AD

RSIC-628

KIRCHHOFF'S THEOREM IN THE ACOUSTICS OF A MOVING MEDIUM

by

L. A. Chernov

Trudy Komissii Po Akustike AN SSSR, 5, 10-22 (1950)

Translated from the Russian

January 1967

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

REDSTONE SCIENTIFIC INFORMATION CENTER
REDSTONE ARSENAL, ALABAMA

JOINTLY SUPPORTED BY

U.S. ARMY MISSILE COMMAND

GEORGE C. MARSHALL SPACE FLIGHT CENTER

Disclaimer

The findings of this report are not to be construed as an official Department of the Army position.

Disposition Instruction

Destroy this report when it is no longer needed. Do not return it to the originator.

5 January 1967 RSIC-628

KIRCHHOFF'S THEOREM IN THE ACOUSTICS OF A MOVING MEDIUM

by

L. A. Chernov

Trudy Komissii Po Akustike AN SSSR, 5, 10-22 (1950)

Translated from the Russian

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Translation Branch
Redstone Scientific Information Center
Research and Development Directorate
U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Kirchhoff's theorem is finding diverse applications in acoustics and optics. Its generalization for the case of a moving medium has been given by D. I. Blokhintsev[1] who limited himself to an examination of a uniform flow. This paper gives a further generalization of this theorem for the case of a potential movement of a medium. The application of the theorem is illustrated with the solution of a problem on the point source of sound, which is in the flow.

1. Derivation of the Auxiliary Lemma

The wave equation of acoustics for the quasisteady potential flow was for the first time given by N. N. Andreyev[2] and somewhat refined by D. I. Blokhintsev[3]. In the refined form, it is as follows:

$$\left(\delta_{\mu\nu} - \frac{u_\mu u_\nu}{c^2}\right) \frac{\partial^2\varphi}{\partial x_\mu \partial x_\nu} - \frac{2u_\mu}{c^2} \frac{\partial^2\varphi}{\partial x_\mu \partial t} - \frac{1}{c^2} \frac{\partial^2\varphi}{\partial t^2}$$

$$+ \left(\frac{\partial \ln \rho}{\partial x_\mu} - \frac{u_\nu}{c^2} \frac{\partial u_\mu}{\partial x_\nu} + \frac{u_\mu u_\nu}{c^2} \frac{\partial \ln c^2}{\partial x_\nu}\right) \frac{\partial\varphi}{\partial x_\mu}$$

$$+ \frac{u_\nu}{c^2} \frac{\partial \ln c^2}{\partial x_\nu} \frac{\partial\varphi}{\partial t} = \frac{1}{c^2} \frac{\partial\Phi}{\partial t} + \frac{u_\mu}{c^2} \frac{\partial\Phi}{\partial x_\mu} - \frac{u_\mu}{c^2} \frac{\partial \ln c^2}{\partial x_\mu} \Phi, \tag{1}$$

where $\delta_{\mu\nu}$ = single matrix

u_μ = speed component along the x_μ axis

ρ = density of the medium

φ = potential of the acoustic speed

Φ = potential of the body force

Here and always henceforth, it is understood that we deal with the summation with respect to two Greek subscripts from 1 to 3.

Equation (1) is approximately valid also for a weakly eddied flow when the dimensionless eddy is small in comparison with the dimensionless speed, i.e.,

$$\frac{|\mathrm{rot}\ u|}{\omega} \ll \frac{u}{c},$$

where ω is the frequency of the sound.

Let us assume that a certain auxiliary function, χ, also satisfies the equation (1), but without the right-hand part:

$$\left(\delta_{\mu\nu} - \frac{u_\mu u_\nu}{c^2}\right) \frac{\partial^2 \chi}{\partial x_\mu \partial x_\nu} - \frac{2u_\mu}{c^2} \frac{\partial^2 \chi}{\partial x_\mu \partial t} - \frac{1}{c^2} \frac{\partial^2 \chi}{\partial t^2}$$

$$+\left(\frac{\partial \ln \rho}{\partial x_\mu} - \frac{u_\nu}{c^2} \frac{\partial u_\mu}{\partial x_\nu} + \frac{u_\mu u_\nu}{c^2} \frac{\partial \ln c^2}{\partial x_\nu}\right) \frac{\partial \chi}{\partial x_\mu} + \frac{u_\nu}{c^2} \frac{\partial \ln c^2}{\partial x_\nu} \cdot \frac{\partial \chi}{\partial t} = 0 \tag{2}$$

Let us prove the lemma; if s is a random closed surface which limits a certain volume, ν, while t_1 and t_2 are two random moments of time, then the following relationship takes place:

$$\oint_s \int_{t_1}^{t_2} \rho \left\{ \left(\delta_{\mu\nu} - \frac{u_\mu u_\nu}{c^2}\right) \left(\chi \frac{\partial \varphi}{\partial x_\nu} - \varphi \frac{\partial \chi}{\partial x_\nu}\right) - \frac{u_\mu}{c^2} \left(\chi \frac{\partial \varphi}{\partial t}\right.\right.$$

$$\left.\left. - \varphi \frac{\partial \chi}{\partial t}\right) \right\} \cos(\bar{n}, x_\mu) dt ds - \int_\nu \left[\frac{\rho u_\mu}{c^2} \left(\chi \frac{\partial \varphi}{\partial x_\mu}\right.\right.$$

$$\left.\left. - \varphi \frac{\partial \chi}{\partial x_\mu}\right) + \frac{\rho}{c^2} \left(\chi \frac{\partial \varphi}{\partial t} - \varphi \frac{\partial \chi}{\partial t}\right)\right]_{t_1}^{t_2} d\nu$$

$$= \iint_{\nu \ t_1}^{t_2} \rho \chi \left(\frac{1}{c^2} \frac{\partial \Phi}{\partial t} + \frac{u_\mu}{c^2} \frac{\partial \Phi}{\partial x_\mu}\right.$$

$$\left. - \frac{u_\mu}{c^2} \frac{\partial \ln c^2}{\partial x_\mu} \Phi\right) dt d\nu. \tag{3}$$

We wish to note, first of all, that equation (1) can be written in a more abbreviated four-dimensional form:

$$\left(\delta_{\mu\nu} - \frac{u_i u_k}{c^2}\right) \frac{\partial^2 \varphi}{\partial x_i \partial x_k} + \left(\frac{\partial \ln \rho}{\partial x_i} - \frac{u_k}{c^2} \frac{\partial u_i}{\partial x_k} + \frac{u_i u_k}{c^2} \frac{\partial \ln c^2}{\partial x_k}\right) \frac{\partial \varphi}{\partial x_i} = F, \tag{4}$$

where the twice repeating Latin subscripts indicate summation from 0 to 3, x_0 indicates the time, t, while the "time component of the speed," u_0, is equal to unity. δ_{ik} differs from the single matrix: $\delta_{ik} = 0$ not only when $i \neq k$, but also when $i = k = 0$ and $\delta_{ik} = 1$ when $i = k \neq 0$. F is the abbreviated designation of the right-hand part of Equation (1).

The proof of the lemma is based on the well-known properties[4] of self-conjugate operators. The operator in the left-hand part of Equation (4) will not be self-conjugate, but it can be made such if the equation is multiplied by ρ (density of the medium). In multiplying Equation (4) by ρ, we introduce the designations:

$$A_{ik} = \rho \left(\delta_{ik} - \frac{u_i u_k}{c^2}\right), \tag{5}$$

$$B_i = \rho\left(\frac{\partial \ln \rho}{\partial x_t} - \frac{u_k}{c^2} \frac{\partial u_i}{\partial x_k} + \frac{u_i u_k}{c^2} \frac{\partial \ln c^2}{\partial x_k}\right). \tag{6}$$

Then it will assume the form:

$$A_{ik} \frac{\partial^2 \varphi}{\partial x_i \partial x_k} + B_i \frac{\partial \varphi}{\partial x_i} = \rho F. \tag{7}$$

If we designate by L the operator in the left-hand part of the last equation, then it is written as follows:

$$L(\varphi) = \rho F. \tag{8}$$

Accordingly, χ will satisfy the equation:

$$L(\chi) = 0. \tag{9}$$

The condition of self-conjugation of the operator, L, is written as follows:

$$\frac{\partial A_{ik}}{\partial x_k} = B_i.$$

It is easy to see that this condition is satisfied in our case. Actually, by differentiating (5) with respect to x_k, we get:

$$\frac{\partial A_{ik}}{\partial x_k} = \frac{\partial}{\partial x_k}\left[\rho\left(\delta_{ik} - \frac{u_i \nu_k}{c^2}\right)\right] = \frac{\partial \rho}{\partial x_k} - \rho\,\frac{u_k}{c^2}\,\frac{\partial u_i}{\partial x_k}$$

$$+ \rho\,\frac{u_i u_k}{c^4}\,\frac{\partial c^2}{\partial x_k} - \frac{u_i}{c^2}\,\frac{\partial}{\partial x_k}\,(\rho u_k).$$

The last member in this equality is converted into zero as a result of the continuity equation which, for the quasisteady flow, has the form of:

$$\frac{\partial\,(\rho u_k)}{\partial x_k} = 0.$$

The first member can be rewritten in the form of $\rho\,\frac{\partial \ln \rho}{\partial x_k}$ and the third in the form of $\rho\,\frac{u_i u_k}{c^2}\,\frac{\partial \ln c^2}{\partial x_k}$, after which the right-hand part of the last equality, as can be seen from a comparison with (6), coincides with B_i.

The self-conjugate operator has the characteristic in that the expression $\chi L(\varphi) - \varphi L(\chi)$, where φ and χ are random functions, represents the divergence of a certain vector C_i, i.e.,

$$\chi L(\varphi) - \varphi L(\chi) = \frac{\partial c_i}{\partial x_i}, \qquad (10)$$

besides, the vector, C_i, in our designations has the following form:

$$C_i = A_{ik}\left(\chi \frac{\partial \varphi}{\partial x_k} - \varphi \frac{\partial \chi}{\partial x_k}\right).$$

Let us now assume that the functions φ and χ satisfy, respectively, Equations (8) and (9). Then, we get from (10):

$$\frac{\partial c_i}{\partial x_i} = \chi \rho F$$

or, in the three-dimensional form,

$$\frac{\partial c_\mu}{\partial x_\mu} + \frac{\partial c_0}{\partial t} = \chi \rho F. \tag{11}$$

By integrating the last equation with respect to time within the limits of t_1 to t_2 and then with respect to volume, ν, and by using the Gauss theorem, we get:

$$\oint_s \int_{t_1}^{t_2} c_\mu \cos\left(\overline{n},\ x_\mu\right) dtds + \int_\nu \left[c_0\right]_{t_1}^{t_2} d\nu = \int_\nu \int_{t_1}^{t_2} \chi \rho F dt d\nu.$$

2. Selection of the Auxiliary Function

Lorenz[5], in deriving the Kirchhoff theorem from the ordinary wave equation, selects the auxiliary function, χ, in the form of a brief pulse which converges to the observation point, P.

It is easy to show that in our case the auxiliary function, χ, which satisfies Equation (2), can also be selected in the form of a converging pulse.

The usual wave equation is symmetric with respect to time. For this reason, by replacing in the solution the time, t, by -t, we also get the solution.

The wave Equation (2) of the acoustics of a moving medium does not possess this symmetry with respect to time. It is disturbed by the members $-\frac{2u_\mu}{c^2}\frac{\partial^2\chi}{\partial x_\mu \partial t}$ and $\frac{u_\nu}{c^2}\frac{\partial \ln c^2}{\partial x_\nu}\frac{\partial\chi}{\partial t}$ which change their sign with the replacement of t by -t. If, for this reason, $\chi(x, y, z, t)$ is the solution of Equation (2), then $\chi(x, y, z, -t)$ will not be its solution. It is easy, however, to see that the function $\chi(x, y, z, -t)$ will satisfy Equation (2), if the direction of the speed of motion of the medium in all the points of the space is changed to the opposite, i.e., if we replace u_μ by $-u_\mu$ in Equation (2). Actually, the members which are nonsymmetric with respect to time will be proportional to the components, u_μ, of the speed of the medium, whereas, the remaining members will either not depend at all on the speeds, u_μ, or will be their quadratic functions. For this reason, the replacement of u_μ by $-u_\mu$ in Equation (2) will lead to a change in the signs only in the members which are nonsymmetric with respect to time. If we now introduce a new variable, $t_1 = -t$, then the nonsymmetric members will once more change their sign and the wave equation will again assume its initial form. Its solution will be the function:

$$\chi(x, y, z, t_1) = \chi(x, y, z, -t).$$

And so, if the function $\chi(x, y, z, t)$ is the solution of the wave Equation (2) in the field of speeds, u_μ, then the function $\chi(x, y, z, -t)$ will be the solution of the same equation in the field of the reverse speeds, $-u_\mu$.

Let us assume now that the function $\chi(x, y, z, -t)$ describes the brief acoustic pulse in the field of reverse speeds, which emanates from the point, P, at the moment of $t = 0$. Then the function $\chi(x, y, z, t)$ will describe the acoustic pulse in the field of speeds, u_μ, which converges into the point, P, at the moment $t = 0$.

Thus, the auxiliary function $\chi(x, y, z, t)$ will always be selected in the form of a pulse which converges to the point P and disappears in it (and consequently also in the entire space) at a definite moment (inasmuch as the start of time counting can be selected at random).

Let us assume that the converging pulse disappears at the moment $t = 0$. If in the relationship (3) the upper limit $t_2 > 0$, then the value of the expression within the rectangular brackets for the upper limit under the sign of the second integral will be equal to zero.

In order to convert to zero the value of this expression for the lower limit, it is sufficient to assume that the acoustic field did not exist forever. If it originated at the moment $t_0 < 0$, then the lower limit should be so selected as to fulfill the inequality $t_1 < t_0$. Then the relationship (3) assumes the form:

$$\oint_s \int_{t_1}^{t_2} \rho\left\{\left(\delta_{\mu\nu} - \frac{u_\mu u_\nu}{c^2}\right)\left(\chi \frac{\partial\varphi}{\partial x_\nu} - \varphi \frac{\partial\chi}{\partial x_\nu}\right) - \frac{u_\mu}{c^2}\left(\chi \frac{\partial\varphi}{\partial t}\right.\right.$$

$$\left.\left. - \varphi \frac{\partial\chi}{\partial t}\right)\right\} \cos(\bar{n}, x_\mu) dtds = \int_\nu \int_{t_1}^{t_2} \rho\chi\left(\frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right.$$

$$\left. + \frac{u_\mu}{c^2}\frac{\partial\Phi}{\partial x_\mu} - \frac{u_\mu}{c^2}\frac{\partial \ln c^2}{\partial x_\mu}\Phi\right) dtds. \qquad (12)$$

By introducing the normal component of the speed,

$u_n = u_\mu \cos(\bar{n}, x_\mu)$,

we rewrite the relationship (12) in the following form:

$$\oint_s \int_{t_1}^{t_2} \rho\left\{\left(\chi \frac{\partial\varphi}{\partial n} - \varphi \frac{\partial\chi}{\partial n}\right) - \frac{u_n u_\nu}{c^2}\left(\chi \frac{\partial\varphi}{\partial x_\nu} - \varphi \frac{\partial\chi}{\partial x_\nu}\right) - \frac{u_n}{c^2}\left(\chi \frac{\partial\varphi}{\partial t}\right.\right.$$

$$\left.\left. - \varphi \frac{\partial\chi}{\partial t}\right)\right\} dtds = \int_\nu \int_{t_1}^{t_2} \rho\chi\left(\frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right.$$

$$\left. + \frac{u_\mu}{c^2}\frac{\partial\Phi}{\partial x_\mu} - \frac{u_\mu}{c^2}\frac{\partial \ln c^2}{\partial x_\mu}\Phi\right) dtds. \qquad (13)$$

3. Uniform Flow

Let us apply the relationship (12), first of all, to a uniform flow. If the axis, x, is directed along the speed of flow and it is considered that in this case ρ = const and $\frac{\partial \ln c^2}{\partial x_\mu} = 0$, then it will assume the form:

$$\int_{t_1}^{t_2} \oint_s \left\{ \left(1 - \frac{u^2}{c^2}\right) \left(\chi \frac{\partial \varphi}{\partial x} - \varphi \frac{\partial \chi}{\partial x}\right) ds_x + \left(\chi \frac{\partial \varphi}{\partial y} + \varphi \frac{\partial \chi}{\partial y}\right) ds_y + \left(\chi \frac{\partial \varphi}{\partial z} - \varphi \frac{\partial \chi}{\partial z}\right) ds_z \right\} dt - \frac{u}{c^2} \oint_s \int_{t_1}^{t_2} \left(\chi \frac{\partial \varphi}{\partial t} - \varphi \frac{\partial \chi}{\partial t}\right) dt ds_x = \frac{1}{c^2} \int_\nu \int_{t_1}^{t_2} \chi \left(\frac{\partial \Phi}{\partial t} + u \frac{\partial \Phi}{\partial x}\right) dt d\nu. \tag{14}$$

The second integral of the left-hand part can be rearranged as follows:

$$\oint_s \int_{t_1}^{t_2} \left(\chi \frac{\partial \varphi}{\partial t} - \varphi \frac{\partial \chi}{\partial t}\right) dt ds_x = 2 \oint_s \int_{t_1}^{t_2} \chi \frac{\partial \varphi}{\partial t} \, dt ds_x - \oint_s [\varphi \chi]_{t_1}^{t_2} ds_x = 2 \oint_s \int_{t_1}^{t_2} \chi \frac{\partial \varphi}{\partial t} \, dt ds_x,$$

because the values of the expression in the rectangular brackets under the sign of the integral for the upper and lower limits are equal to zero.

By introducing the new variables,

$$x' = \frac{x}{\sqrt{1 - \frac{u^2}{c^2}}}, \quad y' = y, \quad z' = z,$$

and accordingly, the projections of the elementary area in the coordinate planes in new variables,

$$ds'_x = ds_x, \quad ds'_y = \frac{ds_y}{\sqrt{1 - \frac{u^2}{c^2}}}, \quad ds'_x = \frac{ds_z}{\sqrt{1 - \frac{u^2}{c^2}}},$$

and also the element of the volume

$$d\nu' = \frac{d\nu}{\sqrt{1 - \frac{u^2}{c^2}}},$$

we rewrite (14) as follows:

$$\sqrt{1 - \frac{u^2}{c^2}} \int_{t_1}^{t_2} \oint_{s'} \left\{ \left(\chi \frac{\partial \varphi}{\partial x'} - \varphi \frac{\partial \chi}{\partial x'} \right) ds'_x + \left(\chi \frac{\partial \varphi}{\partial y'} - \varphi \frac{\partial \chi}{\partial y'} \right) ds'_y \right.$$

$$\left. + \left(\chi \frac{\partial \varphi}{\partial z'} - \varphi \frac{\partial \chi}{\partial z'} \right) ds'_z \right\} dt - \frac{2u}{c^2} \oint_{s'} \int_{t_1}^{t_2} \chi \frac{\partial \varphi}{\partial t} dt ds'_x.$$

$$= \frac{1}{c^2} \sqrt{1 - \frac{u^2}{c^2}} \int_{\nu'} \int_{t_1}^{t} \chi \frac{\partial \Phi}{\partial t} dt d\nu' + \frac{u}{c^2} \int_{\nu'} \int_{t_1}^{t_2} \chi \frac{\partial \Phi}{\partial x'} dt d\nu', \tag{15}$$

or by designating the normal to the surface s' by n' in a more abbreviated form:

$$\sqrt{1-\frac{u^2}{c^2}}\oint_{s'}\int_{t_1}^{t_2}\left(\chi\frac{\partial\varphi}{\partial n'}-\varphi\frac{\partial\chi}{\partial n'}\right)dtds'-\frac{2u}{c^2}\oint_{s'}\int_{t_1}^{t_2}\chi\frac{\partial\varphi}{\partial t}\,dtds'_x$$

$$=\frac{1}{c^2}\sqrt{1-\frac{u^2}{c^2}}\int_{\nu'}\int_{t_1}^{t_2}\chi\frac{\partial\Phi}{\partial t}\,dtd\nu'+\frac{u}{c^2}\int_{\nu'}\int_{t_1}^{t_2}\chi\frac{\partial\Phi}{\partial x'}\,dtd\nu'. \tag{16}$$

In the case of uniform flow, the converging pulse is described by the δ-function,

$$\chi=\frac{\delta\left(t+\frac{R}{c}\right)}{r'} \tag{17}$$

where $r'=\sqrt{x'^2+y'^2+z'^2}$, and

$$R=\frac{\frac{ux'}{c}+r'}{\sqrt{1-\frac{u^2}{c^2}}} \tag{18}$$

besides, the integration should be applied also to the surface of an infinitely small sphere r' = const with the center in the observation point, P.

By repeating the calculations which the reader can find in the book of Lorenz[5], we convert the left-hand part of (16) to the form of

$$\sqrt{1-\frac{u^2}{c^2}}\oint_{s'}\left\{\frac{1}{r'}\left(\frac{\partial\varphi}{\partial n'}\right)_{t=-\frac{R}{c}}-\frac{\partial}{\partial n'}\left(\frac{1}{r'}\right)\varphi_{t=-\frac{R}{c}}+\frac{1}{cr'}\frac{\partial R}{\partial n'}\left(\frac{\partial\varphi}{\partial t}\right)_{t=-\frac{R}{c}}\right\}ds'$$

$$-\frac{2n}{c^2}\oint_{s'}\frac{1}{r'}\left(\frac{\partial\Phi}{\partial t}\right)_{t=-\frac{R}{c}}ds'_x-4\pi\sqrt{1-\frac{u^2}{c^2}}\,\varphi_P(0).$$

By utilizing the well-known characteristic of the δ-function in the right-hand part of the relationship (16) and by transferring the start of the time counting to the point, τ, we get the Kirchhoff theorem:

$$4\pi\sqrt{1-\frac{u^2}{c^2}}\,\varphi_P(\tau) = \sqrt{1-\frac{u^2}{c^2}}\oint_{s'}\left\{\frac{1}{r'}\left[\frac{\partial\varphi}{\partial n'}\right] - \frac{\partial}{\partial r'}\left(\frac{1}{r'}\right)[\varphi] + \frac{1}{cr'}\frac{\partial R}{\partial n'}\left[\frac{\partial\varphi}{\partial t}\right]\right\}ds' - \frac{2u}{c^2}\int_{s'}\frac{1}{r'}\left[\frac{\partial\varphi}{\partial t}\right]ds'_x$$

$$-\frac{1}{c^2}\sqrt{1-\frac{u^2}{c^2}}\int_{\nu'}\frac{1}{r'}\left[\frac{\partial\Phi}{\partial t}\right]d\nu' - \frac{u}{c^2}\int_{\nu'}\frac{1}{r'}\left[\frac{\partial\Phi}{\partial x'}\right]d\nu'. \qquad (19)$$

In Formula (19) the rectangular brackets indicate that the magnitude within them are taken for the moment $t = \tau - R/c$. This formula coincides with the formula obtained by D. I. Blokhintsev.

Assuming that there are no body forces ($\Phi = 0$) and that the boundary values depend harmonically on time:

$$\varphi = \psi e^{i\omega t}, \quad k = \frac{\omega}{c},$$

we get

$$4\pi\sqrt{1-\frac{u^2}{c^2}}\,\psi_P = \sqrt{1-\frac{u^2}{c^2}}\oint_{s'}\left\{\frac{\partial\psi}{\partial n'}\frac{e^{-ikR}}{r'} - \psi\frac{\partial}{\partial n'}\left(\frac{e^{-ikR}}{r'}\right)\right\}ds'$$

$$-\frac{2iku}{c}\oint_{s'}\psi\frac{e^{-ikR}}{r'}ds'_x. \qquad (20)$$

4. Nonuniform Flow

The sources of acoustic waves can be either variable body forces or vibrating solid bodies. Solid bodies which are in the flow disturb its uniformity in opposition to body forces. For this reason, the problem of the emission of vibrating solid bodies, which is of greatest interest, cannot be solved with the help of the Kirchhoff theorem for uniform flow, even in the case when the only reason for the disturbance of the flow uniformity is the vibrating solid bodies themselves. For this reason, we should return to an examination of the nonuniform flow. We will assume that the body forces (which are of lesser interest) are lacking. This will make it possible to simplify somewhat the formulas.

Integration in the relationship (13) should be applied to the surface of the solid bodies and the distant surface. If all the vibrating bodies are concentrated in the end region of the space, then the surface can be selected so far that the acoustic waves which originate at the moment $t_0 < 0$ will not succeed in getting to it at the moment, t_2. For this reason, the integral with respect to the distant surface will change to zero. The integral with respect to the surface of the solid bodies is also strongly simplified because the normal component, u_n, of the speed of flow at the surface of the solid body is equal to zero ($u_n = 0$). The relationship (13) assumes the form:

$$\int_{s} \int_{t_1}^{t_2} \rho \left(\chi \frac{\partial \varphi}{\partial n} - \varphi \frac{\partial \chi}{\partial n} \right) dtds = 0, \tag{21}$$

where the integration should be applied along the surface of all the solid bodies.

In order to determine the acoustic potential in a certain observation point, P, it is necessary to select the auxiliary function in the form of a converging pulse which satisfies Equation (2). Within an infinitely small volume, which encompasses the point, P, the flow can be considered uniform, and having directed the axis, x, along the speed of flow in the point, P, make use of the prior expression (17) for a converging pulse. The integral with respect to an infinitely small surface, which encompasses the point, P, is calculated just as in the case of uniform flow and within the limit is equal to:

$$-4\pi\rho_0\sqrt{1-\frac{u^2}{c^2}}\,\varphi_P(0),$$

where ρ_0 and u_0 are, respectively, the density and speed of the medium in the observation point, P. The Kirchhoff theorem assumes the following simple form:

$$4\pi\rho_0\sqrt{1-\frac{u_0^2}{c^2}}\,\varphi_P(0) = \oint_s \int_{t_1}^{t_2} \rho\left(\chi\frac{\partial\varphi}{\partial n} - \varphi\frac{\partial\chi}{\partial n}\right) dtds. \qquad (22)$$

The right-hand part of this relationship has the same form as in the case of an immobile medium. However, the form of the auxiliary function, χ, which is selected in the form of a pulse that converges to the observation point, P, will be entirely different. The problem of finding this function, which reduces itself to finding a solution of Equation (2) in the form of a converging point, apparently represents considerable mathematical difficulties in the case of nonuniform flow.

By moving the start of time counting to the point, τ, we get:

$$4\pi\rho_0\sqrt{1-\frac{u_0^2}{c^2}}\,\varphi_P(\tau) = \oint_s \int_{t_1}^{t_2} \rho\left(\chi\frac{\partial\varphi}{\partial n} - \varphi\frac{\partial\chi}{\partial n}\right) dtds, \qquad (23)$$

where $t_2 > \tau$, $t_1 < t_0 + \tau$, if $t_0 + \tau$ is the moment of the origin of acoustic vibrations. These conditions are, in particular, satisfied by the infinite limits $t_1 = -\infty$ and $t_2 = +\infty$. The function, χ, represents a solution of Equation (2) in the form of a pulse that converges to the point of observation, P, at the moment, τ.

5. Point Source

A point source of sound can be realized in the form of a pulsating small sphere of very small dimensions or a body force with a spherical symmetry, concentrated in a small volume. In a quiescent medium, such sources, for the same power, create entirely the same sound fields. The question naturally arises: will the fields of the sources also be alike in a moving medium? Moreover, even with respect to the pulsating small sphere, two assumptions can be advanced: 1) the small sphere is covered by the flow; 2) the small sphere is permeable to the flow, which apparently is fundamentally also impossible. In the first case, the uniformity of the flow will be disturbed by the small sphere and in the second case, the uniformity of the flow will not be disturbed by the small sphere. We can justly pose the question: will the fields created by the permeable and impermeable small sphere be the same or different? Or, in other words: will the nature of the flow in the immediate vicinity of the small sphere affect the sound field far from the small sphere?

Thus, we should solve the problem of three different point sources: that of a small sphere washed by the flow, permeable small sphere, and body force.

a. Small Sphere Washed by the Flow

Let us assume that the pulsating small sphere of radius a is washed by a uniform flow with a speed u_0. The uniformity of the flow will be disturbed in a certain region which encompasses the small sphere, the dimensions of which are of the same order as the dimensions of the small sphere. The pulse which converges to the observation point, P, will no longer be given in this region by the expression (17) which is valid for uniform flow. If, however, the dimensions of the small sphere are small in comparison with the distance to the observation point, P, as well as in comparison with the wavelength, then the change in the form of the pulse can be disregarded in this region. Therefore, by substituting in Equation (23) the expression for the function, χ, as given by Equation (17), then we get

$$4\pi\rho_0\sqrt{1-\frac{u_0^2}{c^2}}\,\psi_P = \oint_s \rho\left\{\frac{\partial\psi}{\partial n}\,\frac{e^{-ikR}}{r'} - \psi\,\frac{\partial}{\partial n}\left(\frac{e^{-ikR}}{r'}\right)\right\}ds,$$

where integration is carried out with respect to the surface of the pulsating small sphere. Since on the surface of the pulsating small sphere, the amplitude of the potential of the acoustic speed, ψ, and the amplitude of the acoustic speed $V = \partial\psi/\partial n$ are constant, then the preceding relationship can be written as follows:

$$4\pi\rho_0\sqrt{1-\frac{u_0^2}{c^2}}\,\psi_P = V\oint \rho\,\frac{e^{-ikR}}{r'}\,ds - \psi\oint \rho\,\frac{\partial}{\partial n}\left(\frac{e^{-ikR}}{r'}\right)ds. \tag{24}$$

In view of the smallness of the dimensions of the small sphere, the subintegral function e^{-ikR}/r' can be considered constant. Carrying it out for the sign of the first integral in (24), we get:

$$\oint \rho ds, \tag{25}$$

relative to u/c from the Bernoulli equation:

$$\rho = \rho_0 + \frac{1}{2} \rho_0 \left(\frac{u_0^2}{c^2} - \frac{u^2}{c^2} \right),$$

where c is the speed of the sound. By substituting here the well-known expression for speed, $u = 3/2(u_0 \sin \vartheta)$, of an incompressible liquid which washes the small sphere, we get:

$$\rho = \rho_0 + \frac{1}{2} \rho_0 \frac{u_0^2}{c^2} \left(1 - \frac{9}{4} \sin^2 \vartheta \right), \tag{26}$$

where ϑ is the angle between the direction of the normal to the surface of the small sphere and the direction of the flow.

By substituting (26) in (25), we get:

$$\oint \rho ds = 2\pi a^2 \int_0^{\pi} \left\{ \rho_0 + \frac{1}{2} \rho_0 \frac{u_0^2}{c^2} \cdot \left(1 - \frac{9}{4} \sin^2 \vartheta \right) \right\} \sin \vartheta \, d\vartheta$$

$$= 4\pi a^2 \rho_0 \left(1 - \frac{1}{4} \frac{u_0^2}{c^2} \right). \tag{27}$$

It is easy to show that the second integral in the relationship (24) is equal to zero. Thus, from the relationship (24), we get the following solution:

$$\psi p = Va^2 \frac{1 - \frac{1}{4} \frac{u_0^2}{c^2}}{\sqrt{1 - \frac{u_0^2}{c^2}}} \frac{e^{-ikR}}{r'} .$$

If we introduce the spherical system of coordinates with the origin in the center of the small sphere, the polar axis of which coincides with the direction of the flow (with the axis x), then the solution assumes the form:

$$\psi_P = Va^2 \frac{1 - \frac{1}{4}\frac{u_0^2}{c^2}}{\sqrt{1 - \frac{u_0^2}{c^2}\sin^2\vartheta}} \frac{e^{-ikR}}{r}. \tag{28}$$

b. Permeable Small Sphere .

The problem of a permeable pulsating small sphere can, without effort, be solved with the aid of the relationship (20). Since the radius of the small sphere is small in comparison with the wavelength and the distance to the observation point, then the right-hand part of this relationship reduces itself to a single member:

$$\psi_P = \frac{1}{4\pi}\frac{e^{-ikR}}{r'}\oint \frac{\partial\psi}{\partial n'} ds'.$$

The last integral is calculated as follows:

$$\oint \frac{\partial\psi}{\partial n'} ds' = \oint\left(\frac{\partial\psi}{\partial x'} ds'_x + \frac{\partial\psi}{\partial y'} ds'_y + \frac{\partial\psi}{\partial z'} ds'_x\right) = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}\oint\left[\left(1 - \frac{u^2}{c^2}\right)\frac{\partial\psi}{\partial x} ds_x + \frac{\partial\psi}{\partial y} ds_y + \frac{\partial\psi}{\partial z} ds_z\right] = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}\oint\left(\frac{\partial\psi}{\partial x} ds_x + \frac{\partial\psi}{\partial y} ds_y + \frac{\partial\psi}{\partial z} ds_z\right) - \frac{\frac{u^2}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}\oint \frac{\partial\psi}{\partial x} ds_x$$

$$= \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}\oint \frac{\partial\psi}{\partial n} ds - \frac{\frac{u^2}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}\oint \frac{\partial\psi}{\partial n}\cos^2\vartheta ds.$$

Considering that

$$\oint \cos^2\vartheta \cdot ds = \frac{4\pi}{3} a^2,$$

we get

$$\psi_P = Va^2 \frac{1 - \frac{1}{3}\frac{u^2}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}\sin^2\vartheta}} \frac{e^{-ikR}}{r}. \tag{29}$$

This solution differs from the solution of (28) for a washed small sphere with a coefficient before the ratio u^2/c^2 in the numerator.

Under the same conditions, the amplitude in the sound field of the washed small sphere will be somewhat greater than in the field of the permeable small sphere.

c. Body Force

Let us assume that the source of the sound waves is a body force concentrated in a small volume, $\Delta\nu$, which has spherical symmetry. The body force does not disturb the uniformity of the flow. For this reason, in order to solve the problem, one can use the Kirchhoff theorem (19) for uniform flow. For a harmonic dependence of the body force on the time

$$\Phi = \Phi_0 e^{i\omega t},$$

the relationship (19) assumes the form:

$$4\pi\sqrt{1 - \frac{u^2}{c^2}}\,\psi_P = -\frac{i\omega}{c^2}\sqrt{1 - \frac{u^2}{c^2}}\,\frac{e^{-ikR}}{r'}\int_{\Delta\nu'} \Phi_0 d\nu'$$

$$- \frac{u^2}{c^2}\frac{e^{-ikR}}{r'}\int_{\Delta\nu'} \frac{\partial\Phi_0}{\partial x'} d\nu'.$$

The second integral, in view of the symmetry of the body force, changes into zero:

$$\int_{\Delta\nu'} \frac{\partial \Phi_0}{\partial x'} d\nu' \quad \int_{\Delta\nu} \frac{\partial \Phi_0}{\partial x} d\nu = 0.$$

The first integral is transformed as follows:

$$\int_{\Delta\nu'} \Phi_0 d\nu' = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \int_{\Delta\nu} \Phi_0 d\nu = \frac{\overline{\Phi}_0 \Delta\nu}{\sqrt{1 - \frac{u^2}{c^2}}},$$

where $\overline{\Phi}_0$ is the average value of the potential of the body force in the volume, $\Delta\nu$.

And we get the final solution:

$$\psi_P = -\frac{ik\overline{\Phi}_0\Delta\nu}{4\pi c} \frac{1}{\sqrt{1 - \frac{u^2}{c^2}\sin^2\vartheta}} \frac{e^{-ikR}}{r}. \tag{30}$$

Thus, the dependence of the amplitude in the sound field on the speed of the flow for a fixed potential of the body force, $\overline{\Phi}_0$, will be different from that for a fixed acoustic speed, V, of the pulsating small sphere (washed or permeable).

In order for the body force to create a field identical with the field of the washed pulsating small sphere, the following condition must be fulfilled:

$$\frac{ik\overline{\Phi}_0\Delta\nu}{4\pi c} = Va^2\left(1 - \frac{1}{4}\frac{u_0^2}{c^2}\right).$$

6. Flow of Energy

The density of the flow of acoustic energy was calculated by the author[6] for a point source of sound in the form of a body force. It is determined by the formula:

$$\Pi = \frac{k^4\rho(\overline{\Phi}_0\Delta\nu)^2}{32\pi^2 c r^2\left(1 - \frac{u^2}{c^2}\sin^2\vartheta\right)^{\frac{3}{2}}}.$$

By using the last relationship of the preceding paragraph and limiting ourselves to the magnitudes of the second order of smallness with respect to u_0^2/c^2 for the washed pulsating small sphere, we get:

$$\Pi = \frac{1}{2}\rho_0 c k^2 V^2 a^4\left(1 + \frac{u_0^2}{c^2} - \frac{3}{2}\frac{u_0^2}{c^2}\cos^2\vartheta\right)\cdot\frac{1}{r^2}.$$

By integrating with respect to all directions, for a complete flow of energy, we get:

$$\Pi' = 2\pi\rho_0 c k^2 V^2 a^4\left(1 + \frac{1}{2}\frac{u_0^2}{c^2}\right).$$

It follows from this formula that the sound transfer of the pulsating small sphere increases with increasing speed, u_0, of the flow, if the amplitude of the pulsations of V remains unchanged. Consequently, in order to maintain the pulsations of the unchanged amplitude, it is necessary to expend more energy the greater the speed of the washing flow.

In conclusion, I express deep gratitude to N. N. Andreyev whose comments on the nonidentity of the washed and permeable sources of sound has served as the impetus for this study.

LITERATURE CITED

1. D. I. Blokhintsev, ZhTF (Journal of Technical Physics), Vol. XV, 1945, p. 71.

2. N. N. Andreyev and I. G. Rusakov, AKUSTIKA DVIZHISHCHEY SREDY (Acoustics of a Moving Medium), GTTI (State Publishing House of Technical-Theoretical Literature), 1934.

3. D. I. Blokhintsev, DAN SSSR (Reports Academy Sciences USSR), Vol. XLV, 1944, p. 343.

4. A. Webster and G. Sege, DIFFERENTIAL EQUATIONS IN PARTIAL DERIVATIVES OF MATHEMATICAL PHYSICS, PART 2, GTTI (State Publishing House of Technical-Theoretical Literature), 1934, p. 111.

5. G. A. Lorenz, THEORY OF ELECTRONS, GTTI (State Publishing House of Technical-Theoretical Literature), 1934, p. 315.

6. B. A. Chernov, ZhTF (Journal of Technical Physics), Vol. XVI, 1946, p. 733.

DISTRIBUTION

	No. of Copies
EXTERNAL	
Air University Library ATTN: AUL3T Maxwell Air Force Base, Alabama 36112	1
U. S. Army Electronics Proving Ground ATTN: Technical Library Fort Huachuca, Arizona 85613	1
U. S. Naval Ordnance Test Station ATTN: Technical Library, Code 753 China Lake, California 93555	1
U. S. Naval Ordnance Laboratory ATTN: Library Corona, California 91720	1
Lawrence Radiation Laboratory ATTN: Technical Information Division P. O. Box 808 Livermore, California 94550	1
Sandia Corporation ATTN: Technical Library P. O. Box 969 Livermore, California 94551	1
U. S. Naval Postgraduate School ATTN: Library Monterey, California 93940	1
Electronic Warfare Laboratory, USAECOM Post Office Box 205 Mountain View, California 94042	1
Jet Propulsion Laboratory ATTN: Library (TDS) 4800 Oak Grove Drive Pasadena, California 91103	2
U. S. Naval Missile Center ATTN: Technical Library, Code N3022 Point Mugu, California 93041	1
U. S. Army Air Defense Command ATTN: ADSX Ent Air Force Base, Colorado 80912	1
Central Intelligence Agency ATTN: OCR/DD-Standard Distribution Washington, D. C. 20505	4
Harry Diamond Laboratories ATTN: Library Washington, D. C. 20438	1
Scientific & Tech. Information Div., NASA ATTN: ATS Washington, D. C. 20546	1

	No. of Copies
U. S. Atomic Energy Commission ATTN: Reports Library, Room G-017 Washington, D. C. 20545	1
U. S. Naval Research Laboratory ATTN: Code 2027 Washington, D. C. 20390	1
Weapons Systems Evaluation Group Washington, D. C. 20305	1
John F. Kennedy Space Center, NASA ATTN: KSC Library, Documents Section Kennedy Space Center, Florida 32899	2
APGC (PGBPS-12) Eglin Air Force Base, Florida 32542	1
U. S. Army CDC Infantry Agency Fort Benning, Georgia 31905	1
Argonne National Laboratory ATTN: Report Section 9700 South Cass Avenue Argonne, Illinois 60440	1
U. S. Army Weapons Command ATTN: AMSWE-RDR Rock Island, Illinois 61201	1
Rock Island Arsenal ATTN: SWERI-RDI Rock Island, Illinois 61201	1
U. S. Army Cmd. & General Staff College ATTN: Acquisitions, Library Division Fort Leavenworth, Kansas 66027	1
Combined Arms Group, USACDC ATTN: Op. Res., P and P Div. Fort Leavenworth, Kansas 66027	1
U. S. Army CDC Armor Agency Fort Knox, Kentucky 40121	1
Michoud Assembly Facility, NASA ATTN: Library, I-MICH-OSD P. O. Box 29300 New Orleans, Louisiana 70129	1
Aberdeen Proving Ground ATTN: Technical Library, Bldg. 313 Aberdeen Proving Ground, Maryland 21005	1
NASA Sci. & Tech. Information Facility ATTN: Acquisitions Branch (S-AK/DL) P. O. Box 33 College Park, Maryland 20740	5
U. S. Army Edgewood Arsenal ATTN: Librarian, Tech. Info. Div. Edgewood Arsenal, Maryland 21010	1

	No. of Copies
National Security Agency ATTN: C3/TDL Fort Meade, Maryland 20755	1
Goddard Space Flight Center, NASA ATTN: Library, Documents Section Greenbelt, Maryland 20771	1
U. S. Naval Propellant Plant ATTN: Technical Library Indian Head, Maryland 20640	1
U. S. Naval Ordnance Laboratory ATTN: Librarian, Eva Liberman Silver Spring, Maryland 20910	1
Air Force Cambridge Research Labs. L. G. Hanscom Field ATTN: CRMXLR/Stop 29 Bedford, Massachusetts 01730	1
Springfield Armory ATTN: SWESP-RE Springfield, Massachusetts 01101	1
U. S. Army Materials Research Agency ATTN: AMXMR-ATL Watertown, Massachusetts 02172	1
Strategic Air Command (OAI) Offutt Air Force Base, Nebraska 68113	1
Picatinny Arsenal, USAMUCOM ATTN: SMUPA-VA6 Dover, New Jersey 07801	1
U. S. Army Electronics Command ATTN: AMSEL-CB Fort Monmouth, New Jersey 07703	1
Sandia Corporation ATTN: Technical Library P. O. Box 5800 Albuquerque, New Mexico 87115	1
ORA(RRRT) Holloman Air Force Base, New Mexico 88330	1
Los Alamos Scientific Laboratory ATTN: Report Library P. O. Box 1663 Los Alamos, New Mexico 87544	1
White Sands Missile Range ATTN: Technical Library White Sands, New Mexico 88002	1
Rome Air Development Center (EMLAL-1) ATTN: Documents Library Griffiss Air Force Base, New York 13440	1

	No. of Copies
Brookhaven National Laboratory Technical Information Division ATTN: Classified Documents Group Upton, Long Island, New York 11973	1
Watervliet Arsenal ATTN: SWEWV-RD Watervliet, New York 12189	1
U. S. Army Research Office (ARO-D) ATTN: CRD-AA-IP Box CM, Duke Station Durham, North Carolina 27706	1
Lewis Research Center, NASA ATTN: Library 21000 Brookpark Road Cleveland, Ohio 44135	1
Systems Engineering Group (RTD) ATTN: SEPIR Wright-Patterson Air Force Base, Ohio 45433	1
U. S. Army Artillery & Missile School ATTN: Guided Missile Department Fort Sill, Oklahoma 73503	1
U. S. Army CDC Artillery Agency ATTN: Library Fort Sill, Oklahoma 73504	1
U. S. Army War College ATTN: Library Carlisle Barracks, Pennsylvania 17013	1
U. S. Naval Air Development Center ATTN: Technical Library Johnsville, Warminster, Pennsylvania 18974	1
Frankford Arsenal ATTN: C-2500-Library Philadelphia, Pennsylvania 19137	1
Div. of Technical Information Ext., USAEC P. O. Box 62 Oak Ridge, Tennessee 37830	1
Oak Ridge National Laboratory ATTN: Central Files P. O. Box X Oak Ridge, Tennessee 37830	1
Air Defense Agency, USACDC ATTN: Library Fort Bliss, Texas 79916	1
U. S. Army Air Defense School ATTN: AKBAAS-DR-R Fort Bliss, Texas 79906	1

	No. of Copies
U. S. Army CDC Nuclear Group Fort Bliss, Texas 79916	1
Manned Spacecraft Center, NASA ATTN: Technical Library, Code BM6 Houston, Texas 77058	1
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
U. S. Army Research Office ATTN: STINFO Division 3045 Columbia Pike Arlington, Virginia 22204	1
U. S. Naval Weapons Laboratory ATTN: Technical Library Dahlgren, Virginia 22448	1
U. S. Army Engineer Res. & Dev. Labs. ATTN: Scientific & Technical Info. Br. Fort Belvoir, Virginia 22060	2
Langley Research Center, NASA ATTN: Library, MS-185 Hampton, Virginia 23365	1
Research Analysis Corporation ATTN: Library McLean, Virginia 22101	1
U. S. Army Tank Automotive Center ATTN: SMOTA-RTS.1 Warren, Michigan 48090	1
Hughes Aircraft Company Electronic Properties Information Center Florence Ave. & Teale St. Culver City, California 90230	1
Atomics International, Div. of NAA Liquid Metals Information Center P. O. Box 309 Canoga Park, California 91305	1
Foreign Technology Division ATTN: Library Wright-Patterson Air Force Base, Ohio 45400	1
Clearinghouse for Federal Scientific and Technical Information U. S. Department of Commerce Springfield, Virginia 22151	1
Foreign Science & Technology Center, USAMC ATTN: Mr. Shapiro Washington, D. C. 20315	3

INTERNAL

	No. of Copies
Headquarters U. S. Army Missile Command Redstone Arsenal, Alabama	
ATTN: AMSMI-D	1
AMSMI-XE, Mr. Lowers	1
AMSMI-XS, Dr. Carter	1
AMSMI-Y	1
AMSMI-R, Mr. McDaniel	1
AMSMI-RAP	1
AMSMI-RBLD	10
USACDC-LnO	1
AMSMI-RBT	8
AMSMI-RB, Mr. Croxton	1
National Aeronautics & Space Administration Marshall Space Flight Center Huntsville, Alabama ATTN: MS-T, Mr. Wiggins	5

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*	2a. REPORT SECURITY CLASSIFICATION
Redstone Scientific Information Center Research and Development Directorate U.S. Army Missile Command Redstone Arsenal, Alabama 35809	Unclassified 2b. GROUP N/A

3. REPORT TITLE

KIRCHHOFF'S THEOREM IN THE ACOUSTICS OF A MOVING MEDIUM
Trudy Komissii Po Akustike AN SSSR, 5, 10-22 (1950)

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Translated from the Russian

5. AUTHOR(S) *(Last name, first name, initial)*

Chernov, L. A.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
5 January 1967	23	6

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
N/A b. PROJECT NO. N/A c. d.	RSIC-628 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* AD ________

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Same as No. 1

13. ABSTRACT

Kirchhoff's theorem is finding diverse applications in acoustics and optics. Its generalization for the case of a moving medium has been given by D. I. Blokhintsev who limited himself to an examination of a uniform flow. This paper presents a further generalization of this theorem for the case of a potential movement of a medium. The application of the theorem is illustrated with the solution of a problem on the point source of sound, which is in the flow.

DD FORM 1 JAN 64 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Gauss theorem						
Kirchhoff theorem						
Uniform flow						
Nonuniform flow						
Acoustic potential						
Bernoulli equation						
Pulsating small sphere						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS), (S), (C), or (U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Idenfiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.