ASTROPHYSICAL EVIDENCE FOR THE DIRECT ELECTRON-NEUTRINO WEAK INTERACTION†

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†A summary of this paper was presented at the 4th Eastern U.S. Theoretical Physics Conference, held at the State University of New York at Stony Brook, November, 1965.

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ABSTRACT

Observational evidence exists that the central stars of planetary nebulae are evolving into white dwarfs. However, a "gap" in the stellar population appears between the faintest central stars (100 $L_\odot$) and the brightest white dwarfs (1 $L_\odot$). Theoretical models of superluminous white dwarfs (Gap stars) are constructed with and without the inclusion of neutrino emission processes.

It is known from previous work that neutrino emission speeds up the evolution of a star. The particular process of importance here is the plasma neutrino process of Adams, Ruderman, and Woo, which depends critically on the existence of the $(e\nu_e)(e\nu_e)$ interaction. We have found that the evolution of stars in the mass range 0.7 $M_\odot$ to 1.1 $M_\odot$ (Chandrasekhar mass limit for white dwarfs = 1.4 $M_\odot$) is accelerated by a factor of 10 to 100, depending on the assumed chemical composition, if plasma neutrino emission is included; below 0.4 $M_\odot$, neutrino emission is unimportant. The more massive Gap stars may be identified observationally on the basis of their very blue color. An upper limit to their evolution time is then obtained from a variety of observational arguments; this upper limit is 5×10^5 years,

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to be compared with the theoretical lifetimes of 2×10^5 years with neutrino emission and 2×10^6 years without it. From these results we can draw the following conclusions: (1) the presently known rate of plasma neutrino energy loss roughly reproduces the astronomical data; (2) the weak interaction coupling constant for the $(e\nu_e)(e\nu_e)$ interaction has a lower limit close to the currently accepted value. Further evidence from the known observational lifetime of central stars and from some crude stellar models indicates that neutrino emission (photoneutrino process) may also be accelerating evolution in the central stars.

I. Introduction

Recent observational work on planetary nebulae has cast new light on our understanding of these objects (1,2,3). The central stars of planetary nebulae are observed to be evolving into white dwarfs. The pre-planetary star ejects a hydrogen-rich shell and the ejected matter forms an expanding nebula. Spectral studies of planetary nebulae have shown that they are expanding at an average rate of 20 km/sec until their intensities and densities are too low to detect. This occurs at a radius of the nebular ring < 0.7 pc (1 parsec = 3.26 light-years). From the average expansion rate and the maximum radius, the lifetime of a nebula in its planetary stage is found to be 3-5 x 10^4 years (1,3). During this short period, the central stars are observed to contract gravitationally from over one solar radius ($R_\odot = 6.9 \times 10^{10}$ cm) to the white dwarf radius ($\sim 0.01\ R_\odot$) while their effective surface temperatures and luminosities change from $T_e \sim 4 \times 10^4\ ^\circ K$ and $L \sim 10^4\ L_\odot$ to $T_e \sim 1 \times 10^5\ ^\circ K$ and $L \sim 10^2\ L_\odot$, respectively ($L_\odot = 4.0 \times 10^{33}$ ergs/sec). Because of the high effective surface temperature, most radiation from the central star is in the ultraviolet region; this ultraviolet radiation then ionizes the nebular ring. When the electrons and ions recombine, photons with wavelengths corresponding to atomic transitions are emitted, becoming the source of radiation of the nebula.

There is no direct way of measuring the mass of a central star. By comparing the galactic equivalent width of the planetary nebulae in the solar neighborhood with Schmidt's model of the Galaxy, the average initial mass of the star plus the nebula is estimated to be approximately 1.2 $M_{\odot}$ (1). The mean nebular mass is found to be 0.2 $M_{\odot}$ (1). Hence the average mass of the central stars is about one solar mass or less. This estimate is also compatible with the average mass of the fainter sequence of white dwarfs (0.7 $M_{\odot}$) into which many central stars must subsequently evolve, and with the average mass of RR Lyrae stars ($\sim$ 0.9 $M_{\odot}$) from which some central stars are believed to have evolved.

The white dwarfs are characterized by their exceedingly high mean density ($10^5 - 10^6$ g/cc) and low luminosity ($\leq 1\ L_{\odot}$). Only a few white dwarfs are observed to have luminosities near 1 $L_{\odot}$. Their effective surface temperatures range from 10^5 $^{\circ}$K downward. At such a high density, electrons are Fermi degenerate. Some time ago Chandrasekhar constructed models of completely degenerate stars (4). From his theory of degenerate white dwarfs, the radius may be directly related to the mass, which tends to a finite limit ($\sim 1.4M_{\odot}$ for white dwarfs composed of elements heavier than hydrogen) as the central density increases indefinitely.

Using Chandrasekhar's models and the observed luminosities and surface temperatures of white dwarfs, T. D. Lee showed that nuclear energy can contribute only a negligible amount of luminosity (5). The almost complete degeneracy will prevent any significant gravitational contraction. Thus the only energy source available is the residual thermal energy of the non-degenerate nuclei in the core of the star; white dwarfs must be in a stage of cooling without further changes in stellar structure.

At the end of their evolutionary track, the central stars have already shrunk down to white dwarf radii. Thus, from an observational standpoint, the subsequent evolutionary track is quite certain.

Figure 1 shows the positions of central stars and white dwarfs in the H-R diagram, which is a plot of stellar luminosity versus stellar effective surface temperature (in logarithmic scale). The dashed line indicates the theoretical evolutionary track calculated in Section II. No central star is definitely observed to have luminosity below

10^2 $L_\odot$, nor does any white dwarf lie above 1 $L_\odot$ (2,3,6). Some blue subdwarfs are believed to lie between the luminosity limits 1 $L_\odot$ and 10^2 $L_\odot$ at surface temperatures corresponding to those near the evolutionary track, but their luminosities and, hence, their exact positions in the H-R diagram are rather uncertain. In analogy with the RR Lyrae gap, we shall hereafter refer to the above-mentioned region in the H-R diagram ($L = 1 - 10^2$ $L_\odot$, $T_e \sim 5 \times 10^5$ ^oK) as the Gap. It should be emphasized that the Gap is more an observational selection of bright stars which are central stars of planetary nebulae and white dwarfs, rather than a reality. The stellar density along the evolutionary track in the H-R diagram is inversely proportional to the rate of evolution along the track. By counting the stars which may possibly lie in the Gap, an upper limit may be placed on the evolution time through the Gap. This observational upper limit will be shown to be shorter than what one would expect merely on the basis of thermal energy radiated from the surface of a star.

Previously Chiu and others have emphasized that neutrino emission processes via the direct electron neutrino interaction ($e\ \nu_e$) ($e\ \nu_e$), which has been predicted in Feynman and Gell-Mann's version of the V-A theory, can drastically affect the rate of stellar evolution. Although the direct electron-neutrino interaction has never been observed in the laboratory, there are good theoretical reasons to believe in its existence. The particular neutrino process of interest here is the plasma neutrino process. Briefly speaking, inside an electron gas the dispersion relation of a photon is given by

$$(\hbar\omega)^2 = (\hbar\,\omega_o)^2 + (\hbar kc)^2 \qquad (1)$$

where ω is the angular frequency and k the wave number vector of the photon, and ω_o is the plasma frequency which is a function of the electron temperature and the electron density. In free space ω_o vanishes. A photon propagating in an electron gas therefore behaves as a particle with a rest mass $\hbar\omega_o/c^2$; such a photon is sometimes called a plasmon. Ordinarily the decay of a free photon into a pair of neutrinos is forbidden by gauge invariance and is also incompatible with both the energy conservation and the momentum conservation laws. Because of the dispersion relation (1), a photon interacting with an electron gas can decay into a pair of neutrinos via the direct electron neutrino coupling without violating the gauge invariance. The plasma process was first suggested by Adams, Ruderman and Woo and the calculation of the energy loss rate was carried out by Adams *et al.* (7) and later by Zaidi (8).

In the following sections, we shall show that, because of the plasma process, the theoretical lifetimes of Gap stars can be significantly reduced to values compatible with the observations.

II. White Dwarfs and Gap Stars

Observationally, at the end of its evolutionary track ($L = 10^2 L_\odot$), the central star of a planetary nebula has already contracted down to the white dwarf radius (1,2,3). Previously, T. D. Lee found that the central temperature corresponding to this luminosity is $\sim 2\times10^8$ °K and the completely degenerate white dwarf model is still valid (5). This is confirmed by our calculations. Thus, we take the structure of a Gap star to be the same as that of a white dwarf except in the envelope where electrons are non-degenerate; but in all cases the mass of the envelope is negligible compared with that of the star. The structure of the envelope is required only to obtain the luminosity, otherwise the thickness of the envelope is also negligible compared with the radius of the star. This was pointed out many years ago by Strømgren. The calculated structure, together with the black-body radiation law, gives us the dashed evolutionary track in the Gap for 0.736 $M_{\odot}$ as shown in Figure 1. The evolutionary track in the central star region is based on models for central stars, and will be discussed in the next section.

The central temperature of a degenerate star is determined by integrating the following equation for the pressure, P, and temperature, T, from the surface inward through the envelope, until the temperature no longer increases significantly on account of the high heat conductivity of the degenerate electrons in the core:

$$\frac{dP}{dT} = \frac{M}{L}\left(\frac{16\pi acG}{3}\right)\frac{T^3}{\kappa} \tag{2}$$

where M is the total mass of the star, L the total optical luminosity of the star, a the Stefan-Boltzmann radiation constant, G the universal gravitational constant, and κ the "Rosseland mean opacity", which is roughly the inverse of the average of the mean free path of photons.

Temperatures for various chemical compositions of the envelope have been calculated by Lee. The following empirical formula fits his calculated results fairly well for an envelope composed of 90 percent helium and 10 percent Russell mixture (heavy elements) up to $T_c = 2 \times 10^8$ °K and $L = 10^2 L_\odot$, in the neighborhood of one solar mass:

$$T_c = 4.41 \times 10^7 \left(\frac{L}{M}\right)^{0.321} \qquad (3)$$

(Observationally, for the sun and normal stars the ratio of helium to heavy elements is about 10.)

Although the radiative transfer properties of a material medium generally depend on the chemical composition, in the case of stars in the upper Gap, the temperature is so high that energy transfer in the envelope is largely determined by the electron scattering process, which is virtually independent of chemical composition and the energy of the photon; hence the uncertainty in the chemical composition of the envelope is relatively unimportant in the upper Gap.

According to Chandrasekhar, the density ρ at a distance r from the center is given by the following differential equation:

$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2 \frac{d\varphi}{d\eta}\right) = -\left(\varphi^2 - \frac{1}{y_o^2}\right)^{3/2}$$

$$r = \frac{7.71\text{x}10^8}{\mu_e\, y_o}\,\eta \tag{4}$$

$$\rho = 9.82\text{x}10^5\, \mu_e\, y_o^2 \left(\varphi^2 - \frac{1}{y_o^2}\right)^{5/2}$$

where μ_e (= A/Z) is taken to be two throughout the whole star on account of the absence of hydrogen, and y_o is the parameter characterizing different models (a larger value of y_o corresponds to a smaller mass).

The energy loss in the form of neutrinos is measured by the neutrino luminosity, L_ν, which is obtained by integrating

$$\frac{dL_\nu(r)}{dr} = 4\pi r^2 Q \quad , \tag{5}$$

where Q is the neutrino energy loss rate (ergs/cc-sec) for the various mechanisms producing neutrinos. It is only necessary to consider the plasma and the photoneutrino process. In the case of the white dwarfs and Gap stars, only plasma neutrinos need be considered. The photoneutrino energy loss rate has been calculated by Chiu and Stabler (9) and by Ritus (10):

$$Q = \rho\, T_8^{8} / \mu_e \tag{6}$$

(non-relativistic and non-degenerate)

$$Q = 1.5 \times 10^{2} \times T_8 \; (\rho/\mu_e)^{2/3} \tag{7}$$

(non-relativistic and degenerate)

The plasma neutrino energy loss rate has been calculated by Adams, Ruderman and Woo (7) and later by Zaidi (8).* The major contribution comes from the transverse rate:

$$Q_t = 1.31 \times 10^{15}\, T_9^{9}\, \underline{x}^{9}\, F(\underline{x}) \tag{8}$$

where

$$F(\underline{x}) = \int_0^\infty \frac{\sinh(\xi)\,\cosh(\xi)}{\exp(x\,\cosh(\xi))-1}\, d\xi \quad , \tag{9}$$

and

$$\underline{x} = \hbar\omega_o / kT$$

and ω_o is the plasma frequency.

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* Zaidi's result is smaller than that calculated by Adams, Ruderman and Woo by a factor of four. In view of the uncertainty, we have used the neutrino energy loss rate calculated by the latter. However, the general conclusion of this paper remains valid in either case.

Table 1 lists the values of log $(L_\nu/L_\odot)$ for nine different masses of degenerate stars at various central temperatures.

The neutrino luminosity and the optical luminosity are plotted in Figures 2 and 3 for the case of Gap stars of mass 0.736 $M_\odot$ and 1.08 $M_\odot$ respectively. We have found that in the temperature range $T_c = 4 \times 10^6$ °K to $T_c = 2 \times 10^7$ °K, which is believed to represent the range of internal temperature in most observed white dwarfs, the neutrino luminosity is smaller than the optical luminosity by several orders of magnitude. At temperatures corresponding to an optical luminosity of 1 $L_\odot$, however, the two luminosities are of the same order, except for the case of Gap stars with masses less than 0.4 $M_\odot$, where the neutrino luminosity is much smaller. At still higher temperatures, corresponding to the Gap region the neutrino luminosity is one to two orders of magnitude higher than the optical luminosity for the case of the larger masses ($M = 0.7$ $M_\odot$ to 1.1 $M_\odot$) as shown in Figures 2 and 3. The neutrino luminosity becomes progressively less important as the mass of the star is decreased. Figure 4 shows the two luminosities for the case $M = 0.405$ $M_\odot$ at temperatures up to 8×10^7 °K where the optical luminosity is around 1 $L_\odot$. At such a temperature the neutrino lumiosity almost reaches its maximum, but this is merely comparable with the photon luminosity.

These results can be understood in terms of the behavior of the energy loss rate of the plasma process. At a given temperature, the plasma neutrino emission rate has a maximum at some critical density; in the temperature region under consideration ($4 - 20 \times 10^7$ °K) the maximum occurs at a density around 10^6 to 10^7 g/cm^3, which corresponds to the density of Gap stars with masses in the range 0.7 $M_\odot$ to 1.1 $M_\odot$. Thus, the plasma process is never effective in dissipating stellar energy from the Gap stars with $M \leq 0.4$ $M_\odot$. At still higher temperatures the finite temperature correction to the structure of white dwarf models becomes important, and this will result in a decrease in the density and hence in the plasma neutrino energy loss. Nevertheless, it can be shown that at these stages the temperatures of the small masses ($\leq 0.4 M_\odot$) are never high enough to produce photoneutrinos comparable with the optical luminosity.

Figure 5 shows neutrino energy loss rate ε_ν (ergs/g /sec) against mass fraction of the star during various stages through the Gap. The irregular behavior is due to the peculiar density dependence of the plasma neutrino rate.

III. Stellar Models for Central Stars of Planetary Nebulae

When the internal temperature is high, the complete-degeneracy approximation breaks down, and Chandrasekhar's models are no longer valid. This occurs when the luminosity of the central star is in the range from 10^2 to 10^4 $L_\odot$ with effective surface temperature around 10^5 ^oK. These stars lie above the Gap in the H-R diagram, as shown in Figure 1. Detailed models have been constructed on the basis of our present theory of the stellar interior and of the available observational data on central stars. The temperature at the center of a central star in its early stages, as obtained from stellar models, is around 3 or 4×10^8 ^oK. This temperature is too low to burn carbon (which requires a temperature of 6×10^8 ^oK) but is high enough so that all the helium or hydrogen would already have been converted into carbon. It is therefore safe to assume that as the central star cools, the only energy source is gravitational contraction, which in our calculations is taken to be uniform throughout the star. As for the chemical composition, we have chosen a pure Russell mixture throughout. To facilitate numerical calculation, we divide the star artificially into three regions, namely: (i) non-degenerate outer envelope, (ii) semi-degenerate inner envelope, and (iii) semi-degenerate isothermal core. In regions (i) and (ii), the temperature gradient is determined by electron scattering, which is the only important opacity at such high temperatures. We have used the proper expressions for Compton scattering in the semi-degenerate and semi-relativistic region, as calculated by Chin (11). On account of semi-degeneracy, the temperature, pressure, and density are related to one another through the Fermi-Dirac functions in regions (i) and (ii).

Using the same numerical techniques as described by Schwarzschild (12), we finally obtain the physical characteristics of models for 1.08 $M_{\odot}$ and 0.736 $M_{\odot}$. These are summarized in the 3rd and 5th columns of Table II. The higher mass is close to the upper limit of the average mass of central stars, while the lower mass is close to the average mass of observed white dwarfs.

Rough estimates of the stellar structure can be made when the central degeneracy parameter is set equal to zero. In this case, we are considering an earlier stage of central-star evolution. Assuming uniform contraction and neglecting the temperature dependence of Compton scattering, we can apply Eddington's standard model (13) to obtain

$$\frac{L}{L_\odot} = \frac{40.6}{\varkappa} \mu^4 \left(\frac{M}{M_\odot}\right)^3$$

$$\rho_c = 76.5 \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right) \tag{10}$$

$$T_c = 1.97\text{x}10^3 \mu \frac{M}{M_\odot} \left(\frac{R}{R_\odot}\right)^{-1} {}^\circ K \quad .$$

The results are listed in the 2nd and 4th columns of Table II, where the optical luminosities are obtained by assuming $\varkappa = 0.12$. In view of the approximations involved, it should be emphasized again that these values are rough estimates.

In central stars, the photoneutrino process is the important neutrino emission mechanism. On account of the uncertainty of photoneutrino emission rates in the partially degenerate region, the neutrino luminosities listed in Table II are valid only within one order of magnitude. The neutrino and optical luminosities are plotted in the extreme right of Figures 2 and 3 at the corresponding central temperatures.

It can be seen that in the early evolutionary stages of a central star, the neutrino luminosity is of the same order of magnitude as the optical luminosity for 1.08 $M_{\odot}$ and becomes less important as the mass is decreased. Higher up on the observed evolutionary track, as shown in Figure 1, photo-neutrinos may play an important role due to the occurrence of higher temperatures in the core.

IV. Cooling Time of Pre-white Dwarfs Through the Gap.

As shown in the previous two sections, neutrino emission is important in a Gap star with mass in the range 0.7 $M_{\odot}$ to 1.1 $M_{\odot}$. In this section, we shall calculate evolutionary times through the Gap with and without neutrino emission. For reasons mentioned before, there should be no nuclear burning as the star evolves through the Gap. Gravitational contraction can be shown to be negligibly small for the higher masses because degeneracy sets in at a much earlier stage than for the smaller masses. Assuming the thermal energy of the nuclei in the core to be the only energy source within a Gap star, we obtain the evolution time (as given by Mestel (14)):

$$\tau = - \frac{3}{2} \frac{k}{M_p} \frac{M}{\langle A \rangle} \int \frac{dT}{L} \quad , \tag{11}$$

where M_p is the mass of a proton and $\langle A \rangle$ is the average mass number of the nuclei. With neutrino emission, however, the above equation should be replaced by

$$\tau = - \frac{3}{2} \frac{k}{M_p} \frac{M}{<A>} \int \frac{dT}{L+L_\nu} \quad . \tag{12}$$

For nuclei in the core, we have assumed that their abundances are represented by the Russell mixture ($<A> = 23.2$). The chemical composition of the envelope, upon which the optical luminosity depends, will be assumed to be composed of 90 percent helium and 10 percent Russell mixture ($Y = 0.9$) in one case and pure Russell mixture ($Y = 0$) in the other. The results are listed in Tables III and IV, and plotted in Figures 5 and 6 for 0.736 $M_\odot$ and 1.08 $M_\odot$, respectively. The results for smaller masses, 0.6 $M_\odot$ and 0.4 $M_\odot$, with $Y = 0$ are those of Hayashi, Hōshi and Sugimoto (13). The lifetimes of the smaller masses are found to be longer than what one would obtain by merely applying Eq. (11) since the energy release due to gravitational contraction was taken into account and the relativistic effect arising from high density was neglected. Both factors tend to prolong the lifetime.

Thus a star of mass 0.7 $M_\odot$ to 1.1 $M_\odot$ evolves through the Gap in about 2×10^6 years without neutrino emission and in 2×10^5 years with neutrino emission. At an intermediate luminosity, $L = 10\ L_\odot$, the two evolution times differ by a factor of one hundred. Replacing the envelope of helium and Russell mixture by an envelope of pure Russell mixture results in practically no change in the time scale without neutrino emission but cuts down the evolution time with neutrino emission to 4×10^4 years. For stars of mass less than $0.4\ M_\odot$, neutrinos will have no effect on the thermal time scale of 4×10^6 years. Summarizing, we have shown that, on the neutrino hypothesis, different time scales exist for the two different mass ranges, namely, 2×10^5 years for the mass range $0.7\ M_\odot$ - $1.1\ M_\odot$ and 4×10^6 years for the mass range $0.2\ M_\odot$ - $0.5\ M_\odot$. Without neutrino emission, however, the time scales for the two mass ranges are nearly the same. Just below the Gap, where the brightest white dwarfs are observed, the difference between the time scales with and without neutrino emission for a star of mass $0.7\ M_\odot$ becomes much smaller, but the difference still amounts to a factor of about 2, as shown by Table IV for the evolution times from $L = 1\ L_\odot$ to $L = 0.1\ L_\odot$.

If the star has not undergone carbon burning in the early central star stage, then the core should be composed mainly of carbon ($\langle A \rangle = 12$) instead of the Russell mixture

($\langle A \rangle = 23.2$), and all the cooling times listed with and without neutrino emission should be multiplied by a factor of 2 accordingly.

V. Comparison with Observations.

To compare our theoretical results with the available observational data, we have drawn up Table V summarizing all the evidence. Details on how the observational lifetimes were obtained will be published elsewhere. It is sufficient here merely to outline the methods and present the results. To do so, we discuss in turn each kind of object in Vorontsov-Velyaminov's (15) famous "blue-white sequence".

a) Central Stars of Planetary Nebulae

The central stars of planetary nebulae have been shown to occupy the luminosity range $10^2\ L_\odot$ to $10^4\ L_\odot$ on the H-R diagram. Contraction through this region to the limiting (Chandrasekhar) radius takes place in 3-5 x 10^4 years (1, 3). Theoretical evolutionary tracks through the central-star region have been computed by Hayashi, Hōshi, and Sugimoto(13) for stars of constant mass equal to 0.6 $M_\odot$ and 0.4 $M_\odot$. In the case of 0.4 $M_\odot$, two extreme assumptions about the energy source and chemical composition were made: (1) uniform gravitational contraction and Russell mixture throughout, (2) a thin hydrogen-burning shell near the surface and an inert helium core inside. Only assumption (1) was applied to the case of 0.6 $M_{\odot}$. Neutrino emission processes were not taken into account, and the lifetime in all cases was 1×10^{6} years. Thus mass loss and other uncertainties should not alter this time scale significantly. Hence a discrepancy of a factor 20-30 exists between the observational and theoretical lifetimes.

Previously, Stothers (16) pointed out that photoneutrino emission could drastically affect the evolution of pre-white dwarfs. It was assumed at that time that the core was completely degenerate (non-contracting). A cooling time was obtained by equating the photoneutrino loss rate to the time rate of change in thermal energy. The temperature was chosen high so that the optical luminosity could be neglected in comparison with the neutrino luminosity. Then a simple analytical formula represented the cooling time, dependent only on the temperature. For a temperature corresponding to the average of Hayashi's central-star models of 0.6 $M_{\odot}$, the neutrino cooling time is about 10^{4}

years. This agrees satisfactorily with the observational lifetime of the central stars.

The neglect of gravitational energy release may be serious, however, since central stars are still contracting objects. The analytical models therefore refer only to the advanced (optically thin nebula) phase, where the optical luminosity shows a drop because of the predominance of thermal cooling over the gravitational energy release. According to the observational data, the optically thin phase dominates the evolution; thus the conclusion reached above remains the same.

The models of central stars calculated in the present paper represent very early stages, and although the photo-neutrino loss is small here, the models suggest the importance of this process in later stages of higher temperature, as in Hayashi's sequence. In the original paper with analytical models, a core mass of 0.4 $M_{\odot}$ was assumed in order to estimate the neutrino luminosity. This mass is by chance equal to the core mass of Hayashi's contracting central-star model (13). Hence the estimate in the original paper may be applied directly to Hayashi's model, and indicates that the neutrino luminosity will exceed the optical luminosity by a factor of more than 10. The consequence must be an increased gravitational contraction. In this connection, Seaton (3) has shown that the gravitational luminosity of central stars, as estimated from their observed parameters, exceeds the optical luminosity by a factor of order 10.

Detailed evolutionary tracks computed for central stars by Vila (17), Rose (private communication), and Salpeter (private communication) substantiate the results of the analytical estimates. However, a precise number for the theoretical lifetime cannot yet be given because of the great uncertainty in the assumed initial model (especially chemical composition) and the neglect of mass ejection.

b) White Dwarfs

No white dwarfs are observed in the Gap region (6). Although there exists a theoretical difference of a factor 2 in the time scale between 0.1 $L_\odot$ and 1 $L_\odot$ for stars of mass 0.7 $M_\odot$, depending on whether neutrino emission is included or omitted, this difference is too small to be detectable observationally. Alternatively, one could compare numbers of stars on the 0.7 $M_\odot$ sequence, in which neutrino emission should have some effect, with numbers of stars on the 0.3 $M_\odot$ sequence, in which neutrino emission should not. However, the birth-rate function into each sequence is unknown, so the comparison cannot be made.

Neither white dwarfs nor the central stars of planetary nebulae belong to the Gap region. The next three types of object, the novae, the U Gem stars, and the ultraviolet dwarfs, are actually members of the Gap population.

c) Cataclysmic Variables

It has long been recognized that the explosive variables known as novae and U Gem stars occupy the hottest portion of the H-R diagram. Only recently, however, has more refined data become available (18). These objects all seem to be close binary systems, containing a red star and a white-dwarf-like companion.

The novae explode either in one-shot outbursts or at intervals of decades to centuries. They brighten typically by 10 magnitudes. From observations at minimum light, the blue (and red) components occupy a tremendous range in luminosity. From a crude empirical mass-luminosity relation (19), those objects lying within the Gap region should have masses of $\sim 0.1 - 0.5\ M_{\odot}$. An observational lifetime, based on their observed numbers in the solar neighborhood and an assumed birth-rate function of their main-sequence progenitors, is several million years. This result agrees with the fact that neutrino emission is unimportant at these masses and with the fact that many of the low-mass blue components may actually lie below the Gap.

The U Gem stars have been called "dwarf novae" on account of their less extreme outbursts. They explode quasi-periodically on a time scale of days or months. Their masses are estimated at $\sim 0.7\ M_\odot$, and their statistical luminosity is $\sim 1\ L_\odot$ (<u>20</u>). In a manner similar to that used for the old novae, the observational lifetime comes out to be several hundred thousand years. Since this number is probably not better than an order of magnitude, it should not be definitely concluded that the theoretical lifetime with neutrinos included agrees better with the observations than the lifetime without neutrinos.

<u>d) Ultraviolet Dwarfs</u>

The single stars which occupy the Gap region belong to a group of stars known as hot subdwarfs (see the review article by Greenstein (<u>21</u>)). In order to isolate the stars of mass $\sim 0.7\ M_\odot$ and higher, which constitute the test of the neutrino theory, we may select the hottest subdwarfs on the basis of color: $U - B < -1.1$ (<u>6</u>). This effectively eliminates all white dwarfs below $1\ L_\odot$ and all low-mass pre-white dwarfs. We now define the Gap stars bluer than $U - B = -1.1$ as <u>ultraviolet dwarfs</u>.

With the help of the (B-V, U-B) diagram, spectroscopic types, and statistical luminosities, an upper limit may be placed on the number of possible ultraviolet dwarfs among the objects measured in blue-star surveys (22). The observed ratio of number density of ultraviolet dwarfs, N_{uv}, to number density of blue "horizontal-branch" progenitors, N_{bHB}/p , may be tied in to the number density of "red giant" stars, N_{RG} , by reference to the composite H-R diagram of globular clusters. The time scale of evolution as a red giant, τ_{RG} , is known from theoretical model calculations. In this connection, Sandage's semiempirically determined red-giant lifetime (23), based on star counts of red giants vis-à-vis main-sequence stars and on theoretical evolution times along the main sequence, can be shown to be very uncertain. Alternatively, the lifetime on the blue horizontal branch, τ_{bHB} , may be obtained directly from theoretical model calculations. The final result for the lifetime of an ultraviolet dwarf is:

$$\tau_{uv} = p \frac{N_{uv}}{N_{bHB}} \tau_{bHB} = p \frac{N_{uv}}{N_{bHB}} \frac{N_{bHB}}{N_{RG}} \tau_{RG} = 5\text{x}10^5 \text{ years.} \quad (13)$$

This may be shown to be an upper limit. Part of the reason is the inclusion of stars with mass greater than 1.1 $M_\odot$ (but below the Chandrasekhar limit) as ultraviolet dwarfs. With or without neutrino emission, such stars should have longer lifetimes than the 0.7 $M_\odot$ variety.

The theoretical lifetimes with which we must compare the observational result should be extremized in order to constitute a test. Accordingly, we adopt a pure carbon core and an envelope composed predominantly of helium, for the neutrino-emitting models. (This composition probably refers to reality.) Hence the theoretical time scale entered in Table V should be multiplied by a factor 2, and will then be an upper limit. Second, we adopt a Russell-mixture core and an envelope of arbitrary composition, for the models without neutrino emission. The theoretical time scale as listed in Table V is then a lower limit.

Clearly, the observational lifetime is compatible only with the assumption of an accelerating mechanism besides ordinary thermal emission in the ultraviolet dwarfs. Nuclear reactions and gravitational contraction would only delay the evolution by providing an additional energy source. Mass ejection must eventually result in the 0.7 $M_\odot$ white dwarfs, and for ordinary thermal cooling, a star of higher mass evolves more slowly. However, mass ejection in itself is a form of luminosity. Clayton (private communication) has suggested that a steady mass loss of 10^{-9} $M_\odot$/year could carry energy away from the star at a rate comparable with the optical luminosity. Nevertheless no mechanism is known whereby this might be accomplished, and the required rate is at least 10^5 times greater than the solar wind. Regarding more certain conclusions about the existence of the foregoing processes in very hot white dwarfs, T. D. Lee (5) and Ledoux and Sauvenier-Goffin (24) have ruled out nuclear reactions on theoretical grounds, and gravitational contraction and any significant mass loss are ruled out on observational grounds (6, 21).

In the absence of any other more satisfactory explanation of the acceleration of evolution in the ultraviolet dwarfs, we suggest that this speed-up in evolution is caused by the plasma neutrino process and that <u>this speed-up is strong astrophysical evidence that the direct electron-neutrino interaction exists in nature</u>. Furthermore, the evidence from the central stars of planetary nebulae supports the view that the photoneutrino process is occurring there, and this fact certainly re-emphasizes the statement just made.

Accordingly, a lower limit for the coupling constant of the direct electron-neutrino interaction can be obtained from the upper limit of the evolution time in the Gap. The lower limit is, tentatively, close to the currently accepted value of g, the weak coupling constant, namely, $gcM_p^2/\hbar^3 = 10^{-5}$. Further observational and theoretical work along these lines should prove to be fruitful.

Acknowledgments

We would like to thank the following persons for helpful discussions or correspondence: Drs. A. Baglin, D. D. Clayton, J. L. Greenstein, R. P. Kraft, W. Rose, M. A. Ruderman, E. E. Salpeter, E. Schatzman, B. Strømgren, C.-H. Woo, and N. J. Woolf. One of us (R.S.) acknowledges the support of an NAS-NRC research associateship.

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TABLE I

Log $(L_\nu/L_\odot)$ at Various Central Temperatures and Masses

$M/M_\odot$ \ T_7	1	2	4	6	8	10	12	15	20
1.33	-7.912	-4.876	-1.943	-0.306	0.811	1.649	2.315	3.103	4.065
1.22	-7.302	-4.303	-1.436	0.149	1.221	2.015	2.633	3.343	4.172
1.08	-6.845	-3.888	-1.087	.443	1.454	2.180	2.728	3.340	4.041
0.887	-6.372	-3.478	-0.779	.636	1.521	2.132	2.585	3.089	3.671
0.736	-6.075	-3.238	-0.653	.628	1.403	1.935	2.330	2.773	...
0.613	-5.847	-3.071	-0.636	.508	1.195	1.667	2.022	2.425	...
0.505	-5.654	-2.958	-0.716	.301	0.914	1.341	1.666	...	...
0.405	-5.483	-2.908	-0.899	0.002	0.553	...	...	...	...
0.220	-5.257	-3.213	-1.740	1.041	...	...	...	...	...

TABLE II

Physical Characteristics of Models for Central Stars of Planetary Nebulae (Pure Russell Mixture)

	$1.08\ M_\odot$		$0.736\ M_\odot$	
$R/R_\odot$	0.09	0.07	0.103	0.07
$\log L/L_\odot$	3.70	3.49	3.18	3.13
$\log L_\nu/L_\odot$	3.56	3.70	1.60	2.20
$\log \rho_c$	5.05	5.17	4.71	5.51
$\log T_c$	8.64	8.61	8.41	8.38
$\log T_e$	5.21	5.21	5.05	5.12

TABLE III

Logarithms of Evolutionary Times from $L = 100\ L_\odot$ to Luminosities indicated. Y represents Helium Content in the Envelope.

$M/M_\odot$		$1\ L_\odot$		$3\ L_\odot$	$5\ L_\odot$	$10\ L_\odot$
		Y=0.9	Y=0	Y=0.9	Y=0.9	Y=0.9
1.33	No ν	6.4	6.4	6.1	5.9	5.7
	ν	6.1	5.3	5.4	5.0	4.5
1.08	No ν	6.4	6.3	6.0	5.8	5.6
	ν	5.7	4.6	4.7	4.3	3.8
0.736	No ν	6.2	6.2	5.9	5.7	5.5
	ν	5.2	4.4	4.5	4.2	...
0.6	No ν	6.2	6.6	5.8	5.7	...
0.4	No ν	6.1	6.5	...	...	...
	ν	6.1	6.5	...	...	...

TABLE IV

Evolutionary Times τ in Years Between Optical Luminosities Inside Brackets. Y represents Helium Content in the Envelope.

		$\tau(1\ L_\odot - 0.3\ L_\odot)$		$\tau(0.3\ L_\odot - 0.1\ L_\odot)$	
		Y=0.9	Y=0	Y=0.9	Y=0
$M=0.736\ M_\odot$	No ν	2.4×10^6	2.4×10^6	4.7×10^6	6×10^6
	ν	7.4×10^5	7.4×10^4	3.1×10^6	2.6×10^5
$M = 0.4\ M_\odot$			2.5×10^6		5×10^6

TABLE V

Comparison with Observation

Object	$L/L_\odot$	$M/M_\odot$	Lifetime (yrs)		
			Observational	Theoretical(ν)	Theoretical(No ν)
Planetary central star	$10^2 - 10^4$	≤ 1	$3\text{-}5 \times 10^4$	$\sim 10^4$	1×10^6
Ultraviolet dwarf	$1 - 10^2$	$\gtrsim 0.7$	$< 5 \times 10^5$	2×10^5	2×10^6
Nova	$1 - 10^2$	$\sim 0.1 - 0.5$	several 10^6	4×10^6	4×10^6
U Gem Star	~ 1	~ 0.7	several 10^5	...	...
White dwarf	< 1	~ 0.7	$> 10^6$	$> 1 \times 10^6$	$> 2 \times 10^6$
		~ 0.3	$> 10^6$	$> 4 \times 10^6$	$> 4 \times 10^6$

FIGURE CAPTIONS

Fig. 1. H-R diagram, adapted from Harman and Seaton (2). Open and filled circles represent some observed central stars of planetary nebulae. Crosses represent some observed white dwarfs. Circles with crosses represent the two evolutionary stages of our model for a central star of 0.736 $M_\odot$. The dotted line represents, schematically, the theoretical evolutionary track in the central star region. The dashed curve represents the evolutionary track of the 0.736 $M_\odot$ star through the Gap. The solid curve on the right refers to the horizontal branch and part of the red-giant branch of a typical globular cluster.

Fig. 2. Neutrino and optical luminosities of a star of 0.736 $M_\odot$. Y represents helium content in the envelope.

Fig. 3. Neutrino and optical luminosities of a star of 1.08 $M_\odot$. Y represents helium content in the envelope.

Fig. 4. Neutrino and optical luminosities of a star of 0.405 $M_\odot$.

Fig. 5. Distributions of neutrino energy loss rates in erg/g-sec as a function of mass fraction for stars at various stages through the Gap.

Fig. 6. Cooling times of a star of 0.736 $M_{\odot}$. Y represents helium content in the envelope. Numbers along the curves indicate the optical luminosities in units of $L_{\odot}$ at corresponding points.

Fig. 7. Cooling times of a star of 1.08 $M_{\odot}$. Y represents helium content in the envelope. Numbers along the curves indicate the optical luminosities in units of $L_{\odot}$ at corresponding points.

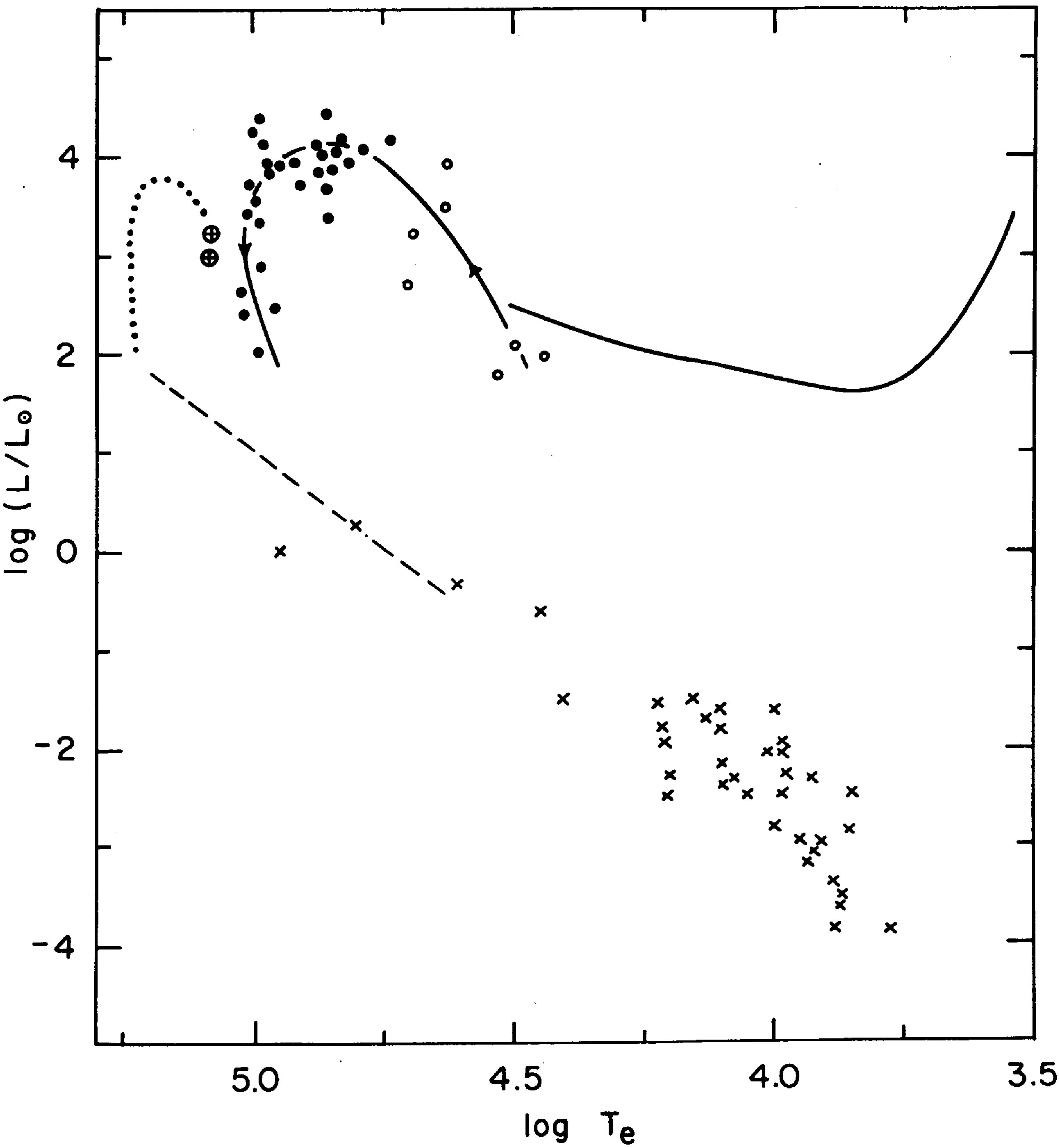

Fig. 1

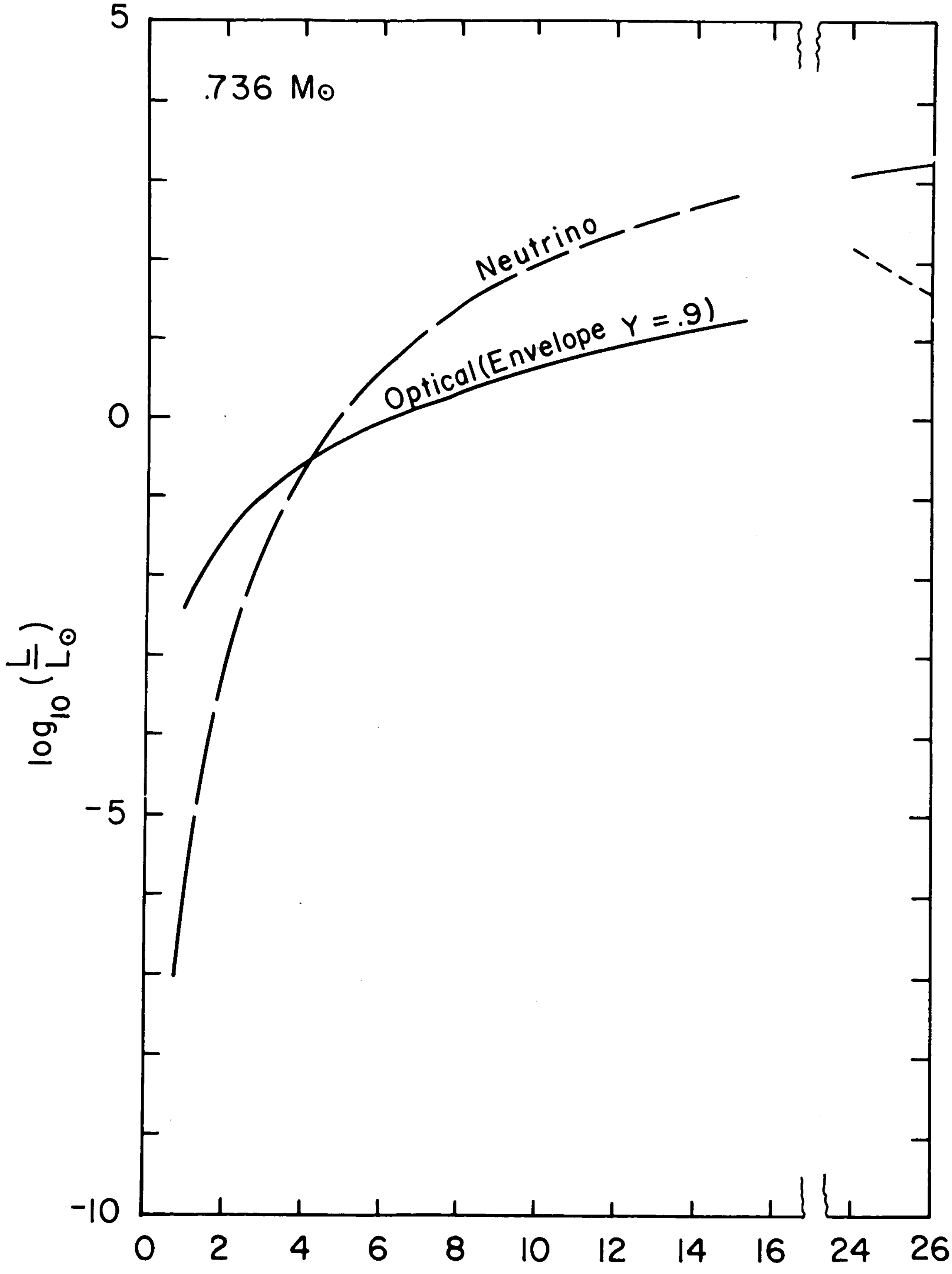
.736 M⊙
Neutrino
Optical(Envelope Y = .9)
5
0
-5
-10
0
2
4
6
8
10
12
14
16
24
26
$\log_{10}(\frac{L}{L_\odot})$
$T_c / 10^7$

Fig. 2

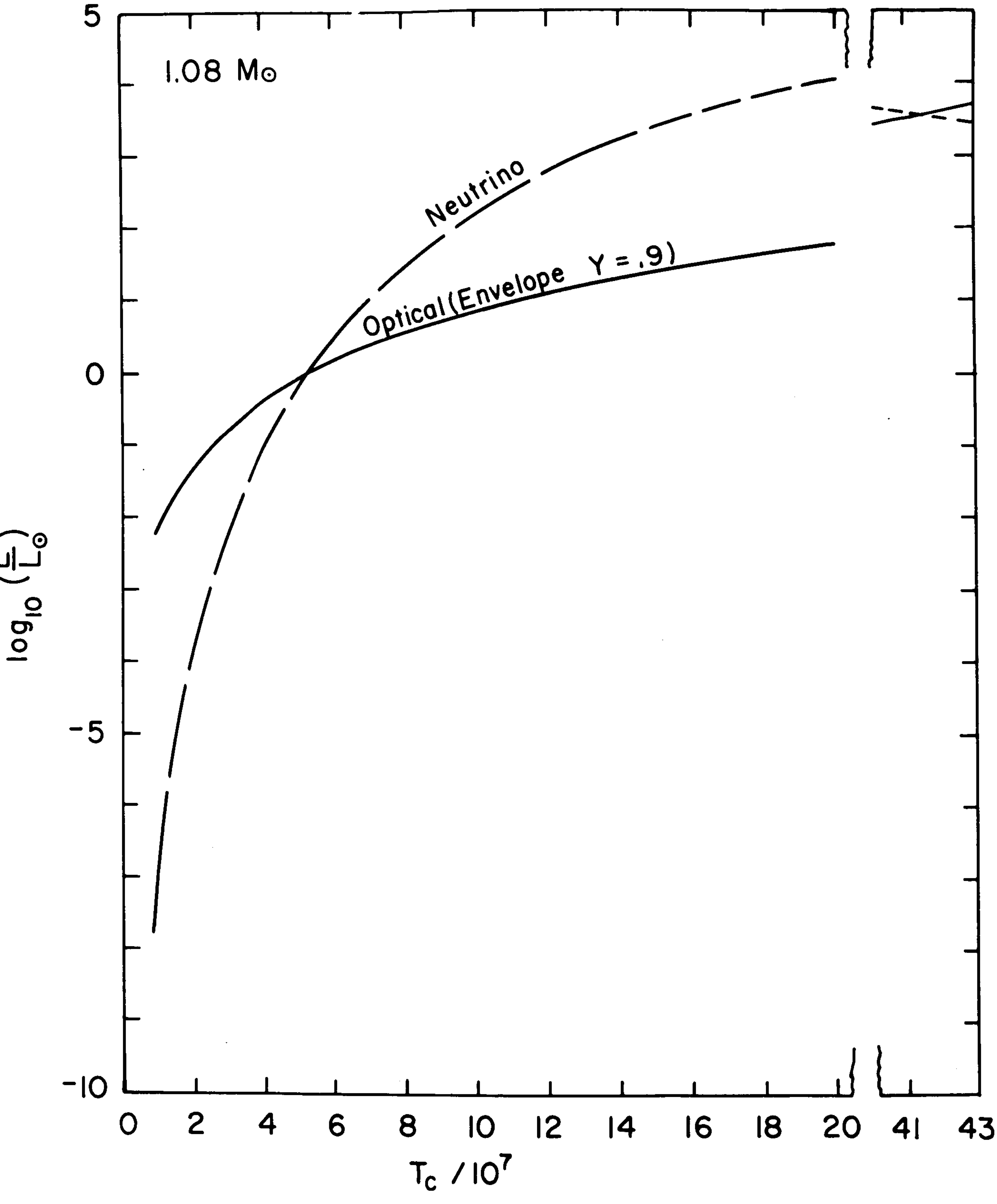
1.08 M⊙
Neutrino
Optical (Envelope Y = .9)
5
0
-5
-10
0
2
4
6
8
10
12
14
16
18
20
41
43
$T_c / 10^7$
$\log_{10}\left(\frac{L}{L_\odot}\right)$

Fig. 3

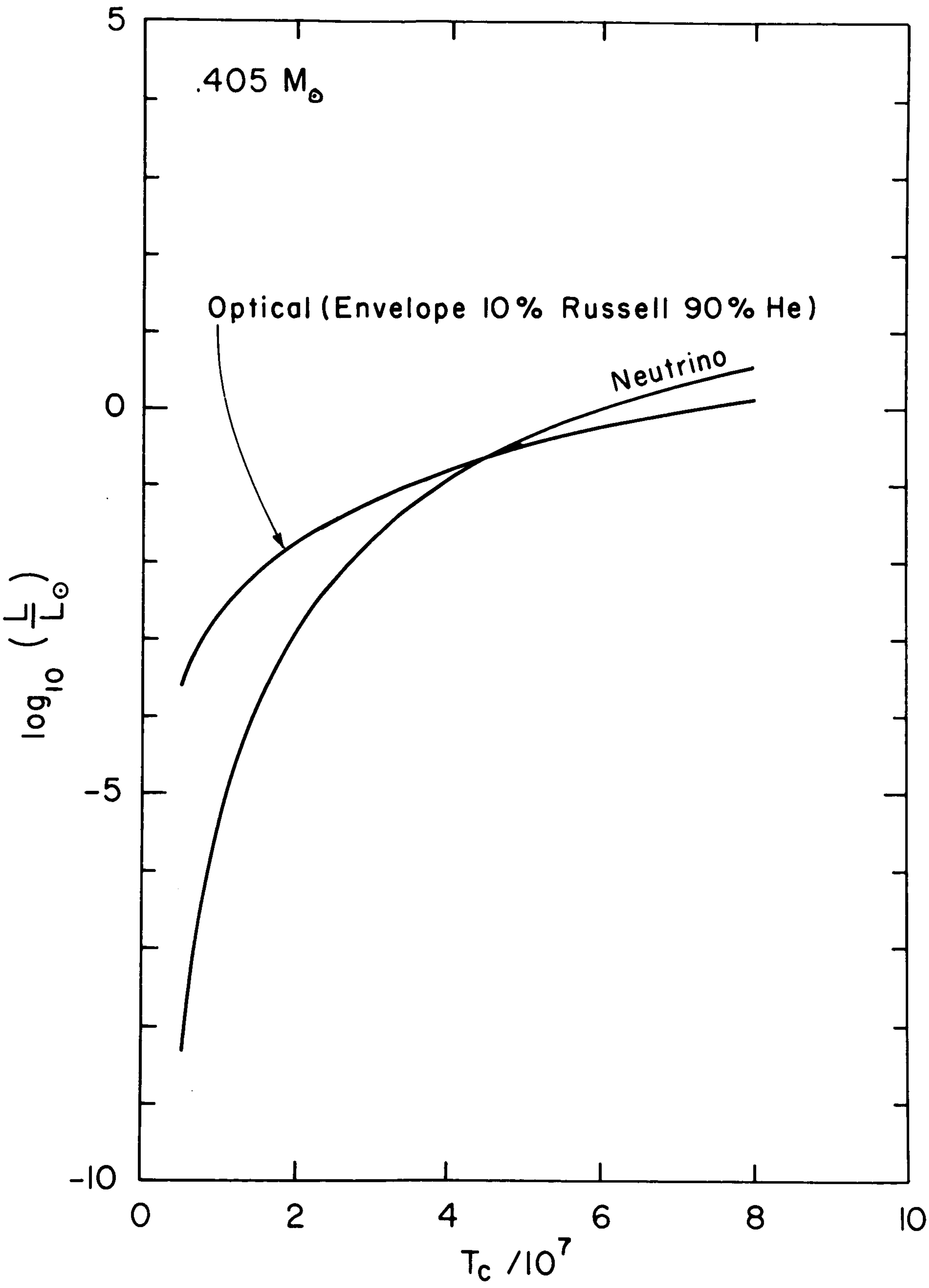
.405 $M_\odot$
Optical (Envelope 10% Russell 90% He)
Neutrino
5
0
-5
-10
0
2
4
6
8
10
$\log_{10}\left(\frac{L}{L_\odot}\right)$
$T_c/10^7$

Fig. 4

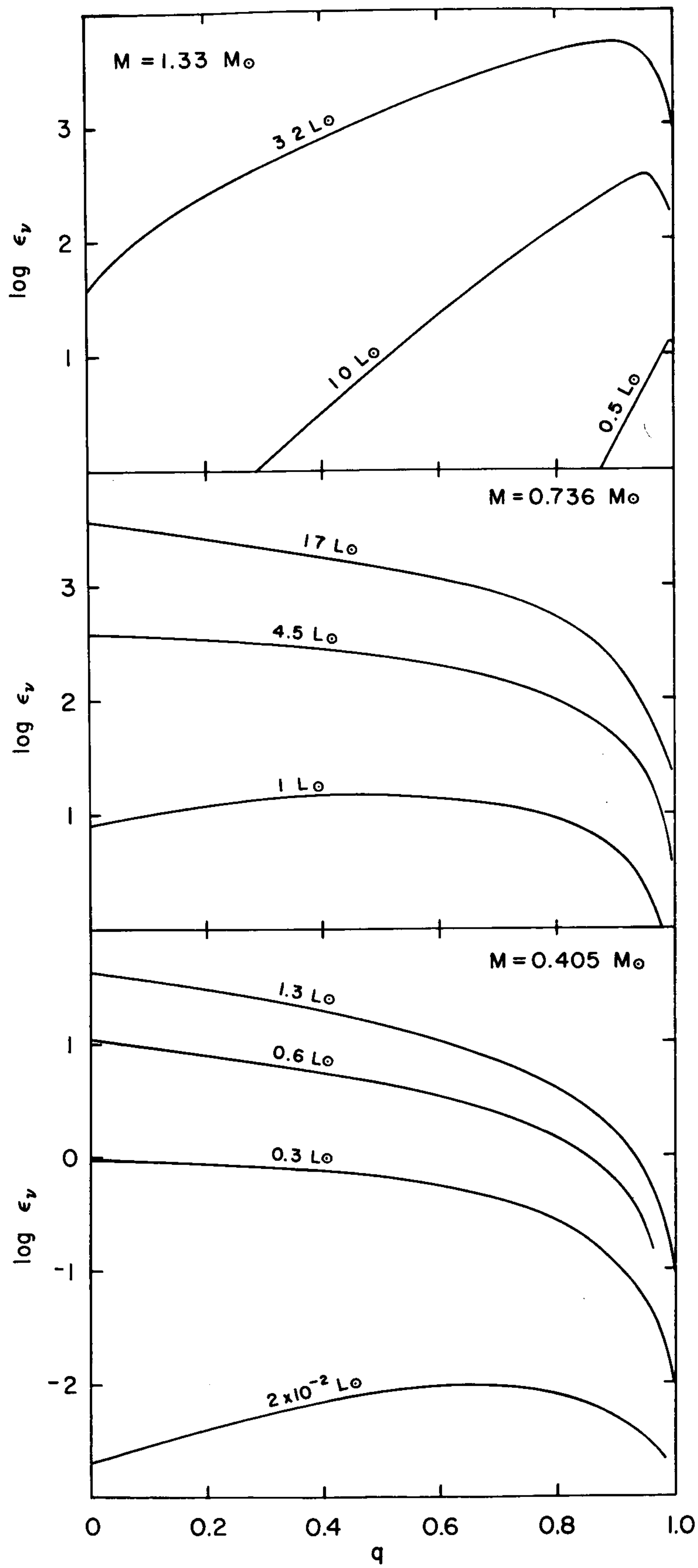
M = 1.33 M⊙
3.2 L⊙
10 L⊙
0.5 L⊙
M = 0.736 M⊙
17 L⊙
4.5 L⊙
1 L⊙
M = 0.405 M⊙
1.3 L⊙
0.6 L⊙
0.3 L⊙
2 x10^-2 L⊙
log εν
q
0
0.2
0.4
0.6
0.8
1.0
1
2
3
-1
-2

Fig. 5

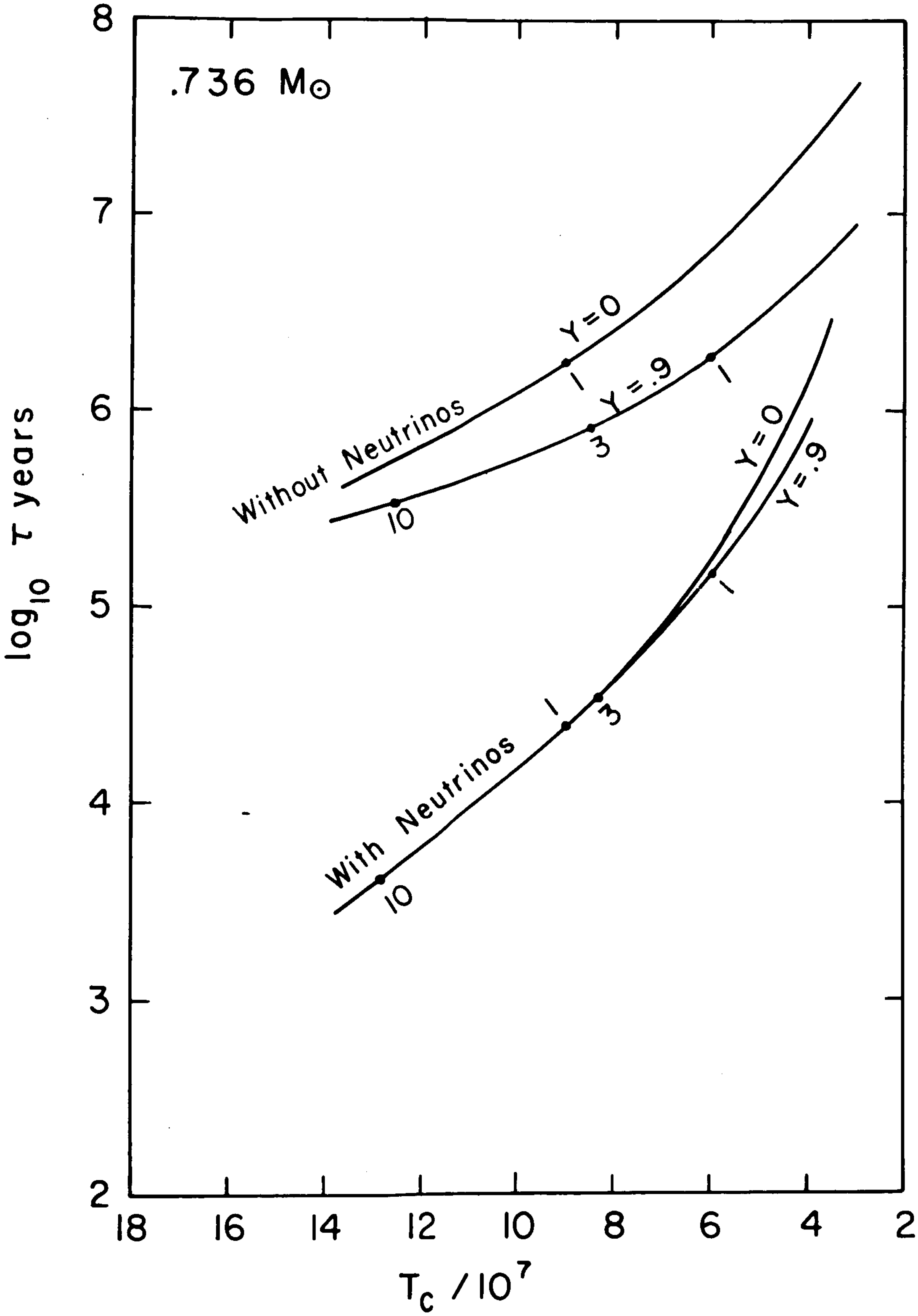
.736 M⊙
Without Neutrinos
With Neutrinos
Y=0
Y=.9
Y=0
Y=.9
10
3
1
1
10
3
1
1
8
7
6
5
4
3
2
18
16
14
12
10
8
6
4
2
$\log_{10} \tau$ years
$T_c/10^7$

Fig. 6

~~31 35594~~
2

3500
27/2/67

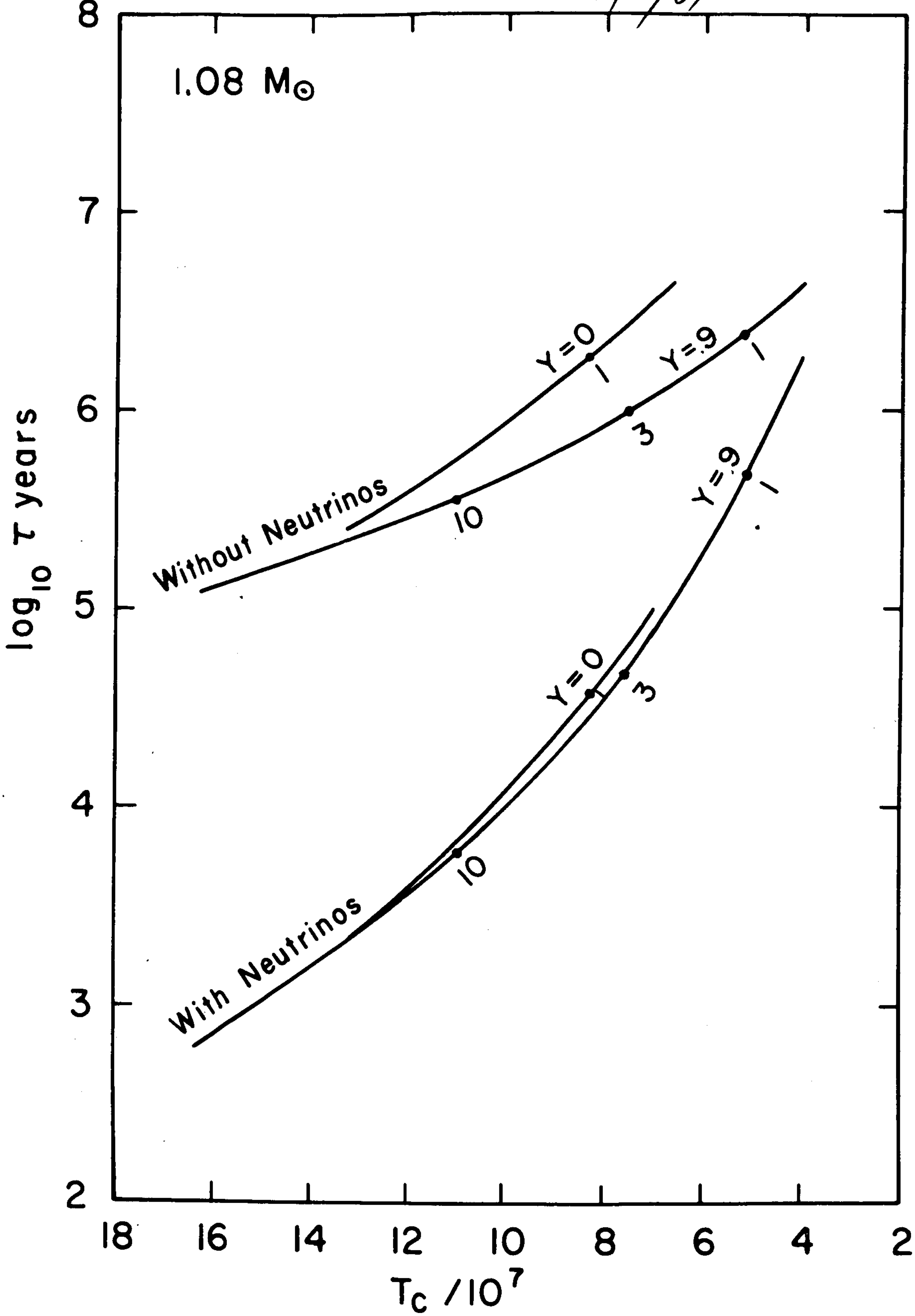

Fig. 7