MINIMUM WEIGHT DESIGN ASPECTS OF STIFFENED CYLINDERS UNDER COMPRESSION

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Symbols

b_r, b_s	ring and stringer spacing, respectively (Fig. 1), in
d	diameter of cylinder, in
d_r, d_s	depth of ring and stringer section, respectively (Fig. 1), in
e_r, e_s	eccentricity of the stiffener centroid with respect to middle surface of skin, in
k	buckling coefficient
m, n	number of buckles along the axial and circumferential directions
t	cylinder skin thickness, in
t_r, t_s	thickness of ring and stringer sections, respectively (Fig. 1), in
u, v, w	displacement in the axial, circumferential and normal directions, in
x, y	curvilinear coordinates
A_r, A_s	area of ring and stringer section, respectively, in^2
B_1, B_2, B_3	axial rigidities of shell-stiffener combinations, lb/in
D_1, D_2, D_3	bending rigidities of shell-stiffener combinations, lb/in
E	Young's modulus, psi
G	shear modulus, psi
I_r, I_s	moment of inertia of ring and stringer section, respectively, about stiffener centroid, in^4
J_r, J_s	torsional constant for ring and stringer section, respectively
L	cylinder length, in
M_{xx}, M_{yy}, M_{xy}, M_{yx}	resultant couples acting on shell element, lb-in/in
N_{xx}, N_{yy}, N_{xy}	membrane stress resultants acting on shell element, lb/in

Symbols (Continued)

R	radius of cylinder, in
Z	shell curvature parameter
α	$m \pi R/L$
β	wave length ratio = $nL/m \pi R$
ϵ_{xx}, ϵ_{yy}, ϵ_{xy}	direct strains acting along shell middle surface
η_r, η_s	additional bending stiffness due to stiffeners
κ_{xx}, κ_{yy}, κ_{xy}	curvature changes about shell middle surface
λ_r, λ_s, λ_{rs}	parameters defined in Eq. (10)
ν	Poisson's ratio
ξ_r, ξ_s	additional axial stiffness terms due to stiffeners
σ	critical stress, psi

MINIMUM WEIGHT DESIGN ASPECTS OF STIFFENED CYLINDERS UNDER COMPRESSION

1. Introduction

The minimum weight design of stiffened cylinders is of fundamental importance in the design of launch vehicles. Since such structures are susceptible to compressive loads, they tend to fail due to instability rather than yielding. Hence, the minimum weight design of these cylinders may be goverened by stability considerations rather than yielding.

The intimate relationship between minimum weight design and the stability of a structure is illustrated in the following axiomatic principle used by designers: if a structure is susceptible to failure in several modes of instability then the minimum weight proportions are achieved when all the possible modes of buckling occur simultaneously. Thus, it is clear that all the pertinent aspects of the instability of a structure must be known in order to achieve a rational minimum design.

In dealing with stability of stiffened cylinders, earlier researches, see for example Refs. 1-3, have ignored the effect of stiffener location on the buckling stress. A minimum weight approach has been considered in Ref. 4 based on these earlier formulations. However, recent experiments at NASA, Ref. 5, and also theoretical predictions reported in Refs. 6 and 7 have shown that both stiffener eccentricity and stiffener location have considerable influence on the buckling stress of stiffened cylinders. Hence, a rigorous approach to minimum weight design of stiffened cylinders must take into account the influence of stiffener eccentricity and location. Section 2 of this report contains a discussion of the pertinent aspects of the stability of stiffened cylinders under compression. Section 3 outlines the problems of minimum weight design and critically examines the current results reported. Section 4 delineates the areas for future work.

2. Stability Modes of Deep-Stiffened Cylinders

Any structure strengthened by extra members such as stiffeners is liable to fail in several stability modes. Thus, the instability may involve the entire structure (general instability) or it may involve a portion of the structure (local instability).

The most general case of a deep-stiffened cylinder includes both stringers and ring frames. Fig. 1 shows a typical grid stiffening system of rectangular section. The types of stabilities involved in this case are three:

1. a local instability of the stiffener flanges
2. a local instability of a panel included by a pair of stringers and a pair of rings
3. the general instability of the entire cylinder inclusive of rings and stringers.

Local Instability

The local instabilities of the deep stiffened cylinder are determined by classical methods, (for example see Ref. 8) and the resulting expressions for the critical stresses are as follows:

$$\sigma_{\text{stiffener}} = (1/24)\pi^2 E\,(1 - \nu^2)^{-1}(t_s/d_s)^2 \tag{1}$$

$$\sigma_{\text{panel}} = (1/3)\pi^2 E\,(1 - \nu^2)^{-1}(t/b_s)^2 \tag{2}$$

where t_s, d_s, b_s are the thickness, depth and the spacing, respectively, of the stringer section and t is the skin thickness.

Governing Equation for General Instability of Deep-Stiffened Cylinders

In order to show the explicit influence of the stringer eccentricity upon the buckling coefficient in the case of deep-stiffened cylinders under compression, it is worthwhile to outline the development of the theory of general instability of deep-stiffened cylinders. This problem was initially studied by Van der Neut (Ref. 9); the outline given herein follows the Baruch-Singer (Ref. 6) formulation.

Assumptions:

The main assumptions of this theory are:

a) The stiffeners are closely spaced so that a "smeared" effect of the stiffeners is considered.

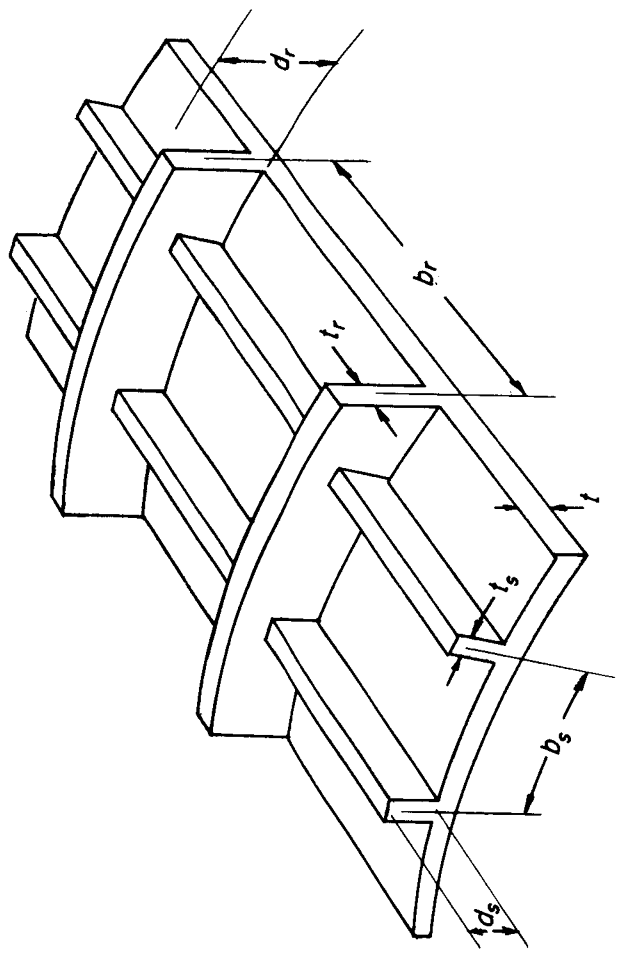

Figure 1 Stiffened Cylinder Configuration

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b) The direct strains of a curved plate element $\epsilon_{xx}, \epsilon_{yy}$ in a two dimensional curvilinear coordinate system x, y, vary linearly across the thickness in the stiffener as well as the sheet. The ϵ_{xx}, ϵ_{yy} in the sheet as well as the stiffener are equal at their point of contact.

c) The stiffeners do not transmit shear. The membrane shear stress resultant N_{xy} is entirely carried by the sheet.

d) The torsional rigidity of the stiffener cross-section is added to that of the sheet.

e) The middle surface of the sheet is chosen as the reference surface for the geometric description of the strain field.

Stress-strain relationship:

As a consequence of these assumptions, the stress resultants N_{xx}, N_{yy}, N_{xy} and the moment resultants M_{xx}, M_{yy}, M_{xy}, M_{yx} acting on a shell element including the stiffeners are related to the middel surface direct strains ϵ_x, ϵ_y, and ϵ_{xy} and middle surface curvature changes κ_x, κ_y, and κ_{xy} as follows:

$$
\begin{aligned}
N_{xx} &= B_1 (\epsilon_x + \nu \epsilon_y + \xi_s \kappa_{xx}) \\
N_{yy} &= B_2 (\epsilon_y + \nu \epsilon_x + \xi_r \kappa_{yy}) \\
N_{xy} &= B_3 (1 - \nu) \epsilon_{xy}
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
M_{xx} &= -D_1 (\kappa_{xx} + \nu \kappa_{yy} + \eta_s \epsilon_x) \\
M_{yy} &= -D_2 (\kappa_{yy} + \nu \kappa_{xx} + \eta_r \epsilon_y) \\
M_{xy} &= -D_3 [(1 - \nu) + K_s] \kappa_{xy} \\
M_{yx} &= -D_3 [(1 - \nu) + K_r] \kappa_{xy}
\end{aligned}
\tag{4}
$$

There are several interesting factors about Eqs. (3) and (4). The terms $(\xi, \eta, K)_{s,r}$ refer to the additional axial, bending and torsional effects induced by the eccentricity of stringers and rings, respectively.

It is of further interest that these terms represent the coupling between membrane and bending terms.

If the stiffening systems were located symmetrically with respect to the sheet or the stiffeners were shallow enough that these coupling terms are of second order of importance, then these terms vanish and the equations reduce to the familiar orthotropic formulation used by Taylor (Ref. 1).

In Eq. (4) we find $M_{xy} \neq M_{yx}$ in this general case, as opposed to the orthotropic formulation.

Strain- displacement relationship:

The direct strain ϵ and the curvature changes κ are related to the displacement u, v, and w, occuring during the buckling process, as follows:

$$
\begin{aligned}
\epsilon_x &= u,_x & \kappa_{xx} &= w,_{xx} \\
\epsilon_y &= v,_y + w/R & \kappa_{yy} &= w,_{yy} \\
2\epsilon_{xy} &= (u,_y + v,_x) & \kappa_{xy} &= w,_{xy}
\end{aligned} \tag{5}
$$

Equilibrium equations:

The equilibrium of the shell element during buckling under the external compressive loading $\overline{N}_x$ is expressed in terms of the induced stress field as follows:

$$N_{xx,x} + N_{xy,y} = 0 \tag{6}$$

$$N_{xy,x} + N_{yy,y} = 0 \tag{7}$$

$$M_{xx,xx} + M_{xy,xy} + M_{yx,xy} + M_{yy,yy} + N_y/R + \overline{N}_x w,_{xx} = 0 \tag{8}$$

By utilizing Eqs. (3), (4), and (5) the equilibrium equations can be written in terms of u, v, and w. By choosing sinusoidal functions for u, v, and w such as

$$
\begin{aligned}
u &= \overline{U} \cos(m\pi x/L) \cos(ny/R) \\
v &= \overline{V} \sin(m\pi x/L) \sin(ny/R) \\
w &= \overline{W} \sin(m\pi x/L) \cos(ny/R)
\end{aligned} \tag{9}
$$

we obtain 3 algebraic equations in $\overline{U}$, $\overline{V}$, $\overline{W}$, the vanishing of whose determinant leads to the stability criterion.

The final expression for the buckling coefficient k, which is defined by

$$k = \overline{N}_x L^2/\pi^2 D$$

is given as follows:

$$k = m^2(1+\beta^2)^2 + m^2(EI_s/b_sD) + m^2\beta^4(EI_r/b_rD) + [(GJ_s/b_sD) + (GJ_r/b_rD)]\,m^2\beta^2$$

$$+ (12Z^2/m^2\pi^4)(1 + \overline{S}\lambda_s + \overline{R}\lambda_r + \overline{S}\,\overline{R}\lambda_{rs})\lambda^{-1} \qquad (10)$$

where

$$\lambda_r = 1 + 2\alpha^2\beta^2(1-\beta^2\nu)(e_r/R) + \alpha^4\beta^4(1+\beta^2)^2(e_r/R)^2$$

$$\lambda_s = 1 + 2\alpha^2(\beta^2-\nu)(e_s/R) + \alpha^4(1+\beta^2)^2(e_s/R)^2$$

$$\lambda_{rs} = 1-\nu^2 + 2\alpha^2\beta^2(1-\nu^2)\,[(e_r/R) + (e_s/R)] + \alpha^4\beta^4[1-\nu^2 + 2\beta^2(1+\nu)](e_r/R)^2$$

$$+ 2\alpha^4\beta^4(1+\nu)^2(e_re_s/R^2) + \alpha^4\beta^2\,[2(1+\nu) + \beta^2(1-\nu^2)]\,(e_s/R)^2$$

$$\lambda = (1+\beta^2)^2 + 2\beta^2(1+\nu)(\overline{R}+\overline{S}) + (1-\nu^2)\,[\overline{S} + 2\beta^2\,\overline{R}\,\overline{S}(1+\nu) + \beta^4\overline{R}]$$

with

$$Z^2 = (L^4/R^2t^2)(1-\nu^2) \qquad D = (Et^3/12)(1-\nu^2)^{-1}$$

$$\overline{S} = (A_s/b_st) \qquad \overline{R} = (A_r/b_rt)$$

$$\alpha = m\pi R/L \qquad \beta = nL/m\pi R$$

The critical value of k is determined by minimizing the expression of Eq. (10) with respect to m and n, which take discrete integer values.

In Eq. (10) the subscripts r and s stand for the ring and stringer quantities respectively. A, I, J are the areal and inertial stiffnesses provided by the stiffeners. The terms e_r, e_s represent the distance of the stiffener centroids from the middle surface of the shell skin. They are positive or negative according as the sign convention adopted for the coordinate system.

Influence of Stiffener Eccentricity on General Instability

It is interesting to see that in Eq. (10), the functions λ_r, λ_s, λ_{rs} contain terms to the first power of e/R which change signs according as they are positive or negative. Thus, the minimum buckling coefficient is influenced considerably by the changes in signs of e/R.

Table 1 displays the computed results from Ref. 7 based on Eq. (10) for moderate length cylinders under compression using Z-type stiffeners. Three cases of stiffening, namely the longitudinal, the ring, and the grid systems have been considered. Table 2 gives the experimental results reported in Ref. 5 for the longitudinally stiffened cylinders.

Table 1

Effect of Stiffener Locations on the Critical Stress: Theoretical Results

Stiffening System	Stiffener Location	$\sigma_{critical}$ (psi)
Stringers and rings	Stringers outside, rings outside	41370
	Stringers outside, rings inside	35740
	Stringers inside, rings inside	27790
Stringers only	Stringers outside	13400
	Stringers inside	6410
Rings only	Rings outside	38260
	Rings inside	13240

Table 2

Effects of Stiffener Location on the Critical Stress: Test Results

Cylinder	Type of Stiffener	Stiffener Location	σ_{psi}
1	Integral Stringer	External	30,500
2	Integral Stringer	Internal	12,900
3	Integral Stringer	External	34,400
4	Integral Stringer	Internal	17,000
5	Z Stringer	External	23,700
6	Z Stringer	Internal	16,000

Table 1 and 2 show that, in general, the outside stiffeners increase the load carrying capacity of the cylinder, compared to the inside stiffeners.

It should, however, be emphasized that the results of Table 1 do no more than merely indicate the trend of the strengthening effect. A clearer understanding of the stiffener effects is obtained when one examines the mechanism of the strengthening due to the stiffener eccentricity. Singer, et al, have discussed this physical mechanism in their paper on the buckling of stiffened cylinders under hydrostatic pressure, Ref. 10. According to them, there are two opposing tendencies present due to the stiffeners: a primary effect, whereby the outside stiffeners increase the actual bending stiffness in the direction of the stiffening and a secondary opposing effect in the orthogonal direction due to Poisson's ratio. In particular cases, the one effect can dominate the other, to give the overall strengthening. In problems of hydrostatic pressure, for example, with ring stiffeners, the secondary effect can dominate so that there is an inversion, that is, inside stiffeners can be more effective than external stiffeners. However, for stringers, in general, the axial membrane forces are much higher than those in the circumferential direction so that this inversion has not been observed.

In view of the above discussion, it is clear that the strengthening effects such as shown in Table 1 can be only tentative. Hence, it is worthwhile to reexamine the problem with a view to single out the effect of eccentricity on the critical stress. In order to simplify an otherwise complex problem, we consider the case of a cylinder stiffened only by stringers. Then in Eq. (10) all terms with the subcript r vanish. Further, in view of results for the stability of longitudinally stiffened moderate length cylinder in axial compression, such as in Ref. 3, we can let m = 1. Hence, Eq. (10) is written as:

$$N = (1+\beta^2)^2 \pi^2 D L^{-2} + \pi^2 E I_s L^{-2} b_s^{-1} + (1/2)\pi^2 E J_s \beta^2 L^{-2} (1+\nu)^{-1} b_s^{-1} + (12/\pi^2) L^2 (1-\nu^2) D R^{-2} t^{-2} (1+\bar{S}\lambda_s)\lambda^{-1} \tag{11}$$

where λ is now given by

$$\lambda = (1+\beta^2)^2 + 2\beta^2(1+\nu)\bar{S} + (1-\nu^2)\bar{S}$$

The geometrical variables that enter into Eq. (11) will be dependent upon the choice of the cross-section chosen for the stiffeners. If a rectangular cross section (Fig. 1)is chosen, the stiffener geometry is governed by t_s, d_s, b_s and the cylinder geometry by L, R and t. In order to reduce the number of stiffener variables to the single parameter of e, the eccentricity, suitable proportions have to be devised for t_s, b_s, d_s. Fortunately, we can utilize the optimization concept of equating the local buckling modes of stringer and panel instability given by Eqs. (1) and (2). By making the single assumption that b_s the stringer spacing be equal to d_s the stringer depth, we find from the elementary geometry of Fig. 2,

$$b_s/R = 2e/R - t/R \tag{12}$$

Hence, the various inertial terms in Eq. (11) can be expressed in terms of the cylinder geometry and e/R the eccentricity ratio. The cylinder geometry is expressible in terms of the familiar curvature parameter, Z, and R/t, the radius to thickness ratio.

Thus Eq. (12) can be written for the specific case of stringers with rectangular cross-section as:

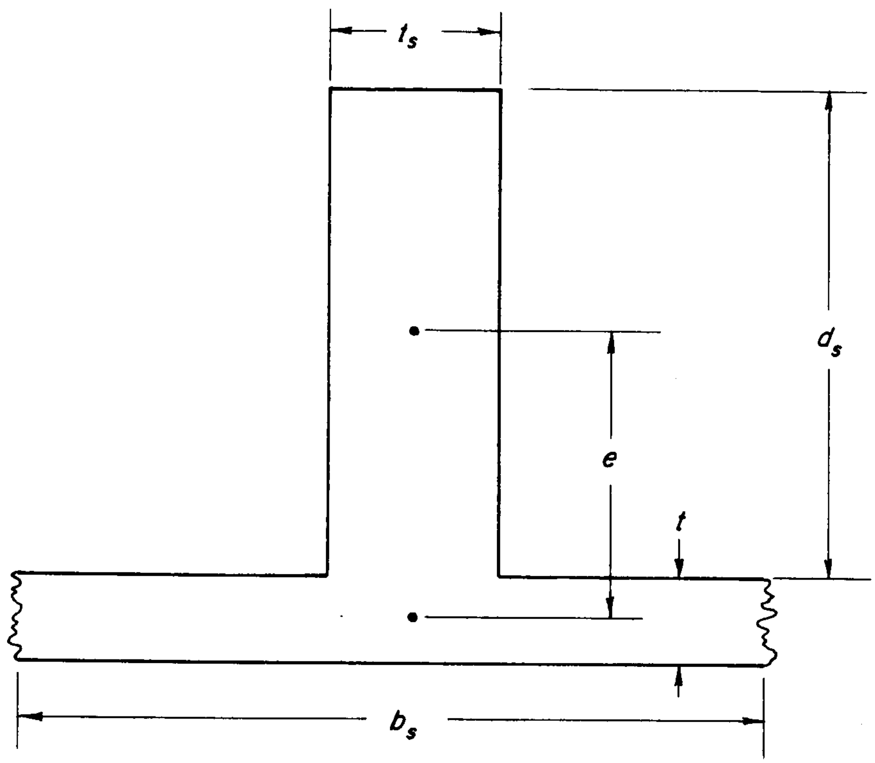

Figure 2 Stringer Section Geometry

$$N/Ed = (\pi^2/24)\ (1 - \nu^2)^{-\frac{1}{2}} (1 + \beta^2)^2\ (R/t)^{-2} Z^{-1} + 4\pi^2 (1 - \nu^2)^{\frac{1}{2}} Z^{-1}\ (R/t)(I_s/b_s d^3)$$

$$+ 2\pi^2\ (1 + \nu)^{-1}\ (1 - \nu^2)^{\frac{1}{2}} \beta^2\ (J_s/b_s d^3)(R/t) Z^{-1}$$

$$+ (1/2\pi^2)\ (1 - \nu^2)^{-\frac{1}{2}} Z (R/t)^{-2}\ (1 + \overline{S}\ \lambda_s)\ \lambda^{-1} \tag{13}$$

with $\quad I_s/b_s d^3 = .117851\ (R/t)^{-1}\ [\ e/R - 0.5\ (R/t)^{-1}\ ]^2$

(14)

and $\quad J_s/b_s d^3 = (R/t)^{-3}\ [.942869 + .055715\ (R/t)^{-1} \{e/R - 0.5\ (R/t)^{-1}\}^{-1}]$

For a cylinder of given Z and R/t, the influence of e/R on the load carrying capacity can be readily evaluated from the above Eq. (13). For very large R/t values, keeping Z constant, it becomes apparent from Eqs. (13) and (14) that N/Ed reduces to

$$N/Ed = C\ Z^{-1}\ (e/R)^2 \tag{15}$$

where C is a numberical constant.

Eq. (15) shows that for larger R/t values and Z constant, the variation of critical stress with e/R is parabolic. That is, there is no difference between outside and inside stiffeners. However, the very interesting fact that emerges from Eq. (15) is that the minimum occurs for e/R = 0 which corresponds to symmetric stiffening. That is, for such extremely thin cylinders, both outside and inside stiffeners are better than symmetric stiffeners.

Fig. 3, shows a plot of N/Ed with e/R for several R/t values. As R/t values decrease, we notice that the minimum of the curve shifts towards the negative e/R values and the inside stiffeners become less and less efficient in strengthening the cylinder. In fact for some R/t values, symmetric stiffening is superior to inside stiffeners.

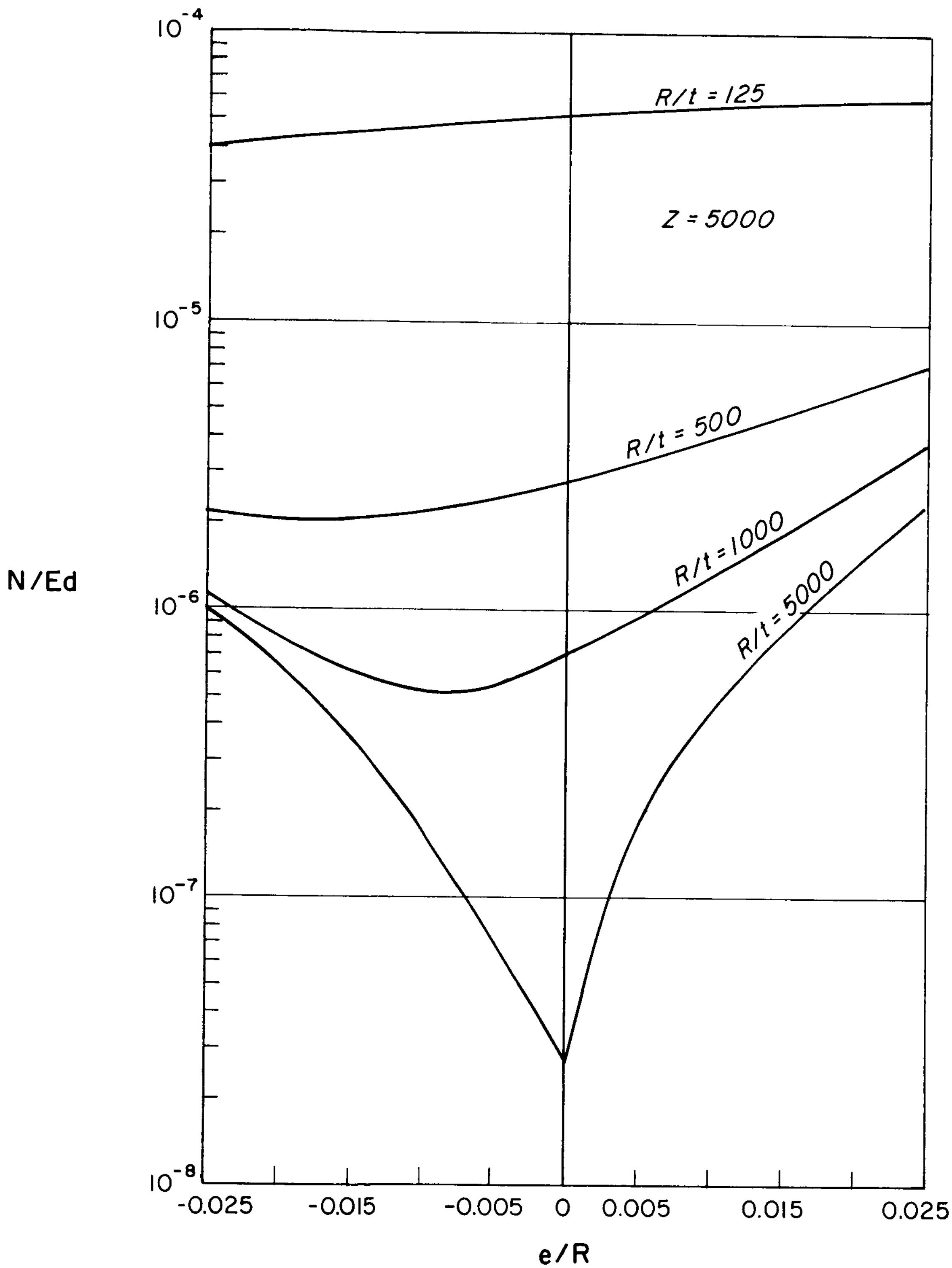

Figure 3 Effect of Eccentricity on the Load Carrying Capacity of Stiffened Cylinders under Compression

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3. Minimum Weight Design Problems of Deep-Stiffened Cylinders

Design Criteria

The problem of minimum weight design is usually posed in the following manner:

"For a cylinder of given length L, diameter d and loading N, with the material constants E and ν also given, what are the optimum cross-sectional proportions so that the structural weight is a minimum?"

The answer to the question is based on the axiom, already noted, that minimum weight proportions are achieved when all the possible structural buckling modes occur simultaneously at the applied stress level. In the case of deep-stiffened cylinders, the effect of the stiffener is an additional geometric criterion as has been noted in Sec. 2, and illustrated in Fig. 3.

Since location of the stiffeners has a definite influence of the strength, it is important to compare the efficiencies of symmetrical and asymmetrical designs. The following discussion provides a qualitative base for making this evaluation.

Symmetric Design vs. Eccentric (one-sided) Designs

This important effect is worth examining in some detail. For illustrative purposes, let us confine our attention to cylinders stiffened only by stringers of rectangular cross-section. In the absence of ring stiffeners, the possible stability modes are: 1) a local panel instability mode which involves the instability of a panel between two stringers 2) a stringer instability mode where there is a single buckle as a flange and 3) the general instability mode involving all the stringers.

Let us assume, to begin with, that a minimum weight design has been achieved with the stringers being symmetrical. This implies that for such a design, the general instability stress and the local flange buckling stress are equal. Now if we were to consider an equal area stiffener, entirely outside the cylinder, we find that the general instability stress level is increased. Hence, if this design were to be of minimum weight, the local instability level should also be raised. In order to achieve this, the geometric parameters to be varied are: the depth of the stiffener and the thickness of the stiffener.

From Eq. (1) the local instability (flange instability) stress level is directly proportional to the square of the flange thickness t_s and inversely proportional to the square of the stiffener depth d_s. Hence, in order to increase the local instability stress there are three possibilities:

1) t_s is increased with d_s constant

2) d_s is decreased with t_s constant

3) t_s is increased and d_s is decreased.

Of the three, the first possibility will generally result in heavier design than the corresponding symmetric case, the second and third may result in designs no lighter than the symmetrical design.

Current Design Results and Their Criticism

In Ref. 11, recent results are reported on the weights of cylinders with different stiffening systems. These are purported to be of minimum or optimum weight, but these designs do not satisfy the fundamental criterion that all modes of stability occur simultaneously and, hence, do not offer a proper basis for comparison. For example, using the nomenclature of Fig. 1 of this report, consider the following data from Ref. 11, for two cases of stringer-stiffened cylinders purported to be minimum weight designs:

Table 3

Data from Ref. 11 for Stringer-Stiffened Cylinders

Case	t (in)	t_s (in)	d_s (in)	b_s (in)	$\sigma_{gen.}$ (psi)	E (psi)	ν
1	.10	.61	.75	7.96	5710	10^7	0.3
2	.10	.34	.75	7.36	6680	10^7	0.3

From Eqs. (1) and (2) herein, utilizing the above data of Table 3, we have the following theoretical elastic buckling stress values:

Case 1 $\sigma_{stringer}$ = 2,989,400 psi σ_{panel} = 5,706 psi

Case 2 $\sigma_{stringer}$ = 928,715 psi σ_{panel} = 6,674 psi

Thus, we see that in Ref. 11 the authors have satisfied the criterion with respect to only one mode besides the general instability mode, which violates the requirement for minimum weight design that all the possible modes of instability be satisfied.

4. Scope for Future Work

In a future program, it is expected that the design study of deep-stiffened cylinders will be conducted on the following lines:

The first study will try to establish the real or meaningful weight efficiency of eccentrically stiffened cylinders over symmetrically stiffened cylinders. This would be a logical extension to the study of minimum weight design of symmetrically stiffened cylinders, Ref. 4.

The second basis would be to seek other cross-sectional shapes such as monolithic and built-up-Z and Y sections which have been found to be more efficient than rectangular cross-sections in the past.

The actual process would be to express the minimum weight parameter, Σ (solidity, the ratio of structural volume to the enclosed volue) in terms of the governing geometrical parameters, given the loading index (N/Ed) for the cylinder.

In the eccentric stiffening system, we have seen, from Section 2, the complex relationship between the geometrical parameters and the buckling stress. Hence, the functional relationship between Σ and the structural parameters may be written as:

$$\Sigma = F\ \left[\left\{ e/d,\ t/d,\ b/d \right\}_{r,\,s};\ R/t,\ Z;\ N/Ed\ \right]$$

Hence, if Σ_{min} is to be obtained for a cylinder of given Z and N/Ed, it is evident a systematic parametric study has to be made with respect to each of the parameters $\left\{ e/d,\ t/d,\ b/d \right\}$ of the stiffeners and R/t of the sheet.

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