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On the Application of Extreme-Value Statistics to Command Oriented Problems

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ABSTRACT

In commanding planetary spacecraft, system constraints allow data rates of only a few bits per second. Also, the accuracy of received information must be high since execution of an improperly received command could disrupt the mission. This report considers the problem of experimentally estimating or verifying error probabilities when the classical error-counting approach is too time consuming to use. The rudiments of extreme-value theory are introduced for the univariate case where the bit-error probability of interest depends on a single variable, and for the bivariate case where the bit-error probability is a function of two dependent variables. Many examples are given, and numerical results are presented. Considerable attention is given to techniques of implementing the theory.

I. INTRODUCTION

The purposes of this report are to discuss the history leading to use of extreme-value theory (EVT) in estimation of statistical parameters of communication systems, detail the basic concepts of EVT and give examples of EVT application in this area, the primary application being error rate estimation. The tone of the report is that of the engineer, as opposed to the mathematician. No attempt has been made to make it mathematically rigorous, and only sufficient mathematics are included to enhance the credibility of the general approach.

II. SOME BASIC QUESTIONS

In nearly all binary communication systems, information is ultimately conveyed by the use of some form of a decision or threshold device. In this type of system the question of accuracy of received information eventually can be, and frequently is, reduced to the concept of a bit error, i.e., the probability of incorrect reception on a particular bit. Thus, given a binary one (zero) and noise as the incoming signal of a threshold type receiver, one basic question becomes: What is the probability of failing to receive a binary one (zero) at the output?

In a coherent system with a transmitted reference, another item of interest is the quality of the received reference. Generally, there is some type of "coherence-loss of coherence" indicator which is used for this purpose. One typical mechanization (*Mariners R* and *C* command systems) of coherent systems employs the loss of coherence indicator to inhibit data reception when the indicator shows the reference to be faulty according to some predetermined criterion. Thus, another question to be answered is: What is the probability that the loss of coherence indicator will inhibit reception? In actuality the reference and data information are usually transmitted through the same medium at the same time and are simultaneously processed by the receiver in somewhat different manners. Often the statistics of the two channels are dependent. (Note that if the statistics are independent, it is a simplified special case of the preceding.) Thus, we can ask: What is the probability of a bit error, given an indication of coherence? Or similarly, given an indication of loss of coherence, what is the probability of a bit error?

In asynchronous systems which depend on the received signal to initiate a processing sequence, the time delay in the processing channel used to derive the initiation signal becomes of interest: e.g., if the delay is too great (due to noise, for example) the system may inherit an unknown time skew between its reference and that of the transmitter. If sufficient, this skew could completely disrupt the decoding scheme. Such an asynchronous system was used on *Rangers VI–IX* command systems and will be described in greater detail in the next section.

The classical, experimental approach to problems of this general type has been that of repeated trials of comparing transmitted and received digital data. For example, using this approach in bit-error testing, the receiver under test is supplied with a prescribed signal-to-noise ratio (SNR), a known bit is transmitted to it, and the receiver output is examined and compared with the value of the bit transmitted. The error rate is defined simply as the ratio of bits in error to total bits transmitted during the test. If either error rates or bit rates are high so that errors accumulate at the rate of 10/ to 20/hr of test time, this approach can give accurate results with high confidence levels in a "reasonable" length of time. However, if error rates are low (say, 10^{-5}) and bit rates are also relatively low (say, 1 bit/sec), then the test time required to experimentally determine such an error rate, with an 80% confidence level less than $\pm 20\%$ wide, is about 1000 hr. Simply to establish if the error rate is less than 10^{-5} at an 80% confidence level requires 45 hr, if no errors are recorded. As bit rates decrease, and/or error rates being measured decrease, the required test time increases even more.

In present day space probes bit rates used in communication with the spacecraft are normally low–1 bit/sec, for example, on both *Ranger* and *Mariner* command systems. Also, reliability of transmitted commands must be high. The maximum error probability acceptable on these two systems is a bit-error rate of 10^{-5}. Several hours of test time are required to establish whether or not the required error rates are obtained at the specified SNR. If it is further desired not only to obtain this one point of data, but also to establish an actual experimental curve of bit-error rates as a function of SNR (perhaps at several combinations of temperatures, power supply voltages, etc.) test time becomes prohibitive. Longer bit times, such as 0.05 bits/sec now being considered, only aggravate the problem. Furthermore, long periods of testing allow variables, some known and some unknown, to influence the system under test. This phenomenon, in turn, leads to highly instrumented test complexes involving large amounts of equipment, manpower, and operating time. A less costly and time consuming approach would obviously be welcome.

III. AMPLITUDE-DISTRIBUTION ANALYSIS

Consider the receiver in Fig. 1. It is not uncommon for the information to be presented to the threshold device in analog fashion. Information is available in the analog signal that is not used in bit-error testing as described previously; for example, one cannot only determine whether or not an error occurred, but also how close it came to occurring. This implies that knowledge of the amplitude distribution of the signal presented to the threshold device at the time at which the threshold detector's output is examined will allow prediction of the probability that any single bit will be in error.

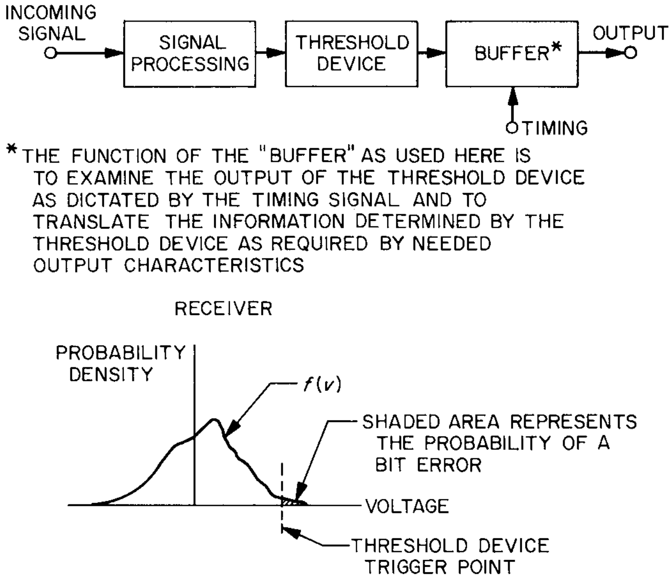


Fig. 1. Typical threshold receiver

To have something more concrete to discuss, consider the system of Fig. 2. This is the bit-detection channel used in *Rangers III–IX* and is, historically, the first system on which the efforts delineated in this report were expended. Basically, the FSK input signal is processed through an analog processing network which results, at point A of Fig. 2, in one dc output for an input of the frequency of the narrow bandpass filter and a second dc voltage for the other FSK input frequency. Of course, both of these dc levels will be perturbed by noise and will shift as a function of input signal-to-noise ratio (SNR) due to signal suppression in the limiter. The Schmitt trigger quantizes the envelope detector output and in this sense serves as the threshold device.

The internal programming of the detector is such that the Schmitt trigger output is sampled and stored at the estimated midpoint of the received bit; no integrating is done other than that accomplished by the filter of the envelope detector. Thus, the data actually used are the behavior of the envelope detector at times other than transitions between bits. The detector relies on the leading edge of the first bit of the incoming command to establish synchronization for the rest of the command. This is an example of the initiation signal mentioned in the preceding section. One point to note is that the sampling of the Schmitt trigger output occurs at a point in time that leaves it essentially uninfluenced by the effects of the normal transitions in the frequency of the FSK input signal. Thus, we can make the statement that the voltage distribution of the steady-state, envelope-detector output controls the error rate. So far as noise is concerned, the statistical properties of the command subsystem are essentially determined by the analog circuitry and command word Schmitt trigger in the detector; thus, the statistical properties of output-signal voltage of the envelope detector, coupled with knowledge of the Schmitt trigger firing voltage, contain sufficient information to indicate the caliber of performance of which the detector is capable–including bit-error rates. Figure 3a is an example of the shape and position of these distributions and how they change as a function of input SNR. Figure 3b details one of the curves of Fig. 3.

An additional example of how amplitude-distribution analyses (ADA) are developed in practice is presented in Fig. 4. The configuration presented is that of a coherent PSK-detection channel; this is basically the scheme used on the *Mariner* 64 command system. In the absence of noise, the matched filter has as its input a signal of $\pm A\,|\cos \omega t|$ which it integrates for one-bit time. At the end of that time the dump and decision circuit dumps the integrator (shorts the capacitor) in preparation for the next bit and examines the direction of the resulting

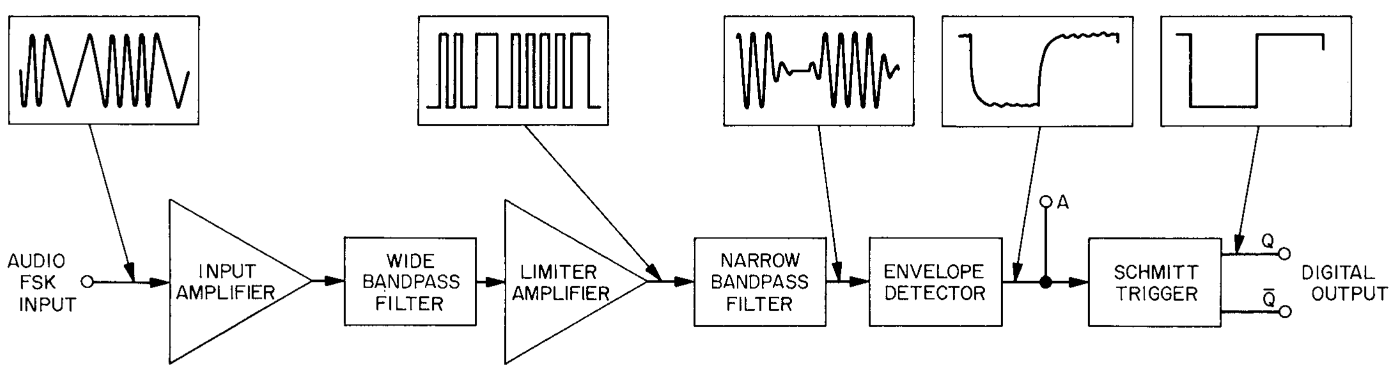


Fig. 2. *Ranger III-IX* command–detection channel

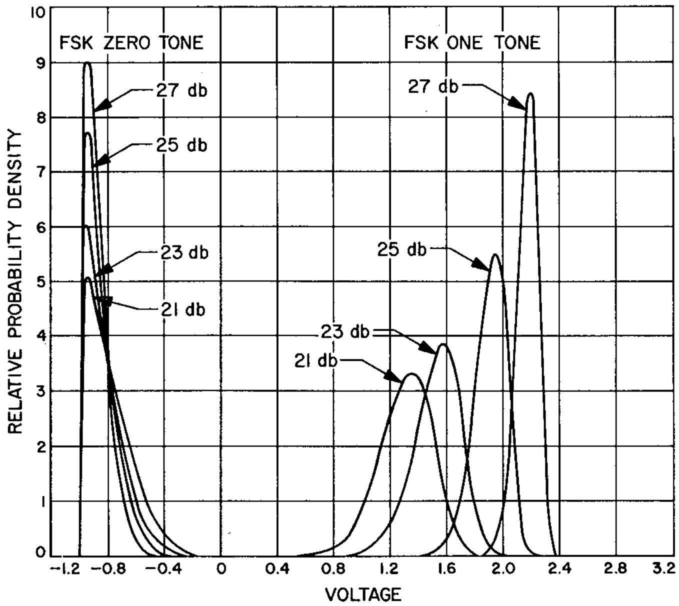


Fig. 3a. Probability density of envelope detector, *Ranger* command–detection channel

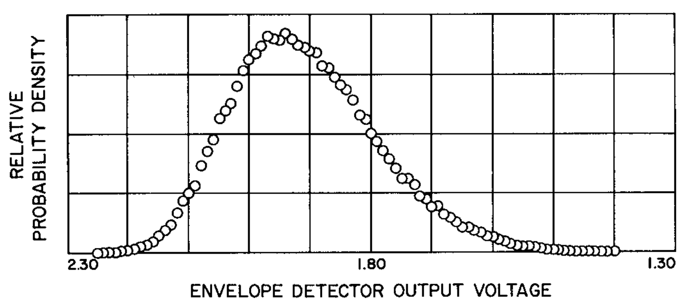


Fig. 3b. Probability density of envelope detector, *Ranger* command–detection channel, FSK one tone—25 db SNR

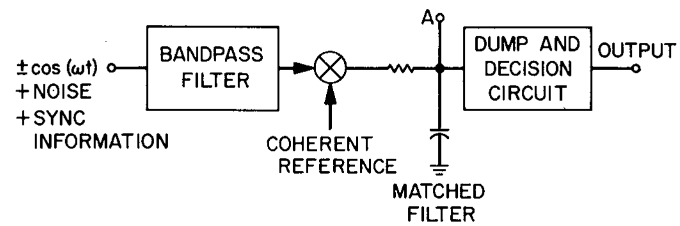


Fig. 4. Coherent PSK–detection channel

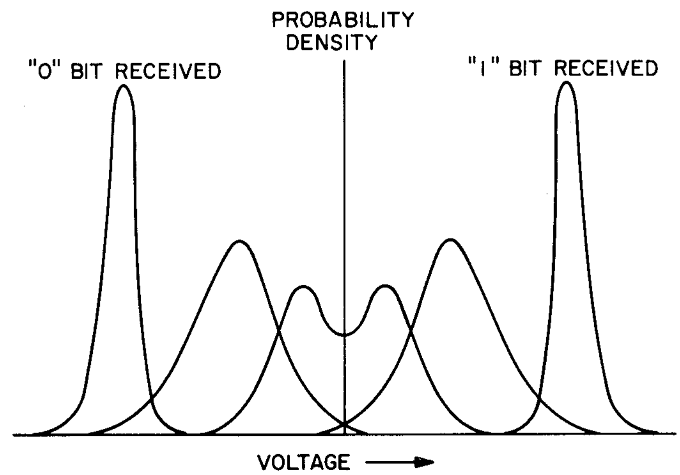


Fig. 5. Probability density of PSK–detection channel and matched-filter output at dump time

transient to determine the type of bit it assumes was transmitted. Thus, the type of bit chosen by the decision circuit is determined by the polarity of the integrator output at dump time.

In this example, the amplitude distribution of the integrator output at dump time becomes of interest in determining statistical behavior. Note that, in contrast to the FSK system of Fig. 2, the voltages of interest occur only at discrete times, i.e., dump times. Figure 5 is a sketch of the shape and position of the distribution of the integrator output at dump time, and how it varies as a function of input SNR.

IV. INTERPRETATION OF EMPIRICAL AMPLITUDE-DISTRIBUTION ANALYSES

These amplitude-distribution analyses can conceivably be used in numerous ways. One of the more obvious ways is a visual examination of a family of ADA curves with the object of comparing these curves with data anticipated and with data obtained from similar tests on prototype equipment–equipment known to perform satisfactorily. Conclusions reached by this approach are arrived at strictly on the basis of engineering judgement and experience.

Another method in which ADA information can be used is to estimate bit-error rates. Conceptually, this method is based on the fact that amplitude-distribution analyses are estimates of probability-density distributions. Consequently, the probability of the variable exceeding the threshold of the decision circuit at any particular time of interest is simply the percent area under the ADA curve where the abscissa has a value greater than threshold. Or in a more concise statement

$$P = \int_{V_T}^{\infty} p_{\mathrm{ADA}}(v)\, dv$$

where

P = probability of error at any one instant,

V_T = value of the variable at the threshold of the decision circuit,

v = variable,

p_{ADA} = probability density function obtained by normalizing the ADA curve so that the total area under the curve is unity.

In the data analyses performed, error rates of interest are on the order of 10^{-3} to 10^{-6}. Thus, the percent area that is of interest is 0.1 to 0.0001%. Since the total area under experimental ADA curves is generally on the order of 10 in.^{2}, direct physical measurement of the area of interest becomes impractical. In fact, the numerical value of the ADA probability density distribution is so small near V_T that data are often not even taken in that area. Practically then, the problem in applying amplitude-distribution analyses to estimating bit-error rates reduces to the following. Given a set of data points in a restricted range, predict with some known accuracy the behavior of the corresponding data outside the range measured.

Considering the command detector of Fig. 2 from a statistical communication point of view, one expects an amplitude-distribution analysis performed on the envelope detector signal to exhibit a near-Gaussian behavior when a tone of the narrow bandpass filter frequency is present at the detector input and near "half-Gaussian" behavior when a tone other than the narrow bandpass filter frequency is present. Indeed, one's expectations are not greatly dampened by a cursory examination of the ADA data plots (Fig. 3). Thus, in an effort to determine the behavior of the data in ranges of voltage where mechanical integration is impractical, attempts have been made to fit the known data by some Gaussian function.

The essence of this approach now becomes: fit the data as best possible with a Gaussian curve and assume the fit behaves properly at all points of interest. The manner of fitting the data and determining precisely what is the "best possible" fit now becomes the problem.

For the record, the following five methods of curve fitting were investigated:

1. Graphical determination of variance and mean by mechanical integration.
2. Mathematical fit of two points with an assumed mean.
3. Mathematical fit of three points.
4. Linearizing of data.
5. Least-square error fit.

The data required to obtain the amplitude-distribution plot in Fig. 3b was recorded in 5 min. Highly controlled, stable conditions can be maintained for such a period with a reasonable degree of effort. The effort involved in maintaining similar conditions for many days or weeks, as mentioned in conjunction with classical error testing, becomes very demanding. This short time required to record the necessary data is one of the most significant factors of the entire ADA approach.

In the cases of primary interest–i.e., error rates on the order of 10^{-5}–the shape of the probability-density curve and the ability to extrapolate data become of great importance if actual bit-error rates are to be estimated because of the small probability density around V_T. However in all the above curve-fitting techniques, an indication was found that the curves had variations from true Gaussian behavior. When Gaussian behavior was assumed, the answers obtained were wrong by several (2 or 3) orders of magnitude if true error rates were near 10^{-5}.

This observation leads naturally to the requirement for more accurate information concerning the amplitude-distribution density, particularly the "tails" of the curves. This information cannot be obtained by x–y plotting of the data as previously indicated, or even by printing it in digitized form unless, of course, the amount of data taken is increased. To well define the tail of the ADA curve requires an amount of data approaching that required for classical bit-error testing. Thus, to truly save test time it is necessary to use some method which allows (1) application of technique to a non-Gaussian (and preferably even undefined) amplitude distribution, (2) extrapolation of observed data beyond the range of data taken. It is in satisfying these two requirements that the branch of mathematics dealing with extreme value statistics becomes important.

V. INTRODUCTION TO EXTREME-VALUE STATISTICS

There have been many articles written about the theory of extreme values. These are scattered throughout scientific literature, have different nomenclature, are somewhat concentrated mathematically and are largely –what is often a handicap from an engineer's point of view–written by mathematicians for other mathematicians.

In addition, with the exception of an application to capacitor failures as a function of voltage and age (Ref. 5) most of the applications of extreme-value theory have been in the fields of actuarial science, climatology and aerodynamics. However it now appears that this theory, which by its very nature is concerned with the uncommon, the extreme, may well have a valuable contribution to make to statistical communications in areas where the uncommon is *precisely* what is of interest.

Grossly, this body of theory is concerned with developing mathematical descriptions of the behavior of the "tails," i.e., extremes, of the ADA's of the previous section, but different techniques and a slightly different approach are used. Fundamentally, this theory defines and allows extrapolation of a processed form of an ADA without detailed knowledge of its shape (univariate extreme-value theory). A second branch of this theory is concerned with the situation where two interdependent data streams are being processed simultaneously and the statistics of one stream affect the processing of the other (bivariate extreme-value theory).

VI. UNIVARIATE EXTREME-VALUE STATISTICS

The basic statement of univariate extreme-value statistics in which we are interested, can be arrived at as follows. Given a set of n independent samples from a data source that forms some cumulative probability function, $F(x)$, we examine the probability $\Phi_n(x)$ that the largest of these samples is less than x. Since the samples are independent, this is simply

$$\Phi_n(x) = F^n(x) \tag{1}$$

Subject to certain restraints on $F(x)$ that are not very limiting in practice (to be detailed later), univariate EVT states that as $n \rightarrow \infty$, $\Phi_n(x)$ *asymptotically approaches* $\exp[-\exp(-\Lambda)]$, where Λ is a linear function of x, i.e.

$$\lim_{n \rightarrow \infty} \Phi_n(x) = \Phi(x) = \exp[-\exp(-\Lambda)] \tag{2}$$

with

$$\Lambda = \alpha(x - u) \tag{3}$$

Here, α and u are constants, and Λ is called the reduced variate. Equation (2) is in fact an equality (Ref. 2) but the asymptotic behavior indicated above [Eq. (2)] is a sufficiently strong statement for our purpose.

In practice what one does is to take a "large" group of data (typically $n = 100$) and find the largest data point, X, within that group. According to Eq. (2) this largest data point will approximately have a double-exponential distribution. To experimentally find this distribution, i.e., the unknown constants of Eq. (3), we proceed as we would with the experimental determination of any distribution; we obtain several, N, groups of data and find the largest data point within each group. These X_i's are then ordered and plotted with some standard technique. This plotting allows estimation of $\Phi(x_0)$ where x_0 is the threshold value of x; thus, $F^n(x_0)$ is known, and $F(x_0)$ is calculable from this.

The above two paragraphs can be restated as follows: In a sample of n independent observations, one of them (or perhaps several identical ones) is the largest. If N such samples are drawn, a distribution of extreme values is obtained, and we are interested in its nature under the condition that n is large. Videlicet, we claim that this distribution of extreme values asymptotically approaches Eq. (2) as n increases without bound.

Introduction of an example may well be appropriate at this point. It will be worked in segments throughout the report as it appears that each segment will be of aid in understanding the subject.

Consider again the system of Fig. 4 introduced in the section, Amplitude-Distribution Analysis. Table 1 lists successive samples taken from the integrator output at dump time with a constant bit type and noise into the detector. If the data of Table 1 are broken into groups of 100 successive data points ($n = 100$), then we have 30 groups ($N = 30$) of 100 data points each. We now search each group for that data point which has the greatest value (indicated by the boxed entries in Table 1). These extremes (one for each group of 100 samples) are tabulated in Table 2.

The basic assertion has been that the data of Table 2 will have a distribution of the form described by Eqs. (2)–(3) for some choice of α and u. Figure 6 plots the data of Table 2 as a cumulative distribution and superimposes on the data points a curve of $\exp[-\exp(-\Lambda)]$ for $\alpha = 0.033363$ and $u = -171.632$ which were chosen by a maximum likelihood technique to be considered in some detail later. The point to notice in Fig. 6 is that there is reasonably good agreement between the curve of Eq. (2) and the data obtained in Table 2.

As an aid to better visualizing the fit, (and indeed fitting by eye if desired) Eq. (2) can be linearized; i.e., if we plot Λ vs $-\ln(-\ln \Phi)$, the data will be a straight line. In fact, we can plot X vs $-\ln(-\ln \Phi)$ and the values of α and u can be estimated from the slope and intercept, respectively, of the straight line. For convenience, extreme-value probability paper is available which uses as axes X in arbitrary units and $-\ln(-\ln \Phi)$ in units of Φ. A sample of the form is given as Fig. 7. Figure 6 is redrawn on extreme-value probability paper in Fig. 8. Note that the data appear to be scattered about the straight line. As a matter of interest, experience has shown that visual fits of a straight line to typical data give surprisingly good results.

Due to the fact that values of $\Phi = 0$ or $\Phi = 1$ cannot be plotted in Fig. 7, the plotting positions tabulated in Table 2 and used in Figs. 6 and 8 were chosen as $i/(N + 1)$ where i is the rank of the data point being plotted, the data having been ordered in increasing value. This particular choice of plotting position has a number of pleasing features. However, this point will not be pursued further in this report since plotting positions are not used in computer processing of data (mathematical fit).

Table 1. List of successive samples taken from the integrator output with a constant bit type and noise into the detector

SAMPLE NUMBER	DATA CHANNEL VALUE	SAMPLE NUMBER	DATA CHANNEL VALUE	SAMPLE NUMBER	DATA CHANNEL VALUE
1	-337.0	75	-236.0	149	-368.0
2	-348.0	76	-325.0	150	-287.0
3	-377.0	77	-401.0	151	-327.0
4	-386.0	78	-319.0	152	-374.0
5	-415.0	79	-399.0	153	-257.0
6	-338.0	80	-258.0	154	-280.0
7	-169.0	81	-237.0	155	-386.0
8	-313.0	82	-298.0	156	-265.0
9	-358.0	83	-284.0	157	-314.0
10	-246.0	84	-360.0	158	-333.0
11	-131.0	85	-369.0	159	-300.0
12	-283.0	86	-324.0	160	-354.0
13	-257.0	87	-396.0	161	-342.0
14	-334.0	88	-231.0	162	-414.0
15	-368.0	89	-297.0	163	-359.0
16	-383.0	90	-330.0	164	-379.0
17	-329.0	91	-366.0	165	-405.0
18	-414.0	92	-305.0	166	-369.0
19	-376.0	93	-287.0	167	-305.0
20	-339.0	94	-371.0	168	-361.0
21	-224.0	95	$\boxed{-95.0}$	169	-268.0
22	-254.0	96	-293.0	170	-308.0
23	-373.0	97	-371.0	171	-398.0
24	-269.0	98	-309.0	172	-318.0
25	-334.0	99	-335.0	173	-360.0
26	-393.0	100	-348.0	174	-422.0
27	-365.0	101	-355.0	175	-230.0
28	-239.0	102	-333.0	176	-309.0
29	-311.0	103	-207.0	177	-244.0
30	-270.0	104	-266.0	178	-222.0
31	-210.0	105	-384.0	179	-334.0
32	-249.0	106	-285.0	180	-352.0
33	-328.0	107	-411.0	181	-351.0
34	-354.0	108	-280.0	182	-262.0
35	-314.0	109	-224.0	183	-342.0
36	-329.0	110	-237.0	184	-248.0
37	-306.0	111	-294.0	185	-324.0
38	-237.0	112	-338.0	186	-309.0
39	-329.0	113	-293.0	187	-320.0
40	-306.0	114	-165.0	188	-312.0
41	-271.0	115	-203.0	189	-307.0
42	-297.0	116	-320.0	190	-337.0
43	-360.0	117	-400.0	191	-145.0
44	-363.0	118	-315.0	192	-333.0
45	-267.0	119	-400.0	193	-265.0
46	-225.0	120	-284.0	194	-353.0
47	-314.0	121	-298.0	195	-374.0
48	-314.0	122	-334.0	196	-266.0
49	-296.0	123	-328.0	197	-364.0
50	-308.0	124	-172.0	198	-253.0
51	-338.0	125	-321.0	199	-341.0
52	-311.0	126	-342.0	200	-334.0
53	-393.0	127	-383.0	201	-315.0
54	-293.0	128	-282.0	202	-317.0
55	-240.0	129	-383.0	203	-318.0
56	-313.0	130	-312.0	204	-336.0
57	-310.0	131	-296.0	205	-337.0
58	-378.0	132	-351.0	206	-302.0
59	-359.0	133	-368.0	207	-331.0
60	-311.0	134	-419.0	208	-298.0
61	-384.0	135	-237.0	209	-354.0
62	-381.0	136	-384.0	210	-369.0
63	-229.0	137	-308.0	211	-381.0
64	-300.0	138	-258.0	212	-298.0
65	-239.0	139	-379.0	213	-299.0
66	-448.0	140	-271.0	214	-269.0
67	-353.0	141	-266.0	215	-308.0
68	-279.0	142	-335.0	216	-315.0
69	-332.0	143	-387.0	217	-319.0
70	-327.0	144	$\boxed{-138.0}$	218	-383.0
71	-281.0	145	-327.0	219	-374.0
72	-342.0	146	-262.0	220	-220.0
73	-399.0	147	-288.0	221	-340.0
74	-277.0	148	-318.0	222	-388.0

Table 1. (Cont'd)

SAMPLE NUMBER	DATA CHANNEL VALUE
223	-360.0
224	-267.0
225	-256.0
226	-431.0
227	-229.0
228	-378.0
229	-350.0
230	-394.0
231	-317.0
232	-348.0
233	-385.0
234	-339.0
235	-343.0
236	-278.0
237	-418.0
238	-295.0
239	-322.0
240	-316.0
241	-336.0
242	-288.0
243	-352.0
244	-384.0
245	-312.0
246	-217.0
247	-379.0
248	-329.0
249	-273.0
250	-373.0
251	-360.0
252	-203.0
253	-253.0
254	-381.0
255	-305.0
256	-356.0
257	-265.0
258	$\boxed{-181.0}$
259	-297.0
260	-343.0
261	-380.0
262	-239.0
263	-365.0
264	-301.0
265	-286.0
266	-282.0
267	-304.0
268	-275.0
269	-376.0
270	-346.0
271	-356.0
272	-297.0
273	-348.0
274	-318.0
275	-339.0
276	-335.0
277	-261.0
278	-334.0
279	-373.0
280	-267.0
281	-274.0
282	-317.0
283	-342.0
284	-297.0
285	-287.0
286	-214.0
287	-304.0
288	-341.0
289	-320.0
290	-285.0
291	-272.0
292	-347.0
293	-254.0
294	-296.0
295	-316.0
296	-217.0
297	-239.0
298	-234.0
299	-313.0
300	-294.0
.	.
.	.
.	.
2961	-313.0
2962	-287.0
2963	-231.0
2964	-314.0
2965	-366.0
2966	-407.0
2967	-358.0
2968	-356.0
2969	-330.0
2970	$\boxed{-170.0}$
2971	-423.0
2972	-319.0
2973	-384.0
2974	-329.0
2975	-302.0
2976	-320.0
2977	-343.0
2978	-304.0
2979	-416.0
2980	-382.0
2981	-356.0
2982	-299.0
2983	-328.0
2984	-286.0
2985	-375.0
2986	-391.0
2987	-325.0
2988	-347.0
2989	-287.0
2990	-338.0
2991	-315.0
2992	-450.0
2993	-394.0
2994	-407.0
2995	-350.0
2996	-423.0
2997	-430.0
2998	-395.0
2999	-453.0
3000	-308.0

Table 2. Extreme data point for each group of 100 samples

Group no.	Chronological extremes	Ordered extremes	Plotting position
1	−95	−211	0.0322581
2	−138	−204	0.0645161
3	−181	−198	0.0967742
4	−158	−197	0.1290323
5	−146	−192	0.1612903
6	−179	−190	0.1935484
6	−192	−185	0.2258065
8	−211	−181	0.2580645
9	−169	−179	0.2903226
10	−198	−174	0.3225806
11	−159	−173	0.3548387
12	−204	−172	0.3870968
13	−197	−171	0.4193548
14	−157	−170	0.4516129
15	−185	−169	0.4838710
16	−173	−159	0.5161290
17	−19	−158	0.5483871
18	−103	−157	0.5806452
19	−108	−153	0.6129032
20	−153	−151	0.6451613
21	−172	−146	0.6774194
22	−112	−138	0.7096774
23	−112	−121	0.7419355
24	−121	−112	0.7741935
25	−171	−112	0.8064516
26	−190	−110	0.8387097
27	−151	−108	0.8709677
28	−110	−103	0.9032258
29	−174	−95	0.9354839
30	−170	−19	0.9677419

From Fig. 8, we see that $\Phi(\text{threshold}) = \Phi(0)$ is 0.99674. But from Eq. (1) and the fact that we had $n = 100$, $\Phi(0) = F^{100}(0) = 0.99674$ so that

$$F(0) = [1 - (1 - 0.99674)]^{1/100} = (1 - 0.00326)^{1/100}$$
$$= 1 - \frac{0.00326}{100} + \cdots \approx 0.9999674.$$

We conclude that for the raw data, the probability that the data will be less than 0, i.e., the probability of a correct bit, is 0.9999674. This means the probability of an error on a single bit is 3.26×10^{-5}. Note that only 3000 data samples were used to make this estimate and that none of them were greater than threshold. Hence, using classical error-counting techniques, no errors would have been observed and the nominal, observed error rate would have been zero. Of course, one would hesitate to say the error rate is zero on the basis of only 3000 data points, so more likely one would make a statement to the effect that the error rate is less than 7.64×10^{-4} with 90% confidence.

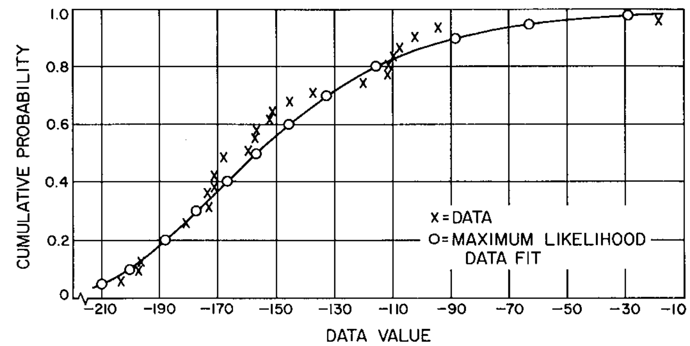


Fig. 6. Cumulative probability of data extremes taken from data of Table 2: plot A

This observation brings up the question of confidence intervals for the EVT estimate of error rate. If we define Λ_0 as the value of Λ at threshold,

$$\Lambda_0 = \alpha(x_0 - u)$$

where x_0 is the threshold in terms of the data, then it can be shown (Ref. 1) that

$$\text{Var } \hat{\Lambda}_0 \approx \frac{6}{N\pi^2}\left[(1 - \gamma + \Lambda_0)^2 + \frac{\pi^2}{6}\right] \tag{4}$$

where γ is Euler's constant, $0.5772 \cdots$. Furthermore, for large N, the maximum likelihood estimators of α and Λ_0 are approximately bivariately normally distributed.

Using Eq. (4) in the example under discussion, we find we can make the statement that the error rate is less than 1.88×10^{-4} with 90% confidence. In terms of a two-sided confidence interval, with 90% confidence, the error rate is between 7.49×10^{-6} and 1.42×10^{-4}. The comparison of upper 90% confidence intervals, i.e., an error rate of less than 7.64×10^{-4} by error counting and 1.88×10^{-4} by EVT methods, gives an indication of one of the prime advantages of the EVT approach to estimation of error rates. Using EVT, we had a meaningful estimate of the error rate, per se, which was totally absent in the error-counting approach and in addition a tighter

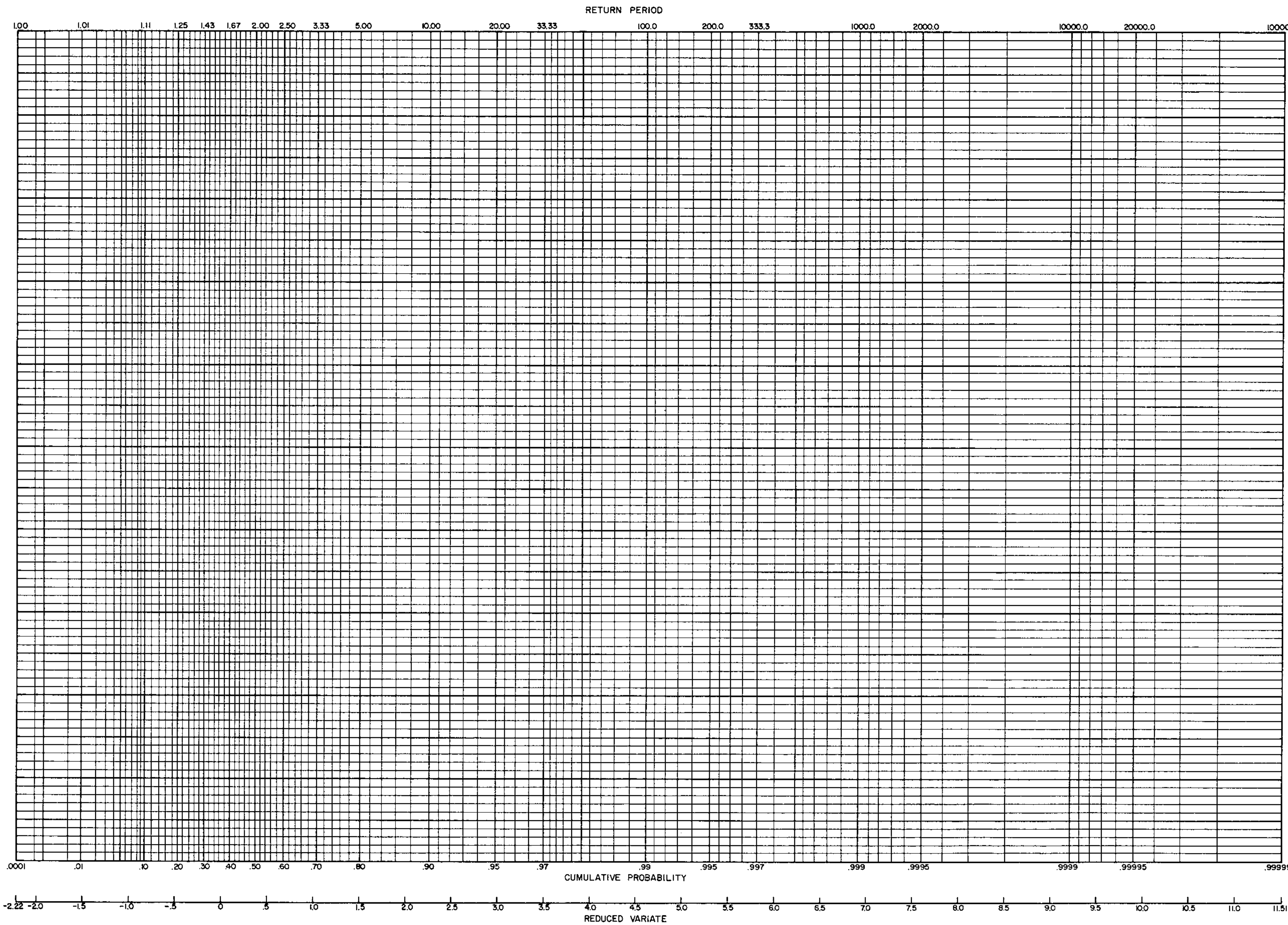


Fig. 7. Extreme-value probability × 100 divisions graph paper

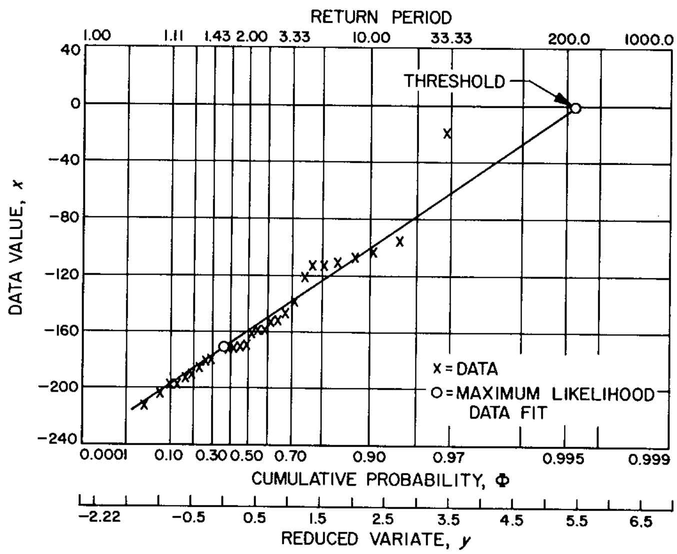


Fig. 8. Cumulative probability of data extremes taken from data of Table 2: plot B

confidence interval on that error rate. Note that if the data were such that the error rate had been lower, the EVT estimate would have been lower and the EVT confidence interval would have followed suit. However, in all likelihood the error-counting method would have counted no errors so that the statements as to error rate and confidence interval would have stayed the same—regardless of how much the error rate decreased!

The source of this improvement comes, of course, from knowledge of not only whether or not an error occurs, but how close it comes to occurring on each bit, i.e., knowledge of the amplitude distribution of the signal behavior just prior to the point at which it is quantized. However, EVT does not use knowledge of the entire distribution but only parts of it. Specifically, we chose $n = 100$ and selected the largest value out of that 100; the other 99% of the data was discarded. This is the price of being able to apply EVT techniques without detailed knowledge of the amplitude distribution of the data being processed.

VII. RESTRICTIONS AND LIMITATIONS ON USE OF UNIVARIATE EXTREME-VALUE THEORY

An implicit restriction used throughout the report is that the mechanism by which a system makes an error is known and can be modeled accurately. In the example above, the system was modeled by noting that the decision circuitry essentially looks at the polarity of the integrator output at dump time. The application of EVT to this system is then predicated on the assumption that this is exactly what happens and the decision circuitry has no biases and makes no errors. It might be pointed out that accurate modeling of the decision making process is not always as straightforward as the samples in Figs. 2 and 4 might lead one to believe. For example, the command receiver in the *Surveyor Block I* spacecraft is a system in which the decision circuitry does not lend itself to being modeled easily. There appears to be a number of interrelated influences involving voltage and time behavior on a bit-by-bit basis as well as a currently not-too-well understood historical influence that some (but not all) bits exert on others. In general, attempts to model this system have led to results that are not accurate to more than a factor of 5 so far as error-rate prediction is concerned.

In the comments leading to Eq. (2), it was pointed out that subject to certain restrictions on $F(x)$,

$$\lim_{n\to\infty} F^n(x) = \Phi(x) = \exp[-\exp(-\Lambda)]$$

The basic restriction on $F(x)$ can be stated in either of two ways: (1)

$$\lim_{x\to\infty} \frac{f(x)}{1 - F(x)} = -\lim_{x\to\infty} \frac{f'(x)}{f(x)}$$

or (2)

$$\lim_{x\to\infty} \left\{ \frac{d}{dx}\left[\frac{1 - F(x)}{f(x)} \right] \right\} = 0$$

where $f(x) = F'(x)$; it can be shown that these two conditions are equivalent. Implicit in this requirement is the need for x to be unlimited in the direction of interest. Grossly, this requires that $F(x)$ have a right-hand tail that is qualitatively like the exponential distribution, $(1 - e^{-x})$. Most of the classical distributions–Gaussian, Rayleigh, etc.,–fall into this category as well as many forms of data encountered in practice. The requirement for x to be unlimited in the direction of interest is frequently ignored by arguing that x can range far beyond values it normally assumes or values near threshold, and that for all practical purposes it can be considered as having unlimited range. If this is not true, EVT techniques can still be used by making the appropriate transformations (Ref. 2).

Again, in the argument leading to Eq. (2) one basic statement used the limit of x as $n \rightarrow \infty$. Obviously, in practice, n is finite, so the question arises as to how large n must be. One would like to keep n as small as possible so that no more data than necessary are used to get the accuracy and confidence intervals desired. The problem can be stated as: Given Nn data points, what is the optimum manner of splitting the data points to get N as large as possible (minimum confidence intervals), thus making n small, but still keeping n large enough so that Eq. (2) is a reasonable approximation?

There seems to be no clear-cut solution to this problem. By experience we have found that it is difficult to construct a reasonable curve of $\Phi = \exp[-\exp(-\Lambda)]$ unless N is at least 20; this is to say nothing of the ballooning confidence intervals for small N's. But minimum sizes for n appear much more elusive, partly, perhaps, because it depends on how "nice" the behavior of $F(x)$ is. In general, especially in cases where little or nothing is known about that behavior of $F(x)$, we have found that $n < 100$ is asking for trouble; however, we have never found $n = 100$ to be insufficient.

If a data source is sampled periodically, the question of how fast to sample becomes a real concern. If the data are sampled too fast, then successive samples are not independent as required for Eq. (1) while if they are sampled much slower than truly necessary, some usable data are lost and required test time is extended. Thus, the question arises: What is the required degree of independence, and how is this to be measured? Consider again the data of Table 1 listing successive samples from the integrator output with a constant bit type and noise into the detector. The degree of independence of successive samples can be indicated as in Fig. 9 which is the normalized autocovariance of 400 samples. Successive samples,

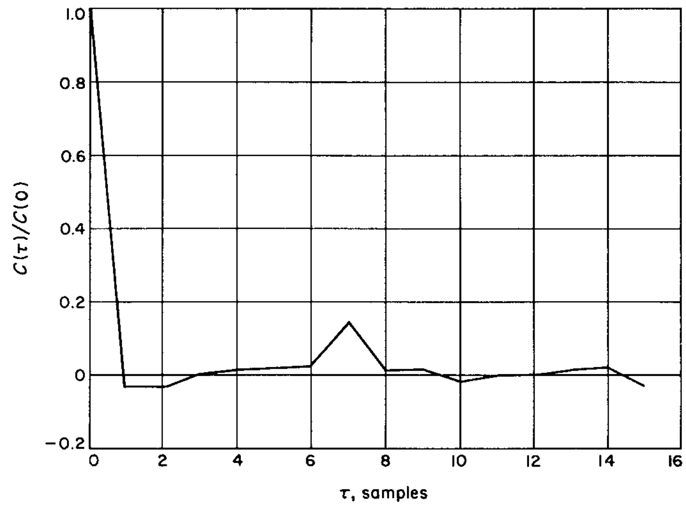


Fig. 9. Normalized autocovariance of independent samples

indicated by $\tau = 1$, have a value of -0.029. In the last analysis, the degree of dependence or independence between successive samples reduces to a subjective judgement, but this approach does serve as a reasonable guide. (For example, Fig. 10 uses data from a different source taken at a high rate so that the samples are "somewhat" dependent.) Data with autocovariances of successive samples as high as 0.6 have been used successfully (but not reliably); however, an upper limit of 0.3 is recommended.

One of the advantages of EVT is that the processed data exhibit some predictable behavior of which we can take advantage. For example, the data of Table 2, processed and plotted in Fig. 8, follow a straight line with

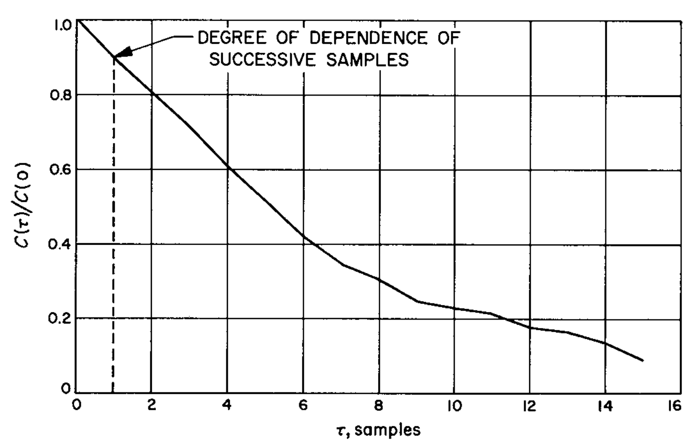


Fig. 10. Normalized autocovariance of dependent samples

some reasonable degree of assurance. Based on this behavior, we extrapolate this line beyond the observed data to make estimates of behavior at threshold (a data value of zero in Fig. 8). This assumes that the data follow the same pattern in regions where they were not measured as in regions where they were. A certain hesitation may be experienced by many people when extrapolation over large values of the variate is required to obtain the desired goal; however, we have never encountered any problems traceable solely to this extrapolation. Perhaps this hesitation can be lessened by noting that extrapolation over large values of the variate required in Eq. (4) increases Λ_0. This results in widening of the confidence intervals pretty much as one would intuitively expect.

It was pointed out earlier that for a given sample size there is some lower limit on error rate beyond which error-counting techniques continue to give the same result. In our specific example, the counted error rate gave a 90% upper confidence level of 7.64×10^{-4}. As long as no errors are counted, it does not matter what the true error rate is–this same result will be obtained. The EVT approach will, however, continue to make estimates of the actual error rate as the error rates decrease, but the confidence intervals will widen.

However, it will be well to consider for a moment the converse problem, i.e., where the error rate increases. Since there seems to be some lower limit on the amount of data that is required in order to apply EVT (2000 to 3000 data points) there will be an error rate at which the confidence intervals for EVT and for error-counting techniques are the same. At greater error rates, the situation will be reversed; i.e., at greater error rates, EVT will require the same amount of data just to be applicable, but error-counting techniques will be able to obtain the same confidence intervals with less data or narrower confidence intervals with the same data. At an error rate of approximately 5×10^{-3}, the confidence intervals arrived at by EVT and classical techniques are the same. Thus, with its more complex instrumentation, application of EVT to estimation of error rates greater than 5×10^{-3} does not appear practical, while at error rates less than 5×10^{-3}, EVT saves test time. Furthermore, the smaller the error rate, the more time these techniques save on a percentage basis.

The preceding sections have dealt with making predictions of maxima from a set of data. Frequently the object of concern is behavior of minima. There is a similar theory of EVT based on minima of extremes (Ref. 2). Rather than introduce unnecessary complexity, minima problems can be treated as maxima problems if all the data (including threshold values, etc.) are multiplied by -1. In fact, this is what was done with the data of Table 1, which lists the mirror images of the raw data. The problem in that instance was to find behavior of minima of data. The data were multiplied by -1, and the maxima within each group were found and processed.

VIII. BIVARIATE EXTREME-VALUE STATISTICS

It was noted in Section II that in a coherent communication system (Fig. 4) the quality, or at least presence, of the received reference or synchronization signal is of interest. The reference and data information are usually transmitted through the same medium at the same time, simultaneously processed by the receiver in somewhat different ways, and one received signal is used in the detection of the second. In view of this, it is not surprising when the statistics of the two channels are dependent. In such cases, error probability estimation is stated in terms of conditional probabilities, such as the probability of a bit error given an indication of coherence. The probability of a bit error is known from univariate EVT, and the probability of an indication of coherence may be obtained similarly by applying univariate EVT techniques to data from the synchronization channel. The problem now is to find the probability of a bit error and an indication of coherence. It is to this case of two dependent channels that we now turn our attention, i.e., bivariate EVT.

Typically, command systems are mechanized to employ an indicator that inhibits data reception when the quality of the received reference degrades below some predetermined criteria. Such a system is presented in Fig. 11.

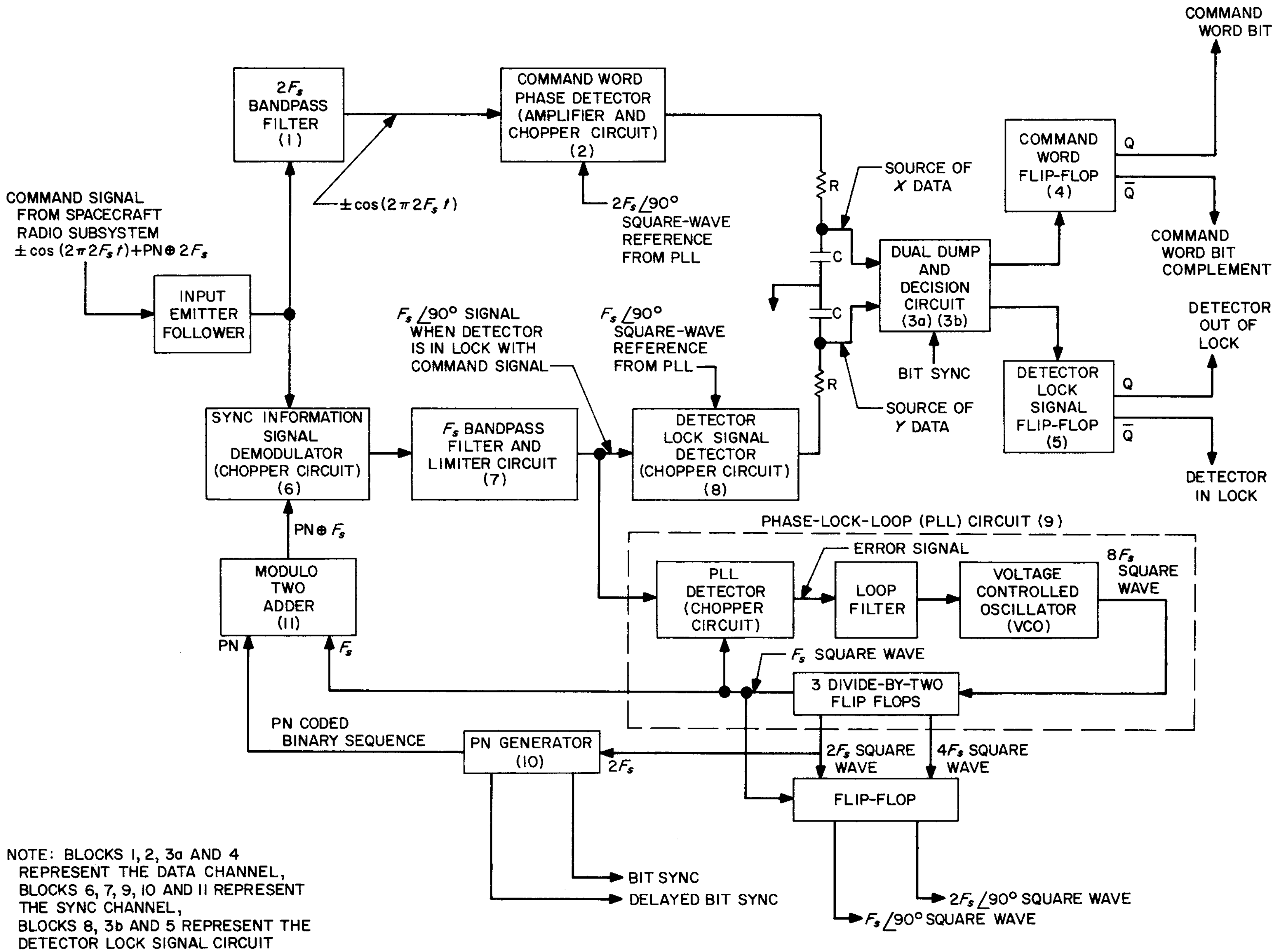


Fig. 11. Two-channel detector functional diagram

As might be suspected from consideration of the univariate case, a sample n-bits long of *pairs* of deviations (x,y) is taken where x is the analog signal in the data channel just prior to quantization, and y is the analog signal of the synchronization channel just prior to quantization. Designating the thresholds of the respective channels as x_o and y_o, we assume the signs so chosen that $x > x_o$ indicates a bit error and $y > y_o$ indicates loss of lock (synchronization). We then record the largest x and the largest y out of the n samples, regardless of whether or not the largest x occurs on the same bit as the largest y. This process gives rise to a new bivariate distribution of random variables X and Y, corresponding to the extremes of the data and synchronization channels, respectively.

From univariate EVT X and Y separately have approximately extreme-value distributions each with α and u parameters, which are estimated from N extremes of groups each of size n as described in Section VI. A linear transformation, $\Lambda = \alpha_\Lambda(X - u_\Lambda)$, $\Omega = \alpha_\Omega(Y - u_\Omega)$ is performed to obtain a pair of random variables (Λ, Ω) which have as their marginal distributions the standardized extreme-value distributions:

$$\Phi(x) = \exp\left(-e^{-\Lambda}\right)$$

$$\Phi(y) = \exp\left(-e^{-\Omega}\right).$$

Note that we have $\Phi(x_0)$ as the probability that n independent bits are all correct and $\Phi(y_0)$ as the probability that all n independent bits have in lock (coherence maintained) indications. Both of these probabilities are calculable from univariate EVT.

We have N independent samples of (Λ, Ω) which we already have used to estimate the α's and u's and these same N samples will be used to estimate the joint distribution of (Λ, Ω) according to a method given in Ref. 3. This joint distribution of Λ and Ω was shown there to be approximately of the form

$$\Psi(x, y) = \exp\left[-\left(e^{-\Lambda} + e^{-\Omega}\right) w(\Lambda - \Omega)\right] \tag{5}$$

where w is a function satisfying some special conditions. For reasons given in Ref. 3, and which are broadly outlined in the following section, we have taken $w(\Lambda - \Omega)$ to be one of functions $w_c(\Lambda - \Omega)$ given by

$$w_c(\Lambda - \Omega) = 1 - c\,\mathrm{sech}^2\left(\frac{\Lambda - \Omega}{2}\right) \tag{6}$$

where c is a parameter between 0 and 1/4. Thus, the "fifth parameter," c, must be estimated instead of an unknown function $w(\Lambda - \Omega)$.

At this point it might be well to reiterate the four preceding paragraphs. Basically, the approach taken is to record pairs of samples from the matched filter outputs of Fig. 11 just before the filter is dumped. Such a set of data might appear as in Table 3, which is an extension of the example begun in Section VI. As in that section, the proper sign convention is adopted so that maxima, rather than minima, univariate EVT is applicable. The data are then broken into groups of n points, n large (typically $n = 100$) and the maximum value recorded within each group for each channel (indicated by the boxed entries in Table 3) is selected as forming a new pair of random variables, X and Y. For this selection of maxima, the data from each channel are treated as if these were data from a univariate EVT problem independent of the other channel. There is no guarantee that the maxima for the two channels will occur on the same sample. The extremes in Table 3 are chronologically listed in Table 4. A linear transformation is performed on the X's and Y's which is identical with that indicated by Eq. (3) and results in a set of new random variables (Λ,Ω). The data from each channel are treated as an independent univariate EVT problem. This yields $\Phi(x)$ and $\Phi(y)$ as indicated previously. Note that of necessity these yields must be the marginal distributions of the joint distribution, Eq. (5).

The basic assertion of bivariate EVT is that the joint distribution of the linearly transformed data, $\Psi(x,y)$, asymptotically approaches Eq. (5) for large n. It is pleasing to notice that Eq. (5) is of the form of the product of the marginal distributions and some modifying function. In fact, after reflection on the form of the bivariate Gaussian distribution, one might hazard a guess (quite correctly!) that the function $w(\Lambda - \Omega)$ denotes some form of correlation. This is particularly apparent when Eqs. (5)–(6) are combined, yielding

$$\Psi(x, y) = \exp -\left[e^{-\Lambda} - c\left(e^{-\Lambda} + e^{-\Omega}\right)\mathrm{sech}^2\left(\frac{\Lambda - \Omega}{2}\right) + e^{-\Omega}\right] \tag{7}$$

It can be shown and has been substantiated in practice that the constant c in Eq. (7) is a very sensitive indicator of correlation between the data from the two channels. As an elementary example, consider the case $c = 0$; then

$$\Psi(x, y) = \exp\left[-\left(e^{-\Lambda} + e^{-\Omega}\right)\right]$$

$$= \Phi(\Lambda)\,\Phi(\Omega),$$

which is frequently taken as the definition of statistical independence.

Table 3. Sample pairs from matched filter outputs just before the filter is dumped

SAMPLE NUMBER	DATA CHANNEL VALUE	SYNC CHANNEL VALUE
1	-337.0	-575.0
2	-348.0	-585.0
3	-377.0	-536.0
4	-386.0	-497.0
5	-415.0	-555.0
6	-338.0	-399.0
7	-169.0	-332.0
8	-313.0	-523.0
9	-358.0	-622.0
10	-246.0	-481.0
11	-131.0	-450.0
12	-283.0	-597.0
13	-257.0	-702.0
14	-334.0	-566.0
15	-368.0	-521.0
16	-383.0	-536.0
17	-329.0	-609.0
18	-414.0	-556.0
19	-376.0	-570.0
20	-339.0	-646.0
21	-224.0	-511.0
22	-254.0	-340.0
23	-373.0	-596.0
24	-269.0	-536.0
25	-334.0	-646.0
26	-393.0	-566.0
27	-365.0	-583.0
28	-239.0	-601.0
29	-311.0	-492.0
30	-270.0	-533.0
31	-210.0	-611.0
32	-249.0	-661.0
33	-328.0	-451.0
34	-354.0	-533.0
35	-314.0	-544.0
36	-329.0	-456.0
37	-306.0	-560.0
38	-237.0	-423.0
39	-329.0	-478.0
40	-306.0	-417.0
41	-271.0	-501.0
42	-297.0	-698.0
43	-360.0	-548.0
44	-363.0	-544.0
45	-267.0	-592.0
46	-225.0	-471.0
47	-314.0	-449.0
48	-314.0	-591.0
49	-296.0	-666.0
50	-308.0	-555.0
51	-338.0	-479.0
52	-311.0	-452.0
53	-393.0	-584.0
54	-293.0	-746.0
55	-240.0	-345.0
56	-313.0	-511.0
57	-310.0	-653.0
58	-378.0	-638.0
59	-359.0	-426.0
60	-311.0	-540.0
61	-384.0	-641.0
62	-381.0	-649.0
63	-229.0	-566.0
64	-300.0	-473.0
65	-239.0	-434.0
66	-448.0	-564.0
67	-353.0	-602.0
68	-279.0	-431.0
69	-332.0	-561.0
70	-327.0	-561.0
71	-281.0	-397.0
72	-342.0	-535.0
73	-399.0	-557.0
74	-277.0	-543.0

Table 3. (Cont'd)

SAMPLE NUMBER	DATA CHANNEL VALUE	SYNC CHANNEL VALUE
75	-236.0	-513.0
76	-325.0	-641.0
77	-401.0	-452.0
78	-319.0	-576.0
79	-399.0	-541.0
80	-258.0	-622.0
81	-237.0	-583.0
82	-298.0	-441.0
83	-284.0	-554.0
84	-360.0	-569.0
85	-369.0	-618.0
86	-324.0	-544.0
87	-396.0	-548.0
88	-231.0	-396.0
89	-297.0	-483.0
90	-330.0	-320.0
91	-366.0	-519.0
92	-305.0	-601.0
93	-287.0	-400.0
94	-371.0	-421.0
95	$\boxed{-95.0}$	$\boxed{-157.0}$
96	-293.0	-543.0
97	-371.0	-429.0
98	-309.0	-416.0
99	-335.0	-601.0
100	-348.0	-537.0
101	-355.0	-641.0
102	-333.0	-526.0
103	-207.0	-414.0
104	-266.0	-601.0
105	-384.0	-602.0
106	-285.0	-446.0
107	-411.0	-580.0
108	-280.0	-542.0
109	-224.0	-373.0
110	-237.0	-493.0
111	-294.0	-545.0
112	-338.0	-556.0
113	-293.0	-722.0
114	-165.0	-394.0
115	-203.0	-421.0
116	-320.0	-516.0
117	-400.0	-441.0
118	-315.0	-545.0
119	-400.0	-536.0
120	-284.0	-621.0
121	-298.0	-545.0
122	-334.0	-527.0
123	-328.0	-617.0
124	-172.0	-716.0
125	-321.0	-662.0
126	-342.0	-567.0
127	-383.0	-471.0
128	-282.0	-614.0
129	-383.0	-702.0
130	-312.0	-609.0
131	-296.0	-516.0
132	-351.0	-437.0
133	-368.0	-603.0
134	-419.0	-693.0
135	-237.0	-582.0
136	-384.0	-627.0
137	-308.0	-462.0
138	-258.0	-532.0
139	-379.0	-473.0
140	-271.0	-659.0
141	-266.0	-576.0
142	-335.0	-656.0
143	-387.0	-488.0
144	$\boxed{-138.0}$	$\boxed{-199.0}$
145	-327.0	-250.0
146	-262.0	-563.0
147	-288.0	-510.0
148	-318.0	-528.0
149	-368.0	-631.0
150	-287.0	-660.0
151	-327.0	-672.0
152	-374.0	-521.0

Table 3. (Cont'd)

SAMPLE NUMBER	DATA CHANNEL VALUE	SYNC CHANNEL VALUE
153	-257.0	-387.0
154	-280.0	-580.0
155	-386.0	-402.0
156	-265.0	-486.0
157	-314.0	-564.0
158	-333.0	-658.0
159	-300.0	-452.0
160	-354.0	-674.0
161	-342.0	-570.0
162	-414.0	-538.0
163	-359.0	-615.0
164	-379.0	-431.0
165	-405.0	-632.0
166	-369.0	-630.0
167	-305.0	-393.0
168	-361.0	-517.0
169	-268.0	-621.0
170	-308.0	-391.0
171	-398.0	-571.0
172	-318.0	-632.0
173	-360.0	-708.0
174	-422.0	-353.0
175	-230.0	-380.0
176	-309.0	-511.0
177	-244.0	-383.0
178	-222.0	-420.0
179	-334.0	-490.0
180	-352.0	-520.0
181	-351.0	-562.0
182	-262.0	-701.0
183	-342.0	-437.0
184	-248.0	-493.0
185	-324.0	-509.0
186	-309.0	-674.0
187	-320.0	-722.0
188	-312.0	-452.0
189	-307.0	-502.0
190	-337.0	-443.0
191	-145.0	-385.0
192	-333.0	-511.0
193	-265.0	-644.0
194	-353.0	-601.0
195	-374.0	-537.0
196	-266.0	-422.0
197	-364.0	-509.0
198	-253.0	-302.0
199	-341.0	-492.0
200	-334.0	-398.0
201	-315.0	-520.0
202	-317.0	-450.0
203	-318.0	-566.0
204	-336.0	-512.0
205	-337.0	-522.0
206	-302.0	-606.0
207	-331.0	-660.0
208	-298.0	-482.0
209	-354.0	-608.0
210	-369.0	-622.0
211	-381.0	-579.0
212	-298.0	-525.0
213	-299.0	-432.0
214	-269.0	-410.0
215	-308.0	-550.0
216	-315.0	-516.0
217	-319.0	-498.0
218	-383.0	-451.0
219	-374.0	-440.0
220	-220.0	-372.0
221	-340.0	-735.0
222	-388.0	-519.0
223	-360.0	-520.0
224	-267.0	-463.0
225	-256.0	-511.0
226	-431.0	-614.0
227	-229.0	-393.0
228	-378.0	-527.0
229	-350.0	-531.0
230	-394.0	-617.0

Table 3. (Cont'd)

SAMPLE NUMBER	DATA CHANNEL VALUE	SYNC CHANNEL VALUE
231	-317.0	-510.0
232	-348.0	-502.0
233	-385.0	-466.0
234	-339.0	-549.0
235	-343.0	-492.0
236	-278.0	-338.0
237	-418.0	-577.0
238	-295.0	-440.0
239	-322.0	-575.0
240	-316.0	-444.0
241	-336.0	-598.0
242	-288.0	-552.0
243	-352.0	-595.0
244	-384.0	-530.0
245	-312.0	-631.0
246	-217.0	-503.0
247	-379.0	-429.0
248	-329.0	-652.0
249	-273.0	-488.0
250	-373.0	-601.0
251	-360.0	-520.0
252	-203.0	-400.0
253	-253.0	-472.0
254	-381.0	-500.0
255	-305.0	-647.0
256	-356.0	-468.0
257	-265.0	-584.0
258	$\boxed{-181.0}$	-510.0
259	-297.0	-532.0
260	-343.0	-661.0
261	-380.0	-657.0
262	-239.0	-564.0
263	-365.0	-620.0
264	-301.0	-610.0
265	-286.0	-447.0
266	-282.0	-615.0
267	-304.0	-512.0
268	-275.0	-513.0
269	-376.0	-510.0
270	-346.0	-570.0
271	-356.0	-519.0
272	-297.0	-427.0
273	-348.0	-553.0
274	-318.0	-559.0
275	-339.0	-494.0
276	-335.0	-467.0
277	-261.0	-471.0
278	-334.0	-721.0
279	-373.0	-428.0
280	-267.0	-658.0
281	-274.0	-352.0
282	-317.0	-595.0
283	-342.0	-471.0
284	-297.0	-652.0
285	-287.0	-556.0
286	-214.0	$\boxed{-321.0}$
287	-304.0	-510.0
288	-341.0	-610.0
289	-320.0	-372.0
290	-285.0	-591.0
291	-272.0	-419.0
292	-347.0	-606.0
293	-254.0	-682.0
294	-296.0	-519.0
295	-316.0	-609.0
296	-217.0	-467.0
297	-239.0	-547.0
298	-234.0	-463.0
299	-313.0	-360.0
300	-294.0	-387.0
301	-317.0	-574.0
302	-224.0	-446.0
303	-340.0	-653.0
304	-202.0	-537.0
305	-350.0	-511.0
⋮	⋮	⋮

Table 3. (Cont'd)

SAMPLE NUMBER	DATA CHANNEL VALUE	SYNC CHANNEL VALUE
2922	-312.0	-441.0
2923	-319.0	-394.0
2924	-319.0	-571.0
2925	-329.0	-395.0
2926	-258.0	-653.0
2927	-263.0	-495.0
2928	-333.0	-560.0
2929	-352.0	-598.0
2930	-268.0	-525.0
2931	-297.0	-450.0
2932	-299.0	-514.0
2933	-296.0	-451.0
2934	-324.0	-465.0
2935	-316.0	-629.0
2936	-284.0	-653.0
2937	-345.0	-504.0
2938	-333.0	-464.0
2939	-248.0	-422.0
2940	-363.0	-645.0
2941	-376.0	-409.0
2942	-277.0	-475.0
2943	-346.0	-569.0
2944	-386.0	-532.0
2945	-302.0	-481.0
2946	-305.0	-526.0
2947	-297.0	-263.0
2948	-287.0	-343.0
2949	-389.0	-601.0
2950	-289.0	-491.0
2951	-328.0	-544.0
2952	-384.0	-521.0
2953	-296.0	-326.0
2954	-343.0	-514.0
2955	-247.0	-314.0
2956	-314.0	-377.0
2957	-312.0	-666.0
2958	-371.0	-426.0
2959	-384.0	-570.0
2960	-428.0	-603.0
2961	-313.0	-550.0
2962	-287.0	-496.0
2963	-231.0	-292.0
2964	-314.0	-581.0
2965	-366.0	-551.0
2966	-407.0	-519.0
2967	-358.0	-418.0
2968	-356.0	-625.0
2969	-330.0	-341.0
2970	$\boxed{-170.0}$	$\boxed{-240.0}$
2971	-423.0	-480.0
2972	-319.0	-577.0
2973	-384.0	-551.0
2974	-329.0	-608.0
2975	-302.0	-602.0
2976	-320.0	-271.0
2977	-343.0	-571.0
2978	-304.0	-490.0
2979	-416.0	-601.0
2980	-382.0	-478.0
2981	-356.0	-568.0
2982	-299.0	-709.0
2983	-328.0	-477.0
2984	-286.0	-387.0
2985	-375.0	-481.0
2986	-391.0	-426.0
2987	-325.0	-495.0
2988	-347.0	-702.0
2989	-287.0	-574.0
2990	-338.0	-591.0
2991	-315.0	-485.0
2992	-450.0	-681.0
2993	-394.0	-547.0
2994	-407.0	-609.0
2995	-350.0	-488.0
2996	-423.0	-518.0
2997	-430.0	-676.0
2998	-395.0	-472.0
2999	-453.0	-421.0
3000	-308.0	-430.0

Table 4. A list of pairs of extremes (X, Y)

Group no.	Data channel value	Synchronization channel value
1	−95	−157
2	−138	−199
3	−181	−321
4	−158	−355
5	−146	−209
6	−179	−331
7	−192	−273
8	−211	−299
9	−169	−274
10	−198	−322
11	−159	−333
12	−204	−300
13	−197	−327
14	−157	−321
15	−185	−304
16	−173	−274
17	−19	−216
18	−103	−366
19	−108	−253
20	−153	−265
21	−172	−293
22	−112	−282
23	−112	−336
24	−121	−238
25	−171	−215
26	−190	−272
27	−151	−326
28	−110	−185
29	−174	−255
30	−170	−240

Although we have indicated how the constants in the marginal distributions are found, the experimental determination of the constant c in Eqs. (6)–(7) has not been considered. In practice, the parameter c is usually estimated—at least initially—by a method first used in Ref. 3. The technique revolves around the relation

$$Pr\{|\Lambda - \Omega| < a\} = \frac{e^a - 1}{e^a + 1} + 2\frac{w'(a)}{w(a)} \tag{8}$$

where a is some positive constant between 1.5 and 2. Equation (8) is derived from Eq. (5) by integration between the proper limits. If we let $\nu_N(a)$ denote the number of times $|\Lambda_i - \Omega_i| < a$ in N samples, then $\nu_N(a)/N$ is an estimate of $Pr\{|\Lambda - \Omega| < a\}$ which is known from Eq. (8). Thus, c satisfies

$$\frac{\nu_N(a)}{N} = \tanh\left(\frac{a}{2}\right) + 2c\frac{\operatorname{sech}^2\left(\frac{a}{2}\right)\tanh\left(\frac{a}{2}\right)}{1 - c\operatorname{sech}^2\left(\frac{a}{2}\right)}$$

This can be solved for c giving an estimate, $\hat{c}$, for c as

$$\hat{c} = \frac{\tanh\left(\frac{a}{2}\right) - \frac{\nu_N(a)}{N}}{-2\operatorname{sech}^2\left[\left(\frac{a}{2}\right)\right]\left[\tanh\left(\frac{a}{2}\right)\right] + \left[\operatorname{sech}^2\left(\frac{a}{2}\right)\right]\left[\tanh\left(\frac{a}{2}\right) - \frac{\nu_N(a)}{N}\right]} \tag{9}$$

In Ref. 3 it is shown that

$$\operatorname{Var}\hat{c} = \frac{\nu_N(a)}{4N^2}\left[1 - \frac{\nu_N(a)}{N}\right]\left[\frac{c\,w^2(a)}{w'(a)}\right]^2 \tag{10}$$

It turns out that the variance of $\hat{c}$ does not depend very much on the value of a, and for $1.5 \le a \le 2.0$, it is approximately twice the variance of the maximum likelihood estimate over much of the range of c. Hence, this estimator is a good one to use to avoid solving the likelihood equations, which for Eq. (7) are indeed formidable.

The processing of the data of Table 4 proceeds in the following manner. Table 4 lists the pairs of random variables (X, Y) obtained by dividing the data into thirty groups each of 100 points. Application of univariate EVT to each channel *independently* results in the parameters

$$\alpha_\Lambda = 0.033363$$
$$u_\Lambda = -171.632$$
$$\alpha_\Omega = 0.022859$$
$$u_\Omega = -302.892$$

Table 5. Normalized pairs of extremes (Λ, Ω)

DATA CHANNEL Λ	SYNC CHANNEL Ω
2.5567	3.3349
1.1221	2.3749
-0.3125	-0.4139
0.4548	-1.1911
0.8552	2.1463
-0.2458	-0.6425
-0.6795	0.6833
-1.3134	0.0890
0.0878	0.6604
-0.8797	-0.4368
0.4214	-0.6882
-1.0799	0.0661
-0.8464	-0.5511
0.4882	-0.4139
-0.4460	-0.0253
-0.0456	0.6604
5.0923	1.9863
2.2898	-1.4426
2.1230	1.1405
0.6216	0.8662
-0.0123	0.2261
1.9895	0.4776
1.9895	-0.7568
1.6892	1.4834
0.0211	2.0091
-0.6128	0.7062
0.6883	-0.5282
2.0562	2.6949
-0.0790	1.0948
0.0544	1.4376

These parameters are then used to normalize the random variables of Table 4 (X, Y) obtaining the set of random variables (Λ, Ω) in Table 5. If we choose a in Eq. (8) to be 1.5, we find from Table 5 that ν_{30} (1.5) is 24 so that we estimate $Pr\{|\Lambda - \Omega| < 1.5\} = 24/30 = 0.80$. Using this in Eq. (9) gives an estimate $\hat{c} = 0.19254$. For this example, we then have

$$\Phi(x) = \exp - [e^{-.033363(x+171.632)}] \tag{11a}$$

$$\Phi(y) = \exp - [e^{-.022859(y+302.892)}] \tag{11b}$$

and

$$\Psi(x,y) = \exp - \Big\{ e^{-.033363(x+171.632)} + e^{-.022859(y+302.892)} - 0.19254\,[e^{-.033363(x+171.632)} + e^{-.022859(y+302.892)}]$$

$$\times \operatorname{sech}^2\left(\frac{-0.033363x - 0.022859y - 12.650036}{2}\right)\Big\} \tag{11c}$$

We now have an expression for $\Psi(x,y)$ and by inserting the thresholds of the two channels, we have the probability that the data channel and the synchronization channel both make correct decisions on each of n bits (since there are n samples per group). Then the probability, p, of any one bit being correct and being accepted—i.e., the synchronization channel giving an in-lock indication—is the n^{th} root of this, or

$$p = \Psi^{1/n}(x_o, y_o)\,. \tag{12a}$$

Since the threshold devices of both channels are essentially polarity sensing devices, the example gives as an initial estimate

$$p = \Psi^{1/100}(x_o, y_o) = \Psi^{1/100}(0,0)$$

$$= (0.996345)^{1/100}$$

$$= 0.9999634$$

There are, of course, three other probabilities of interest. These are (1) q, the probability of receiving and rejecting a correct bit, (2) r, the probability of receiving and accepting an incorrect bit and (3) s, the probability of receiving and rejecting an incorrect bit. Of course, $p + q + r + s = 1$. The interrelation of these four probabilities can be visualized with the aid of Fig. 12 giving

$$q = \Phi^{1/100}(x_0) - p \tag{12b}$$

$$r = \Phi^{1/100}(y_0) - p \tag{12c}$$

$$s = 1 - p - q - r \tag{12d}$$

Using the four parameters α_Λ, α_Ω, u_Λ and u_Ω listed above and applying univariate EVT to each channel independently gives $\Phi^{1/100}(x_0)$, the probability of a correct bit, and $\Phi^{1/100}(y_0)$, the probability of an in-lock indication, respectively, as

$$\Phi^{1/100}(x_0) = 0.9999674$$

$$\Phi^{1/100}(y_0) = 0.9999902$$

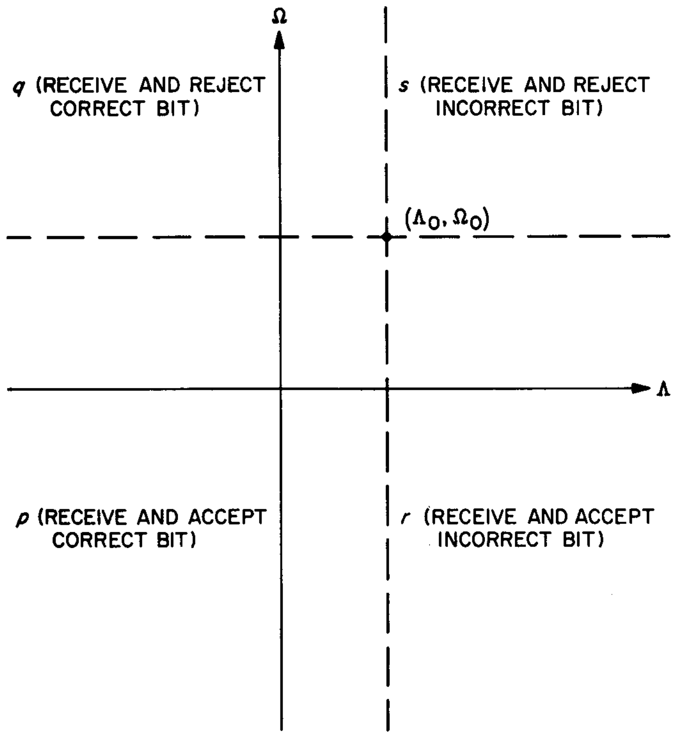


Fig. 12. The four bivariate probability regions

Hence, we compute for the initial estimate:

$$q = \Phi^{1/100}(x_0) - p = 4.02 \times 10^{-6}$$

$$r = \Phi^{1/100}(y_0) - p = 2.68 \times 10^{-5}$$

$$s = 1 - p - q - r = 5.82 \times 10^{-6}$$

As indicated earlier, the maximum likelihood equations for Eq. (7) are quite difficult even to derive, let alone solve. The approach taken by us has been to use a numerical technique based on successive iterations of the mixed second partial derivative of Eq. (7). This type of maximum likelihood technique is described in more detail in Section X. Applying this technique to the above example results in the following parameters:

$$\alpha_\Lambda = 0.033500 \qquad u_\Lambda = -173.178$$

$$\alpha_\Omega = 0.022290 \qquad u_\Omega = -300.845$$

$$c = 0.139145$$

Using these parameters, we recalculate the final results as:

$$\Phi^{1/100}(x_0) = 0.9999698$$

$$\Phi^{1/100}(y_0) = 0.9999878$$

$$p = 0.9999624$$

$$q = 7.39 \times 10^{-6}$$

$$r = 2.54 \times 10^{-5}$$

$$s = 4.85 \times 10^{-6}$$

In no case are any of the changes large ones.

It is interesting to note that while the probability of making an error on any particular bit is 3.02×10^{-5}, the probability of making a bit error given an in-lock indication is $2.54 \times 10^{-5}/0.9999878 \approx 2.54 \times 10^{-5}$, a slight decrease. One is now in a position to begin questioning the system design and asking for tradeoffs. For example, by biasing the lock indicator so that it is more likely to indicate out-of-lock, one would expect changes in the conditional probabilities calculated above. To this end, Table 6 was constructed.

We see from the table that the conditional probability of a bit error is decreased by a factor of 2 as the lock-channel bias is decreased to -252; but the probability of an out-of-lock is simultaneously increased by a factor of 300. This may or may not be acceptable, but the point is that the tradeoffs are quantitatively known. Furthermore, these tradeoffs were arrived at without recourse to hardware changes. The only change was that of x_0 and y_0 in Eqs. (11)–(12) and the reevaluation of the probabilities of interest! Here we have a striking example of the fact that *extreme value techniques can be used as a design tool,* as well as for analysis after design.

The question now arises as to whether or not the data fit obtained by using the techniques outlined above does in fact represent the data in accordance with the assertion in Eqs. (5)–(7). To aid in visualizing such a fit, Fig. 13 shows the density corresponding to Eq. (7) along with the experimental data fitted by the density. It might be well to point out that the data presented in Fig. 13

Table 6. Conditional bit-error rates as a function of threshold

	Bias[1] (data, synchronization)		
	0, 0	0, −157	0, −252
Pr (bit error)	3.02×10^{-5}	3.02×10^{-5}	3.02×10^{-5}
Pr (bit error given in-lock)	2.54×10^{-5}	1.46×10^{-5}	1.35×10^{-5}
Pr (out-of-lock)	1.22×10^{-5}	4.05×10^{-4}	3.36×10^{-3}
[1] Relative units.			

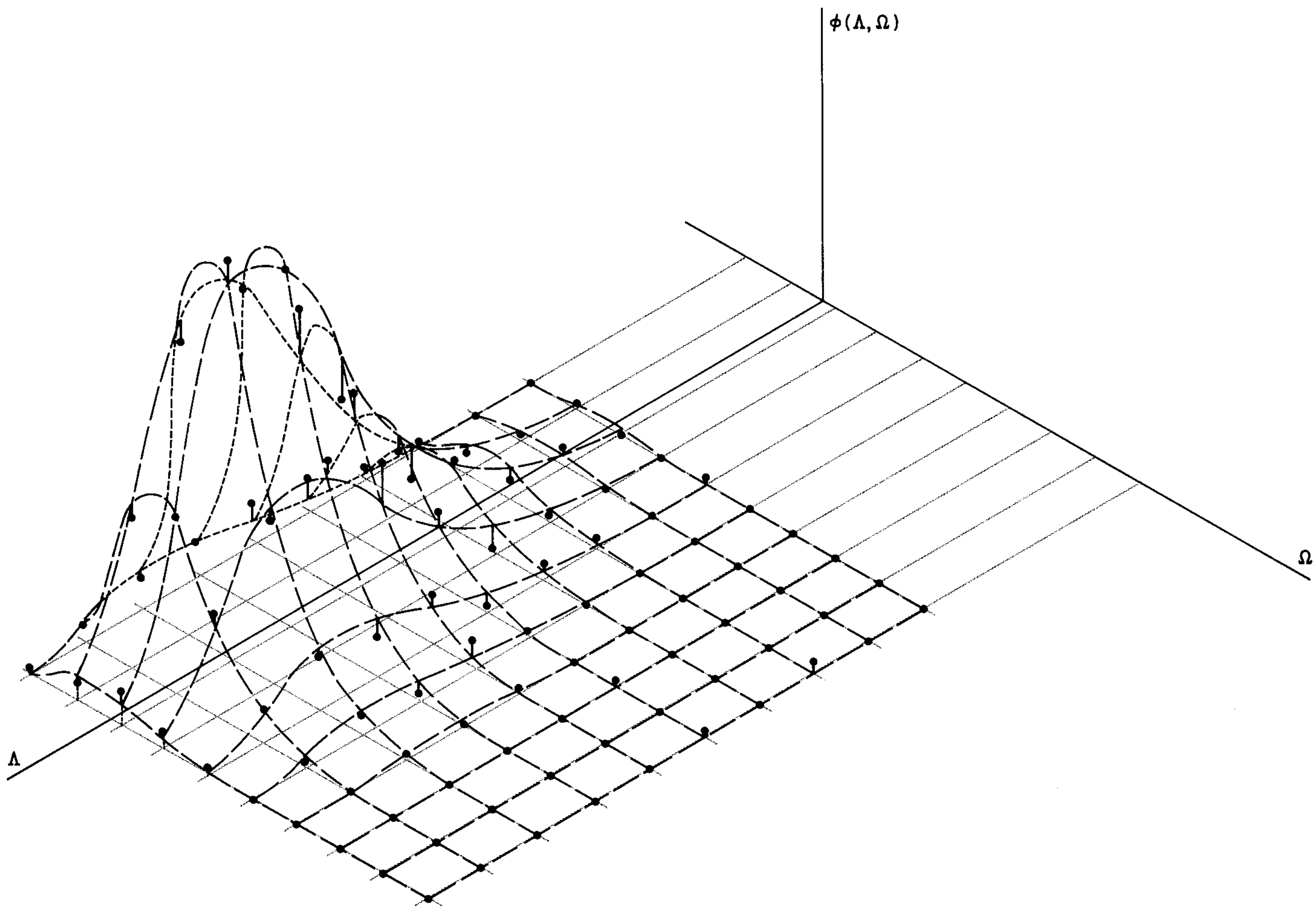


Fig. 13. Bivariate probability density of data extremes

are not the same as those used in the example above. As opposed to the 3000 data points used in the example, Fig. 13 represents 70,000 data points so that the experimental density would be smoother. As in the example, though, 3000 data points are a sufficient number to apply the technique. The significant fact is that Fig. 13 demonstrates reasonably good agreement between the experimental data and the fit obtained. Unlike the univariate case, the experimental data are difficult to plot, and visual fits of the data are not easily made or interpreted. Little effort has been expended along these lines and no success has been encountered.

IX. RESTRICTIONS AND LIMITATIONS ON THE USE OF BIVARIATE EXTREME-VALUE THEORY

Many of the restrictions on the use of bivariate EVT can be traced to those of univariate EVT. Of course, it is necessary to be able to model the system accurately, and it must be valid to apply univariate EVT to each of the two variables independently. Successive samples were assumed independent as in the univariate case, and the same criteria of independence can be applied to each channel as with the univariate case.

However, in the step from Eqs. (5) to (7), i.e., choosing $w(\Lambda - \Omega)$, another restriction unique to bivariate EVT is encountered. With the level of understanding that we presently have, it appears there is a large family of functions that could be used for $w(\Lambda - \Omega)$. Each of these functions satisfies all constraints known to exist on $w(\Lambda - \Omega)$. While the known constraints do not completely specify $w(\Lambda - \Omega)$, they are sufficient that the probabilities calculated from $\Psi(x, y)$ do not appear to depend drastically on the choice of the function $w(\Lambda - \Omega)$ as long as it is chosen within these constraints. In view of this fact, the function in Eq. (6), $w_c(\Lambda - \Omega)$, was selected from among the family of w's as one having nice mathematical properties. Specifically, $w(\Lambda - \Omega)$ was chosen so that it depended on a single constant. Thus, the parameter c of Eq. (7) is estimated rather than the entire function $w(\Lambda - \Omega)$.

A note of caution should be interjected at this point. The value of c is restricted so that $0 \leq c \leq 1/4$. When $c = 0$, the data from the two channels are uncorrelated, as pointed out in Section VIII. However, the case of $c = 1/4$ does not correspond to complete correlation. When $c > 1/4$, the function $\Psi(x, y)$ in Eq. (7) ceases to be a valid probability function, i.e., $\Psi(x, y)$ violates one of the basic axioms of probability theory, namely that $\Psi(x, y)$ must be a non-decreasing function. The value of $c = 1/4$ corresponds to a linear correlation coefficient between the extremes of the data, ρ, of $2/3$. Since ρ is much easier to calculate than c, this fact is of considerable aid in applying bivariate EVT where high correlation between the channels exists. Our experience has been that few systems exhibit c's even approaching $1/4$. Usually c remains below 0.2, with ρ remaining below 0.4. Except in artificially constructed cases, we have had no difficulties with large values of c. On the other hand, small values of c are not uncommon. When the data are uncorrelated, the maximum likelihood estimate of c is negative and must be held at zero. This analysis is considered again in Section X.

X. DATA-PROCESSING TECHNIQUES

The purposes of this section are to discuss in detail the various processing techniques which we have used to compute both the univariate and bivariate extreme-value statistics, to present mathematical descriptions where necessary, and to enumerate specific approaches used to overcome difficulties encountered in processing the data. All computations were accomplished by a FORTRAN program written for an SDS-920 computer. This section of the report is heavily slanted toward the computer program. Appendix A describes the capabilities and limitations of the program itself, Appendix B contains a table of nomenclature of the program, a simplified flow diagram and a program listing. Appendix C contains a copy of the sample output of the program using the example discussed throughout this report.

For simplicity, the discussion which follows will be geared to one channel only, and we have arbitrarily selected the data channel. It should be kept in mind that identical procedures must be applied to the second channel in bivariate statistics, as well as further computations on both channels. These procedures will be described later.

The development of EVT statistics in this report has been concerned with predicting bit-error rates of maxima from a set of data. In many instances we are concerned with minima EVT, that is, with data where $x < x_0$ denotes a bit error and/or $y < y_0$ indicates a loss of synchronization. The data-processing technique we have used to handle this condition is to multiply any such data, including the corresponding threshold, by -1 so that maxima EVT is applicable.

The extremes used to estimate the statistics of the data are obtained as follows. The data are divided into N groups of n points each. The maximum value, X_i, from each group of n points is then found, and using these N maxima we proceed to calculate the univariate EVT statistics. Having plotted the N maxima (Fig. 8) we see that a straight line could be fitted by eye. However, we desire a mathematical fit based on some minimizing criteria, and to this end we use a maximum likelihood fit.

To obtain an initial estimate for the parameters α_Λ and u_Λ we must first know the expected mean, μ_e, and expected standard deviation, σ_e, which are calculable (see Ref. 2) as

$$\mu_e = \frac{1}{N}\sum_{i=1}^{N} - \ln\left(-\ln\frac{i}{N+1}\right) \qquad (13a)$$

$$\sigma_e = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[-\ln\left(-\ln\frac{i}{N+1}\right) - \mu_e\right]^2} \qquad (13b)$$

Note that as written, Eq. (13b) requires two sequential computations; the first computes μ_e and the second calculates σ_e. To reduce processing time, another form for computing standard deviations is employed. Specifically,

$$\sigma_e = \sqrt{\left\{\frac{1}{N}\sum_{i=1}^{N}\left[-\ln\left(-\ln\frac{i}{N+1}\right)\right]^2\right\} - \mu_e^2} \qquad (14)$$

Equation (14) calculates σ_e in one computation during which two sums are formed, the sum of the individual terms and the sum of the squares of the individual terms. From the former sum we easily obtain μ_e, and direct substitution into Eq. (14) yields σ_e.

It is shown in Ref. 2 that if μ_Λ denotes the mean and σ_Λ the standard deviation of the data channel maxima then as a first estimate α_Λ and u_Λ can be calculated as

$$\alpha_\Lambda = \frac{\sigma_e}{\sigma_\Lambda}$$

$$u_\Lambda = \mu_\Lambda - \frac{\mu_e}{\alpha_\Lambda}$$

Knowing the values of α_Λ, u_Λ, and x_o, the threshold of the data channel, we obtain initial estimates for $\Phi(x_0)$ and $\Phi^{1/n}(x_0)$ using Eq. (2).

To proceed with a maximum likelihood fit, we first change parameters in the extreme value distribution of Eq. (2) to a set of parameters more suited to our purpose. Specifically, we are interested in the probability that a random variable having an extreme-value distribution will not exceed the threshold x_o, rather than in the parameters α_Λ and u_Λ. This probability has been previously defined as

$$\Phi(x_o) = \exp - [-\exp(-\Lambda_o)] \qquad (15a)$$

where

$$\Lambda_o = \alpha_\Lambda(x_o - u_\Lambda) \qquad (15b)$$

We write $\Phi(x)$ in terms of α_Λ and $\Phi(x_o)$ rather than α_Λ and u_Λ as

$$\Phi(x) = \exp - [\exp - (\alpha_\Lambda(x - x_o) + \Lambda_o)] \qquad (16)$$

which has the unknown parameters α_Λ and Λ_o.

We now obtain the maximum likelihood estimators of α_Λ and Λ_o. Let $X_1, X_2, \cdots, X_N$ represent the N data channel maxima. Since the density function $\phi(x)$ of Eq. (16) has the form

$$\phi(x) = \frac{d\,\Phi(x)}{dx} = \alpha_\Lambda \Phi(x) \exp[-(\alpha_\Lambda(x - x_o) + \Lambda_o)]$$

the likelihood function, L, for this sample is

$$\begin{aligned} L(X_1, \cdots, X_N, \alpha_\Lambda, \Lambda_o) &= \prod_{i=1}^{N} \alpha_\Lambda \Phi(X_i) \exp[-(\alpha_\Lambda(X_i - x_o) + \Lambda_o)] \\ &= \alpha_\Lambda^N \left[\exp - \left(\alpha_\Lambda \sum_{i=1}^{N}(X_i - x_o) + N\Lambda_o\right)\right] \exp - \left(\sum_{i=1}^{N} \exp - (\alpha_\Lambda(X_i - x_o) + \Lambda_o)\right) \end{aligned} \qquad (17)$$

To maximize Eq. (17), or equivalently, to maximize the logarithm of Eq. (17), we differentiate *ln L* (since it has a simpler form) and obtain

$$\ln L = N \ln \alpha_\Lambda - N\alpha_\Lambda(\mu_\Lambda - x_o) - N\Lambda_o - \sum_{i=1}^{N} \exp - (\alpha_\Lambda(X_i - x_o) + \Lambda_o)$$

$$\frac{\partial \ln L}{\partial \alpha_\Lambda} = \frac{N}{\alpha_\Lambda} - N(\mu_\Lambda - x_o) + \sum_{i=1}^{N}(X_i - x_o)\exp - (\alpha_\Lambda(X_i - x_o) + \Lambda_o) \qquad (18a)$$

$$\frac{\partial\, ln\, L}{\partial \Lambda_o} = -N + \sum_{i=1}^{N} \exp - (\alpha_\Lambda(X_i - x_o) + \Lambda_o) \qquad (18b)$$

To solve for the maximum likelihood estimators of α_Λ and Λ_o, $\hat{\alpha}_\Lambda$ and $\hat{\Lambda}_o$, we set

$$\frac{\partial \ln L}{\partial \alpha_\Lambda} = \frac{\partial \ln L}{\partial \Lambda_o} = 0$$

When set equal to zero, Eqs. (18a) and (18b) do not have a closed-form solution; a numerical technique, the Newton-Raphson method for solving systems of equations, is used to find good approximations to $\hat{\alpha}_\Lambda$ and $\hat{\Lambda}_o$ (Ref. 4). Numerically, we proceed as follows. Using the initial estimates of α_Λ and Λ_o as the arguments for the partial derivatives of $\ln L$, we compute better estimates to α_Λ and Λ_o, say $\alpha_\Lambda^{(1)}$ and $\Lambda_o^{(1)}$ and calculate the corresponding values of $\partial \ln L/\partial \alpha_\Lambda^{(1)}$ and $\partial \ln L/\partial \Lambda_o^{(1)}$. If both of these values are greater than or equal to a specified limit (we have used 10^{-5}) we repeat the procedure and calculate $\alpha_\Lambda^{(2)}$ and $\Lambda_o^{(2)}$, obtaining still better estimates. This iterative procedure continues until Eqs. (18a) and (18b) both have values less than our specified limit. At this point we take the values of $\alpha_\Lambda^{(i)}$ and $\Lambda_o^{(i)}$ to be the maximum likelihood estimators, $\hat{\alpha}_\Lambda$ and $\hat{\Lambda}_o$.

To complete the univariate EVT application we compute the statistics $\Phi(x_o)$ and $\Phi^{1/n}(x_o)$ from Eq. (2) using the maximum likelihood estimators, obtain a new estimate for u_Λ by substituting $\hat{\alpha}_\Lambda$ and $\hat{\Lambda}_o$ into Eq. (15), and proceed to find confidence intervals for the predicted bit-error rate. In computing the confidence intervals we use the fact that the maximum likelihood estimators $\hat{\alpha}_\Lambda$ and $\hat{\Lambda}_o$ are approximately bivariately normally distributed for large N (Ref. 1). If, for example, a 99% confidence interval is desired, the quantile of order 0.99 of the unit-variance normal distribution is 2.576 (that is, a unit normal variate is less than ± 2.576 with 0.99 probability). Thus, using Eq. (4) we set

$$\Lambda_o^* = \hat{\Lambda}_o \pm 2.576\,(\mathrm{var}\,\hat{\Lambda}_o)$$

and compute the two-sided 99% confidence interval for the predicted bit error rate by computing $1 - \Phi^{1/n}(x_o)$ for these two values of Λ_o^*. The data processing program repeats the above procedure to also obtain the 95, 90, 80, and 70% confidence intervals.

Having calculated univariate EVT statistics for each channel, we now proceed to bivariate calculations. We use the univariate maximum likelihood estimators of α_Λ, Λ_o, α_Ω, and Ω_o to linearly transform the N pairs of random variables (X_i, Y_i) obtaining the pairs (Λ_i, Ω_i) where

$$\Lambda_i = \alpha_\Lambda (X_i - u_\Lambda)$$

$$\Omega_i = \alpha_\Omega\,(Y_i - u_\Omega)$$

The parameter c is initially estimated by using Eqs. (8)–(9) in which $pr\{\,|\Lambda_i - \Omega_i| < a\}$ is approximated by $\nu_N(a)/N$ where $\nu_N(a)$ denotes the number of times $|\Lambda_i - \Omega_i| < a$ in N samples; we restrict a so that $1.5 \le a \le 2.0$. Using this value of c we compute the initial bivariate statistics $\Psi(x_o, y_o)$, p, q, r, and s as described by Eqs. (7) and (12a–d).

At this point it seems wise to interject a few comments concerning the different forms of Eqs. (5), (6) and (9) which appear in the computer program.

Eq. (6)

$$w\,(\Lambda - \Omega) = 1 - c\,\mathrm{sech}^2\left(\frac{\Lambda - \Omega}{2}\right)$$

can be written as

$$w\,(\Lambda - \Omega) = 1 - \frac{4ce^{(\Lambda-\Omega)}}{(1 + e^{(\Lambda-\Omega)})^2} \tag{19}$$

The program uses Eq. (19) in calculating c, so that Eq. (9) is rewritten as

$$\hat{c} = \frac{\tanh\left(\dfrac{a}{2}\right) - \dfrac{\nu_N(a)}{N}}{\dfrac{8(e^a - e^{3a})}{(1+e^a)^4} + \dfrac{4e^a}{(1+e^a)^2}\left(\tanh\left(\dfrac{a}{2}\right) - \dfrac{\nu_N(a)}{N}\right)}$$

As stated previously the bivariate experimental data, in comparison to that of the univariate case, are difficult to plot and do not allow a visual fit of the data which is either easily made or interpreted. Once again, a mathematical fit based on some minimizing criteria is desirable. A maximum likelihood approach to calculate the estimators, $\hat{\alpha}_\Lambda$, $\hat{u}_\Lambda$, $\hat{\alpha}_\Omega$, $\hat{u}_\Omega$, and $\hat{c}$, such as the one used in calculating the univariate maximum likelihood estimators is not feasible. The approach we have taken is based on the likelihood function of $\Psi(x, y)$. Let the density function of $\Psi(x, y)$ be represented by $\psi(x, y)$ where

$$\psi\,(x, y) = \frac{\partial^2 \Psi(x, y)}{\partial x, \partial y}$$

By straightforward calculations

$$\psi(x,y) = \left\{\left[\alpha_\Lambda g(z)e^{-\Lambda} - \frac{\alpha_\Lambda}{2} g'(z)(e^{-\Lambda}+e^{-\Omega})\right]\left[\alpha_\Omega g(z)e^{-\Omega} + \frac{\alpha_\Omega}{2} g'(z)(e^{-\Lambda}+e^{-\Omega})\right] + \left[-\frac{\alpha_\Lambda\alpha_\Omega}{2} g'(z)(e^{-\Lambda}-e^{-\Omega}) + \frac{\alpha_\Lambda\alpha_\Omega}{4} g''(z)(e^{-\Lambda}+e^{-\Omega})\right]\right\}\cdot\Psi(x,y) \tag{20}$$

where

$$\Lambda = \alpha_\Lambda (x - u_\Lambda)$$

$$\Omega = \alpha_\Omega (y - u_\Omega)$$

$$z = \frac{\Lambda - \Omega}{2}$$

$$g(z) = 1 - c\,\mathrm{sech}^2 z$$

$$g'(z) = 2c\,\mathrm{sech}^2 z \tanh z$$

$$g''(z) = 2c\,\mathrm{sech}^4 z - 4c\,\mathrm{sech}^2 z \tanh^2 z$$

$$\Psi(x,y) = \exp\left[-(e^{-\Lambda}+e^{-\Omega})\,g(z)\right]$$

If we let $(X_1, Y_1)\cdots(X_N, Y_N)$ represent the N pairs of random variables, the likelihood function for this sample is

$$L((X_1,Y_1),\cdots,(X_N Y_N);\alpha_\Lambda,u_\Lambda,\alpha_\Omega,u_\Omega,c) = \prod_{i=1}^{N}\psi(X_i,Y_i) \tag{21}$$

To proceed as in the univariate case would require that we minimize ln L which would necessitate finding the five first partial derivatives of ln L with respect to α_Λ, u_Λ, α_Ω, u_Ω, and c, equating these equations to zero, and solving these five simultaneous equations for the maximum likelihood estimators.

In lieu of the difficulties presented by the above approach, we have employed a numerical method based on the assumption that the bivariate surface is "nice." This assumption has been shown to be valid in all the various examples we have tried. The method can be described as a parabola fitting procedure on the five parameters.

We begin by selecting c as the first parameter to be varied since α_Λ, u_Λ, α_Ω, and u_Ω have been estimated by the univariate maximum likelihood fit. Holding the other four parameters constant, we obtain two other values of ln L near c; that is, we set

$$\xi_1 = c$$

$$\xi_2 = c - .01\,|c|$$

$$\xi_3 = c + .01\,|c|$$

and use these values to compute three corresponding values, ζ_1, ζ_2, and ζ_3, of ln L; that is

$$\zeta_i = \ln L\left((X_1,Y_1),\cdots,(X_N,Y_N);\alpha_\Lambda,u_\Lambda,\alpha_\Omega,u_\Omega,\xi_i\right) = \sum_{i=1}^{N}\ln\psi(X_i,Y_i) \qquad i = 1,2,3$$

To fit a parabola through the points (ξ_1, ζ_1), (ξ_2, ζ_2), and (ξ_3, ζ_3) we solve the three simultaneous linear equations

$$\zeta_i = A\xi_i^2 + B\xi_i + C \qquad i = 1,2,3 \tag{22}$$

for the coefficients A and B using Cramer's rule. The vertex, v, of the parabola fitted to these points will be

$$v = -\frac{B}{2A}$$

If this newly computed vertex differs from the previous vertex by some specified limit (we have been using 0.01%) then we consider the procedure to have converged. If the two successive vertices do not satisfy this condition, we determine new points for another attempt at fitting a parabola, as illustrated in Fig. 14. We iteratively fit parabolas in this manner until the above difference condition is satisfied, that is, until convergence is achieved. The last vertex calculated is now used as a better approximation to c.

Having found a better approximation to c we apply this same method to α_Λ, α_Ω, u_Λ, and u_Ω in that order. When all five parameters have been estimated, one iteration is considered to be done. (The total number of iterations is variable in the program of Appendix A.) These newly estimated parameters are now used to re-calculate the bivariate statistics of interest.

Several difficulties which we encountered warrant special mention. Due to the numerical capacity of the computer (approximately twelve decimal digits) some overflow problems occurred when using Cramer's rule to solve the system of equations used in the parabola fitting

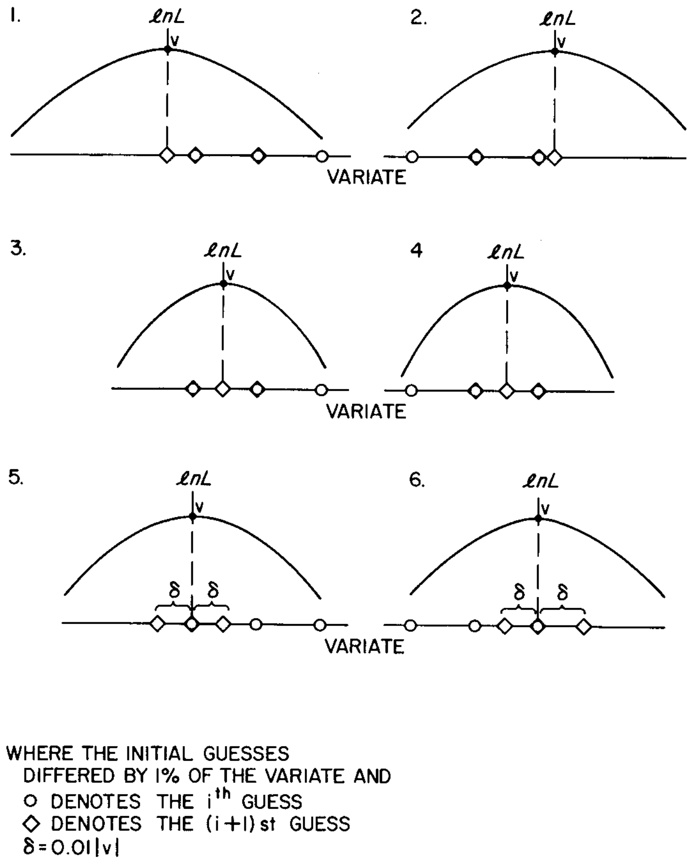


Fig. 14. Details of bivariate iterative maximum likelihood fit

procedure. This problem is alleviated by performing a translation of axes so that (ξ_1, ζ_1) becomes the origin of the new coordinate system. This new coordinate system is used to compute the vertex of the parabola and to determine the new set of points for the next parabola fit. All other calculations are performed in the original coordinate system.

Another problem occurred when taking the n^{th} root of the various cumulative probabilities, $\Phi(x_o)$, $\Phi(y_o)$, and $\Psi(x_o, y_o)$. The first method we employed was to compute $\Phi^{1/n}(x_o)$ for example, as

$$\Phi^{1/n}(x_o) = \exp\left[\frac{1}{n}\ln\Phi(x_o)\right]$$

However, in cases where we were concerned with small error rates, it was found that the round-off errors propagated by the two program library routines, "exp" and "log," occasionally affected our results significantly. A second method of series expansion accurate to the eleventh decimal digit is incorporated in the program. $\Phi^{1/n}(x_o)$ is calculated as

$$\Phi^{1/n}(x_o) = [1 - \{1 - \Phi(x_o)\}]^{1/n} = 1 - \frac{[1 - \Phi(x_o)]}{n} + \cdots$$

For purposes of comparison the program computes $\Phi^{1/n}(x_o)$ using both methods. $\Phi^{1/n}(x_o)$ always assumes the value computed by the second method, except in instances where the second method overflows, due to the capacity of the computer.

Further explanations concerning the data processing program are necessary at this point. As stated in Section IX on the restrictions and limitations of bivariate EVT, the parameter c is restricted to the range $0 \leq c \leq 1/4$. After the last complete iteration of the bivariate maximum likelihood parabola fit, the program determines whether or not c lies in the above closed interval. If not, c is modified so that if $c < 0$ then c is set equal to 0 and similarly if $c > 1/4$ then c is set equal to $1/4$. These modifications occur prior to the final calculation of the bivariate statistics, thereby assuring that c satisfies the restrictions placed on it by the bivariate theory.

As an additional feature the data-processing program computes the correlation coefficients between the data and between the extremes of the data of the two channels. Neither correlation coefficient is used in EVT statistics. However, the correlation coefficient between the data is an aid to evaluation of the data of the entire test and the correlation coefficient between the extremes of the data gives us an easily calculable indication of anticipated behavior of the parameter c, which is of considerable aid in applying bivariate EVT in cases where high correlation exists. Using the notation explained earlier in this section, we compute, for example, the correlation coefficient ρ between the extremes of the two channels as

$$\rho = \frac{\sum_{i=1}^{N}(X_i - \mu_\Lambda)(Y_i - \mu_\Omega)}{N\sigma_\Lambda\sigma_\Omega}$$

As in the computation of the standard deviations discussed above, this form of ρ requires two passes over the data. Processing time is reduced by using the equivalent form

$$\rho = \frac{N\sum_{i=1}^{N}X_iY_i - \sum_{i=1}^{N}X_i\sum_{i=1}^{N}Y_i}{\sqrt{\left[N\sum_{i=1}^{N}(X_i)^2 - (\sum_{i=1}^{N}X_i)^2\right]\left[N\sum_{i=1}^{N}(Y_i)^2 - (\sum_{i=1}^{N}Y_i)^2\right]}}$$

NOMENCLATURE

This table, although not intended to be complete, identifies the major parameters used throughout the report. A nomenclature of the data-processing program is given in Appendix B.

ADA	Amplitude distribution analysis
c	An indicator of correlation between pairs of data extremes. A basic parameter of $\Psi(x, y)$
EVT	Extreme value theory
$f(x)$	Derivative of $F(x)$ with respect to x
$f'(x)$	Derivative of $f(x)$ with respect to x
$F(x)$	Cumulative probability as a function of x
$L(\ )$	Likelihood function
n	The number of samples per group
N	The number of groups
p	The probability of receiving and accepting a correct bit
q	The probability of receiving and rejecting a correct bit
r	The probability of receiving and accepting an incorrect bit
s	The probability of receiving and rejecting an incorrect bit
SNR	Signal to noise ratio
u	The mean of $\Phi(x)$
u_Λ	That u associated with the source of x data
u_Ω	That u associated with the source of y data
$w(\Lambda - \Omega)$	A class of correlation functions
$w_c(\Lambda - \Omega)$	A particular $w(\Lambda - \Omega)$
x	Basic, measured variable of one channel
x_0	Threshold value of x
X_i	Largest value of x within the i^{th} group of data; X has the same units as x
y	Basic measured value of second channel
y_0	Threshold value of y
Y_i	Largest value of y with the i^{th} group of data; Y has the same units as y
α	A measure of concentration of $\Phi(x)$ about u
α_Λ	That α associated with the source of x data
α_Ω	That α associated with the source of y data
γ	Euler's constant (0.5772 . . .)
Λ	Reduced variate; $\Lambda = \alpha (X - u)$
Λ_0	The value of Λ at x_0 (threshold)
μ_e	Expected mean of extremes of data
μ_Λ	Mean of extremes associated with x data, X_i
$\nu_N(a)$	The number of times $\lvert \Lambda_i - \Omega_i \rvert < a$ in N pairs of samples
ρ	The linear correlation coefficient between extremes of pairs of data samples
σ_e	Expected standard deviation of extremes of data
σ_Ω	Standard deviation of extremes associated with x data, X_i
$\phi(x)$	Density function corresponding to $\Phi(x)$
$\Phi(x)$	$\lim_{n \to \infty} \Phi_n(x)$
$\Phi_n(x)$	The probability that in a set of n independent samples the largest sample is less than x
$\psi(x, y)$	Density function correspond to $\Psi(x, y)$
$\Psi(x, y)$	The asymptotic expression for large n of the probability that in a set of n independent pairs of samples, the largest sample from one member of the pair is less than x and that the largest sample from the other member is less than y
Ω	Reduced variate; $\Omega = \alpha_\Omega (Y - u_\Omega)$

APPENDIX A

Data-Processing Program for Bivariate EVT Statistics

The data-processing program which computes the bivariate EVT statistics is written in FORTRAN, with the exception of one subroutine which is written in SYMBOL. The program is based on the capacity of an SDS-920 computer with an 8000-word memory. It was the authors' intention to develop as flexible a program as possible. As a result, the program devised is capable of processing 700 extremes for each channel, i.e., 700 pairs of random variables. Because of the small memory size of the computer, the numerous extremes we wanted to be able to process, and the program flexibility we desired to incorporate, we were required to divide the program into three sub-programs or links, only one of which remains in the memory at any one time.

The first link of the program computes the univariate EVT statistics for each channel independently, i.e., it performs the following functions for each channel:

1. If necessary, multiplies the raw data by -1 so that maxima EVT is applicable
2. Splits the initial data matrix into N groups of n points each
3. Finds the maximum value within each group
4. Computes the parameters α and u
5. Computes the univariate EVT statistics
6. Computes the confidence intervals for the predicted error rates

In addition, this first link computes the mean, standard deviation and a form of signal-to-noise ratio for the raw data of each channel. It also computes the correlation coefficients between the data and between the extremes of the data, and computes the classical, i.e., error-counting, probabilities corresponding to the probabilities p, q, r, and s of Eq. (12).

Link two is incorporated in the program as a supplement to the univariate statistics. It orders the extremes of each channel in increasing value, prints the unordered extremes, the ordered extremes and their respective plotting positions, and offers the operator an option of obtaining a plot of the data on a Cal-Comp plotter, coupled to the computer (Appendix C). If a plot is desired, this link scales the range of the channel maxima so that it coincides with the smaller dimension of the Cal-Comp plotting paper (10 × 16 in.) and so that the threshold may also be plotted on the graph. Using the data channel, for example, this link plots the N scaled, ordered channel extremes, X_i, vs $-\ln[-\ln(i/(N+1))]$ where i is the rank of the ordered extreme by drawing a +. The latter coordinate is measured along the linear reduced variate scale which runs parallel to the non-linear cumulative probability scale (Fig. 7). The routine also plots the reduced variate at threshold and two points of the regression equation

$$x = u_\Lambda + \frac{\text{reduced variate}}{\alpha_\Lambda}$$

These last three points are denoted by the mark □ on the plot.

The third link of the program computes the bivariate EVT statistics. It performs the following functions:

1. Computes an initial guess for the parameter c
2. Performs a variable number of iterations during which a parabola fit is calculated in each of the c, α_Λ, α_Ω, u_Λ, u_Ω planes
3. Computes bivariate EVT statistics whenever specified.

The program is blocked into these three links and their respective subroutines in the following manner:

Link 1.	— Univariate EVT
UMAXLIK	— Computes the univariate maximum likelihood estimators
CONFINT	— Computes the confidence intervals for predicted error rates
Link 2.	— Univariate EVT
ORDER	— Orders and prints the channel extremes
GRAPH	— Provides a linearized univariate EVT plot

Link 3. — Bivariate EVT

BEVT — Computes bivariate EVT statistics

BMAXLIK — Computes the value of the bivariate EVT likelihood function

PARAFIT — Fits a parabola through three given points and solves for the vertex

HELP — Determines new points for successive parabola fits

Each of the above links, except for the subroutine GRAPH is written in SDS FORTRAN II. GRAPH is coded in SDS symbolic programming language, SYMBOL.

All operational directions are typed on the console typewriter during execution of the program. These directions indicate options which are available, and explain the various inputs which the operator must supply. Some elaboration on these options and required inputs seems appropriate here.

The operator has the following options, all of which are controlled by the four breakpoint switches on the console:

1. Breakpoints one and two, respectively, control the need for multiplication by -1 of the raw data from the data and synchronization channels, i.e., whether or not it is necessary to convert the data so that maxima EVT is applicable
2. Breakpoint three controls whether or not link two will be used. If it is used, breakpoints one and two are used again to determine whether or not the operator desires linearized univariate EVT plots of the respective channels
3. Breakpoint four controls whether or not link three will be used, i.e., whether the program will proceed to compute bivariate statistics, or will terminate execution at the end of univariate calculations. If link three is used, breakpoints three and four are used again to offer further options on completion of bivariate calculations. Breakpoint three gives the option of changing the value of the variable a used in Eq. (8) and breakpoint four, the option of changing the thresholds of the two channels. These last options may be used either individually or simultaneously. If any one of the options is used, the program computes bivariate statistics based on the changed inputs. If neither option is used, execution is terminated and control is transferred back to the top of the program (link one).

All program inputs must be typed according to the format specifications of the operational directions mentioned above. Link one requires that the operator input, via the typewriter, the following variables in this order:

1. The test number
2. The number of groups
3. The number of samples per group
4. The data channel threshold
5. The synchronization channel threshold
6. The univariate maximum likelihood fit error limit

Link two takes all its inputs from link one. Link three initially requires the following additional typewritten inputs in this order:

1. The total number of iterations desired for the bivariate maximum likelihood fit
2. The number of iterations to be performed before bivariate statistics are computed, e.g., if the first input = 8 and this input = 2, then 8 iterations will take place, but bivariate statistics will be calculated and printed after each second iteration
3. Value of the variable a used to compute the initial estimate of the parameter c {Eq. (8)}
4. The bivariate maximum likelihood fit error limit

It should be noted that the various tests which we executed were stored on magnetic tape with format fixed by convention, so that the data were input via READ TAPE commands. This input procedure would need to be changed if data were used which were recorded under any other convention.

APPENDIX B

Data-Processing Program Nomenclature, Simplified Flow Diagram, and Listing

This appendix contains a table of nomenclature for the data-processing program (Table B-1), a simplified flow diagram of the program (Fig. B-1), and a complete listing of the program segmented into three links, each having its respective subroutines (Table B-2). Since various portions of this program were written at different times, the nomenclature varies from link to link. In an attempt to alleviate any confusion which might exist, the table of nomenclature lists all important program variables according to the links in which they are used, and enumerates any equivalent names that might be used to represent the same variables throughout the rest of the program. This table also states restrictions which must be placed on certain variables for successful execution of the program. It references specific variables according to sections, appendixes or equations of this report which might clarify their usage.

Table B-1. Nomenclature of the data-processing program

Variable	Equivalent names	Definition	Restrictions	References
		Link 1		
ADC1		Storage array for data-channel data		
ADC2		Storage array for synchronization-channel data		
MAX1	IDEXT	Array for data-channel extremes		
MAX2	ISEXT	Array for synchronization-channel extremes		
ALPHA1	ALPHAD	Parameter alpha for data channel		
U1	UD	Parameter u for data channel		
ALPHA2	ALPHAS	Parameter alpha for synchronization channel		
U2	US	Parameter u for synchronization channel		
T1	TD	Data-channel threshold		
T2	TS	Synchronization-channel threshold		
ITN		Test number		
NG	NQ	Number of groups (extremes)	$0 < NG \leq 700$	
NDP	NDS	Number of points/group	$0 < NDP$	
XMEAN		Mean of data-channel data and also of extremes of the data		Section X
SX		Standard deviation of data-channel data and also of extremes of the data		Section X
YMEAN		Mean of synchronization-channel data and also of extremes of the data		Section X
SY		Standard deviation of synchronization-channel data and also of extremes of the data		Section X
EMEAN		Expected mean		Eq. (13a)
SIGMA		Expected standard deviation		Eq. (13b)
CC		Correlation coefficient		Section X
PER		Classical probability of an error		Appendix A
POUT		Classical probability of an out-of-lock		Appendix A
PNEIN		Classical probability of no error and an in-lock		Appendix A

Table B-1. (Cont'd)

Variable	Equivalent names	Definition	Restrictions	References
		Link 1 (Cont'd)		
PNEOUT		Classical probability of no error and an out-of-lock		Appendix A
PEIN		Classical probability of an error and an in-lock		Appendix A
PEOUT		Classical probability of an error and an out-of-lock		Appendix A
SNR		Signal-to-noise ratio		Appendix A
YT1	YTD	Data-channel reduced variate at threshold		Eq. (15b)
YT2	YTS	Synchronization-channel reduced variate at threshold		
PCT1		Data-channel cumulative probability at threshold		Eq. (15a)
PCT2		Synchronization-channel cumulative probability at threshold		
PCT1NDP		Predicted bit-error rate for data channel		Section VI
PCT2NDP		Predicted out-of-lock rate for synchronization channel		Section VI
ERROR		Error limit for univariate maximum-likelihood fit	ERROR $>$ 0	Section X
		Subroutine UMAXLIK		
F		Univariate maximum-likelihood equation		Eq. (18a)
G		Univariate maximum-likelihood equation		Eq. (18b)
FI		Partial derivative of F with respect to α_Λ and then α_Ω		Section X
GI		Partial derivative of G with respect to Λ_0 and then Ω_0		Section X.
DFDN		Mixed partial derivative of F and G with respect to α_Λ and Λ_0 and then α_Ω and Ω_0		Section X
		Subroutine CONFINT		
Z		Array containing the quantiles of order 99, 95, 90, 80, and 70 of the unit variance normal distribution for computation of confidence intervals		Section X
HOLD		Variance of the reduced variate at threshold		Eq. (4)
BETA1		Upper confidence limit		Section X
BETA2		Lower confidence limit		Section X
IPCENT		Percent confidence		

Table B-1. (Cont'd)

Variable	Equivalent names	Definition	Restrictions	References
		Link 2		
MATRIX		Storage array to order channel extremes and to store coordinates to be plotted		
XI		Scale factor		Section X
MAX		Reduced variate at threshold		
MIN		Scaled value of threshold		
I		A point of regression equation		Appendix A
J		A point of regression equation		Appendix A
		Subroutine ORDER		
IARRAY		Unordered extremes		
JARRAY		Ordered extremes		
HOLD		Plotting position with respect to non-linear cumulative probability scale		Appendix A
		Link 3		
NIT		Number of iterations for bivariate maximum-likelihood fit	$0 < \text{NIT}$	Appendix A
NBEVT		Number of iterations to occur before computation of bivariate statistics	$0 < \text{NBEVT} \leq \text{NIT}$	Appendix A
A		Strip estimator used to compute initial value of c	$1.5 \leq A \leq 2.0$	Section VIII
C		Bivariate EVT parameter c		Section VIII
DUA		Derivative of $1\text{-}c\ \text{sech}^2\left(\frac{a}{2}\right)$ with respect to a		Section VIII
F		Normalized data-channel extremes		Section VIII
G		Normalized synchronization-channel extremes		Section VIII
ERROR		Error limit for bivariate maximum-likelihood fit	$\text{ERROR} > 0$	Section X
COUNT		Number of times normalized variables fall within strip		Section VIII
II		Bivariate maximum-likelihood fit iteration number		
X1 X2 X3		Varied values of bivariate maximum-likelihood estimators		Section X
Y1 Y2 Y3		Corresponding values of the bivariate-likelihood function		Section X
VERTEX		Vertex of parabola fitted to the points (X1,Y1), (X2,Y2), (X3,Y3)		Section X

Table B-1. (Cont'd)

Variable	Equivalent names	Definition	Restrictions	References
Subroutine BEVT				
P		Probability of a correct bit being received and accepted		Section VIII
Q		Probability of a correct bit being received and rejected		Section VIII
R		Probability of an incorrect bit being received and accepted		Section VIII
S		Probability of an incorrect bit being received and rejected		Section VIII
PP		Probability of a correct command of length *NDP* being received and accepted		Section VIII
QQ		Probability of a correct command of length *NDP* being received and rejected		Section VIII
RR		Probability of an incorrect command of length *NDP* being received and accepted		Section VIII
SS		Probability of an incorrect command of length *NDP* being received and rejected		Section VIII
SERIES		Value of the NDP^{th} root of cumulative probabilities as computed by series expansion		Section X
VARC		Variance of parameter c		Eq. (10)
GU		Probability of a correct bit		Section VIII
GV		Probability of an in-lock on any one bit		Section VIII
Subroutine BMAXLIK				
VARC		Parameter c		Section X
U		$\{\alpha_\Lambda(X - u_\Lambda) - \alpha_\Omega(Y - u_\Omega)\}/2.0$		Section X
SECH2		$\mathrm{Sech}^2\, u$		Section X
TANH		$\mathrm{Tanh}\, u$		Section X
WU		$g(u)$		Section X
WU1		$dg(u)/du$		Section X
WU2		$d^2g(u)/du^2$		Section X
EX		$\Phi(x)$		Section X
EY		$\Phi(y)$		Section X
PROD		Value of bivariate maximum-likelihood function		Eq. (20)
Subroutine PARAFIT				
(P1X,P1Y) (P2X,P2Y) (P3X,P3Y)		Translated coordinates for parabola fit		Section X
AA		Coefficient *A* of parabola equation		Section X
BB		Coefficient *B* of parabola equation		Section X

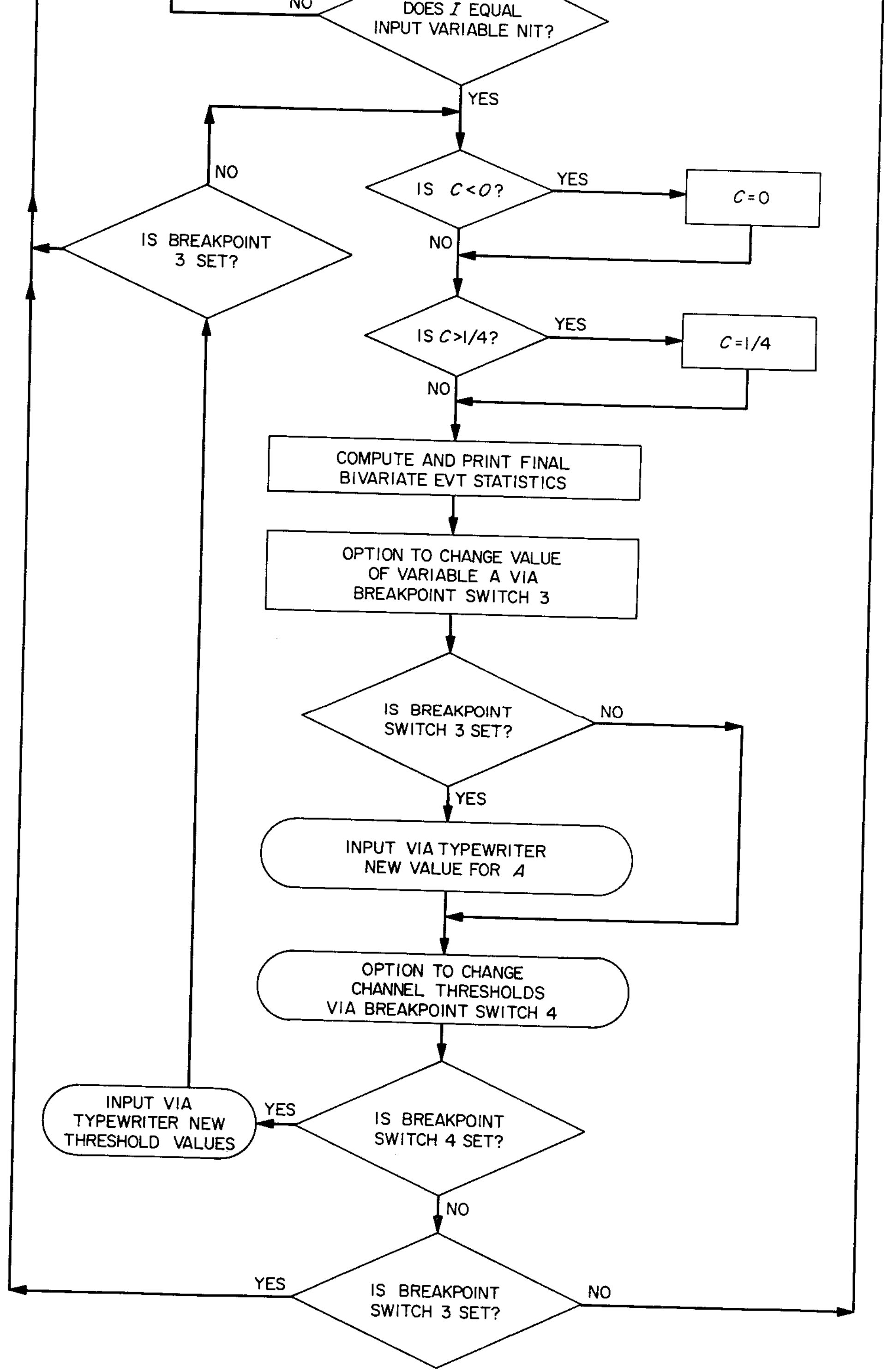
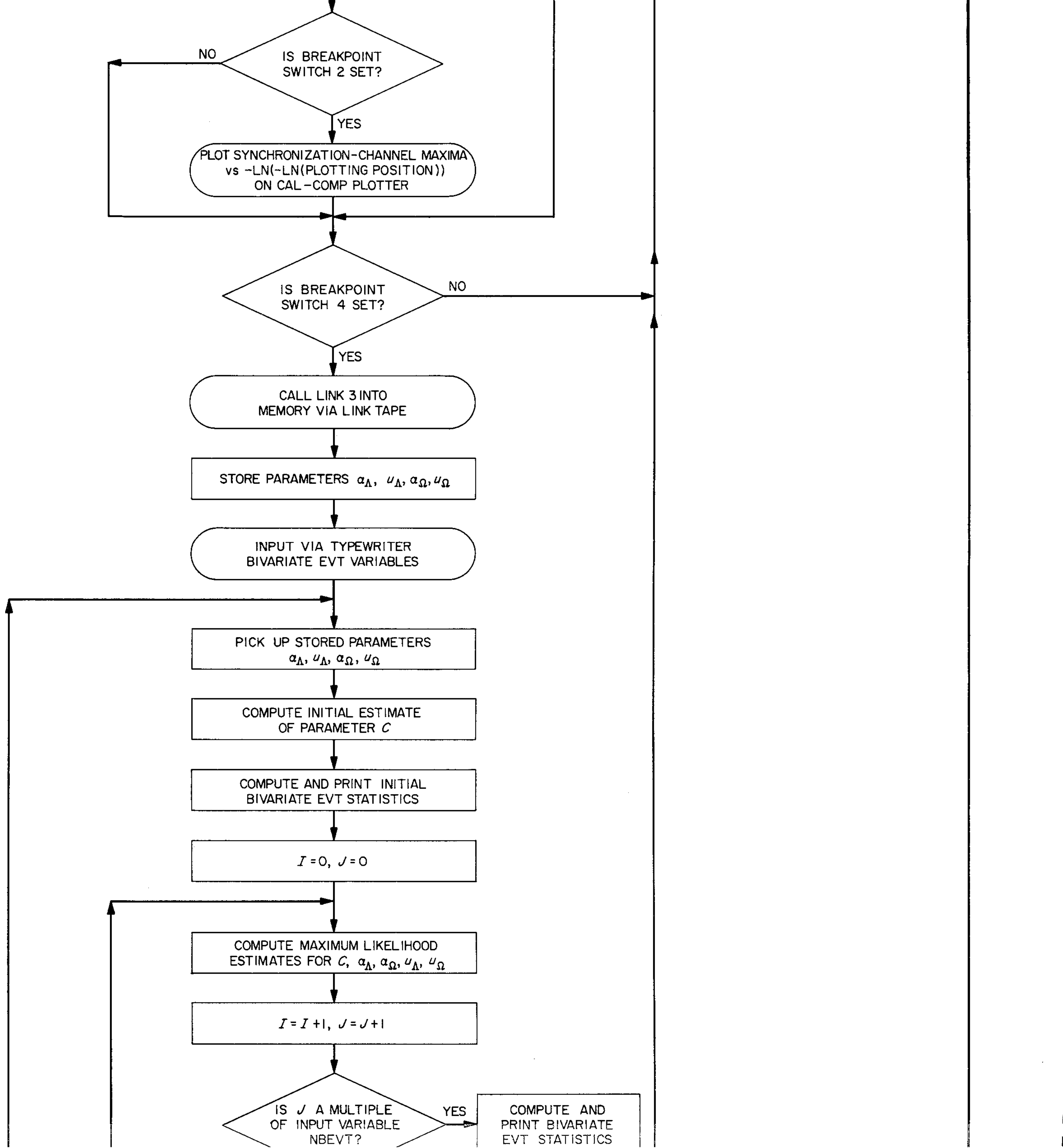
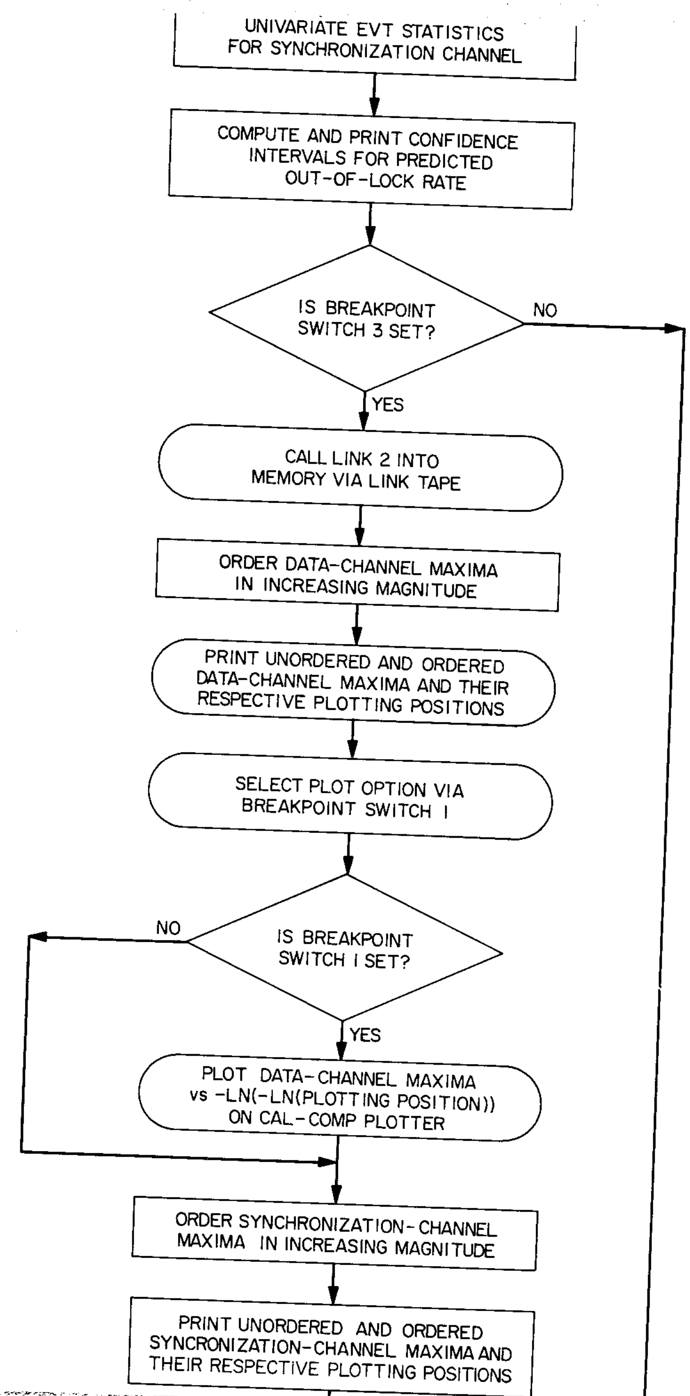
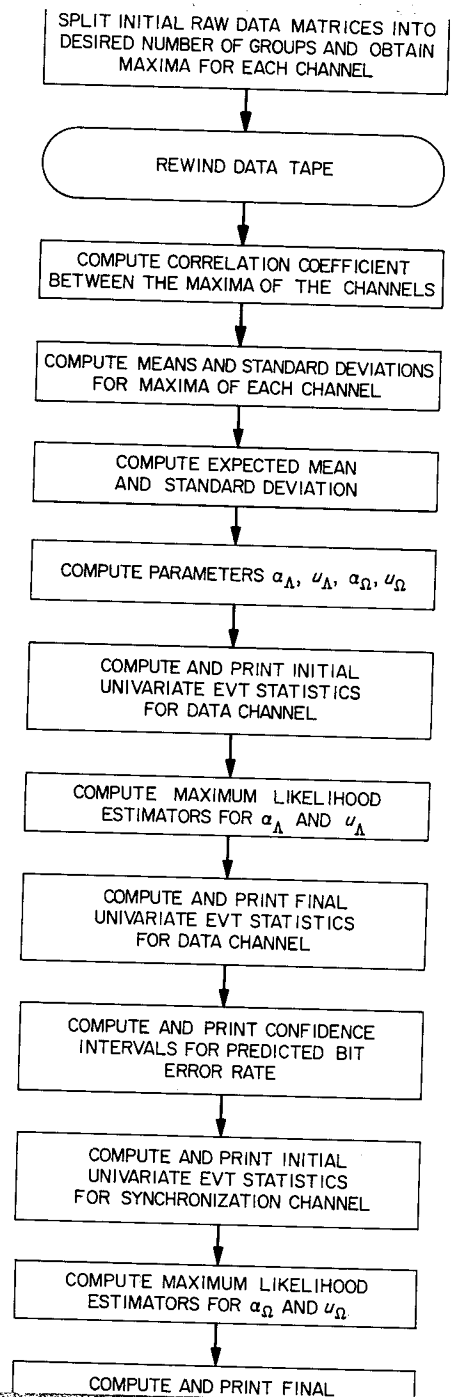


Fig. B-1. Simplified flow diagram for data-processing program


IS BREAKPOINT SWITCH 2 SET?
NO
YES
PLOT SYNCHRONIZATION-CHANNEL MAXIMA vs -LN(-LN(PLOTTING POSITION)) ON CAL-COMP PLOTTER
IS BREAKPOINT SWITCH 4 SET?
NO
YES
CALL LINK 3 INTO MEMORY VIA LINK TAPE
STORE PARAMETERS α_Λ, u_Λ, α_Ω, u_Ω
INPUT VIA TYPEWRITER BIVARIATE EVT VARIABLES
PICK UP STORED PARAMETERS α_Λ, u_Λ, α_Ω, u_Ω
COMPUTE INITIAL ESTIMATE OF PARAMETER C
COMPUTE AND PRINT INITIAL BIVARIATE EVT STATISTICS
$I=0$, $J=0$
COMPUTE MAXIMUM LIKELIHOOD ESTIMATES FOR C, α_Λ, α_Ω, u_Λ, u_Ω
$I=I+1$, $J=J+1$
IS J A MULTIPLE OF INPUT VARIABLE NBEVT?
YES
COMPUTE AND PRINT BIVARIATE EVT STATISTICS


UNIVARIATE EVT STATISTICS FOR SYNCHRONIZATION CHANNEL
COMPUTE AND PRINT CONFIDENCE INTERVALS FOR PREDICTED OUT-OF-LOCK RATE
IS BREAKPOINT SWITCH 3 SET?
NO
YES
CALL LINK 2 INTO MEMORY VIA LINK TAPE
ORDER DATA-CHANNEL MAXIMA IN INCREASING MAGNITUDE
PRINT UNORDERED AND ORDERED DATA-CHANNEL MAXIMA AND THEIR RESPECTIVE PLOTTING POSITIONS
SELECT PLOT OPTION VIA BREAKPOINT SWITCH I
IS BREAKPOINT SWITCH I SET?
NO
YES
PLOT DATA-CHANNEL MAXIMA vs -LN(-LN(PLOTTING POSITION)) ON CAL-COMP PLOTTER
ORDER SYNCHRONIZATION-CHANNEL MAXIMA IN INCREASING MAGNITUDE
PRINT UNORDERED AND ORDERED SYNCRONIZATION-CHANNEL MAXIMA AND THEIR RESPECTIVE PLOTTING POSITIONS


SPLIT INITIAL RAW DATA MATRICES INTO DESIRED NUMBER OF GROUPS AND OBTAIN MAXIMA FOR EACH CHANNEL
REWIND DATA TAPE
COMPUTE CORRELATION COEFFICIENT BETWEEN THE MAXIMA OF THE CHANNELS
COMPUTE MEANS AND STANDARD DEVIATIONS FOR MAXIMA OF EACH CHANNEL
COMPUTE EXPECTED MEAN AND STANDARD DEVIATION
COMPUTE PARAMETERS α_Λ, u_Λ, α_Ω, u_Ω
COMPUTE AND PRINT INITIAL UNIVARIATE EVT STATISTICS FOR DATA CHANNEL
COMPUTE MAXIMUM LIKELIHOOD ESTIMATORS FOR α_Λ AND u_Λ
COMPUTE AND PRINT FINAL UNIVARIATE EVT STATISTICS FOR DATA CHANNEL
COMPUTE AND PRINT CONFIDENCE INTERVALS FOR PREDICTED BIT ERROR RATE
COMPUTE AND PRINT INITIAL UNIVARIATE EVT STATISTICS FOR SYNCHRONIZATION CHANNEL
COMPUTE MAXIMUM LIKELIHOOD ESTIMATORS FOR α_Ω AND u_Ω
COMPUTE AND PRINT FINAL

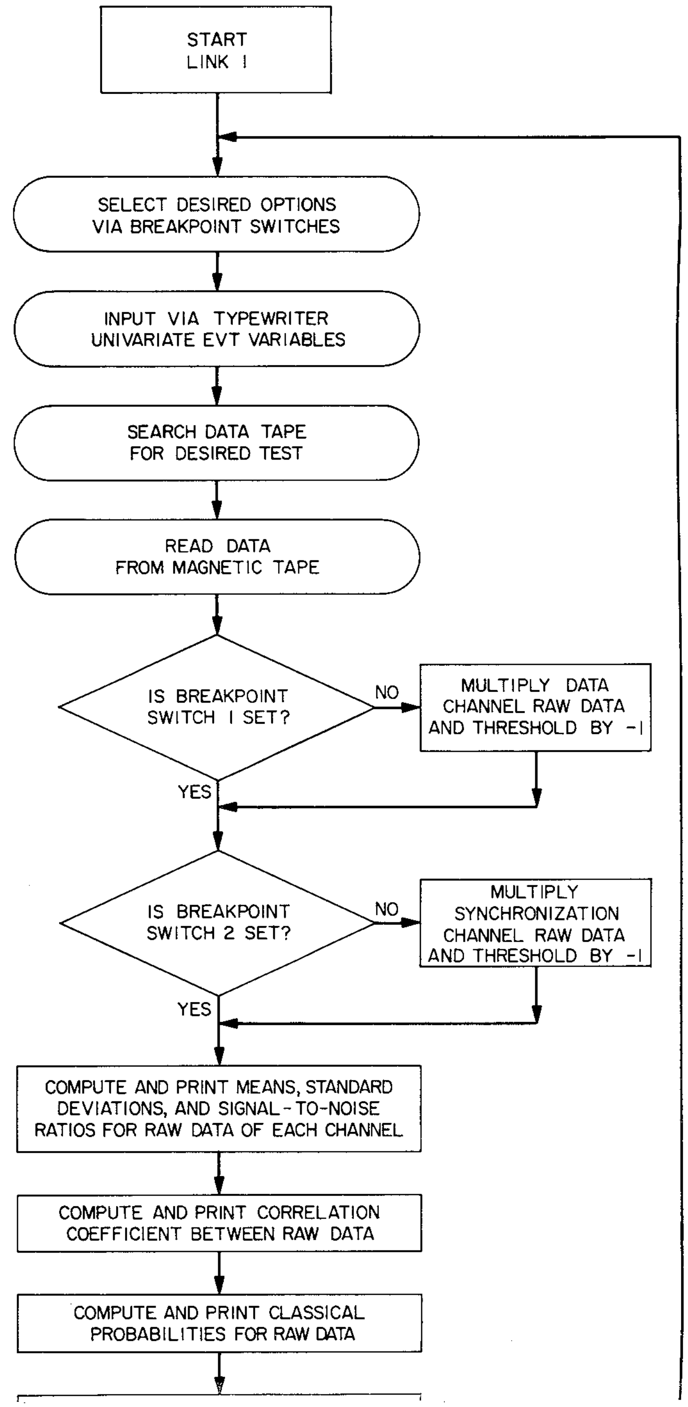

START
LINK 1
SELECT DESIRED OPTIONS VIA BREAKPOINT SWITCHES
INPUT VIA TYPEWRITER UNIVARIATE EVT VARIABLES
SEARCH DATA TAPE FOR DESIRED TEST
READ DATA FROM MAGNETIC TAPE
IS BREAKPOINT SWITCH 1 SET?
NO
MULTIPLY DATA CHANNEL RAW DATA AND THRESHOLD BY -1
YES
IS BREAKPOINT SWITCH 2 SET?
NO
MULTIPLY SYNCHRONIZATION CHANNEL RAW DATA AND THRESHOLD BY -1
YES
COMPUTE AND PRINT MEANS, STANDARD DEVIATIONS, AND SIGNAL-TO-NOISE RATIOS FOR RAW DATA OF EACH CHANNEL
COMPUTE AND PRINT CORRELATION COEFFICIENT BETWEEN RAW DATA
COMPUTE AND PRINT CLASSICAL PROBABILITIES FOR RAW DATA

Table B-2. Listing of the data-processing program

```
C MASTER BIVARIATE EXTREME VALUE PROGRAM FOR TWO CHANNEL DATA...........
C.........................SANDRA LURIE...................................
C.........................JULY  1966.....................................
C LINK(1) OF THE PROGRAM.................................................
C USES FORTRAN SUBROUTINES UMAXLIK (UNIVARIATE MAXIMUM LIKELIHOOD FIT)..
C                          CONFINT (CONFIDENCE INTERVALS)...............
C UNIVARIATE EXTREME VALUE CALCULATIONS..................................
      DIMENSION IADC1(500),IADC2(500),MAX1(700),MAX2(700)
      COMMON MAX1,MAX2,ALPHA1,ALPHA2,U1,U2,NG,NDP,T1,T2,ITN,ERROR
C TYPES OPERATIONAL DIRECTIONS AND ACCEPTS TYPEWRITER INPUTS............
    1 TYPE 900
      PAUSE
      TYPE 930
      TYPE 901
    2 ACCEPT 902,ITN,NG,NDP,T1,T2,ERROR
C SEARCHES DATA TAPE FOR DESIRED TEST....................................
    3 READ TAPE 1,L,I,I,I,ITAB,TEMP,TEMP,TEMP,TEMP,TEMP,TEMP
      IF(L-ITN)4,7,6
    4 DO 5 J=1,ITAB
    5 READ TAPE 1
      GO TO 3
    6 REWIND 1
      GO TO 3
C CALCULATES THE MEANS, DEVIATIONS, SIGNAL TO NOISE RATIOS, AND THE
C   CORRELATION COEFFICIENT OF THE TOTAL SAMPLE.........................
C COMPUTES CLASSICAL PROBABILITIES.......................................
C CONVERTS ALL DATA SO THAT MAXIMA EVT IS USED..........................
C SPLITS THE INITIAL DATA MATRIX INTO NG GROUPS EACH OF NDP SAMPLES.....
C FINDS GROUP EXTREMES (MAXIMA) FOR EACH CHANNEL AND THEIR CORRELATION
C   COEFFICIENT..........................................................
    7 XMEAN=0.
      YMEAN=0.
      SX=0.
      SY=0.
      SUMXY=0.
      PER=0.
      POUT=0.
      PNEIN=0.
```

Table B-2. (Cont'd)

```
      PNEØUT=0.
      PEIN=0.
      PEØUT=0.
      JTAB=0
      LARGE1=-1000000
      LARGE2=LARGE1
      READ TAPE 1,(IADC1(J),J=1,500)
      READ TAPE 1,(IADC2(J),J=1,500)
      JTAB=JTAB+2
      IF(SENSE SWITCH 1)11,9
    9 T1=-T1
      DØ 10 J=1,500
   10 IADC1(J)=-IADC1(J)
   11 IF(SENSE SWITCH 2)14,12
   12 T2=-T2
      DØ 13 J=1,500
   13 IADC2(J)=-IADC2(J)
   14 II=1
      K=1
   15 III=0
   16 III=III+1
      XMEAN=XMEAN+IADC1(II)
      SX=SX+(IADC1(II))**2
      YMEAN=YMEAN+IADC2(II)
      SY=SY+(IADC2(II))**2
      SUMXY=SUMXY+(IADC1(II))*(IADC2(II))
      IF(IADC1(II)-T1)41,41,31
   31 PER=PER+1.0
      IF(IADC2(II)-T2)33,33,32
   32 PØUT=PØUT+1.0
      PEØUT=PEØUT+1.0
      GØ TØ 38
   33 PEIN=PEIN+1.0
      GØ TØ 38
   41 IF(IADC2(II)-T2)37,37,34
   34 PØUT=PØUT+1.0
      PNEØUT=PNEØUT+1.0
      GØ TØ 38
   37 PNEIN=PNEIN+1.0
```

Table B-2. (Cont'd)

```
38 IF(IADC1(II)-LARGE1)18,18,17
17 LARGE1=IADC1(II)
18 IF(IADC2(II)-LARGE2)20,20,19
19 LARGE2=IADC2(II)
20 IF(III-NDP)21,30,30
21 II=II+1
   IF(II-500)16,16,22
22 READ TAPE 1,(IADC1(J),J=1,500)
   READ TAPE 1,(IADC2(J),J=1,500)
   JTAB=JTAB+2
85 IF(SENSE SWITCH 1)25,23
23 DO 24 J=1,500
24 IADC1(J)=-IADC1(J)
25 IF(SENSE SWITCH 2)28,26
26 DO 27 J=1,500
27 IADC2(J)=-IADC2(J)
28 II=1
   GO TO 16
30 MAX1(K)=LARGE1
   MAX2(K)=LARGE2
   LARGE1=-1000000
   LARGE2=LARGE1
   III=0
   K=K+1
   IF(K-NG)21,21,35
35 DO 36 I=1,JTAB+1
36 BACKSPACE 1
   K=NG*NDP
   CC=(K*SUMXY-XMEAN*YMEAN)/(SQRT((K*SX-XMEAN**2)*(K*SY-YMEAN**2)))
   XMEAN=XMEAN/K
   YMEAN=YMEAN/K
   SX=SQRT((SX/K)-XMEAN**2)
   SY=SQRT((SY/K)-YMEAN**2)
   PRINT 903,ITN
   SNR=XMEAN/SX
   DB=0.4342945*20.0*(ALOG(ABSF(SNR)))
   PRINT 904
   PRINT 906,K,XMEAN,SX,SNR,DB
   SNR=YMEAN/SY
```

Table B-2. (Cont'd)

```
      DB=0.4342945*20.0*(ALOG(ABSF(SNR)))
      PRINT 905
      PRINT 906,K,YMEAN,SY,SNR,DB
      PRINT 925,K,CC
      XMEAN=0.
      SX=0.
      YMEAN=0.
      SY=0.
      SUMXY=0.
      DO 50 I=1,NG
      XMEAN=XMEAN+MAX1(I)
      SX=SX+(MAX1(I))**2
      YMEAN=YMEAN+MAX2(I)
      SY=SY+(MAX2(I))**2
   50 SUMXY=SUMXY+(MAX1(I))*(MAX2(I))
      CC=(NG*SUMXY-XMEAN*YMEAN)/(SQRT((NG*SX-XMEAN**2)*(NG*SY-YMEAN**2))
     1)
      PRINT 926,NG,NDP,CC
      HOLD=PER
      PER=PER/K
      PRINT 903,ITN
      PRINT 914,PER
      PRINT 920,HOLD
      HOLD=POUT
      POUT=POUT/K
      PRINT 915,POUT
      PRINT 920,HOLD
      HOLD=PNEIN
      PNEIN=PNEIN/K
      PRINT 916,PNEIN
      PRINT 920,HOLD
      HOLD=PNEOUT
      PNEOUT=PNEOUT/K
      PRINT 917,PNEOUT
      PRINT 920,HOLD
      HOLD=PEIN
      PEIN=PEIN/K
      PRINT 918,PEIN
      PRINT 920,HOLD
```

Table B-2. (Cont'd)

```
      HOLD=PEOUT
      PEOUT=PEOUT/K
      PRINT 919,PEOUT
      PRINT 920,HOLD
C COMPUTES MARGINAL EVT DISTRIBUTIONS FOR EACH CHANNEL..................
C COMPUTES MEANS AND DEVIATIONS OF CHANNEL EXTREMES.....................
  100 XMEAN=XMEAN/NG
      YMEAN=YMEAN/NG
      SX=SQRT((SX/NG)-XMEAN**2)
      SY=SQRT((SY/NG)-YMEAN**2)
C COMPUTES EXPECTED MEAN AND DEVIATION..................................
      EMEAN=0.
      SIGMA=0.
      DO 110 I=1,NG
      HOLD=-ALOG(-ALOG(I/(NG+1.0)))
      EMEAN=EMEAN+HOLD
  110 SIGMA=SIGMA+HOLD**2
      EMEAN=EMEAN/NG
      SIGMA=SQRT((SIGMA/NG)-EMEAN**2)
C COMPUTES FOR EACH CHANNEL THE LINEARIZATION PARAMETERS (ALPHA AND U),
C   THE INITIAL REDUCED VARIATE AND CUMULATIVE PROBABILITY AT THRESHOLD
C   AND THE INITIAL ESTIMATE OF THE PREDICTED ERROR RATE...............
      ALPHA1=SIGMA/SX
      ALPHA2=SIGMA/SY
      U1=XMEAN-EMEAN/ALPHA1
      U2=YMEAN-EMEAN/ALPHA2
      YT1=(T1-U1)*ALPHA1
      YT2=(T2-U2)*ALPHA2
      PCT1=EXPF(-EXPF(-YT1))
      PCT1NDP=1-EXPF((ALOG(PCT1))/NDP)
      PCT2=EXPF(-EXPF(-YT2))
      PCT2NDP=1-EXPF((ALOG(PCT2))/NDP)
C COMPUTES THE UNIVARIATE MAXIMUM LIKELIHOOD FIT VIA SUBROUTINE UMAXLIK.
C COMPUTES THE UNIVARIATE EVT STATISTICS................................
C COMPUTES CONFIDENCE INTERVALS VIA SUBROUTINE CONFINT.................
      PRINT 903,ITN
      PRINT 907,NG,NDP,ERROR
      PRINT 904
      PRINT 908
```

Table B-2. (Cont'd)

```
      HOLD=1/ALPHA1
      PRINT 910,T1,ALPHA1,U1,U1,HOLD,YT1,PCT1NDP,PCT1
      CALL UMAXLIK(MAX1,T1,ALPHA1,NG,YT1,XMEAN,ERROR)
      PCT1=EXPF(-EXPF(-YT1))
      PCT1NDP=1-EXPF((ALOG(PCT1))/NDP)
      U1=T1-(YT1/ALPHA1)
      PRINT 903,ITN
      PRINT 904
      PRINT 909
      TEMP=1/ALPHA1
      PRINT 910,T1,ALPHA1,U1,U1,TEMP,YT1,PCT1NDP,PCT1
  125 CALL CONFINT(YT1,NG,NDP)
      PRINT 903,ITN
      PRINT 907,NG,NDP,ERROR
      PRINT 905
      PRINT 908
      HOLD=1/ALPHA2
      PRINT 910,T2,ALPHA2,U2,U2,HOLD,YT2,PCT2NDP,PCT2
      CALL UMAXLIK(MAX2,T2,ALPHA2,NG,YT2,YMEAN,ERROR)
      PCT2=EXPF(-EXPF(-YT2))
      PCT2NDP=1-EXPF((ALOG(PCT2))/NDP)
      U2=T2-(YT2/ALPHA2)
      PRINT 903,ITN
      PRINT 905
      PRINT 909
      TEMP=1/ALPHA2
      PRINT 910,T2,ALPHA2,U2,U2,TEMP,YT2,PCT2NDP,PCT2
  127 CALL CONFINT(YT2,NG,NDP)
      IF(SENSE SWITCH 3)130,131
  130 CALL LINK(2)
  131 IF(SENSE SWITCH 4)132,1
  132 CALL LINK(3)
  900 FORMAT(/$SET BP1 IF LOOKING FOR A MAXIMUM FOR ADC-1.$/3X,$RESET BP
     11 IF LOOKING FOR A MINIMUM.$/$SET BP2 IF LOOKING FOR A MAXIMUM FOR
     2 ADC-2.$/3X,$RESET BP2 IF LOOKING FOR A MINIMUM.$/$SET BP3 FOR PRI
     3NTOUT OF CHANNEL EXTREMES AND OPTION TO OBTAIN A GUMBEL PLOT.$/$SE
     4T BP4 FOR BIVARIATE ANALYSIS.$/$CLEAR HALT.$/)
  901 FORMAT(/$TYPE IN FORMAT (3I4,3F12.5)$,/$  ITN--TEST NO.$/$  NG--NO
     1. OF GROUPS$/$  NDP--NO. OF SAMPLES/GROUP$/$  T1,T2--ADC1,ADC2 THR
```

Table B-2. (Cont'd)

```
   2ESHOLDS$/$  ERROR--ERROR FOR UNIVARIATE MAXIMUM LIKELIHOOD FITS//)
902 FORMAT(3I4,3F12.5)
903 FORMAT(1H1,38X,$UNIVARIATE EXTREME VALUES/46X,$TEST$,I4//)
904 FORMAT(/$FOR ADC-1$/)
905 FORMAT(/$FOR ADC-2$/)
906 FORMAT($  BASED ON THE TOTAL SAMPLE SIZE = $,I6,$ SAMPLES$/5X,$MEA
   1N = $,E20.12/5X,$STANDARD DEVIATION = $,E20.12/5X,$SIGNAL TO NOISE
   2 RATIO = $,E20.12,$  =  $,E20.12,$  DB.$//)
907 FORMAT(/$THERE ARE $,I5,$ GROUPS OF $,I5, $ SAMPLES EACH.$//$ERROR
   1 FOR UNIVARIATE MAXIMUM LIKELIHOOD FIT = $,E20.12/)
908 FORMAT($  VALUES BEFORE UNIVARIATE MAXIMUM LIKELIHOOD FIT$//)
909 FORMAT($  VALUES AFTER UNIVARIATE MAXIMUM LIKELIHOOD FIT$)
910 FORMAT(/5X $THRESHOLD = $,E20.12//5X,$ALPHA = $,E20.12/5X,$U = $,E
   120.12//5X,$THE REGRESSION EQUATION = $,F20.7,$ + $,F12.7,$ Y$/5X$R
   2EDUCED VARIATE AT TRIGGER LEVEL =$E20.12//5X,$PREDICTED BIT ERROR
   3RATE = $,E20.12/5X,$CUMULATIVE PROBABILITY AT TRIGGER LEVEL = $,
   4E20.12)
914 FORMAT(////$CLASSICAL PROBABILITIES$//$PROBABILITY OF A BIT ERROR
   1= $,E20.12)
915 FORMAT($PROBABILITY OF AN OUT OF LOCK = $,E20.12)
916 FORMAT($PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED
   1  = $,E20.12)
917 FORMAT($PROBABILITY OF A CORRECT BIT BEING RECEIVED AND REJECTED
   1  = $,E20.12)
918 FORMAT($PROBABILITY OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTE
   1D = $,E20.12)
919 FORMAT($PROBABILITY OF AN INCORRECT BIT BEING RECEIVED AND REJECTE
   1D = $,E20.12)
920 FORMAT($  NUMBER OF OCCURENCES = $,F12.1/)
925 FORMAT(///$BASED ON $I6$  RAW DATA SAMPLES, THE CORRELATION COEFFI
   1CIENT = $,E20.12)
926 FORMAT(/$BASED ON EXTREMES OF $I4$  GROUPS OF $I4$  SAMPLES, THE C
   1ORRELATION COEFFICIENT = $,E20.12)
930 FORMAT(/$IF AN ERROR IS MADE WHILE TYPING INPUTS, DO THE FOLLOWING
   1$/$  1.  PUT RUN-IDLE-STEP (R-I-S) SWITCH TO IDLE$/$  2.  SET REGI
   2STER KNOB TO C$/$  3.  PUSH START$/$  4.  FILL REGISTER DISPLAY WI
   3TH A  BRU 03522  COMMAND,$/8X$THAT IS, WITH THE OCTAL NUMBER 00103
   4522$/$  5.  PUT R-I-S SWITCH TO RUN$/$  6.  RETYPE INPUTS$//)
    END
```

Table B-2. (Cont'd)

```
      SUBROUTINE UMAXLIK(IARRAY,T,ALPHA,NQ,YT,ZMEAN,ERR)
C COMPUTES UNIVARIATE MAXIMUM LIKELIHOOD FIT............................
C OBTAINS MAXIMUM LIKELIHOOD ESTIMATORS OF ALPHA AND THE REDUCED VARIATE
C   AT THRESHOLD BY SOLVING
C          D(LOG L)                AND            D(LOG L)
C   1.   ------------   =  0             2.   ------------   =  0
C          DALPHA                                    DU
C   WHERE
C       LOG L = NG*LOG(ALPHA)-NG*ALPHA*(XMEAN-THRESHOLD)-NG*YT
C                   -SUMMATION(EXP(-(ALPHA*(X(I)-THRESHOLD)+YT)))
C   AND I = 1,2,-----,NG.............................................
C USES NEWTON-RAPHSON METHOD FOR SYSTEMS OF EQUATIONS...................
      DIMENSION IARRAY(700)
  199 F=NQ/ALPHA-NQ*(ZMEAN-T)
      FI=-NQ/(ALPHA**2)
      DFDN=-F
      DO 200 I=1,NQ
      HOLD=(IARRAY(I)-T)*(EXPF(-1.0*(ALPHA*(IARRAY(I)-T)+YT)))
      F=F+HOLD
  200 FI=FI-(IARRAY(I)-T)*HOLD
      DFDN=-F-DFDN
  205 G=-NQ
      GI=0.
      DO 210 I=1,NQ
      HOLD=EXPF(-1.0*(ALPHA*(IARRAY(I)-T)+YT))
      G=G+HOLD
  210 GI=GI-HOLD
      HOLD=FI*GI-DFDN**2
      ALPHA=ALPHA-(GI*F-DFDN*G)/HOLD
      YT=YT-(-DFDN*F+FI*G)/HOLD
      F=NQ/ALPHA-NQ*(ZMEAN-T)
      G=-NQ
      DO 215 I=1,NQ
      HOLD=EXPF(-1.0*(ALPHA*(IARRAY(I)-T)+YT))
      F=F+(IARRAY(I)-T)*HOLD
  215 G=G+HOLD
      IF(ABSF(F)-ERR)220,199,199
  220 IF(ABSF(G)-ERR)225,199,199
  225 RETURN
      END
```

Table B-2. (Cont'd)

```
      SUBROUTINE CONFINT(YT,NQ,NDS)
C COMPUTES TWO-SIDED CONFIDENCE INTERVALS FOR PREDICTED ERROR RATES.....
C COMPUTES 99, 95, 90, 80, AND 70 PERCENT CONFIDENCE INTERVALS..........
      DIMENSION Z(5)
      Z(1)=2.575991
      Z(2)=1.960101
      Z(3)=1.644731
      Z(4)=1.281561
      Z(5)=1.036435
      HOLD=SQRT((6/(NQ*9.869604))*((1-.57721566+YT)**2+9.869604/6.0))
      PRINT 960
      DO 350 I=1,5
      BETA1=1-EXPF((-EXPF(-YT-HOLD*Z(I)))/NDS)
      BETA2=1-EXPF((-EXPF(-YT+HOLD*Z(I)))/NDS)
      GO TO (341,342,343,344,345),I
  341 IPCENT=99
      GO TO 350
  342 IPCENT=95
      GO TO 350
  343 IPCENT=90
      GO TO 350
  344 IPCENT=80
      GO TO 350
  345 IPCENT=70
  350 PRINT 961,IPCENT,BETA1,BETA2
  960 FORMAT(///$PERCENT CONFIDENCE$, 23X,$CONFIDENCE INTERVAL FOR PREDI
     1CTED BIT ERROR RATE$/)
  961 FORMAT(I10,24X,E20.12,24X,E20.12)
      RETURN
      END
ΔEOF
```

Table B-2. (Cont'd)

```
C MASTER BIVARIATE EXTREME VALUE PROGRAM FOR TWO CHANNEL DATA...........
C.........................SANDRA LURIE..................................
C.........................JULY  1966....................................
C LINK(2) OF THE PROGRAM.................................................
C USES FORTRAN SUBROUTINE ORDER (ORDERS CHANNEL EXTREMES) AND
C       SYMBOL SUBROUTINE GRAPH (PRODUCES A GUMBEL PLOT ON THE PLOTTER).
C UNIVARIATE EXTREME VALUE CALCULATIONS..................................
      DIMENSION MAX1(700),MAX2(700),MATRIX(1402)
      COMMON MAX1,MAX2,ALPHA1,ALPHA2,U1,U2,NG,NDP,T1,T2,ITN,ERROR
C ORDERS ADC-1 EXTREMES IN INCREASING MAGNITUDE..........................
C PRINTS ADC-1 EXTREMES AND THEIR PLOTTING POSITIONS.....................
   50 PRINT 903,ITN
      PRINT 904
      CALL ORDER(MAX1,MATRIX,NG)
C OPTION TO OBTAIN ADC-1 GUMBEL PLOT.....................................
      TYPE 912
      TYPE 911
      PAUSE
      IF (SENSE SWITCH 1)49,60
C SCALES EXTREMES TO FIT EXTREME VALUE PROBABILITY PAPER.................
C CONVERTS PLOTTING POSITIONS TO REDUCED VARIATE SCALE...................
   49 ALPHA=ALPHA1
      U=U1
      YT=(T1-U1)*ALPHA1
      T=T1
      IHOLD=0
   51 DO 52 I=NG,1,-1
      K=2*I+1
   52 MATRIX(K)=-MATRIX(I)
      IF(MATRIX(2*NG+1)-T)56,57,57
   56 IF(MATRIX(3)-T)70,72,72
   70 HOLD=ABSF(T-MATRIX(2*NG+1))
      XI=1000.0/HOLD
      MATRIX(1)=T*XI
      GO TO 59
   72 HOLD=ABSF(MATRIX(2*NG+1)-(MATRIX(3)+5))
      GO TO 58
   57 HOLD=ABSF(T-(MATRIX(3)+5))
```

Table B-2. (Cont'd)

```
   58 XI=1000.0/HOLD
   54 MATRIX(1)=(MATRIX(3)+5)*XI
   59 MATRIX(2)=-300
      J=0
      DO 55 I=3,2*NG+2,2
      MATRIX(I)=XI*MATRIX(I)
      J=J+1
   55 MATRIX(I+1)=100*(-ALOG(-ALOG(J/(NG+1.0))))
      MAX=100*YT
      MIN=-XI*T
      I=-XI*U
      J=-(XI*(U+1.0/ALPHA))
      K=2*NG+2
      CALL GRAPH(K,MATRIX,MAX,MIN,I,J)
      IF(IHOLD-1)60,100,60
C ORDERS ADC-2 EXTREMES IN INCREASING MAGNITUDE........................
C PRINTS ADC-2 EXTREMES AND THEIR PLOTTING POSITIONS...................
   60 PRINT 903,ITN
      PRINT 905
      CALL ORDER(MAX2,MATRIX,NG)
C OPTION TO OBTAIN ADC-2 GUMBEL PLOT...................................
      TYPE 913
      TYPE 911
      PAUSE
      IF(SENSE SWITCH 2)61,100
   61 ALPHA=ALPHA2
      U=U2
      T=T2
      YT=(T2-U2)*ALPHA2
      IHOLD=1
      GO TO 51
  100 IF(SENSE SWITCH 4)101,102
  101 CALL LINK(3)
  102 CALL LINK(1)
  903 FORMAT(1H1,38X,$UNIVARIATE EXTREME VALUES$/46X,$TEST$,I4//)
  904 FORMAT(/$FOR ADC-1$/)
  905 FORMAT(/$FOR ADC-2$/)
  911 FORMAT($  IF SET, POSITION PLOTTER PEN AT BOTTOM RIGHT-HAND CORNER
     1 OF GRAPH PAPER$/$CLEAR HALT$/)
  912 FORMAT(/$SET BP1 FOR ADC-1 GUMBEL PLOT$)
  913 FORMAT(/$SET BP2 FOR ADC-2 GUMBEL PLOT$)
      END
```

Table B-2. (Cont'd)

```
      SUBROUTINE ORDER(IARRAY,JARRAY,NQ)
C ORDERS EXTREMES OF EACH CHANNEL IN INCREASING MAGNITUDE...............
C PRINTS UNORDERED AND ORDERED CHANNEL EXTREMES AND THEIR GUMBEL
C   PLOTTING POSITIONS..................................................
      DIMENSION IARRAY(700),JARRAY(1000)
      DO 75 J=1,NQ
   75 JARRAY(J)=IARRAY(J)
      DO 85 J=1,NQ-1
      MIN=JARRAY(J)
      DO 85 I=J+1,NQ
      IF(MIN-JARRAY(I))85,85,80
   80 MIN=JARRAY(I)
      JARRAY(I)=JARRAY(J)
      JARRAY(J)=MIN
   85 CONTINUE
      PRINT 950
      DO 90 J=1,NQ
      HOLD=J/(NQ+1.0)
   90 PRINT 951,J,IARRAY(J),JARRAY(J),HOLD
  950 FORMAT ($GROUP NUMBER$,6X,$UNORDERED EXTREMES$,10X,$ORDERED EXTREM
     1ES$,12X,$PLOTTING POSITION$/)
  951 FORMAT(I7,I22,I27,18X,E17.10)
      RETURN
      END
```

Table B-2. (Cont'd)

```
* GRAPHS A LINEARIZED EVT PLOT ON THE CAL-COMP PLOTTER..................
* PLOTTER PEN MUST BE POSITIONED IN THE BOTTOM RIGHT-HAND CORNER........
XSD     OPD      010000000
$GRAPH PZE
* STORES ADDRESSES OF THE SUBROUTINE PARAMETERS.........................
        BRM      201SYS
        XSD      NUM                 ADDRESS OF SAMPLE SIZE
        XSD      POINT               BEGINNING ADDRESS OF COORDINATE ARRAY
        XSD      RVT                 ADDRESS OF THE REDUCED VARIATE
        XSD      THRES               ADDRESS OF THE THRESHOLD
        XSD      LINE0               ADDRESS OF REGRESSION EQUATION POINT
        XSD      LINE1               ADDRESS OF REGRESSION EQUATION POINT
        BRM      202SYS
* PLOTS THE CHANNEL EXTREMES VS. THEIR PLOTTING POSITIONS WHICH HAVE
*   BEEN LINEARIZED TO THE REDUCED VARIATE SCALE........................
        LDX      =00040000
NEXT    LDA      *POINT+1
        STA      XHOLD               SAVES VALUE OF CHANNEL EXTREME
        BRX      $+1
        LDA      *POINT+1
        STA      YHOLD               SAVES VALUE OF PLOTTING POSITION
        BRX      $+1
* DETERMINES INCREMENT ALONG THE CHANNEL EXTREMES AXIS..................
* MOVES PEN ALONG CHANNEL EXTREMES AXIS.................................
        CLA
        STA      COUNT
        LDA      XHOLD
        SUB      *POINT+1
        STA      TEMP
        SKE      ZERO
        BRU      $+2
        BRU      B
A       EOM      00064
        MIW      PYUP
        EOM      14000
        SKS      21000
        BRU      $-1
        MIN      COUNT
```

Table B-2. (Cont'd)

```
        LDA     TEMP
        SKE     COUNT
        BRU     A
B       BRX     $+1
* DETERMINES INCREMENT ALONG THE REDUCED VARIATE AXIS...................
* MOVES PEN ALONG REDUCED VARIATE AXIS..................................
        CLA
        STA     COUNT
        LDA     *POINT+1
        SUB     YHOLD
        STA     TEMP
        SKE     ZERO
        BRU     C
        BRU     $+10
C       EOM     00064
        MIW     PXUP
        EOM     14000
        SKS     21000
        BRU     $-1
        MIN     COUNT
        LDA     TEMP
        SKE     COUNT
        BRU     C
        CLA
        STA     COUNT
        BRM     UPX                     BRANCH TO ROUTINE WHICH PLOTS A +
        BRU     E-2
* ROUTINE WHICH PLOTS COORDINATES BY USING THE MARK + ..................
UPX     PZE                             POSITIONS PEN FOR VERTICAL BAR OF +
D       EOM     00064
        MIW     PYUP
        EOM     14000
        SKS     21000
        BRU     $-1
        MIN     COUNT
        LDA     =5
        SKE     COUNT
        BRU     D
        BRR     UPX
```

Table B-2. (Cont'd)

```
         CLA
         STA      COUNT
E        EOM      00064              DRAWS VERTICAL BAR OF +
         MIW      MYDO
         EOM      14000
         SKS      21000
         BRU      $-1
         MIN      COUNT
         LDA      =10
         SKE      COUNT
         BRU      E
         CLA
         STA      COUNT
         BRM      UPX
         CLA
         STA      COUNT
         BRM      LEFTX
         BRU      G-2
LEFTX    PZE                         POSITIONS PEN FOR HORIZONTAL BAR OF +
F        EOM      00064
         MIW      MXUP
         EOM      14000
         SKS      21000
         BRU      $-1
         MIN      COUNT
         LDA      =5
         SKE      COUNT
         BRU      F
         BRR      LEFTX
         CLA
         STA      COUNT
G        EOM      00064              DRAWS HORIZONTAL BAR OF +
         MIW      PXDO
         EOM      14000
         SKS      21000
         BRU      $-1
         MIN      COUNT
         LDA      =10
         SKE      COUNT
```

Table B-2. (Cont'd)

```
        BRU     G
        CLA
        STA     COUNT
        BRM     LEFTX
* TESTS TO SEE IF ALL COORDINATES HAVE BEEN PLOTTED.....................
        CXA
        ADD     ONE
        LDB     =037777
        SKM     *NUM
        BRU     $+4
        SUB     TWO
        CAX
        BRU     REST                ALL COORDINATES HAVE BEEN PLOTTED
        SUB     TWO
        CAX
        BRU     NEXT                NOT ALL COORDINATES HAVE BEEN PLOTTED
* PLOTS THE FOLLOWING THREE COORDINATES BY DRAWING A SQUARE.............
* PLOTS THE REDUCED VARIATE AT THRESHOLD................................
REST    LDA     *POINT+1
        STA     XHOLD               SAVES VALUE OF LAST CHANNEL EXTREME
        LDA     *THRES
        STA     YHOLD               SAVES VALUE OF THRESHOLD
        BRM     ABSCIS              BRANCH TO PEN POSITIONING ROUTINE
        BRX     $+1
        LDA     *POINT+1
        STA     XHOLD               SAVES VALUE OF LAST PLOTTING POSITION
        LDA     *RVT
        STA     YHOLD               SAVES VALUE OF REDUCED VARIATE
        BRM     ORD                 BRANCH TO PEN POSITIONING ROUTINE
        BRM     MARK                BRANCH TO SQUARE DRAWING ROUTINE
* PLOTS POINT OF REGRESSION EQUATION WHEN REDUCED VARIATE = 0...........
        LDA     *THRES
        STA     XHOLD               SAVES VALUE OF THRESHOLD
        LDA     *LINEO
        STA     YHOLD               SAVES VALUE OF REGRESSION EQUATION
        BRM     ABSCIS              BRANCH TO PEN POSITIONING ROUTINE
        LDA     *RVT
        STA     XHOLD               SAVES VALUE OF REDUCED VARIATE
        CLA
```

Table B-2. (Cont'd)

```
       STA     YHOLD                    SAVES VALUE OF O FOR REDUCED VARIATE
       BRM     ORD                      BRANCH TO PEN POSITIONING ROUTINE
       BRM     MARK                     BRANCH TO SQUARE DRAWING ROUTINE
* PLOTS POINT OF REGRESSION EQUATION WHEN REDUCED VARIATE = 1...........
       LDA     *LINEO
       STA     XHOLD                    SAVES LAST VALUE OF REGRESSION EQTN.
       LDA     *LINE1
       STA     YHOLD                    SAVES NEW VALUE OF REGRESSION EQTN.
       BRM     ABSCIS                   BRANCH TO PEN POSITIONING ROUTINE
* MOVES PEN ALONG REDUCED VARIATE AXIS SO THAT REDUCED VARIATE = 1......
       CLA
       STA     COUNT
ZZ     EOM     00064
       MIW     PXUP
       EOM     14000
       SKS     21000
       BRU     $-1
       MIN     COUNT
       LDA     =105
       SKE     COUNT
       BRU     ZZ
       BRM     MARK                     BRANCH TO SQURE DRAWING ROUTINE
       BRR     GRAPH                    RETURN TO MAIN PROGRAM
* DETERMINES INCREMENT ALONG THE CHANNEL EXTREMES AXIS..................
* MOVES PEN ALONG CHANNEL EXTREMES AXIS.................................
ABSCIS PZE
       CLA
       STA     COUNT
       LDA     XHOLD
       SUB     YHOLD
       STA     TEMP
       SKE     ZERO
       BRU     $+2
       BRR     ABSCIS
       SKN     TEMP
       BRU     $+3
       CNA
       STA     TEMP
       LDA     YHOLD
```

Table B-2. (Cont'd)

```
        SKG     XHOLD
        BRU     UP
H       EOM     00064
        MIW     MYUP
        EOM     14000
        SKS     21000
        BRU     $-1
        MIN     COUNT
        LDA     TEMP
        SKE     COUNT
        BRU     H
        BRR     ABSCIS
UP      EOM     00064
        MIW     PYUP
        EOM     14000
        SKS     21000
        BRU     $-1
        MIN     COUNT
        LDA     TEMP
        SKE     COUNT
        BRU     UP
        BRR     ABSCIS
* DETERMINES INCREMENT ALONG THE REDUCED VARIATE AXIS..................
* MOVES PEN ALONG REDUCED VARIATE AXIS.................................
ORD     PZE
        CLA
        STA     COUNT
        LDA     XHOLD
        SUB     YHOLD
        STA     TEMP
        SKE     ZERO
        BRU     $+11
        EOM     00064
        MIW     PXUP
        EOM     14000
        SKS     21000
        BRU     $-1
        MIN     COUNT
        LDA     =5
```

Table B-2. (Cont'd)

```
        SKE     COUNT
        BRU     $-8
        BRR     ORD
        SKN     TEMP
        BRU     $+3
        CNA
        STA     TEMP
        LDA     XHOLD
        SKG     YHOLD
        BRU     RIGHT
        LDA     =5
        STA     COUNT
Q       EOM     00064
        MIW     MXUP
        EOM     14000
        SKS     21000
        BRU     $-1
        MIN     COUNT
        LDA     TEMP
        SKE     COUNT
        BRU     Q
        BRR     ORD
RIGHT   LDA     =5
        ADM     TEMP
        EOM     00064
        MIW     PXUP
        EOM     14000
        SKS     21000
        BRU     $-1
        MIN     COUNT
        LDA     COUNT
        SKE     TEMP
        BRU     RIGHT+2
        BRR     ORD
* ROUTINE WHICH DRAWS A SQUARE.........................................
MARK    PZE
        CLA
        STA     COUNT
P       EOM     00064
```

Table B-2. (Cont'd)

```
          MIW    PYD0
          E0M    14000
          SKS    21000
          BRU    $-1
          MIN    C0UNT
          LDA    =5
          SKE    C0UNT
          BRU    P
          CLA
          STA    C0UNT
R         E0M    00064
          MIW    MXD0
          E0M    14000
          SKS    21000
          BRU    $-1
          MIN    C0UNT
          LDA    =10
          SKE    C0UNT
          BRU    R
          CLA
          STA    C0UNT
S         E0M    00064
          MIW    MYD0
          E0M    14000
          SKS    21000
          BRU    $-1
          MIN    C0UNT
          LDA    =10
          SKE    C0UNT
          BRU    S
          CLA
          STA    C0UNT
T         E0M    00064
          MIW    PXD0
          E0M    14000
          SKS    21000
          BRU    $-1
          MIN    C0UNT
          LDA    =10
```

Table B-2. (Cont'd)

```
         SKE     COUNT
         BRU     T
         CLA
         STA     COUNT
Z        EOM     00064
         MIW     PYDO
         EOM     14000
         SKS     21000
         BRU     $-1
         MIN     COUNT
         LDA     =5
         SKE     COUNT
         BRU     Z
         CLA
         STA     COUNT
         BRM     LEFTX
         BRR     MARK
NUM      RES     2
POINT    RES     2
RVT      RES     2
THRES    RES     2
LINE0    RES     2
LINE1    RES     2
ZERO     PZE
ONE      DATA    1
TWO      DATA    2
XHOLD    PZE
YHOLD    PZE
TEMP     PZE
COUNT    PZE
PXUP     DATA    042000000          PEN UP, +X DIRECTION
PYUP     DATA    012000000          PEN UP, +Y DIRECTION
MXUP     DATA    022000000          PEN UP, -X DIRECTION
MYUP     DATA    006000000          PEN UP, -Y DIRECTION
PXDO     DATA    041000000          PEN DOWN, +X DIRECTION
PYDO     DATA    011000000          PEN DOWN, +Y DIRECTION
MXDO     DATA    021000000          PEN DOWN, -X DIRECTION
MYDO     DATA    005000000          PEN DOWN, -Y DIRECTION
         END
```

Table B-2. (Cont'd)

```
C MASTER BIVARIATE EXTREME VALUE PROGRAM FOR TWO CHANNEL DATA...........
C.........................SANDRA LURIE...................................
C.........................JULY  1966.....................................
C LINK(3) OF THE PROGRAM.................................................
C USES FORTRAN SUBROUTINES  BEVT (BIVARIATE EXTREME VALUE CALCULATIONS).
C                           BMAXLIK (BIVARIATE MAXIMUM LIKELIHOOD FIT)..
C                           PARAFIT (PARABOLA FIT)......................
C                           HELP (NEW POINT DETERMINATION FOR PARAFIT)..
C BIVARIATE EXTREME VALUE CALCULATIONS...................................
      DIMENSION MAX1(700),MAX2(700),IDEXT(700),ISEXT(700)
      COMMON MAX1,MAX2,ALPHA1,ALPHA2,U1,U2,NG,NDP,T1,T2,ITN,ERROR
      EQUIVALENCE (MAX1,IDEXT),(MAX2,ISEXT),(ALPHA1,ALPHAD)
     1(ALPHA2,ALPHAS),(U1,UD),(U2,US),(T1,TD),(T2,TS),(NDP,NDS)
C STORES PARAMETERS......................................................
      FI=ALPHAD
      GI=ALPHAS
      DFDN=UD
      ALPHA=US
      T=TD
      YT=TS
C TYPES OPERATIONAL DIRECTIONS AND ACCEPTS TYPEWRITER INPUTS............
  300 TYPE 913
      TYPE 904
  301 ACCEPT 905,NIT,NBEVT,A,ERROR
C PICKS UP PARAMETERS....................................................
  310 ALPHAD=FI
      ALPHAS=GI
      UD=DFDN
      US=ALPHA
      TD=T
      TS=YT
C ESTIMATES C PARAMETER BY STRIP METHOD..................................
      COUNT=0.
      DO 350 I=1,NG
      F=(IDEXT(I)-UD)*ALPHAD
      G=(ISEXT(I)-US)*ALPHAS
  330 IF(ABSF(F-G)-A)340,350,350
  340 COUNT=COUNT+1.0
```

Table B-2. (Cont'd)

```
  350 CONTINUE
      PRINT 907,ITN
      PRINT 910,NG,NDS,TD,TS,A,COUNT
      PRINT 911,NIT,NBEVT,ERROR
      COUNT=COUNT/NG
      DUA=(4.0*(EXPF(A)-EXPF(3.0*A)))/(1.0+EXPF(A))**4
      HOLD=(EXPF(A/2.0)-EXPF(-A/2.0))/(EXPF(A/2.0)+EXPF(-A/2.0))-COUNT
      C=HOLD/(2.0*DUA+((4.0*EXPF(A))/(1.0+EXPF(A))**2)*HOLD)
C COMPUTES INITIAL BIVARIATE EVT STATISTICS...........................
  370 II=0
      CALL BEVT(ITN,NG,NDS,COUNT,A,DUA,ALPHAD,ALPHAS,UD,US,C,TD,TS,II)
C BIVARIATE MAXIMUM LIKELIHOOD FIT.....................................
C FITS A PARABOLA THROUGH EACH OF THE PARAMETERS C, ALPHA1, ALPHA2, U1,
C   AND U2.............................................................
      H=0.01
      HOLD=0.
      DO 700 I=1,NIT,NBEVT
      DO 600 J=1,NBEVT
C VARIES C PARAMETER AND PERFORMS A PARABOLA FIT.......................
      X1=C
      X2=C-H*ABSF(C)
      X3=C+H*ABSF(C)
  501 CALL BMAXLIK(ALPHAD,ALPHAS,UD,US,X1,Y1,IDEXT,ISEXT,NG)
  502 CALL BMAXLIK(ALPHAD,ALPHAS,UD,US,X2,Y2,IDEXT,ISEXT,NG)
  503 CALL BMAXLIK(ALPHAD,ALPHAS,UD,US,X3,Y3,IDEXT,ISEXT,NG)
      CALL PARAFIT(X1,Y1,X2,Y2,X3,Y3,VERTEX)
      CALL HELP(HOLD,X1,Y1,X2,Y2,X3,Y3,VERTEX,ERROR,H)
      IF(HOLD-0.0)505,510,505
  505 GO TO (503,502,501),Y3
  510 C=VERTEX
C VARIES ALPHA1 PARAMETER AND PERFORMS A PARABOLA FIT..................
      X1=ALPHAD
      X2=ALPHAD-H*ABSF(ALPHAD)
      X3=ALPHAD+H*ABSF(ALPHAD)
  511 CALL BMAXLIK(X1,ALPHAS,UD,US,C,Y1,IDEXT,ISEXT,NG)
  512 CALL BMAXLIK(X2,ALPHAS,UD,US,C,Y2,IDEXT,ISEXT,NG)
  513 CALL BMAXLIK(X3,ALPHAS,UD,US,C,Y3,IDEXT,ISEXT,NG)
      CALL PARAFIT(X1,Y1,X2,Y2,X3,Y3,VERTEX)
      CALL HELP(HOLD,X1,Y1,X2,Y2,X3,Y3,VERTEX,ERROR,H)
```

Table B-2. (Cont'd)

```
      IF(HOLD-0.0)515,520,515
  515 GO TO (513,512,511),Y3
  520 ALPHAD=VERTEX
C VARIES ALPHA2 PARAMETER AND PERFORMS A PARABOLA FIT..................
      X1=ALPHAS
      X2=ALPHAS-H*ABSF(ALPHAS)
      X3=ALPHAS+H*ABSF(ALPHAS)
  521 CALL BMAXLIK(ALPHAD,X1,UD,US,C,Y1,IDEXT,ISEXT,NG)
  522 CALL BMAXLIK(ALPHAD,X2,UD,US,C,Y2,IDEXT,ISEXT,NG)
  523 CALL BMAXLIK(ALPHAD,X3,UD,US,C,Y3,IDEXT,ISEXT,NG)
      CALL PARAFIT(X1,Y1,X2,Y2,X3,Y3,VERTEX)
      CALL HELP(HOLD,X1,Y1,X2,Y2,X3,Y3,VERTEX,ERROR,H)
      IF(HOLD-0.0)525,529,525
  525 GO TO (523,522,521),Y3
  529 ALPHAS=VERTEX
C VARIES U1 PARAMETER AND PERFORMS A PARABOLA FIT......................
  530 X1=UD
      X2=UD-H*ABSF(UD)
      X3=UD+H*ABSF(UD)
  531 CALL BMAXLIK(ALPHAD,ALPHAS,X1,US,C,Y1,IDEXT,ISEXT,NG)
  532 CALL BMAXLIK(ALPHAD,ALPHAS,X2,US,C,Y2,IDEXT,ISEXT,NG)
  533 CALL BMAXLIK(ALPHAD,ALPHAS,X3,US,C,Y3,IDEXT,ISEXT,NG)
      CALL PARAFIT(X1,Y1,X2,Y2,X3,Y3,VERTEX)
      CALL HELP(HOLD,X1,Y1,X2,Y2,X3,Y3,VERTEX,ERROR,H)
      IF(HOLD-0.0)535,540,535
  535 GO TO (533,532,531),Y3
  540 UD=VERTEX
C VARIES U2 PARAMETER AND PERFORMS A PARABOLA FIT......................
      X1=US
      X2=US-H*ABSF(US)
      X3=US+H*ABSF(US)
  541 CALL BMAXLIK(ALPHAD,ALPHAS,UD,X1,C,Y1,IDEXT,ISEXT,NG)
  542 CALL BMAXLIK(ALPHAD,ALPHAS,UD,X2,C,Y2,IDEXT,ISEXT,NG)
  543 CALL BMAXLIK(ALPHAD,ALPHAS,UD,X3,C,Y3,IDEXT,ISEXT,NG)
      CALL PARAFIT(X1,Y1,X2,Y2,X3,Y3,VERTEX)
      CALL HELP(HOLD,X1,Y1,X2,Y2,X3,Y3,VERTEX,ERROR,H)
      IF(HOLD-0.0)545,550,545
  545 GO TO (543,542,541),Y3
  550 US=VERTEX
```

Table B-2. (Cont'd)

```
      II=II+1
  600 CONTINUE
      IF(II-NIT)650,625,625
C ON THE LAST ITERATION REPLACES C BY ZERO IF C IS LESS THAN ZERO AND
C    BY 0.25 IF C IS GREATER THAN 0.25.....................................
  625 IF(C)630,650,626
  626 IF(C-0.25)650,650,627
  627 HOLD=0.25
  630 CALL BEVT(ITN,NG,NDS,COUNT,A,DUA,ALPHAD,ALPHAS,UD,US,HOLD,TD,TS,
     1II)
      GO TO 700
  650 CALL BEVT(ITN,NG,NDS,COUNT,A,DUA,ALPHAD,ALPHAS,UD,US,C,TD,TS,II)
  700 CONTINUE
      IF(C)705,725,710
  705 PRINT 912,C
      GO TO 725
  710 IF(C-0.25)725,725,715
  715 PRINT 914,C
C OPTIONS TO ALLOW CHANGING OF THE PARAMETER A AND THE THRESHOLDS OF THE
C    TWO CHANNELS..........................................................
  725 TYPE 915
      PAUSE
      IF(SENSE SWITCH 3)730,750
C INPUT NEW VALUE FOR PARAMETER A..........................................
  730 TYPE 916
  731 ACCEPT 917,A
  750 IF(SENSE SWITCH 4)760,770
C INPUT NEW VALUES FOR CHANNEL THRESHOLDS..................................
  760 TYPE 918
  761 ACCEPT 919,T,YT
      TD=T
      TS=YT
      IF(SENSE SWITCH 3)310,762
  762 PRINT 907,ITN
      X=COUNT*NG
      PRINT 910,NG,NDS,TD,TS,A,X
      GO TO 625
  770 IF(SENSE SWITCH 3)310,800
C JOB DONE, RETURN CONTROL TO LINK (1).....................................
```

Table B-2. (Cont'd)

```
800 TYPE 906
    CALL LINK(1)
904 FORMAT(/$INPUT IN FORMAT 2I10,2F15.5$/$  NIT--ITERATIONS FOR BEVT
   1MAXIMUM LIKELIHOOD FITS$/$  NBEVT--ITERATIONS BEFORE EACH BEVT PROB
   2ABILITY CALCULATION$/$  A--STRIP ESTIMATE PARAMETERS/4X$A MUST BE
   3IN THE CLOSED INTERVAL 1.5 TO 2.0$/$  ERROR--ERROR FOR BIVARIATE M
   4AXIMUM LIKELIHOOD FIT$/)
905 FORMAT(2I10,2F15.5)
906 FORMAT(/$JOB DONE.  READY NEW INPUT.$/)
907 FORMAT(1H1,38X$BIVARIATE EXTREME VALUE$/46X,$TEST$,I4//)
910 FORMAT(////$THERE ARE $,I5,$ GROUPS OF $,I5,$ DATA POINTS EACH.$//
   1/$ADC1 CHANNEL THRESHOLD = $,F10.5//$ADC2 CHANNEL THRESHOLD = $,F1
   20.5///$A = $,F10.5//$ABS(XADC1(N)-XADC2(N)) LESS THAN A OCCURS $F6
   3.2$ TIMES.$//)
911 FORMAT(/I3$ ITERATIONS TO BE PERFORMED.$//$BIVARIATE CALCULATIONS
   1WILL OCCUR EVERY $,I2,$ ITERATIONS.$//// $ERROR ESTIMATE FOR MAXIM
   2UM LIKELIHOOD FIT = $,E20.12)
912 FORMAT(///$ON THE LAST ITERATION C WAS NEGATIVE, C = $,E20.12/$FOR
   1 THE PRECEDING BIVARIATE COMPUTATIONS C = 0.0$)
913 FORMAT(/$IF AN ERROR IS MADE WHILE TYPING INPUTS, DO THE FOLLOWING
   1$/$  1.  PUT RUN-IDLE-STEP (R-I-S) SWITCH TO IDLE$/$  2.  SET REGI
   2STER KNOB TO C$/$  3.  PUSH START$/$  4.  FILL REGISTER DISPLAY WI
   3TH A  BRU 03531  COMMAND,$/8X$THAT IS, WITH THE OCTAL NUMBER 00103
   4531$/$  5.  PUT R-I-S SWITCH TO RUN$/$  6.  RETYPE INPUTS$//)
914 FORMAT(///$ON THE LAST ITERATION C WAS GREATER THAN 0.25, C = $,
   1E20.12/$FOR THE PRECEDING BIVARIATE COMPUTATION C = 0.25$)
915 FORMAT(/$SET BP3 TO CHANGE THE VALUE OF PARAMETER A$/$SET BP4 TO C
   1HANGE THE CHANNEL THRESHOLDS$//$IF NEITHER BREAKPOINT IS SET, CONT
   2ROL TRANSFERS TO LINK(1)$/$CLEAR HALT TO PROCEED$/)
916 FORMAT(/$INPUT THE NEW VALUE FOR A IN FORMAT F10.5$/$  IF AN ERROR
   1 IS MADE WHILE TYPING, REPEAT THE 6 STEPS LISTED ABOVE,$/$    EXCE
   2PT IN STEP 4 FILL THE REGISTER DISPLAY WITH A BRU 05013 COMMAND,$/
   3$     THAT IS, WITH THE OCTAL NUMBER 00105013$/)
917 FORMAT(F10.5)
918 FORMAT(/$INPUT IN FORMAT 2F10.5$/$  NEW ADC-1 THRESHOLD VALUE$/$
   1NEW ADC-2 THRESHOLD VALUE$/$IF AN ERROR IS MADE WHILE TYPING, REPE
   2AT THE 6 STEPS LISTED ABOVE,$/$  EXCEPT IN STEP 4 FILL THE REGISTE
   3R DISPLAY WITH A BRU 05026 COMMAND,$/$  THAT IS, WITH THE OCTAL NU
   4MBER 00105026$/)
919 FORMAT(2F10.5)
    END
```

Table B-2. (Cont'd)

```
      SUBROUTINE BEVT(IBITN,IBNG,IBNDS,BCOUNT,BA,BDUA,BALPHAD,BALPHAS,BU
     1D,BUS,BC,BTD,BTS,IIB)
C PERFORMS BIVARIATE CALCULATIONS.........................................
      ROOTX=1.0/IBNDS
      WA=1.0-(4.0*BC*EXPF(BA))/(1.0+EXPF(BA))**2
      VARC=(1.0/(4.0*IBNG))*BCOUNT*(1.0-BCOUNT)*(((WA**2)/BDUA)**2)
C RE-CALCULATES UNIVARIATE EVT STATISTICS FOR ADC1......................
      YTD=(BTD-BUD)*BALPHAD
      PCT=EXPF(-EXPF(-YTD))
      RTPCT=EXPF((ALOG(PCT))/IBNDS)
      N=1
      TEMP=1.0-PCT
      GO TO 850
  801 GU=SERIES
      PCTNDP=1.0-GU
      N=N+1
      PRINT 920,IBITN,IIB
      PRINT 921,BALPHAD,BUD,BALPHAS,BUS,BC,VARC
      PRINT 922
      PRINT 923,PCTNDP,PCT,RTPCT,GU
C RE-CALCULATES UNIVARIATE EVT STATISTICS FOR ADC2......................
      YTS=(BTS-BUS)*BALPHAS
      PCT=EXPF(-EXPF(-YTS))
      RTPCT=EXPF((ALOG(PCT))/IBNDS)
      TEMP=1.0-PCT
      GO TO 850
  802 GV=SERIES
      PCTNDP=1.0-GV
      N=N+1
      PRINT 924
      PRINT 923,PCTNDP,PCT,RTPCT,GV
C COMPUTES BIVARIATE EVT STATISTICS.......................................
      TEMP=YTD-YTS
      WZ=1.0-(4.0*BC*EXPF(TEMP))/(1.0+EXPF(TEMP))**2
      PR=EXPF(-(EXPF(-YTD)+EXPF(-YTS))*WZ)
      PRINT 920,IBITN,IIB
      PRI=EXPF((1.0/IBNDS)*ALOG(PR))
      RTPCT=PRI
```

Table B-2. (Cont'd)

```
      TEMP=1.0-PR
      GO TO 850
  803 P=SERIES
      PRINT 926,PRI,P
      Q=GU-P
      R=GV-P
      S=1.0-P-Q-R
      PP=P**IBNDS
      QQ=(P+Q)**IBNDS-PP
      RR=(P+R)**IBNDS-PP
      SS=1.0+PP-(P+Q)**IBNDS-(P+R)**IBNDS
      PRINT 927,P,Q,R
      PRINT 928,S,IBNDS,PP,QQ,RR,SS
      RETURN
C CALCULATES THE XTH ROOT OF THE CUMULATIVE PROBABILITY, WHERE X IS THE
C   RECIPROCAL OF THE NUMBER OF DATA SAMPLES/GROUP, BY SERIES EXPANSION
C   WHICH IS ACCURATE TO THE 11TH DECIMAL PLACE.  IF THIS PROCEDURE
C   OVERFLOWS, THAT IS, IF THE NUMERICAL CAPACITY OF THE COMPUTER IS
C   EXCEEDED, THE SERIES VALUE IS REPLACED BY THE VALUE OBTAINED BY
C   USING LOGARITHMS.............................................
  850 ROOTY=ROOTX
      I=1
      FACT=I
      SERIES=1.0-(ROOTX*TEMP)/FACT
  855 ROOT=ROOTY*(ROOTX-I)
      ROOTY=ROOT
      FACT=FACT*(I+1.0)
      I=I+1
      HANG=ABSF((ROOT*(TEMP**I))/FACT)
      SERIES=SERIES-HANG
      IF(ABSF(SERIES)-1.0)859,856,856
  856 SERIES=RTPCT
      GO TO 860
  859 IF(HANG-0.00000000001)860,860,855
  860 GO TO (801,802,803),N
  920 FORMAT(1H1,38X,$BIVARIATE EXTREME VALUES$/46X,$TEST$I4//$ITERATIONS$
     1,I4//)
  921 FORMAT(///$FOR THE FOLLOWING CALCULATIONS:$//5X,$ALPHA1 = $,E20.12
     1,15X,$U1 = $,E20.12/5X,$ALPHA2 = $,E20.12,15X,$U2 = $,E20.12/5X,$C
```

Table B-2. (Cont'd)

```
   2       = $,E20.12,15X,$VARIANCE ØF C = $,E20.12)
922 FØRMAT(///$FØR ADC-1$/)
923 FØRMAT(/5X,$PREDICTED ERRØR RATE =$,E21.12,/5X,$CUMULATIVE PRØBABI
   1LITY AT TRIGGER LEVEL = $,E20.12,/5X,$THE NDP RØØT ØF THE CUMULATI
   2VE PRØBABILITY:$/7X,$BY LØGS    = $,E20.12,/7X,$BY SERIES = $,E20.1
   32)
924 FØRMAT(///$FØR ADC-2$/)
926 FØRMAT(//$THE PRØBABILITY ØF A CØRRECT BIT BEING RECEIVED AND ACCE
   1PTED:$/7X,$BY LØGS    = $,E20.12,/7X,$BY SERIES = $,E20.12)
927 FØRMAT(///$PRØBABILITY P ØF A CØRRECT BIT BEING RECEIVED AND ACCEP
   1TED    = $,E20.12/$PRØBABILITY Q ØF A CØRRECT BIT BEING RECEIVED A
   2ND REJECTED     = $,E20.12/$PRØBABILITY R ØF AN INCØRRECT BIT BEING
   3 RECEIVED AND ACCEPTED = $,E20.12)
928 FØRMAT($PRØBABILITY S ØF AN INCØRRECT BIT BEING RECEIVED AND REJEC
   1TED = $,E20.12///$FØR A CØMMAND ØF LENGTH = $,I5$ BITS:$//5X,$P =
   2$,E20.12/5X,$Q = $,E20.12/5X,$R = $,E20.12/5X,$S = $,E20.12)
    END
```

Table B-2. (Cont'd)

```
      SUBROUTINE BMAXLIK(VARA1,VARA2,VARU1,VARU2,VARC,PROD,MATDAT,MATSYN
     1,NQ)
C COMPUTES THE BIVARIATE MAXIMUM LIKELIHOOD FUNCTION....................
C COMPUTES THE SUM OF LN(DF/DXDY) FOR ALL PAIRS OF THE CHANNEL EXTREMES.
      DIMENSION MATDAT(700),MATSYN(700)
      PROD=0.0
      DO 775 J=1,NQ
      U=(VARA1*(MATDAT(J)-VARU1)-VARA2*(MATSYN(J)-VARU2))/2.0
      SECH2=(2.0/(EXPF(U)+EXPF(-U)))**2
      TANH=(EXPF(U)-EXPF(-U))/(EXPF(U)+EXPF(-U))
      WU=1.0-VARC*SECH2
      WU1=2.0*VARC*SECH2*TANH
      WU2=2.0*VARC*SECH2*(3.0*SECH2-2.0)
      EX=EXPF(-VARA1*(MATDAT(J)-VARU1))
      EY=EXPF(-VARA2*(MATSYN(J)-VARU2))
      TEMP=VARA1*WU*EX-(VARA1/2.0)*WU1*(EX+EY)
      TEMP=TEMP*(VARA2*WU*EY+(VARA2/2.0)*WU1*(EX+EY))
      TEMP=TEMP+(VARA1*VARA2/4.0)*WU2*(EX+EY)-(VARA1*VARA2/2.0)*WU1*(EX-
     1EY)
      TEMP=TEMP*(EXPF(-(EX+EY)*WU))
      PROD=PROD+ALOG(TEMP)
  775 CONTINUE
      RETURN
      END


      SUBROUTINE PARAFIT(P1X,P1Y,P2X,P2Y,P3X,P3Y,VERT)
C FITS A PARABOLA THROUGH THE POINTS (X1,Y1),(X2,Y2),(X3,Y3)............
C FINDS THE VERTEX OF THE PARABOLA......................................
      H1=P2X
      H2=P2Y
      P1X=P1X-H1
      P3X=P3X-H1
      P2X=0.
      P1Y=P1Y-H2
      P3Y=P3Y-H2
      P2Y=0.
      DET=(P1X**2)*(P2X-P3X)-(P2X**2)*(P1X-P3X)+(P3X**2)*(P1X-P2X)
      AA=(P1Y*(P2X-P3X)-P2Y*(P1X-P3X)+P3Y*(P1X-P2X))/DET
      BB=((P1X**2)*(P2Y-P3Y)-(P2X**2)*(P1Y-P3Y)+(P3X**2)*(P1Y-P2Y))/DET
      VERT=-BB/(2.0*AA)+H1
      P1X=P1X+H1
      P3X=P3X+H1
      P2X=H1
      P1Y=P1Y+H2
      P3Y=P3Y+H2
      P2Y=H2
      RETURN
      END
```

Table B-2. (Cont'd)

```
      SUBROUTINE HELP(SAVE,P1X,P1Y,P2X,P2Y,P3X,P3Y,VERT,ERR,STEP)
C DETERMINES NEW POINTS FOR SUCCESSIVE PARABOLA FITS....................
      IF(ABSF(SAVE/VERT)-(1.0-ERR))706,705,703
  703 IF(ABSF(SAVE/VERT)-(1.0+ERR))705,705,706
  705 SAVE=0.
      RETURN
  706 SAVE=VERT
      IF(VERT-P2X)708,720,707
  707 IF(VERT-P3X)725,720,709
  708 P3X=P1X
      P1X=P2X
      P2X=VERT
      P1Y=P2Y
      P3Y=2.0
      RETURN
  709 P2X=P1X
      P1X=P3X
      P3X=VERT
      P2Y=P1Y
      P1Y=P3Y
      P3Y=1.0
      RETURN
  720 P1X=VERT
      P2X=P1X-STEP*ABSF(P1X)
      P3X=P1X+STEP*ABSF(P1X)
      GO TO 750
  725 IF(VERT-P1X)730,740,735
  730 P3X=P1X
      P1X=VERT
      GO TO 750
  735 P2X=P1X
      P1X=VERT
      GO TO 750
  740 P1X=VERT
      P2X=P1X-STEP*ABSF(P1X)
      P3X=P1X+STEP*ABSF(P1X)
  750 P3Y=3.0
      RETURN
      END
ΛEOF
```

APPENDIX C

Data-Processing Program Sample Output and Operational Directions

This appendix contains a sample output of the data-processing program (Table C-1, and Figs. C-1 and C-2) and a set of operational directions which were typed on the console typewriter during program execution (Table C-2). These directions illustrate the various options which are available and detail the required typewriter inputs for this sample output. The output contains both the univariate and bivariate EVT statistics of the example discussed throughout the report. It also includes the linearized univariate EVT plots for each channel, as plotted on the Cal-Comp plotter, the statistics obtained by biasing the lock indicator (Section VIII) and the statistics obtained by changing the value of the strip estimator, a, from 1.5 to 2.0. Approximately one and one-half hours of SDS-920 computer time were needed to obtain all of the output contained in this appendix.

Table C-1. Sample output of the data-processing program

```
                              UNIVARIATE EXTREME VALUE
                                   TEST   1


FOR ADC-1

  BASED ON THE TOTAL SAMPLE SIZE =    3000 SAMPLES
     MEAN =  -0.316310333334E 03
     STANDARD DEVIATION =   0.555328613801E 02
     SIGNAL TO NOISE RATIO =  -0.569591275280E 01   =    0.151112671965E 02  DB.


FOR ADC-2

  BASED ON THE TOTAL SAMPLE SIZE =    3000 SAMPLES
     MEAN =  -0.519096000001E 03
     STANDARD DEVIATION =   0.897556430027E 02
     SIGNAL TO NOISE RATIO =  -0.578343581123E 01   =    0.152437190316E 02  DB.



BASED ON   3000  RAW DATA SAMPLES, THE CORRELATION COEFFICIENT =   0.227415805883E 00

BASED ON EXTREMES OF   30  GROUPS OF  100  SAMPLES, THE CORRELATION COEFFICIENT =   0.385838213564E 00
```

Table C-1. (Cont'd)

UNIVARIATE EXTREME VALUE
TEST 1

```
CLASSICAL PROBABILITIES

PROBABILITY OF A BIT ERROR =   0.000000000000E 00
  NUMBER OF OCCURENCES =          0.0

PROBABILITY OF AN OUT OF LOCK =   0.000000000000E 00
  NUMBER OF OCCURENCES =          0.0

PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED     =   0.100000000000E 01
  NUMBER OF OCCURENCES =       3000.0

PROBABILITY OF A CORRECT BIT BEING RECEIVED AND REJECTED     =   0.000000000000E 00
  NUMBER OF OCCURENCES =          0.0

PROBABILITY OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTED =   0.000000000000E 00
  NUMBER OF OCCURENCES =          0.0

PROBABILITY OF AN INCORRECT BIT BEING RECEIVED AND REJECTED =   0.000000000000E 00
  NUMBER OF OCCURENCES =          0.0
```

Table C-1. (Cont'd)

```
                                        UNIVARIATE EXTREME VALUE
                                               TEST    1


THERE ARE     30 GROUPS OF    100 SAMPLES EACH.

ERROR FOR UNIVARIATE MAXIMUM LIKELIHOOD FIT =    0.999999999995E-05


FOR ADC-1

  VALUES BEFORE UNIVARIATE MAXIMUM LIKELIHOOD FIT


     THRESHOLD =    0.000000000000E 00

     ALPHA =   0.273034555335E-01
     U =  -0.173239308614E 03

     THE REGRESSION EQUATION =          -173.2393086 +   36.6254007 Y
     REDUCED VARIATE AT TRIGGER LEVEL =  0.473003175939E 01

     PREDICTED BIT ERROR RATE =   0.882580352481E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.991212645731E 00
```

Table C-1. (Cont'd)

```
                 UNIVARIATE EXTREME VALUE
                        TEST    1


FOR ADC-1

  VALUES AFTER UNIVARIATE MAXIMUM LIKELIHOOD FIT

     THRESHOLD =    0.000000000000E 00

     ALPHA =    0.333626955910E-01
     U =   -0.171631574073E 03

     THE REGRESSION EQUATION =          -171.6315741 +    29.9735972 Y
     REDUCED VARIATE AT TRIGGER LEVEL =  0.572609195960E 01

     PREDICTED BIT ERROR RATE =   0.325973887811E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =    0.996745515564E 00
```

PERCENT CONFIDENCE	CONFIDENCE INTERVAL FOR PREDICTED BIT ERROR RATE	
99	0.325743167195E-05	0.326163200952E-03
95	0.564985384699E-05	0.188061923836E-03
90	0.749035098124E-05	0.141855307447E-03
80	0.103640413726E-04	0.102524347312E-03
70	0.129037507576E-04	0.823461596155E-04

Table C-1. (Cont'd)

```
                                    UNIVARIATE EXTREME VALUE
                                           TEST   1


THERE ARE    30 GROUPS OF   100 SAMPLES EACH.

ERROR FOR UNIVARIATE MAXIMUM LIKELIHOOD FIT =   0.999999999995E-05

FOR ADC-2

  VALUES BEFORE UNIVARIATE MAXIMUM LIKELIHOOD FIT


     THRESHOLD =   0.000000000000E 00

     ALPHA =   0.213265757956E-01
     U =  -0.303176656604E 03

     THE REGRESSION EQUATION =         -303.1766566 +   46.8898528 Y
     REDUCED VARIATE AT TRIGGER LEVEL =  0.646571994651E 01

     PREDICTED BIT ERROR RATE =   0.155585948959E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.998445339021E 00
```

Table C-1. (Cont'd)

UNIVARIATE EXTREME VALUE
TEST 1

FOR ADC-2

VALUES AFTER UNIVARIATE MAXIMUM LIKELIHOOD FIT

THRESHOLD = 0.000000000000E 00

ALPHA = 0.228594114902E-01
U = -0.302891621230E 03

THE REGRESSION EQUATION = -302.8916212 + 43.7456581 Y
REDUCED VARIATE AT TRIGGER LEVEL = 0.692392420667E 01

PREDICTED BIT ERROR RATE = 0.983958307188E-05
CUMULATIVE PROBABILITY AT TRIGGER LEVEL = 0.999016522837E 00

PERCENT CONFIDENCE	CONFIDENCE INTERVAL FOR PREDICTED BIT ERROR RATE	
99	0.638705387246E-06	0.151580003148E-03
95	0.122815981740E-05	0.788302859291E-04
90	0.171655119629E-05	0.564017100259E-04
80	0.252405152423E-05	0.383575970772E-04
70	0.327428278979E-05	0.295688587357E-04

Table C-1. (Cont'd)

UNIVARIATE EXTREME VALUE
TEST 1

FOR ADC-1

GROUP NUMBER	UNORDERED EXTREMES	ORDERED EXTREMES	PLOTTING POSITION
1	-95	-211	0.3225806451E-01
2	-138	-204	0.6451612903E-01
3	-181	-198	0.9677419354E-01
4	-158	-197	0.1290322581E 00
5	-146	-192	0.1612903226E 00
6	-179	-190	0.1935483871E 00
7	-192	-185	0.2258064516E 00
8	-211	-181	0.2580645161E 00
9	-169	-179	0.2903225806E 00
10	-198	-174	0.3225806452E 00
11	-159	-173	0.3548387097E 00
12	-204	-172	0.3870967742E 00
13	-197	-171	0.4193548387E 00
14	-157	-170	0.4516129032E 00
15	-185	-169	0.4838709677E 00
16	-173	-159	0.5161290323E 00
17	-19	-158	0.5483870968E 00
18	-103	-157	0.5806451613E 00
19	-108	-153	0.6129032258E 00
20	-153	-151	0.6451612903E 00
21	-172	-146	0.6774193548E 00
22	-112	-138	0.7096774194E 00
23	-112	-121	0.7419354839E 00
24	-121	-112	0.7741935484E 00
25	-171	-112	0.8064516129E 00
26	-190	-110	0.8387096774E 00
27	-151	-108	0.8709677419E 00
28	-110	-103	0.9032258065E 00
29	-174	-95	0.9354838710E 00
30	-170	-19	0.9677419355E 00

Table C-1. (Cont'd)

UNIVARIATE EXTREME VALUE
TEST 1

FOR ADC-2

GROUP NUMBER	UNORDERED EXTREMES	ORDERED EXTREMES	PLOTTING POSITION
1	-157	-366	0.3225806451E-01
2	-199	-355	0.6451612903E-01
3	-321	-336	0.9677419354E-01
4	-355	-333	0.1290322581E 00
5	-209	-331	0.1612903226E 00
6	-331	-327	0.1935483871E 00
7	-273	-326	0.2258064516E 00
8	-299	-322	0.2580645161E 00
9	-274	-321	0.2903225806E 00
10	-322	-321	0.3225806452E 00
11	-333	-304	0.3548387097E 00
12	-300	-300	0.3870967742E 00
13	-327	-299	0.4193548387E 00
14	-321	-293	0.4516129032E 00
15	-304	-282	0.4838709677E 00
16	-274	-274	0.5161290323E 00
17	-216	-274	0.5483870968E 00
18	-366	-273	0.5806451613E 00
19	-253	-272	0.6129032258E 00
20	-265	-265	0.6451612903E 00
21	-293	-255	0.6774193548E 00
22	-282	-253	0.7096774194E 00
23	-336	-240	0.7419354839E 00
24	-238	-238	0.7741935484E 00
25	-215	-216	0.8064516129E 00
26	-272	-215	0.8387096774E 00
27	-326	-209	0.8709677419E 00
28	-185	-199	0.9032258065E 00
29	-255	-185	0.9354838710E 00
30	-240	-157	0.9677419355E 00

Table C-1. (Cont'd)

```
                                 BIVARIATE EXTREME VALUE
                                        TEST    1



THERE ARE     30 GROUPS OF    100 DATA POINTS EACH.

ADC1 CHANNEL THRESHOLD =     0.00000

ADC2 CHANNEL THRESHOLD =     0.00000

A =     1.50000

ABS[XADC1[N]-XADC2[N]] LESS THAN A OCCURS  24.00 TIMES.


  8 ITERATIONS TO BE PERFORMED.

BIVARIATE CALCULATIONS WILL OCCUR EVERY  4 ITERATIONS.


ERROR ESTIMATE FOR MAXIMUM LIKELIHOOD FIT =    0.999999999998E-04
```

Table C-1. (Cont'd)

```
                                   BIVARIATE EXTREME VALUE
                                        TEST    1

ITERATION   0



FOR THE FOLLOWING CALCULATIONS:

     ALPHA1 =    0.333626955910E-01                    U1 =   -0.171631574073E 03
     ALPHA2 =    0.228594114902E-01                    U2 =   -0.302891621230E 03
     C      =    0.192540375740E 00                    VARIANCE OF C =    0.570001688422E-02


FOR ADC-1

     PREDICTED ERROR RATE =    0.325973815051E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.996745515564E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999967402611E 00
       BY SERIES  =    0.999967402619E 00


FOR ADC-2

     PREDICTED ERROR RATE =    0.983955396805E-05
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.999016522837E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999990160421E 00
       BY SERIES  =    0.999990160446E 00
```

Table C-1. (Cont'd)

```
                         BIVARIATE EXTREME VALUE
                                TEST    1

ITERATION    0


THE PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED:
       BY LOGS    =    0.999963384158E 00
       BY SERIES  =    0.999963384173E 00


PROBABILITY P OF A CORRECT BIT BEING RECEIVED AND ACCEPTED     =   0.999963384173E 00
PROBABILITY Q OF A CORRECT BIT BEING RECEIVED AND REJECTED     =   0.401844590669E-05
PROBABILITY R OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTED  =   0.267762734438E-04
PROBABILITY S OF AN INCORRECT BIT BEING RECEIVED AND REJECTED  =   0.582110806135E-05

FOR A COMMAND OF LENGTH =   100 BITS:

     P =    0.996345045878E 00
     Q =    0.400470169552E-03
     R =    0.267147780323E-02
     S =    0.583006149099E-03
```

Table C-1. (Cont'd)

```
                              BIVARIATE EXTREME VALUE
                                     TEST   1

ITERATION   4


FOR THE FOLLOWING CALCULATIONS:

     ALPHA1 =    0.335083643738E-01                  U1 =   -0.173182352723E 03
     ALPHA2 =    0.222935228479E-01                  U2 =   -0.300849995527E 03
     C      =    0.139019911072E 00                  VARIANCE OF C =    0.656807094691E-02


FOR ADC-1

     PREDICTED ERROR RATE =    0.301826585200E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.996986238017E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999969817323E 00
       BY SERIES =    0.9[illegible]17342E 00


FOR ADC-2

     PREDICTED ERROR RATE =    0.122230994748E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.998778428755E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999987776886E 00
       BY SERIES =    0.999987776904E 00
```

Table C-1. (Cont'd)

```
                                   BIVARIATE EXTREME VALUE
                                          TEST    1
ITERATION    4



THE PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED:
       BY LOGS    =    0.999962432274E 00
       BY SERIES =    0.999962432288E 00


PROBABILITY P OF A CORRECT BIT BEING RECEIVED AND ACCEPTED     =    0.999962432288E 00
PROBABILITY Q OF A CORRECT BIT BEING RECEIVED AND REJECTED     =    0.738505332265E-05
PROBABILITY R OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTED =    0.253446160059E-04
PROBABILITY S OF AN INCORRECT BIT BEING RECEIVED AND REJECTED =    0.483804251416E-05


FOR A COMMAND OF LENGTH =    100 BITS:

     P =    0.996250206335E 00
     Q =    0.736032772693E-03
     R =    0.252822333277E-02
     S =    0.485537559143E-03
```

Table C-1. (Cont'd)

```
                                        BIVARIATE EXTREME VALUE
                                              TEST    1

ITERATION    8



FOR THE FOLLOWING CALCULATIONS:

     ALPHA1 =    0.334996709373E-01                       U1 =   -0.173177517458E 03
     ALPHA2 =    0.222900564506E-01                       U2 =   -0.300844906796E 03
     C      =    0.139164561566E 00                       VARIANCE OF C =    0.656559905016E-02


FOR ADC-1


     PREDICTED ERROR RATE =    0.302330372505E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =    0.996981215878E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999969766952E 00
       BY SERIES  =    0.999969766966E 00


FOR ADC-2


     PREDICTED ERROR RATE =    0.122372439364E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =    0.998777016288E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999987762745E 00
       BY SERIES  =    0.999987762756E 00
```

Table C-1. (Cont'd)

```
                                    BIVARIATE EXTREME VALUE
                                         TEST   1

ITERATION   8


THE PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED:
       BY LOGS    =    0.999962379112E 00
       BY SERIES =    0.999962379126E 00


PROBABILITY P OF A CORRECT BIT BEING RECEIVED AND ACCEPTED     =    0.999962379126E 00
PROBABILITY Q OF A CORRECT BIT BEING RECEIVED AND REJECTED     =    0.738784001441E-05
PROBABILITY R OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTED =    0.253836296906E-04
PROBABILITY S OF AN INCORRECT BIT BEING RECEIVED AND REJECTED =    0.484940755996E-05

FOR A COMMAND OF LENGTH =   100 BITS:

     P =    0.996244909889E 00
     Q =    0.736306745239E-03
     R =    0.253210665323E-02
     S =    0.486676712171E-03
```

Table C-1. (Cont'd)

BIVARIATE EXTREME VALUE
TEST 1

```
THERE ARE    30 GROUPS OF   100 DATA POINTS EACH.

ADC1 CHANNEL THRESHOLD =    0.00000

ADC2 CHANNEL THRESHOLD = -157.00000

A =    1.50000

ABS[XADC1[N]-XADC2[N]] LESS THAN A OCCURS  24.00 TIMES.
```

Table C-1. (Cont'd)

```
                                  BIVARIATE EXTREME VALUE
                                         TEST   1

ITERATIØN   8



FØR THE FØLLØWING CALCULATIØNS:

     ALPHA1 =    0.334996709373E-01                  U1 =   -0.173177517458E 03
     ALPHA2 =    0.222900564506E-01                  U2 =   -0.300844906796E 03
     C      =    0.139164561566E 00                  VARIANCE ØF C =   0.656559905016E-02


FØR ADC-1


     PREDICTED ERRØR RATE =    0.302330372505E-04
     CUMULATIVE PRØBABILITY AT TRIGGER LEVEL =   0.996981215878E 00
     THE NDP RØØT ØF THE CUMULATIVE PRØBABILITY:
       BY LØGS   =    0.999969766952E 00
       BY SERIES =    0.999969766966E 00


FØR ADC-2


     PREDICTED ERRØR RATE =    0.404975566198E-03
     CUMULATIVE PRØBABILITY AT TRIGGER LEVEL =   0.960303633117E 00
     THE NDP RØØT ØF THE CUMULATIVE PRØBABILITY:
       BY LØGS   =    0.999595024419E 00
       BY SERIES =    0.999595024434E 00
```

Table C-1. (Cont'd)

```
                                         BIVARIATE EXTREME VALUE
                                               TEST    1

ITERATION    8



THE PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED:
        BY LOGS    =    0.999580457734E 00
        BY SERIES =    0.999580457745E 00


PROBABILITY P OF A CORRECT BIT BEING RECEIVED AND ACCEPTED     =    0.999580457745E 00
PROBABILITY Q OF A CORRECT BIT BEING RECEIVED AND REJECTED     =    0.389309221645E-03
PROBABILITY R OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTED =    0.145666890603E-04
PROBABILITY S OF AN INCORRECT BIT BEING RECEIVED AND REJECTED =    0.156663481902E-04

FOR A COMMAND OF LENGTH =    100 BITS:

     P =    0.958905231812E 00
     Q =    0.380759848230E-01
     R =    0.139840217889E-02
     S =    0.162038119015E-02
```

Table C-1. (Cont'd)

```
                                          BIVARIATE EXTREME VALUE
                                                 TEST    1



THERE ARE     30 GROUPS OF    100 DATA POINTS EACH.

ADC1 CHANNEL THRESHOLD =     0.00000

ADC2 CHANNEL THRESHOLD = -252.00000

A =     1.50000

ABS[XADC1[N]-XADC2[N]] LESS THAN A OCCURS  24.00 TIMES.
```

Table C-1. (Cont'd)

```
                                   BIVARIATE EXTREME VALUE
                                          TEST   1

ITERATION   8



FOR THE FOLLOWING CALCULATIONS:

     ALPHA1 =    0.334996709373E-01                  U1 =   -0.173177517458E 03
     ALPHA2 =    0.222900564506E-01                  U2 =   -0.300844906796E 03
     C      =    0.139164561566E 00                  VARIANCE OF C =    0.656559905016E-02


FOR ADC-1


     PREDICTED ERROR RATE =    0.302330372505E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.996981215878E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999969766952E 00
       BY SERIES =     0.999969766966E 00


FOR ADC-2


     PREDICTED ERROR RATE =    0.336069115292E-02
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.714169394018E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.996639308803E 00
       BY SERIES =     0.996639308851E 00
```

Table C-1. (Cont'd)

```
                                        BIVARIATE EXTREME VALUE
                                             TEST    1

ITERATIØN    8



THE PRØBABILITY ØF A CØRRECT BIT BEING RECEIVED AND ACCEPTED:
        BY LØGS   =   0.996625800886E 00
        BY SERIES =   0.996625800919E 00


PRØBABILITY P ØF A CØRRECT BIT BEING RECEIVED AND ACCEPTED    =   0.996625800919E 00
PRØBABILITY Q ØF A CØRRECT BIT BEING RECEIVED AND REJECTED    =   0.334396604739E-02
PRØBABILITY R ØF AN INCØRRECT BIT BEING RECEIVED AND ACCEPTED =   0.135079317260E-04
PRØBABILITY S ØF AN INCØRRECT BIT BEING RECEIVED AND REJECTED =   0.167251055245E-04

FØR A CØMMAND ØF LENGTH =   100 BITS:

      P =   0.713202096908E 00
      Q =   0.283779119727E 00
      R =   0.967299005424E-03
      S =   0.205148436361E-02
```

Table C-1. (Cont'd)

```
                         BIVARIATE EXTREME VALUE
                              TEST    1


THERE ARE     30 GROUPS OF    100 DATA POINTS EACH.

ADC1 CHANNEL THRESHOLD =     0.00000

ADC2 CHANNEL THRESHOLD =     0.00000

A =    2.00000

ABS(XADC1(N)-XADC2(N)) LESS THAN A OCCURS  27.00 TIMES.


  8 ITERATIONS TO BE PERFORMED.

BIVARIATE CALCULATIONS WILL OCCUR EVERY  4 ITERATIONS.


ERROR ESTIMATE FOR MAXIMUM LIKELIHOOD FIT =   0.999999999998E-04
```

Table C-1. (Cont'd)

```
                         BIVARIATE EXTREME VALUE
                              TEST    1

ITERATION    0



FOR THE FOLLOWING CALCULATIONS:

     ALPHA1 =    0.333626955910E-01             U1 =   -0.171631574073E 03
     ALPHA2 =    0.228594114902E-01             U2 =   -0.302891621230E 03
     C      =    0.198338358297E 00             VARIANCE OF C =    0.517705896666E-02


FOR ADC-1

     PREDICTED ERROR RATE =    0.325973815051E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =    0.996745515564E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999967402611E 00
       BY SERIES =    0.999967402619E 00


FOR ADC-2

     PREDICTED ERROR RATE =    0.983955396805E-05
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =    0.999016522837E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999990160421E 00
       BY SERIES =    0.999990160446E 00
```

Table C-1. (Cont'd)

BIVARIATE EXTREME VALUE
TEST 1

ITERATION 0

THE PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED:
BY LOGS = 0.999963559440E 00
BY SERIES = 0.999963559458E 00

PROBABILITY P OF A CORRECT BIT BEING RECEIVED AND ACCEPTED = 0.999963559458E 00
PROBABILITY Q OF A CORRECT BIT BEING RECEIVED AND REJECTED = 0.384316081181E-05
PROBABILITY R OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTED = 0.266009883489E-04
PROBABILITY S OF AN INCORRECT BIT BEING RECEIVED AND REJECTED = 0.599639315624E-05

FOR A COMMAND OF LENGTH = 100 BITS:

P = 0.996362511112E 00
Q = 0.383004935429E-03
R = 0.265401256911E-02
S = 0.600471386860E-03

Table C-1. (Cont'd)

```
                         BIVARIATE EXTREME VALUE
                               TEST   1

ITERATION    4



FOR THE FOLLOWING CALCULATIONS:

     ALPHA1 =    0.335083649913E-01              U1 =   -0.173182336089E 03
     ALPHA2 =    0.222935173401E-01              U2 =   -0.300849992007E 03
     C      =    0.139019899004E 00              VARIANCE OF C =    0.576318288108E-02


FOR ADC-1


     PREDICTED ERROR RATE =    0.301826730719E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.996986236663E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999969817312E 00
       BY SERIES =    0.999969817327E 00


FOR ADC-2


     PREDICTED ERROR RATE =    0.122231213026E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.998778426638E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999987776868E 00
       BY SERIES =    0.999987776882E 00
```

Table C-1. (Cont'd)

```
                                   BIVARIATE EXTREME VALUE
                                           TEST    1

ITERATION    4



THE PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED:
        BY LOGS    =   0.999962432245E 00
        BY SERIES  =   0.999962432255E 00


PROBABILITY P OF A CORRECT BIT BEING RECEIVED AND ACCEPTED     =   0.999962432255E 00
PROBABILITY Q OF A CORRECT BIT BEING RECEIVED AND REJECTED     =   0.738507151254E-05
PROBABILITY R OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTED  =   0.253446269198E-04
PROBABILITY S OF AN INCORRECT BIT BEING RECEIVED AND REJECTED  =   0.483804979012E-05

FOR A COMMAND OF LENGTH =    100 BITS:

     P =    0.996250203072E 00
     Q =    0.736034584406E-03
     R =    0.252822441689E-02
     S =    0.485537922941E-03
```

Table C-1. (Cont'd)

```
                              BIVARIATE EXTREME VALUE
                                     TEST   1

ITERATION   8



FOR THE FOLLOWING CALCULATIONS:

     ALPHA1 =    0.334999273013E-01                  U1 =   -0.173177831350E 03
     ALPHA2 =    0.222900797034E-01                  U2 =   -0.300844671669E 03
     C      =    0.139164789797E 00                  VARIANCE OF C =    0.576169327848E-02


FOR ADC-1

     PREDICTED ERROR RATE =    0.302313710562E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.996981381388E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999969768614E 00
       BY SERIES =    0.999969768629E 00


FOR ADC-2

     PREDICTED ERROR RATE =    0.122372221085E-04
     CUMULATIVE PROBABILITY AT TRIGGER LEVEL =   0.998777018431E 00
     THE NDP ROOT OF THE CUMULATIVE PROBABILITY:
       BY LOGS    =    0.999987762767E 00
       BY SERIES =    0.999987762778E 00
```

Table C-1. (Cont'd)

```
                                BIVARIATE EXTREME VALUE
                                       TEST    1

ITERATION    8


THE PROBABILITY OF A CORRECT BIT BEING RECEIVED AND ACCEPTED:
       BY LOGS    =    0.999962380716E 00
       BY SERIES =    0.999962380734E 00


PROBABILITY P OF A CORRECT BIT BEING RECEIVED AND ACCEPTED     =    0.999962380734E 00
PROBABILITY Q OF A CORRECT BIT BEING RECEIVED AND REJECTED     =    0.738789458410E-05
PROBABILITY R OF AN INCORRECT BIT BEING RECEIVED AND ACCEPTED =    0.253820435318E-04
PROBABILITY S OF AN INCORRECT BIT BEING RECEIVED AND REJECTED =    0.484933116240E-05


FOR A COMMAND OF LENGTH =    100 BITS:

     P =    0.996245070091E 00
     Q =    0.736312296794E-03
     R =    0.253194863034E-02
     S =    0.486668985104E-03
```

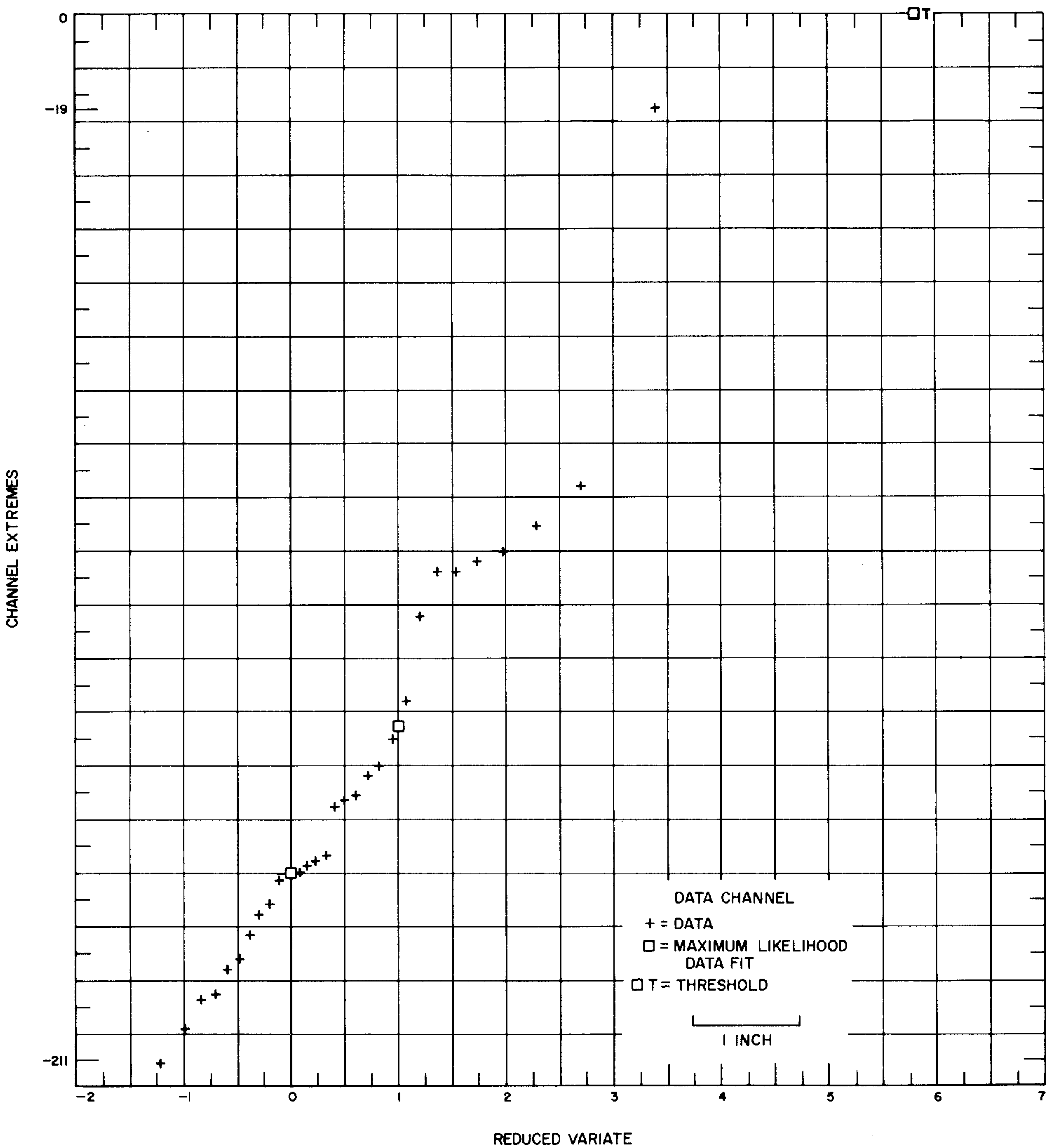


Fig. C-1. Computer plot for data channel

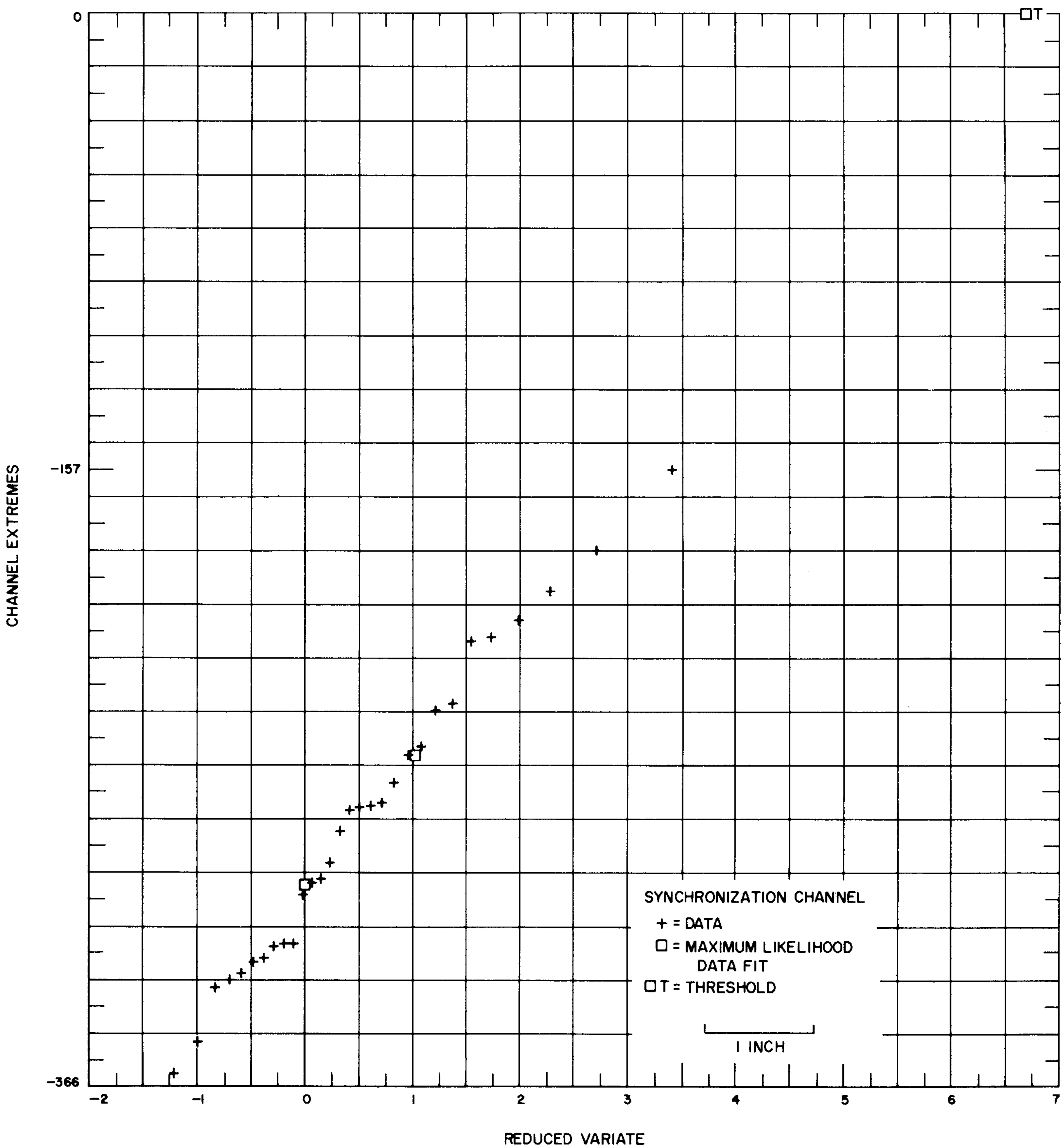


Fig. C-2. Computer plot for synchronization channel

Table C-2. Operational directions of the data-processing program

```
SET BP1 IF LOOKING FOR A MAXIMUM FOR ADC-1.
   RESET BP1 IF LOOKING FOR A MINIMUM.
SET BP2 IF LOOKING FOR A MAXIMUM FOR ADC-2.
   RESET BP2 IF LOOKING FOR A MINIMUM.
SET BP3 FOR PRINTOUT OF CHANNEL EXTREMES AND OPTION TO OBTAIN A GUMBEL PLOT.
SET BP4 FOR BIVARIATE ANALYSIS.
CLEAR HALT.


IF AN ERROR IS MADE WHILE TYPING INPUTS, DO THE FOLLOWING
  1.  PUT RUN-IDLE-STEP [R-I-S] SWITCH TO IDLE
  2.  SET REGISTER KNOB TO C
  3.  PUSH START
  4.  FILL REGISTER DISPLAY WITH A  BRU Ø3522  COMMAND,
        THAT IS, WITH THE OCTAL NUMBER ØØ1Ø3522
  5.  PUT R-I-S SWITCH TO RUN
  6.  RETYPE INPUTS


TYPE IN FORMAT [3I4,3F12.5]
  ITN--TEST NO.
  NG--NO. OF GROUPS
  NDP--NO. OF SAMPLES/GROUP
  T1,T2--ADC1,ADC2 THRESHOLDS
  ERROR--ERROR FOR UNIVARITE MAXIMUM LIKELIHOOD FIT

1,3Ø,1ØØ,Ø.Ø,Ø.Ø,Ø.ØØØØ1,

SET BP1 FOR ADC-1 GUMBEL PLOT
  IF SET, POSITION PLOTTER PEN AT BOTTOM RIGHT-HAND CORNER OF GRAPH PAPER
CLEAR HALT


SET BP2 FOR ADC-2 GUMBEL PLOT
  IF SET, POSITION PLOTTER PEN AT BOTTOM RIGHT-HAND CORNER OF GRAPH PAPER
CLEAR HALT


IF AN ERROR IS MADE WHILE TYPING INPUTS, DO THE FOLLOWING
  1.  PUT RUN-IDLE-STEP [R-I-S] SWITCH TO IDLE
  2.  SET REGISTER KNOB TO C
  3.  PUSH START
  4.  FILL REGISTER DISPLAY WITH A  BRU Ø3531  COMMAND,
        THAT IS, WITH THE OCTAL NUMBER ØØ1Ø3531
  5.  PUT R-I-S SWITCH TO RUN
  6.  RETYPE INPUTS


INPUT IN FORMAT 2I1Ø,2F15.5
  NIT--ITERATIONS FOR BEVT MAXIMUM LIKELIHOOD FIT
  NBEVT--ITERATIONS BEFORE EACH BEVT PROBABILITY CALCULATION
  A--STRIP ESTIMATE PARAMETER
    A MUST BE IN THE CLOSED INTERVAL 1.5 TO 2.Ø
  ERROR--ERROR FOR BIVARIATE MAXIMUM LIKELIHOOD FIT

8,4,1.5,Ø.ØØØ1,
```

Table C-2. (Cont'd)

```
SET BP3 TO CHANGE THE VALUE OF PARAMETER A
SET BP4 TO CHANGE THE CHANNEL THRESHOLDS

IF NEITHER BREAKPOINT IS SET, CONTROL TRANSFERS TO LINK[1]
CLEAR HALT TO PROCEED


INPUT IN FORMAT 2F1Ø.5
  NEW ADC-1 THRESHOLD VALUE
  NEW ADC-2 THRESHOLD VALUE
IF AN ERROR IS MADE WHILE TYPING, REPEAT THE 6 STEPS LISTED ABOVE,
  EXCEPT IN STEP 4 FILL THE REGISTER DISPLAY WITH A BRU Ø5Ø26 COMMAND,
  THAT IS, WITH THE OCTAL NUMBER ØØ1Ø5Ø26

Ø.Ø,-157.Ø,

SET BP3 TO CHANGE THE VALUE OF PARAMETER A
SET BP4 TO CHANGE THE CHANNEL THRESHOLDS

IF NEITHER BREAKPOINT IS SET, CONTROL TRANSFERS TO LINK[1]
CLEAR HALT TO PROCEED


INPUT IN FORMAT 2F1Ø.5
  NEW ADC-1 THRESHOLD VALUE
  NEW ADC-2 THRESHOLD VALUE
IF AN ERROR IS MADE WHILE TYPING, REPEAT THE 6 STEPS LISTED ABOVE,
  EXCEPT IN STEP 4 FILL THE REGISTER DISPLAY WITH A BRU Ø5Ø26 COMMAND,
  THAT IS, WITH THE OCTAL NUMBER ØØ1Ø5Ø26

Ø.Ø,-252.Ø,

SET BP3 TO CHANGE THE VALUE OF PARAMETER A
SET BP4 TO CHANGE THE CHANNEL THRESHOLDS

IF NEITHER BREAKPOINT IS SET, CONTROL TRANSFERS TO LINK[1]
CLEAR HALT TO PROCEED


INPUT THE NEW VALUE FOR A IN FORMAT F1Ø.5
  IF AN ERROR IS MADE WHILE TYPING, REPEAT THE 6 STEPS LISTED ABOVE,
    EXCEPT IN STEP 4 FILL THE REGISTER DISPLAY WITH A BRU Ø5Ø13 COMMAND,
    THAT IS, WITH THE OCTAL NUMBER ØØ1Ø5Ø13

2.Ø,

INPUT IN FORMAT 2F1Ø.5
  NEW ADC-1 THRESHOLD VALUE
  NEW ADC-2 THRESHOLD VALUE
IF AN ERROR IS MADE WHILE TYPING, REPEAT THE 6 STEPS LISTED ABOVE,
  EXCEPT IN STEP 4 FILL THE REGISTER DISPLAY WITH A BRU Ø5Ø26 COMMAND,
  THAT IS, WITH THE OCTAL NUMBER ØØ1Ø5Ø26

Ø.Ø,Ø.Ø,
```

Table C-2. (Cont'd)

```
SET BP3 TO CHANGE THE VALUE OF PARAMETER A
SET BP4 TO CHANGE THE CHANNEL THRESHOLDS

IF NEITHER BREAKPOINT IS SET, CONTROL TRANSFERS TO LINK[1]
CLEAR HALT TO PROCEED


JOB DONE.  READY NEW INPUT.


SET BP1 IF LOOKING FOR A MAXIMUM FOR ADC-1.
   RESET BP1 IF LOOKING FOR A MINIMUM.
SET BP2 IF LOOKING FOR A MAXIMUM FOR ADC-2.
   RESET BP2 IF LOOKING FOR A MINIMUM.
SET BP3 FOR PRINTOUT OF CHANNEL EXTREMES AND OPTION TO OBTAIN A GUMBEL PLOT.
SET BP4 FOR BIVARIATE ANALYSIS.
CLEAR HALT.
```

REFERENCES

1. Posner, E. C., *The Application of Extreme-Value Theory to Error-Probability Estimation in the* Ranger *Block III Command Detector*, Technical Report 32-705. Jet Propulsion Laboratory, Pasadena, Calif., Jan. 15, 1965.
2. Gumbel, F. J., *Statistics of Extremes*. Columbia University Press, New York, 1958.
3. Rodemich, E., "Joint Asymptotic Distributions," in *Supporting Research and Advanced Development*, Space Programs Summary 37-36, volume IV, pp. 317–323. Jet Propulsion Laboratory, Pasadena, Calif., 1966.
4. Hildebrand, F. B., *Introduction to Numerical Analysis*. McGraw-Hill Book Company, Inc., New York, 1956.
5. Epstein, B., and Brooks, H., "The Theory of Extreme Values and Its Implications in the Study of the Dielectric Strength of Paper Capacitors," *J. Appl. Phys.*, vol. 19, pp. 544–550, June 1948.

BIBLIOGRAPHY

Ashlock, J. C., and Posner, E. C., "Application of the Statistical Theory of Extreme Values to Spacecraft Receivers," in *Proceedings of IEEE National Telemetry Conference*, May 10–12, 1966, pp. 167–172, Raytheon Company, Bedford Publishing Department, Boston, April 1966.

Ashlock, J. C., and Posner, E. C., *Application of the Statistical Theory of Extreme Values to Spacecraft Receivers*, Technical Report 32-737, Jet Propulsion Laboratory, Pasadena, Calif., May 15, 1965.

Ashlock, J. C., and Posner, E. C., "Error Probability Estimation Using Bivariate Extreme Value Theory," in *Supplement to IEEE Transactions on Aerospace and Electronic Systems*, Volume AES-2, Number 4, pp. 463–468. Institute of Electrical and Electronic Engineers, New York, July 1966.

Darling, D. A., "The Kolmogorov-Smirnov, Cramér-Von Mises Tests," *Ann. Math. Statist.*, vol. 28, pp. 823–835, 1951.

Dodd, E. L., "The Greatest and the Least Variate Under General Laws of Error," *Trans. Am. Math. Soc.*, vol. 25, pp. 525–539, 1923.

Doob, J. L., "Heuristic Approach to the Kolmogorov-Smirnov Theorems," *Ann. Math. Statist.*, vol. 20, pp. 393–403, 1949.

Epstein, B., "Application of the Theory of Extreme Values in Fracture Problems," *Am. Statist. Ass. J.*, pp. 403–412, Sept. 1948 (Presented at the 107th Annual Meeting of the American Statistical Association, December 30, 1947).

Fisher, R. A., "On the Mathematical Foundations of Theoretical Statistics," *Phil. Trans. Roy. Soc. London* [A], vol. 222, pp. 309–368, 1922.

Fisher, R. A., and Tippett, L. H. C., "Limiting Forms of the Frequency Distribution of the Largest or Smallest Member of a Sample," *Proc. Cambridge Phil. Soc.*, vol. 24, pt. 2, pp. 180–190, April 1928.

Gumbel, E. J., "Bivariate Exponential Distributions," *Am. Statist. Ass. J.*, pp. 698–707, December 1960.

Gumbel, E. J., "Bivariate Logistic Distributions," *Am. Statist. Ass. J.*, pp. 335–349, June 1961.

Gumbel, E. J., "Les Valeurs Extrêmes des Distributions Statistiques," *Ann. de l'Institut Henri Poincaré*, vol. V, pp. 115–158, 1935 (in French).

Gumbel, E. J., and Greenwood, J. A., "Table of the Asymptotic Distribution of the Second Extreme," *Ann. Math. Statist.*, vol. 22, pp. 121–124, March 1951.

Gumbel, E. J., and von Schelling, H., "The Distribution of the Number of Exceedances," *Ann. Math. Statist.*, vol. 21, pp. 247–262, June 1950.

Kimball, B. F., "An Approximation to the Sampling Variance of an Estimated Maximum Value of Given Frequency Based on Fit of Doubly Exponential Distribution of Maximum Values," *Ann. Math. Statist.*, vol. 20, pp. 110–113, March 1949.

Kimball, B. F., "Limited Type of Primary Probability Distribution Applied to Annual Maximum Flood Flows," *Ann. Math. Statist.*, vol. 13, pp. 318–325, Sept. 1942.

Lieblein, J., *A New Method of Analyzing Extreme-Value Data*, NACA TN 3053. National Advisory Committee for Aeronautics, National Bureau of Standards, Washington, January 1954.

Miller, L. H., "Table of Percentage Points of Kolmogorov Statistics," *Am. Statist. Ass. J.*, pp. 111–121, March 1956.

Plackett, R. L., "A Class of Bivariate Distributions," *Am. Statist. Ass. J.*, pp. 516–522, June 1965.

Posner, E. C., "Extension of Extreme-Value Theory for Error-Probability Estimation," in *Supporting Research and Advanced Development*, Space Programs Summary 37-28, Volume IV, pp. 213–219, Jet Propulsion Laboratory, Pasadena, Calif., Aug. 31, 1964.

Posner, E. C., and Zohar, S., "A Distribution Free Goodness-of-Fit Test for Use in Extreme-Value Theory," in *Supporting Research and Advanced Development*, Space Programs Summary, 37-29, Volume IV, pp. 336–339. Jet Propulsion Laboratory, Pasadena, Calif., Oct. 31, 1964.

Sibuya, M., "Bivariate Extreme Statistics, I," *Ann. Inst. Statist. Math.* (Tokyo), vol. 11, pp. 195–210, 1960.