GRADUATE AERONAUTICAL LABORATORIES
CALIFORNIA INSTITUTE OF TECHNOLOGY

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on

The Buckling of Cylindrical Shells

NASA Grant NsG 18-59

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Firestone Flight Sciences Laboratory

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[redacted]

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THE BUCKLING OF CYLINDRICAL SHELLS

A. Buckling of Initially Imperfect Cylindrical Shells

A series of tests on the buckling of electroformed cylindrical shells under uniform axial compression with a local initial imperfection has been completed. The imperfection consists of a circular area which is positioned half way between the ends of the shell. The imperfection is shown in figure 1.

The data from these tests is shown in Table 1 and plotted in figure 2. This data is represented as a buckling load P_{cr} divided by the classical load, $P_{c\ell}$ versus the size of the imperfection a, divided by the radius of the shell R. The radius to thickness ratio for the shells tested varied from 1050 to 1250. A plot of load versus depth d of the imperfection would have the same trend as figure 2. While the data have a great deal of scatter in the middle range of a/R, the trend is pretty well established. It is interesting to note that even for very large imperfections, 47 per cent of the classical load can still be obtained.

The largest imperfections tested to date are of the order of the size of the buckled wave form. In the future this will be carried out to larger imperfections to determine if a large drop in the buckling load occurs when the imperfection exceeds the size of the buckled wave form. In addition the effect of an outward imperfection will be studied.

In addition to the imperfection tests on the electroformed shells, some tests have been carried out using shells formed from Mylar. The imperfection is imposed on the shell surface using an external pointer. These tests have the advantage that one shell can be used for the whole series of tests. The character of the results for this type of testing is shown in figure 3. As the figure shows, the buckling load is unaffected by the imposed imperfection until a certain critical depth of imperfection is reached. After this point the buckling load drops and as the imperfection size is increased the curve splits into a general instability and local instability or one wave buckling. The interesting thing to note is that the one wave or local instability is an extension of what appeared to be general instability.

With this information, it was decided to determine the deflection of the shell before buckling to see if the buckling point could be anticipated from the deflection of the shell before buckling occurred. These data have been obtained and are being reduced. A complete report on this project will be furnished in the near future.

B. <u>Influence of Testing Machine on the Buckling Load</u>

Some of the experimental data taken in the past on the buckling of the electroformed cylindrical shells under uniform axial compression has been correlated with respect to the type of testing machine used in the testing. Three different types of loading apparatus have been used to perform these tests.

1. Controlled displacement type of testing machine
2. Center point load from load ring
3. Air pressure loading diaphragm

The first type of loading device has the advantage that the load distribution can be adjusted during the loading.

The parameter used to correlate the data was the stiffness of the testing apparatus (assuming that it was linear) divided by the stiffness of the unbuckled shell. In addition, the buckling load as determined by an energy criterion was also calculated. The result of this calculation and the available experimental data is shown in figure 4, where the buckling load P_{cr} divided by the classical load $P_{c\ell}$ is plotted against the testing machine stiffness K divided by the test shell stiffness K_s. As As the figure shows, the data is independent of the testing machine stiffness but the energy criterion is heavily dependent upon this parameter. A more complete report on this correlation will be furnished in the future.

C. <u>Buckling of Conical Shells - Experimental Work</u>

The design and the construction of the new small controlled end displacement type testing machine has been completed. Its design incorporates the following special features:

1) By the use of matched pairs of high precision pre-loaded thrust bearings the expected axial elastic displacement of the testing machine under load was kept to a minimum.

2) By means of an attachment, consisting of an inductance type pick-up mounted on a traverse that can travel both up and down and in the circumferential direction, we have the capability of determining the initial imperfections of the test specimens and their subsequent deformation under loading.

The development of the production of conical shells by the electroforming process is continuing. When switching from cylindrical to conical shells considerable difficulty has been encountered in trying to obtain a uniform thickness distribution along the generator of the shell. This problem has been solved partially by proper placement of the copper anodes. So far cones with half angles of 5°, 10°, and 15° have been produced with satisfactory thickness distributions (see figure 5). However cones with half angle of 20° show a very marked and undesirable nonlinear increase in thickness toward the smaller end (see figure 6). Work continues on this problem.

As soon as the production of conical shells by the electroforming process has been worked out plans call for a series of buckling tests under axial compression keeping the major diameter and the ratio of length over major diameter of the test specimens constant and varying their half angles from 5° - 25°. Comparison of the results of these experiments with the results of previous tests on similarly built cylindrical shells carried out at this same institute should show up the effect of the half angle α on the critical buckling load.

D. The Stability of Cylindrical Shells under Axisymmetric Moving Loads

The work in this area is concerned with axisymmetric moving loads on cylindrical shells.

The stability of a thin elastic cylindrical shell subjected to a moving load was first investigated in reference 1. In that paper the stability of the steady state[1.] response of an infinite length cylinder subjected to a class of axisymmetric loads moving with a constant velocity was considered. Sufficient conditions for instability were

1. The term "steady state" as used here refers to a time invariant wave form in a coordinate system moving with the load.

developed for load velocities less than the minimum propagation velocity of undistorted sinusoidal wave trains in the shell[2], and a method suitable for determining an upper bound on the transition from stability to instability in this velocity range was presented. The results of the analysis, when applied to a uniform radial line load (ring load) traveling with a constant velocity, indicated a marked dependence of stability on load velocity.

Recently the analysis of this problem has been extended to include both necessary and sufficient conditions for instability. (These further results will be reported in a future publication.) As an application of the theory, the moving ring and decayed step (shock-wave) type loads were considered. The results obtained indicate that the procedure outlined previously for obtaining an upper bound on the transition from stability to instability predicts the transition itself.

Typical results of this study as applied to the decayed step wave is given in graphical form by figures 7 and 8. Explanation is as follows: Figure 7 is a plot of the static critical magnitude[3] (in nondimensional form) of an exponentially decayed step wave as a function of a decay parameter, b. Figure 8 is a typical interaction curve indicating the effect of velocity and decay parameter for a radius to thickness ratio of 100. As $b \rightarrow \infty$ in these plots one obtains a ring load, and as $b \rightarrow o$ a step wave.

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2. This velocity, denoted by V_L in figures 1 - 5, is of the order of 400-2000 f. p. s. for radius to thickness ratios of 1000-40 respectively.

3. The word "critical" refers to the magnitude of P for which a transition from stability to instability occurs.

Note that as the load velocity approaches V_{co}, in either the ring or decayed step cases, the shell becomes unstable under an infinitesimal load magnitude. This is the result of a resonance condition of the axisymmetric response at $V = V_{co}$. The effect of damping (resulting from a fluid-shell interaction for example) is expected to have a considerable influence on this resonance effect, and is presently under study.

E. Dynamic Buckling of Thin Shells

A theoretical study is underway that has, as a final objective, a feasible method of predicting the magnitude of dynamically applied loads (with fixed spatial distribution) that lead to a post-buckled state of the shell. At present several very elementary models are under consideration. It is too early, however, to report on progress.

In addition, an experimental program is being planned in the area of dynamic loading of shells that exhibit a large drop off of load in the post buckled range. This investigation will be conducted simultaneously with the theoretical investigation so that the two programs can complement one another.

F. Buckling of Cylindrical Shells with Random Imperfections

The buckling of a thin, elastic cylindrical shell with a random spatial distribution of geometrical imperfections of random amplitude is presently under study. The shell loading is assumed as a constant axial load. Several types of imperfection distribution are being considered. The investigation is a theoretical one.

REFERENCES

1. Hegemier, G. A.: "Instability of Cylindrical Shells Subjected to Axisymmetric Moving Loads", Journal of Applied Mechanics, Paper No. 65-4PMW-35.

TABLE I

Shell	R/t	d/t	a/R	P_{cr}/P_{cl}
1	1150	.00	.000	.68
2	1050	.00	.000	.75
3	1130	.35	.025	.64
4	1170	1.84	.056	.51
5	1180	3.30	.072	.53
6	1200	3.30	.072	.54
7	1210	3.34	.075	.48
8	1240	4.40	.084	.42
9	1250	4.50	.084	.36
10	1180	5.33	.095	.56
11	1200	6.00	.099	.43
12	1180	7.65	.114	.59
13	1190	7.90	.115	.52
14	1170	8.70	.119	.46
15	1210	9.10	.119	.42
16	1060	13.20	.159	.48
17	1170	14.60	.156	.46
18	1150	19.25	.182	.48

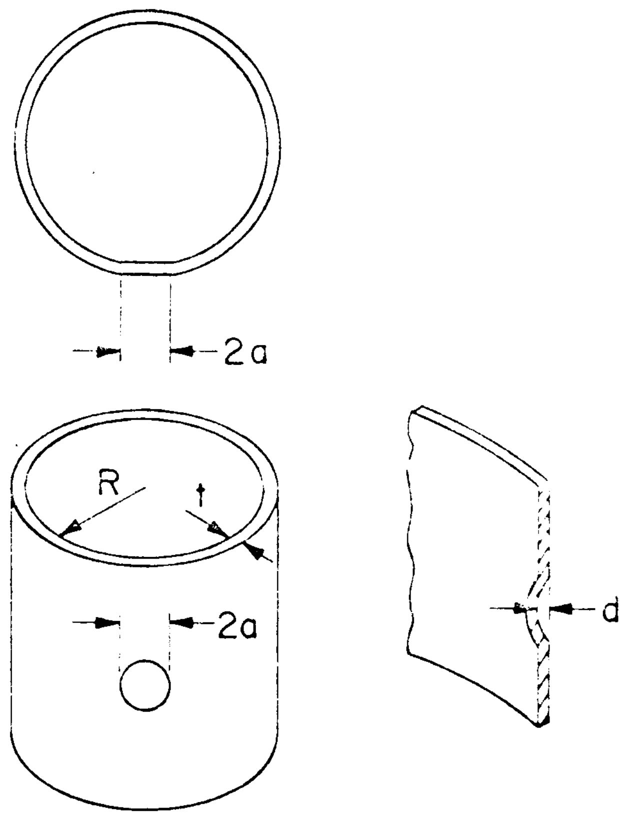


Figure 1

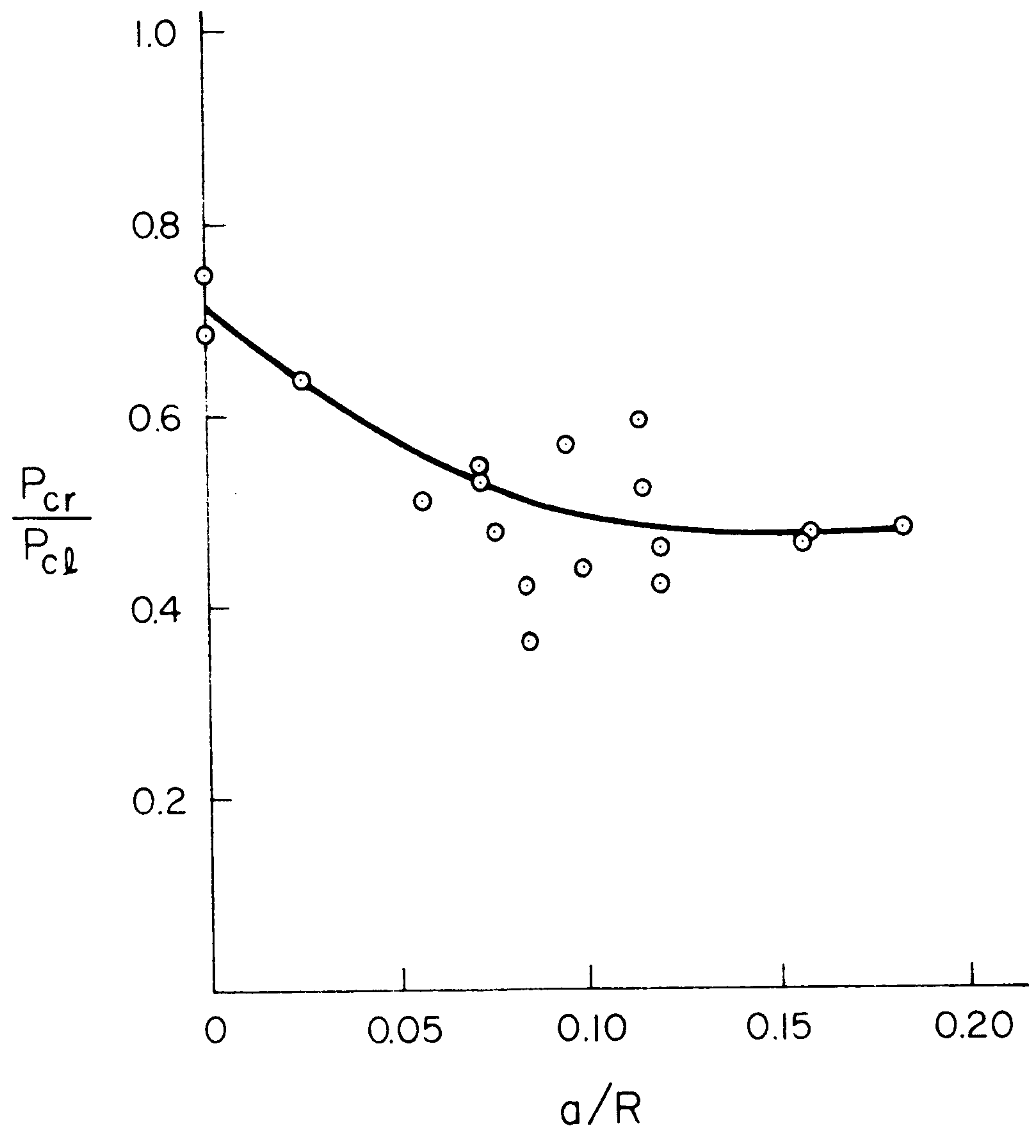
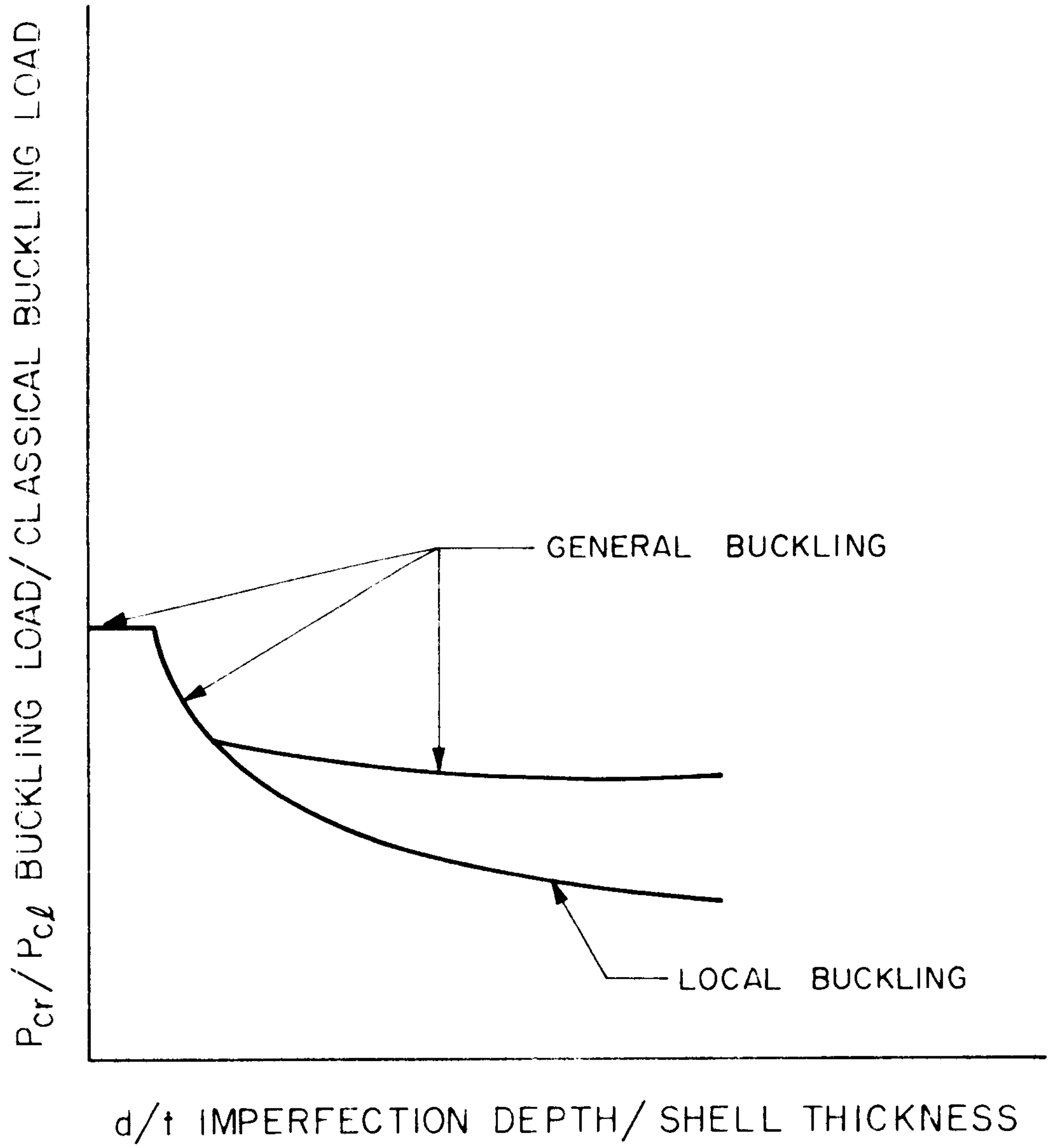


Figure 2



GENERAL BUCKLING
LOCAL BUCKLING
$P_{cr}/P_{c\ell}$ BUCKLING LOAD/CLASSICAL BUCKLING LOAD
d/t IMPERFECTION DEPTH/SHELL THICKNESS

Figure 3

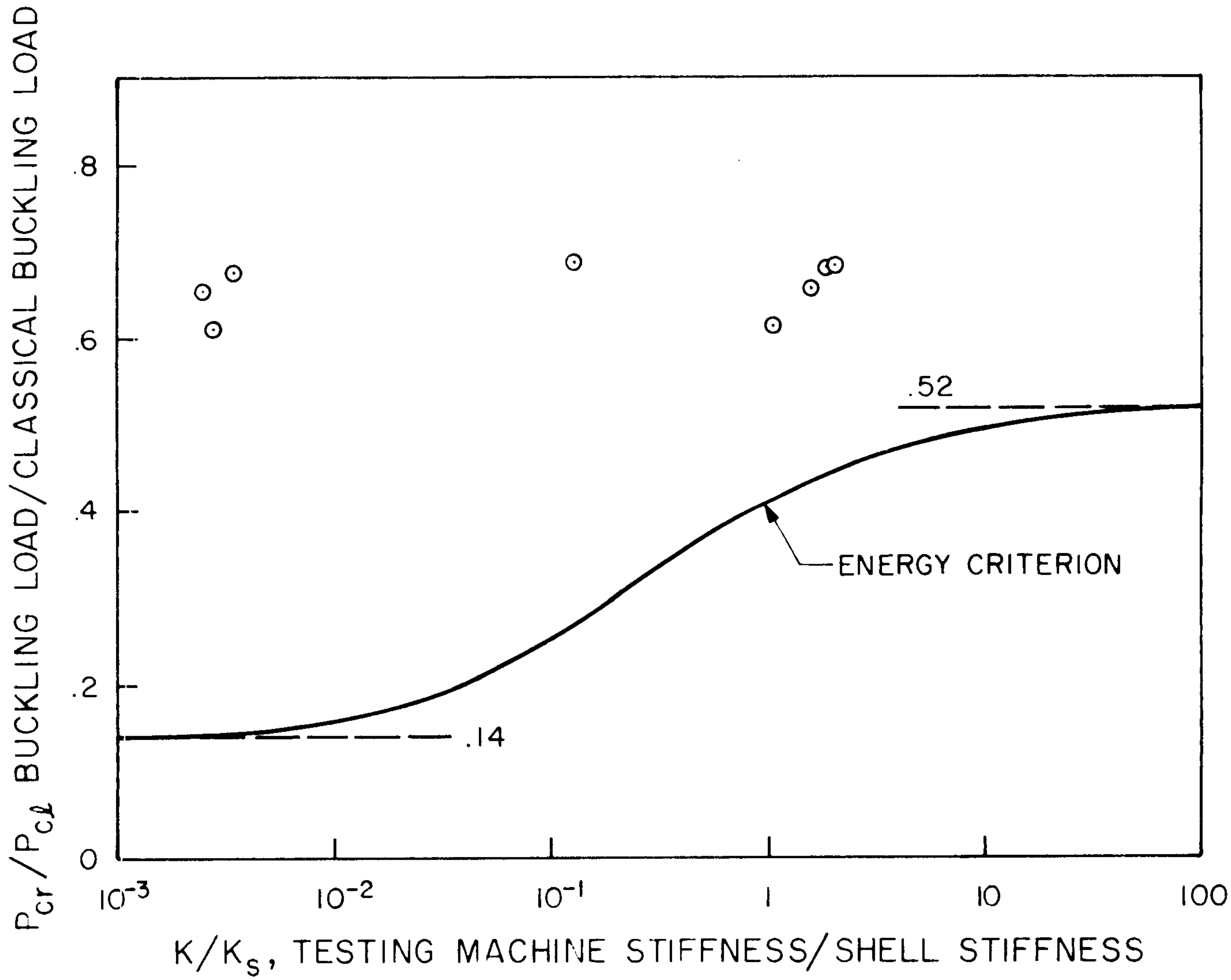

.8
.6
.4
.2
0
$P_{cr}/P_{c\ell}$ BUCKLING LOAD/CLASSICAL BUCKLING LOAD
10^{-3}
10^{-2}
10^{-1}
1
10
100
K/K_s, TESTING MACHINE STIFFNESS/SHELL STIFFNESS
.52
.14
ENERGY CRITERION

Figure 4

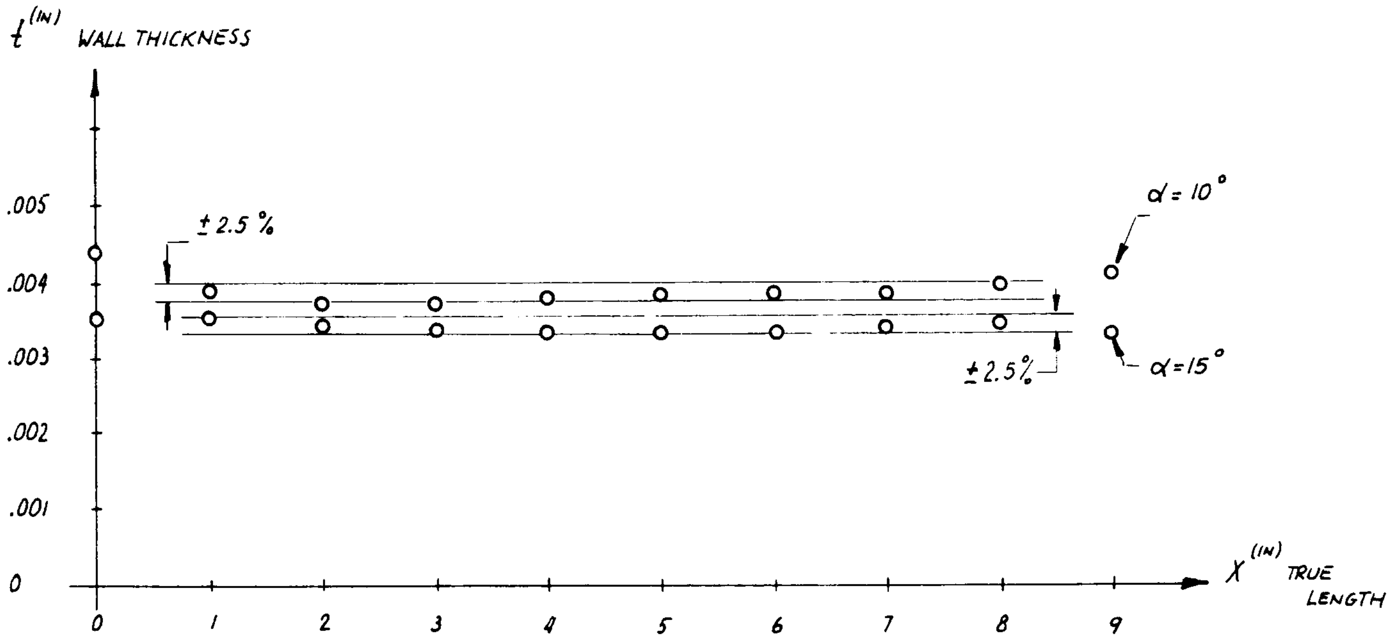
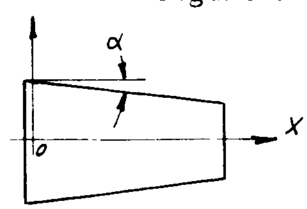


Figure 5



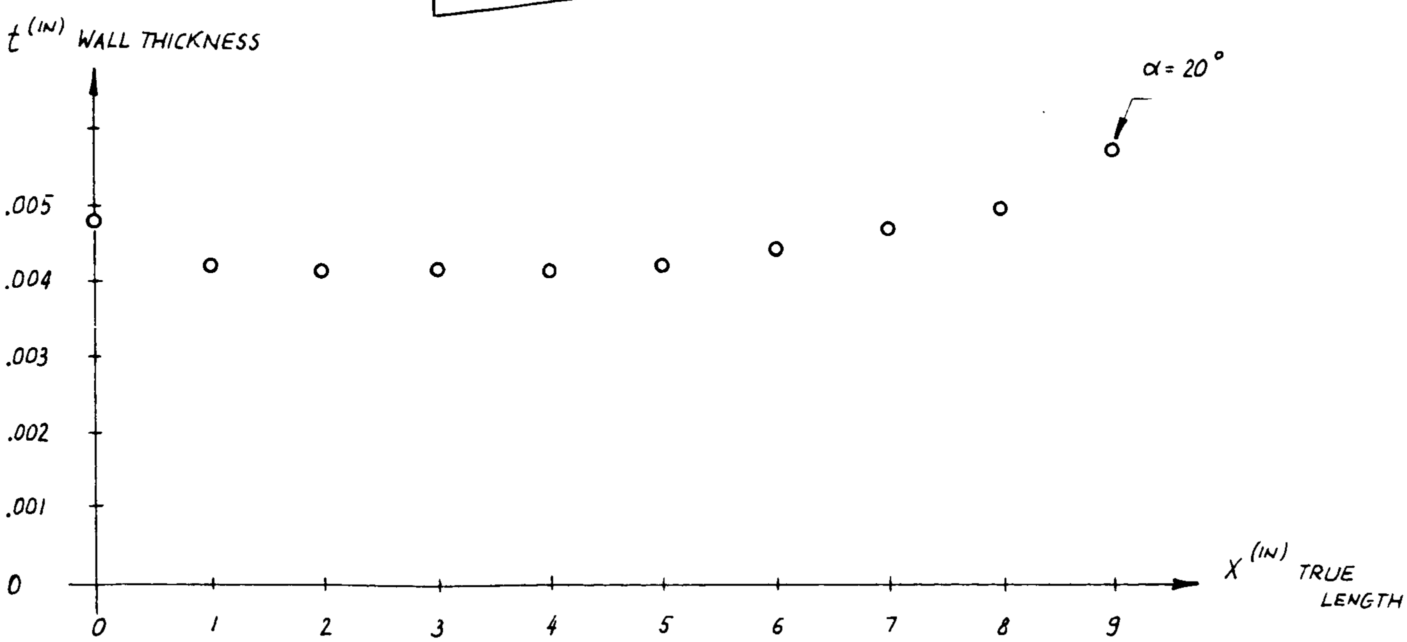


Figure 6

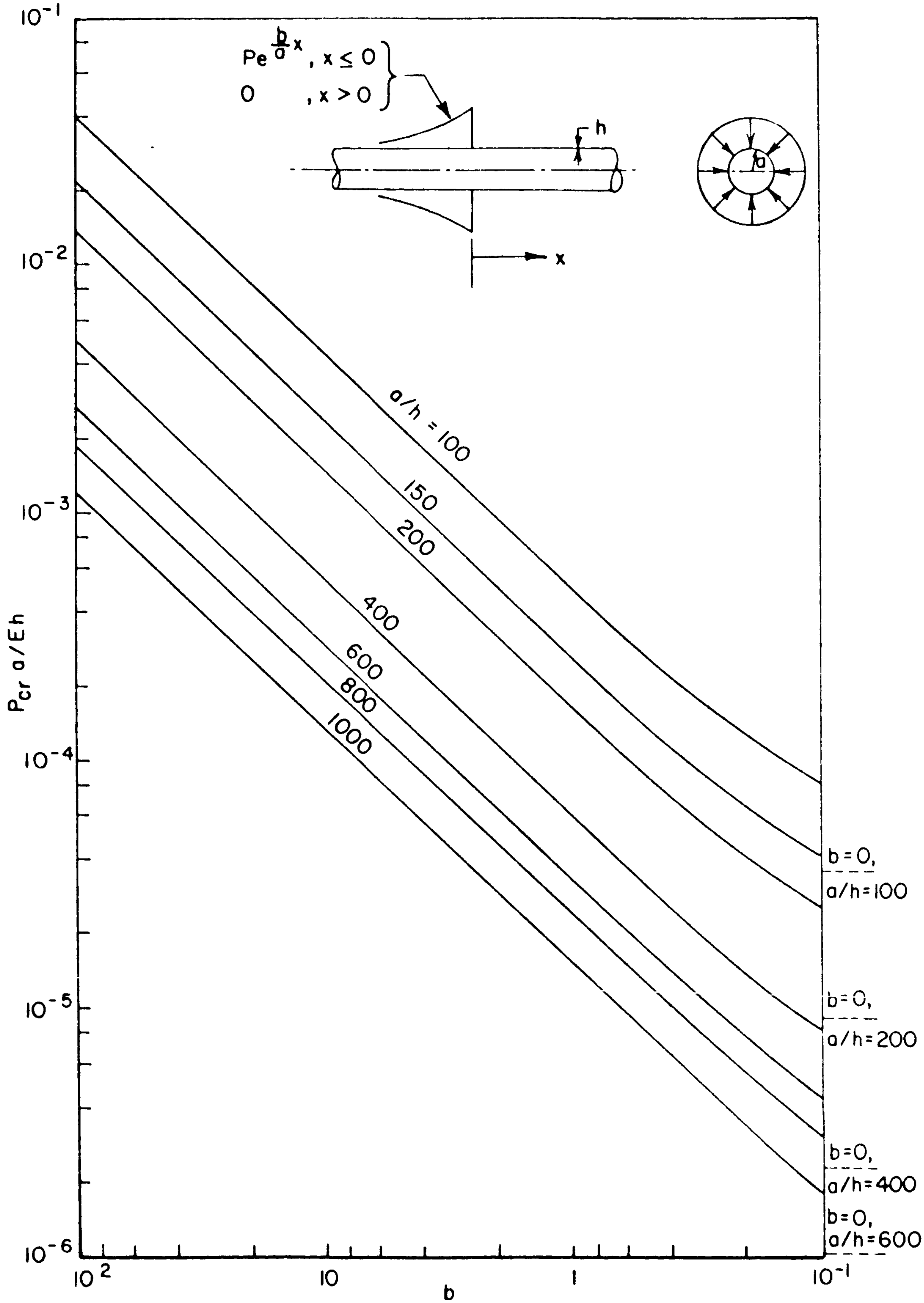

$Pe^{\frac{b}{a}x}$, $x \le 0$
0 , $x > 0$
h
a
x
$P_{cr}\, a/Eh$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
a/h = 100
150
200
400
600
800
1000
b=0,
a/h=100
b=0,
a/h=200
b=0,
a/h=400
b=0,
a/h=600
10^2
10
1
10^{-1}
b

Figure 7

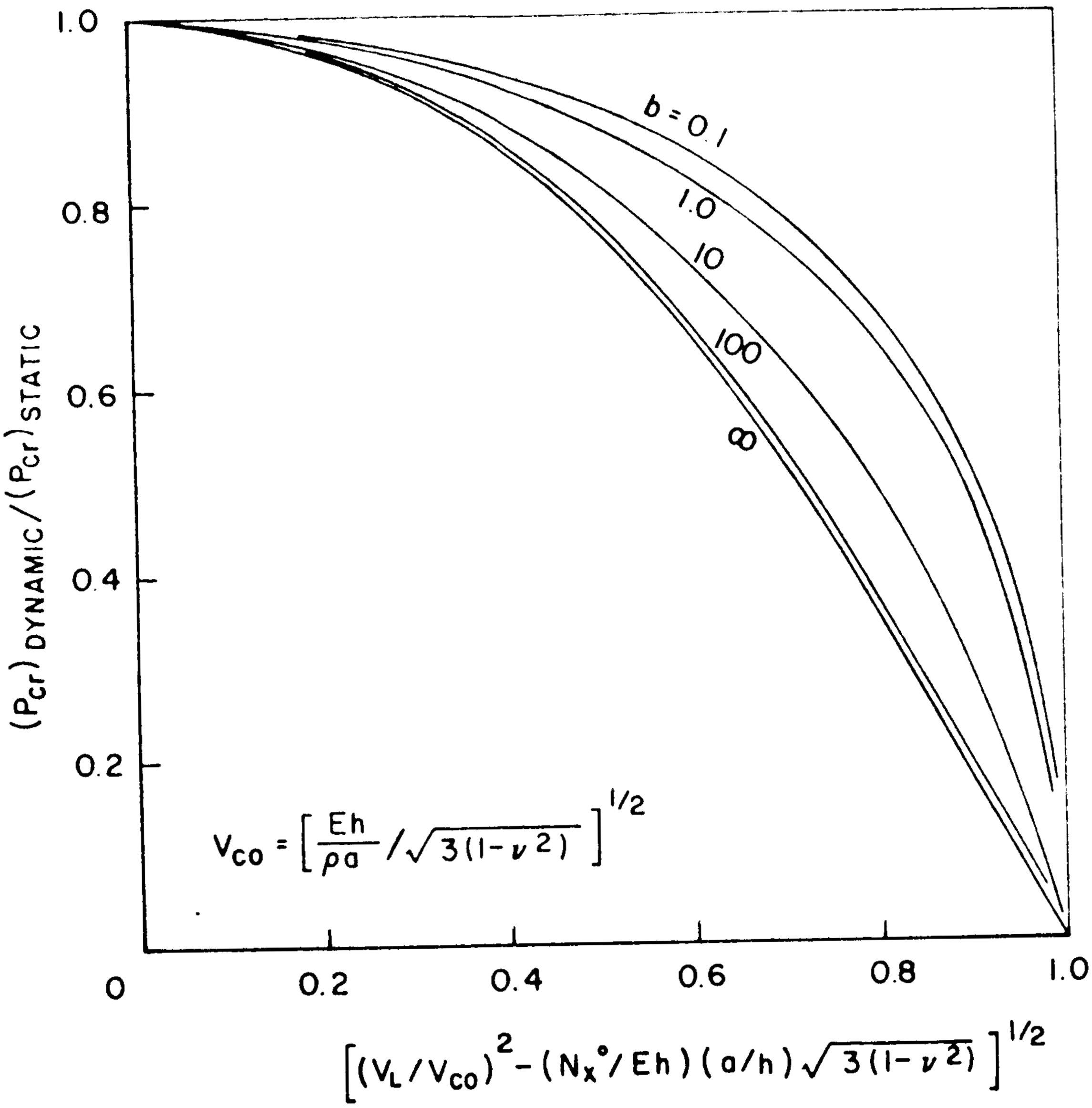

1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
b = 0.1
1.0
10
100
∞
$(P_{cr})_{DYNAMIC}/(P_{cr})_{STATIC}$
$V_{co} = \left[\frac{Eh}{\rho a}/\sqrt{3(1-\nu^2)}\right]^{1/2}$
$\left[(V_L/V_{co})^2-(N_x^\circ/Eh)(a/h)\sqrt{3(1-\nu^2)}\right]^{1/2}$

Figure 8