NASA GRANT NO. NGR-44-006-045

(Optimum Hypersonic Lifting Wings)

PROGRESS REPORT (*)

by

ANGELO MIELE(**)

cde / 1 Page 5 / 81789 CR / Cat 01

N67 18133

HC $ 3.00 MF .65

The object of this investigation is a general analysis of optimum two-dimensional or three-dimensional lifting wings in hypersonic flow. The methods of the theory of maxima and minima and those of the calculus of variations in one or two independent variables are used in order to maximize the lift-to-drag ratio for given geometric and aerodynamic constraints. This quantity is important in that the range and the maneuverability of a hypersonic cruise vehicle, a hypersonic glide vehicle, and a reentry vehicle increase linearly with it. Since flat-top wings are naturally suited to produce high lift-to-drag ratios at hypersonic speeds, particular attention is devoted to these wings under the assumptions that the free-stream velocity is parallel to the flat top, the pressure distribution is modified Newtonian, and the surface-averaged skin-friction coefficient is constant.

Direct Methods. The class of two-dimensional wings whose chordwise contour is a power law is investigated in Ref. 1, and the combination of power law exponent and thickness ratio maximizing the lift-to-drag ratio is determined. First, unconstrained

---

(*) Period July 1, 1966 through December 31, 1966.

(**) Professor of Astronautics and Director of the Aero-Astronautics Group, Department of Mechanical and Aerospace Engineering and Materials Science, Rice University, Houston, Texas.

configurations are considered, and it is shown that the optimum thickness ratio is such that the skin-friction drag is one-third of the total drag. For a surface-averaged skin-friction coefficient $C_f = 10^{-3}$, the maximum lift-to-drag ratio is $E = 5.29$ and corresponds to a wedge of thickness ratio $\tau = 0.126$. Next, constrained configurations are considered, that is, conditions are imposed on the length, the thickness, the enclosed area, and the position of the center of pressure. For each combination of constraints, an appropriate similarity parameter is introduced, and the optimum power law exponent, thickness ratio, and lift-to-drag ratio are determined as functions of the similarity parameter.

Indirect Methods in One Independent Variable. The class of two-dimensional wings whose chordwise contour is arbitrary is investigated in Refs. 2 and 3. First, it is assumed that the lift is a prescribed quantity (Ref. 2), and the necessary conditions to be satisfied by a minimum drag airfoil are derived for conditions imposed on the length, the thickness, the profile area, and the pitching moment. Then, the following particular cases are analyzed: (a) given lift, (b) given lift and chord length, (c) given lift and thickness, (d) given lift and profile area, and (e) given lift, pitching moment, and chord length. In all of these cases, analytical expressions are presented for the geometry of the optimum airfoil and the aerodynamic drag. Next, the lift is regarded to be unconstrained (Ref. 3), and the necessary conditions to be satisfied by a maximum lift-to-drag ratio airfoil are derived for conditions imposed on the length, the thickness, and the profile area. Once more, several particular cases are studied and, for each of these, analytical expressions are determined for the optimum shape and the maximum lift-to-drag ratio.

Indirect Methods in Two Independent Variables. The class of three-dimensional wings whose chordwise and spanwise contours are arbitrary is investigated in Refs. 4 and 5.

First, it is assumed that the lift is prescribed (Ref. 4), and the necessary conditions to be satisfied by a minimum drag wing are derived for (a) unconstrained volume and (b) given volume. For case (a), the optimum wing has a constant chordwise slope and a trailing edge thickness distribution similar to the chord distribution. While the planform area is uniquely determined, the chord distribution is not. In other words, there exist an infinite number of chord distributions yielding the same maximum value of the lift-to-drag ratio. For case (b), two solutions are possible depending on the value of the volume-lift parameter, a parameter directly proportional to the volume and inversely proportional to the lift squared. If the volume-lift parameter is greater than a certain critical value, the optimum wing is identical with that of case (a). If the volume-lift parameter is smaller than the critical value, the optimum wing has a constant chord and a constant trailing edge thickness. Also, the chordwise slope is constant in the spanwise sense but not in the chordwise sense. Finally, the maximum lift-to-drag ratio decreases as the volume-lift parameter decreases.

Next, the lift is regarded to be unconstrained, and the necessary conditions to be satisfied by a maximum lift-to-drag ratio wing are derived for (a) unconstrained volume and (b) given volume. For case (a), the optimum wing surface is unique; it has a constant chordwise slope and a trailing edge thickness distribution similar to the chord distribution. For case (b), the chordwise slope of the optimum wing is constant in the spanwise sense but not in the chordwise sense. A one-parameter family of extremal solutions exists, depending on the value of the volume parameter: this parameter is directly proportional to the volume and inversely proportional to the span and the root chord squared. If the volume parameter exceeds a certain critical value, the optimum wing is convex. If the volume parameter is equal to the critical value, the optimum wing is identical with that of case (a). Finally, if the volume parameter is smaller than the critical value, the optimum wing is slightly concave.

REPORTS

1. MIELE, A. and WILSON, W.L., Two-Dimensional, Power-Law Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 23, 1966.

2. HULL, D.G., Two-Dimensional, Lifting Wings of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 24, 1966.

3. HULL, D.G., Two-Dimensional Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 25, 1966.

4. MIELE, A. and HULL, D.G., Three-Dimensional, Lifting Wings of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 26, 1966.

5. MIELE, A. and HULL, D.G., Three-Dimensional Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 27, 1966.