2.77	344
2.77	7.10	.501	8.00	.364	.364	.585	.577	1.08	2.77	180
2.77	7.36	.514	8.00	.472	.472	.490	.703	1.22	2.77	173
2.77	7.50	.512	8.00	.780	.780	.210	1.46	1.97	2.77	174

ALUMINUM 2017 PROJECTILES @ ALUMINUM 2024T3 TARGETS L/D#10.7

2.77	7.16	.087	10.7	.496	.372	.575	.781	.868	2.77	328
2.77	7.27	.085	10.7	.779	.584	.400	1.09	1.17	2.77	329
2.77	7.35	.112	10.7	.400	.300	.625	.650	.762	2.77	345
2.77	3.23	.170	10.7	.699	.524	.450	1.01	1.18	2.77	297
2.77	3.75	.170	10.7	1.26	.948	.050	1.32	1.49	2.77	299
2.77	4.24	.172	10.7	.608	.456	.515	.921	1.09	2.77	298
2.77	4.27	.171	10.7	1.33	1.00	.000	1.33	1.50	2.77	280
2.77	4.27	.171	10.7	.651	.488	.480	.963	1.13	2.77	246
2.77	4.28	.223	10.7	.491	.368	.580	.777	1.00	2.77	140
2.77	4.31	.171	10.7	.768	.576	.400	1.08	1.25	2.77	136
2.77	4.33	.170	10.7	.912	.684	.290	1.18	1.35	2.77	030
2.77	4.42	.200	10.7	.496	.372	.570	.779	.979	2.77	024
2.77	5.86	.171	10.7	.416	.312	.620	.674	.845	2.77	244
2.77	6.19	.171	10.7	.475	.356	.590	.755	.926	2.77	455
2.77	6.16	.171	10.7	.485	.364	.590	.771	.942	2.77	367
2.77	7.27	.224	10.7	.229	.172	.725	.395	.619	2.77	346
2.77	7.32	.170	10.7	.496	.372	.575	.781	.951	2.77	348
2.77	7.32	.171	10.7	.309	.232	.670	.516	.687	2.77	358
2.77	7.42	.171	10.7	.309	.232	.670	.517	.688	2.77	306
2.77	7.44	.171	10.7	.491	.368	.580	.776	.947	2.77	369
2.77	7.45	.171	10.7	.277	.208	.680	.465	.636	2.77	326
2.77	7.48	.224	10.7	.416	.312	.620	.674	.898	2.77	360
2.77	7.53	.171	10.7	.304	.228	.670	.508	.679	2.77	391
2.77	7.53	.173	10.7	.619	.464	.510	.935	1.11	2.77	231
2.77	7.59	.171	10.7	.256	.192	.690	.433	.604	2.77	390
2.77	7.99	.171	10.7	.384	.228	.670	.641	.812	2.77	212

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.77	1.14	.256	10.7	.165	.016	.740	.287	.543	2.77	419
2.77	2.21	.256	10.7	.528	.050	.735	.916	1.17	2.77	418
2.77	1.23	.336	10.7	.155	.015	.740	.270	.606	2.77	420
2.77	3.90	.336	10.7	.160	.120	.715	.274	.610	2.77	462
2.77	4.56	.341	10.7	.336	.252	.660	.558	.899	2.77	021
2.77	4.61	.341	10.7	.325	.244	.660	.540	.881	2.77	435
2.77	4.63	.336	10.7	.251	.188	.690	.424	.760	2.77	374
2.77	4.65	.336	10.7	.325	.244	.660	.540	.876	2.77	274
2.77	5.12	.336	10.7	.251	.188	.690	.424	.760	2.77	373
2.77	6.04	.336	10.7	.261	.196	.690	.441	.777	2.77	243
2.77	6.10	.336	10.7	.139	.104	.720	.239	.575	2.77	461
2.77	6.19	.336	10.7	.192	.144	.705	.327	.663	2.77	442
2.77	6.22	.341	10.7	.176	.132	.710	.301	.642	2.77	366
2.77	6.23	.336	10.7	.416	.312	.620	.674	1.01	2.77	247
2.77	6.32	.336	10.7	.277	.208	.680	.465	.801	2.77	244
2.77	7.33	.341	10.7	.261	.196	.685	.440	.781	2.77	357
2.77	7.44	.331	10.7	.133	.100	.720	.229	.560	2.77	370
2.77	7.51	.347	10.7	.299	.224	.670	.495	.842	2.77	230
2.77	7.55	.336	10.7	.469	.352	.595	.748	1.08	2.77	359
2.77	7.64	.336	10.7	.240	.180	.695	.407	.743	2.77	269
2.77	4.13	.378	10.7	.405	.304	.625	.658	1.04	2.77	334
2.77	4.27	.378	10.7	.549	.412	.545	.848	1.23	2.77	336
2.77	4.28	.364	10.7	.373	.280	.630	.608	.972	2.77	022
2.77	4.28	.378	10.7	.400	.300	.625	1.03	2.77		335
2.77	4.31	.373	10.7	.277	.208	.680	.465	.838	2.77	138
2.77	4.37	.378	10.7	.304	.228	.670	.508	.886	2.77	208
2.77	6.10	.378	10.7	.309	.232	.670	.516	.894	2.77	333
2.77	6.13	.362	10.7	.315	.236	.670	.526	.888	2.77	331
2.77	6.15	.378	10.7	.267	.200	.685	.450	.828	2.77	332
2.77	7.33	.373	10.7	.187	.140	.710	.320	.693	2.77	347
2.77	7.83	.378	10.7	.176	.132	.720	.303	.681	2.77	209
2.77	7.71	.686	10.7	.208	.156	.700	1.04	1.73	2.77	211

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
ALUMINUM 2017 PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#15.97										
2.79	1.10	0.00	15.97	.477	1.00	1.00	.477	.477	2.80	4
2.79	6.88	0.00	15.97	3.459	.866	1.13	3.909	3.909	2.80	1316
2.79	7.38	0.00	15.97	1.997	1.00	1.00	1.997	1.997	2.80	1329
2.79	7.38	0.00	15.97	2.855	1.00	1.00	2.855	2.855	2.80	1319
2.79	7.41	0.00	15.97	.953	1.00	1.00	.953	.953	2.80	1320
2.79	7.44	0.00	15.97	2.208	.921	1.075	2.374	2.374	2.80	1330
2.79	7.47	0.00	15.97	2.148	.538	1.43	3.072	3.072	2.80	1332
2.79	7.50	0.00	15.97	1.00	1.00	1.00	1.00	1.00	2.80	1328
2.79	7.50	0.00	15.97	2.994	1.00	1.00	2.994	2.994	2.80	1331
2.79	7.53	0.00	3.99	2.028	.411	1.55	3.143	3.143	2.80	1366
2.79	7.56	0.00	3.99	2.053	.447	1.52	3.121	3.121	2.80	1353
2.79	7.59	0.00	15.97	2.855	1.00	1.00	2.855	2.855	2.80	1314
2.79	.72	.0915	15.97	.0487	.024	1.735	.0845	.176	2.80	5
2.79	1.15	.0915	15.97	.2563	.128	1.71	.4383	.530	2.80	1
2.79	1.89	.0915	15.97	.4796	.120	1.72	.8249	.916	2.80	9
2.79	2.68	.0915	15.97	.7343	.184	1.69	1.241	1.332	2.80	13
2.79	2.91	.0915	15.97	.8569	.429	1.53	1.311	1.403	2.80	947
2.79	2.91	.0915	15.97	.8632	.432	1.525	1.316	1.408	2.80	916
2.79	3.00	.0915	15.97	.8947	.448	1.52	1.3599	1.451	2.80	992
2.79	4.82	.0915	15.97	.6871	.3441	1.60	1.099	1.191	2.80	914
2.79	6.89	.0915	15.97	.4009	.201	1.69	.678	.769	2.80	920
2.79	7.52	.099	15.97	.3994	.555	1.425	.5691	.668	2.80	1670
2.79	7.56	.0915	15.97	.1211	.0606	1.73	.2095	.301	2.80	948
2.79	7.56	.0915	15.97	.1279	.0638	1.73	.2204	.312	2.80	978
2.79	7.65	.0915	15.97	.1682	.0843	1.725	.2901	.382	2.80	980
2.79	7.68	.160	15.97	.134	.134	1.71	.229	.389	2.80	878
2.79	1.09	.190	15.97	.1682	.084	1.725	.2901	.480	2.80	2
2.79	1.90	.190	15.97	.4796	.343	1.60	.7674	.957	2.80	54
2.79	1.90	.190	15.97	.8066	.807	1.185	.9558	1.146	2.80	52
2.79	1.90	.190	15.97	.5676	.474	1.50	.8514	1.041	2.80	53
2.79	1.94	.190	15.97	.3443	.086	1.725	.5939	.784	2.80	10
2.79	2.07	.190	15.97	.3758	.094	1.72	.6464	.836	2.80	14
2.79	2.40	.190	15.97	.9261	.773	1.21	1.121	1.311	2.80	41
2.79	2.52	.190	15.97	.7186	.513	1.47	1.056	1.246	2.80	42
2.79	3.11	.190	15.97	.7123	.178	1.695	1.207	1.397	2.80	1118

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.79	3.26	.190	15.97	.5833	.146	1.71	.9974	1.187	2.80	1117
2.79	3.29	.190	15.97	.2940	.074	1.725	.5072	.697	2.80	1119
2.79	3.38	.190	15.97	.6871	.172	1.700	1.168	1.358	2.80	1120
2.79	3.76	.190	15.97	.3694	.185	1.69	.6243	.814	2.80	997
2.79	3.78	.190	15.97	.5299	.265	1.65	.8743	1.064	2.80	918
2.79	3.87	.190	15.97	.5047	.253	1.66	.8378	1.028	2.80	993
2.79	3.99	.190	15.97	.8412	.421	1.53	1.287	1.477	2.80	950
2.79	4.57	.190	15.97	.5173	.259	1.66	.8587	1.049	2.80	976
2.79	4.72	.190	15.97	.4167	.2087	1.69	.7042	.894	2.80	913
2.79	5.18	.190	15.97	.6714	.168	1.70	1.141	1.331	2.80	1389
2.79	5.43	.192	15.97	.4371	.219	1.68	.7343	.926	2.80	1564
2.79	5.61	.190	15.97	.3192	.080	1.725	.5506	.741	2.80	1123
2.79	5.61	.190	15.97	.3821	.096	1.72	.6572	.847	2.80	1121
2.79	5.64	.190	15.97	.1840	.046	1.735	.3192	.509	2.80	1122
2.79	5.78	.190	15.97	.0550	.014	1.74	.0957	.286	2.80	1124
2.79	6.28	.190	15.97	.2154	.108	1.72	.370	.560	2.80	919
2.79	6.40	.190	15.97	.2248	.113	1.72	.387	.577	2.80	951
2.79	6.61	.190	15.97	.1525	.0764	1.725	.2631	.453	2.80	977
2.79	6.61	.190	15.97	.1745	.044	1.735	.3028	.493	2.80	1236
2.79	7.10	.190	15.97	.1588	.040	1.735	.2755	.466	2.80	1237
2.79	7.19	.190	15.97	.1588	.040	1.735	.2755	.466	2.80	1239
2.79	7.19	.190	15.97	.1053	.026	1.735	.1827	.373	2.80	1238
2.79	7.26	.190	15.97	.0802	.020	1.740	.1395	.330	2.80	1242
2.79	7.29	.190	15.97	.0896	.0224	1.74	.156	.346	2.80	903
2.79	7.29	.190	15.97	.0330	.008	1.74	.0574	.247	2.80	1128
2.79	7.32	.190	15.97	.1211	.030	1.735	.2101	.400	2.80	1127
2.79	7.38	.190	15.97	.1934	.048	1.730	.3346	.525	2.80	1241
2.79	7.41	.190	15.97	.0802	.054	1.73	.1387	.329	2.80	723
2.79	7.46	.190	15.97	.0173	.0043	1.74	.030	.220	2.80	902
2.79	7.50	.192	15.97	.2547	.064	1.73	.4406	.633	2.80	1541
2.79	7.50	.190	15.97	.0645	.016	1.74	.1122	.302	2.80	1125
2.79	7.50	.201	15.97	.8679	.868	1.120	.9720	1.173	2.67	1671
2.79	7.53	.190	15.97	.1745	.058	1.73	.3019	.492	2.80	1338
2.79	7.54	.201	15.97	.5031	1.00	1.000	.5031	.704	2.67	1672
2.79	7.56	.190	15.97	.2091	.052	1.73	.3617	.552	2.80	1396
2.79	7.56	.192	15.97	.1918	.266	1.650	.3165	.508	2.80	1669
2.79	7.56	.192	15.97	.3805	.380	1.57	.5974	.789	2.80	1543
2.79	7.59	.192	15.97	.2704	.135	1.71	.4624	.654	2.80	1542
2.79	7.60	.190	15.97	.0802	.0201	1.74	.140	.330	2.80	904
2.79	7.68	.190	15.97	.1525	.038	1.735	.2646	.455	2.67	1390

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.79	7.93	.190	15.97	.0236	.012	1.74	.0411	.231	2.80	722
2.79	1.10	.306	15.97	.0472	.024	1.735	.0819	.388	2.80	3
2.79	1.87	.306	15.97	.2610	.131	1.71	.4463	.752	2.80	11
2.79	2.47	.306	15.97	.4874	.407	1.55	.7555	1.061	2.80	46
2.79	2.59	.306	15.97	.5031	.503	1.475	.7421	1.048	2.80	45
2.79	2.72	.306	15.97	.3270	.164	1.70	.5559	.862	2.80	15
2.79	2.78	.306	15.97	.0283	.0071	1.74	.0492	.355	2.80	910
2.79	3.62	.319	16.66	.1246	.060	1.73	.2156	.535	2.80	1056
2.79	4.76	.306	15.97	.2327	.1165	1.72	.4002	.706	2.80	912
2.79	5.43	.306	15.97	.2557&	1.00&	1.00	.2557&	.563&	2.80	726
2.79	5.43	.306	15.97	.1447	.0362	1.74	.2518	.558	2.80	911
2.79	6.80	.306	15.97	.0943	.024	1.735	.1636	.470	2.80	1649
2.79	6.89	.306	15.97	.0535	.027	1.735	.0928	.399	2.80	1157
2.79	7.16	.319	15.97	.0066	.003	1.74	.0115	.330	2.80	1055
2.79	7.50	.306	15.97	.0472	.0118	1.74	.0821	.388	2.80	909
2.79	7.80	.306	15.97	.0472	.0118	1.74	.0821	.388	2.80	905
2.79	1.92	.477	15.97	.2107	.106	1.72	.3624	.839	2.80	12
2.79	2.12	.477	15.97	.2421	.303	1.625	.3934	.870	2.80	49
2.79	2.14	.477	15.97	.0597	.060	1.73	.1033	.580	2.80	50
2.79	2.75	.477	15.97	.1792	.090	1.72	.3082	.785	2.80	16
2.79	2.85	.480	15.97	.1447	.0724	1.725	.250	.730	2.80	917
2.79	4.72	.480	15.97	.0975	.0488	1.73	.1687	.649	2.80	915
2.79	5.18	.480	15.97	.0723	.0362	1.74	.126	.606	2.80	921
2.79	5.21	.477	15.97	.2358	.059	1.73	.4079	.885	2.80	1398
2.79	7.56	.480	15.97	.1352	.0677	1.73	.2339	.714	2.80	979
2.79	7.62	.477	15.97	.1698	.042	1.735	.2946	.772	2.80	1397
2.79	7.50	1.904	15.97	.0157	.016	1.74	.0273	1.931	2.80	1321
2.79	7.47	1.904	15.97	0.00	0.00	1.74	0.00	1.904	2.80	1323
2.79	7.65	1.997	15.97	.1352	.135	1.71	.2312	2.228	2.80	1368

ALUMINUM 2017 PROJECTILES @ ALUMINUM 2024T3 TARGETS L/D#16.0

2.77	1.01	.040	16.0	.240	.015	.740	.418	.458	2.77	413
2.77	1.48	.040	16.0	.480	.030	.730	.830	.870	2.77	411
2.77	2.56	.040	16.0	.880	.055	.735	1.53	1.57	2.77	410
2.77	3.12	.040	16.0	1.00	.063	.730	1.730	1.770	2.77	325

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.77	3.81	.040	16.0	1.68	.840	.145	1.92	1.96	2.77	057
2.77	4.69	.040	16.0	1.33	.083	.715	2.28	2.32	2.77	152
2.77	5.12	.040	16.0	1.23	.077	.735	2.13	2.17	2.77	104
2.77	5.97	.040	16.0	1.06	.067	.730	1.83	1.87	2.77	150
2.77	7.01	.040	16.0	1.03	.516	.465	1.51	1.55	2.77	051
2.77	7.50	.040	16.0	.880	.055	.735	1.53	1.57	2.77	151
2.77	4.18	.128	16.0	.824	.052	.730	1.43	1.56	2.77	429
2.77	4.56	.128	16.0	.792	.396	.560	1.24	1.37	2.77	078
2.77	4.63	.128	16.0	.736	.368	.585	1.167	1.295	2.77	071
2.77	4.66	.128	16.0	.848	.424	.535	1.30	1.43	2.77	308
2.77	5.26	.128	16.0	.584	.036	.735	1.01	1.14	2.77	428
2.77	6.16	.128	16.0	.424	.212	.680	.712	.840	2.77	076
2.77	6.18	.128	16.0	.424	.026	.735	.736	.864	2.77	425
2.77	7.13	.128	16.0	.368	.184	.690	.622	.750	2.77	443
2.77	7.47	.128	16.0	.152	.010	.740	.264	.392	2.77	426
2.77	7.92	.128	16.0	.120	.008	.740	.209	.337	2.77	431
2.77	.922	.256	16.0	.064	.004	.740	.111	.367	2.77	281
2.77	1.37	.256	16.0	.120	.060	.730	.208	.464	2.77	416
2.77	1.49	.256	16.0	.248	.124	.720	.426	.682	2.77	415
2.77	1.54	.256	16.0	.256	.016	.740	.445	.701	2.77	277
2.77	1.62	.256	16.0	.216	.014	.740	.375	.631	2.77	288
2.77	2.20	.256	16.0	.480	.030	.730	.830	1.09	2.77	287
2.77	2.70	.256	16.0	.437	.041	.735	.758	1.01	2.77	084
2.77	3.17	.256	16.0	.536	.268	.645	.882	1.14	2.77	430
2.77	3.40	.256	16.0	.496	.031	.730	.858	1.11	2.77	271
2.77	4.00	.256	16.0	.456	.228	.670	.762	1.02	2.77	307
2.77	4.63	.256	16.0	.384	.024	.735	.666	.922	2.77	272
2.77	4.80	.256	16.0	.395	.037	.735	.685	.941	2.77	080
2.77	4.88	.256	16.0	.384	.192	.690	.649	.905	2.77	072
2.77	5.18	.256	16.0	.288	.144	.710	.492	.748	2.77	073
2.77	5.39	.256	16.0	.283	.027	.740	.492	.748	2.77	081
2.77	5.53	.256	16.0	.328	.020	.735	.569	.825	2.77	273
2.77	5.78	.256	16.0	.280	.140	.710	.479	.735	2.77	394
2.77	5.94	.256	16.0	.202	.019	.690	.341	.597	2.77	082
2.77	6.04	.256	16.0	.200	.112	.720	.344	.600	2.77	432
2.77	6.23	.256	16.0	.208	.013	.740	.362	.618	2.77	289
2.77	6.23	.256	16.0	.224	.112	.720	.385	.641	2.77	243
2.77	6.28	.256	16.0	.184	.092	.720	.316	.572	2.77	077

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.77	6.49	.256	16.0	.128	.012	.740	.223	.479	2.77	083
2.77	6.77	.256	16.0	.176	.017	.690	.297	.553	2.77	085
2.77	6.98	.256	16.0	.144	.014	.740	.251	.507	2.77	086
2.77	7.53	.256	16.0	.080	.040	.730	.138	.394	2.77	464
2.77	7.57	.256	16.0	.072	.036	.735	.125	.610	2.77	465
2.77	7.70	.256	16.0	.072	.036	.735	.125	.381	2.77	463
2.77	1.37	.384	16.0	.128	.008	.740	.223	.607	2.77	409
2.77	2.21	.384	16.0	.384	.024	.735	.666	1.05	2.77	408
2.77	3.26	.384	16.0	.400	.025	.735	.694	1.08	2.77	323
2.77	4.15	.384	16.0	.288	.018	.740	.501	.885	2.77	407
2.77	[illegible]	.384	16.0	.208	.013	.740	.362	.746	2.77	091
2.77	5.46	.384	16.0	.208	.013	.710	.356	.740	2.77	088
2.77	5.61	.384	16.0	.192	.012	.740	.334	.718	2.77	089
2.77	6.45	.384	16.0	.192	.012	.740	.334	.718	2.77	090
2.77	7.01	.384	16.0	.160	.010	.740	.278	.662	2.77	092
2.77	7.59	.384	16.0	.072	.005	.740	.125	.509	2.77	087
2.77	1.57	.504	16.0	.192	.012	.740	.334	.838	2.77	422
2.77	2.11	.504	16.0	.224	.014	.740	.390	.894	2.77	407
2.77	2.58	.504	16.0	.320	.020	.735	.555	1.06	2.77	421
2.77	3.08	.504	16.0	.312	.020	.735	.541	1.04	2.77	405
2.77	3.73	.504	16.0	.256	.128	.715	.439	.943	2.77	393
2.77	3.75	.504	16.0	.240	.120	.720	.413	.917	2.77	060
2.77	3.87	.504	16.0	.288	.144	.710	.492	.996	2.77	392
2.77	4.14	.504	16.0	.224	.014	.740	.390	.894	2.77	424
2.77	4.14	.504	16.0	.272	.017	.740	.473	.977	2.77	409
2.77	4.85	.504	16.0	.328	.020	.685	.553	1.06	2.77	284
2.77	5.56	.504	16.0	.208	.013	.740	.362	.866	2.77	285
2.77	5.73	.504	16.0	.160	.080	.735	.278	.782	2.77	061
2.77	6.06	.504	16.0	.200	.100	.720	.344	.848	2.77	440
2.77	6.10	.504	16.0	.144	.009	.740	.251	.755	2.77	404
2.77	6.17	.504	16.0	.088	.044	.730	.134	.638	2.77	242
2.77	6.66	.504	16.0	.144	.009	.740	.251	.755	2.77	286
2.77	7.19	.504	16.0	.072	.005	.740	.125	.629	2.77	423
2.77	7.25	.504	16.0	.240	.120	.720	.413	.917	2.77	063
2.77	7.31	.512	16.0	.384	.192	.690	.649	1.16	2.77	305
2.77	7.32	.504	16.0	.088	.006	.735	.153	.657	2.77	293
2.77	7.47	.504	16.0	.128	.008	.740	.223	.727	2.77	287
2.77	7.57	.504	16.0	.072	.005	.740	.125	.629	2.77	422

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
2.77	7.59	.504	16.0	.096	.048	.730	.166	.670	2.77	441
2.77	7.60	.504	16.0	.184	.092	.720	.316	.820	2.77	395
2.77	7.60	.504	16.0	.160	.080	.725	.276	.780	2.77	396
2.77	7.68	.504	16.0	.056	.004	.740	.097	.601	2.77	413
2.77	7.83	.504	16.0	.784	.049	.735	1.36	1.86	2.77	288
2.77	7.89	.504	16.0	.056	.004	.740	.097	.601	2.77	412

ALUMINUM 2017 PROJECTILES @ ALUMINUM 2024T3 TARGETS L/D#21.3

2.77	6.05	.171	21.3	.331	.248	.660	.549	.720	2.77	009
2.77	5.36	.171	21.3	.416	.312	.620	.674	.845	2.77	007

ALUMINUM 2017 PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#32.0

2.79	4.69	.091	31.95	.5459	.273	1.640	.8953	.986	2.80	1233
2.79	6.59	.091	31.95	.4642	.232	1.670	.7752	.866	2.80	1234
2.79	4.36	.183	31.95	.4623	.058	1.730	.7998	.983	2.80	1231
2.79	6.16	.183	31.95	.0786	.039	1.735	.1364	.319	2.80	1232
2.79	6.50	.183	31.95	.0975	.049	1.730	.1687	.352	2.80	1235
2.79	7.41	.192	32.08	.1572	.781	1.20	.1886	.381	2.80	1546
2.79	7.78	.182	31.95	.3031	1.192	1.25	.379	.561	2.80	973
2.79	7.78	.182	31.95	.0031	.0031	1.74	.0054	.187	2.80	974
2.79	7.87	.182	31.95	.0344	.1351	1.71	.0588	.241	2.80	975
2.79	7.89	.182	31.95	.0156	.031	1.74	.0271	.209	2.80	972
2.77	4.67	.512	32.0	.096	.024	.735	.167	.679	2.77	070

BUMPERED TARGET DATA

References 11 and 13

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
CADMIUM PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#12.7										
8.67	3.44	.590	12.7	.0120	.008	1.74	.0209	.611	2.80	1048
CADMIUM PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#16.0										
8.67	2.77	.075	16.0	.112	.056	.735	.194	.269	2.77	512
8.67	3.88	.075	16.0	.128	.064	.730	.221	.296	2.77	508
8.67	6.52	.075	16.0	.008	.004	.740	.014	.089	2.77	502
8.67	5.43	.101	16.0	.760	.380	.570	1.19	1.29	2.77	495
8.67	6.54	.100	16.0	.848	.424	.580	1.34	1.44	2.77	503
8.67	2.71	.120	16.0	.040	.020	.740	.070	.190	2.77	513
8.67	3.89	.120	16.0	.048	.024	.740	.084	.204	2.77	509
8.67	3.06	.200	16.0	1.06	.532	.490	1.58	1.78	2.77	506
8.67	3.87	.200	16.0	.288	.114	.720	.496	.696	2.77	511
8.67	5.49	.202	16.0	.360	.180	.690	.608	.810	2.77	496
8.67	6.58	.200	16.0	.336	.168	.700	.571	.771	2.77	500
8.67	3.55	.321	15.97	.2233	.112	1.72	.3841	.705	2.80	1222
8.67	3.60	.321	15.97	.1038	.026	1.735	.1801	.501	2.80	1230
8.67	3.69	.321	15.98	.1132	.028	1.735	.1964	.517	2.80	1154
8.67	3.87	.321	15.97	.1289	.032	1.735	.2236	.545	2.80	1156
8.67	5.18	.321	15.97	.0560	.014	1.74	.0974	.418	2.80	1453
8.67	5.34	.321	15.97	.0723	.018	1.74	.1258	.447	2.80	999
8.67	5.61	.321	15.97	.0409	.010	1.74	.0712	.392	2.80	1019
8.67	6.40	.321	15.97	.1038	.104	1.72	.1785	.500	2.80	1711
8.67	6.49	.321	15.97	.2956	.198	1.69	.4996	.821	2.80	1713
8.67	3.54	.351	16.0	.216	.108	.720	.372	.723	2.77	507
8.67	4.07	.351	16.0	.232	.116	.720	.400	.751	2.77	510

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
8.67	5.46	.351	16.0	.136	.068	.725	.235	.586	2.77	497
8.67	6.58	.351	16.0	.120	.060	.730	.208	.559	2.77	501
8.67	5.97	.487	15.97	.5377	.510	1.470	.7904	1.277	2.80	1625
8.67	6.00	.487	15.97	.4969	.988	1.010	.5018	.989	2.80	1622
8.67	3.18	.618	15.97	.0708	.018	1.74	.1232	.741	2.80	1046
8.67	5.12	.618	15.97	.0802	.020	1.74	.1395	.758	2.80	1155
8.67	5.12	.618	15.97	.0550	.028	1.735	.0954	.713	2.80	1223
8.67	5.38	.618	15.97	0.00	0.00	1.74	0.00	.618	2.80	1324
8.67	5.48	.618	15.97	.0079	.002	1.74	.0137	.632	2.80	1044
8.67	5.70	.618	15.97	.0399	.010	1.74	.0694	.687	2.80	1326
8.67	6.40	.618	15.97	0.00	0.00	1.74	0.00	.618	2.80	1454
8.67	6.40	.618	15.97	.0173	.004	1.74	.0301	.648	2.80	1045
8.67	6.46	.618	15.97	.0802	.040	1.735	.1391	.757	2.80	1049
8.67	6.49	.618	15.97	.0393	.020	1.74	.0684	.686	2.80	1047
8.67	6.53	.618	15.97	0.00	0.00	1.74	0.00	.618	2.80	1455
8.67	6.53	.618	15.97	0.00	0.00	1.74	0.00	.618	2.80	1457
8.67	5.46	.751	16.0	.088	.044	.730	.152	.903	2.77	498
8.67	3.69	1.188	15.97	.2390	.060	1.73	.4135	1.601	2.80	1580
8.67	5.18	1.188	15.97	0.00	0.00	1.74	0.00	1.188	2.80	1581
8.67	6.31	1.188	15.97	.1289	.129	1.710	.2204	1.408	2.80	1700
8.67	6.34	1.188	15.97	0.00	0.00	1.74	0.00	1.188	2.80	1582
8.67	6.40	1.188	15.97	0.00	0.00	1.74	0.00	1.188	2.80	1712

CADMIUM PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#19.84

8.67	3.84	.768	19.84	.0488	.078	1.725	.0842	.852	2.80	1153
8.67	6.43	.774	19.84	0.00	0.00	1.74	0.00	.774	2.80	1583

CADMIUM PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#59.4

8.67	4.27	.768	59.38	0.00	0.00	1.74	0.00	.768	2.80	1456

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
PYREX PROJECTILES @ ALUMINUM 2024T3 TARGETS L/D#16.0										
2.23	5.49	.080	16.0	.616	.308	.625	1.00	1.08	2.77	033
2.23	4.01	.080	16.0	.432	.216	.680	.726	.806	2.77	035
2.23	3.87	.080	16.0	.600	.300	.630	.978	1.06	2.77	036
2.23	6.86	.080	16.0	.864	.432	.530	1.32	1.40	2.77	037
2.23	5.44	.080	16.0	.520	.260	.660	.863	.943	2.77	038
2.23	6.40	.080	16.0	.440	.220	.680	.739	.819	2.77	040
2.23	3.38	.080	16.0	.352	.176	.700	.598	.678	2.77	042
2.23	6.31	.080	16.0	.824	.412	.550	1.28	2.16	2.77	050
2.23	4.75	.080	16.0	.640	.320	.620	1.04	1.92	2.77	045
2.23	7.32	.080	16.0	.648	.324	.620	1.05	1.93	2.77	043
2.23	4.92	.190	15.97	.3192	.160	1.700	.5426	.733	2.80	1224
2.23	3.63	.190	15.97	.1588	.0795	1.725	.2739	.464	2.80	994
2.23	6.58	.190	15.97	.1211	.0606	1.73	.2095	.400	2.80	995
PYREX PROJECTILES @ ALUMINUM 2024T3 TARGETS L/D#32.0										
2.23	3.90	.512	32.0	.158	.396	.560	.246	.758	2.77	068
2.23	6.00	.512	32.0	.120	.300	.630	.196	.708	2.77	069
STEEL PROJECTILES @ ALUMINUM 2024T3 TARGETS L/D#16.0										
7.85	6.36	.504	16.0	.952	.476	.490	1.42	1.92	2.77	259
7.85	4.88	.504	16.0	1.10	.548	.425	1.57	2.07	2.77	260
7.85	7.03	.566	16.0	.456	.228	.670	.762	1.33	2.77	401
7.85	7.07	.566	16.0	.728	.364	.585	1.15	1.72	2.77	400
7.85	7.13	.566	16.0	.464	.232	.670	.775	1.34	2.77	399
7.85	6.96	1.03	16.0	.272	.136	.710	.465	1.49	2.77	301
7.85	7.07	1.02	16.0	.432	.216	.680	.726	1.75	2.77	302
7.85	7.01	1.00	16.0	.192	.096	.725	.331	1.33	2.77	303
7.85	7.10	1.02	16.0	.216	.108	.720	.372	1.39	2.77	304

RHO P	VEL	T1 EQ	L/D	H/D	H/T2	FACT	TP/D	T/D	RHO T	NO.
	COPPER PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#24.4									
8.90	4.17	.468	24.4	1.096	.359	1.59	1.743	2.211	2.80	1051
8.90	7.10	.468	24.42	.4760	.078	1.725	.8211	1.289	2.80	1348
8.90	7.10	.468	24.4	.5000	.164	1.70	.8500	1.318	2.80	1054
	TITANIUM PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#19.39									
4.54	7.25	.371	19.39	.3855	.159	1.70	.6554	1.026	2.80	1053
4.54	3.49	.371	19.39	.7252	.929	1.625	1.178	1.549	2.80	1052
	NYLON PROJECTILES @ ALUMINUM 7075T6 TARGETS L/D#12.12									
1.14	4.11	.146	12.12	.1384	.091	1.72	.2380	.384	2.80	1575
1.14	7.50	.146	12.12	.1265	.083	1.725	.2182	.364	2.80	1561
1.14	2.92	.232	12.12	.0286	.019	1.74	.0498	.282	2.80	1114
1.14	6.10	.232	12.12	.0358	.024	1.74	.0623	.294	2.80	1115
1.14	6.31	.232	12.12	.0716	.024	1.74	.1246	.357	2.80	1349
1.14	6.80	.232	12.12	.0597	.0394	1.735	.1036	.336	2.80	1116
1.14	7.13	.232	12.12	.0668	.022	1.74	.1162	.348	2.80	1371

APPENDIX D

STATISTICAL METHOD

The data analyzed in this study represent experimental observations collected from independently conducted experiments involving a variety of combinations of projectile and target materials. As a result, the data could not be treated by the methods of any one of the systematically planned experimental designs in Analysis of Variance. Thus, the effects of each dimensionless variable and the random errors affecting it are not as accurately or as easily estimated as they would have been in a planned experiment. Based on these considerations, regression and correlation methods were used to conduct an objective analysis of the relation of the variables of this model.

Regression models enable the effects of various factors to be assessed from experimental data even when the experiment does not follow a prescribed pattern, or when the variables affecting the results cannot be controlled in a manner so as to make a designed experiment possible. Thus, regression analysis provides a means, based on probabilistic considerations for testing the concordance between theory and observation.

The objective of this study is to develop a method for estimating the dimensionless depth of penetration variable, $\frac{h}{d}$, from the data associated with statistically significant independent variables. Through a statistical analysis, a mathematical quantification of the functional relationships of these variables may be obtained and a basis for the discussion of the uncertainty of the relationships may be established.

General Assumptions of Regression Analysis: Multivariate regression is an extension of single variable regression. The following underlying assumptions are the basis for linear regression analysis:

a) For every fixed value of independent variables $(\pi_2, \pi_3 \ldots \pi_{23})$, the dependent variable π_1 is normally distributed.

b) The mean value of π_1 is a linear function of $\pi_2, \pi_3, \ldots \pi_{23}$, and may be expressed as $\bar{\pi}_1 = \beta_1 + \beta_2 (\pi_2 - \bar{\pi}_2) + \ldots + \beta_{23} (\pi_{23} - \bar{\pi}_{23})$ where $\bar{\pi}_i$ denotes the expected value of the ith series.

c) The variance of π_1 is independent of the values of $(\pi_2, \pi_3, \ldots, \pi_{23})$ so that the variance

$$V \left\{ \pi_i \;\middle|\; \pi_2, \pi_3, \ldots, \pi_{23} \right\} = \sigma^2,$$

or it is proportional to a given function so that

$$V \left\{ \pi_1 \;\middle|\; (\pi_2, \pi_3, \ldots, \pi_{23}) \right\} = \sigma^2 h^2(\pi_2, \ldots, \pi_{23}).$$

d) The observations are stochastically independent, i.e., the values $(\pi_{2i}, \pi_{3i}, \ldots, \pi_{23i})$ are independent of $(\pi_{2j}, \pi_{3j}, \ldots, \pi_{23j})$, $i \neq j$. In the nonlinear case, the variables $(\pi_2, \ldots, \pi_{23}, \pi_1)$ are replaced by new variables $(g_2, \ldots, g_{23}, f(\pi_2)$, where $g_i = g_i(\pi_2, \ldots, \pi_{23})$ $i = 2, \ldots, 23$ and $f(\pi_1)$ are given functions.

Assumption b) above becomes, $\bar{\pi}_1 = \beta_1 + \beta_2 (g_2 - \bar{g}_2 + \beta_3(g_3 - \bar{g}_3) + \ldots + \beta_{23}(g_{23} - \bar{g}_{23})$ where $\bar{g}_i = \frac{1}{N} \sum_{h=1}^{N} g_i(\pi_{2h}, \pi_{3h}, \ldots, \pi_{23h})$

The variance (assumption c) becomes

$$V\left\{ f(\pi_1) \;\middle|\; g_2,\ldots\ldots,g_{23}\right) = \sigma^2 h^2(g_2,\ldots\ldots\ldots,g_{23}).$$

In regression analysis, the dependent variable π_1 is a stochastic variable whose distribution function depends on the π_i, $i = 2,\ldots\ldots,23$. In particular, the population mean of π_1 will be a function of the π_i, $i=2,\ldots\ldots,23$, e.g., $\overline{\pi}_1 = f(\pi_i) = f'(\pi_i, \beta_1)$, $i=2,\ldots\ldots,23$. The functional form of $\overline{\pi}_1$ is preselected and estimates of the coefficients in the preselected equation are determined from the data. No assumptions are made regarding the distributions of the π_i, $i = 2,\ldots\ldots,23$, in the regression analysis.

<u>General Assumptions in Correlation Analysis</u>: In correlation theory, all variables (the dependent and all independent variables), are assumed to be stochastic. The association between any two variables, say (X_1, X_2), is described by a two-dimensional probability density function. In principle, the association is characterized by calculating estimates of the parameters of this two-dimensional distribution. This characteristic is often supplemented as in this study, by a regression analysis in which the mean value of X_1 is determined as a function of X_2, either because variations in the size of X_2 are the cause of variations in X_1 or because the regression curve furnishes a suitable description of the association between the two variables for certain practical purposes.

<u>The Development of a Regression Surface</u>:

<u>Notation</u>: The notation appearing in the following section is summarized below:

X_1	=	Observations of the dependent variable, $\ln \pi_1 = \ln(h/d)$
$X_2, X_3, \ldots, X_{23}$	=	Observation of the independent variables denoting $\ln(\pi_j)$, $j = 2, \ldots.23$.
x_{ij}	=	The jth observation of the ith variable measured from the expected value of the ith series so that $x_{ij} = (X_{ij} - \bar{X}_i)$ In general, a lower case x_i represents deviations from the mean of the X_i series.
X_i	=	In general, this term represents the actual observed values of the ith series.
$\sum$	=	The summation sign. The usual practice of indexing the summation sign will be omitted in order to simplify the notation herein. The general inference is that the summation is taken over <u>all data points</u>.
$\sum x_1^2$	=	The total variation in the dependent variable series, i.e., $\sum_{j=1}^{N} (X_{1j} - \overline{X_1})^2$
$\sum x^2_{c1\cdot 2,\ldots,m}$	=	The variation in the dependent variable series "explained" by the regression surface in the independent variables x_j, $j = 2, \ldots\ldots, m$.
$\sum x^2_{s1\cdot 2,\ldots,m}$	=	The variation in the dependent variables series "unexplained" by the regression surface.
$\sum x^2_{c1\cdot[\ ]}$	=	The same definition as above, yet the brackets will be used when <u>all</u> of the independent variables are implied as in the above definition $\sum x^2_{c1\cdot 2,\ldots,m}$.

$a_{1\cdot 2,3,\ldots\ldots,m}$ = The theoretical value of the regressive surface when $X_2, X_3, \ldots\ldots, X_m$ are equated to zero.

$b_{1,i\cdot j,k,\ldots,m}$ = The partical regression coefficient of the ith variable in the regression surface $X_{c1\cdot[\;]} = a_{1\cdot[\;]} + b_{1,2\cdot 3,\ldots m}X_2 + b_{1,3\cdot 2,4,\ldots\ldots,m}X_3 + \ldots\ldots\ldots + b_{1,m\cdot 2,3,\ldots\ldots,(m-1)}X_m$.

$R^2_{1\cdot[\;]}$ = The coefficient of multiple determination. The proportion of total variation in the dependent variable which is explained by the calculated regression surface. R is the coefficient of multiple correlation.

$r^2_{1,2\cdot 3,4,\ldots,m}$ = The coefficient of partial determination; the additional variation in X_1 explained by X_2 expressed as a proportion of variation in X_1 which was unexplained by $X_3, X_4, \ldots . X_m$ before the introduction of X_2.

$s^2_{1\cdot[\;]}$ = The estimate of variance of the calculated regression surface.

Least Squares Criterion: The functional relationship of the independent variable π_1 and the other 22 chosen variables in the model has been hypothesized to be of the form[1]

$$X_{c1\cdot 2,3,\ldots\ldots,23} = \ln\pi_1 = \ln\beta_1 + \sum_{i=2}^{23}\beta_{1,i\cdot[\;]}\ln\pi_1 \qquad \text{(D-1)}$$

[1]This equation is a transformation of $\pi_1 = \beta_1 \prod^{23} \pi_i\,\beta^i$. It is recognized that the β's of (D-1) only approximate the β's of this product form when fitted by the least squares criterion.

or in generalized terms

$$X_{c1\cdot 2,3,\ldots,23} = a_{1\cdot 2,3,\ldots,23} + \sum_{i=2}^{23} \beta_{1,i\cdot[\ ]} X_i \tag{D-2}$$

Where the $\beta_{1,i\cdot[\ ]}$, the partial regression coefficients denote respectively the regression coefficients between the dependent variable X_1 and the independent variables X_i.

In order for the regression surface to satisfy the least squares criterion, estimates of the constant term $a_{1\cdot 2,3,\ldots,23}$ and the partial regression coefficients $\beta_{1,i\cdot[\ ]}$ must be calculated so as to minimize the sum of the squared vertical deviations measured from the data points to the regression surface. The estimates of the 23 coefficients in the regression surface are obtained by minimizing,

$$S = \sum_{i=1}^{N} (X_{1i} - X_{c1\cdot 2,3,\ldots,23})^2$$

$$= \sum_{i=1}^{N} \left[X_{1i} - (a_{1\cdot 2,3,\ldots,23} + \beta_{1,2\cdot[\ ]} X_2 + \ldots\ldots + \beta_{1,2,3\cdot[\ ]} X_{23} \right]^2 \tag{D-3}$$

The $a_{1\cdot 2,3,\ldots,23}$ and $\beta_{1,i\cdot[\ ]}$ which minimize S are found by differentiating S with respect to each coefficient.

This process results in the following k normal equations:[2,3]

[2] Within this text, all $\sum$'s are taken over the N components of data unless otherwise noted.

[3] In the set of equations (D-4), $a_{1\cdot 2,3,\ldots,23}$ and the $b_{1,i\cdot[\ ]}$ represent statistical estimates of β_1 and $\beta_{1,i}$.

$$
\begin{aligned}
&Na_{1\cdot 2,3,\ldots,23} + b_{1,2\cdot 3,4,\ldots,23} \sum X_2 + b_{1,3\cdot 2,4,\ldots,23} \sum X_3 + \ldots\ldots\ldots \\
&+ b_{1,23\cdot 2,3,\ldots,22} \sum X_{23} = \sum X_1 \\
&A_{1\cdot 2,3,\ldots,23} \sum X_2 + b_{1,2\cdot 3,4,\ldots,23} \sum X_2^2 + b_{1,3\cdot 2,4,\ldots,23} \sum X_2 X_3 + \ldots\ldots \\
&+ b_{1,23\cdot 2,3,\ldots,22} = \sum X_2 X_{23} = \sum X_1 X_2 \qquad\qquad \text{(D-4)} \\
&\qquad\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
&A_{1\cdot 2,3,\ldots,23} \sum X_{23} + b_{1,2\cdot 3,4,\ldots,23} \sum X_2 X_{23} + b_{1,3\cdot 2,4,\ldots,23} \sum X_3 X_{24} + \\
&\ldots\ldots + b_{1,23\cdot 2,3,\ldots,22} \sum X_{23}^2 = \sum X_1 X_{23}
\end{aligned}
$$

Computational difficulties are somewhat diminished if deviations from the expected value of each independent variable series, $x_i = (X_i - \bar{X}_i)$, are considered rather than the observed values X_i. In the two dimensional case, this procedure has the effect of shifting the origin of the data to $(\bar{X}_1, \bar{X}_2)$ from the origin (0, 0). Since $\sum x_1 = \sum x_2 = \sum x_3 = \ldots\ldots = \sum x_{23} = 0$, the first row and the first column in the set of equations (D-4) are eliminated and the regression surface satisfies

$$x_{c1\cdot 2,3,\ldots,23} = b_{1,2\cdot 3,4,\ldots,23} x_2 + b_{1,3\cdot 2,4\ldots,23} x_3 + \cdots + b_{1,23\cdot 2,3,\ldots,22} x_{23}$$

The set of normal equations for this regression surface is

$$b_{1,2\cdot 3,4,\ldots,23} \sum x_2^2 + b_{1,3\cdot 2,4,\ldots,23} \sum x_2 x_3 + \cdots + b_{1,23\cdot 2,3,\ldots,22} \sum x_2 x_{23} = \sum x_1 x_2$$

$$b_{1,2\cdot 3,4,\ldots,23} \sum x_2 x_3 + b_{1,3\cdot 2,4,\ldots,23} \sum x_3^2 + \cdots + b_{1,23\cdot 2,3,\ldots,22} \sum x_3 x_{23} = \sum x_1 x_3$$

$$\vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad \text{(D-5)}$$

$$b_{1,2\cdot 3,4,\ldots,23} \sum x_2 x_{23} + b_{1,3\cdot 2,4,\ldots,23} \sum x_3 x_{23} + \cdots + b_{1,23\cdot 2,3,\ldots,22} \sum x_{23}^2 = \sum x_1 x_{23}.$$

In matrix notation, (D-5) may be expressed as

$$\begin{bmatrix} \sum x_2^2 & \sum x_2 x_3 & \cdots & \sum x_2 x_{23} \\ \sum x_2 x_3 & \sum x_3^2 & \cdots & \sum x_3 x_{23} \\ \vdots & \vdots & & \vdots \\ \sum x_2 x_{23} & \sum x_3 x_{23} & & \sum x_{23}^2 \end{bmatrix} \begin{bmatrix} b_{1,2\cdot 3,4,\ldots,23} \\ b_{1,3\cdot 2,4,\ldots,23} \\ \vdots \\ b_{1,23\cdot 2,3,\ldots,22} \end{bmatrix} = \begin{bmatrix} \sum x_1 x_2 \\ \sum x_1 x_3 \\ \vdots \\ \sum x_1 x_{23} \end{bmatrix} \qquad \text{(D-6}$$

Allowing A to represent the matrix of cross-product terms, B to represent the column vector of regression coefficients and C to represent the column vector of cross-product terms involving the dependent variable x_1, the solution vector is seen to be

$$B = A^{-1}\,C \qquad \text{(D-7)}$$

As A is a symmetric matrix, its inverse A^{-1} is also symmetric. Many digital programming methods are available for solving the inverse of symmetric A. The method used for this study was a double-precision Gauss reduction procedure.

Explained and Unexplained Variation: At this point, it is desirable to discuss the concept of explained and unexplained variation and to illustrate the equivalency of considering either observed values or deviations of values from their respective means by considering a simple two-variable linear case. Consider

$$X_{c1\cdot2} = a_{1\cdot2} + \beta_{1\cdot2} X_2$$

In regression analysis, it is usual to select values of the independent variable and record measurements of the dependent variable. Usually due to measurement error, several different values are recorded for the dependent variable. This regression may be graphically illustrated as in Figure D-1.

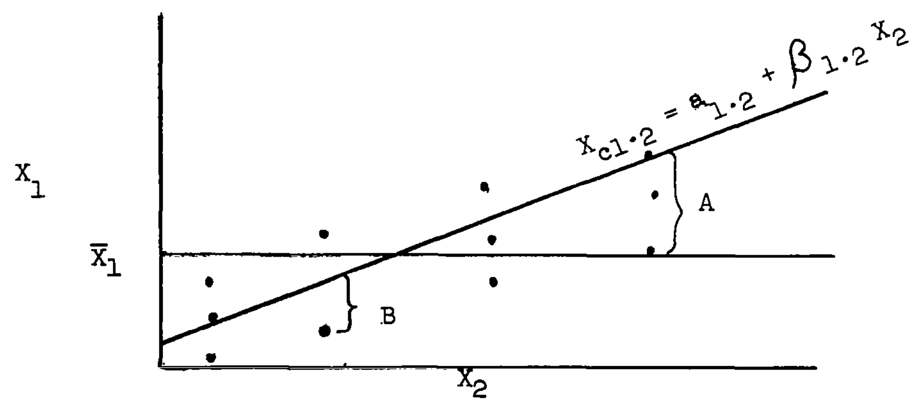

FIGURE D-1

GRAPHIC ILLUSTRATION OF TWO VARIABLE LINEAR REGRESSION

The sums of squares of the deviations of the X_1 values from their mean, $\sum(X_{1i} - \bar{X}_1)^2$, is termed total variation.

This total variation may be partitioned into two parts: (a) that which has been "explained" by the curve of relationship and (b) that variation which the regression function does not explain. These definitions are algebraically expressed as,

$\sum(X_1 - \bar{X}_1)^2$ = Total variation in the dependent variable series,

$\sum(X_{c1 \cdot 2} - \bar{X}_1)^2$ = Explained variation (as explained by the Reg. Surface),

$\sum(X_1 - X_{c1 \cdot 2})^2$ = Unexplained variation.

In Figure D-1, the distance A represents one of the N terms in the summation denoting explained variation. The distance B represents a similar component of unexplained variation.

A fundamentally important fact is that in simple and in multivariable regression, (see Proof 1, of this Appendix)

$$\sum(X_1 - \bar{X}_1)^2 = \sum(X_{c1 \cdot 2} - \bar{X}_1)^2 + \sum(X_1 - X_{c1 \cdot 2})^2 \tag{D-8}$$

or that total variation = explained variation + unexplained variation. In our notation for squared deviations, the symbol x^2 is used so that equation (D-8) may be written as

$$\sum x_1^2 = \sum x_{c1 \cdot 2}^2 + \sum x_{s1 \cdot 2}^2 \tag{D-9}$$

By (D-4), the normal equations for this case are,

$$\sum X_1 = Na_{1\cdot 2} + b_{1\cdot 2} \sum X_2,$$
$$\sum X_1X_2 = a_{1\cdot 2}\sum X_2 + b_{1\cdot 2}\sum X_2^2 \qquad \text{(D-10)}$$

Algebraic solutions to (D-10) are,

$$a_{1\cdot 2} = \overline{X}_1 - b_{1\cdot 2}\,\overline{X}_2\,,$$
$$b_{1\cdot 2} = \sum X_1X_2 - a_{1\cdot 2} \sum X_2 \Big/ \sum X_2^2. \qquad \text{(D-11)}$$

If we consider the regression surface as $x_{c1\cdot 2} = b_{1\cdot 2}\, x_2$ which utilizes deviations from the expected values, the normal equations are,

$$\sum x_1 = Na_{1\cdot 2} + b_{1\cdot 2} \sum x_2,$$
$$\sum x_1x_2 = a_{1\cdot 2}\sum x_2 + b_{1\cdot 2}\sum x_2^2.$$

But since $\sum x_1 = \sum x_2 = 0$, then a = 0 and $b_{1\cdot 2} = \sum x_1x_2 \Big/ \sum x_2^2$.

In order to show that in either case (origin at (0,0) or $(\overline{X}_1, \overline{X}_2)$), $b_{1\cdot 2}$ is the same, it is necessary to show that the two derived expressions above for $b_{1\cdot 2}$ in the respective cases are equal. This proof is shown in Proof 2 of this Appendix. With this result, we have shown that using data in terms of deviations from the means of each series does not destroy the method and facilitates computations by reducing the number of normal equations to be solved.

The Relationship of the Coefficient of Correlation and the Regression Coefficient:

It is apparent that $b_{1\cdot 2}$ is the slope of the regression surface $x_{c1\cdot 2} = b_{1\cdot 2}x_2$ when X_1 and X_2 are measured as deviations from their respective means.

The expression for $b_{1\cdot 2}$ in this single case is

$$b_{1\cdot 2} = \sum x_1 x_2 \Big/ \sum x_2^2 \tag{D-12}$$

Since the values of the variables may be distributed over different ranges, it is convenient to normalize them, i.e., calculating $(X_i - \bar{X}_i)\big/\sigma_{X_i}$ for each series. Normalizing $b_{1\cdot 2}$ in Equation (D-12), results in

$$\frac{\sum \left(\frac{x_1}{s_{X_1}}\right)\left(\frac{x_2}{s_{X_2}}\right)}{\sum \left(\frac{x_2}{s_{X_2}}\right)^2} = \frac{\sum x_1 x_2}{s_{X_1} s_{X_2}} \cdot \frac{s_{X_2}^2}{\sum x_2^2}$$

$$= \frac{\sum x_1 x_2}{\sum x_2^2} \div \frac{s_{X_1}}{s_{X_2}}$$

$$= \frac{\sum x_1 x_2}{\sqrt{x_1^2 \, x_2^2}} \tag{D-13}$$

Since $s_{X_1}^2 = \sum x_1^2 \Big/ (N-1)$ and $s_{X_2}^2 = \sum x_2^2 \Big/ (N-1)$, Equation (D-13) is seen to be the simple coefficient of correlation. Thus, the following relation holds

$$r_{1\cdot 2} = b_{1\cdot 2} \left(\frac{s_{X_2}}{s_{X_1}}\right) \tag{D-14}$$

Therefore, the coefficient of correlation $r_{1\cdot 2}$ is the slope of the regression surface when both series are expressed in multiples of their respective standard deviations.

Method of Calculation: In Proof 4 of this Appendix, a general derivation is given for the calculation of explained variation. This equation is:

$$\sum x^2_{c1\cdot i,j,\dots,t} = b_{1,i\cdot j,k,\dots,t} \sum x_1 x_i + b_{1,j\cdot i,k,\dots,t} \sum x_1 x_i + \dots\dots\dots + b_{1,t\cdot i,j,\dots,(t-1)} \sum x_1 x_t. \tag{D-15}$$

Thus, in the simple two-variable case, the explained variation is

$$\sum x^2_{c1\cdot 2} = b_{1\cdot 2} \sum x_1 x_2.$$

Through the relationship given by D-9 , the unexplained variation is

$$\sum x^2_{s1\cdot 2} = \sum x_1^2 - \sum x^2_{c1\cdot 2}.$$

The standard error of estimate in the two variable case is defined as

$$s_{1\cdot 2} = \sqrt{\sum x^2_{s1\cdot 2} \Big/ (N - 1)}. \tag{D-16}$$

Exactly the same principles are involved in multi-variable linear regression as in single variable regression. As seen by (D-3), the regression coefficients are of a slightly different form in multi-variable regression and are termed partial regression coefficients. For example, the coefficient $b_{1,i\cdot j,k,l,\dots,t}$ represent the effect on X_1 of a unit change in variable X_i when all other variables $X_j, X_k, \dots\dots, X_t$, are evaluated at their expected values. In multivariable regression, the standard error of estimate is analogous to (D-16) so that

$$s_{1\cdot 2,\dots,t} = \sqrt{\sum x^2_{s1\cdot 2,3,\dots,t} \Big/ (N-k-1)}$$

where k is the number of independent variables in the regression analysis.

The coefficient of multiple determination is defined as

$$R^2_{1\cdot i,\ldots,t} = \frac{\sum x^2_{c1\cdot i,\ldots,t}}{\sum x_1^2} \tag{D-18}$$

This coefficient states the proportion of total variation which is accounted for or is "explained" by the regression surface. As additional pertinent independent variables are brought into the problem, $R^2_{1\cdot 2,3,\ldots}$ approaches 1.0 and $s_{1\cdot 2,3,\ldots}$. approaches zero. If all pertinent variables are included, only experimental measurement error prevents making perfect estimates. The coefficient of multiple determination affords a criterion for the selection of a particular regression surface. A predetermined minimum tolerable R^2 may be selected, say 90%, and any one of the infinitely many possible functional forms of equations which does surpass this figure may be hypothesized to be an empirical law. The level of R^2 selected depends, of course, on the experimenter's knowledge of the process under investigation and how much experimental error he believes will be introduced in the measurements of the dependent variable.

<u>Functional Dependency in the Independent Variable</u>: The term, independent variable, used to denote all variables other than the dependent variable is a misnomer, of course. In almost every case where large numbers of variables are involved, certain variables will interact with others in a mathematical covariance sense. A second primary goal in regression analysis is to select individual or groups of variables that act independently from others. This disclosure will aid in testing a new theory, and will provide

a firm basis for either the exclusion or inclusion of a variable in the analysis.

The simplest measure of relationship between two variables is the correlation coefficient. This statistic is a measurement of the square root of the proportion of total variation in the dependent variable explained by an independent variable or is, (See Proof 3),

$$r_{1\cdot 2} = \sqrt{\frac{\sum x_{c1\cdot 2}^2}{\sum x_1^2}} \tag{D-19}$$

This simple or gross correlation is influenced by interaction effects of other variables pertinent to the behavior of the dependent variable, thus presents a somewhat distorted picture of the relationship of two variables.

The relationships of the variables can be more extensively investigated by using the partial correlation theory. The partial correlation coefficient is a measure of the covariance between the dependent variable and one independent variable, when the influence of all other independent variables theoretically has been removed. These coefficients show the relative importance of the different independent variables in explaining variations in the dependent variable. More precisely, the coefficient of partial correlation is the square root of the ratio of the increase in the variation of the computed explained variation resulting from the introduction of a new variable, and the variation that had not been explained before introducing that variable. This definition is based on the following philosophy:

The use of any pertinent independent variable, say X_2, results in a certain amount of explained variation, indicated by $\sum x^2_{c1\cdot 2}$, but some of the variation in X_1 remains unexplained, which is indicated by $\sum x^2_{s1\cdot 2}$. Introducing variable X_3 in addition to X_2 will increase the proportion of explained variation if X_3 is germane to the problem. The explained variation calculated from a regression surface in variables X_2 and X_3, denoted by $\sum x^2_{c1\cdot 2,3}$ will exceed $\sum x^2_{c1\cdot 2}$ if X_3 is a significant variable. Subsequently, the relative importance of any variable may be ascertained simply by comparing the explained variation of a regression surface using that variable to the explained variation associated with a surface which omits that variable.

Several computational methods are available for the partial correlation coefficient. Among these are

$$r_{1,i\cdot j,k,\ldots,t} = \frac{r_{1,i\cdot j,k,\ldots,(t-1)} - \left[\left(r_{1,k\cdot j,\ldots(t-1)}\right)\left(r_{i,k\cdot 1,j,\ldots,(t-1)}\right)\right]}{\sqrt{\left(1 - r^2_{1,k\cdot j,\ldots,(t-1)}\right)\left(1 - r^2_{i,k\cdot 1,j,\ldots,(t-1)}\right)}}, \quad \text{(D-20}$$

and the analysis of variance formula

$$r_{1,i\cdot j,\ldots,t} = \sqrt{\frac{\sum x^2_{c1\cdot i,j,\ldots,t} - \sum x^2_{c1\cdot j,\ldots,t}}{\sum x^2_1 - \sum x^2_{ci\cdot j,k,\ldots,t}}} \quad \text{(D-21}$$

The coefficient of partial correlation $r_{1,i\cdot j,\ldots,t}$ takes the sign of $\beta_{1,i\cdot j,\ldots,t}$ in the regression surface. The subscript $1,i\cdot j,\ldots,t$ for this coefficient, indicates that this is the correlation between the dependent variable X_1 and the independent variable X_i when all other independent variables are taken as their respective mean values. The major importance of the partial correlation coefficient is that it indicates the relative importance of the independent variables in explaining variations in the dependent variable. The relative importance of each variable and its associated statistical test of significance (the F test) will indicate whether the variable should be included in the hypothesized empirical law of penetration.

The varying degree of covariance among pertinent variables is a violation of the basic underlying assumption of independence in the regression variables. The consequence of this violation will be discussed in the following situation.

In a two independent variable regression analysis, the linear regression surface is of the form

$$X_{c1\cdot2,3} = a_{1\cdot2,3} + \beta_{1,2\cdot3} X_2 + \beta_{1,3\cdot2} X_3$$

The basic assumptions of regression analysis imply that X_2 and X_3 are independent and the data points of each observation (X_{2i}, X_{3i}, X_{1i}) are distributed about a plane in the $X_2, X_3, X_1)$ coordinate system. If we have a situation in which X_2 is an approximately linear function of X_3 so that

$$X_2 \cong K + CX_3$$

then the observations (X_{2i}, X_{3i}, X_{1i}), $i=1,\ldots,N$, are situated approximately in a plane perpendicular to the (X_2, X_3) plane along the line $X_2 = K + CX_3$. The X_1 values are then approximately normally distributed about the line in space which forms the intersection between the plane

$$X_{c1\cdot 2,3} = a_{1\cdot 2,3} + \beta_{1,2\cdot 3}X_2 + \beta_{1,3\cdot 2}X_3$$

and the vertical plane through $X_2 = K + CX_3$.

The regression analysis will formally lead to a solution; however, it is quite clear from a geometrical viewpoint that the calculated plane is unstable because all the values of the dependent variable are distributed about an approximate line rather than in a plane. Many other planes intersecting this line will exhibit nearly as good statistical prediction capability; thus in this regard, the calculated regression surface is termed statistically unstable.

This fact does not, of course, destroy the uniqueness of the surface calculated using least squares criterion or the ranking of the variables adjudged to be significant to the penetration process. Having chosen the proper functional form, the calculated hypersurface will be characterized by a large coefficient of multiple correlation and a relatively low variance of estimate - the desired characteristics of a regression surface.

When applying regression analysis, it is important to have a clear understanding of the degree of covariance between the "independent" variables. In the model presented here, all of the variables were assumed to act independently. It is known, however, that certain approximate relationships exist in the engineering properties for certain materials. Since at this time, very little information concerning these relationships is available, the assumption of independence was made.

In respect to the above considerations, all of the calculated surfaces must be regarded as unstable in the sense just described since there are relative degrees of covariance exhibited between the pertinent variables. Just as in the case illustrated of two "independent" variables where one variable is related to the other through an approximate relationship, it must be concluded that other hypersurfaces will closely approximate π_1 within the range of the variables considered.

One further topic must be discussed which pertains to the establishment of variable relationships in this study. The basic assumption that a correlation analysis of two variables is characterized by a two-dimensional density function has been discussed. The assumption implies that there exists a continuous two-dimensional density function relating the dependent variable π_1 to any other independent variable, say π_{16}. Furthermore, this distribution must be Gaussian in order to apply the F test of significance used herein. One of the basic properties of a two-dimensional Gaussian density function is that it is continuous. Thus, any value (X_1, X_{16}) is obtainable. However, it is apparent that in considering experiments of a limited number of projectile and target material combinations, only a very limited number of different values of π_{16} will be found. The number of different values of π_{16} obtained and the spread of these values then is a direct function of the chosen experiments. The histogram of the distribution of π_{16} may depict a highly skewed distribution resulting from analyzing experimental information.

TESTS OF SIGNIFICANCE

Partial Correlation Coefficients: Two alternate methods are available to test the statistical significance of the partial correlation coefficient. The first method makes use of the t Statistic, where

$$t = \sqrt{\frac{r^2_{1,m\cdot 2,3,\ldots,(m-1)} \Big/ (N-m)}{1-r^2_{1,m\cdot 2,3,\ldots,(m-1)}}}$$

Since a coefficient of partial determination relates the proportion that the additional explained variation attributable to a given independent variable bears to the unexplained variation before the use of that independent variable, it is the usual procedure to test the hypothesis that the coefficient equals zero. When $r_{ij\cdot k,l,\ldots,m} = 0$, the t statistic is distributed as Student's t distribution with (N-M) degrees of freedom.

An analysis of variance technique makes use of the following F statistic.

$$F = \frac{\sum x^2_{c1\cdot 2,3,\ldots,m} - \sum x^2_{c1\cdot 2,3,\ldots,(m-1)}}{\sum x_1^2 - \sum x^2_{c1\cdot 2,3,\ldots,m}}$$

If the calculated F value exceeds the tabulated α % F value* (with 1 and (N-m) degrees of freedom), the partial correlation coefficient is adjudged to be statistically significant to the α % level.

---

*α is a commonly used notation denoting the level of significance chosen for a statistical test of hypothesis.

Coefficient of Multiple Correlation: To ascertain whether a multiple coefficient of determination significantly exceeds zero, we use an F test similar to the one just described. In general form, we may use either

$$F = \frac{R^2_{c1\cdot 2,\ldots,m}/(m-1)}{(1-R^2_{c1\cdot 2,3,\ldots,m})/(N-m)}$$

or

$$F = \frac{\sum x^2_{c1\cdot 2,3,\ldots,m}/(m-1)}{\sum x^2_{s1\cdot 2,3,\ldots,m}/(N-m)}$$

Again, if the calculated value of F exceeds that of the α % tabulated value of F (with (M-1) and (N-m) degrees of freedom), the regression surface is adjudged to be statistically significant in assessing the dependent variable.

The Regression Coefficient: The estimates $\left(b_{1,2\cdot 3,4,\ldots,m},\ b_{1,3\cdot 2,4,\ldots,m}\right.$ $\left.\ldots,\ b_{1,m\cdot 2,3,\ldots,(m-1)}\right)$ are normally distributed with means, $\left(\beta_{1,2\cdot 3,4,\ldots,m},\ \beta_{1,3\cdot 2,4,\ldots,m},\ ----,\ \beta_{1,m\cdot 2,3,\ldots,(m-1)}\right)$, and variances

$$V(b_{1,i\cdot j,\ldots,m}) = C_{(i-1)(i-1)}\,\sigma^2,\ i=2,3,\ldots,m.$$

Here $C_{(i-1)(i-1)}$ is the appropriate element of Matrix A^{-1} and σ^2 is the estimate of the variance of the regression surface.

A marginal test for any specified hypothetical value of $\beta_{1,i\cdot[\ ]}$ may be made by the t test where

$$t = \frac{b_{1,i\cdot[\ ]} - \beta_{1,i\cdot[\ ]}}{s_{b_{1,i\cdot[\ ]}}}$$

where $s_{b_{1,i\cdot[\ ]}}$ is the standard deviation of the respective partial correlation coefficient as given above. The t statistic (D-22) is distributed as Student's t with N-m-1 degrees of freedom.

PROOFS

Proof 1: Total Variation = Explained Variation + Unexplained Variation

or $$\sum x_1^2 = \sum x^2_{c1\cdot 2} + \sum x^2_{s1\cdot 2} \qquad \text{(D-23)}$$

The two dimensional case is considered here. Expressions for each term in (D-23) and the terms arranged as in (D-8) are shown to be equal.

(1). $$\sum x_1^2 = \sum X_1^2 - \bar{X}_1 \sum X_1$$

Proof: $$\sum x_1^2 = \sum (X_1 - \bar{X}_1)^2$$
$$= \sum (X_1^2 - 2X_1\bar{X}_1 + \bar{X}_1^2)$$
$$= \sum X_1^2 - 2\bar{X}_1 \sum X_1 + \sum \bar{X}_1^2$$
$$= \sum X_1^2 - 2\bar{X}_1 \sum X_1 + N\bar{X}_1^2$$

Since $\bar{X}_1^2 = (\sum X_1/N)^2$, then

$$\sum x_1^2 = \sum X_1^2 - \bar{X}_1 \sum X_1. \qquad \text{(D-24)}$$

(2). $$\sum x^2_{c1\cdot 2} = \sum (X_{c1\cdot 2} - \bar{X}_1)^2$$

Proof: $$\sum x^2_{c1\cdot 2} = \sum (X^2_{c1\cdot 2} - 2X_{c1\cdot 2}\bar{X}_1 + \bar{X}^2)$$
$$= \sum X^2_{c1\cdot 2} - 2\bar{X}_1 \sum X_{c1\cdot 2} + \sum \bar{X}_1^2$$

Since $\bar{X}_{c1\cdot 2} = \bar{X}_1$ and $\sum X_{c1\cdot 2} = \sum X_1$ because of the least square properties, then

$$\sum x^2_{c1\cdot 2} = \sum X^2_{c1\cdot 2} - \bar{X}_1 \sum X_1 \qquad \text{(D-25)}$$

3. $$\sum x^2_{s1\cdot 2} = \sum x_1^2 - \sum x^2_{c1\cdot 2}$$

Proof: $$\sum x^2_{s1\cdot 2} = \sum (x_1 - x_{c1\cdot 2})^2$$

$$= \sum (x_1^2 - 2X_1X_{c1\cdot 2} + x^2_{c1\cdot 2})$$

$$= \sum x_1^2 - 2\sum x_1x_{c1\cdot 2} + \sum x^2_{c1\cdot 2}.$$

Since $X_{c1\cdot 2} = a + bX_2$, then $\sum X_1X_{c1\cdot 2} = \sum \left| X_1(a+bX_2) \right|$

$$= \sum (aX_1 + bX_1X_2)$$

$$= a\sum X_1 + b\sum X_1X_2$$

$$= \sum X^2_{c1\cdot 2}$$

Therefore $\sum x^2_{s1\cdot 2} = \sum x_1^2 - 2\sum x_1x_{c1\cdot 2} + \sum x^2_{c1\cdot 2}$ becomes

$$\sum x^2_{s1\cdot 2} = \sum x_1^2 - \sum x^2_{c1\cdot 2} \tag{D-26}$$

Thus, combining (D-24), (D-25), and (D-26), the result becomes

$$\sum x_1^2 = \sum x^2_{c1\cdot 2} + \sum x^2_{s1\cdot 2} \quad \text{for}$$

$$\sum x_1^2 - \bar{x}_1\sum x_1 \equiv \sum x^2_{c1\cdot 2} - \bar{x}_1\sum x_1 + \sum x_1^2 - \sum x^2_{c1\cdot 2}$$

It is apparent that in 2 and 3, the expansion of respective equations will result in equivalent expression [(D-25), (D-26)] because of the least squares properties. Thus, in general

$$\sum x_1^2 = \sum x^2_{c1\cdot 2,\ldots,} + \sum x^2_{s1\cdot 2,\ldots,} \tag{D-27}$$

Proof 2: $b_{1\cdot 2}$ is equivalent in either regression surface,

$$x_{c1\cdot 2} = b_{1\cdot 2}x_2$$

or

$$X_{c1\cdot 2} = a_{1\cdot 2} + b_{1\cdot 2}X_2.$$

From the expression $X_{c1\cdot 2} = a_{1\cdot 2} + b_{1\cdot 2}X_2$, it was shown that

$$a_{1\cdot 2} = \bar{X}_1 - b_{1\cdot 2}\overline{X_2}, \qquad \text{from (page D-11)},$$

and $$b_{1\cdot 2} = \sum X_1X_2 - a_{1\cdot 2}\sum X_2 / \sum X_2^2, \qquad \text{from (page D-11)},$$

From the expression $x_{c1\cdot 2} = b_{1\cdot 2}\sum x^2$, the following holds

$$b_{1\cdot 2} = \sum x_1x_2 / \sum x_2^2, \qquad \text{from (page D-11)}.$$

It is necessary to show that

$$\frac{\sum X_1X_2 - a_{1\cdot 2}\sum X_2}{\sum X_2^2} = b_{1\cdot 2} = \frac{\sum x_1x_2}{\sum x_2^2} \tag{D-28}$$

Equation (D-28) may be written as

$$\frac{\sum X_1X_2 - a_{1\cdot 2}\sum X_2}{\sum X_2^2} = \frac{\sum X_1X_2}{\sum X_2^2} - \frac{\left[(\bar{X}_1 - b_{1\cdot 2}X_2)(\sum X_2)\right]}{\sum X_2^2}$$

$$= \frac{\sum X_1X_2}{\sum X_2^2} - \frac{X_1\sum X_2}{\sum X_2^2} + \frac{b_{1\cdot 2}\bar{X}_2\sum X_2}{\sum X_2^2}$$

From this,

$$\frac{\sum X_1X_2}{\sum X_2^2} - \frac{\bar{X}_1\sum X_2}{\sum X_2^2} = \left[1 - \frac{X_2\sum X_2}{\sum X^2}\right] \quad b_{1\cdot 2}$$

or $$\frac{\sum x_1x_2 - \bar{x}_1 \sum x_2}{\sum x_2^2 - \bar{x}_2 \sum x_2} = b_{1\cdot 2}$$

It is easily shown that the numerator of the above equals $\sum x_1x_2$ and the denominator equals x_2^2 by expanding the terms of the two respective quantities; thus, $b_{1\cdot 2}$ is equivalent in both cases of regression since this results in an identity to the right hand sides of equation (D-28).

Proof 3: In the one independent variable case,

$$r^2_{1\cdot 2} = \frac{\sum x_{c1\cdot 2}}{\sum x_1^2} = \frac{\text{Explained Variation}}{\text{Total Variation}} . \qquad \text{(D-28a)}$$

Since $x_{c1\cdot 2} = b_{1\cdot 2}x_2$, then

$$\frac{\sum x^2_{c1\cdot 2}}{\sum x_1^2} = \frac{\sum (b_{1\cdot 2}x_2)^2}{\sum x_1^2} = \frac{b^2_{1\cdot 2}\sum x_2^2}{\sum x_1^2}$$

$$= \frac{\left[\frac{\sum x_1x_2}{\sum x_2^2}\right]^2 \sum x_2^2}{\sum x_1^2} = \frac{\left(\sum x_1x_2\right)^2}{\sum x_2^2 \sum x_1^2} = r^2_{1\cdot 2}$$

Proof 4: A General Method of Finding the Expression for Explained Variation.

1. For the <u>one</u> Independent Variable case, $X_{c1\cdot 2} = a_{1\cdot 2} + b_{1\cdot 2} X_2$,

   We seek expressions for $\sum X^2_{c1\cdot 2}$ and $\sum x^2_{c1\cdot 2}$. So that

$$\sum x^2_{c1\cdot 2} = \sum (a_{1\cdot 2} + b_{1\cdot 2} X_2)^2$$

$$= \sum \left[a^2_{1\cdot 2} + 2a_{1\cdot 2} b_{1\cdot 2} X_2 + b^2_{1\cdot 2} X^2_2 \right]$$

$$= Na^2_{1\cdot 2} + 2a_{1\cdot 2} b_{1\cdot 2} \sum X_2 + b^2_{1\cdot 2} \sum X^2_2$$

$$= a_{1\cdot 2} (Na_{1\cdot 2} + b_{1\cdot 2} \sum X_2) + b_{1\cdot 2} (a_{1\cdot 2} \sum X_2 + b_{1\cdot 2} \sum X^2_2)$$

Notice that the normal equations for this case are:

$$\text{(1)} \quad \sum X_1 = Na_{1\cdot 2} + b_{1\cdot 2} \sum X_2,$$

$$\text{(2)} \quad \sum X_1 X_2 = a_{1\cdot 2} \sum X_2 + b_{1\cdot 2} \sum X^2_2.$$

Using (1) and (2), the above reduces to

$$\sum X^2_{c1\cdot 2} = a_{1\cdot 2} \sum X_1 + b_{1\cdot 2} \sum X_1 X_2 \tag{D-29}$$

It was shown in (D-25) that,

$$\sum x^2_{c1\cdot 2} = \sum X^2_{c1\cdot 2} - \overline{X_1} \sum X_1$$

but from (D-29), we see that

$\sum x^2_{c1\ 2} = (a_{1\cdot 2} \sum X_1 + b_{1\cdot 2} \sum X_1 X_2) - \overline{X}_1 \sum X_1.$ Since $a_{1\cdot 2} = 0$, then

$$\sum x^2_{c1\cdot 2} = b_{1\cdot 2} \sum x_1 x_2 \ . \tag{D-30}$$

2. For a three dimensional case, $X_{c1\cdot 23} = a_{1\cdot 2,3} + b_{1,2\cdot 3} X_2 + b_{1,3\cdot 2} X_3$

We seek expressions for $\sum X^2_{c1\cdot2,3}$ and $\sum x^2_{c1\cdot2,3}$.

$$\textcircled{1}\ \sum X^2_{c1\cdot2,3} = \sum(a_{1\cdot2,3} + b_{1,2\cdot3}X_2 + b_{1,3\cdot2}X_3)^2$$

$$= \sum\left[(a_{1\cdot2,3} + b_{1,2\cdot3}X_2) + (b_{1,3\cdot2}X_3)\right]^2$$

$$= \sum\left[(a_{1\cdot2,3} + b_{1,2\cdot3}X_2)^2 + 2(a_{1\cdot2,3} + b_{1,2\cdot3}X_3)(b_{1,3\cdot2}X_3) + (b_{1,3\cdot2}X_3)^2\right]$$

$$= \sum\left[a^2_{1\cdot2,3} + 2a_{1\cdot2,3}b_{1,2\cdot3}X_2 + b^2_{1,2\cdot3}X_2^2 + 2a_{1\cdot2,3}b_{1,3\cdot2}X_3 + 2b_{1,2\cdot3}X_2b_{1,3\cdot2}X_3 + b^2_{1,3\cdot2}X_3^2\right]$$

$$= Na^2_{1\cdot2,3} + 2a_{1\cdot2,3}b_{1,2\cdot3}\sum X_2 + b^2_{1,2\cdot3}\sum X_2^2 + 2a_{1\cdot2,3}b_{1,3\cdot2}\sum X_3 + 2b_{1,2\cdot3}b_{1,3\cdot2}\sum X_2X_3 + b^2_{1,3\cdot2}\sum X_3^2$$

$$= a_{1\cdot2,3}(Na_{1\cdot2,3} + b_{1,2\cdot3}\sum X_2 + b_{1,3\cdot2}\sum X_3) + b_{1,2\cdot3}(a_{1\cdot2,3}\sum X_2 + b_{1,2\cdot3}\sum X_2^2 + b_{1,3\cdot2}\sum X_2X_3) + b_{1,3\cdot2}(a_{1\cdot2,3}\sum X_3 + b_{1,2\cdot3}\sum X_2X_3 + b_{1,3\cdot2}\sum X_3^2)$$

The normal equations for this case are:

$$\textcircled{1}\ \sum X_1 = Na_{1\cdot2,3} + b_{1,2\cdot3}\sum X_2 + b_{1,3\cdot2}\sum X_3,$$

$$(2) \quad \sum X_1X_2 = a_{1\cdot 2,3} \sum X_2 + b_{1,2\cdot 3} \sum X_2^2 + b_{1,3.2} \sum X_2X_3,$$

$$(3) \quad \sum X_1X_3 = a_{1\cdot 2,3} \sum X_3 + b_{1,2\cdot 3} \sum X_2X_3 + b_{1,3\cdot 2} \sum X_3^2.$$

And by substitution, we have

$$\sum X_{c1\cdot 23}^2 = a_{1\cdot 2,3} \sum X_1 + b_{1,2\cdot 3} (\sum X_1X_2) + b_{1,3\cdot 2} (\sum X_1X_3). \qquad \text{(D-31)}$$

Now to obtain $\sum x_{c1\cdot 23}^2$, we note that

$$\sum x_{c1\cdot 23}^2 = \sum X_{c1\cdot 23}^2 - \bar{X}_1 \sum X_1$$

Using (D-31) $\sum x_{c1\cdot 2,3}^2 = a_{1\cdot 2,3} \sum X_1 + b_{1,2\cdot 3} \sum X_1X_2 + b_{1,3\cdot 2} \sum X_1X_3 - \bar{X}_1 \sum X_1;$

but $a_{1\cdot 23} = \bar{X}_1 - b_{1,2\cdot 3} \bar{X}_2 - b_{1,3\cdot 2} X_3$.

$$\text{Thus, } \sum x_{c1\cdot 23}^2 = (\bar{X}_1 - b_{1,2\cdot 3}\bar{X}_2 - b_{1,3\cdot 2}\bar{X}_3) \sum X_1 + b_{1,2\cdot 3} \sum X_1X_2 + b_{1,3\cdot 2} \sum X_1X_3 - \bar{X}_1 \sum X_1$$

$$= -b_{1,2\cdot 3} \bar{X}_2 \sum X_1 - b_{1,3\cdot 2}\bar{X}_3 \sum X_1 + b_{1,2\cdot 3} \sum X_1X_2 + b_{1,3\cdot 2} \sum X_1X_3$$

$$= -Nb_{1,2\cdot 3}\bar{X}_2\bar{X}_1 - Nb_{1,3\cdot 2}\bar{X}_3\bar{X}_1 + b_{1,2\cdot 3} \sum X_1X_2 + b_{1,3\cdot 2} \sum X_1X_3 =$$

$$= b_{1,2\cdot 3}(\sum X_1X_2 - N\bar{X}_1\bar{X}_2) + b_{1,3\cdot 2} (\sum X_1X_3 - N\bar{X}_1\bar{X}_3$$

$$= b_{1,2\cdot 3} \sum x_1x_2 + b_{1,3\cdot 2} \sum x_1x_3. \qquad \text{(D-32)}$$

A general proof by induction may be given to show that any explained variation may be given by,

$$\sum x^2_{c1\cdot i,j,k,\ldots,} = b_{1,i\cdot j,k,\ldots,}\sum x_1x_i + b_{1,j\cdot i,k,\ldots,}\sum x_1x_j + b_{1,k\cdot i,j,\ldots,}\cdot\sum x_1x_k + ,\ldots,$$

$$\text{or}\quad \sum x^2_{c1\cdot 2,3,\ldots,} = a_{1\cdot 2,3,\ldots,}\sum x_1 + b_{1,2\cdot 3,4,\ldots,}\sum x_1x_2 + b_{1,3\cdot 2,4,\ldots,}\sum x_1x_3 + \ldots \quad \text{(D-33)}$$