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Turbulence by Electrostatic Fluctuations

by

C.M. Tchen

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Abstract

The spectra of turbulence and density fluctuations in a plasma are derived from a hydrodynamic description. The turbulent spectrum is governed by a turbulent cascade process, a viscous dissipation, and a diffusion by electrostatic fluctuations. The density spectrum is governed by a density cascade process and a dissipation by collisions. The strong and weak interactions between waves are considered in the 2 cascade processes. The integral equations for the spectral functions are solved for the following three subranges: inertial-convection subrange, viscous-diffusive subrange, and inertial-diffusive subrange.

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I. INTRODUCTION

The investigation of the equilibrium spectrum of hydrodynamic turbulence has been initiated by Kolmogoroff[1] and Heisenberg[2]. Later theoretical and experimental efforts have confirmed their results. The essential feature of the problem of hydrodynamic turbulence is the cascade process, and was elaborated semi-empirically[2] on the basis of the Navier-Stokes equation.

In a plasma, the fluctuations in density of the charged particles set up electrostatic fluctuations, and consequently add a diffusion to the above cascade process of the momentum transfer. In this way, one may conceive that the hydrodynamic equation of Navier-Stokes may be still valid, when such an electrostatic process of diffusion is incorporated. On the other hand, the density fluctuations should follow a classical diffusion equation. It is hoped that such a system of equations will describe plasmas of high ionization where the electrostatic process is important, as well as plasmas of low ionization where the electrostatic process is negligible in the momentum transfer. In the latter case experiments of turbulence in a weakly ionized gas (0.001% ionization) has been made by Granatstein, etc.[3].

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[1] A.N. Kolmogoroff, C.R. Acad. Sci. (USSR) **30**, 301(1941); **32**, 16(1941).

[2] W. Heisenberg, 2. f. Phys. **124**, 628(1948).

[3] V.L. Granatstein, S.J. Buchsbaum and D.S. Bugnolo, Phys. Rev. Letters **16**, 504(1966).

In the following pages, we propose to use the above system of equations for describing a plasma of high and low ionizations, and to derive the spectral functions of turbulence and density fluctuations.

II. FUNDAMENTAL EQUATIONS

We use the Navier-Stokes equation for describing the turbulent fluctuations in velocity $\underset{\sim}{u}$ of the plasma; the collisions between the ionized particles and the neutral host gas are written in the form of a kinematic viscosity ν. We have

$$\left(\frac{\partial}{\partial t} + \underset{\sim}{u} \frac{\partial}{\partial \underset{\sim}{x}} \right) \underset{\sim}{u} = \frac{e}{m} \underset{\sim}{E} + \nu \frac{\partial^2 \underset{\sim}{u}}{\partial \underset{\sim}{x}^2} + \underset{\sim}{X} \quad . \tag{1}$$

Here $\underset{\sim}{X}$ is a stochastic force (noise), representing fluctuations in pressure and brownian movements. The pressure is lumped in this way, because it is relatively unimportant in the study of energy spectrum. Moreover we shall assume

$$< X \, u > = 0 \quad .$$

Further m is the mass, n is the fluctuation in number-density, $\underset{\sim}{E}$ is the self-consistent electric field, satisfying the Poisson equation

$$\frac{\partial \underset{\sim}{E}}{\partial \underset{\sim}{x}} = 4\pi \, e \, n , \tag{2}$$

and e is the charge: $+|e|$ for ions and $-|e|$ for electrons. Sometimes it is convenient to introduce an electric potential φ, such that

$$\underset{\sim}{E} = - \partial\varphi / \partial \underset{\sim}{x} .$$

A diffusion equation is used for describing the fluctuations in concentration or number-density of a plasma:

$$\frac{\partial n}{\partial t} + \underset{\sim}{u} \frac{\partial n}{\partial \underset{\sim}{x}} = \lambda \frac{\partial^2 n}{\partial \underset{\sim}{x}^2} , \tag{3}$$

where λ is the molecular diffusion coefficient.

When we introduce the Fourier transform

$$\underset{\sim}{u}(t,\underset{\sim}{x}) = \int_{-\infty}^{\infty} d\underset{\sim}{k} \; \underset{\sim}{u}(t,\underset{\sim}{k}) \; e^{i \underset{\sim}{k}\underset{\sim}{x}} ,$$

and similar formulas for n and $\underset{\sim}{E}$, Eqs. (1) - (3) are transposed to

$$\left(\frac{\partial}{\partial t} + 2\nu k^2 \right) < u_i(t, \underset{\sim}{k}) \, u_i(t, -\underset{\sim}{k}) >$$

$$= -\int_{-\infty}^{\infty} d\underset{\sim}{k}' \; ik_j' < u_j(\underset{\sim}{k} - \underset{\sim}{k}') \, u_i(\underset{\sim}{k}') \, u_i(-\underset{\sim}{k}) >$$

$$+ \frac{e}{m} < E_j(t,\underset{\sim}{k}) \, u_j(t, -\underset{\sim}{k}) > + \text{ c. c. } , \tag{4}$$

$$\left(\frac{\partial}{\partial t} + 2\lambda\, k^2\right) < n(t,\underset{\sim}{k})\; n(t,-\underset{\sim}{k}) > \;=\; -\int_{-\infty}^{\infty} d\underset{\sim}{k}'\; ik_j{}' < u_j(\underset{\sim}{k} - \underset{\sim}{k}')\; n(\underset{\sim}{k}')\; n(-\underset{\sim}{k}) >$$

$$+ \text{ c. c. }, \qquad (5)$$

$$\frac{e}{m}\underset{\sim}{E} \;=\; -\frac{i\underset{\sim}{k}}{k^2}\,\varpi^2 n \quad , \qquad (6)$$

$$-i\underset{\sim}{k}\,\varphi = \underset{\sim}{E} \;,$$

where

$$\varpi^2 \;=\; 4\pi e^2/m \quad ,$$

and (c. c.) represents similar complex conjugate terms. It is noted that the equations for $\underset{\sim}{u}$ and n have been written in terms of energy.

III. SPECTRAL DECOMPOSITION

We introduce the following spectral functions:

$$\tfrac{1}{2} < u^2 > \;=\; \int_0^{\infty} dk \;\; F(k) \quad ,$$

$$\tfrac{1}{2} < n^2 > \;=\; \int_0^{\infty} dk \;\; G(k) \quad ,$$

$$\tfrac{1}{2}\left(\frac{e}{m}\right)^2 < E^2 > \;=\; \int_0^{\infty} dk \;\; G_E(k) \;,$$

$$\tfrac{1}{2}\left(\frac{e}{m}\right)^2 < \varphi^2 > \;=\; \int_0^{\infty} dk \;\; G_{\varphi}(k) \;.$$

Evidently G_φ , G_E and G are related through formulas (6):

$$G_E = \frac{\varpi^4}{k^2} G \quad ; \qquad G_\varphi = \frac{\varpi^4}{k^4} G \quad . \tag{7}$$

Further we introduce the notations of vorticities:

$$R(k) = 2 \int_0^\infty dk \; k^2 F \quad ,$$

$$J(k) = 2 \int_0^\infty dk \; k^2 G \quad .$$

The Fourier components and the spectral functions are related by

$$\begin{aligned} \int_0^k dk \; F(k) &= \kappa \int_0^k d\underset{\sim}{k} < \underset{\sim}{u}(\underset{\sim}{k}) \, \underset{\sim}{u}(-\underset{\sim}{k}) > \quad , \\ \int_0^k dk \; G(k) &= \kappa \int_0^k d\underset{\sim}{k} < n(\underset{\sim}{k}) \, n(-\underset{\sim}{k}) > \quad ; \end{aligned} \tag{8}$$

where

$$\kappa = \tfrac{1}{2}(2\pi/X)^3 \quad ,$$

and $2X$ is the length of truncation

$$-X \leq \underset{\sim}{x} \leq + X$$

of the Fourier decomposition of $\underset{\sim}{u}(\underset{\sim}{x})$ and $n(\underset{\sim}{x})$. On the right hand sides of (8), there are triple integrals

$$\int_0^k d\underset{\sim}{k}$$

in a sphere of radius k.

In terms of those new notations, the energy equations (4) and (5) become

$$-\frac{\partial}{\partial t}\int_0^k dk\ F(k) = \nu R(k) + T(k) + D_E(k) \quad ,$$

$$-\frac{\partial}{\partial t}\int_0^k dk\ G(k) = \lambda J(k) + W(k) \quad ; \tag{9}$$

where

$$T(k) = \kappa \int_0^k d\underset{\sim}{k} \int_{-\infty}^{\infty} d\underset{\sim}{k}'\ ik_j' < u_j(\underset{\sim}{k}-\underset{\sim}{k}')\ u_i(\underset{\sim}{k}')\ u_i(-\underset{\sim}{k}) + u_j(-\underset{\sim}{k}-\underset{\sim}{k}')\ u_i(\underset{\sim}{k}')\ u_i(\underset{\sim}{k}) > , \tag{10a}$$

$$W(k) = \kappa \int_0^k d\underset{\sim}{k} \int_{-\infty}^{\infty} d\underset{\sim}{k}'\ ik_j' < u_j(\underset{\sim}{k}-\underset{\sim}{k}')\ n(\underset{\sim}{k}')\ n(-\underset{\sim}{k}) + u_j(-\underset{\sim}{k}-\underset{\sim}{k}')\ n(\underset{\sim}{k}')\ n(\underset{\sim}{k}) > , \tag{10b}$$

and

$$D_E(k) = \kappa \int_0^k d\underset{\sim}{k}\ \frac{e}{m} < \underset{\sim}{E}(\underset{\sim}{k})\ \underset{\sim}{u}(-\underset{\sim}{k}) + \underset{\sim}{E}(-\underset{\sim}{k})\ \underset{\sim}{u}(\underset{\sim}{k}) > . \tag{10c}$$

The three functions T, W and D_E will be evaluated by approximate methods in the following section.

IV. CASCADE PROCESS IN STRONG INTERACTION AND SPECTRA IN THE INVISCID SUBRANGES

We introduce a phenomenological theory based on an extension of the mixing-length hypothesis. The latter states that from the correlation between a concentration n and a velocity $\underset{\sim}{u}$ of the turbulent medium, in which the concentration is embedded, there results a transport of concentration by turbulence, determined by two factors: (i) a macroscopic density gradient of a large scale, and (ii) a diffusion coefficient of the turbulent medium $(u\ell)$, contributed by smaller eddies. If we distinguish two groups of eddies: a group of large eddies $\underset{\sim}{k}$, carrying a density gradient $-i\underset{\sim}{k}\, n(\underset{\sim}{k})$, and a group of smaller eddies $\underset{\sim}{k}'(k' > k)$, contributing to a diffusion coefficient

$$(u\ell)_{k' > k} \quad ,$$

then the mixing of the two waves results in a transport of concentration by turbulence

$$n(\underset{\sim}{k}')\, u_j(\underset{\sim}{k}-\underset{\sim}{k}') = -\,(u\ell)_{k' > k}\, ik_j\, n(\underset{\sim}{k}) \; .$$

Also we have

$$ik_j'\, n(\underset{\sim}{k}')\, u_j(\underset{\sim}{k}-\underset{\sim}{k}') = (u\ell)_{k' > k}\, k^2\, n(\underset{\sim}{k}) \quad . \tag{11a}$$

Similarly

$$ik_j'\, u_i(\underset{\sim}{k}')\, u_j(\underset{\sim}{k}-\underset{\sim}{k}') = (u\ell)_{k' > k}\, k^2\, u_i(\underset{\sim}{k}) \quad . \tag{11b}$$

Upon substitution of relations (11) into expressions (10), we find

$$T(k) = \nu_k R(k) ,$$
$$W(k) = \nu_k J(k) , \tag{12}$$

and

$$\nu_k = \int_k^\infty d\underset{\sim}{k}' \ (u\ell)_{k'} ,$$
$$= \chi \int_k^\infty dk \ (F/k^3)^{\frac{1}{2}} , \tag{13}$$

from dimensional reasonings[2]. Here χ is a numerical constant.

We may remark that $(u\ell)$, which is the basis of ν_k, is a diffusion by velocity fluctuations, while

$$\frac{e}{m} < E \ \mathbf{u} > ,$$

which forms the basis of D_E, is a diffusion by electrostatic fluctuations. Thus we can find the structure of $D_E(k)$ by writing

$$D_E(k) = \text{const} \int_0^k dk \ \ G_E^{\alpha} \ k^{\beta} ,$$

where α and β are found to be:

$$\alpha = -\beta = 3/4 ,$$

by satisfying the dimensional conditions. Hence

$$D_E(k) = \chi_1 \int_0^k dk \, (G/k^3)^{3/4} . \tag{14}$$

where χ_1 = numerical constant ϖ^3 .

Upon substitution of (13) and (14) into (9), (10c) and (12), we can rewrite Eqs.(9) in the form:

$$-\frac{\partial}{\partial t} \int_0^k dk \, F(k) = (\nu + \nu_k) \, R(k) + \chi_1 \int_0^k dk \, (G/k^3)^{3/4} , \tag{15a}$$

and

$$-\frac{\partial}{\partial t} \int_0^k dk \, G(k) = (\lambda + \nu_k) \, J(k) . \tag{15b}$$

If we introduce the notations

$$\varepsilon_\nu = \nu < (\partial u_i/\partial x_j)^2 > ,$$

$$\varepsilon_\lambda = \lambda < (\partial n/\partial x_j)^2 > ,$$

then, for equilibrium spectra, Equations (15) reduce to

$$(\nu + \nu_k)\, R(k) \; - \; \chi_1 \int_k^{\infty} dk\, (G/k^3)^{3/4} = \epsilon_\nu \quad , \tag{16a}$$

$$(\lambda + \nu_k)\, J(k) \; = \; \epsilon_\lambda \tag{16b}$$

These are the 2 basic equations for determining the spectral functions of turbulence and concentration (or electric field) under the conditions of strong interaction.

We shall investigate the spectral laws in the inviscid subranges, also called the inertial and convective subranges, where

$$\nu_k >> \nu \quad ,$$

$$\nu_k >> \lambda \quad .$$

In these subranges, the system of equations (15) reduces to

$$\nu_k\, R(k) \; = \; \chi_1 \int_k^{\infty} dk\, (G/k^3)^{3/4} \qquad ,$$

and

$$\nu_k J_k \; = \; \epsilon_\lambda \qquad .$$

After substitution for ν_k from the second equation, and after differentiation, the first equation of the system becomes:

$$\epsilon_\lambda k^2 F = \chi_1 [k^2 G \int_k^\infty dk\ (G/k^3)^{3/4} - (G/k^3)^{3/4} J(k)] .$$

The second term between the brackets is negligible, as the inertial subrange of the G spectrum does not contribute much to the vorticity $J(k)$. Hence we obtain

$$\epsilon_\lambda F = \chi_1 G \int_k^\infty dk\ (G/k^3)^{3/4} . \tag{17}$$

The new system of equations (17) and (16b) yield the following solutions:

$$F = A k^{-3} , \tag{18a}$$

and

$$G = B k^{-1} ; \tag{18b}$$

where

$$A = (a^4 b^7)^{2/15} ,$$

$$B = (a^{-1} b^2)^{4/15} ;$$

$$a = \chi_1/2\epsilon_\lambda ,$$

$$b = 2\epsilon_\lambda/\chi .$$

From formulas (7) and (18b), we find

$$G_{\varphi} = B\, \varpi^{4}\, k^{-5} \quad . \tag{18c}$$

It is to be noted that the dissipation rate ϵ_{λ} alone determines the inertial and convective subranges of both F and G spectra. The turbulent spectrum F in formula (18a) drops faster than the $k^{-5/3}$ law (Kolmogoroff law), because here the cascade is drained by an additional diffusion from electrostatic fluctuations.

V. SPECTRA IN THE VISCOUS AND DIFFUSIVE SUBRANGES WITH STRONG INTERACTION BETWEEN TURBULENCE AND ELECTROSTATIC FIELD

In the viscous and diffusive subranges, we have

$$\nu_k << \nu \quad \text{and} \quad \nu_k << \lambda \quad .$$

The viscous and diffusive dissipations being dominant at large k , the diffusion by electrostatic fluctuations is negligible, and the system of equations (16) reduce to

$$\begin{aligned} (\nu + \nu_k)\, R(k) &= \epsilon_{\nu} \quad , \\ (\lambda + \nu_k)\, J(k) &= \epsilon_{\lambda} \quad . \end{aligned} \tag{19}$$

A differentiation gives

$$2\nu\, k^2 F - \chi\, (F/k^3)^{\frac{1}{2}}\, R(k \to \infty) = 0 \quad ,$$

$$2\lambda\, k^2 G - \chi\, (F/k^3)^{\frac{1}{2}}\, J(k \to \infty) = 0 \quad .$$

The solutions are

$$F = (\chi\, \epsilon_\nu / 2\nu^2)^2\, k^{-7} \quad ,$$

and

$$G = \frac{\chi\, \epsilon_\nu}{2\nu^2}\, \frac{\chi\, \epsilon_\lambda}{2\lambda^2}\, k^{-7} \ . \qquad (20)$$

With those solutions (20), it can be verified that the electrostatic diffusion

$$\chi_1 \int_k^\infty dk\ (G/k^3)^{3/4}$$

is indeed small, as compared with the cascade flow

$$\nu_k\, R(k \rightarrow \infty) \quad ,$$

in order to justify such an assumption used in **Eqs.**(19).

The first of the solutions (20) is in agreement with the viscous law of the turbulent spectrum found by Heisenberg[2] .

VI. WEAK INTERACTION BETWEEN TURBULENCE AND ELECTROSTATIC FIELD

The cascade process based on the mixing-length requires a condition of strong interaction. For a weak interaction, we can rewrite Eq. (3) in its Fourier transform:

$$\frac{\partial n(\underset{\sim}{k})}{\partial t} + \int_{-\infty}^{\infty} d\underset{\sim}{k}' i k_j' \, u_j(\underset{\sim}{k}-\underset{\sim}{k}') \, n(\underset{\sim}{k}') = -\lambda k^2 \, n(\underset{\sim}{k}) \quad .$$

By multiplying each member by its complex conjugate, and assuming a stationary process we have

$$\lambda^2 \, k^4 < n(\underset{\sim}{k}) \, n(-\underset{\sim}{k})> = \int\int_{-\infty}^{\infty} d\underset{\sim}{k}' \, d\underset{\sim}{k}'' \, k_i' \, k_j'' < u_i(\underset{\sim}{k}-\underset{\sim}{k}') \, u_j(-\underset{\sim}{k}+\underset{\sim}{k}'') \, n(\underset{\sim}{k}') \, n(-\underset{\sim}{k}'') > \quad .$$

A weak interaction between $\underset{\sim}{u}$ and n may be assumed, when the spectrum of n undergoes a diffusive dissipation, while the spectrum of u is maintained in its inertial cascade process. This occurs when we have

$$\nu << \lambda \quad .$$

The case of weak interaction is relevant to a plasma where a weak concentration of ionized particles are carried by a strong turbulent fluid. The latter fluid can be considered as neutral, the weak electrostatic diffusion being negligible.

For a weak interaction, the arguments of $\underset{\sim}{u}$ and n in the integrand are of different order of magnitude:

$$\underset{\sim}{k}-\underset{\sim}{k}' >> \underset{\sim}{k}' \quad ,$$

and

$$\underset{\sim}{k}'-\underset{\sim}{k}'' >> \underset{\sim}{k}'' \quad ;$$

or

$$\underset{\sim}{k} >> \underset{\sim}{k}' \quad , \qquad\qquad \underset{\sim}{k} >> \underset{\sim}{k}'' \quad .$$

Consequently we can write

$$< u_i(\underset{\sim}{k}-\underset{\sim}{k}') \; u_j(-\underset{\sim}{k} + \underset{\sim}{k}'') \; n(k') \; n(-\underset{\sim}{k}'') >$$

$$= \; < u_i(\underset{\sim}{k}) \; u_j(-\underset{\sim}{k}) > < n(\underset{\sim}{k}') \; n(-\underset{\sim}{k}'')> \quad .$$

Hence

$$\lambda^2 \; k^4 < n(\underset{\sim}{k}) \; n(-\underset{\sim}{k}) >$$

$$= \; < u_i(\underset{\sim}{k}) \; u_j(-\underset{\sim}{k}) > < \int_{-\infty}^{\infty} d\underset{\sim}{k}' \; k_i' \; n(k') \int_{-\infty}^{\infty} d\underset{\sim}{k}'' \; k_j'' \; n(-\underset{\sim}{k}'') > \quad ,$$

$$= \; \frac{1}{3} < u_i(\underset{\sim}{k}) \; u_i(-\underset{\sim}{k}) > < (\partial n/\partial x_j)^2 > ,$$

for isotropic turbulence. The spectral functions follow

$$\lambda^2 k^4 G = \frac{1}{3} J_\infty F \quad ; \tag{21a}$$

where

$$J_\infty = J(k = \infty) ,$$
$$= < (\partial n/\partial x_j)^2 > .$$

Equation (21a) can be transformed into the following familiar form:

$$(\lambda + \overline{\nu}_k) J(k) = \overline{\epsilon}_\lambda \quad , \tag{21b}$$

with the following new values of $\overline{\nu}_k$ and $\overline{\epsilon}_\lambda$:

$$\overline{\nu}_k = \frac{2}{3\lambda} \int_k^\infty dk\ F/k^2 \quad , \tag{21c}$$

and

$$\overline{\epsilon}_\lambda = \overline{\nu}_{k=0}\ J_\infty \quad .$$

In the above transformation, we have neglected the term

$$-2\ \overline{\nu}_k \int_k^\infty dk\ k^2\ G \quad ,$$

as compared to

$$-2\bar{\nu}_k \int_0^k dk\, k^2 G \quad ,$$

for large k .

When the G spectrum undergoes a diffusive dissipation, while the F spectrum is in the inertial cascade process (inertial-diffusive subranges), the solution of Eq. (21a) is

$$G = \frac{J_\infty}{3\lambda^2} k^{-4} F \quad .$$

If the F spectrum assumes the Kolmogoroff law of the inertial subrange, without diffusion by electrostatic fluctuation, then

$$F = A_1 k^{-5/3} \quad , \tag{22a}$$

and

$$G = \frac{J_\infty}{3\lambda^2} A_1 k^{-17/3} \quad ; \tag{22b}$$

where

$$A_1 = (8\epsilon_\nu/9\chi)^{2/3} \quad .$$

The contribution of the diffusion term

$$\chi_1 \int_k^{\infty} dk\ (G/k^3)^{3/4} \quad ,$$

for G in the dissipative subrange at large k , is negligible.

However, it is to be remarked that, when a strong interaction could still exist between the turbulence and the electrostatic field, in the diffusive subrange of the G spectrum, especially near the lower k part of the subrange, then instead of Eq. (21b), we would use Eq. (16b). After differentiating the latter equation, we would get

$$-\chi\ (F/k^3)^{\frac{1}{2}}\ J(k \to \infty) + 2\lambda\ k^2 G = 0 \ ,$$

with the solution:

$$G = \frac{\chi \epsilon_\lambda}{2\lambda^2}\ A_1^{\frac{1}{2}}\ k^{-13/3} \quad . \tag{23}$$

This law (23) was used by Granatstein, Buchsbaum and Bugnolo[3] to interpret the initial portion of the diffusive subrange of the G spectrum in a weakly ionized plasma.

VII. CONCLUSION

The spectrum of turbulence is governed by a turbulent cascade process, a viscous dissipation, and a diffusion by electrostatic fluctuations. The density spectrum which characterizes the distribution of density or electrostatic fluctuations, is governed by a density cascade process, and a dissipation by collisions. The turbulent cascade is controlled by a strong interaction between waves, while the density cascade may be controlled by a strong interaction or a weak interaction between the density and the velocity waves. The mechanism of strong interaction is formulated by a phenomenological theory, based upon the mixing-length hypothesis extended to waves. The mechanism of weak interaction is formulated by degenerating the fourth order correlation in density and velocity fluctuations into a product of two second order correlations. The equations determining the spectra are derived in the form of integral equations. The solutions for the following cases are found:

Case (a). In a plasma of high ionization, as characterized by the dominant role of the electrostatic diffusion in the turbulent spectrum, the intervals of the inviscid (inertial and convective) subranges of the two spectra coincide, when

$$\nu \cong \lambda \quad ,$$

and for values of k, such that

$$\nu_k >> \nu \quad , \quad \nu_k >> \lambda \quad .$$

Since the intervals of the inviscid subranges coincide, and the ionization is high, a strong interaction should govern the cascade, and the spectra are found to be

$$F = \text{const} \quad k^{-3} \quad ,$$

and

$$G = \text{const} \; k^{-1} \quad ,$$

see (18a) and (18b). The turbulent spectrum drops faster than the 5/3-law of Kolmogoroff, because the cascade is accompanied by a sink in the form of electrostatic diffusion.

Case (b). In the collisional (viscous and diffusional) subranges of the two spectra, for values of k such that

$$\nu_k << \nu \quad , \quad \nu_k << \lambda \quad ,$$

the spectra follow the k^{-7} law, see (20):

$$F = \text{const} \quad k^{-7} \quad ,$$

and

$$G = \text{const} \quad k^{-7} \quad .$$

The electrostatic diffusion at such high values of k is negligible. Thus the same results hold for plasmas of high and low ionizations.

Case (c). Consider a plasma with a low ionization, where a small number of charged particles diffuse in a turbulent field of high Reynolds numbers. Moreover, the electron temperature is much larger than the gas temperature, so that

$$\nu << \lambda \quad .$$

It follows that the density spectrum cannot maintain a sizable convective subrange, but noticeably will undergo a molecular diffusive regime, with

$$\nu_k >> \nu \quad \text{and} \quad \bar{\nu}_k << \lambda \quad ,$$

while the turbulent diffusivity $\bar{\nu}_k$, see (21c), is being provided by the turbulent spectrum maintained at the inertial subrange. Because of such different regimes, the density cascade is contributed by waves in weak interactions, see (21b) and (21c). It follow that the density spectrum in the diffusive subrange will be

$$G = \text{const} \;\; k^{-17/3} \quad ,$$

see (22b).