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CONCERNING ONE NOTION ABOUT THE ANOMALOUS GRAVITATIONAL FIELD

by

A. A. Mikhaylov

[USSR]

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CONCERNING ONE NOTION ABOUT THE ANOMALOUS GRAVITATIONAL FIELD *

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by A. A. Mikhaylov

ABSTRACT

This work deals with the representation of the Earth's anomalous gravitational field by proper assortment of values of anomalies on the sphere, such that the perturbing potential be fully determined only by these values, thus avoiding complex calculation processes.

The problem, as stated by M. S. Molodenskiy, is resolved constructively in that it constitutes at the same time its algorithm.

Some conclusions are also derived in respect to the theory of the geoid's shape, the problem that may be considered as boundary value only in the first approximation.

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In the works [1, 2], the perturbing potential T of the normal (standard) Earth is considered in the form of the sum of the parts T and δT.** The first of them is determined by the generalized Stokes formula [2] at any outward point and the second is found in the form of the potential of the simple layer; a certain integral equation is obtained for the corresponding density $\delta\phi$ [2]. In order to avoid the solution of this equation, M. S. Molodenskiy has stated the following problem: to select such values of anomalies on the sphere σ that they give everywhere on the surface S of the normal Earth G the assigned anomalous field values. Could not

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* OB ODNOM PREDSTAVLENIY ANOMAL'NOGO GRAVITATSIONNOGO POLYA

** We made use of the designations of work [2].

one assort such values of anomalies on the sphere (for example, by way of probes or the working out of general methods of analytical sequence) that all the residual anomalies δg become zero everywhere on S? In other words, is it not possible to construct on the surface of the sphere a system of anomalies that would determine alongside with the field of the normal Earth the outer gravitational field coinciding on the surface S (and, therefore, within S) with the gravitational field of the normal Earth? Then $\delta\phi$ and consequently, δT would be zero. Therefore, the perturbing potential would be fully determined only through the thus assorted anomalies and we would omit the complex computation process of the density $\delta\phi$ of the auxiliary surface layer [2].

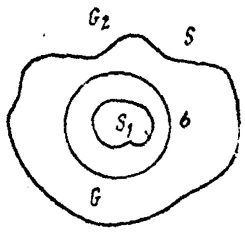

Figure 1

In the present note, put forth by M. S. Molodenskiy, the question is resolved positively. The evidence brought forth below is constructive in the sense that the demonstration is at the same time the algorithm of the problem to be solved.

A function T(S) is preassigned on a known surface S of the normal Earth G (it is assumed that the equation of this surface is known). It is required to find such values $t(\tau)$ on the sphere σ so that the solution of the problem

$$\Delta T' = 0 \text{ in } G_1;$$

$$T'|_\sigma = t(\tau);$$

$$\text{at infinity } T' = 0,$$

where G_1, which is a region relative to the sphere σ, on the surface S, took the preassigned values of T(S). It is natural to assume $\sigma \in G$ (Figure 1).

Let us demonstrate the following assumption. For any $\varepsilon > 0$ and $T'(S) \in L_2$, there will be found a bounded function $t(\varepsilon, \tau, T)$ defined on σ, such that the inequality

$$|T(M) - T'(M)| < \varepsilon \qquad (1)$$

be valid for any point M of the region G_2, whereas on the surface S

$$\left\{\iint [T(S)-T'(S)]^2\,dS\right\}^{1/2}<\varepsilon, \tag{2}$$

where G_2 is a region external with respect to S, T is the perturbing potential corresponding to the function T(S), that is, the solution of the problem $\Delta T = 0$ in G_1, $T|_S = T(S)$, $T = 0$ at infinity, and T'(S) is the solution of the problem

$$\Delta T'(M)=0 \text{ in } G_1;\quad T'(M)|_\sigma = t(\varepsilon,\tau,T);\ T'(\infty)=0. \tag{3}$$

Let S_1 be a closed Lyapunov surface lying inside the sphere but not tangent to it. We shall examine the set of functions $\left\{\frac{1}{r(M_i,M)}\right\}$, where $M\in S$; the points $M_i\in S_1$ are elements of a denumerable set of densely disposed points everywhere on the surface S_1; $r(M_i, M)$ is the distance between the points M_i and M. The linear independence and the completeness of this set on the surface S are demonstrated in [3, 4].

We shall bring forth still one more proof of the completeness of this set. Let $T(S)\in L_2$ be orthogonal to all functions of the set $\left\{\frac{1}{r(M_i,M)}\right\}$. We shall show then T(S) = 0. Let us consider the function

$$\iint_S T(S)\frac{1}{r(M_i,S)}\,dS,\quad T(S)\in L_2,\quad M_i\in S_1. \tag{4}$$

This continuous function takes zero values on the everywhere absolutely dense set of points $M_i\in S_1$, therefore

$$\iint_S T(S)\frac{1}{r(M,S)}\,dS\equiv 0,\quad M\in S_1. \tag{5}$$

But since the potential of the simple layer (4) is harmonical within S_1, on the strength of (5), it is identically equal to zero and within S_1 and consequently [5], the density $T(S)\in L_2$ of the simple layer is zero. We obtained that for any $\varepsilon > 0$ such an N will be found that the linear combination

$$\varphi(\varepsilon,S,T)=\sum_{i=1}^{N} a_i\frac{1}{r(M_i,S)} \tag{6}$$

approximates T(S) in the sense of L_2 metric with a precision ε

$$\left\{\iint [T(S)-\varphi(\varepsilon,S)]^2\,dS\right\}^{1/2}<\varepsilon. \tag{7}$$

If we orthonormalize beforehand the set $\left\{\frac{1}{r(M_i,S)}\right\}$, we may then take for the coefficients a_i the Fourier coefficients of the function T(S) according to the orthonormalized set $\{\psi_i(S)\}$

$a_i=\iint T(S)\psi_i(S)\,dS,$ where $\psi_i(S)=\sum_{k=1}^{i} A_{k,i}\frac{1}{r(M_i,S)}$;

$A_{k,i}$ are the coefficients of orthonormalization. It is known that in this case, inequality (7) will be satisfied. Let us examine on the sphere σ the bounded function (the boundedness follows from the fact that the surfaces σ and S_2 are not tangent)

$$t(\varepsilon, \tau, T) = \left(\sum_{i=1}^{N} a_i \frac{1}{r(M_i, M)}\right)\Big|_\sigma \tag{8}$$

and resolve for this function the outer Dirichlet problem (3). On the strength of the uniqueness of the solution of this problem, we shall have

$$T'(M) = \sum_{i=1}^{N} a_i \frac{1}{r(M_i, M)},$$

and (2) is thus demonstrated.

To demonstrate (1) we shall examine in G_2 the difference $r(M) = T(M) - T'(M)$. As the difference of the two harmonic functions, $r(M)$ is the solution of the problem

$$\Delta r(M) = 0 \qquad G_1; \quad r(M)\,|_S = T(S) - T'(S); \quad r(\infty) = 0.$$

We shall represent the solution of this problem with the aid of the Green function

$$r(M) = \iint_S \frac{\partial G}{\partial n}\,[T(S) - T'(S)]\,dS. \tag{9}$$

Applying to (9) the Schwartz inequality and taking into account the finiteness for any point $M \in G_2, M \in S$ of the integral

$$\iint_S \left[\frac{\partial G}{\partial n}\right]^2 dS,$$

we shall obtain (1).

Therefore, we obtained that if on the known surface S the boundary values T(S) of the perturbing potential T are known, one can select such a function (6) that it will approximate T(S) in the sense of L_2 metric and at any point M beyond S the expression $\sum_{i=1}^{N} a_i \frac{1}{r(M_i, M)}$ will coincide to any degree of precision with the perturbing potential.

One may take for the coefficients a_i the Fourier coefficients by the orthonormalized set $\psi_i(S)$ (it follows from the linear dependence of the set $\left\{\frac{1}{r(M_i, S)}\right\}$ that the orthonormalization is fulfilled). This is why the necessity of computing the function $t(\varepsilon, \tau, T)$ on the sphere σ drops off, although we may for the sake of control compute the expansion of function $t(\varepsilon, \tau, T)$, defined by relation (8), by spherical functions and to verify the presence of spherical harmonics of the first order (as was shown in [2], the amplitude of these harmonics must be a negligibly small quantity).

We must note in conclusion that the main difficulties in the investigations of the Earth's shape by theory arise when determining the surface S of the normal Earth and the limit values of the perturbing potential T, for g is not a harmonic function and T is not measured directly. This is why, as validly noted on page 5 in [2], the problem of determining the geoid's shape may be considered as a boundary value problem only in the first approximation.

* * * T H E   E N D * * *

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1145 19th Street, N.W.
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Translated by
DIANE A. ROBERTS
Principal Investigator
ANDRE L. BRICHANT