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# A HARMONIC $L_4$ ORBIT FOR THE VERY RESTRICTED FOUR-BODY PROBLEM




BY

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GODDARD SPACE FLIGHT CENTER

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## ABSTRACT

A method of general perturbations utilizing Chebyshev series is used to investigate motion in the vicinity of the $L_4$ triangular point of the earth-moon system. The model used is that of the very restricted four body problem for the earth-moon-sun system. A harmonic orbit, in the numerical sense, with respect to a rotating coordinate frame centered at $L_4$ is found. The period of this harmonic orbit, being equal to the period of the disturbing force, is the same as the moon's synodic period. This orbit remains within 6860 km of the $L_4$ point. It describes two different size loops about $L_4$, the smaller one traversed in 36 percent of the period. The disturbing force, being nearly periodic with half the moon's synodic period, gives rise to another orbit about $L_4$ which is nearly periodic with half the synodic period of the moon. This orbit remains within 4574 km of the $L_4$ point for twelve periods investigated. Deviations from the mid orbit during this time is less than 381 km.

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## 1. INTRODUCTION

In some recent literature interest has been shown in the problem of the influence of the sun on motion close to the libration points of the earth-moon system as well as motion about the earth and the moon itself. One possible model for the earth-moon-sun system in which the problem might be considered has been proposed by Su-Shu Huang (1960), who called it the "very restricted four-body problem." Here the earth and moon describe circular orbits relative to one another, and their center of mass describes a circular orbit around the sun; all these orbits are Keplerian, lie in a plane, and no perturbations are considered. Using this model Huang studied the motion of a fourth body of an infinitesimal mass in a similar manner as in the restricted three-body problem. He concludes this model gives a general idea of where the fourth body could or could not go under given initial conditions when they are no longer very near the earth. Using a similar model Cronin et al. (1964) proved that under certain conditions the fourth body has a periodic motion, relative to a rotating coordinate frame, near each of the libration points of the restricted three-body problem. Their proof is based upon assumptions concerning the masses and distances of the bodies which are not satisfied by the earth-moon-sun system.

Siferd (1965) used Huang's model for the earth-moon-sun system to generate some periodic orbits. Using a numerical integration procedure, the equations of motion for the very restricted four-body problem were iterated upon utilizing a digital computer until some periodic orbits were obtained. By this technique eight periodic orbits, in the numerical sense, with a respect to a rotating coordinate system were found. Three periodic orbits were around the earth, three were around the moon, and two were around the earth-moon libration point ($L_1$). No periodic orbits near the triangular points were obtained.

Danby (1965) investigated the influence of the sun on motion close to the triangular points of the earth-moon system. He felt the very restricted four-body model inadequate for his investigation and therefore used a model in which the secular perturbations of the moon due to the sun were retained. The results may be said to strengthen the hope that stable motion around the triangular points of the earth-moon system is possible. Other investigators include Tapley, et al. (1963 and 1965) who used a model similar to the very restricted four-body model except the moon's orbit is inclined with respect to the earth-sun plane. The equations of motion for a particle near the triangular points of the earth-moon system are numerically integrated on a digital computer for various initial conditions. One result indicates that a particle placed initially at a triangular point ($L_4$) with zero relative velocity has an envelope of motion, centered at $L_4$, going through a mode of expansion to a value of one earth-moon distance for the major axis followed by a mode of contraction to a value of 1/8 earth-moon distance for the major axis. The envelope repeats this sequence several times during the 2500-day period investigated. The nature of these data suggests that such a motion may persist for a period of time much longer than that considered in the study.

The present paper uses the very restricted four-body problem model as proposed by Huang for the earth-moon-sun system. The merits of this model for the earth-moon-sun system are still in doubt; however, it has been used in this study as a first attempt to find orbits which remain near triangular points of the earth-moon system. Using a technique proposed by Carpenter (1966), a harmonic orbit in the numerical sense, with respect to the rotating coordinate frame and about the $L_4$ triangular point of the earth-moon system is found. In addition, a nearly periodic orbit having half the period of the harmonic orbit is obtained.

## 2. THE MODEL AND COORDINATE SYSTEM

Consider an infinitesimal body of mass, m in a system of three bodies $m_1$, $m_2$, and $m_3$ which are the sun, earth, and moon respectively. Further assume that all four bodies remain in a plane so arranged that the center of mass, B of $m_2$ and $m_3$ is revolving around the center of mass, 0, of the entire system in a circular orbit and $m_2$ and $m_3$ themselves are revolving around B also in circular orbits. Table I indicates the numerical values used for this model, and Figure 1 shows the geometry.

TABLE I
Physical Characteristics of the Model and System

| ITEM | REMARKS | SUN | MOON | REFERENCE ORBIT |
|---|---|---|---|---|
| Semimajor axis | Earth radii | 23454.87 | 60.0 | 59.75609983 |
| Eccentricity | Dimensionless | 0.0 | 0.0 | 0.0 |
| Argument of perigee | Degrees | 0.0 | 0.0 | 0.0 |
| Inclination | Degrees, with respect to X, Y plane | 0.0 | 0.0 | 0.0 |
| Longitude of the ascending node | Degrees | 0.0 | 0.0 | 0.0 |
| Mean anomaly at epoch | Degrees | 0.0 | 0.0 | 60.0 |
| Mass ratio | Body with respect to earth | 332951.29 | 0.012294830 | — |
| Mean motion | Radians per hour | $7.16754988 \times 10^{-4}$ | $9.659527869 \times 10^{-3}$ | $9.659527869 \times 10^{-3}$ |

Time is zero and epoch is defined as the instant the center of mass of the sun crosses the positive X axis.

Synodic period of the moon is taken to be 702.5992263 hours.

Earth's gravitational parameter, $k_e^2$, is taken to be 19.9094165 (earth radii)$^3$/hr$^2$.

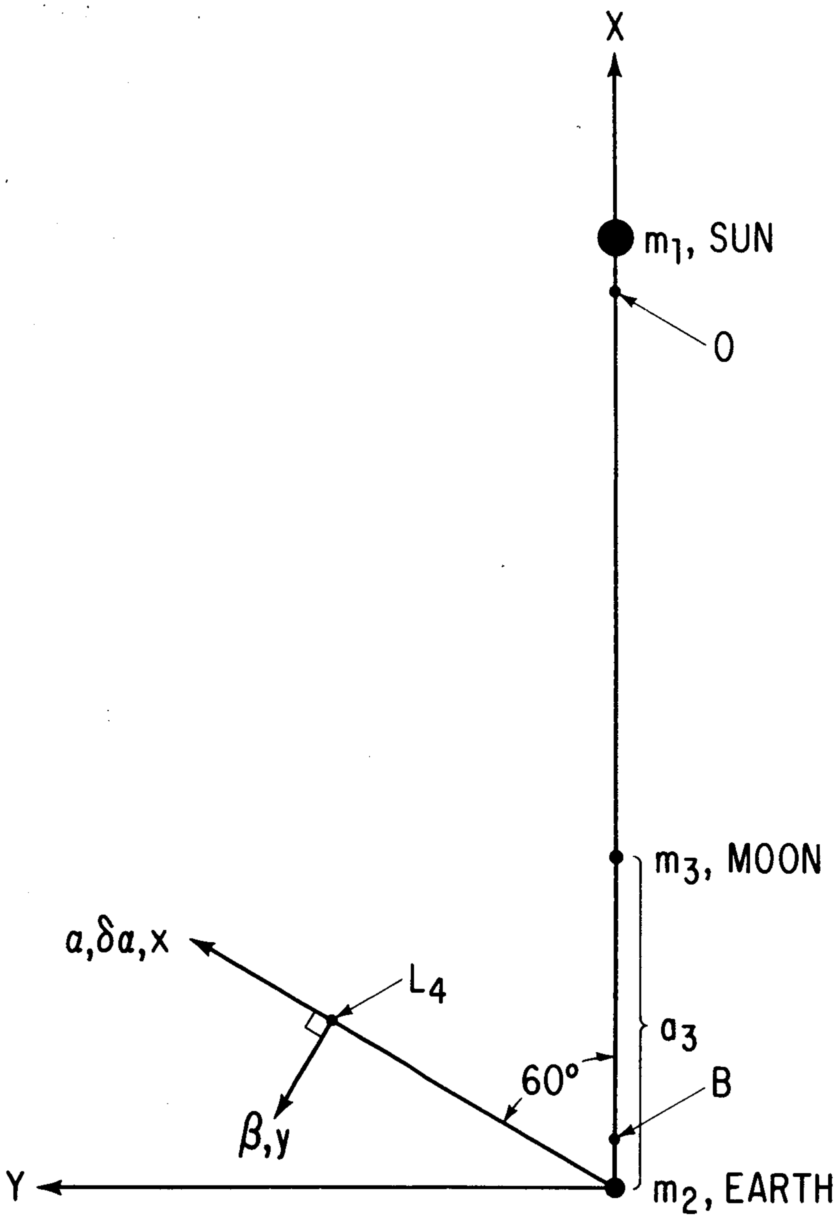


Figure 1. Geometry of the Coordinate System at Epoch

The right angle X, Y axis system with its origin at the center of mass of the earth is rotating at a uniform rate so as to keep the center of mass of the moon on the X axis. The masses m, $m_1$, $m_2$, and $m_3$ are in the X, Y plane. A point $L_4$, in the X, Y plane, is 60° from the X axis at lunar distance, $a_3$, and in advance of the moon's position. This point corresponds to a triangular point of the earth-moon system three body problem.

## 3. THE HARMONIC ORBIT

Using the model described the motion of an infinitesimal mass, m in the vicinity of the $L_4$ point was investigated with the purpose of obtaining a harmonic orbit in the numerical sense. This harmonic orbit has a period equal to the period of the disturbing force, which for this model is the moon's synodic period. Musen's (1963) method is applied with the perturbations represented in Chebyshev series as proposed by Carpenter (1966). In this method the geocentric position vector, $\vec{r}$, of the mass m near the $L_4$ triangular point is given by

$$\vec{r} = (1+\alpha)\vec{r}_o + \beta\,\vec{w}$$

where $\alpha$ and $\beta$ are the components of the perturbations, $\vec{r}_o$ is the position vector in the fixed reference ellipse, a is the semimajor axis of the reference ellipse and

$$\vec{w} = \frac{1}{n}\,\frac{d\vec{r}_o}{dt}$$

n being the mean motion in the reference ellipse and t the time. The functions $\alpha$ and $\beta$ can be represented by uniformly convergent series in the interval $-1 \leq \tau \leq 1$ by

$$\alpha(\tau) = \sum_{k=0}^{\infty}{}' \alpha_k\, T_k(\tau)$$

$$\beta(\tau) = \sum_{k=0}^{\infty}{}' \beta_k\, T_k(\tau)$$

where the prime on the summation sign is used to indicate that the first term is to be factored by one-half. The $T_k(\tau)$ are the Chebyshev polynomials defined by

$$T_k(\tau) = \cos\left[k\cos^{-1}\tau\right]$$

where the coefficients of these polynomials are given by $\alpha_k$ and $\beta_k$.

The synodic period for the model being utilized is given by the equation

$$P = \frac{2\pi}{n_3 - n_1}$$

where $n_1$ and $n_3$ are the mean motions of the sun and moon respectively. The dimensionless time $\tau$ is related to time t from epoch by

$$\tau = \frac{2t}{P}$$

where t is zero at the epoch which is defined as the first time the mass $m_1$ crosses the positive X axis.

Starting with initial conditions at $\tau = 0$ in the X, Y plane and near the $L_4$ point, an orbit was generated by using the Chebyshev polynomial procedure. Initial conditions corresponding to $\tau$ of -1 and 1 were thus obtained. Using numerical partial derivatives from this orbit an iteration scheme was employed to match the initial conditions at $\tau$ of -1 and 1. After several iterations this was accomplished, and the results are shown in Table II. The $\alpha'$ and $\beta'$ values, shown in this table, are derivatives of $\alpha$ and $\beta$ with respect to nt. Relative geocentric errors in position and velocity are indicated by the differences in Table II. These differences indicate agreement in ten significant figures which correspond to changes from $\tau = -1$ to $\tau = 1$ of 0.2 meters in the position vector,

TABLE II

Initial Conditions at $\tau = -1, 0, 1$ for the Harmonic Orbit

| $\tau$ | $\alpha(\tau) \times 10^3$ | $\beta(\tau) \times 10^3$ | $\alpha'(\tau) \times 10^3$ | $\beta'(\tau) \times 10^2$ |
|---|---|---|---|---|
| -1 | 8.90721044 | 1.43123468 | 7.84005032 | -1.341882564 |
| 0 | 4.41178027 | 16.95842648 | 14.7369698 | -0.507181064 |
| 1 | 8.90721028 | 1.43123517 | 7.84005038 | -1.341882534 |
| Differences | | | | |
| $\alpha(+1) - \alpha(-1) = -1.6 \times 10^{-10}$ | | | $\alpha'(+1) - \alpha'(-1) = 6.0 \times 10^{-11}$ | |
| $\beta(+1) - \beta(-1) = 4.9 \times 10^{-10}$ | | | $\beta'(+1) - \beta'(-1) = 3.0 \times 10^{-10}$ | |

$\vec{r}$, and $5 \times 10^{-7}$ meters per second in the velocity vector, $\dot{\vec{r}}$. From the numer- point of view it seems adequate to call this a harmonic orbit.

Chebyshev coefficients for this orbit are given in Table III. Using the initial conditions at epoch this orbit was extended for a total of twelve synodic periods ($-1 \leq \tau \leq 25$). This was done as a further check to insure the accuracy of the orbit. For this time period agreement with the initial orbit was eight significant figures in position and velocity. This is within the anticipated accuracy of the calculation. It was decided to plot the harmonic orbit with respect to a rotating coordinate system centered at $L_4$. Referring to Figure 2, a geocentric vector, $\vec{r}_o$, directed toward $L_4$ has a magnitude, a, defined by

$$a^3 = \frac{k_e^2 m_2}{n^2}$$

TABLE III
Chebyshev Coefficients for the Harmonic Orbit

| k | $\alpha_k \times 10^6$ | $\beta_k \times 10^6$ |
|---|---|---|
| 0 | 9233.8688 | 271.0928 |
| 1 | -466.7498 | 4715.9669 |
| 2 | 4193.3102 | -2590.7864 |
| 3 | -2839.4208 | -2764.2264 |
| 4 | 1530.9949 | 7775.0747 |
| 5 | 4927.4893 | -3235.6624 |
| 6 | -1877.9548 | -5000.6993 |
| 7 | -1940.7497 | 1552.0981 |
| 8 | 507.1676 | 1271.4117 |
| 9 | 356.8288 | -301.5204 |
| 10 | -67.8275 | -172.1400 |
| 11 | -41.2120 | 36.8027 |
| 12 | 4.4834 | 13.0026 |
| 13 | 4.3726 | -3.8665 |
| 14 | 0.1851 | -0.0154 |
| 15 | -0.6623 | 0.4549 |
| 16 | -0.0905 | -0.1980 |
| 17 | 0.1213 | -0.0468 |
| 18 | 0.0042 | 0.0448 |
| 19 | -0.0196 | -0.0019 |
| 20 | 0.0048 | -0.0069 |
| 21 | 0.0024 | 0.0029 |
| 22 | -0.0021 | 0.0007 |
| 23 | -0.0001 | -0.0009 |
| 24 | 0.0005 | 0.0000 |
| 25 | -0.0001 | 0.0002 |
| 26 | -0.0001 | 0.0000 |
| 27 | 0.0000 | 0.0000 |

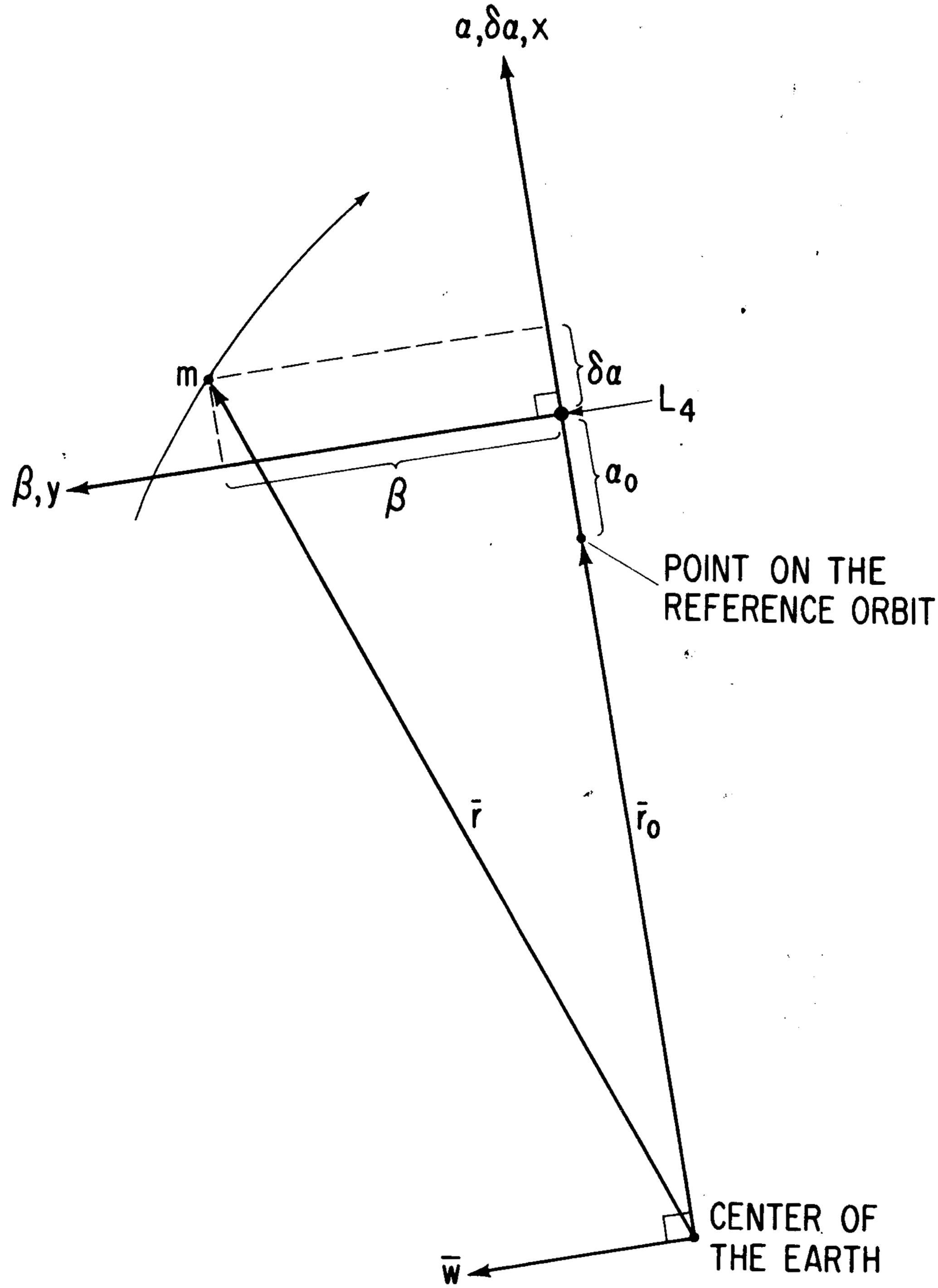


Figure 2. Coordinate Systems near $L_4$

where a is the semimajor axis of the reference orbit, $k_e^2$ is the earth's gravitational parameter and n is equal to the mean motion of the moon, $n_3$. The $\alpha$, $\beta$ coordinate system has its origin at $\vec{r}_0$ with $\alpha$ directed along $\vec{r}_0$ and $\beta$ at right angles to $\alpha$ in the direction of motion in the reference orbit.

A $\delta\alpha$, $\beta$ coordinate system has its origin at $(1 + \alpha_0)\ \vec{r}_0$ which corresponds to $L_4$. The value of $\alpha_0$ is given by the quantity $(a_3 - a)/a$ where $a_3$, the moon's semi-major axis, is taken to be 60 earth radii.

The $\delta\alpha$ component is directed along $\vec{r}_0$ and $\beta$ is the same component previously defined but translated parallel to itself to the $L_4$ point. Utilizing the data given in Table III the harmonic $L_4$ orbit is plotted, see Figure 3, using the $\delta\alpha$,

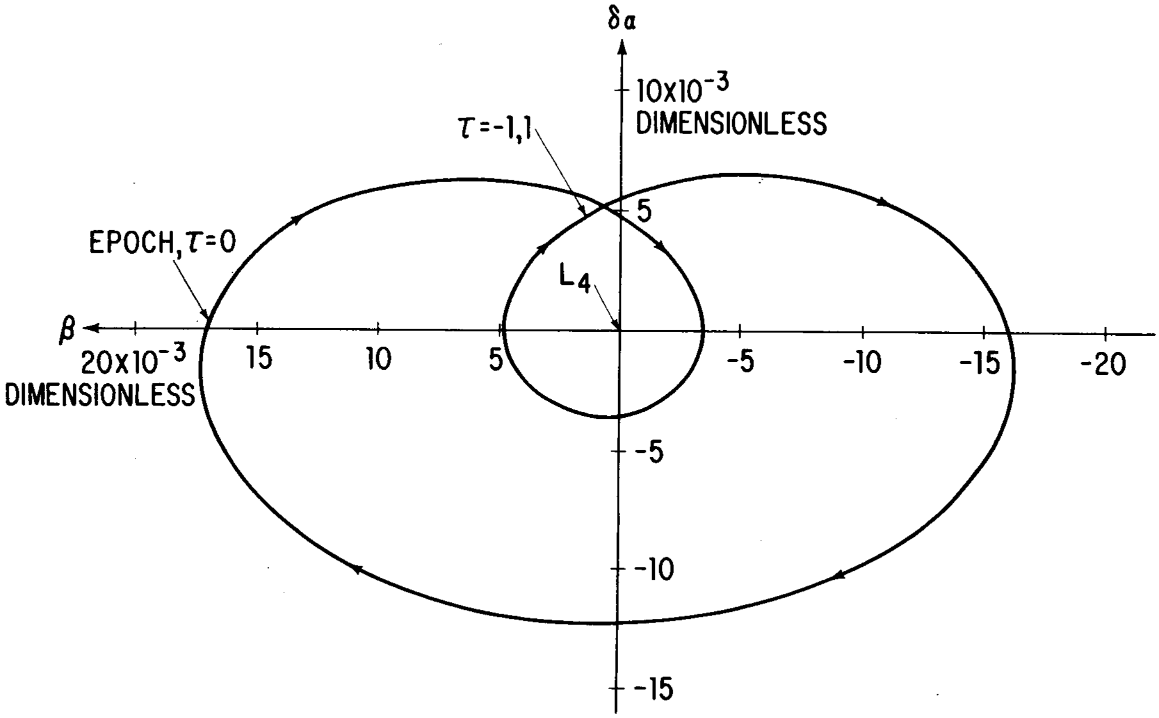


Figure 3. A Harmonic $L_4$ Orbit

$\beta$ coordinate axes. Units for this plot are dimensionless with respect to the semimajor axis of the reference orbit. The harmonic orbit remains close to the $L_4$ point having a deviation of less than 1.8 percent (6860 km) of the earth-moon distance. This orbit is seen to describe two different size loops about $L_4$, the time in the smaller loop is 36 percent of the period. Conversion to the usual x, y coordinate system where x and y are centered at $L_4$ parallel to $\delta\alpha$, $\beta$ but have dimensionless units with respect to the semimajor axis of the moon is accomplished by the transformation

$$x = \delta\alpha \frac{a}{a_3}$$

and

$$y = \beta \frac{a}{a_3}$$

Since the ratio $(a_3/a)$ is near unity there would be no noticeable difference by replacing $\delta\alpha$ and $\beta$ by x and y respectively in Figure 3, this matter being brought to attention for purposes of calculation.

## 4. A NEARLY PERIODIC ORBIT

Since the disturbing force is nearly periodic with half the synodic period of the moon, it is possible to find orbits which are nearly periodic with half the moon's synodic period. One such orbit was obtained by approximately matching initial conditions at $\tau$ of -1 and zero. The initial conditions are shown in Table IV. If the orbit were periodic with half the moon's synodic period, initial conditions at $\tau$ of -1 and zero would match with those at $\tau = 1$. The agreement in

TABLE IV

Initial Conditions at $\tau = -1, 0, 1$ for the Nearly Periodic Orbit

| $\tau$ | $\alpha(\tau) \times 10^3$ | $\beta(\tau) \times 10^3$ | $\alpha'(\tau) \times 10^2$ | $\beta'(\tau) \times 10^3$ |
|---|---|---|---|---|
| -1 | 6.67701926 | 9.31381378 | 1.132682180 | -9.25579538 |
| 0 | 6.67701357 | 9.31377754 | 1.132682017 | -9.25577168 |
| 1 | 6.65506332 | 9.24996973 | 1.128066502 | -9.18511281 |
| Differences | | | | |
| $\alpha(0) - \alpha(-1) = -5.69 \times 10^{-9}$ | | | $\alpha'(0) - \alpha'(-1) = -1.63 \times 10^{-9}$ | |
| $\beta(0) - \beta(-1) = -3.62 \times 10^{-8}$ | | | $\beta'(0) - \beta'(-1) = 2.37 \times 10^{-8}$ | |
| $\alpha(0) - \alpha(+1) = 2.19 \times 10^{-5}$ | | | $\alpha'(0) - \alpha'(+1) = 4.615 \times 10^{-5}$ | |
| $\beta(0) - \beta(+1) = 6.38 \times 10^{-5}$ | | | $\beta'(0) - \beta'(+1) = -7.06 \times 10^{-5}$ | |

initial conditions between $\tau$ of -1 and zero is eight significant figures in position and velocity; however, between $\tau$ of zero and one there is only five significant figure agreement. Further reduction of this difference between $\tau$ of -1 and zero tended to increase the differences between zero and one. A plot of this nearly periodic $L_4$ orbit in the $\delta\alpha$, $\beta$ coordinate system for $\tau$ between minus one and one is given in Figure 4. Also shown is the envelope of the orbit for six synodic periods ($-6 \leq \tau \leq 6$). During this time interval the orbit remains within 1.2 percent (4574 km) of the earth-moon distance from $L_4$. Deviations from the $\tau$ of minus one to one orbit are seen to be less than one tenth of a percent (381 km) of the earth-moon distance. During longer time intervals the deviations are expected to increase.

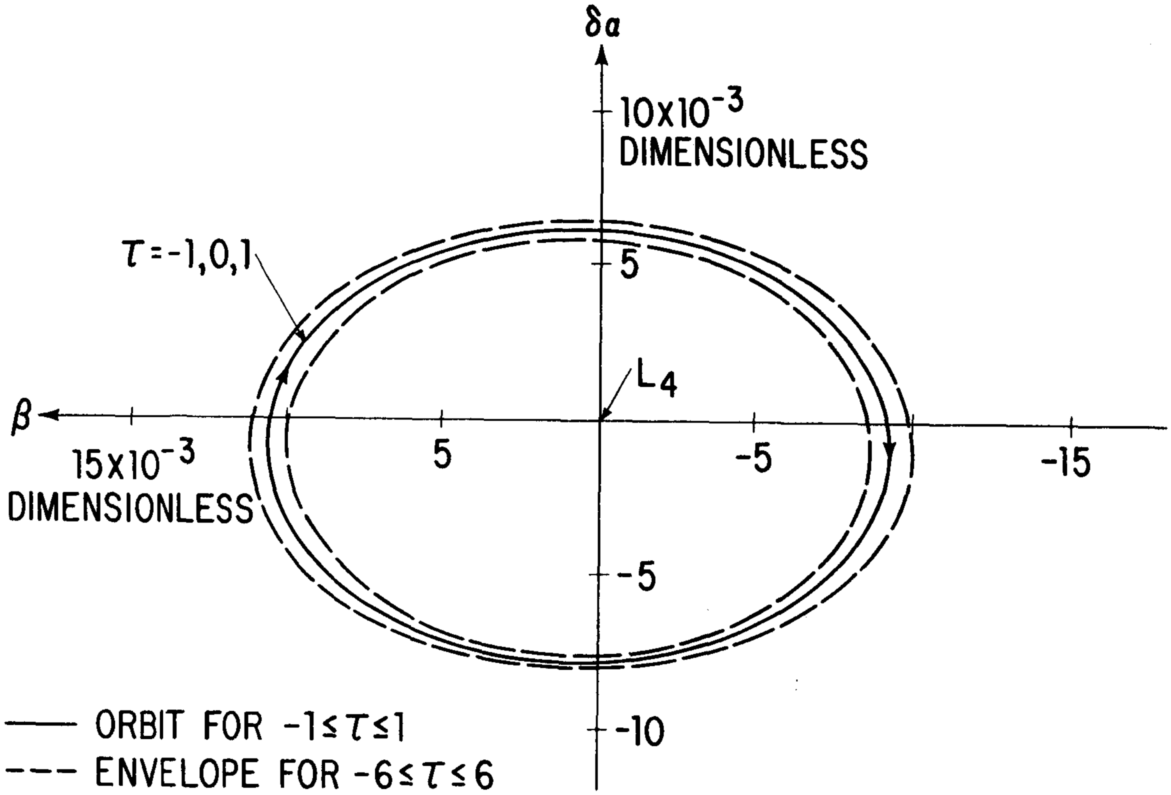


Figure 4. A Nearly Periodic $L_4$ Orbit

Using the Chebyshev polynomial approach, motion in the vicinity of $L_4$ can be investigated for other models of the earth-moon-sun system. One of the more interesting models would include the moon moving in an eccentric orbit. With this model some insight may be gained as to the importance of the moon's eccentricity on motion near $L_4$. The method is not restricted to simple models, e.g., it is possible to study motion near $L_4$ using the actual motions of the principal bodies taken from a general theory or from an ephemeris.

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# Appendix A

## Derivations

1. Derivation of system of equations for matching initial condition at $\tau$ of -1 and 1.

Let $\alpha(-1)$, $\beta(-1)$, $\alpha'(-1)$, $\beta'(-1)$ and $\alpha(1)$, $\beta(1)$, $\alpha'(1)$, $\beta'(1)$ be the values at $\tau = -1$ and $+1$ respectively. If small changes are made in each of the parameters at $\tau = -1$ these will result in changes in the parameters at $\tau = +1$ and the ratios will give

$$\frac{\partial\alpha(1)}{\partial\alpha(-1)}, \frac{\partial\alpha(1)}{\partial\beta(-1)}, \frac{\partial\alpha(1)}{\partial\alpha'(-1)}, \frac{\partial\alpha(1)}{\partial\beta'(-1)}, \frac{\partial\beta(1)}{\partial\alpha(-1)}, \frac{\partial\beta(1)}{\partial\beta(-1)}, \cdots \text{ etc.}$$

for a total of 16 numerical partial derivatives. Now suppose $\alpha(-1)$, $\beta(-1)$, $\alpha'(-1)$, and $\beta'(-1)$ are replaced by $\alpha(-1) + \delta\alpha$ , $\beta(-1) + \delta\beta$ , $\alpha'(-1) + \delta\alpha'$ , $\beta'(-1) + \delta\beta'$ then to first order the new values at $\tau = 1$ will be

$$\alpha(1) + \frac{\partial\alpha(1)}{\partial\alpha(-1)}\delta\alpha + \frac{\partial\alpha(1)}{\partial\beta(-1)}\delta\beta + \frac{\partial\alpha(1)}{\partial\alpha'(-1)}\delta\alpha' + \frac{\partial\alpha(1)}{\partial\beta'(-1)}\delta\beta'$$

$$\beta(1) + \frac{\partial\beta(1)}{\partial\alpha(-1)}\delta\alpha + \frac{\partial\beta(1)}{\partial\beta(-1)}\delta\beta + \frac{\partial\beta(1)}{\partial\alpha'(-1)}\delta\alpha' + \frac{\partial\beta(1)}{\partial\beta'(-1)}\delta\beta'$$

$$\alpha'(1) + \frac{\partial\alpha'(1)}{\partial\alpha(-1)}\delta\alpha + \frac{\partial\alpha'(1)}{\partial\beta(-1)}\delta\beta + \frac{\partial\alpha'(1)}{\partial\alpha'(-1)}\delta\alpha' + \frac{\partial\alpha'(1)}{\partial\beta'(-1)}\delta\beta'$$

$$\beta'(1) + \frac{\partial\beta'(1)}{\partial\alpha(-1)}\,\delta\alpha + \frac{\partial\beta'(1)}{\partial\beta(-1)}\,\delta\beta + \frac{\partial\beta'(1)}{\partial\alpha'(-1)}\,\delta\alpha' + \frac{\partial\beta'(1)}{\partial\beta'(-1)}\,\delta\beta' \;.$$

The object is to find $\delta\alpha$, $\delta\beta$, $\delta\alpha'$, $\delta\beta'$ such that the values at $\tau = \pm 1$ are equal. These $\delta$ corrections will be the solution of the following system of equations

$$\left[1 - \frac{\partial\alpha(1)}{\partial\alpha(-1)}\right]\delta\alpha - \frac{\partial\alpha(1)}{\partial\beta(-1)}\,\delta\beta - \frac{\partial\alpha(1)}{\partial\alpha'(-1)}\,\delta\alpha' - \frac{\partial\alpha(1)}{\partial\beta'(-1)}\,\delta\beta' = \alpha(1) - \alpha(-1) \equiv \Delta\alpha$$

$$-\frac{\partial\beta(1)}{\partial\alpha(-1)}\,\delta\alpha + \left[1 - \frac{\partial\beta(1)}{\partial\beta(-1)}\right]\delta\beta - \frac{\partial\beta(1)}{\partial\alpha'(-1)}\,\delta\alpha' - \frac{\partial\beta(1)}{\partial\beta'(-1)}\,\delta\beta' = \beta(1) - \beta(-1) \equiv \Delta\beta$$

$$-\frac{\partial\alpha'(1)}{\partial\alpha(-1)}\,\delta\alpha - \frac{\partial\alpha'(1)}{\partial\beta(-1)}\,\delta\beta + \left[1 - \frac{\partial\alpha'(1)}{\partial\alpha'(-1)}\right]\delta\alpha' - \frac{\partial\alpha'(1)}{\partial\beta'(-1)}\,\delta\beta' = \alpha'(1) - \alpha'(-1) \equiv \Delta\alpha'$$

$$-\frac{\partial\beta'(1)}{\partial\alpha(-1)}\,\delta\alpha - \frac{\partial\beta'(1)}{\partial\beta(-1)}\,\delta\beta - \frac{\partial\beta'(1)}{\partial\alpha'(-1)}\,\delta\alpha' + \left[1 - \frac{\partial\beta'(1)}{\partial\beta'(-1)}\right]\delta\beta' = \beta'(1) - \beta'(-1) \equiv \Delta\beta'$$

In the above system of equations the unknown are

$\delta\alpha$ , $\delta\beta$, $\delta\alpha'$ , and $\delta\beta'$ thus the equations were inverted numerically to find these unknows. Denoting the matrix of partial derivatives by P where

$$\mathrm{P} \equiv \begin{bmatrix} \frac{\partial\alpha(1)}{\partial\alpha(-1)} & \frac{\partial\alpha(1)}{\partial\beta(-1)} & \frac{\partial\alpha(1)}{\partial\alpha'(-1)} & \frac{\partial\alpha(1)}{\partial\beta'(-1)} \\ \frac{\partial\beta(1)}{\partial\alpha(-1)} & \frac{\partial\beta(1)}{\partial\beta(-1)} & \frac{\partial\beta(1)}{\partial\alpha'(-1)} & \frac{\partial\beta(1)}{\partial\beta'(-1)} \\ \frac{\partial\alpha'(1)}{\partial\alpha(-1)} & \frac{\partial\alpha'(1)}{\partial\beta(-1)} & \frac{\partial\alpha'(1)}{\partial\alpha'(-1)} & \frac{\partial\alpha'(1)}{\partial\beta'(-1)} \\ \frac{\partial\beta'(1)}{\partial\alpha(-1)} & \frac{\partial\beta'(1)}{\partial\beta(-1)} & \frac{\partial\beta'(1)}{\partial\alpha'(-1)} & \frac{\partial\beta'(1)}{\partial\beta'(-1)} \end{bmatrix}$$

and define the matrices

$$[\delta A] \equiv \begin{bmatrix} \delta\alpha \\ \delta\beta \\ \delta\alpha' \\ \delta\beta' \end{bmatrix} \quad ; \quad [\Delta A] \equiv \begin{bmatrix} \Delta\alpha \\ \Delta\beta \\ \Delta\alpha' \\ \Delta\beta' \end{bmatrix}$$

The system of equations can be written as

$$(I - P)\,[\delta A] = [\Delta A]$$

where I is the unit matrix.

The desired inverse of this system of equations is therefore

$$[\delta A] = (I - P)^{-1}\,[\Delta A]$$

For the harmonic orbit the P matrix was found to be

$$P = \begin{bmatrix} -6.519783 & -0.233560 & -1.091679 & -3.707652 \\ -19.707221 & 0.9242190 & 4.682355 & -10.352294 \\ -0.773883 & 0.297094 & 0.436769 & -0.225023 \\ 12.07645 & 0.37297 & 1.72651 & 6.93592 \end{bmatrix}$$

2. Derivation of the velocity equation

The position vector, $\vec{r}$, of a body is given by

$$\vec{r} = (1 + \alpha)\,\vec{r}_0 + \beta\vec{w} + \gamma\, a\vec{R}$$

where $\alpha$, $\beta$, $\gamma$ are the components of the perturbations $\vec{r}_0$, is the position vector in the fixed reference ellipse, a is the semimajor axis of the reference ellipse, $\vec{R}$ is the unit vector in the direction of the angular momentum of the motion in the reference ellipse and

$$\vec{w} = \frac{1}{n} \frac{d\vec{r}_0}{dt}$$

n being the mean motion of the reference ellipse and t the time. Let prime denote derivative with respect to nt and take the derivative of the position vector, thus

$$\vec{r}\,' = \alpha' \vec{r}_0 + \beta' \vec{w} + \gamma' a\vec{R} + (1 + \alpha)\, \vec{r}_0^{\,\prime} + \beta \vec{w}\,'$$

Since $\vec{w} \equiv \vec{r}_0^{\,\prime}$ and $\vec{w}\,' = 1/n^2 \; d^2\vec{r}_0/dt^2$ where Keplerian motion is given by

$$\frac{d^2\vec{r}_0}{dt^2} = -\mu^2 \frac{\vec{r}_0}{r_0^{\,3}} = -n^2 a^3 \frac{\vec{r}_0}{r_0^{\,3}}$$

then $\vec{w}\,' = -(a/r_0)^3 \vec{r}_0$ making these substitutions

$$\vec{r}\,' = \left[\alpha' - \beta\left(\frac{a}{r_0}\right)^3\right] \vec{r}_0 + (\beta' + 1 + \alpha)\, \vec{w} + \gamma' a\vec{R}$$

3. Converting relative geocentric errors in position and velocity into errors in meters and meters per second. From Table II the errors were found to be

$$\Delta\alpha = -1.6 \times 10^{-10} \qquad \Delta\alpha' = 6.0 \times 10^{-11}$$

$$\Delta\beta = 4.9 \times 10^{-10} \qquad \Delta\beta' = 3.0 \times 10^{-10}$$

For motion in the plane the position vector is given by

$$\vec{r} = (1 + \alpha)\,\vec{r}_0 + \beta\vec{w}$$

in which $\vec{r}_0$ and $\vec{w}$ have the magnitude of the semimajor axis of the reference orbit, a = 59.756099826 earth radii. The position error vector is given by

$$\Delta\vec{r} = \Delta\alpha\,\vec{r}_0 + \Delta\beta\vec{w}$$

which has a magnitude of

$$\begin{aligned} |\Delta\vec{r}| &= a\sqrt{(\Delta\alpha)^2 + (\Delta\beta)^2} \\ &= (59.756099826)(6378165.)\sqrt{(-1.6 \times 10^{-10})^2 + (4.9 \times 10^{-10})^2} \\ &= 0.196 \text{ meters} \end{aligned}$$

For motion in the plane the velocity vector is given by

$$\vec{r}\,' = \left[\alpha' - \beta\left(\frac{a}{r_0}\right)^3\right]\vec{r}_0 + (\beta' + 1 + \alpha)\,\vec{w}$$

since the reference orbit is circular $a \equiv r_0$ and the velocity error vector is given by

$$\Delta\vec{r}\,' = (\Delta\alpha' - \Delta\beta)\,\vec{r}_0 + (\Delta\beta' + \Delta\alpha)\,\vec{w}$$

which has a magnitude of

$$|n\Delta\vec{r}\,'| = an\sqrt{(\Delta\alpha' - \Delta\beta)^2 + (\Delta\beta' + \Delta\alpha)^2}$$

From Table I

$$n = 9.6595278 \times 10^{-3} \text{ rad per hr.}$$

$$= 2.683202 \times 10^{-6} \text{ rad per sec.}$$

$$|n\Delta\vec{r}\,'| = [(59.756099826)(6378165)]$$

$$[2.683202 \times 10^{-6}] \sqrt{[(0.6 - 4.9) \times 10^{-10}]^2 + [(3.0 - 1.6) \times 10^{-10}]^2}$$

$$= 4.62 \times 10^{-7} \text{ meters per second .}$$

## Appendix B

## Tables of Chebyshev Coefficients

### Table B-1

### Nearly Periodic Orbit for One Synodic Period.

| k | $\alpha_k \times 10^6$ | $\beta_k \times 10^6$ | k | $\alpha_k \times 10^6$ | $\beta_k \times 10^6$ |
|---|---|---|---|---|---|
| 0 | 7894.6238 | 5171.6046 | 1 | -2593.3642 | 2089.0217 |
| 2 | 2013.9442 | 4931.6485 | 3 | -369.8085 | 279.5289 |
| 4 | 2200.5436 | 5439.2947 | 5 | 4551.9664 | -3710.5870 |
| 6 | -1947.8405 | -4775.9305 | 7 | -1918.8083 | 1574.1473 |
| 8 | 518.3504 | 1259.8947 | 9 | 356.4568 | -295.3571 |
| 10 | -71.6512 | -171.6326 | 11 | -41.2018 | 34.3535 |
| 12 | 5.4041 | 13.0195 | 13 | 4.3107 | -3.3862 |
| 14 | 0.0552 | -0.0650 | 15 | -0.6192 | 0.4024 |
| 16 | -0.0864 | -0.1716 | 17 | 0.1040 | -0.0474 |
| 18 | 0.0079 | 0.0363 | 19 | -0.0151 | 0.0001 |
| 20 | 0.0033 | -0.0051 | 21 | 0.0017 | 0.0022 |
| 22 | -0.0017 | 0.0005 | 23 | -0.0001 | -0.0008 |
| 24 | 0.0004 | 0.0000 | 25 | -0.0000 | 0.0002 |
| 26 | -0.0001 | -0.0000 | 27 | 0.0000 | -0.0000 |

Note: These coefficients are approximately the same as those obtained for a model having the sun in a circular orbit about the mid-point of the earth-partical line.

Table B-2

**Nearly Periodic Orbit for 6.26246 Synodic Periods (4400 hours).**

| k | $\alpha_k \times 10^6$ | $\beta_k \times 10^6$ | k | $\alpha_k \times 10^6$ | $\beta_k \times 10^6$ |
|---|---|---|---|---|---|
| 0 | 6963.9090 | 2506.5141 | 1 | 1267.7887 | -873.0461 |
| 2 | 585.5650 | 1302.5396 | 3 | 1363.4846 | -963.0635 |
| 4 | 485.7971 | 984.3795 | 5 | 1507.1638 | -1119.4409 |
| 6 | 291.5963 | 210.0752 | 7 | 1519.0140 | -1318.8197 |
| 8 | -44.4945 | -372.7752 | 9 | 1401.6762 | -1331.1329 |
| 10 | -485.9954 | -1189.4363 | 11 | 973.8196 | -968.5940 |
| 12 | -880.8550 | -1840.0052 | 13 | 204.6290 | -116.2743 |
| 14 | -927.9831 | -2103.0428 | 15 | -844.5835 | 860.5722 |
| 16 | -447.8515 | -1357.0757 | 17 | -1659.5763 | 1278.7061 |
| 18 | 314.9301 | 788.3247 | 19 | -1141.2482 | 895.9795 |
| 20 | 918.8582 | 2348.2517 | 21 | 563.7213 | -407.1751 |
| 22 | 620.6477 | 1447.3271 | 23 | 1703.5065 | -1421.5062 |
| 24 | -551.9072 | -1325.9619 | 25 | 561.2978 | -450.2972 |
| 26 | -951.8150 | -2345.6191 | 27 | -1636.8243 | 1334.8778 |
| 28 | 336.1807 | 827.0519 | 29 | -800.7523 | 654.1480 |
| 30 | 1014.6487 | 2492.3961 | 31 | 1895.4409 | -1547.6910 |
| 32 | -699.5445 | -1714.7432 | 33 | -85.2864 | 70.5438 |
| 34 | -616.3778 | -1513.0319 | 35 | -1942.5146 | 1588.8578 |
| 36 | 1369.8313 | 3353.7886 | 37 | 2412.2744 | -1974.8528 |
| 38 | -1241.4901 | -3042.0659 | 39 | -1755.7531 | 1435.0543 |
| 40 | 757.6500 | 1861.6671 | 41 | 932.4472 | -761.0120 |
| 42 | -356.3634 | -873.4677 | 43 | -390.6775 | 321.0836 |
| 44 | 137.8553 | 333.1282 | 45 | 136.0663 | -113.2943 |
| 46 | -43.3800 | -105.5421 | 47 | -41.2262 | 33.0096 |
| 48 | 10.1023 | 28.8901 | 49 | 9.8678 | -6.0441 |
| 50 | -1.8861 | -7.9597 | 51 | -1.5876 | 0.4239 |
| 52 | 0.8858 | 1.4722 | 53 | 1.2066 | -2.0122 |
| 54 | -1.4598 | 0.7220 | 55 | -0.6756 | 3.0527 |
| 56 | 2.4475 | 0.0990 | 57 | -0.8579 | -2.2271 |
| 58 | -2.4662 | -0.9416 | 59 | 0.7778 | 1.5086 |
| 60 | 1.1769 | 0.0209 | 61 | 0.6425 | -1.4536 |
| 62 | 0.0358 | 1.0218 | 63 | -1.0284 | 1.0925 |
| 64 | -0.0981 | -0.5620 | 65 | -0.1118 | -0.1593 |
| 66 | -0.4879 | -0.6467 | 67 | 1.2122 | -0.5766 |
| 68 | 0.6546 | 1.0474 | 69 | -1.1221 | 0.5551 |
| 70 | -0.1295 | -0.2827 | 71 | 0.1550 | -0.0122 |
| 72 | -0.5853 | -0.8301 | 73 | 0.8586 | -0.5366 |
| 74 | 0.9276 | 1.4402 | 75 | -1.4591 | 0.8118 |
| 76 | -0.8324 | -1.3886 | 77 | 1.4867 | -0.7753 |
| 78 | 0.6022 | 1.0401 | 79 | -1.0035 | 0.5336 |
| 80 | -0.4319 | -0.6962 | 81 | 0.4786 | -0.2828 |
| 82 | 0.2578 | 0.3899 | 83 | -0.2935 | 0.1519 |
| 84 | -0.0663 | -0.1624 | 85 | 0.1653 | -0.0761 |
| 86 | 0.0776 | 0.0779 | 87 | 0.0130 | 0.0276 |
| 88 | 0.0055 | -0.0249 | 89 | 0.0864 | -0.0112 |
| 90 | 0.0243 | -0.0070 | 91 | 0.0668 | 0.0057 |
| 92 | 0.0203 | -0.0043 | 93 | 0.0755 | -0.0003 |
| 94 | 0.0403 | -0.0008 | 95 | 0.0544 | 0.0079 |
| 96 | 0.0552 | -0.0161 | 97 | -0.0038 | 0.0026 |
| 98 | 0.0542 | -0.0173 | 99 | -0.0954 | -0.0182 |
| 100 | 0.0074 | -0.0018 | 101 | -0.0958 | -0.0132 |
| 102 | -0.0796 | 0.0422 | 103 | 0.1021 | 0.0162 |
| 104 | -0.0135 | 0.0087 | 105 | 0.1468 | 0.0236 |
| 106 | 0.1400 | -0.0895 | 107 | -0.2996 | -0.0586 |
| 108 | -0.1851 | 0.1075 | 109 | 0.2457 | 0.0530 |
| 110 | 0.0866 | -0.0719 | 111 | -0.1260 | -0.0321 |
| 112 | -0.0489 | 0.0329 | 113 | 0.0475 | 0.0206 |
| 114 | 0.0108 | -0.0111 | 115 | -0.0126 | -0.0077 |
| 116 | 0.0209 | 0.0028 | 117 | 0.0032 | 0.0065 |
| 118 | 0.0141 | -0.0007 | 119 | -0.0008 | -0.0067 |
| 120 | 0.0063 | -0.0001 | 121 | -0.0009 | -0.0013 |
| 122 | -0.0197 | 0.0001 | 123 | -0.0004 | -0.0017 |
| 124 | -0.0190 | 0.0002 | 125 | 0.0008 | 0.0046 |
| 126 | 0.0032 | -0.0001 | 127 | 0.0000 | 0.0033 |
| 128 | 0.0251 | -0.0000 | 129 | -0.0002 | -0.0028 |
| 130 | 0.0068 | -0.0000 | 131 | -0.0006 | -0.0053 |
| 132 | -0.0250 | 0.0002 | 133 | 0.0005 | 0.0025 |
| 134 | -0.0098 | 0.0000 | 135 | 0.0008 | 0.0060 |
| 136 | 0.0283 | -0.0002 | 137 | -0.0006 | -0.0042 |
| 138 | 0.0014 | -0.0001 | 139 | -0.0007 | -0.0051 |
| 140 | -0.0315 | 0.0002 | 141 | 0.0007 | 0.0081 |
| 142 | 0.0223 | -0.0001 | 143 | -0.0001 | -0.0014 |
| 144 | 0.0125 | -0.0001 | 145 | -0.0005 | -0.0076 |
| 146 | -0.0390 | 0.0003 | 147 | 0.0012 | 0.0120 |
| 148 | 0.0437 | -0.0003 | 149 | -0.0011 | -0.0111 |
| 150 | -0.0338 | 0.0002 | 151 | 0.0004 | 0.0077 |
| 152 | 0.0207 | -0.0001 | 153 | -0.0003 | -0.0043 |
| 154 | -0.0103 | 0.0001 | 155 | 0.0005 | 0.0020 |
| 156 | 0.0044 | -0.0001 | 157 | -0.0001 | -0.0009 |
| 158 | -0.0018 | -0.0000 | 159 | -0.0003 | 0.0003 |
| 160 | 0.0006 | 0.0001 | 161 | 0.0000 | -0.0001 |

**Table B-3**

**Harmonic Orbit for Seven Synodic Periods.**

| k | $\alpha_k \times 10^6$ | $\beta_k \times 10^6$ | k | $\alpha_k \times 10^6$ | $\beta_k \times 10^6$ |
|---|---|---|---|---|---|
| 0 | 7457.2159 | 810.6086 | 1 | -146.2737 | 1921.1129 |
| 2 | 1205.6185 | -500.8064 | 3 | -224.7309 | 1623.8637 |
| 4 | 1394.2414 | -722.6099 | 5 | -428.5244 | 924.6166 |
| 6 | 1564.0341 | -761.5695 | 7 | -760.0051 | -245.9927 |
| 8 | 1463.3128 | -48.8037 | 9 | -966.9615 | -1597.7711 |
| 10 | 841.1339 | 1826.0628 | 11 | -394.3421 | -2190.0430 |
| 12 | -138.0494 | 3797.1057 | 13 | 1405.5604 | -1068.2085 |
| 14 | -573.9662 | 2559.3025 | 15 | 2931.6451 | 587.8964 |
| 16 | 46.4183 | -3005.8293 | 17 | 1049.7570 | -477.3923 |
| 18 | -166.0784 | -4544.2480 | 19 | -2081.8566 | -1803.0256 |
| 20 | -1769.4628 | 2083.0325 | 21 | 119.1789 | 3110.2725 |
| 22 | 624.9484 | -2196.4307 | 23 | -2058.7499 | 34.2711 |
| 24 | 561.8066 | 3171.5108 | 25 | 678.7966 | 32.8783 |
| 26 | 704.3463 | 1175.6787 | 27 | 1565.6424 | -1448.8345 |
| 28 | -572.0417 | -1272.6793 | 29 | 442.0562 | -330.1177 |
| 30 | -867.4497 | -2159.4794 | 31 | -1649.9648 | 1347.2723 |
| 32 | 460.5920 | 1134.1503 | 33 | -479.6221 | 398.4775 |
| 34 | 873.7834 | 2146.8321 | 35 | 1872.3926 | -1531.3793 |
| 36 | -840.0863 | -2060.5426 | 37 | -515.0913 | 420.1151 |
| 38 | -340.4363 | -834.8463 | 39 | -1536.3100 | 1254.2673 |
| 40 | 1228.2742 | 3014.9955 | 41 | 2346.5570 | -1923.5473 |
| 42 | -1285.7716 | -3160.8588 | 43 | -1930.7972 | 1586.8700 |
| 44 | 882.4341 | 2166.2507 | 45 | 1140.5773 | -932.6490 |
| 46 | -461.4496 | -1130.6934 | 47 | -533.3931 | 432.2833 |
| 48 | 194.4513 | 477.1455 | 49 | 206.5843 | -171.3318 |
| 50 | -68.3608 | -170.3621 | 51 | -66.9316 | 60.4684 |
| 52 | 20.9647 | 52.8676 | 53 | 19.4166 | -15.7319 |
| 54 | -6.0197 | -13.2520 | 55 | -5.8660 | -1.1356 |
| 56 | 2.3297 | 2.8936 | 57 | 0.7112 | 3.6051 |
| 58 | -1.1327 | -1.5754 | 59 | 0.6108 | 1.2709 |
| 60 | -0.7226 | 0.3364 | 61 | 0.6086 | -6.6775 |
| 62 | 2.0755 | 0.8289 | 63 | -0.7801 | 8.0019 |
| 64 | -1.5574 | -0.0827 | 65 | -0.5831 | -5.7733 |
| 66 | 0.4457 | -0.9704 | 67 | 0.9449 | 3.3120 |
| 68 | -0.3348 | 0.4266 | 69 | 0.2556 | -2.1731 |
| 70 | 0.7828 | 0.7733 | 71 | -1.0809 | 1.5228 |
| 72 | -0.6363 | -0.8429 | 73 | 0.4054 | -0.5742 |
| 74 | -0.1065 | -0.2232 | 75 | 0.8127 | -0.3245 |
| 76 | 0.6327 | 1.0732 | 77 | -1.1764 | 0.5902 |
| 78 | -0.4940 | -0.8443 | 79 | 0.4232 | -0.2123 |
| 80 | -0.0828 | -0.1386 | 81 | 0.6938 | -0.3732 |
| 82 | 0.6253 | 1.0494 | 83 | -1.3982 | 0.7573 |
| 84 | -0.8525 | -1.4063 | 85 | 1.4817 | -0.8174 |
| 86 | 0.7809 | 1.2646 | 87 | -1.1785 | 0.6598 |
| 88 | -0.5671 | -0.9125 | 89 | 0.7856 | -0.4349 |
| 90 | 0.3412 | 0.5694 | 91 | -0.4601 | 0.2382 |
| 92 | -0.1706 | -0.3133 | 93 | 0.2342 | -0.1113 |
| 94 | 0.0776 | 0.1444 | 95 | -0.0942 | 0.0542 |
| 96 | -0.0438 | -0.0467 | 97 | -0.0224 | -0.0346 |
| 98 | 0.0291 | 0.0049 | 99 | 0.0001 | 0.0192 |
| 100 | -0.0101 | 0.0003 | 101 | 0.0044 | -0.0005 |
| 102 | -0.0075 | 0.0099 | 103 | -0.0158 | -0.0108 |
| 104 | 0.0126 | -0.0198 | 105 | 0.0227 | 0.0108 |
| 106 | -0.0088 | 0.0230 | 107 | -0.0228 | -0.0073 |
| 108 | 0.0066 | -0.0205 | 109 | 0.0183 | 0.0075 |
| 110 | -0.0092 | 0.0151 | 111 | -0.0124 | -0.0098 |
| 112 | 0.0109 | -0.0096 | 113 | 0.0073 | 0.0088 |
| 114 | -0.0072 | 0.0053 | 115 | -0.0039 | -0.0034 |
| 116 | -0.0000 | -0.0027 | 117 | 0.0019 | -0.0026 |
| 118 | 0.0053 | 0.0012 | 119 | -0.0009 | 0.0051 |
| 120 | -0.0053 | -0.0005 | 121 | 0.0003 | -0.0030 |
| 122 | 0.0012 | 0.0001 | 123 | -0.0000 | -0.0012 |
| 124 | 0.0038 | -0.0001 | 125 | 0.0000 | 0.0049 |
| 126 | -0.0071 | 0.0002 | 127 | -0.0002 | -0.0067 |
| 128 | 0.0080 | -0.0002 | 129 | 0.0002 | 0.0066 |
| 130 | -0.0071 | 0.0002 | 131 | -0.0000 | -0.0054 |
| 132 | 0.0053 | -0.0000 | 133 | -0.0001 | 0.0038 |
| 134 | -0.0035 | -0.0001 | 135 | 0.0001 | -0.0023 |
| 136 | 0.0020 | 0.0000 | 137 | 0.0000 | 0.0013 |
| 138 | -0.0010 | 0.0001 | 139 | -0.0001 | -0.0006 |
| 140 | 0.0005 | -0.0001 | 141 | 0.0001 | 0.0002 |
| 142 | -0.0002 | 0.0001 | 143 | -0.0000 | -0.0001 |
| 144 | 0.0001 | 0.0000 | 145 | -0.0001 | 0.0001 |
| 146 | -0.0001 | -0.0001 | 147 | 0.0001 | -0.0001 |
| 148 | 0.0001 | 0.0001 | 149 | -0.0001 | 0.0001 |
| 150 | -0.0001 | -0.0001 | 151 | 0.0001 | -0.0001 |
| 152 | 0.0001 | 0.0001 | 153 | -0.0001 | 0.0001 |
| 154 | -0.0000 | -0.0000 | 155 | 0.0001 | -0.0000 |
| 156 | 0.0000 | 0.0000 | 157 | -0.0001 | 0.0000 |
| 158 | -0.0000 | -0.0000 | 159 | 0.0000 | 0.0000 |

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# Appendix C

## Computer Programs

### Table C-1

### Listing of CHBY, Subroutines, and a Sample Case.

```
$JOB   1203P003 126 M32RK M3081C1234
$EXECUTE          IBJOB
$IBJOB            SOURCE,MAP,GO
$IBFTC CHBY       DECK,M94,XR7
C       MODIFIED FOR THE FOUR BODY PROBLEM
C ****** MOON MUST BE THE FIRST DISTURBING BODY CONSIDERED IN THIS
C                  VERSION OF THE PROGRAM *****
C  CARPENTER, CHBY PROGRAM, COMPUTES PLANETARY PERTURBATIONS (ALPHA,
C  BETA, GAMMA) IN CHEBYSHEV SERIES
      DOUBLE PRECISION EKK,XJ,V,SA,SB,SE,SN,SGZ,SM,EP,P,Q,R,EK,RAD,
     1PI,TWOPI,EP2,SO2,SI2,CO2,SGZ2,SMINNR,SO,SI,CO,SN2,A,B,C,THET,DLL,
     2E,COSC,SINC,ESE,ECE,BK,DCOS,DSIN,COS2C,SIN2C,X,VAL,VALZ,FAC,ZK,
     3TWOSE,CN1,CN2,CN3,CN4,DN1,DN2,CNN,SAB,TRATIO,DXZR,DSQRT,TEPOCH,
     4PEPOCH,DTCHEB,PTCHEB,EPS1
      COMMON  X,EKK,NTRM,N1,XJ,V,SA,SB,SE,SN,SGZ,SM,EP,P,Q,R,IPRINT,NTIM
      COMMON TRATIO,DXZR,ZK
      DIMENSION  X( 701,8) ,AA(6),ZK(6),       V(5),ITITLE(12)
      DIMENSION A(3),B(3),C(3),P(3),Q(3),R(3),VAL(3,2,3),VALZ(3,2)
    1 FORMAT (4F15.7)
    2 FORMAT (1H08X1HX17X4HTHET41X1HV/1P7D18.10)
    3 FORMAT (80(1H3))
    4 FORMAT(34H0THE MOTION OF G IN DTCHEB DAYS IS,
    411PD25.15,8H DEGREES)
    5 FORMAT (I8,6P6F12.4)
    6 FORMAT (24H0VALUE OF SERIES AT X=-1 /1P6D20.12/
   61         24H0VALUE OF SERIES AT X= 0 /1P6D20.12/
   62         24H0VALUE OF SERIES AT X=+1 /1P6D20.12)
    7 FORMAT (26H0 VALUE OF SERIES AT EPOCH/ 11X1HA19X1HB19X
   71    1HC16X7HA PRIME13X7HB PRIME13X7HC PRIME//)
    8 FORMAT (1P6D20.12)
    9 FORMAT (/2(7X1HK1X11HALPHA*1.E6 2X10HBETA*1.E6 1X11HGAMMA*1.E6 )//
   91)
   10 FORMAT (2(I8,6P3F12.4))
   12 FORMAT (10H CARPENTER)
   13 FORMAT (22H0INTEGRATION CONSTANTS/1H0,9X2HZK,I1,5(17X2HZK,I1))
   15 FORMAT (9I8)
   17 FORMAT (8H0 TEPOCH12X6HDTCHEB12X6HPEPOCH12X6HPTCHEB13X4HEPS1/5D18.
  111)
   22 FORMAT (17H0TEST ZERO VALUES/1P6D20.12)
   25 FORMAT (5H0   K55X2HN113X2HN213X2HN312X4HCOSE11X4HSINE//)
   26 FORMAT (I5,48X,1P5E15.7)
   29 FORMAT (26H0 INITIAL CONDITIONS AT X=,F3.0/11X1HA19X1HB19X
  291    1HC16X7HA PRIME13X7HB PRIME13X7HC PRIME//)
   32 FORMAT (7H0VALID ,F10.1,6H THRU ,F10.1)
   33 FORMAT (4D18.11)
   34 FORMAT (I8,0P3F12.0,10X,1P3E15.7)
   36 FORMAT (48HJ      N1     NTRM     ITRM  IPRINT  IPUNCH      IC)
   37 FORMAT (28H0REFERENCE ELLIPTIC ELEMENTS/
  371         1H013X2HSA23X2HSE22X3HSO222X3HSI2/1P4D25.15)
   38 FORMAT (1H012X3HCO223X2HSM22X4HSGZ220X6HSMINNR/1P4D25.15)
   39 FORMAT (1H013X2HSB22X3HSN2/1P2D25.15)
   41 FORMAT(12A6)
   43 FORMAT(1H1)
   52 FORMAT (64H0VECTORIAL CONSTANTS REFERRED TO EQUATOR AND MEAN EQUIN
  5210X 1950.0/
  522         1H010X1HP19X1HQ19X1HR19X1HA19X1HB19X1HC/(1P6D20.12))
      PI=3.141592653589793
      TWOPI=2.*PI
```

```
      RAD=TWOPI/360.
C
C  EK IS THE GAUSSIAN CONSTANT
C
C     EK=0.01720209895D+00
C THE NEXT EK IS FOR EARTH(REF. ITEM) UNITS FOR EK SQ ARE ER CUBE/HR SQ.
      EK=4.461996918D+00
C
C  EP2 IS NEWCOMB OBLIQUITY FOR JAN 0 1950, 23DEG 26MIN 44.84SEC
C
      EP2=23.44578888888889
      EP=EP2*RAD
C
C  ENTRY POINT FOR SUCCESSIVE CASES
C
   55 CONTINUE
      DO 59 I=6,8
      DO 59 J=1, 701
   59 X (J,I)=0.
      READ  (5,41) ITITLE
C
C  ITITLE IS 72 BCD CHARACTERS USED AS A TITLE FOR THE RUN
C
      READ  (5,15) N1,NTRM,ITRM,IPRINT,IPUNCH,IC
C
C  N1+1       NUMBER OF SPECIAL VALUES
C  NTRM       NUMBER OF COEFFICIENTS (NOT TO EXCEED 701)
C  ITRM       NUMBER OF ITERATIONS FOR THIS RUN
C  IPRINT     =1 PRINT RESULTS ONLY
C             =2 PRINT SPECIAL VALUES AND EXPANSION COEFFICIENTS
C             =3 PRINT TEST QUANTITIES IN FFD SUBROUTINE
C  IPUNCH     =1 DO NOT PUNCH COEFFICIENTS ON CARDS
C             =2 PUNCH COEFFICIENTS
C  IC         =1 INITIAL CONDITIONS AT X=-1
C             =2 INITIAL CONDITIONS AT X= 0
C             =3 INITIAL CONDITIONS AT X=+1
C
      READ (5,33) TEPOCH,DTCHEB, EPS1
C
C  TEPOCH     JULIAN DATE OF THE EPOCH
C  DTCHEB     TIME INTERVAL IN DAYS FOR VALIDITY OF THEORY
C  EPS1       CRITERION FOR TRUNCATING COMPUTED SERIES (AFTER STATEMENT 410)
C
      READ (5,33) PEPOCH,PTCHEB
C
C  PEPOCH, PTCHEB ARE EPOCH AND TIME INTERVAL FOR DISTURBING PLANETS
C
      ASSIGN 570 TO NPUNCH
      IF (IPUNCH-1)62,62,60
   60 PUNCH 12
      PUNCH 3
      PUNCH 3
      PUNCH 3
      IF (ITRM-1) 61,61,62
   61 ASSIGN 568 TO NPUNCH
   62 CONTINUE
      READ (5,33) SA,SE,SO2,SI2,CO2,SM,SGZ2,SMINNR
C
C  NOTATION FOR ELEMENTS OF DISTURBED PLANET
C  SA         SEMI-MAJOR AXIS IN EARTH RADII
C  SE         ECCENTRICITY
```

Table C-1 (Continued)

```
C   SO2         ARGUMENT OF PERIHELION (DEGREES)
C   SI          INCLINATION (DEGREES) WITH RESPECT TO ECLIPTIC
C             A VALUE OF 23.445788888 PUTS BODY IN ECLIPTIC PLANE
C   CO2         LONGITUDE OF ASCENDING NODE (DEGREES)
C   SM          MASS OF DISTURBED PLANET IN UNITS OF EARTH MASS
C   SGZ2        MEAN ANOMALY AT EPOCH (DEGREES)
C   THE NEXT CARD IS NOT NEEDED WHEN EARTH IS PRIMARY
C   SMINNR      MASS OF INNER PLANETS (TO BE ADDED TO SOLAR MASS)
C
      READ (5,33)ZK
C
C   ZK ARE INTEGRATION CONSTANTS USED FOR COMPUTING SPECIAL VALUES
C
      SO=SO2*RAD
      SI=SI2*RAD
      CO=CO2*RAD
      SGZ=SGZ2*RAD
      SN=EK*DSQRT(1.+SMINNR+SM)/SA**1.5
      EKK=.5*SN*DTCHEB
      TRATIO=DTCHEB/PTCHEB
      DXZR=(TEPOCH-PEPOCH)*2./PTCHEB
      SN2=SN/RAD
      SB=SA*DSQRT(1.-SE**2)
      SAB=SA/SB
      WRITE (6,43)
      WRITE (6,41)ITITLE
      WRITE (6,36)
      WRITE (6,15) N1,NTRM,ITRM,IPRINT,IPUNCH,IC
      WRITE (6,37) SA,SE,SO2,SI2
      WRITE (6,38) CO2,SM,SGZ2,SMINNR
      WRITE (6,39) SB,SN2
      WRITE (6,17)TEPOCH,DTCHEB,PEPOCH,PTCHEB,EPS1
      XJ1=TEPOCH-DTCHEB/2.
      XJ2=TEPOCH+DTCHEB/2.
      WRITE (6,32) XJ1,XJ2
      BK=2.*EKK/RAD
      WRITE(6,4)BK
C
C   VALZ ARE THE INITIAL CONDITIONS
C
      READ (5,33) ((VALZ(I,J),I=1,3),J=1,2)
      XJ=IC-2
      WRITE (6,29) XJ
      WRITE (6,8) ((VALZ(I,J),I=1,3),J=1,2)
      WRITE (6,13)(I,I=1,6)
      WRITE (6,8)ZK
C                                                                  PQRDP2
C     CALL PQRDP2(SO,SI,CO,P,Q,R,EP)
      XJ=0.
      CALL PQRDP2(SO,SI,CO,P,Q,R,XJ)
      DO 65 I=1,3
      A(I)=SA*P(I)
      B(I)=SB*Q(I)
   65 C(I)=SA*R(I)
      WRITE (6,52) (P(I),Q(I),R(I),A(I),B(I),C(I),I=1,3)
C
C   READ INPUT PERTURBATIONS
C
      NTIM=0
      WRITE (6,9)
   70 READ  (5,5) K,(AA(I),I=1,6)
```

Table C-1 (Continued)

```
C
C   EACH CARD CONTAINS THE COEFFICIENTS OF T SUB K OF X AND T SUB K+1
C   OF X FROM THE PREVIOUS ESTIMATES OF ALPHA, BETA, GAMMA
C
      KP=K+1
      WRITE (6,10)K,(AA(I),I=1,3),KP,(AA(I),I=4,6)
      IF (K-1001) 80,92,92
   80 K1=K+1
      KP=K+2
      IF (K1-700) 82,82,70
   82 CONTINUE
      NTIM=KP
      DO  90 I=1,3
      J=I+3
      X(K1,I+5)=AA(I)
      X(KP,I+5)=AA(J)
   90 CONTINUE
      GO TO 70
   92 CONTINUE
      ITYPE=1
C
C   CALL FFD TO READ DISTURBING PLANET DATA
C
      CALL FFD (ITYPE)
      ITTRM=1
C
C   ENTRY POINT FOR SUCCESSIVE ITERATIONS
C
   94 ITYPE=2
      DO 96 I=1,5
      DO 96 J=1, 701
   96 X (J,I)=0.
C
C   THE FOLLOWING OPERATIONS THRU STATEMENT 230 GIVE THE CHEBYSHEV
C   EXPANSIONS OF THE DISTURBING FORCES.  THE EXPANDED QUANTITIES ARE
C       V(1)           V(2)           V(3)          V(4)          V(5)
C      M1*ADR       2.*M2*ADR       M3*ADR        COS(E)        SIN(E)
C
C   WHERE M1, M2, M3 ARE THE SUBSCRIPTED QUANTITIES FROM THE EQUATIONS
C   FOR THE PERTURBATIONS, ADR IS THE FACTOR SMALL A/ SMALL R SUB ZERO,
C   AND E IS THE ECCENTRIC ANOMALY IN THE REFERENCE ELLIPSE
C
      XJ=-1.D+0
C
C   CALL FFD FOR FIRST SET OF SPECIAL VALUES
C
      CALL FFD (ITYPE)
      THET=PI
      IF (IPRINT-2)110,100,100
  100 CONTINUE
      WRITE (6,2)XJ ,THET ,(V(I),I=1,5)
  110 CONTINUE
      DO 200 I=1,5
      V(I)=.5D+0*V(I)
  200 CONTINUE
C
C   COEFF ADDS CONTRIBUTIONS OF SPECIAL VALUES TO SERIES COEFFICIENTS
C
      CALL COEFF (XJ,V)
      EN1=N1
      DLL=PI/EN1
```

Table C-1 (Continued)

```
      DO 210 K=2,N1
      THET=THET-DLL
      XJ=DCOS(THET)
C
C  CALL FFD FOR INTERIOR POINTS
C
      CALL FFD (ITYPE)
C
      CALL COEFF (XJ,V)
      IF (IPRINT-2)210,202,202
  202 WRITE (6,2)XJ ,THET ,(V(I),I=1,5)
  210 CONTINUE
      XJ=+1.D+0
C
C  CALL FFD FOR  LAST SET OF SPECIAL VALUES
C
      CALL FFD (ITYPE)
      IF (IPRINT-2)214,212,212
  212 CONTINUE
      THET=0.
      WRITE (6,2)XJ ,THET ,(V(I),I=1,5)
  214 CONTINUE
      DO 220 I=1,5
      V(I)=.5D+0*V(I)
  220 CONTINUE
C
      CALL COEFF (XJ,V)
      FAC=2./EN1
      N2=NTRM-10
      DO 230 K=1,NTRM
      DO 230 I=1,5
      X(K,I)=X(K,I)*FAC
  230 CONTINUE
      IF (IPRINT-2)364,240,240
  240 CONTINUE
      WRITE (6,43)
      WRITE (6,41) ITITLE
      WRITE (6,25)
      DO 340 K=1,NTRM
      KP=K-1
      WRITE (6,26) KP,(X(K,I),I=1,5)
  340 CONTINUE
C
C  TEST ZERO VALUES OF X(K,1) THRU X(K,5)
C
      DO 362 I=1,5
      ZK(I)=.5*X(1,I)
      BK=1.
      DO 362 K=3,N2,2
      BK=-BK
      ZK(I)=ZK(I)+BK*X(K,I)
  362 CONTINUE
      WRITE (6,22)(ZK(I),I=1,5)
C
C  PERFORM SERIES INTEGRATIONS (THRU STATEMENT 410).  THE PREVIOUS
C  VALUES OF THE PERTURBATION COEFFICIENTS ARE NO LONGER NEEDED. THE NEW
C  SERIES OCCUPY THE OLD LOCATIONS (X(K,6), X(K,7), X(K,8))
C
  364 TWOSE=2.*SE
      X(1,4)=X(1,4)-TWOSE
C
```

```
C  MLTPLY (I1,I2,I3) CAUSES THE SERIES X(K,I1) AND X(K,I2) TO BE
C  MULTIPLIED AND THE PRODUCT TO BE STORED IN X(K,I3)
C
      CALL MLTPLY (4,1,8)
      X(1,4)=X(1,4)+TWOSE
      CALL MLTPLY (5,2,7)
      DO 370 K=1,N2
      X(K,8)=X(K,8)-X(K,7)
  370 CONTINUE
C
C  NTGRT (I1,I2) CAUSES THE SERIES X(K,I1) TO BE INTEGRATED WITH RESPECT
C  TO NT AND THE INTEGRAL TO BE STORED IN X(K,I2)
C
      CALL NTGRT (8,7)
      CALL MLTPLY (5,7,6)
      CALL MLTPLY (5,1,8)
      CALL MLTPLY (4,2,7)
      DO 380 K=1,N2
      X(K,8)=-X(K,8)-X(K,7)
  380 CONTINUE
      CALL NTGRT (8,7)
      X(1,4)=X(1,4)-TWOSE
      CALL MLTPLY (4,7,8)
      X(1,4)=X(1,4)+TWOSE
      DO 390 K=1,N2
      X(K,6)=X(K,6)+X(K,8)
      X(K,1)=-SE*X(K,4)
  390 CONTINUE
      X(1,1)=2.+X(1,1)
      CALL MLTPLY (1,2,7)
      CALL NTGRT (7,8)
      DO 400 K=1,N2
      X(K,6)=X(K,6)+X(K,8)
      X(K,8)=.5*X(K,8)-2.*X(K,6)
  400 CONTINUE
C
C  ALPHA IS NOW IN X(K,6)
      CALL NTGRT (8,7)
C
C  BETA  IS NOW IN X(K,7)
      X(1,4)=X(1,4)-TWOSE
      CALL MLTPLY (3,4,1)
      CALL NTGRT (1,2)
      CALL MLTPLY (2,5,8)
      CALL MLTPLY (3,5,1)
      CALL NTGRT (1,2)
      CALL MLTPLY (2,4,1)
      DO 410 K=1,N2
      X(K,8)=X(K,8)-X(K,1)
  410 CONTINUE
C
C  GAMMA IS NOW IN X(K,8)
C
C  ESTABLISH TRUNCATION POINT, NTIM, FROM MAGNITUDES OF COEFFICIENTS
C
      EPTST=EPS1/1.E10
      NTIM=NTRM+1
      DO 414 K=1,NTRM
      NTIM=NTIM-1
      DO 412 I=1,3
      IF (ABS(X(NTIM,I+5))-EPTST) 412,416,416
```

Table C-1 (Continued)

```
  412 CONTINUE
  414 CONTINUE
  416 CONTINUE
      WRITE (6,43)
      WRITE (6,41)ITITLE
C
C  PRINT COEFFICIENTS
C
  418 CONTINUE
      WRITE (6,9)
      DO 460 K=1,NTIM,2
      K1=K-1
      L=K+1
      DO 430 I=1,3
      J=I+3
      AA(I)=X(K,I+5)
      AA(J)=X(L,I+5)
  430 CONTINUE
      WRITE (6,10)K1,(AA(I),I=1,3),K,(AA(I),I=4,6)
  460 CONTINUE
C
C  EVALUATE SERIES AT EPOCH
C
      DO 500 I=1,3
      DO 500 J=1,2
      DO 500 K=1,3
      VAL(I,J,K)=0.
  500 CONTINUE
      DO 520 I=1,3
      IS=I+5
      VAL(I,1,1)=.5*X(1,IS)
      VAL(I,1,2)=.5*X(1,IS)
      VAL(I,1,3)=.5*X(1,IS)
      VAL(I,2,1)=0.
      VAL(I,2,2)=0.
      VAL(I,2,3)=0.
      BK=1.
      FK=1.
      DO 510 K=3,N2,2
      BK=-BK
      FK= FK+2.
      VAL(I,1,1)=VAL(I,1,1)-X(K-1,IS)+X(K,IS)
      VAL(I,1,2)=VAL(I,1,2)        +BK*X(K,IS)
      VAL(I,1,3)=VAL(I,1,3)+X(K-1,IS)+X(K,IS)
      CN1=(FK-2.)**2
      CN2=(FK-1.)**2
      CN3=-BK*(FK-2.)
      VAL(I,2,1)=VAL(I,2,1)+CN1*X(K-1,IS)-CN2*X(K,IS)
      VAL(I,2,2)=VAL(I,2,2)+CN3*X(K-1,IS)
      VAL(I,2,3)=VAL(I,2,3)+CN1*X(K-1,IS)+CN2*X(K,IS)
  510 CONTINUE
      VAL(I,2,1)=VAL(I,2,1)/EKK
      VAL(I,2,2)=VAL(I,2,2)/EKK
      VAL(I,2,3)=VAL(I,2,3)/EKK
  520 CONTINUE
      WRITE (6,41) ITITLE
      WRITE (6,7)
      WRITE (6,6) (((VAL(I,J,K),I=1,3),J=1,2),K=1,3)
      XJ=IC-2
      WRITE (6,29) XJ
      WRITE (6, 8)((VALZ(I,J),I=1,3),J=1,2)
```

Table C-1 (Continued)

```
C
C  CONSTANTS OF INTEGRATION FOR INITIAL CONDITIONS
C
      XJ=IC-2
      CN1=SGZ+EKK*XJ
      E=CN1
      SINC=DSIN(E)
      COSC=DCOS(E)
      DO 542 I=1,3
      ESE=SE*SINC
      ECE=1.-SE*COSC
      BK=(CN1-E+ESE)/ECE
      E=E+BK-.5*BK*BK*ESE/ECE
      SINC=DSIN(E)
      COSC=DCOS(E)
  542 CONTINUE
      CN1=COSC/(1.-SE*COSC)
      CN3=SINC/(1.-SE*COSC)
      CN2=-SINC
      CN4=COSC-SE
      DN1=VALZ(3,1)-VAL(3,1,IC)
      DN2=VALZ(3,2)-VAL(3,2,IC)
      ZK(5)=DN1*CN1+DN2*CN2
      ZK(6)=DN1*CN3+DN2*CN4
      DN1=VALZ(1,1)-VAL(1,1,IC)
      DN2=VALZ(1,2)-VAL(1,2,IC)
      ZK(3)=2.*DN1+VALZ(2,2)-VAL(2,2,IC)
      DN1=DN1-2.*ZK(3)
      ZK(1)=DN1*CN1+DN2*CN2
      ZK(2)=DN1*CN3+DN2*CN4
      SIN2C=2.*SINC*COSC
      COS2C=1.-2.*SINC**2
      CNN=-2.+SE*SE
      DN1=VALZ(2,1)-VAL(2,1,IC)
      DN2=-3.*(SE*ZK(1)-ZK(3))*EKK*XJ
      ZK(4)=DN1+DN2                 +(-CNN*SINC-.5*SE*SIN2C)*ZK(1)
     1                              +( -2.*COSC+.5*SE*COS2C)*ZK(2)
      WRITE (6,13)(I,I=1,6)
      WRITE (6, 8)(ZK(I),I=1,6)
C
C  ADD CONTRIBUTIONS FROM CONSTANTS TO SERIES COEFFICIENTS
C
      X(1,7)=X(1,7)+2.*ZK(4)
      X(2,7)=X(2,7)+3.*(SE*ZK(1)-ZK(3))*EKK
      X(1,6)=X(1,6)+4.*ZK(3)-TWOSE*ZK(1)
      X(1,8)=X(1,8)            -TWOSE*ZK(5)
      X(1,4)=X(1,4)+TWOSE
      CALL MLTPLY (4,4,1)
      X(1,1)=X(1,1)-1.
C  X(K,1) IS .5*COS(2E)
      CALL MLTPLY (4,5,2)
C  X(K,2) IS .5*SIN(2E)
      DO 550 K=1,N2
      X(K,6)=X(K,6)+ZK(1)*X(K,4)+ZK(2)*X(K,5)
      X(K,8)=X(K,8)+ZK(5)*X(K,4)+ZK(6)*X(K,5)
      X(K,7)=X(K,7)+ZK(1)*(CNN*X(K,5)+SE*X(K,2))
     1              +ZK(2)*( 2.*X(K,4)-SE*X(K,1))
  550 CONTINUE
C
C  PRINT AND PUNCH COEFFICIENTS
C
```

```
      WRITE (6,43)
      WRITE (6,41)ITITLE
      WRITE (6,9)
      DO 570 K=1,NTIM,2
      K1=K-1
      L=K+1
      DO 564 I=1,3
      J=I+3
      AA(I)=X(K,I+5)
      AA(J)=X(L,I+5)
  564 CONTINUE
      WRITE (6,10)K1,(AA(I),I=1,3),K,(AA(I),I=4,6)
      GO TO NPUNCH ,(568,570)
  568 PUNCH      5,K1,(AA(I),I=1,6)
  570 CONTINUE
C
C  EVALUATE SERIES AT EPOCH (SHOULD EQUAL INITIAL CONDITIONS)
C
      DO 580 I=1,3
      DO 580 J=1,2
      DO 580 K=1,3
      VAL(I,J,K)=0.
  580 CONTINUE
      DO 600 I=1,3
      IS=I+5
      VAL(I,1,1)=.5*X(1,IS)
      VAL(I,1,2)=.5*X(1,IS)
      VAL(I,1,3)=.5*X(1,IS)
      VAL(I,2,1)=0.
      VAL(I,2,2)=0.
      VAL(I,2,3)=0.
      BK=1.
      FK=1.
      DO 590 K=3,N2,2
      BK=-BK
      FK= FK+2.
      VAL(I,1,1)=VAL(I,1,1)-X(K-1,IS)+X(K,IS)
      VAL(I,1,2)=VAL(I,1,2)        +BK*X(K,IS)
      VAL(I,1,3)=VAL(I,1,3)+X(K-1,IS)+X(K,IS)
      CN1=(FK-2.)**2
      CN2=(FK-1.)**2
      CN3=-BK*(FK-2.)
      VAL(I,2,1)=VAL(I,2,1)+CN1*X(K-1,IS)-CN2*X(K,IS)
      VAL(I,2,2)=VAL(I,2,2)+CN3*X(K-1,IS)
      VAL(I,2,3)=VAL(I,2,3)+CN1*X(K-1,IS)+CN2*X(K,IS)
  590 CONTINUE
      VAL(I,2,1)=VAL(I,2,1)/EKK
      VAL(I,2,2)=VAL(I,2,2)/EKK
      VAL(I,2,3)=VAL(I,2,3)/EKK
  600 CONTINUE
      WRITE (6,41) ITITLE
      WRITE (6,7)
      WRITE (6,6) (((VAL(I,J,K),I=1,3),J=1,2),K=1,3)
      DO 602 I=1,6
      ZK(I)=0.
  602 CONTINUE
      ITTRM=ITTRM+1
      IF (ITTRM-ITRM) 94,610,604
  604 PUNCH 3
      PUNCH 3
      PUNCH 3
```

```
      GO TO 55
  610 IF (IPUNCH-1) 94,94,620
  620 ASSIGN 568 TO NPUNCH
      GO TO 94
      END
$IBFTC FFD     DECK,M94,XR7
      SUBROUTINE  FFD (ITYPE)
C  SUBROUTINE FOR COMPUTING SPECIAL VALUES OF DISTURBING FORCE
      DOUBLE PRECISION EKK,XJ,V,SA,SB,SE,SN,SGZ,SM,EP,P,Q,R,EK,RAD,X,
     1SAB,P7,P9,P11,P13,SA2,C11,C31,SAP,SEP,SOP2,SIP2,COP2,SMP2,SGZP2,
     2SMINNR,SOP,COP,SIP,SGZP,SNP,SNP2,PPP,QPP,RPP,SBP,AP,BP,CP,PP,QP,
     3RP,SMP,SMPA2,EKP,ENT,SG,E,SNE,CSE,DCOS,DSIN,DSQRT,ESE,ECE,BK,COSE,
     4SINE,SR,SR2,SR3,ADR,TADR,SRZ,S1,S2,EX,SRV,F,ENTP,SGP,CK1,CK2,CK3,
     5SPZ,RHO2,SRP2,SPV,RHO,CRHO,CSRP,TRATIO,DXZR,XJP,ZK,CS1,CS2,SS2,
     6HALFSE,SEE,BARY,BR
      DOUBLE PRECISION PK1,PK2,PK3,SRZDS
      COMMON  X,EKK,NTRM,N1,XJ,V,SA,SB,SE,SN,SGZ,SM,EP,P,Q,R,IPRINT,NTIM
      COMMON TRATIO,DXZR,ZK
      DIMENSION SEP(9),SGZP(9),PPP(3),QPP(3),RPP(3),AP(3,9),BP(3,9),
     1CP(3,9),PP(3),QP(3),RP(3),SMPA2(9),X( 701,8),   Y(3,1200),   AA(6),
     2NTPM(9),SRZ(2),SW(2),S1(2),S2(2),PERT(3),S(3),SRV(3),F(3),EKP(9),
     3SPZ(3),SWZ(3),SPV(3),RHO(3),ITITLE(12),P(3),Q(3),R(3),V(5),
     4NFIRST(9),ZK(6),BR(3)
    1 FORMAT (12A6)
    2 FORMAT (1H1,12A6,4H  M=,I2)
    3 FORMAT (4D18.11)
    4 FORMAT (12HJSMALL OMEGAF15.7,F13.7,15H ) ECLIPTIC AND5X7HSMALL E
    41   F12.8/12H CAP    OMEGAF15.7,F13.7,15H ) MEAN EQUINOX5X7HSMALL A
    42   F15.7,6X2HER/8H SMALL IF19.7,F13.7,11H )     1950.09X7HSMALL N
    43   E16.8,8H DEG/HR /7H G ZEROF20.7,F13.7,20X7HSMALL ME16.8)
    5 FORMAT (1HK9X1HA16X1HB16X1HC13X6H    P   10X6H    Q   10X7H    R    //)
    6 FORMAT (6F17.8,15H ) EQUATOR  AND/
   61          6F17.8,15H ) MEAN EQUINOX/
   62          6F17.8,12H )     1950.0)
    7 FORMAT (1H0)
    8 FORMAT (6F17.8,15H ) P,Q,R,SYSTEM/
   81          6F17.8, 9H )      OF/
   82          6F17.8,15H )  COORDINATES)
    9 FORMAT (I8,6P6F12.4)
   10 FORMAT (/2(7X1HK1X11HALPHA*1.E6 2X10HBETA*1.E6 1X11HGAMMA*1.E6 )6X
   1017HSTORAGE//)
   11 FORMAT (58H0  K    A**2*F(I) SUM     ALPHA,BETA,GAMMA    E,COS(E),SIN
   111(E)6X6HSRV(I)12X6HSRZ(I)13X5HSW(I))
   12 FORMAT (1H0,I3,1PD18.10,E18.7,5D18.10)
   13 FORMAT      (4X,1PD18.10,E18.7,5D18.10)
   14 FORMAT (25H0  M    DISTURBING PLANETS94X6HRHO(I))
   16 FORMAT (2(I8,6P3F12.4),2I8)
   17 FORMAT (1H017X7HDEGREES7X7HRADIANS)
   18 FORMAT (6H SNP =1PD24.15,6HRAD/HR)
      IF (ITYPE-2)100,300,300
C
C  COMPUTE CONSTANTS AND READ DISTURBING PLANET DATA
C
  100 CONTINUE
      KEPITR=3
      IF (SE-0.25)104,102,102
  102 KEPITR=4
  104 CONTINUE
      RAD=2.*3.141592653589793/360.
      SAB=SA/SB
C THE NEXT EK IS FOR EARTH(REF. ITEM) UNITS FOR EK SQ ARE ER CUBE/HR SQ.
```

```
      EK=4.461996918D+00
      P 7= 7./ 6.
      P 9= 9./ 8.
      P11=11./10.
      P13=13./12.
      SA2=SA*SA
      HALFSE=SE*.5
      SEE=-2.+SE**2
      C11=1.5*SA2
      C31=-15.*SA2/8.
      M=0
      K2=0
C
C  ENTRY FOR SUCCESSIVE DISTURBING PLANETS
C
  110 CONTINUE
      M=M+1
      READ (5,1)ITITLE
      WRITE(6,2)ITITLE,M
C ****** MOON MUST BE THE FIRST DISTURBING BODY CONSIDERED IN THIS
C                  VERSION OF THE PROGRAM ******
      READ (5,3) SAP,SEP(M),SOP2,SIP2,COP2,SMP2,SGZP2,SMINNR
C
C  ELEMENTS OF DISTURBING PLANETS IN SAME FORM AS DISTURBED PLANET
C
      SOP=SOP2    *RAD
      COP=COP2    *RAD
      SIP=SIP2    *RAD
      SGZP(M)=SGZP2    *RAD
      SNP=EK*DSQRT(1.+SMINNR+SMP2)/SAP**1.5
       SNP2  =SNP       /RAD
      WRITE (6,17)
      WRITE(6,4) SOP2,SOP,SEP(M),COP2,COP,SAP,SIP2,SIP,SNP2,SGZP2,
     1SGZP(M),SMP2
      WRITE (6,18) SNP
C
      CALL PQRDP2 (SOP,SIP,COP,PPP,QPP,RPP,EP)
      SBP=SAP   *DSQRT (1.-SEP(M)**2)
      DO 120 I=1,3
      AP(I,M)=SAP    *PPP(I)
      BP(I,M)=SBP    *QPP(I)
      CP(I,M)=SAP    *RPP(I)
  120 CONTINUE
      WRITE(6,5)
      WRITE(6,6)(AP(I,M),BP(I,M),CP(I,M),PPP(I),QPP(I),RPP(I),I=1,3)
      DO 130 I=1,3
      PP(I   )=0.
      QP(I   )=0.
      RP(I   )=0.
  130 CONTINUE
      DO 140 J=1,3
      PP(1   )=PP(1   )+P(J)*PPP(J)
      QP(1   )=QP(1   )+P(J)*QPP(J)
      RP(1   )=RP(1   )+P(J)*RPP(J)
      PP(2   )=PP(2   )+Q(J)*PPP(J)
      QP(2   )=QP(2   )+Q(J)*QPP(J)
      RP(2   )=RP(2   )+Q(J)*RPP(J)
      PP(3   )=PP(3   )+R(J)*PPP(J)
      QP(3   )=QP(3   )+R(J)*QPP(J)
      RP(3   )=RP(3   )+R(J)*RPP(J)
  140 CONTINUE
```

```
      DO 150 I=1,3
      AP(I,M)=SAP    *PP(I)
      BP(I,M)=SBP    *QP(I)
      CP(I,M)=SAP    *RP(I)
  150 CONTINUE
      WRITE(6,7)
      WRITE(6,8)(AP(I,M),BP(I,M),CP(I,M),PP(I   ),QP(I   ),RP(I   ),I=1,3)
      SMP=SMP2/(1.+SMINNR+SM)
      SMPA2(M)=SA2*SMP
      EKP(M)=EKK*SNP   /(SN*TRATIO)
C
C  READ PERTURBATIONS.  ALL COEFFICIENTS STORED IN Y ARRAY
C
      NFIRST(M)=K2+1
      NTPM(M)=0
      WRITE(6,10)
  190 READ (5,9) K,(AA(I),I=1,6)
      KK=K+1
      KP=K+NFIRST(M)
      KP1=KP+1
      WRITE(6,16)K,(AA(I),I=1,3),KK,(AA(I),I=4,6),KP,KP1
      IF (K-1001)200,110,220
  200 K1=KP
      K2=KP1
      IF (K2-1200)202,202,190
  202 CONTINUE
      NTPM(M)=K+2
      DO 210 I=1,3
      J=I+3
      Y(I,K1)   =AA(I)
      Y(I,K2)   =AA(J)
  210 CONTINUE
      GO TO 190
  220 NPLNET=M
      BARY=SMPA2(1)/(SA2+SMPA2(1))
      K=0
      RETURN
C
C  COMPUTE SPECIAL VALUES
C
  300 CONTINUE
      K=K+1
C
C  COMPUTE POSITION VECTOR OF DISTURBED PLANET
C
      ENT=EKK*XJ
      SG=SGZ+ENT
      E=SG
      SNE=DSIN(SG )
      CSE=DCOS(SG )
      DO 310 I=1,KEPITR
      ESE=SNE*SE
      ECE=1.-CSE*SE
      BK=(SG -E+ESE)/ECE
      E=E+BK-.5*BK*BK*ESE/ECE
      CSE=DCOS(E)
      SNE=DSIN(E)
  310 CONTINUE
      COSE=CSE
      SINE=SNE
      CS1=COSE-SE
```

```
      CS2=2.*COSE-SE*(.5-SINE**2)
      SS2=SEE*SINE+SE*(SINE*COSE+3.*ENT)
      V(4)=COSE
      V(5)=SINE
      SR     =SA*(1.-SE*COSE)
      SR2=SR*SR
      SR3=SR*SR2
      ADR=SA/SR
      TADR=2.*ADR
      SRZ(1)=SA*CS1
      SRZ(2)=SB*SINE
      SW(1)=-SA*ADR*SINE
      SW(2)= SB*ADR*COSE
      S1(1)= ADR*COSE
      S1(2)= ADR*SINE*SAB
      S2(1)= -TADR*SINE
      S2(2)=  TADR*CS1*SAB
      ALPHA=2.*ZK(3)+ZK(1)*CS1+ZK(2)*SINE
      GAMMA=          ZK(5)*CS1+ZK(6)*SINE
      BETA =ZK(4)+ZK(1)*SS2+ZK(2)*CS2-3.*ENT*ZK(3)
C
C  NTIM IS THE NUMBER OF TERMS IN THE SERIES FOR THE PERTURBATIONS
C
      EX=2.*XJ
      IF (NTIM-1)370,330,340
  330 ALPHA=.5*X(1,6)    +ALPHA
      BETA =.5*X(1,7)    +BETA
      GAMMA=.5*X(1,8)    +GAMMA
      GO TO 370
  340 CONTINUE
      DO 360 IS=6,8
      B2=0.
      B1=0.
      NTR=NTIM-1
      DO 350 I=1,NTR
      I1=NTR-I+2
      BZ=EX*B1-B2+X(I1,IS)
      B2=B1
      B1=BZ
  350 CONTINUE
      JS=IS-5
      PERT(JS)=.5*(EX*B1-2.*B2+X(1,IS))
  360 CONTINUE
      ALPHA=PERT(1)      +ALPHA
      BETA =PERT(2)      +BETA
      GAMMA=PERT(3)      +GAMMA
  370 CONTINUE
      S(1)=ALPHA*SRZ(1)+BETA*SW(1)
      S(2)=ALPHA*SRZ(2)+BETA*SW(2)
      S(3)=GAMMA*SA
C
C  EXPAND SOLAR TERMS IN POWERS OF PERTURBATIONS
C
      SQ=S(1)*S(1)+S(2)*S(2)+S(3)*S(3)
      SRV(1)=SRZ(1)+S(1)
      SRV(2)=SRZ(2)+S(2)
      SRV(3)=        S(3)
      DEL=(2.*(SRZ(1)*S(1)+SRZ(2)*S(2))+SQ)/SR2
C     C1=C11*SQ/(SR2*SR3)
C     C2=C11*DEL/SR3
C     C3=C31*DEL**2*(1.-P7*DEL*(1.-P9*DEL*(1.-P11*DEL*(1.-P13*DEL))))/SR
```

```
C      13
C       F(1)=C1*SRZ(1)+C2*S(1)+C3*SRV(1)
C       F(2)=C1*SRZ(2)+C2*S(2)+C3*SRV(2)
C       F(3)=          C2*S(3)+C3*SRV(3)
C      ********** F(1),F(2),F(3) CHANGED TO THOSE SHOWN BELOW*******
C       PK1,PK2,PK3,SRZDS,F1,F2,F3  ARE DOUBLE PRECISION
        PK1=SA2/(SR3*(1.+DEL)**1.5)
        PK2=SA2/SR3
        PK3=3.*PK2/SR2
        SRZDS=SRZ(1)*S(1)+SRZ(2)*S(2)
        F(1)=(-PK1+PK2)*SRV(1)-PK3*SRZ(1)*SRZDS
        F(2)=(-PK1+PK2)*SRV(2)-PK3*SRZ(2)*SRZDS
        F(3)=(-PK1+PK2)*SRV(3)
        IF (IPRINT-3) 372,371,371
  371   WRITE (6,11)
        WRITE (6,12) K,F(1),ALPHA,    E,SRV(1),SRZ(1),SW(1)
        WRITE (6,13)   F(2), BETA,COSE,SRV(2),SRZ(2),SW(2)
        WRITE (6,13)   F(3),GAMMA,SINE,SRV(3),SRZ(3),SW(3)
  372   CONTINUE
C
C   NEXT TAKE THE DISTURBING PLANETS
C
        IF (IPRINT-3)376,374,374
  374   WRITE (6,14)
  376   XJP=XJ*TRATIO+DXZR
        BR(1)=0.
        BR(2)=0.
        BR(3)=0.
        EX=2.*XJP
        DO 480 M=1,NPLNET
        ENTP=EKP(M)*XJP
        SGP=SGZP(M)+ENTP
        E=SGP
        SNE=DSIN(SGP)
        CSE=DCOS(SGP)
        DO 380 I=1,3
        ESE=SNE*SEP(M)
        ECE=1.-CSE*SEP(M)
        BK=(SGP-E+ESE)/ECE
        E=E+BK-.5*BK*BK*ESE/ECE
        CSE=DCOS(E)
        SNE=DSIN(E)
  380   CONTINUE
        CK1=CSE-SEP(M)
        CK2=-SNE/(1.-SEP(M)*CSE)
        CK3=CSE/(1.-SEP(M)*CSE)
        DO 390 I=1,3
        SPZ(I)=AP(I,M)*CK1+BP(I,M)*SNE
        SWZ(I)=AP(I,M)*CK2+BP(I,M)*CK3
  390   CONTINUE
        IF  (NTPM(M)-1)400,410,420
  400   ALP=0.
        BEP=0.
        GAP=0.
        GO TO 450
  410   JPP=NFIRST(M)
        ALP=.5*Y(1,JPP)
        BEP=.5*Y(2,JPP)
        GAP=.5*Y(3,JPP)
        GO TO 450
  420   CONTINUE
```

Table C-1 (Continued)

```
      NTP=NTPM(M)-1
      DO 440 IS=1,3
      B2=0.
      B1=0.
      DO 430 I=1,NTP
      I1=NTP-I+2
      JPP=NFIRST(M)+I1-1
      BZ=EX*B1-B2+Y(IS,JPP)
      B2=B1
      B1=BZ
  430 CONTINUE
      JPP=NFIRST(M)
      PERT(IS)=.5*(EX*B1-2.*B2+Y(IS,JPP))
  440 CONTINUE
      ALP=PERT(1)
      BEP=PERT(2)
      GAP=PERT(3)
  450 CONTINUE
      RHO2=0.
      SRP2=0.
      DO 460 I=1,3
      SPV(I)=(1.+ALP)*SPZ(I)+BEP*SWZ(I)+GAP*CP(I,M)+BR(I)
      RHO(I)=SPV(I)-SRV(I)
      RHO2=RHO2+RHO(I)*RHO(I)
      SRP2=SRP2+SPV(I)*SPV(I)
  460 CONTINUE
      CRHO=SMPA2(M)/(RHO2*DSQRT(RHO2))
      CSRP=SMPA2(M)/(SRP2*DSQRT(SRP2))
      DO 470 I=1,3
      BR(I)=BARY*SPV(I)
      F(I)=F(I)+CRHO*RHO(I)-CSRP*SPV(I)
  470 CONTINUE
      IF (IPRINT-3)480,471,471
  471 WRITE (6,12) M,F(1),ALP,  E,SPV(1),SPZ(1),SWZ(1),RHO(1)
      WRITE (6,13)   F(2),BEP,CSE,SPV(2),SPZ(2),SWZ(2),RHO(2)
      WRITE (6,13)   F(3),GAP,SNE,SPV(3),SPZ(3),SWZ(3),RHO(3)
  480 CONTINUE
      V(1)  =F(1)*S1(1)+F(2)*S1(2)
      V(2)  =F(1)*S2(1)+F(2)*S2(2)
      V(3)  =F(3)
      RETURN
      END
$IBFTC PQRDP2  NODECK,M94,XR7
      SUBROUTINE PQRDP2(SO,SI,CO,P,Q,R,EP)
C     COMPUTE VECTORS P,Q,R
      DOUBLE PRECISION SO,SI,CO,P,Q,R,EP,EPC,EPS,CCO,SCO,CSO,SSO,
     X    CSI,SSI,TP1,TP2,DCOS,DSIN
      DIMENSION P(3),Q(3),R(3)
C EP IS OBLIQUITY OF ECLIPTIC
      EPC=DCOS(EP)
      EPS=DSIN(EP)
      CCO=DCOS(CO)
      SCO=DSIN(CO)
      CSO=DCOS(SO)
      SSO=DSIN(SO)
      CSI=DCOS(SI)
      SSI=DSIN(SI)
      P(1)=-CSI*SSO*SCO+CSO*CCO
      TP1 =+CSI*SSO*CCO+CSO*SCO
      TP2 =+SSI*SSO
      P(2)=EPC*TP1-EPS*TP2
```

```
      P(3)=EPS*TP1+EPC*TP2
      Q(1)=-CSI*CSO*SCO-SSO*CCO
      TP1 = CSI*CSO*CCO-SSO*SCO
      TP2 = SSI*CSO
      Q(2)=EPC*TP1-EPS*TP2
      Q(3)=EPS*TP1+EPC*TP2
      R(1)= SSI*SCO
      TP1 =-SSI*CCO
      TP2 = CSI
      R(2)=EPC*TP1-EPS*TP2
      R(3)=EPS*TP1+EPC*TP2
      RETURN
      END
$IBFTC COEFF   DECK,M94,XR7
      SUBROUTINE COEFF
C  COMPUTE CONTRIBUTIONS OF SPECIAL VALUES TO SERIES COEFFICIENTS
      DOUBLE PRECISION XJ,V,X, TZ,T1,T2,TWOX,EKK
      DIMENSION V(5),X( 701,8)
      COMMON  X,EKK,NTRM,N1,XJ,V
      TZ=1.
      T1=XJ
      DO 100 I=1,5
      X (1,I)=X (1,I)+V(I)
      X (2,I)=X (2,I)+V(I)*T1
  100 CONTINUE
      TWOX=2.*XJ
      DO 120 K=3,NTRM
      T2=TWOX*T1-TZ
      DO 110 I=1,5
      X (K,I)=X (K,I)+V(I)*T2
  110 CONTINUE
      TZ=T1
      T1=T2
  120 CONTINUE
      RETURN
      END
$IBFTC MLTPLY  DECK,M94,XR7
      SUBROUTINE MLTPLY (I1,I2,I3)
C  PERFORM CHEBYSHEV SERIES MULTIPLICATION
C THE FACTORS ARE IN X(K,J1) AND X(K,J2). THE PRODUCT GOES IN X(K,J3).
      COMMON X( 701,8),EKK,N
      DOUBLE PRECISION EKK,X,EX
      J1=I1
      J2=I2
      J3=I3
      DO 100 K=1, 701
      X(K,J3)=0.
  100 CONTINUE
      N1=N
      DO 130 K=1,N1
        EX    =0.
      IM=N-K+1
      DO 120 I=1,IM
      L=I+K-1
        EX    =  EX    +X(I,J1)*X(L,J2)+X(L,J1)*X(I,J2)
  120 CONTINUE
      X(K,J3)=.5*EX
  130 CONTINUE
      X(1,J3)=X(1,J3)-.5*X(1,J1)*X(1,J2)
      DO 150 K=3,N1
        EX    =0.
      IM=K-1
      DO 140 I=2,IM
      L=K-I+1
        EX    =  EX    +X(I,J1)*X(L,J2)
  140 CONTINUE
      X(K,J3)=.5*EX   +X(K,J3)
  150 CONTINUE
      RETURN
      END
$IBFTC NTGRT   DECK,M94,XR7
      SUBROUTINE NTGRT (J1,J2)
C  PERFORM CHEBYSHEV SERIES INTEGRATION
C THE INTEGRAND IS STORED IN X(K,J1) AND THE INTEGRAL IN X(K,J2)
C THE FACTOR EKK COMES FROM   D(NT) = EKK * D(X)
      COMMON X( 701,8),EKK,N
      DOUBLE PRECISION EKK,X,EK2,EK
      I=J1
      J=J2
      X(1,J)=0.
      EK2=2./EKK
      EK=0.
      DO 100 K=2,N
      EK=EK + EK2
      X(K,J)= (X(K-1,I)-X(K+1,I))/EK
  100 CONTINUE
      X(N+1,J)= X(N,I)/(EK+EK2)
      RETURN
      END
```

## Table C-1 (Continued)

```
$DATA
 VERY RESTRICTED 4 BODY PROB. HARMONIC ORBIT RUN NO. 10060 B
      60       60       1       1       2       2
    0.0        D+0702.59922630241D+0    0.5        D+0
    0.0        D+0702.59922630241D+0    0.5        D+0
 59.756099826 D+00 0.0           D+00  0.0000      D+0 23.445788888 D+00
  0.0         D+00 0.0           D+00    60.0     D+00  0.0         D+00


4.411780267734 D-3 1.695842648237D-2       0.0     D+0 1.473696984961D-2
-5.071810642160D-3      0.0      D+0
       0   9233.8688    271.0929    -0.0000   -466.7498   4715.9670     0.0000
       2   4193.3102  -2590.7863    -0.0000  -2839.4208  -2764.2263    -0.0000
       4   1530.9949   7775.0747     0.0000   4927.4893  -3235.6624    -0.0000
       6  -1877.9548  -5000.6993     0.0000  -1940.7497   1552.0981     0.0000
       8    507.1676   1271.4117    -0.0000    356.8288   -301.5204     0.0000
      10    -67.8275   -172.1400     0.0000    -41.2120     36.8027    -0.0000
      12      4.4834     13.0026     0.0000      4.3726     -3.8665     0.0000
      14      0.1851     -0.0154    -0.0000     -0.6623      0.4549    -0.0000
      16     -0.0905     -0.1980     0.0000      0.1213     -0.0468    -0.0000
      18      0.0042      0.0448    -0.0000     -0.0196     -0.0019     0.0000
      20      0.0048     -0.0069    -0.0000      0.0024      0.0029    -0.0000
      22     -0.0021      0.0007     0.0000     -0.0001     -0.0009     0.0000
      24      0.0005      0.0000    -0.0000     -0.0001      0.0002     0.0000
      26     -0.0001     -0.0000     0.0000      0.0000     -0.0000    -0.0000
    1001
 MOON ELEMENTS
 60.0         D+00 0.0           D+00    0.0      D+00  0.0         D+00MOON 1
  0.0         D+00  .012294830   D+00    0.0      D+00  0.0         D+00MOON 2
    1001
 SUN ELEMENTS
 23454.87     D+00 0.0           D+00    0.0      D+00  0.0         D+00SUN 1
  0.0         D+00  332951.290   D+00    0.0      D+00  0.0         D+00SUN 2
    1002
```

Table C-1 (Continued)

```
VERY RESTRICTED 4 BODY PROB. HARMONIC ORBIT RUN NO. 10060 B

    N1     NTRM     ITRM   IPRINT   IPUNCH      IC
    60      60        1       1        2         2

REFERENCE ELLIPTIC ELEMENTS

          SA                          SE                         SO2                          SI2
  5.975609982600000D 01    0.                          0.                         2.3445788888000000D 01

          CO2                         SM                         SGZ2                        SMINNR
  0.                       0.                          6.0000000000000000D 01   0.

          SB                          SN2
  5.975609982600000D 01    5.534501789698960D-01

 TEPOCH              DTCHEB              PEPOCH              PTCHEB           EPS1
0.                 0.702599226300 03 0.                 0.702599226300 03 0.500000000000 00

VALID     -351.3 THRU      351.3

THE MOTION OF G IN DTCHEB DAYS IS    3.888536675411791D 02 DEGREES

 INITIAL CONDITIONS AT X= 0.
          A                  B                    C                    A PRIME               B PRIME              C PRIME

 4.411780267734D-03  1.695842648237D-02  0.                      1.473696984961D-02 -5.071810642160D-03  0.

INTEGRATION CONSTANTS

        ZK1                ZK2                ZK3                ZK4                ZK5                ZK6
-0.                -0.                -0.                -0.                -0.                -0.

VECTORIAL CONSTANTS REFERRED TO EQUATOR AND MEAN EQUINOX 1950.0

          P                  Q                   R                    A                     B                     C
 1.000000000000D 00 -0.                  0.                   5.975609982600D 01 -0.                    0.
 0.                   9.174369452164D-01 -3.978812028131D-01  0.                    5.482245368241D 01 -2.377582887419D 01
 0.                   3.978812028131D-01  9.174369452164D-01  0.                    2.377582887419D 01  5.482245368241D 01
```

| K | ALPHA*1.E6 | BETA*1.E6 | GAMMA*1.E6 | K | ALPHA*1.E6 | BETA*1.E6 | GAMMA*1.E6 |
|---|---|---|---|---|---|---|---|
| 0 | 9233.8688 | 271.0929 | -0. | 1 | -466.7498 | 4715.9670 | 0. |
| 2 | 4193.3101 | -2590.7863 | -0. | 3 | -2839.4208 | -2764.2263 | -0. |
| 4 | 1530.9949 | 7775.0746 | 0. | 5 | 4927.4893 | -3235.6624 | -0. |
| 6 | -1877.9548 | -5000.6992 | 0. | 7 | -1940.7497 | 1552.0981 | 0. |
| 8 | 507.1676 | 1271.4117 | -0. | 9 | 356.8288 | -301.5204 | 0. |
| 10 | -67.8275 | -172.1400 | 0. | 11 | -41.2120 | 36.8027 | -0. |
| 12 | 4.4834 | 13.0026 | 0. | 13 | 4.3726 | -3.8665 | 0. |
| 14 | 0.1851 | -0.0154 | -0. | 15 | -0.6623 | 0.4549 | -0. |
| 16 | -0.0905 | -0.1980 | 0. | 17 | 0.1213 | -0.0468 | -0. |
| 18 | 0.0042 | 0.0448 | -0. | 19 | -0.0196 | -0.0019 | 0. |
| 20 | 0.0048 | -0.0069 | -0. | 21 | 0.0024 | 0.0029 | -0. |
| 22 | -0.0021 | 0.0007 | 0. | 23 | -0.0001 | -0.0009 | 0. |
| 24 | 0.0005 | 0. | -0. | 25 | -0.0001 | 0.0002 | 0. |
| 26 | -0.0001 | -0. | 0. | 27 | 0. | -0. | -0. |
| 1001 | -0. | -0. | -0. | 1002 | -0. | -0. | -0. |

Table C-1 (Continued)

```
MOON ELEMENTS                                                                         M= 1

                  DEGREES        RADIANS

SMALL OMEGA       0.             0.            ) ECLIPTIC AND     SMALL E  0.
CAP   OMEGA       0.             0.            ) MEAN EQUINOX     SMALL A     60.0000000      ER
SMALL I           0.             0.            )   1950.0         SMALL N  0.55345017E 00 DEG/HR
G ZERO            0.             0.                               SMALL M  0.12294830E-01
SNP =    9.65952786867233D-03RAD/HR

         A               B               C              P              Q              R

    60.00000000      -0.              0.            1.00000000     -0.              0.          ) EQUATOR  AND
     0.              55.04621649    -23.87287211    0.              0.91743694    -0.39788120 ) MEAN EQUINOX
     0.              23.87287211     55.04621649    0.              0.39788120     0.91743694 )    1950.0

    60.00000000       0.              0.            1.00000000      0.              0.          ) P,Q,R,SYSTEM
     0.              60.00000000     -0.00000000    0.              1.00000000    -0.00000000 )      OF
     0.               0.00000000     60.00000000    0.              0.00000000     1.00000000 )  COORDINATES

     K ALPHA*1.E6    BETA*1.E6   GAMMA*1.E6      K ALPHA*1.E6   BETA*1.E6   GAMMA*1.E6       STORAGE

   1001      -0.          -0.          -0.      1002      -0.          -0.          -0.       1002     1003
SUN ELEMENTS                                                                          M= 2

                  DEGREES        RADIANS

SMALL OMEGA       0.             0.            ) ECLIPTIC AND     SMALL E  0.
CAP   OMEGA       0.             0.            ) MEAN EQUINOX     SMALL A  23454.8698730      ER
SMALL I           0.             0.            )   1950.0         SMALL N  0.41067035E-01 DEG/HR
G ZERO            0.             0.                               SMALL M  0.33295128E 06
SNP =    7.16754987789702 4D-04RAD/HR

         A               B               C              P              Q              R

 23454.86987305      -0.              0.            1.00000000     -0.              0.          ) EQUATOR  AND
     0.           21518.36425781  -9332.25183105    0.              0.91743694    -0.39788120 ) MEAN EQUINOX
     0.            9332.25183105  21518.36425781    0.              0.39788120     0.91743694 )    1950.0

 23454.86987305       0.              0.            1.00000000      0.              0.          ) P,Q,R,SYSTEM
     0.           23454.86987305     -0.00000036    0.              1.00000000    -0.00000000 )      OF
     0.               0.00000036  23454.86987305    0.              0.00000000     1.00000000 )  COORDINATES

     K ALPHA*1.E6    BETA*1.E6   GAMMA*1.E6      K ALPHA*1.E6   BETA*1.E6   GAMMA*1.E6       STORAGE

   1002      -0.          -0.          -0.      1003      -0.          -0.          -0.       1003     1004
```

Table C-1 (Continued)

VERY RESTRICTED 4 BODY PROB. HARMONIC ORBIT RUN NO. 10060 B

| K | ALPHA*1.E6 | BETA*1.E6 | GAMMA*1.E6 | K | ALPHA*1.E6 | BETA*1.E6 | GAMMA*1.E6 |
|---|---|---|---|---|---|---|---|
| 0 | -14539.8375 | 0. | -0.0000 | 1 | -763.3034 | 64896.2329 | -0.0000 |
| 2 | 4074.7054 | -1057.8167 | -0.0000 | 3 | -2232.4767 | -2576.3893 | 0.0000 |
| 4 | 1578.4518 | 7161.6935 | -0.0000 | 5 | 4811.4085 | -3271.5867 | -0.0000 |
| 6 | -1883.4308 | -4929.9221 | 0.0000 | 7 | -1931.6861 | 1554.9031 | 0.0000 |
| 8 | 507.4779 | 1267.4013 | -0.0000 | 9 | 356.4379 | -301.6413 | 0.0000 |
| 10 | -67.8380 | -172.0037 | 0.0000 | 11 | -41.2013 | 36.8060 | -0.0000 |
| 12 | 4.4837 | 12.9995 | 0.0000 | 13 | 4.3724 | -3.8666 | 0.0000 |
| 14 | 0.1851 | -0.0154 | -0.0000 | 15 | -0.6623 | 0.4549 | -0.0000 |
| 16 | -0.0935 | -0.1980 | 0.0000 | 17 | 0.1213 | -0.0468 | -0.0000 |
| 18 | 0.0042 | 0.0448 | 0.0000 | 19 | -0.0196 | -0.0019 | 0.0000 |
| 20 | 0.0048 | -0.0069 | -0.0000 | 21 | 0.0024 | 0.0029 | -0.0000 |
| 22 | -0.0021 | 0.0007 | 0.0000 | 23 | -0.0001 | -0.0009 | 0.0000 |
| 24 | 0.0005 | 0.0000 | -0.0000 | 25 | -0.0001 | 0.0002 | 0.0000 |
| 26 | -0.0001 | -0.0000 | -0.0000 | 27 | 0.0000 | -0.0000 | -0.0000 |

VERY RESTRICTED 4 BODY PROB. HARMONIC ORBIT RUN NO. 10060 B

VALUE OF SERIES AT EPOCH

| | A | B | C | A PRIME | B PRIME | C PRIME |
|---|---|---|---|---|---|---|
| VALUE OF SERIES AT X=-1 | -3.2589658528200D-03 | -5.8052690202150D-02 | -1.8562488918040D-14 | 8.5977341893630D-03 | 4.9929900722690D-03 | 3.9836324006430D-16 |
| VALUE OF SERIES AT X= 0 | -7.3032143742600D-03 | 1.4601602053610D-02 | -2.9826603993830D-14 | 1.3922181680130D-02 | 1.2437641781880D-02 | -9.1515167080130D-15 |
| VALUE OF SERIES AT X=+1 | -2.8529677795090D-03 | 6.2617044513760D-02 | -2.5781358409930D-14 | 8.6605571267400D-03 | 4.1809938600940D-03 | -3.3114033604830D-14 |

INITIAL CONDITIONS AT X= 0.

| A | B | C | A PRIME | B PRIME | C PRIME |
|---|---|---|---|---|---|
| 4.4117802677340D-03 | 1.6558426482370D-02 | 0. | 1.4736969849610D-02 | -5.0718106421600D-03 | 0. |

INTEGRATION CONSTANTS

| ZK1 | ZK2 | ZK3 | ZK4 | ZK5 | ZK6 |
|---|---|---|---|---|---|
| -7.6866679241770D-04 | 2.9820640039740D-04 | 5.9205368599440D-03 | 7.2724808980070D-04 | 6.9878560446180D-15 | 3.0406355121280D-14 |

Table C-1 (Continued)

VERY RESTRICTED 4 BODY PROB. HARMONIC ORBIT RUN NO. 10060 B

| K | ALPHA*1.E6 | BETA*1.E6 | GAMMA*1.E6 | K | ALPHA*1.E6 | BETA*1.E6 | GAMMA*1.E6 |
|---|---|---|---|---|---|---|---|
| 0 | 9233.8688 | 271.0929 | -0.0000 | 1 | -466.7498 | 4715.9670 | 0.0000 |
| 2 | 4193.3101 | -2590.7863 | -0.0000 | 3 | -2839.4208 | -2764.2263 | -0.0000 |
| 4 | 1530.9949 | 7775.0747 | 0.0000 | 5 | 4927.4893 | -3235.6624 | -0.0000 |
| 6 | -1877.9548 | -5000.6993 | 0.0000 | 7 | -1940.7497 | 1552.0981 | 0.0000 |
| 8 | 507.1676 | 1271.4117 | -0.0000 | 9 | 356.8288 | -301.5204 | 0.0000 |
| 10 | -67.8275 | -172.1400 | 0.0000 | 11 | -41.2120 | 36.8027 | -0.0000 |
| 12 | 4.4834 | 13.0026 | 0.0000 | 13 | 4.3726 | -3.8665 | 0.0000 |
| 14 | 0.1851 | -0.0154 | -0.0000 | 15 | -0.6623 | 0.4549 | -0.0000 |
| 16 | -0.0905 | -0.1980 | 0.0000 | 17 | 0.1213 | -0.0468 | -0.0000 |
| 18 | 0.0042 | 0.0448 | 0.0000 | 19 | -0.0196 | -0.0019 | 0.0000 |
| 20 | 0.0048 | -0.0069 | -0.0000 | 21 | 0.0024 | 0.0029 | -0.0000 |
| 22 | -0.0021 | 0.0007 | 0.0000 | 23 | -0.0001 | -0.0009 | 0.0000 |
| 24 | 0.0005 | 0.0000 | -0.0000 | 25 | -0.0001 | 0.0002 | 0.0000 |
| 26 | -0.0001 | -0.0000 | -0.0000 | 27 | 0.0000 | -0.0000 | -0.0000 |

VERY RESTRICTED 4 BODY PROB. HARMONIC ORBIT RUN NO. 10060 B

VALUE OF SERIES AT EPOCH

| | A | B | C | A PRIME | B PRIME | C PRIME |
|---|---|---|---|---|---|---|
| VALUE OF SERIES AT X=-1 | 8.9072104040880-03 | 1.4312345881080-03 | -4.5168517447400-14 | 7.8400508548390-03 | -1.3418825581650-02 | -1.5895678915910-14 |
| VALUE OF SERIES AT X= 0 | 4.4117802677340-03 | 1.6958426482370-02 | 3.4327219082030-28 | 1.4736969849610-02 | -5.0718106421600-03 | 2.8204422676310-27 |
| VALUE OF SERIES AT X=+1 | 8.9072101511380-03 | 1.4312354421590-03 | -5.6947468449280-14 | 7.8400502308010-03 | -1.3418825141230-02 | -3.4545867699580-14 |

Table C-2

Listing of INVT, Subroutines, and a Sample Case.

```
$JOB   1203P002 126 M32RK M3081C1234
*EXECUTE        IBJOB
$IBJOB          SOURCE,MAP,GO
$IBFTC INVT     DECK,M94,XR7
C
      DIMENSION CM(4,1),BM(4,4)
    1 FORMAT(4D18.1)
   23 FORMAT (1HL13X2HZ114X2HZ214X2HZ314X2HZ414X2HZ514X2HZ6/4X6P6F16.8)
   29 FORMAT (17HONORMAL EQUATIONS/1X)
   30 FORMAT(1P4E15.7,5X,1PE15.7)
   43 FORMAT(1H1)
   41 FORMAT(12A6)
      READ(5,1) ((BM(I,J),I=1,4),J=1,4)
      READ(5,1) (CM(I,1),I=1,4)
      WRITE (6,43)
      WRITE (6,41) ITITLE
      WRITE (6,29)
      WRITE(6,30) ((BM(I,J),J=1,4),CM(I,1),I=1,4)
      CALL MATINV(BM,4,CM,1,DETERM)
      Z1=CM(1,1)
      Z2=CM(2,1)
      Z3=CM(3,1)
      Z4=CM(4,1)
      WRITE (6,23) Z1,Z2,Z3,Z4,Z5,Z6
      RETURN
      END
$IBFTC MATINV  DECK,M94,XR7
      SUBROUTINE MATINV(A,N,B,M,DETERM)
C
      DIMENSION IPIVOT(4),A(4,4),B(4,1),INDEX(4,2),PIVOT(4)
C
C     INITIALIZATION
C
   10 DETERM=1.0
   15 DO 20 J=1,N
   20 IPIVOT(J)=0
   30 DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVOT(J)-1) 60,105,60
   60 DO 100 K=1,N
   70 IF (IPIVOT(K)-1) 80,100,740
   80 IF (ABS (AMAX)-ABS (A(J,K))) 85,100,100
   85 IROW=J
   90 ICOLUM=K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLUM) 140,260,140
  140 DETERM=-DETERM
  150 DO 200 L=1,N
```

Table C-2 (Continued)

```
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  205 IF(M) 260,260,210
  210 DO 250 L=1,M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
  310 PIVOT(I)=A(ICOLUM,ICOLUM)
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
  355 IF(M) 380,380,360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400,550,400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550,550,460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I
  620 IF (INDEX(L,1)-INDEX(L,2)) 630,710,630
  630 JROW=INDEX(L,1)
  640 JCOLUM=INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLUM)
  700 A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
      DO 800 I=1,N
      J=N+1-I
  800 DETERM=DETERM*PIVOT(J)
  740 RETURN
      END
```

Table C-2 (Continued)

```
$DATA
 7.6519783        D0 19.707211     D0 0.773883      D0 -12.07645     D0 A
 0.23356          D0 0.075781      D0 -0.297094     D0 -0.37297      D0 B
 1.091679         D0 -4.682355     D0 0.563231      D0 -1.72651      D0 A PRIME
 3.707652         D0 10.352294     D0 0.225023      D0 -5.93592      D0 B PRIME
 0.000530009    D-6 0.001245303  D-6 0.000355989  D-6 -0.0008261   D-6 CONST
```

```
00000

NORMAL EQUATIONS

 7.6519783E 00  2.3356000E-01  1.0916790E 00  3.7076520E 00        5.3000899E-10
 1.9707211E 01  7.5781000E-02 -4.6823550E 00  1.0352294E 01        1.2453030E-09
 7.7388299E-01 -2.9709400E-01  5.6323100E-01  2.2502300E-01        3.5598900E-10
-1.2076450E 01 -3.7297000E-01 -1.7265100E 00 -5.9359199E 00       -8.2609999E-10

            Z1            Z2            Z3            Z4            Z5            Z6
    0.00012539   -0.00081334    0.00006408   -0.00008347   -0.00000000   -0.00000000
```

Table C-3

Listing of CHVL, Subroutines, and a Sample Case.

```
$JOB   1203P003 126 M32RK M3081C1234
$EXECUTE         IBJOB
$IBJOB           SOURCE,MAP,GO
$IBFTC CHVL      DECK,M94,XR7
C
      DOUBLE PRECISION SA,SE,SO2,SO,SI2,SI,CO2,CO,SM,SGZ2,SGZ,PI,TWOPI,
     X   RAD,EK,EP2,EP,SB,TIME,SG,E,SINE,COSE,ESE,ECE,BK,  RVZ,RNUM,
     X   DELRHO,P,Q,R,SN2,SN,DSQRT,DCOS,DSIN,DELTAE,DELTAA,DELTAG,
     X   DELTAI,DELCO,DELSO,DELTAN,SANEW,SENEW,SONEW,SINEW,CONEW,
     X   SGZNEW,SNNEW ,A,B,C,DELTAT,TZERO,XJD,DTCHEB,SMINNR,EEK,TAU,
     X   EX,DEL1,DEL2,DEL3,ASTAR,BSTAR,GSTAR,DIFF
      DIMENSION XG(3,1001)
      DIMENSION A(3),B(3),C(3)
      DIMENSION H(6),         W(3),RV(3),RNUM(3, 501),DIFF(3,201,10)
      DIMENSION           ITITLE(12)
      DIMENSION AA(7),BB(7,7),BM(4,4),CM(4,1)
      DIMENSION   P(3),Q(3),R(3),        DELRHO(3),RVZ(3)
    1 FORMAT (4D18.1)
    2 FORMAT (26H0INITIAL CONDITION CHANGES/1P6E15.7)
    3 FORMAT (1X)
    4 FORMAT (10X3D20.10)
    5 FORMAT (I8,6P6F12.4)
    6 FORMAT (3F14.8)
    7 FORMAT (12H1DIFFERENCES)
    8 FORMAT (1PD19.10,10E11.2)
    9 FORMAT (/2(7X1HK1X11HALPHA*1.E102X10HBETA*1.E101X11HGAMMA*1.E10)//
   91)
   10 FORMAT (2(I8,6P3F12.4))
   14 FORMAT (1H015X5HALPHA7X5HBETA 7X5HGAMMA6X6HALPHA*6X6HBETA* 6X6HGAM
  141MA*7X2HDA10X2HDB10X2HDC)
   15 FORMAT (14I5)
   17 FORMAT (8HKDELTA TF12.7/6H TZEROF14.7/3H EPF17.7/8H JUL DAYF10.1,
  1718H DTCHEB F9.0)
   18 FORMAT (F10.1,6P9F12.4)
   19 FORMAT (21H0    JD     TAU,SG2,E25X5HRZERO9X1HW11X1HR13X4HRNUM11X5H
  191A,B,C6X8HA*,B*,C*4X8HDA,DB,DC/0PF10.1,F12.8,3F12.7,1PD18.10,6P3F12
  192.4)
   20 FORMAT (10X0PF12.4,3F12.7,1PD18.10,6P3F12.4)
   21 FORMAT (32HKNEW ELEMENTS FOR DISTURBED BODY)
   23 FORMAT (1HL13X2HZ114X2HZ214X2HZ314X2HZ414X2HZ514X2HZ6/4X6P6F16.8)
   24 FORMAT (22HL    CHANGE IN ELEMENTS//11X2HSA16X2HSE16X
     X   2HSO16X2HSI16X2HCO16X3HSGZ15X2HSN/7E18.8)
   27 FORMAT (24H0ROOT MEAN SQUARE ERRORS/1H08X5HALPHA10X5H BETA10X5HGAM
  271MA/6P6F15.6)
   28 FORMAT (6P6F12.6)
   29 FORMAT (17H0NORMAL EQUATIONS/1X)
   30 FORMAT (1P7E15.7)
   31 FORMAT (24H0MAXIMUM ABSOLUTE ERRORS/1H08X5HALPHA10X5H BETA10X5HGAM
  311MA/6P6F15.6)
   33 FORMAT (4D18.11)
   36 FORMAT (26H0NTIME IPRINT IELE ID IPRM)
   41 FORMAT(12A6)
   43 FORMAT(1H1)
   44 FORMAT (8HJSMALL A/D24.16)
   45 FORMAT (8HJSMALL E/D24.16)
   46 FORMAT (12HJSMALL OMEGA/D24.16)
   47 FORMAT (8HJSMALL I/D24.16)
   48 FORMAT (10HJCAP OMEGA/D24.16)
   49 FORMAT (8HJSMALL M/D24.16)
   50 FORMAT (7HJG ZERO/D24.16)
   51 FORMAT (8HJSMALL N/D24.16)
```

```
   52 FORMAT (1HJ10X1HP19X1HQ19X1HR19X1HA19X1HB19X1HC/(1P6D20.12))
   53 FORMAT (7HJSMINNR/D24.16)
      PI=3.141592653589793
      TWOPI=2.*PI
      RAD=TWOPI/360.
   55 CONTINUE
      DO 59 I=1,3
      DO 59 J=1,1001
   59 XG(I,J)=0.
      READ  (5,41) ITITLE
      READ  (5,15) NTIME,IPRINT,IELE,ID,IPRM
C
C     NTIME IS NUMBER OF TABULATED DATES
C     IPRINT CONTROLS PRINT
C     IELE = 0 FOR REGULAR RUN(2 ITERATIONS. , = NO. OF ITERATIONS FOR OTHER RUNS
C     ID GREATER THAN ZERO FOR TEST  OF COORDINATE DIFFERENCES
C     IPRM = 0 FOR SUN AS PRIMARY,  GREATER THAN ZERO FOR EARTH PRIMARY
C
      READ (5,33) SA,SE,SO2,SI2,CO2,SM,SGZ2,SMINNR
      READ (5,1) DELTAT,TZERO,XJD,DTCHEB
C     TZERO IS STARTING TIME FOR EVALUATION COUNTED IN DAYS FROM EPOCH
C     DELTAT IS STEP INTERVAL FOR EVALUATION
C     XJD IS JULIAN DATE OF TZERO
C     DTCHEB IS TOTAL INTERVAL OF VALIDITY OF SERIES IN DAYS
      READ (5,28) Z1,Z2,Z3,Z4,Z5,Z6
      SO=SO2*RAD
      SI=SI2*RAD
      CO=CO2*RAD
      SGZ=SGZ2*RAD
      EK=0.01720209895D+00
      EP2=23.4457888889
      EP=EP2*RAD
C     IF(IPRM)SUN,SUN,EARTH
      IF (IPRM) 72,72,70
   70 EK=4.461996918D+0
      EP=0.0
   72 CONTINUE
      SN=EK*DSQRT(1.+SMINNR+SM)/SA**1.5
      SN2=SN/RAD
      WRITE (6,43)
      WRITE (6,41)ITITLE
      WRITE (6,36)
      WRITE (6,15) NTIME,IPRINT,IELE,ID,IPRM
      WRITE (6,44)SA
      WRITE (6,45)SE
      WRITE (6,46)SO2
      WRITE (6,47)SI2
      WRITE (6,48)CO2
      WRITE (6,49)SM
      WRITE (6,50)SGZ2
      WRITE (6,51)SN2
      WRITE (6,53)SMINNR
      WRITE (6,17)DELTAT,TZERO,EP2,XJD,DTCHEB
      WRITE (6,23)Z1,Z2,Z3,Z4,Z5,Z6
      WRITE (6,43)
      WRITE (6,41) ITITLE
C
C     READ CHEBYCHEV SERIES PERTURBATION TERMS
C
      WRITE (6,9)
      NTRM=0
```

```
   90 READ  (5, 5) K,(H(I),I=1,6)
      KP=K+1
      WRITE (6,10) K,(H(I),I=1,3),KP,(H(I),I=4,6)
      IF (K-1001) 100,120,120
  100 K1=K+1
      KP=K+2
      NTRM=KP
      DO 110 I=1,3
      J=I+3
      XG(I,K1)=H(I)
      XG(I,KP)=H(J)
  110 CONTINUE
      GO TO 90
  120 CONTINUE
C
C  READ GENERAL PERTURBATIONS TERMS
C
C  READ COORDINATES
C
C  CARDS REMOVED TO MODIFY FOR RES. 3 BDY. PROB.
  160 CONTINUE
  165 CONTINUE
      ITRMAX=2
      IF (IELE) 184,184,182
  182 ITRMAX=IELE
  184 CONTINUE
C
C  ENTRY FOR COMPARISON
C
      ITRATN=0
  185 CONTINUE
      SB=SA*DSQRT(1.-SE**2)
C                                                                      PQRDP2
      CALL PQRDP2(SO,SI,CO,P,Q,R,EP)
      DO 186 I=1,3
      A(I)=SA*P(I)
      B(I)=SB*Q(I)
      C(I)=SA*R(I)
  186 CONTINUE
      WRITE (6,52) (P(I),Q(I),R(I),A(I),B(I),C(I),I=1,3)
      ASSIGN 322 TO NS1
  188 WRITE (6,43)
      WRITE (6,41) ITITLE
      ITRATN=ITRATN+1
      IF (IPRINT-1)192,192,190
  190 ASSIGN 323 TO NS1
      GO TO 194
  192 WRITE (6,14)
  194 CONTINUE
C
C     ZERO OUT BB(I,J).
C
      DO 196 I=1,7
      DO 196 J=1,7
  196 BB(I,J)=0.
      AAA=0.
      BBB=0.
      GGG=0.
      DAMX=0.
      DBMX=0.
      DGMX=0.
```

```
C
C     EQUALLY SPACED TIME INTERVALS DO LOOP.
C
      NTR2=NTRM+1
      NTR =NTRM-1
      EEK=2./DTCHEB
      DAY=XJD-DELTAT
      TIME=-DELTAT+TZERO
      DO 324 N=1,NTIME
      TIME=TIME+DELTAT
      TAU=EEK*TIME
      DAY=DAY+DELTAT
      SNT=SN*TIME
      SG=SGZ+SN*TIME
      SG2=SG/RAD
C
C     COMPUTE ECCENTRIC ANOMALY.
C
      E=SG
      SINE=DSIN(SG)
      COSE=DCOS(SG)
      SING=SINE
      COSG=COSE
      DO 198 M=1,3
      ESE=SE*SINE
      ECE=1.-SE*COSE
      BK=(SG-E+ESE)/ECE
      E=E+BK-.5*BK*BK*ESE/ECE
      COSE=DCOS(E)
      SINE=DSIN(E)
  198 CONTINUE
      COS2E=1.-2.*SINE**2
      SIN2E=2.*SINE*COSE
      C1=1./(1.-SE*COSE)
      E2=E/RAD
      ALPHA=0.
      BETA=0.
      GAMMA=0.
C
C  EVALUATE CHEBYCHEV SERIES
C
      EX=2.*TAU
      IF (NTR) 240,200,210
  200 ALPHA=.5*XG(1,1)
      BETA =.5*XG(2,1)
      GAMMA=.5*XG(3,1)
      GO TO 240
  210 CONTINUE
      DO 230 I=1,3
      B2=0.
      B1=0.
      DO 220 K=1,NTR
      I1=NTR2-K
      BZ=EX*B1-B2+XG(I,I1)
      B2=B1
      B1=BZ
  220 CONTINUE
      H(I)=.5*(EX*B1-2.*B2+XG(I,1))
  230 CONTINUE
      ALPHA=H(1)
      BETA =H(2)
```

```
      GAMMA=H(3)
  240 CONTINUE
C
C  EVALUATE GENERAL PERTURBATIONS SERIES
C
C
C     ADD IN CONSTANTS OF INTERGRATION.
C
      ALPHA=ALPHA+2.*Z3+Z1*(COSE-SE)+Z2*SINE
      BETA=BETA+Z4+Z1*(3.*SE*SNT-(2.-SE**2)*SINE+.5*SE*SIN2E)+
     X     Z2*(2.*COSE-.5*SE*COS2E)+Z3*(-3.*SNT)
      GAMMA=GAMMA+Z5*(COSE-SE)+Z6*SINE
C
C     COMPUTE PERTURBATION VECTOR.
C
      DO 295 I=1,3
      RVZ(I)=P(I)*SA*(COSE-SE)+Q(I)*SB*SINE
      W(I)=C1*(-SINE*A(I)+COSE*B(I))
      RV(I)=(1.+ALPHA)*RVZ(I)+BETA*W(I)+GAMMA*C(I)
  295 CONTINUE
C   CARDS HAVE REMOVED AND ADDED TO MODIFY FOR RES. 3 BDY. PROB
      ASTAR=4081.5946 E-6
      BSTAR=0.
      GSTAR=0.
C
C     COMPUTE BB(I,J).
C
      AA(1)=ALPHA-ASTAR
      AMA=AA(1)
      DAMX=AMAX1(DAMX,ABS(AMA))
      AAA=AAA+AMA*AMA
      AA(2)=COSE-SE
      AA(3)=SINE
      AA(4)=2.
      DO 310 J=1,4
      DO 310 I=J,4
  310 BB(I,J)=BB(I,J)+AA(I)*AA(J)
      AA(1)=BETA-BSTAR
      BMB=AA(1)
      DBMX=AMAX1(DBMX,ABS(BMB))
      BBB=BBB+BMB*BMB
      AA(2)=3.*SE*SNT-(2.-SE**2)*SINE+.5*SE*SIN2E
      AA(3)=2.*COSE-.5*SE*COS2E
      AA(4)=-3.*SNT
      AA(5)=1.
      DO 315 J=1,5
      DO 315 I=J,5
  315 BB(I,J)=BB(I,J)+AA(I)*AA(J)
      AA(1)=GAMMA-GSTAR
      GMG=AA(1)
      DGMX=AMAX1(DGMX,ABS(GMG))
      GGG=GGG+GMG*GMG
      AA(6)=COSE-SE
      AA(7)=SINE
      BB(1,1)=BB(1,1)+AA(1)*AA(1)
      BB(6,1)=BB(6,1)+AA(6)*AA(1)
      BB(7,1)=BB(7,1)+AA(7)*AA(1)
      DO 320 J=6,7
      DO 320 I=J,7
  320 BB(I,J)=BB(I,J)+AA(I)*AA(J)
      ATA=ALPHA
```

Table C-3 (Continued)

```
      BTA= BETA
      GTA=GAMMA
      ATS=ASTAR
      BTS=BSTAR
      GTS=GSTAR
      DAT=  AMA
      DBT=  BMB
      DGT=  GMG
      GO TO NS1,(322,323)
  322 WRITE (6,18) DAY,ATA,BTA,GTA,ATS,BTS,GTS,DAT,DBT,DGT
      GO TO 324
  323 CONTINUE
      WRITE (6,19) DAY,TAU,RVZ(1),W(1),RV(1),RNUM(1,N),ATA,ATS,DAT
      WRITE (6,20)     SG2,RVZ(2),W(2),RV(2),RNUM(2,N),BTA,BTS,DBT
      WRITE (6,20)      E2,RVZ(3),W(3),RV(3),RNUM(3,N),GTA,GTS,DGT
  324 CONTINUE
      DO 325 J=3,5
      IJ=J-1
      DO 325 I=2,IJ
  325 BB(I,J)=BB(J,I)
      BB(6,7)=BB(7,6)
      DO 330 I=1,4
      CM(I,1)=-BB(I+1,1)
      DO 330 J=1,4
  330 BM(I,J)=BB(I+1,J+1)
      WRITE (6,43)
      WRITE (6,41) ITITLE
      WRITE (6,29)
      WRITE (6,30)((BB(I,J),I=2,7),BB(J,1),J=2,7)
      CALL MATINV(BM,4,CM,1,DETERM)
      Z1=CM(1,1)                                                      +Z1
      Z2=CM(2,1)                                                      +Z2
      Z3=CM(3,1)                                                      +Z3
      Z4=CM(4,1)                                                      +Z4
      Z5=(BB(7,7)*BB(6,1)-BB(7,1)*BB(6,7))/(BB(7,6)*BB(6,7)
     X   -BB(7,7)*BB(6,6))                                            +Z5
      Z6=(BB(7,1)*BB(6,6)-BB(6,1)*BB(7,6))/(BB(6,7)*BB(7,6)
     X   -BB(7,7)*BB(6,6))                                            +Z6
      WRITE (6,23) Z1,Z2,Z3,Z4,Z5,Z6
C
C     COMPUTE CHANGE IN ELEMENTS.
C
      CON1=SQRT(1.-SE**2)
      DELTAP=Z6/CON1
      DELTAQ=-Z5
      DELTAR=-CON1*Z2/SE
      DELTAE=-(1.-SE**2)*Z1
      DELTAA=SA*(2.*Z3-2.*SE*Z1)
      DELTAG=Z4+(2.+SE**2)*Z2/(2.*SE)
      COSSO=COS(SO)
      SINSO=SIN(SO)
      DELTAI=DELTAP*COSSO-DELTAQ*SINSO
      COSSI=COS(SI)
      SINSI=SIN(SI)
      DELCO=(DELTAP*SINSO+DELTAQ*COSSO)/SINSI
      DELSO=DELTAR-COSSI*DELCO
      SANEW=SA+DELTAA
      SENEW=SE+DELTAE
      SONEW=SO+DELSO
      SINEW=SI+DELTAI
      CONEW=CO+DELCO
```

Table C-3 (Continued)

```
      SGZNEW=SGZ+DELTAG
      SNNEW=EK*DSQRT(1.+SMINNR+SM)/SANEW**1.5
      DELTAN=SNNEW-SN
      DELTAI=DELTAI/RAD
      DELSO=DELSO/RAD
      DELCO=DELCO/RAD
      DELTAG=DELTAG/RAD
      DELTAN=DELTAN/RAD
      SI2=SINEW/RAD
      SO2=SONEW/RAD
      CO2=CONEW/RAD
      SGZ2=SGZNEW/RAD
      SN2=SNNEW/RAD
      ENTIME=NTIME
      AAA=SQRT(AAA/ENTIME)
      BBB=SQRT(BBB/ENTIME)
      GGG=SQRT(GGG/ENTIME)
      WRITE (6,31) DAMX,DBMX,DGMX
      WRITE (6,27) AAA,BBB,GGG
      WRITE (6,24) DELTAA,DELTAE,DELSO,DELTAI,
     X      DELCO,DELTAG,DELTAN
      WRITE (6,21)
      WRITE (6,44)SANEW
      WRITE (6,45)SENEW
      WRITE (6,46)SO2
      WRITE (6,47)SI2
      WRITE (6,48)CO2
      WRITE (6,49)SM
      WRITE (6,50)SGZ2
      WRITE (6,51)SN2
      WRITE (6,53)SMINNR
C
C     COMPUTE CHANGES IN INITIAL CONDITIONS
C
      SG=SGZ
      E=SG
      SINE=DSIN(SG)
      COSE=DCOS(SG)
      DO 340 M=1,3
      ESE=SE*SINE
      ECE=1.-SE*COSE
      BK=(SG-E+ESE)/ECE
      E=E+BK-.5*BK*BK*ESE/ECE
      COSE=DCOS(E)
      SINE=DSIN(E)
  340 CONTINUE
      COS2E=1.-2.*SINE**2
      SIN2E=2.*SINE*COSE
      DA=2.*Z3+Z1*(COSE-SE)+Z2*SINE
      DG=       Z5*(COSE-SE)+Z6*SINE
      DB=Z4+Z1*(-(2.-SE*SE)*SINE+.5*SE*SIN2E)+Z2*(2.*COSE-.5*SE*COS2E)
      ADR=1./(1.-SE*COSE)
      DAP=ADR*(-Z1*SINE+Z2*COSE)
      DGP=ADR*(-Z5*SINE+Z6*COSE)
      DBP=3.*(SE*Z1-Z3)+ADR*((-(2.-SE*SE)*COSE+SE*COS2E)*Z1
     1                     +(- 2.        *SINE+SE*SIN2E)*Z2)
      WRITE (6,2) DA,DB,DG,DAP,DBP,DGP
      IF (ITRATN-ITRMAX) 188,55,55
      END
$IBFTC PQRDP2  DECK,M94,XR7
      SUBROUTINE PQRDP2(SO,SI,CO,P,Q,R,EP)
```

```
C      COMPUTE VECTORS P,Q,R
       DOUBLE PRECISION SO,SI,CO,P,Q,R,EP,EPC,EPS,CCO,SCO,CSO,SSO,
     X     CSI,SSI,TP1,TP2,DCOS,DSIN
       DIMENSION P(3),Q(3),R(3)
C EP IS OBLIQUITY OF ECLIPTIC
       EPC=DCOS(EP)
       EPS=DSIN(EP)
       CCO=DCOS(CO)
       SCO=DSIN(CO)
       CSO=DCOS(SO)
       SSO=DSIN(SO)
       CSI=DCOS(SI)
       SSI=DSIN(SI)
       P(1)=-CSI*SSO*SCO+CSO*CCO
       TP1 =+CSI*SSO*CCO+CSO*SCO
       TP2 =+SSI*SSO
       P(2)=EPC*TP1-EPS*TP2
       P(3)=EPS*TP1+EPC*TP2
       Q(1)=-CSI*CSO*SCO-SSO*CCO
       TP1 = CSI*CSO*CCO-SSO*SCO
       TP2 = SSI*CSO
       Q(2)=EPC*TP1-EPS*TP2
       Q(3)=EPS*TP1+EPC*TP2
       R(1)= SSI*SCO
       TP1 =-SSI*CCO
       TP2 = CSI
       R(2)=EPC*TP1-EPS*TP2
       R(3)=EPS*TP1+EPC*TP2
       RETURN
       END
$IBFTC MATINV  DECK,M94,XR7
       SUBROUTINE MATINV(A,N,B,M,DETERM)
C
       DIMENSION IPIVOT(4),A(4,4),B(4,1),INDEX(4,2),PIVOT(4)
C
C      INITIALIZATION
C
   10 DETERM=1.0
   15 DO 20 J=1,N
   20 IPIVOT(J)=0
   30 DO 550 I=1,N
C
C      SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVOT(J)-1) 60,105,60
   60 DO 100 K=1,N
   70 IF (IPIVOT(K)-1) 80,100,740
   80 IF (ABS (AMAX)-ABS (A(J,K))) 85,100,100
   85 IROW=J
   90 ICOLUM=K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C      INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLUM) 140,260,140
  140 DETERM=-DETERM
```

Table C-3 (Continued)

```
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  205 IF(M) 260,260,210
  210 DO 250 L=1,M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
  310 PIVOT(I)=A(ICOLUM,ICOLUM)
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
  355 IF(M) 380,380,360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400,550,400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550,550,460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I
  620 IF (INDEX(L,1)-INDEX(L,2)) 630,710,630
  630 JROW=INDEX(L,1)
  640 JCOLUM=INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLUM)
  700 A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
      DO 800 I=1,N
      J=N+1-I
  800 DETERM=DETERM*PIVOT(J)
  740 RETURN
      END
```

## Table C-3 (Continued)

```
$DATA
 VERY RES 4 BDY PROB
   51     1     0     0     1
 59.756099826 D+00 0.0            D+00      0.0       D+00 23.445788888 D+00
  0.0         D+00 0.0            D+00     60.0       D+00  0.0         D+00
14.051984526048D+0-351.2996131512D+0-351.2996131512D+0702.59922630241D+0

        0   9233.8688    271.0929    -0.0000   -466.7498   4715.9670    0.0000
        2   4193.3101  -2590.7863    -0.0000  -2839.4208  -2764.2263   -0.0000
        4   1530.9949   7775.0747     0.0000   4927.4893  -3235.6624   -0.0000
        6  -1877.9548  -5000.6993     0.0000  -1940.7497   1552.0981    0.0000
        8    507.1676   1271.4117    -0.0000    356.8288   -301.5204    0.0000
       10    -67.8275   -172.1400     0.0000    -41.2120     36.8027   -0.0000
       12      4.4834     13.0026     0.0000      4.3726     -3.8665    0.0000
       14      0.1851     -0.0154    -0.0000     -0.6623      0.4549   -0.0000
       16     -0.0905     -0.1980     0.0000      0.1213     -0.0468   -0.0000
       18      0.0042      0.0448     0.0000     -0.0196     -0.0019    0.0000
       20      0.0048     -0.0069    -0.0000      0.0024      0.0029   -0.0000
       22     -0.0021      0.0007     0.0000     -0.0001     -0.0009    0.0000
       24      0.0005      0.0000    -0.0000     -0.0001      0.0002    0.0000
       26     -0.0001     -0.0000    -0.0000      0.0000     -0.0000   -0.0000
     1001
```

Table C-3 (Continued)

```
VERY RES 4 BDY PROB

NTIME IPRINT IELE ID IPRM
  51    1     0    0   1

SMALL A
 0.59756099825999999D 02

SMALL E
 0.

SMALL OMEGA
 0.

SMALL I
 0.23445788888000000D 02

CAP OMEGA
 0.

SMALL M
 0.

G ZERO
 0.60000000000000000D 02

SMALL N
 0.55345017896989960D 00

SMINNR
 0.

DELTA T  14.0519844
TZERO  -351.2996101
EP       23.4457889
JUL DAY    -351.3 DTCHEB      703.

            Z1           Z2           Z3           Z4           Z5           Z6
      -0.          -0.          -0.          -0.          -0.          -0.
```

Table C-3 (Continued)

VERY RES 4 BDY PROB

| K | ALPHA*1.E10 | BETA*1.E10 | GAMMA*1.E10 | K | ALPHA*1.E10 | BETA*1.E10 | GAMMA*1.E10 |
|---|---|---|---|---|---|---|---|
| 0 | 9233.8688 | 271.0929 | -0. | 1 | -466.7498 | 4715.9670 | 0. |
| 2 | 4193.3100 | -2590.7863 | -0. | 3 | -2839.4208 | -2764.2263 | -0. |
| 4 | 1530.9949 | 7775.0746 | 0. | 5 | 4927.4893 | -3235.6624 | -0. |
| 6 | -1877.9548 | -5000.6992 | 0. | 7 | -1940.7497 | 1552.0981 | 0. |
| 8 | 507.1676 | 1271.4117 | -0. | 9 | 356.8288 | -301.5204 | 0. |
| 10 | -67.8275 | -172.1400 | 0. | 11 | -41.2120 | 36.8027 | -0. |
| 12 | 4.4834 | 13.0026 | 0. | 13 | 4.3726 | -3.8665 | 0. |
| 14 | 0.1851 | -0.0154 | -0. | 15 | -0.6623 | 0.4549 | -0. |
| 16 | -0.0905 | -0.1980 | 0. | 17 | 0.1213 | -0.0468 | -0. |
| 18 | 0.0042 | 0.0448 | 0. | 19 | -0.0196 | -0.0019 | 0. |
| 20 | 0.0048 | -0.0069 | -0. | 21 | 0.0024 | 0.0029 | -0. |
| 22 | -0.0021 | 0.0007 | 0. | 23 | -0.0001 | -0.0009 | 0. |
| 24 | 0.0005 | 0. | -0. | 25 | -0.0001 | 0.0002 | 0. |
| 26 | -0.0001 | -0. | -0. | 27 | 0. | -0. | -0. |
| 1001 | -0. | -0. | -0. | 1002 | -0. | -0. | -0. |

| P | Q | R | A | B | C |
|---|---|---|---|---|---|
| 1.000000000000D 00 | -0. | 0. | 5.975609982600D 01 | -0. | 0. |
| 0. | 9.174369452164D-01 | -3.978812028131D-01 | 0. | 5.482245368241D 01 | -2.377582887419D 01 |
| 0. | 3.978812028131D-01 | 9.174369452164D-01 | 0. | 2.377582887419D 01 | 5.482245368241D 01 |

Table C-3 (Continued)

VERY RES 4 BDY PROB

| | ALPHA | BETA | GAMMA | ALPHA. | BETA. | GAMMA. | DA | DB | DC |
|---|---|---|---|---|---|---|---|---|---|
| -351.3 | 8907.2100 | 1431.2343 | -0. | 4081.5946 | 0. | 0. | 4825.6154 | 1431.2343 | -0. |
| -337.2 | 9828.8690 | -583.8700 | -0. | 4081.5946 | 0. | 0. | 5747.2744 | -583.8700 | -0. |
| -323.2 | 10414.1664 | -2913.4038 | -0. | 4081.5946 | 0. | 0. | 6332.5719 | -2913.4038 | -0. |
| -309.1 | 10601.7914 | -5429.6097 | -0. | 4081.5946 | 0. | 0. | 6520.1968 | -5429.6097 | -0. |
| -295.1 | 10354.6448 | -7986.6226 | -0. | 4081.5946 | 0. | 0. | 6273.0502 | -7986.6226 | -0. |
| -281.0 | 9662.6432 | -10429.3237 | -0. | 4081.5946 | 0. | 0. | 5581.0486 | -10429.3237 | -0. |
| -267.0 | 8543.7819 | -12602.8796 | -0. | 4081.5946 | 0. | 0. | 4462.1873 | -12602.8796 | -0. |
| -252.9 | 7043.3772 | -14362.3915 | -0. | 4081.5946 | 0. | 0. | 2961.7826 | -14362.3915 | -0. |
| -238.9 | 5231.5306 | -15582.0791 | -0. | 4081.5946 | 0. | 0. | 1149.9361 | -15582.0791 | -0. |
| -224.8 | 3198.9886 | -16163.4325 | -0. | 4081.5946 | 0. | 0. | -882.6060 | -16163.4325 | -0. |
| -210.8 | 1051.6794 | -16041.8275 | -0. | 4081.5946 | 0. | 0. | -3029.9152 | -16041.8275 | -0. |
| -196.7 | -1095.7040 | -15191.1671 | -0. | 4081.5946 | 0. | 0. | -5177.2985 | -15191.1671 | -0. |
| -182.7 | -3126.6482 | -13626.2252 | -0. | 4081.5946 | 0. | 0. | -7208.2427 | -13626.2252 | -0. |
| -168.6 | -4929.8422 | -11402.5062 | -0. | 4081.5946 | 0. | 0. | -9011.4368 | -11402.5062 | -0. |
| -154.6 | -6405.8409 | -8613.5691 | -0. | 4081.5946 | 0. | 0. | -10487.4354 | -8613.5691 | -0. |
| -140.5 | -7472.9694 | -5385.9485 | -0. | 4081.5946 | 0. | 0. | -11554.5640 | -5385.9485 | -0. |
| -126.5 | -8072.1340 | -1871.9507 | -0. | 4081.5946 | 0. | 0. | -12153.7285 | -1871.9507 | -0. |
| -112.4 | -8170.2771 | 1759.2257 | -0. | 4081.5946 | 0. | 0. | -12251.8716 | 1759.2257 | -0. |
| -98.4 | -7762.2983 | 5331.4853 | -0. | 4081.5946 | 0. | 0. | -11843.8928 | 5331.4853 | -0. |
| -84.3 | -6871.3475 | 8672.2853 | -0. | 4081.5946 | 0. | 0. | -10952.9420 | 8672.2853 | -0. |
| -70.3 | -5547.4689 | 11622.8514 | -0. | 4081.5946 | 0. | 0. | -9629.0634 | 11622.8514 | -0. |
| -56.2 | -3864.6728 | 14047.4832 | -0. | 4081.5946 | 0. | 0. | -7946.2673 | 14047.4832 | -0. |
| -42.2 | -1916.5740 | 15841.3521 | -0. | 4081.5946 | 0. | 0. | -5998.1685 | 15841.3521 | -0. |
| -28.1 | 189.1622 | 16936.3000 | -0. | 4081.5946 | 0. | 0. | -3892.4324 | 16936.3000 | -0. |
| -14.1 | 2337.2603 | 17304.3032 | -0. | 4081.5946 | 0. | 0. | -1744.3343 | 17304.3032 | -0. |

UNDRFLOW AT 04321 IN MQ

UNDRFLOW AT 04321 IN AC AND MQ.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 4411.7801 | 16958.4253 | -0. | 4081.5946 | 0. | 0. | 330.1855 | 16958.4253 | -0. |
| 14.1 | 6303.1393 | 15951.2462 | -0. | 4081.5946 | 0. | 0. | 2221.5447 | 15951.2462 | -0. |
| 28.1 | 7914.7635 | 14370.9156 | -0. | 4081.5946 | 0. | 0. | 3833.1689 | 14370.9156 | -0. |
| 42.2 | 9168.9255 | 12335.1232 | -0. | 4081.5946 | 0. | 0. | 5087.3310 | 12335.1232 | -0. |
| 56.2 | 10011.3802 | 9983.3750 | -0. | 4081.5946 | 0. | 0. | 5929.7857 | 9983.3750 | -0. |
| 70.3 | 10414.4795 | 7468.0771 | -0. | 4081.5946 | 0. | 0. | 6332.8849 | 7468.0771 | -0. |
| 84.3 | 10378.5580 | 4944.9709 | -0. | 4081.5946 | 0. | 0. | 6296.9634 | 4944.9709 | -0. |
| 98.4 | 9931.4973 | 2563.5008 | -0. | 4081.5946 | 0. | 0. | 5849.9028 | 2563.5008 | -0. |
| 112.4 | 9126.5093 | 457.6874 | -0. | 4081.5946 | 0. | 0. | 5044.9147 | 457.6874 | -0. |
| 126.5 | 8038.2999 | -1261.9449 | -0. | 4081.5946 | 0. | 0. | 3956.7054 | -1261.9449 | -0. |
| 140.5 | 6757.8859 | -2514.9038 | -0. | 4081.5946 | 0. | 0. | 2676.2914 | -2514.9038 | -0. |
| 154.6 | 5386.4245 | -3255.3340 | -0. | 4081.5946 | 0. | 0. | 1304.8300 | -3255.3340 | -0. |
| 168.6 | 4028.4736 | -3474.4025 | -0. | 4081.5946 | 0. | 0. | -53.1210 | -3474.4025 | -0. |
| 182.7 | 2785.1252 | -3200.3128 | -0. | 4081.5946 | 0. | 0. | -1296.4693 | -3200.3128 | -0. |
| 196.7 | 1747.4498 | -2495.9347 | -0. | 4081.5946 | 0. | 0. | -2334.1448 | -2495.9347 | -0. |
| 210.8 | 990.6554 | -1454.1897 | -0. | 4081.5946 | 0. | 0. | -3090.9392 | -1454.1897 | -0. |
| 224.8 | 569.3068 | -191.4781 | -0. | 4081.5946 | 0. | 0. | -3512.2878 | -191.4781 | -0. |
| 238.9 | 513.8798 | 1160.4330 | -0. | 4081.5946 | 0. | 0. | -3567.7148 | 1160.4330 | -0. |
| 252.9 | 828.8414 | 2463.5352 | -0. | 4081.5946 | 0. | 0. | -3252.7531 | 2463.5352 | -0. |
| 267.0 | 1492.3617 | 3583.1169 | -0. | 4081.5946 | 0. | 0. | -2589.2328 | 3583.1169 | -0. |
| 281.0 | 2457.6724 | 4397.0856 | -0. | 4081.5946 | 0. | 0. | -1623.9222 | 4397.0856 | -0. |
| 295.1 | 3655.9992 | 4804.4601 | -0. | 4081.5946 | 0. | 0. | -425.5954 | 4804.4601 | -0. |
| 309.1 | 5000.9090 | 4732.4773 | -0. | 4081.5946 | 0. | 0. | 919.3144 | 4732.4773 | -0. |
| 323.2 | 6393.8224 | 4141.8582 | -0. | 4081.5946 | 0. | 0. | 2312.2279 | 4141.8582 | -0. |
| 337.2 | 7730.3699 | 3029.9048 | -0. | 4081.5946 | 0. | 0. | 3648.7754 | 3029.9048 | -0. |
| 351.3 | 8907.2097 | 1431.2359 | -0. | 4081.5946 | 0. | 0. | 4825.6151 | 1431.2359 | -0. |

Table C-3 (Continued)

```
VERY RES 4 BDY PROB

NORMAL EQUATIONS

 1.3080690E 02 -5.7277803E 00  1.2684667E 02  8.0260466E 00  0.             0.            -5.7491702E-01
-5.7277803E 00  1.2419303E 02  2.1970499E 02 -4.6338440E 00  0.             0.             5.0715097E-02
 1.2684667E 02  2.1970499E 02  2.0362799E 03 -2.0861626E-05  0.             0.            -1.2233527E 00
 8.0260466E 00 -4.6338440E 00 -2.0861626E-05  5.1000000E 01  0.             0.             3.1688653E-02
 0.             0.             0.             0.             2.4397685E 01  1.9092602E 00  0.
 0.             0.             0.             0.             1.9092602E 00  2.6602307E 01  0.

           Z1             Z2             Z3             Z4             Z5             Z6
     3969.79125977  -1113.39509583    473.61677551  -1347.24845886     -0.            -0.

MAXIMUM ABSOLUTE ERRORS

        ALPHA          BETA          GAMMA
  12251.871582   17304.303223       0.

ROOT MEAN SQUARE ERRORS

        ALPHA          BETA          GAMMA
   5891.728271    9413.185303       0.

   CHANGE IN ELEMENTS

          SA                SE                SO                SI                CO                SGZ               SN
   0.56602982E-01   -0.39697912E-02   0.63792840E-01   -0.               -0.                -0.20477733E-00   -0.78543980E-03

NEW ELEMENTS FOR DISTURBED BODY

SMALL A
 0.5981270280865686D 02

SMALL E
-0.3969791287090629D-02

SMALL OMEGA
 0.6379284043366512D-01

SMALL I
 0.2344578888799999D 02

CAP OMEGA
 0.

SMALL M
 0.

G ZERO
 0.5979522266775598D 02

SMALL N
 0.5526647391691009D 00

SMINNR
 0.

INITIAL CONDITION CHANGES
 1.9679007E-03 -9.3365237E-03 -0.            -3.9946376E-03 -3.4621847E-03  0.
```