THE NONSINGULAR ORIGIN OF THE UNIVERSE

by

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In an expanding universe the average energy density diminishes with time. The further we go back in time the greater was the density, and according to current theory the universe originated from (or passed through) a space-time singularity of infinite density some 10^{10} years ago[1].

Attempts to quantize the gravitational field encounter both formal and conceptual difficulties[2]. Failing a rational theory we adopt qualitative arguments and show that it is unlikely that the universe originated as a singularity.

Consider first particles of mass m for which the gravitational coupling constant is unity:

$$G\, m^2/\hbar c = 1\ . \tag{1}$$

Their Compton wavelength $\lambda = \hbar/mc$ equals their gravitational length $G\, m/c^2$, or

$$\lambda = (G\hbar/c^3)^{1/2} \sim 10^{-20}\, \lambda_n \tag{2}$$

$$m = (\hbar c/G)^{1/2} \sim 10^{20}\, m_n \tag{3}$$

where $\lambda_n = \hbar/m_n c$ and m_n is the nucleon mass. So far the particles are of arbitrary size. If now their density approaches $\rho = m/\lambda^3$, or

$$\rho = c^5/G^2\hbar \sim 10^{95}\ \mathrm{gcm}^{-3} \tag{4}$$

the quantum fluctuations of the metric assume their maximum value[3,4]. According to (1) they should be strongly interacting; but at the density (4) they are "ghost" particles enclosed in their own metric, and at the most interact only feebly with each other.

Consider now what happens when the universe has the very high density (4). For simplicity let all particles be identical, of energy mc^2. (The main conclusions are little affected for a spectrum of fermions and bosons in which the Fermi and thermal energies are comparable and equal to mc^2.) The age[5] t of the universe at this stage is $\tau \sim (\rho G)^{-1/2}$. But, from (4)

$$(G\rho)^{-1/2} = (G\hbar/c^5) = \lambda/c \tag{5}$$

and therefore the age

$$\tau \sim \lambda/c \sim 10^{-44} \text{ sec} \tag{6}$$

is the time required for light to travel the interparticle distance λ. Neighboring particles recede from each other at the velocity (almost) of light, and the observable universe[6] has a radius of $\lambda \sim 10^{-33}$ cm.

When the age t of the universe is large compared with τ it is possible to construct in the usual way cosmological models of nested hypersurfaces orthogonal to time. This is because the unavoidable uncertainties nevertheless permit $\Delta t \ll t$. But at time $t = \tau$ the uncertainty principle[4] $\Delta E \Delta t \sim \hbar$ can be cast into the form

$$\Delta E \Delta t \sim E\tau \tag{7}$$

where $E = mc^2$, $\tau \sim (G\rho)^{-1/2} \doteq \hbar/E$. For $\Delta E \sim E$, it follows that $\Delta t \sim \tau$, and the spatial hypersurface is smeared throughout the age of the universe. The universe now verges on the edge of indescribable conditions in which the notion of a space-time manifold is untenable. Thus, it is impossible to trace the origin of the universe back to a space-time singularity of infinite density at $t = 0$, for at time τ the universe dissolves into a "phantom" metric of ghost particles.

REFERENCES

[1]H. P. Robertson, Rev. Mod. Phys. 5, 62 (1935). For more recent work and references see S. W. Hawking, Phys. Rev. Letters 15, 689 (1965).

[2]See J. L. Anderson, Gravitation and Relativity (ed. H. Y. Chiu and W. F. Hoffmann, Benjamin, New York, 1964) p. 279, for a discussion and references.

[3]J. A. Wheeler, Geometrodynamics (Academic Press, New York, 1962). Also, L. D. Landau, Niels Bohr and the Development of Physics (ed. W. Pauli, Pergamon, London, 1955) suggests a cut-off energy of mc^2 in electrodynamics.

[4]If $g_{\mu\nu}$ are the metric coefficients, one might hazard that the uncertainty relations are of the form

$$\Delta(g_{oo}^{1/2}E)\Delta(g_{oo}^{1/2}t) \sim \hbar$$

$$\Delta(\sqrt{-g_{xx}}p^x)\Delta(\sqrt{-g_{xx}}x) \sim \hbar$$

but $\Delta g_{oo} \sim -\Delta g_{xx} \sim Gm/c^2\lambda = 1$, and for $\Delta E \sim E$, $\Delta t \sim t$ the usual relations: $\Delta E\Delta t \sim \hbar$, $\Delta p^x\Delta x \sim \hbar$ follow. The extent of the fluctuation is λ in space and λ/c in time.

[5]We have: $\dot{R}^2 = 8\Pi G\rho R^2/3$ (the curvature constant is negligible), and for $\rho \propto R^{-4}$: $t = (32\Pi G\rho/3)^{-1/2}$.

[6]Assuming the present density is 10^{-30} g cm^{-3} and $R \sim 10^{28}$ cm, and also $\rho \propto R^{-3}$ for $\rho < m_n/\lambda^3{}_n \sim 10^{16}$ g cm^{-3}, and $\rho \propto R^{-4}$ for $\rho > m_n/\lambda^3{}_n$, it follows that at the density (4) the radius of curvature of a closed universe is $R \sim 10^{-7}$ cm.