MODIFICATIONS TO THE THEORY OF THE DIFFERENTIAL ABSORPTION EXPERIMENT

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Electro-magnetic waves of medium frequency, which are partially reflected from irregularities in the mesosphere, were first used to measure electron densities by Gardner and Pawsey in 1953. The method used is discussed at some length in the original paper, and later by Fejer and Vice (1959), and Belrose and Burke (1964).

If the receiver outputs for the ordinary and extraordinary components are A_o, A_e; the corresponding Fresnel reflection coefficients are R_o, R_e; the absorption coefficients are K_o, K_e, and the electron density at height h is N(h), then

$$A_{o,e} = Y_{o,e} \cdot R_{o,e} \exp\left(-\int_o^h 2 \frac{K_{o,e}}{N(h)} \, N(h) dh\right) \tag{1}$$

Providing the transfer function, $Y_{o,e}$, of the receiving equipment is identical for both components, equation (1) gives

$$\frac{A_e}{A_o} = \frac{R_e}{R_o} \exp\left(- \int_o^h 2 \frac{(K_e - K_o)}{N(h)} N(h)dh\right) \tag{2}$$

The ratio R_e/R_o (as defined by Gardner and Pawsey and other workers), and $\frac{(K_e - K_o)}{N(h)}$, may be calculated from electro-magnetic theory. In the lower mesosphere the collision frequency is of the order of the wave frequency, and it is necessary to use the generalized theory, where the energy dependence of the collision frequency is considered (Phelps and Pack, 1959; Sen and Wyller, 1960). It may be shown that for a probing frequency of 2.4 Mc/s, both R_e/R_o and $(K_e - K_o)/N(h)$ are essentially independent of N(h) for the conditions encountered in the mid-latitude mesosphere. The errors involved in this assumption are small and are not important to the argument which follows.

The modification proposed here involves the Fresnel reflection coefficient of the irregularities. This is defined as

$$R = \frac{n_2 - n_1}{n_2 + n_1}$$ [redacted]

where n_1, n_2 are the refractive indices for an electro-magnetic wave, in regions of the ionosphere where the refractive index changes rapidly within one wavelength. The validity of this latter assumption depends on the nature of these irregularities in the mesosphere. Unfortunately, rocket measurements of relevant variables from 60-90 km are not generally sensitive enough to show the small changes in refractive index which are required for the observed reflections. It is to be hoped that future rocket-born experiments will give information as to the change in refractive index with height.

The most plausible explanation for these irregularities is due to Hines (1960), who described them in terms of internal atmospheric gravity waves. He suggested that the partial reflections from 60-75 km were due to irregularities which were caused directly by these gravity waves; and above 80 km were due to turbulence caused by the waves. Observations of the partial reflections seem to bear out this hypothesis. The lowest echoes show seasonal variations in occurrence, and slow, highly correlated (dependent) fading of the two reflected components at a given height; while the stronger echoes above 80 km show more rapid fading, often of uncorrelated (non-dependent) nature. These latter echoes suggest irregularities smaller than the Fresnel zone in horizontal extent. Booker's (1959) paper on the scattering of electro-magnetic waves from ionospheric irregularities, although lacking in generality, suggested that the definition of the reflection coefficient (equation 3) holds even under these conditions, providing the irregularities produce similar reflected wave spectra for ordinary and extraordinary components. This is a reasonable assumption if the irregularities in refractive index are due to turbulence.

Experimental evidence obtained by using a gated integration device on the two reflected components, has shown that the measured amplitude ratio of dependent and independent fading signals is the same within experimental error. It therefore seems justified with present experimental and theoretical evidence to use the Fresnel reflection coefficient for studies of the partial reflections from the mesosphere.

Returning to the definition of R, where the refractive index tends to unity, and $n_2 - n_1 = \delta n$ is small,

$$R_o \doteq \delta n_o / 2 \tag{4}$$

and

$$R_o / R_e \doteq \delta n_o / \delta n_e \tag{5}$$

Since $n_o{}^2$ is a function of electron density N(h), and collision frequency ν_m, then

$$n_{o,e}{}^2 = f(\nu_m, N(h)) \tag{6}$$

and

$$\delta n_{o,e} = \tfrac{1}{2} \left(\frac{\partial f}{\partial \nu_m} \delta \nu_m + \frac{\partial f}{\partial N(h)} \delta N(h) \right) \quad (7)$$

The definition of R used by previous workers assumed that the change in collision frequency, $\delta \nu_m$, was zero. However, if the echoes are due to irregularities in the atmospheric density profile, which are caused by internal gravity waves, there will be an associated pressure change and hence a change in collision frequency within each irregularity.

The changes in $\log_e (R_e/R_o)$ for electron-density irregularities, and combined electron-density and collision frequency irregularities (Fig. 1) are shown in figure 1. A change in N(h) of 1-10 per cent is required for the observed echoes, and changes in ν_m of 1-10 per cent are theoretically predicted, so these curves are quite realistic for the lower mesosphere. It is evident that the value of (R_e/R_o) is strongly dependent upon the relative magnitudes of $\delta \nu_m / \nu_m$ and $\delta N(h)/N(h)$, and hence requires a knowledge of the physical nature of the irregularity. This was clearly unnecessary with the earlier treatment, when the independence of (R_e/R_o) upon the size of the electron density irregularity enabled an explicit expression for N(h) to be obtained. It has also been found that the dependence of R_e/R_o on $\delta\nu_m/\nu_m$ decreases with height, and becomes quite small above 80 km. At these heights the error introduced into electron density calculations by observational techniques would usually be larger than that due to the omission of the collision frequency term from equation 7.

The error introduced by ignoring the collision frequency irregularities at the lower heights (60-80 km) is significant however.

(Figure 2) This is demonstrated by figure 2, which presents electron density measurements made at Birdling's Flat, New Zealand, during the winter of 1964. The three curves were obtained from the same experimental data, by substituting three different values of the collision frequency irregularity ($\delta\nu_m/\nu_m$) into the expression for the reflection coefficients. Further theoretical and experimental work is intended, to help clarify the basis of the reflection mechanism.

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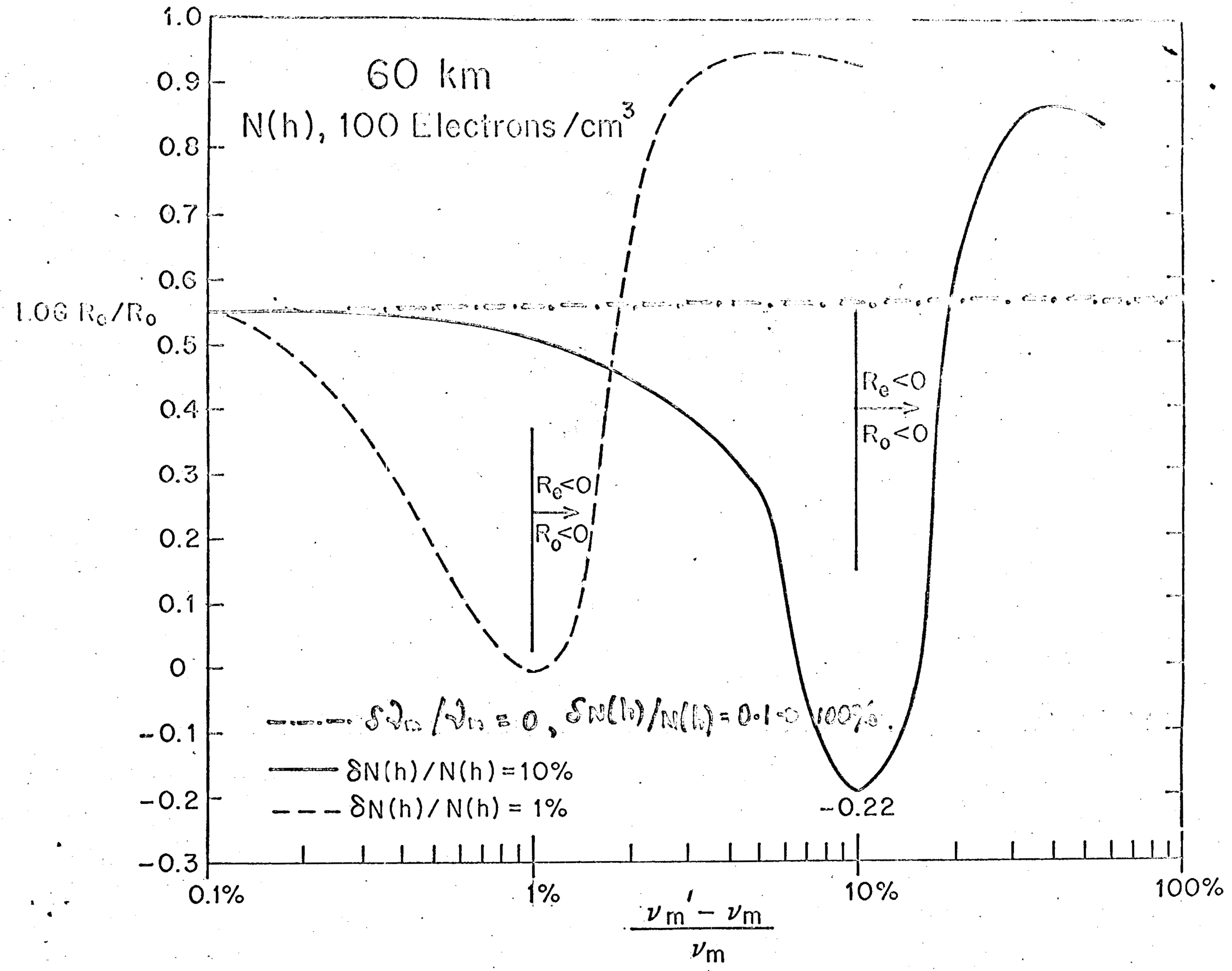

60 km
N(h), 100 Electrons/cm^3
LOG R_e/R_o
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.1%
1%
10%
100%
$\frac{\nu_m' - \nu_m}{\nu_m}$
$R_e<0$
$R_o<0$
$R_e<0$
$R_o<0$
-0.22
$\delta\nu_m/\nu_m = 0$, $\delta N(h)/N(h) = 0.1 \to 100\%$.
$\delta N(h)/N(h) = 10\%$
$\delta N(h)/N(h) = 1\%$

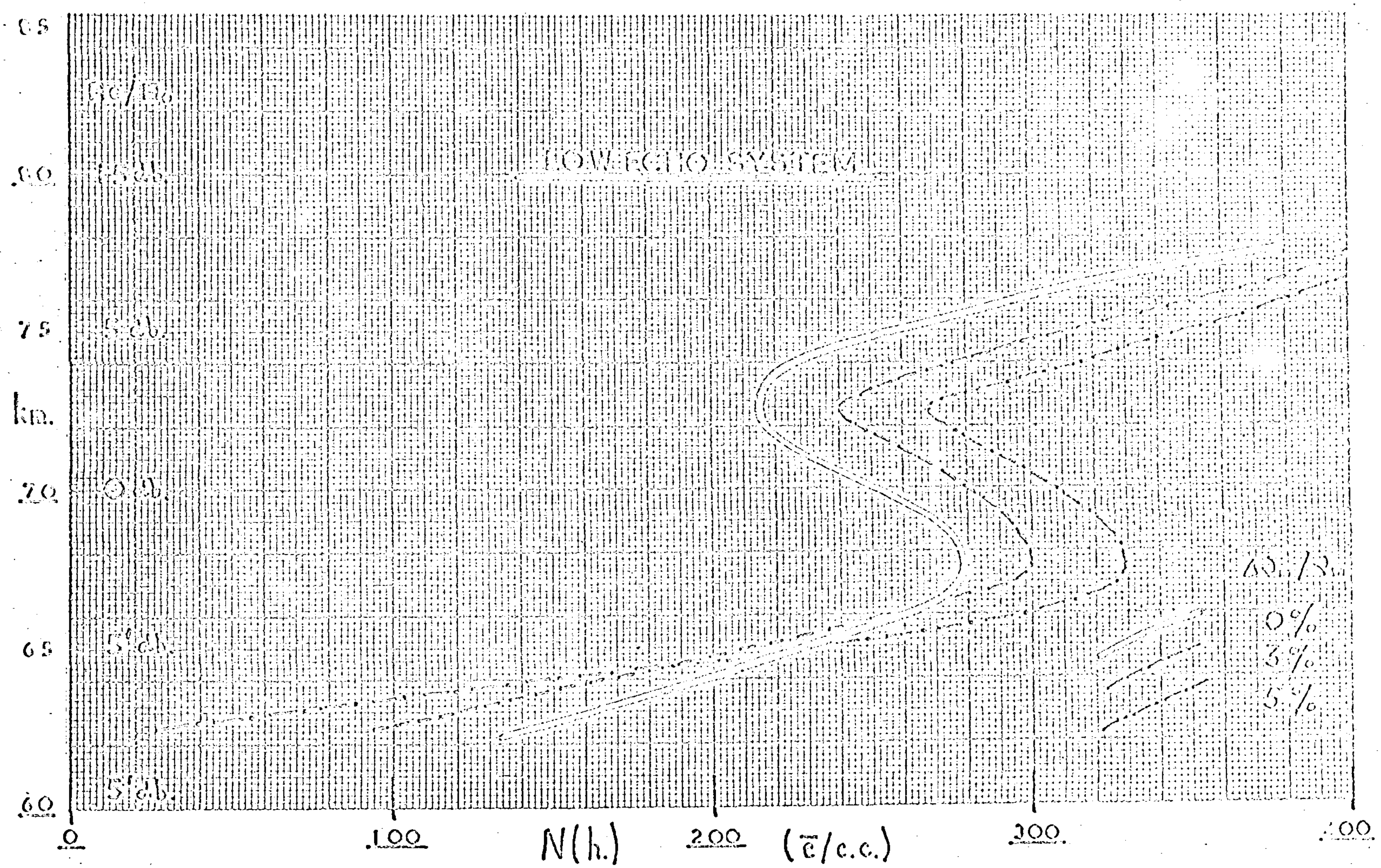

FIGURE 2. N(h) ; $\Delta\nu_0/\nu_0$ 0,3,5% ; $\Delta N(h)/N(h)$ 10%.