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THERMODYNAMIC AND TURBINE CHARACTERISTICS OF HYDROGEN-FUELED OPEN-CYCLE AUXILIARY SPACE POWER SYSTEMS

by Michael R. Vanco

Lewis Research Center

Cleveland, Ohio

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SUMMARY

An analytical study was conducted to determine the characteristics of turbine-driven hydrogen-fueled auxiliary space power systems. The oxidizers considered for these systems were oxygen and fluorine. The thermodynamic characteristics were examined in terms of specific propellant consumption, oxidizer-to-fuel ratio, and specific volume consumption. The turbine characteristics required by the thermodynamic conditions of interest were then studied in terms of stage number, blade speed, flow area, and stress.

Because high-turbine-pressure ratios can be obtained by venting the exhaust to space, system ideal specific propellant consumptions of less than 1 pound per horsepower-hour can be achieved. Ideal specific propellant consumption decreases to a minimum and then increases as turbine-inlet temperature increases. The optimum temperature exceeds 5000° R. If the turbine-inlet temperature is restricted to 2500° R by material considerations, there are significant increases in specific propellant consumption and specific volume consumption over those at the optimum temperature. An improvement in specific propellant consumption is obtained by using a recuperated system; there is, however, no significant change in the specific volume consumption.

For the thermodynamic conditions of interest, turbines for this application are characterized by low blade-jet speed ratios and low flow rates. These features imply relatively low efficiencies. With moderate blade speeds (about 1200 ft/sec), many turbine stages are needed. A turbine design with relatively few stages requires quite high blade speeds (about 2400 ft/sec). In order to achieve the desirable high blade speeds with current superalloys, turbine-disk temperatures must be restricted to less than 1500° F. The low flow rates necessitate either partial-admission or intermittant-flow operation. Appropriate turbine types for this application include a multiple-reentry design for continuous-flow operation at moderate speeds or a velocity-compounded design for operation at high speeds. The efficiency levels achievable for turbines designed particularly for this application are still undetermined.

INTRODUCTION

Chemically-fueled auxiliary-power systems are attractive for short-time, low-power space applications. Possible uses for these systems could include power for driving vehicles or for operating mechanical equipment and tools on the lunar surface or for emergency use on space vehicles. The fuel source for these systems could be either supercritical liquids or boiloff gases from rocket propulsion tanks. Since hydrogen is an attractive fuel for primary propulsion for future missions, open-cycle, hydrogen-fueled auxiliary space power systems are of interest. The oxidants being considered for these missions are oxygen and fluorine.

For low-power applications, two systems can be considered, the reciprocating internal-combustion engine and a turbine-powered system. The internal-combustion engine system (1 to 5 hp) has been examined in reference 1 for use with hydrogen and oxygen. This power system consists basically of a reciprocating engine and a recuperator. The specific propellant consumption achieved for this system using high-pressure propellants (300 and 700 psig for hydrogen and oxygen, respectively) was 1.6 pounds per horsepower-hour for a power level of 4.5 horsepower. When the source of fuel is relatively low-pressure boiloff gases, the system needs two compressors to bring the propellant to the required system pressure level. The turbine-powered system consists of a combustor and a turbine; it may or may not have a recuperator. This system can operate at low gas pressures without the need for compressors.

To determine the characteristics of a turbine-driven system, an analytical study was conducted. The thermodynamic characteristics of the system are examined in terms of specific propellant consumption by both weight and volume, and oxidizer-to-fuel ratio as a function of system propellant, pressure ratio, and temperature. Recuperated and nonrecuperated systems are given consideration. The turbine characteristics required by the thermodynamic conditions of interest are then discussed.

THERMODYNAMIC CHARACTERISTICS

This section presents a description of the cycle and the effect of the cycle parameters on system characteristics. The symbols used in the analysis are presented in appendix A, and the equations used for obtaining these characteristics are given in appendix B.

Cycle Description

The simplest cycle for a system using chemical fuels is an open cycle. A schematic

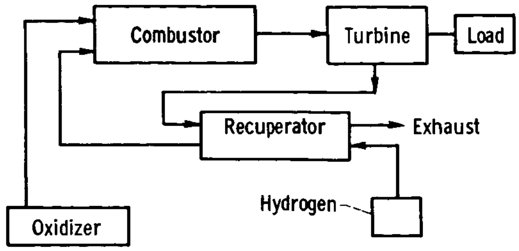


Figure 1. - Schematic diagram of hydrogen-fueled open-cycle power system.

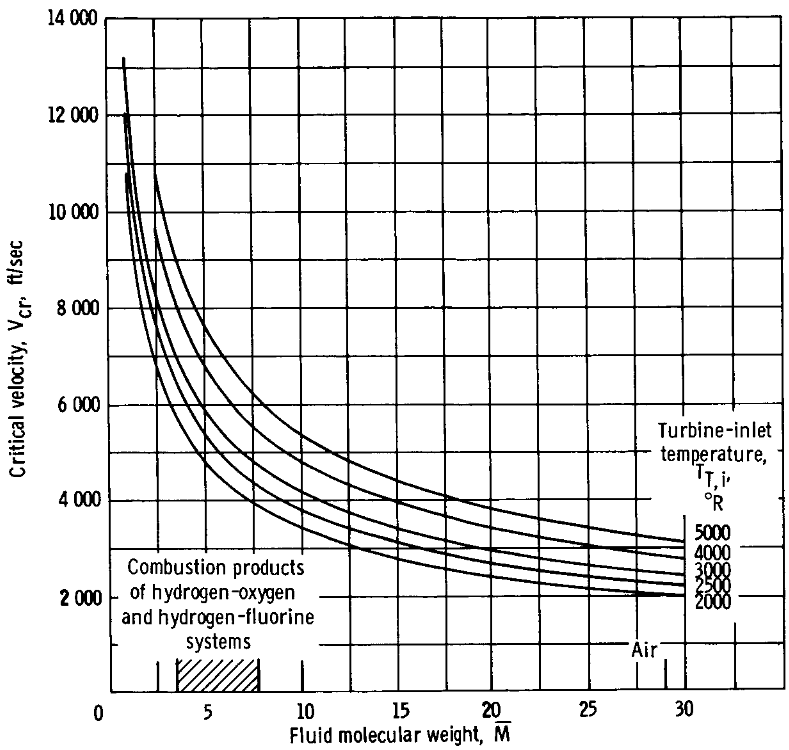


Figure 2. - Effect of molecular weight on critical velocity.

diagram of a hydrogen-fueled recuperated open cycle is shown in figure 1. This system consists of propellant (oxidizer and fuel) tanks, a combustor, a turbine, and a recuperator. The fuel from the fuel tank is heated in the recuperator by the turbine exit gases. The preheated fuel then enters the combustor and is burned with the oxidizer from the oxidizer tank. Expansion of the hot combustion products in the turbine produces the required power to drive the system load. The turbine exit gases are then cooled in the recuperator and exhausted to space. If no recuperator is present, the fuel is fed directly to the combustor and the turbine exit gases exhaust directly to space.

Effect of Cycle Parameters on System Characteristics

The characteristics of an open-cycle system are examined in terms of specific propellant consumption by both weight and volume, and oxidizer-to-fuel ratio as a function of system fluid and pressure ratio, temperature, and recuperator effectiveness. According to standard convention, specific propellant consumption by weight is herein called specific propellant consumption; consumption by volume is called specific volume consumption.

Fluid and pressure ratio. - The effect of molecular weight on critical velocity (eq. (B1b)), a measure of the available energy of a fluid, is shown in figure 2 where critical velocity is plotted against molecular weight for several turbine-inlet temperatures. Critical velocity decreases as the molecular weight increases. Thus, the high-available-energy fluids are those that have low molecular weights. Hydrogen is a fuel in this category. With oxygen and fluorine as oxidizers, hydrogen-rich systems produce

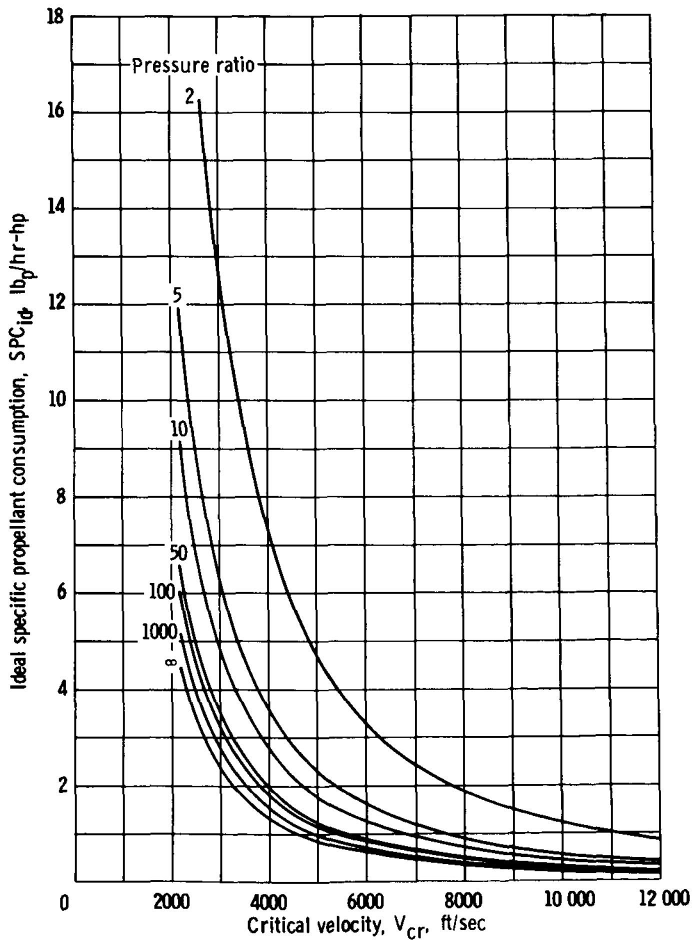


Figure 3. - Effect of critical velocity and pressure ratio on ideal specific propellant consumption.

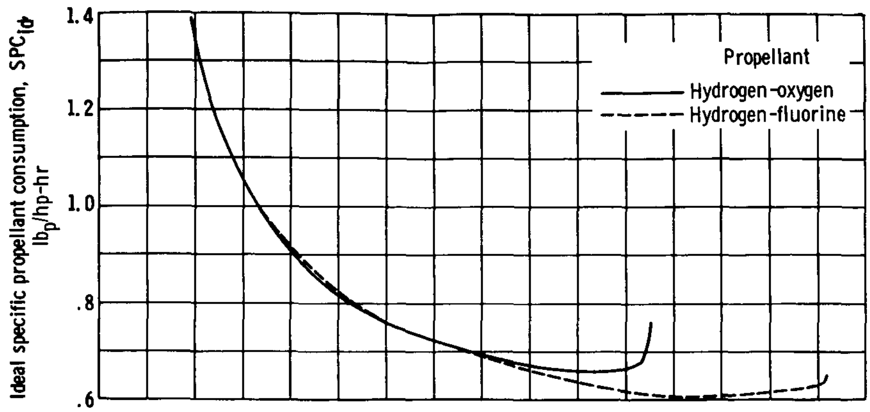
combustion products with molecular weights in the range of about 3.5 to 7.5. The critical velocities for these combustion products are 2 to 3 times that of air; the associated kinetic energies that can be converted to shaft work, therefore, are 4 to 9 times that of air. Figure 2 also shows that an increase in the temperature of a fluid increases the energy of that fluid.

The effect of critical velocity and pressure ratio on ideal specific propellant consumption (eq. (B2e)) is presented in figure 3, where ideal specific propellant consumption is plotted against critical velocity with turbine pressure ratio as a parameter. For a given turbine pressure ratio, the ideal specific propellant consumption decreases as the critical velocity increases. This decrease in specific propellant consumption is caused by the increase in kinetic energy per pound of fluid that is available to do work in the turbine. For a given critical velocity, the ideal specific propellant consumption decreases as the turbine pressure ratio increases. Therefore, by using high-energy fluids, ideal specific propellant consumptions of less than 1 pound per horsepower-hour can be achieved with high turbine pressure ratios. Since the system being considered would vent its exhaust to space, high turbine pressure ratios, on the order of 100 to 1000, can be achieved readily.

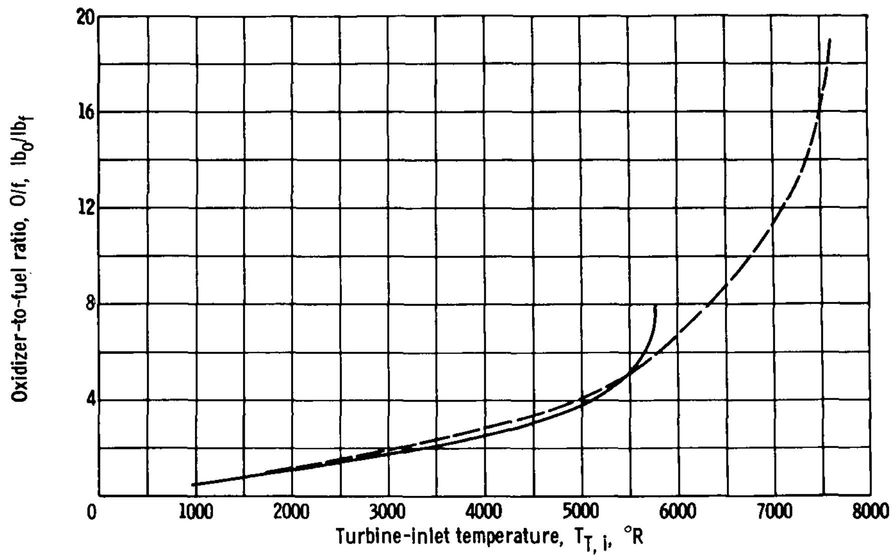
Temperature. - The effect of turbine-inlet temperature on ideal specific propellant consumption, on oxidizer-to-fuel ratio, and on ideal specific volume consumption and the effect of oxidizer-to-fuel ratio on specific volume are shown in this section. The combustion properties used for the calculations of specific propellant consumption and oxidizer-to-fuel ratio for hydrogen-oxygen and hydrogen-fluorine systems were obtained from the equilibrium composition program described in reference 2, using the inlet conditions stated in figure 4. The pure gas properties were obtained from reference 3. The turbine pressure ratio used for this and all subsequent discussions was 375.

The effect of turbine-inlet temperature on ideal specific propellant consumption is shown in figure 4(a) where ideal specific propellant consumption is plotted against turbine-inlet temperature. The ideal specific propellant consumption decreases to a minimum and then increases as turbine-inlet temperature increases. This effect is the result of a decrease in heat capacity of the combustion gases due to the change in the oxidizer-to-fuel ratio (fig. 4(b)) as the combustion (turbine inlet) temperature increases. Minimum ideal specific propellant consumption occurred at a turbine-inlet temperature of 5125° R for the hydrogen-oxygen system and 6250° R for the hydrogen-fluorine system. If the turbine-inlet temperature is restricted to 2500° R by material considerations, the increase in ideal specific propellant consumption over the minimum values is 25.2 percent for the hydrogen-oxygen system and 36.4 percent for the hydrogen-fluorine system.

The oxidizer-to-fuel ratios required to achieve the various turbine-inlet temperatures are presented in figure 4(b). For these fuel-rich systems, the oxidizer-to-fuel ratios increase as turbine-inlet temperature increases. The oxidizer-to-fuel ratios



(a) Ideal specific propellant consumption.



(b) Oxidizer-to-fuel ratio.

Figure 4. - Effect of turbine-inlet temperature on ideal specific propellant consumption and oxidizer-to-fuel ratio. Inlet conditions: hydrogen gas at 54° R, oxygen gas at 180° R, and fluorine gas at 180° R.

corresponding to 2500° R and to the optimum temperatures (minimum specific propellant consumption) are 1.4 and 4.05 for the hydrogen-oxygen system and 1.5 and 7.6 for the hydrogen-fluorine system.

The effect of oxidizer-to-fuel ratio on the component and total volumes of liquid (eqs. (B3)) associated with each pound of propellant (hereafter termed specific volumes) is shown in figure 5. The predominant factor determining liquid volume requirements is the high specific volume of the pure hydrogen, which causes a high proportion of hydrogen by volume in the mixture. As the proportion of hydrogen decreases with increasing oxidizer-to-fuel ratio, the total as well as the hydrogen specific volume is significantly reduced.

Ideal specific volume consumption (eq. (B4)) is plotted against turbine-inlet temper-

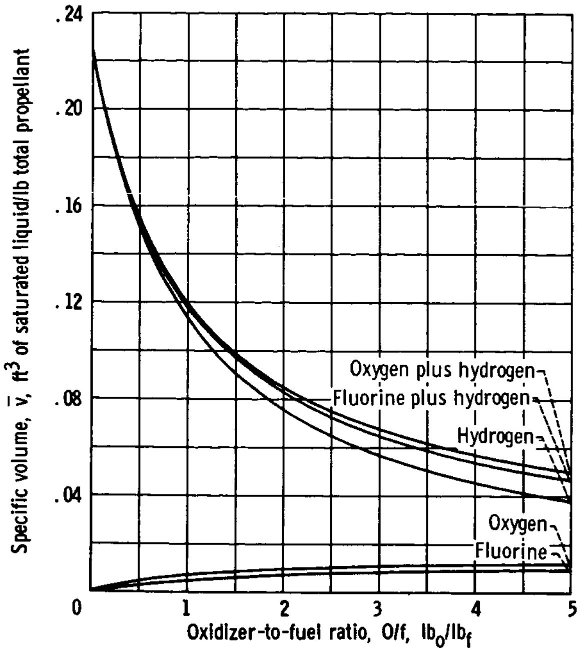


Figure 5. - Effect of oxidizer-to-fuel ratio on specific volume. Oxygen temperature, 162.2° R, specific volume, 0.0140 cubic foot per pound; fluorine temperature, 154° R, specific volume, 0.0106 cubic foot per pound; hydrogen temperature, 36.7° R, specific volume, 0.227 cubic foot per pound.

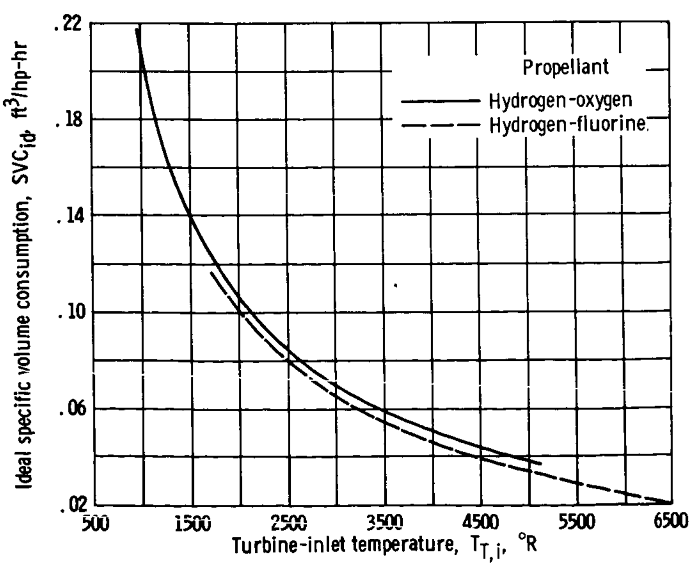


Figure 6. - Effect of turbine-inlet temperature on ideal specific volume consumption.

ature in figure 6. As can be seen from figure 6, ideal specific volume consumption decreases as turbine-inlet temperature increases. The significant decrease in ideal specific volume consumption is caused by the combination of the decrease in specific propellant consumption (fig. 4(a)) and the reduction in the propellant specific volume (fig. 5) associated with the increasing oxidizer-to-fuel ratio. If the turbine inlet-temperature is restricted to 2500° R by material considerations, an increase in propellant tankage is indicated by the increase in ideal specific volume consumption over the optimum temperature values of 128 percent for the hydrogen-oxygen system and 264 percent for the hydrogen-fluorine system.

Recuperator effectiveness. - The effect of the recuperator on specific propellant consumption, oxidizer-to-fuel ratio, and specific volume consumption is given for turbine-inlet temperatures of 2500° and 4500° R. For this case, actual specific propellant consumption is used instead of ideal specific propellant consumption in order to obtain realistic turbine-exit temperatures (eq. (B5a)) resulting in realistic recuperator performance. The turbine efficiency used for these calculations was 50 percent.

The effect of the recuperator on specific propellant consumption is presented in figure 7(a), where specific propellant consumption is plotted

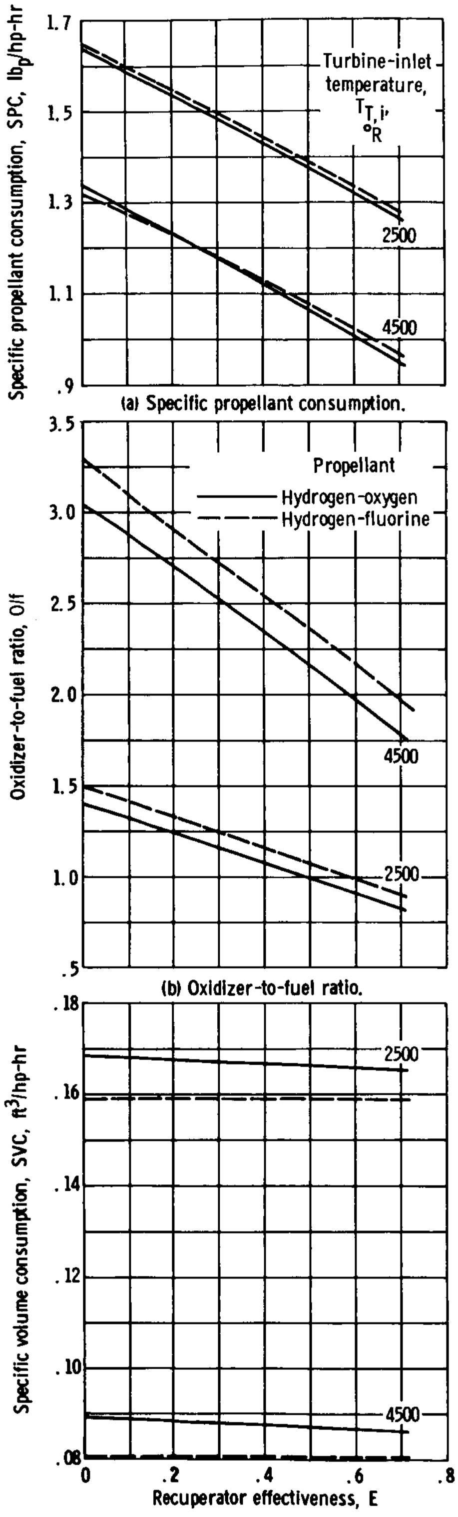


Figure 7. - Effect of recuperator on thermodynamic performance. Turbine efficiency, 0.50.

against recuperator effectiveness. Specific propellant consumption decreases as recuperator effectiveness increases. The use of a recuperator results in a smaller temperature rise in the combustor. This reduction in temperature rise is accompanied by a decrease in oxidizer-to-fuel ratio (fig. 7(b)). This decreasing oxygen-to-fuel ratio increases the specific heat of the combustion products, thereby decreasing specific propellant consumption. For example, for the hydrogen-oxygen system at turbine-inlet temperatures of 2500° and 4500° R, 19.5 percent and 24.6 percent reductions in specific propellant consumption are obtained by using a recuperator with an effectiveness of 0.6.

The effect of the recuperator on specific volume consumption is shown in figure 7(c). No change in specific volume consumption with an increase in recuperator effectiveness occurs for the hydrogen-fluorine system, and only a very slight decrease occurs for the hydrogen-oxygen system. The decrease in specific propellant consumption caused by the increase in recuperator effectiveness is offset by the increase in propellant specific volume associated with the decrease in oxidizer-to-fuel ratio. The net result, therefore, is no significant change in propellant tankage requirement with increasing recuperator effectiveness.

In general, it can be seen from figures 4 to 7 that either fluorine or oxygen could be used for the open-cycle system without any significant difference in the thermodynamic characteristics.

TURBINE CHARACTERISTICS

The turbine characteristics are discussed in terms of aerodynamic loading, flow area, and stress. Since the thermodynamic conditions are approximately the same for systems using either oxidizer

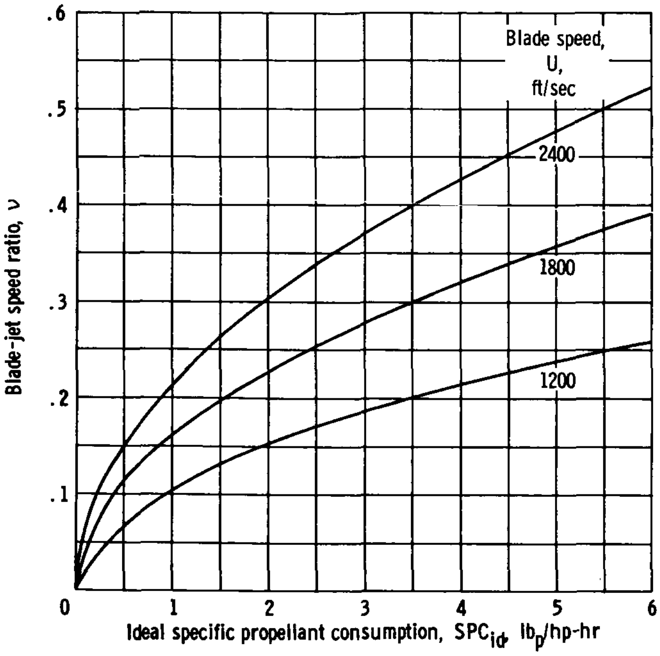


Figure 8. - Effect of ideal specific propellant consumption on blade-jet speed ratio.

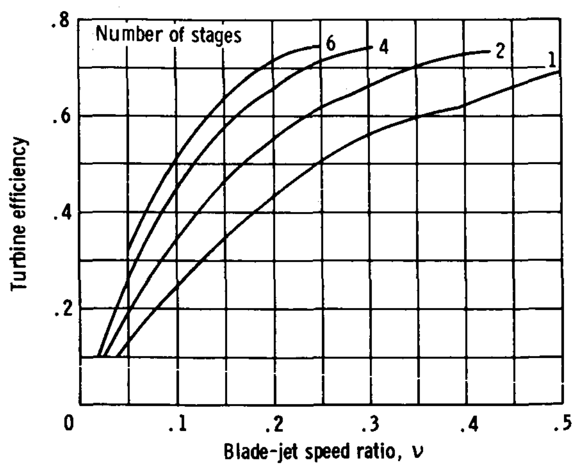


Figure 9. - Effect of blade-jet speed ratio on turbine efficiency. Reynolds number, 10 000; ratio of axial kinetic energy, exit to inlet, 1.0; constant of proportionality, 0.4; nozzle angle, 20°.

of interest at a given turbine-inlet temperature to 4500° R, the turbine characteristics are correspondingly similar. The turbine characteristics are shown for a turbine-inlet temperature of 2500° R and when a molecular weight is required, the hydrogen-oxygen propellant is used.

Aerodynamics

The effect of ideal specific propellant consumption on the aerodynamic loading parameter blade-jet speed ratio (eq. (B6a)) is shown in figure 8, where blade-jet speed ratio is plotted against ideal specific propellant consumption for blade speeds of 1200, 1800, and 2400 feet per second. As can be seen from figure 8, blade-jet speed ratio increases as both ideal specific propellant consumption and blade speed increase. The low ideal specific propellant consumptions of interest for auxiliary-power systems, therefore, result in low blade-jet speed ratios. For example, for an ideal specific propellant consumption of 0.8, blade-jet speed ratios of about 0.1 to 0.2 are encountered.

The effect of blade-jet speed ratio on turbine efficiency is presented in figure 9 where turbine efficiency is plotted against blade-jet speed ratio with stage number as a parameter. The method used for the calculation of these curves, which represent the performance of large conservatively designed full-admission turbines, was obtained from reference 4. Figure 9 shows that turbine efficiency increases as blade-jet speed ratio increases and that for a given blade-jet speed ratio, efficiency increases with stage number. For the blade-jet speed ratios of interest, the associated turbine efficiencies even for the favorable conditions represented by the curves are relatively low, in the range of 22 to 42 percent for a single-stage turbine and about 10 percentage points higher for two stages. Figures 8 and 9 show that to achieve reasonable efficiencies with moderate blade speeds of about 1200 feet per second, many stages are required. Even for the higher blade speeds of about 2400 feet per second, two or more stages are required. Note that for this application there are factors, which are subsequently discussed, that impose penalties on the efficiency values shown in figure 9.

Flow Area

Effect of specific propellant consumption and pressure-to-power ratio on nozzle-throat area (eq. (B7c)) is presented in figure 10, where nozzle-throat area is plotted against specific propellant consumption for several ratios of combustor chamber pressure to turbine output power. Figure 10 shows that the nozzle-throat area is directly proportional to specific propellant consumption for a given pressure-to-power ratio. For the pressure-to-power ratios shown, the nozzle-throat areas are quite small especially for the low specific propellant consumptions of interest. For example, for a pressure-to-power ratio of 5 and a specific propellant consumption of 1.64, the nozzle-throat area is 0.0214 square inch, which is the area of a hole slightly less than 3/16 inch in diameter. These small areas imply the necessity for partial-admission turbines.

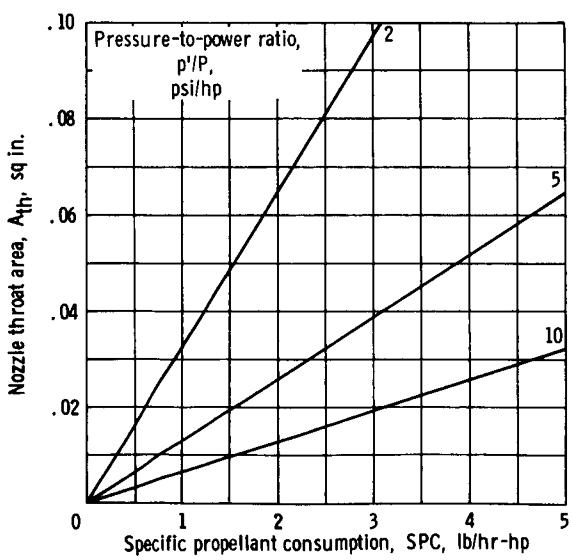


Figure 10. - Effect of pressure-to-power ratio and specific propellant consumption on nozzle-throat area.

The nozzle arcs of admission (eq. (B8d)) for convergent-choked nozzles are presented in figure 11, where nozzle arc of admission is plotted against turbine-tip diameter for several hub-to-tip radius ratios. Figure 11 shows that the arcs of admission for a convergent-choked nozzle are very small, on the order of a few percent of the annulus. As the turbine arc of admission is reduced from a full circle, additional losses associated with partial-admission operation are encountered (ref. 5). These losses, especially in multistage turbines, can impose severe

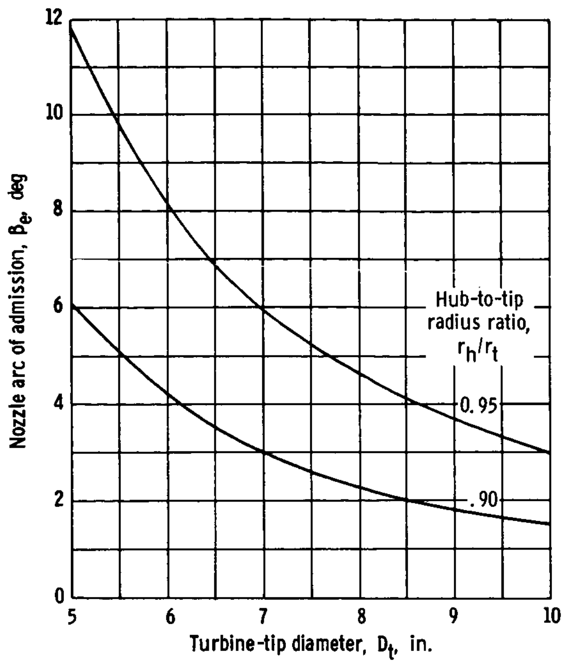


Figure 11. - Effect of turbine-tip diameter on nozzle arc of admission. Turbine-inlet temperature, 2500° R; pressure-to-power ratio, 5 pounds per square inch per horsepower; specific propellant consumption, 1.64 pounds propellant per horsepower-hour; nozzle angle, 20°.

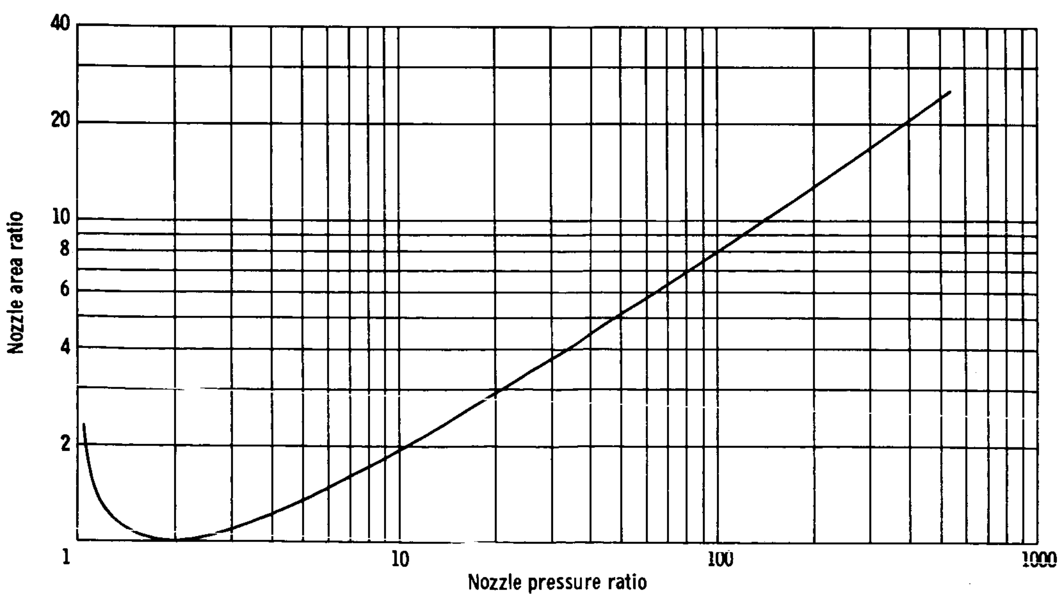


Figure 12. - Effect of nozzle pressure ratio on nozzle area ratio.

reductions on the full-admission efficiencies shown in figure 9 (p. 9).

These arcs of admission for cases requiring only a few stages can be increased by using a high-pressure ratio across the first nozzle, thus obtaining supersonic flow. This effect is shown in figure 12, where nozzle area ratio is plotted against nozzle pressure ratio. Figure 12 shows that an increase in pressure ratio from 1.89 (the critical value) to 100 increases the arc of admission by a factor of 8.

The preceding flow-area discussion indicating the need for partial-admission operation refers to the usual situation where flow through the turbine is continuous. It is possible, however, to eliminate the need for partial admission through the use of intermittant flow. The time-averaged flow rate would remain the same as for continuous flow, but the higher flow rates during the flow intervals would permit full-admission operation. The percentage of total time that is devoted to flow depends on the available flow area; a case requiring a 45° arc of admission (1/8 of total area) with continuous flow, for example, would require flow for only 1/8 of the total time. Since the turbine accelerates during the flow period and decelerates during the nonflow period, a speed excursion occurs during each cycle; the magnitude of this speed excursion can be controlled by the frequency of the flow cycle. Although the turbine-performance penalties associated with partial admission are eliminated, the intermittant-flow system must cope with the problems associated with the continuous cycling of the flow. An intermittant-flow system for a related application is discussed in reference 6.

Stress

Turbine stress limitations will be discussed in terms of disk stress to density ratio because, for the small blade heights being considered for this application, disk stress is more critical than blade-hub stress. Turbine disk stress-to-density ratio can be related to blade-tip speed, hub-to-tip radius ratio, and disk taper by the equations of reference 7 (eqs. (B9)).

The effect of the allowable disk stress-to-density ratio and disk taper on the allowable blade-tip speed is presented in figure 13, where blade-tip speed is plotted against disk stress-to-density ratio with disk taper as a parameter. The limiting disk-stress-to-density-ratio range of the current superalloys is also shown in this figure for disk temperatures of 1000° and 1500° F. As can be seen from figure 13, in order to achieve high blade speeds with the current materials, relatively low disk temperatures are required. For example, at a blade speed of 2400 feet per second, disk temperatures less than 1500° F are required if the current superalloys are to be used. Even at a disk temperature of 1000° F, the turbine disks must have high tapers, thus resulting in heavy rotor weights. These stress characteristics, therefore, can impose critical restrictions

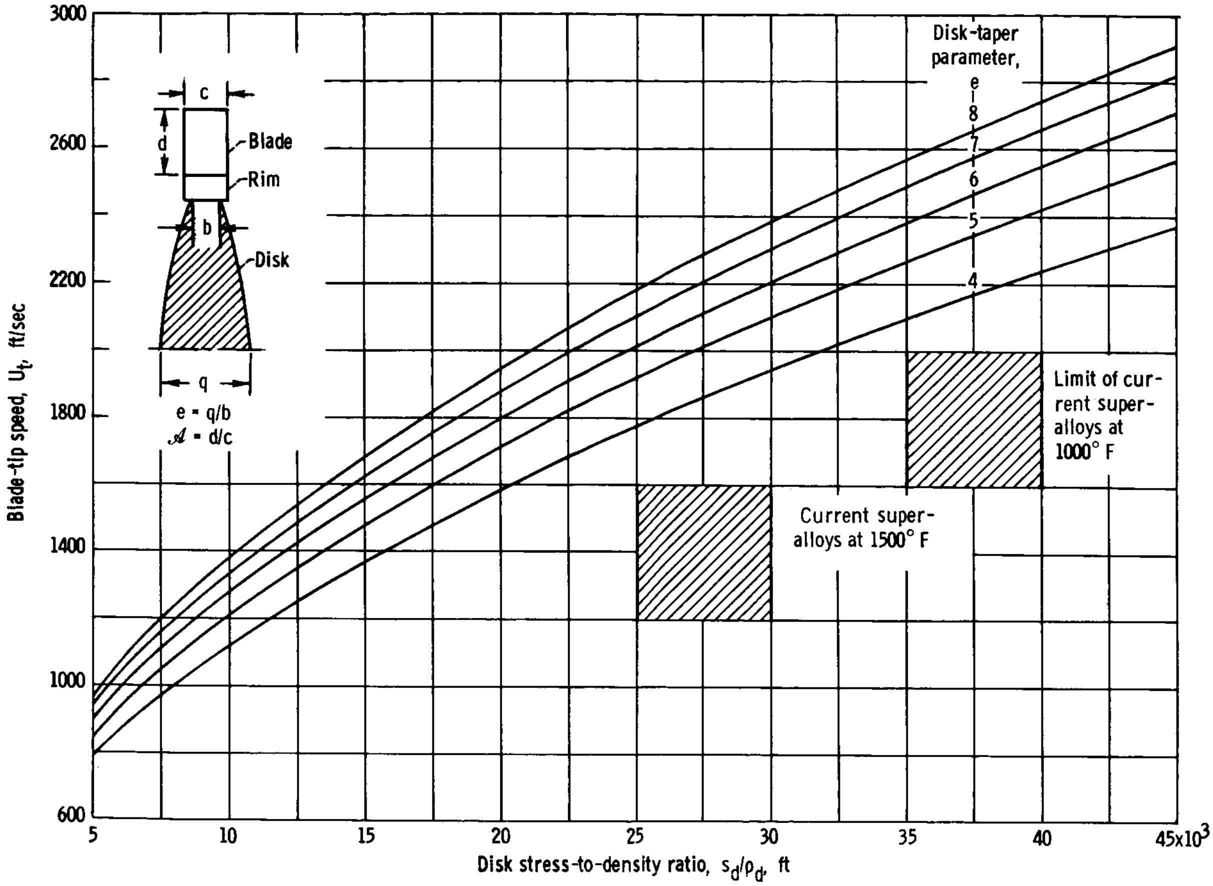


Figure 13. - Effect of disk stress-to-density ratio and disk taper parameter on blade-tip speed. Hub-to-tip radius ratio, 0.92; aspect ratio, 1.0.

with respect to temperature or blade speed if the disk temperature cannot be kept low. In order to achieve lighter, more conventional disks or to operate at temperature levels approaching maximum thermodynamic performance or at blade speeds yielding compact high-efficiency turbines, materials with stress-to-density ratios superior to those superalloys are required. Either coated refractory metals or low-density nonmetallics could provide such improvements, if these can be developed.

Design

The previous discussion has shown that turbines for this application are characterized by low blade-jet speed ratios and low flow rates. Low blade-jet speed ratios infer that many stages are needed if the blade speed is to be kept moderate, and that high blade speed is required if the turbine is to be designed with only a few stages. The low flow rates result in requirements for either partial-admission or intermittant-flow operation.

These characteristics suggest certain types of turbine designs as possible candidates for this application. Where many stages are required, a conventional multistage turbine appears to be the prime candidate with intermittant flow and also is a possibility with a continuous partial-admission design. Another possibility with partial admission is a multiple reentry design, where as many as 4 to 6 stages may be accomodated on a single wheel as a result of the small areas required for the flow. For the high blade-speed turbine with only a few stages, a velocity-compounded design appears appropriate. With partial admission, the largest permissible arc of admission, as indicated in figure 12, is obtained in this case because all the available pressure drop is expanded across the first nozzle.

It must be emphasized that, consistent with the characteristics imposed by the application, the turbine types discussed are all relatively low-efficiency units. Actual levels of efficiency achievable for turbines designed particularly for this application are still undetermined.

SUMMARY OF RESULTS

An analytical study was conducted to determine the characteristics of turbine-driven hydrogen-fueled auxiliary space power systems. The oxidizers considered for these systems were oxygen and fluorine. The thermodynamic characteristics were examined in terms of specific propellant consumption, oxidizer-to-fuel ratio, and specific volume consumption as a function of system fluid, pressure ratio, temperature, and recuperator effectiveness. The turbine characteristics required by the thermodynamic conditions of interest were then studied in terms of stage number, blade speed, flow area, and stress. The pertinent results of this analysis are as follows:

1. With high turbine pressure ratios being obtainable by venting the exhaust to space, system ideal specific propellant consumptions less than 1 pound per horsepower-hour can be achieved.

2. Ideal specific propellant consumption decreases to a minimum and then increases as turbine-inlet temperature increases. Minimum ideal specific propellant consumption for the propellant conditions considered occurs at turbine-inlet temperatures greater than 5000° R. If the turbine-inlet temperature is restricted to 2500° R by material considerations, the increase in ideal specific propellant consumption over the minimum values are 25.2 percent for the hydrogen-oxygen system and 36.4 percent for the hydrogen-fluorine system. The corresponding increases in ideal specific volume consumption are 128 percent for the hydrogen-oxygen system and 264 percent for the hydrogen-fluorine system.

3. An improvement in specific propellant consumption is obtained by using a recuperated system; there is, however, no significant change in the specific volume consumption. At a turbine-inlet temperature of 2500° R for the hydrogen-oxygen system, for example, a 19.5 percent reduction in specific propellant consumption is obtained by using a recuperator with an effectiveness of 0.6.

4. For the thermodynamic conditions of interest, the turbine blade-jet speed ratios are relatively low, thus implying low efficiencies. With moderate blade speeds of about 1200 feet per second, many stages are required. Even for the higher blade speeds of about 2400 feet per second, two or more stages are required.

5. For the pressure-to-power ratios of interest, the required flow areas in the turbine are very small, thus resulting in arcs of admission of only a few percent with continuous-flow operation. For designs requiring only a few stages, these arcs of admission can be increased by using a high-pressure ratio across the first nozzle. It is possible to eliminate the need for partial admission through the use of intermittant flow.

6. The turbine-disk stresses can impose critical restrictions with respect to the achievement of the high speeds required for turbine designs having only a few stages. At a blade speed of 2400 feet per second, for example, disk temperatures less than 1500° F are required if the current superalloys are to be used. Even at a disk temperature of 1000° F, the turbine disks must have high tapers, thus resulting in heavy weights.

7. The turbine characteristics suggest current types of turbine designs as possible candidates for this application. Where many stages and partial admission are required, a multiple reentry design, where as many as 4 or 6 stages can be accommodated on a single wheel, could be utilized. For the high blade-speed turbine with only a few stages,

a velocity-compounded design appears appropriate. The efficiency levels achievable for turbines of these types designed particularly for this application, are still undetermined.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 26, 1966,
128-31-02-25-22.

APPENDIX A

SYMBOLS

A cross-sectional flow area, sq in.

a speed of sound, ft/sec

$\mathcal{A}$ aspect ratio, d/c

b width at narrowest portion of disk, in.

c blade width, in.

c_p specific heat, Btu/(lb)($^{\circ}$R)

D diameter, in.

d blade height, in.

E recuperator effectiveness

e disk-taper parameter, q/b

f fuel

g gravitational constant, 32.17 ft/sec^2

Δh enthalpy drop across the turbine, Btu/lb

J mechanical equivalent of heat, 778 (ft)(lb)/Btu

L nozzle width, in.

M Mach number, V/a

$\overline{M}$ molecular weight

O oxidizer

P power, hp

p pressure, psia

p_r turbine pressure ratio

q width of disk at centerline, in.

R gas constant, 1545/$\overline{M}$, (ft)(lb)/(lb)($^{\circ}$R)

R_0 universal gas constant, 1545 (ft)(lb)/($^{\circ}$R)(mole)

r radius, in.

SPC specific propellant consumption, lb_p/(hp)(hr)

SVC specific volume consumption, ft_p^3/(hp)(hr)

s stress, lb/sq ft

T temperature, $^{\circ}$R

U blade speed, ft/sec

V velocity, ft/sec

V_j ideal jet speed, $\sqrt{2gJ\Delta h_{id}}$, ft/sec

v specific volume, ft^3/lb

$\overline{v}$ specific volume, ft^3/lb propellant

w weight flow, lb/hr

α nozzle angle, deg

β arc of admission, deg

γ specific-heat ratio

ν blade-jet speed ratio, U/V_j

ρ density, lb/ft^3

Subscripts:

b blade

cr critical

d disk

e exit

f	fuel
h	hub
i	inlet
id	ideal
min	minimum
m	mean
o	oxidizer
p	propellant
T	turbine
t	tip
th	throat

Superscripts:

*	critical
'	total

APPENDIX B

ANALYSIS EQUATIONS

Presented in this appendix are the equations used to calculate the thermodynamic and turbine characteristics.

Thermodynamic Characteristics

Critical velocity. - The velocity can be expressed as

$$V = aM \tag{B1}$$

The critical velocity is obtained when flow Mach number $M = 1$. Therefore,

$$V_{cr} = a^* = \sqrt{g\gamma RT^*} \tag{B1a}$$

For isentropic flow,

$$T^* = T'\left(\frac{2}{\gamma + 1}\right)$$

Therefore,

$$V_{cr} = \sqrt{\frac{2g\gamma R_o T'}{(\gamma + 1)\overline{M}}} \tag{B1b}$$

Specific propellant consumption. - Specific propellant consumption is defined as propellant weight flow divided by the output power.

$$SPC = \frac{w_p}{P} \tag{B2}$$

The power can be expressed as

$$P = w_p \, \Delta h\left(\frac{1}{2545}\right) \tag{B2a}$$

Therefore,

$$SPC = \frac{2545}{\Delta h} \tag{B2b}$$

and

$$SPC_{id} = \frac{2545}{\Delta h_{id}} \tag{B2c}$$

where Δh_{id} for a perfect gas is

$$\Delta h_{id} = \frac{\gamma}{\gamma - 1} \frac{R}{J} T' \left[1 - p_r^{(1-\gamma)/\gamma}\right] \tag{B2d}$$

where the pressure ratio p_r is greater than 1. Ideal work can be expressed in terms of critical velocity by combining equations (B2d) and (B1b).

$$\Delta h_{id} = \frac{\gamma + 1}{\gamma - 1} \frac{V_{cr}^2}{2gJ} \left[1 - p_r^{(1-\gamma)/\gamma}\right] \tag{B2e}$$

Specific volume. - The specific volumes of the oxidizer and fuel in terms of cubic feet per pound of propellant are

$$\overline{v}_o = v_o \left(\frac{O/f}{1 + O/f}\right) \tag{B3a}$$

$$\overline{v}_o = v_f \left(\frac{1}{1 + O/f}\right) \tag{B3b}$$

The specific volume of the propellant is

$$\overline{v}_p = \overline{v}_o + \overline{v}_f \tag{B3c}$$

Specific volume consumption. - The specific volume consumption is defined as the propellant volume flow divided by the output power. Therefore, specific volume consumption is

$$SVC = \overline{v}_p(SPC) \tag{B4}$$

Recuperator effectiveness. - The recuperator effectiveness is defined as the actual rate of heat transfer to the maximum possible rate of heat transfer. Therefore, for the recuperator in this report, the recuperator effectiveness is

$$E = \frac{(wc_p)_f (T_{f,e} - T_{f,i})}{(wc_p)_{min} (T_{T,e} - T_{f,i})} \tag{B5}$$

Since $(wc_p)_{min} = (wc_p)_f$,

$$E = \frac{T_{f,e} - T_{f,i}}{T_{T,e} - T_{f,i}} \tag{B5a}$$

Turbine Characteristics

Blade-jet speed ratio. - The definition of blade-jet speed ratio is

$$\nu = \frac{U}{\sqrt{2gJ\Delta h_{id}}} \tag{B6}$$

Substitution of equation (B2c) into (B6) yields

$$\nu = \frac{U}{\sqrt{5090gJ/SPC_{id}}} \tag{B6a}$$

Nozzle throat area. - The continuity equation is

$$w = \frac{3600}{144} \rho AV = 3600 \frac{pAaM}{RT} \tag{B7}$$

At the throat, $M = 1$, and the throat area is

$$A_{th} = \frac{wRT}{3600(a^*)p^*} \tag{B7a}$$

Since $a^* = \sqrt{\gamma g R T^*}$ and $T^* = T' \left[2/(\gamma + 1)\right]$ and $p^* = p' \left[2/(\gamma + 1)\right]^{\gamma/(\gamma-1)}$ for isentropic flow,

$$A_{th} = \frac{w\sqrt{RT' \frac{2}{\gamma + 1}}}{3600\left(\sqrt{\gamma g}\right)p'\left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma-1)}} \tag{B7b}$$

Substituting equation (B2a), weight flow, into equation (B7b) yields

$$A_{th} = \frac{0.707\sqrt{RT'\left(\frac{2}{\gamma + 1}\right)}SPC}{\left(\frac{p'}{P}\right)\left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma-1)}\sqrt{\gamma g}} \tag{B7c}$$

Nozzle arc of admission. - The nozzle-throat area can be expressed as

$$A_{th} = Ld \tag{B8a}$$

where d is the blade height and L is the nozzle width. The blade height can be expressed as

$$d = r_t\left(1 - \frac{r_h}{r_t}\right) \tag{B8b}$$

The nozzle width is

$$L = \frac{A_{th}}{r_t\left(1 - \frac{r_h}{r_t}\right)} \tag{B8c}$$

Therefore, the arc of admission at the exit is

$$\beta_e = \frac{L(360^o)}{\pi D_m \sin \alpha} \tag{B8d}$$

where α is the nozzle angle.

Allowable blade-tip speed. - The allowable blade-tip speed can be related to the ratio of turbine disk stress to density s_d/ρ_d, hub-to-tip radius ratio r_h/r_t, and disk taper e by the equations of reference 7.

$$U_t^2 = \frac{s_b}{\rho_b} \frac{2g}{1 - \left(\frac{r_h}{r_t}\right)^2} \tag{B9a}$$

and

$$\ln e = \frac{\frac{\rho_d}{\rho_b} \frac{s_b}{s_d} \left(\frac{1 + \frac{r_h}{r_t}}{1 - \frac{r_h}{r_t}} - 1 - \frac{2}{\mathcal{A}} \right)^2}{4 \left(\frac{1 + \frac{r_h}{r_t}}{1 - \frac{r_h}{r_t}} \right)} \tag{B9b}$$

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