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SOME FREE VIBRATION AND DYNAMIC RESPONSE
PROBLEMS ASSOCIATED WITH CENTRIFUGALLY
STABILIZED DISK AND SHELL STRUCTURES

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INTRODUCTION

This progress report covers the period from August 15, 1966, to February 15, 1967. The end of this six month period was nominally the expiration date of the grant. However, as personally discussed during the principal investigator's visit to NASA Headquarters in December, and formally requested by a letter, dated January 6, 1967, to Dr. R.H. Wilson with a copy to Miss Beverly Hungerford, the duration of the grant will be extended, with no additional funding, for one year. The extension of the grant is for the purpose of allowing a research assistant to be employed for a full one-year period.

SUMMARY OF PROGRESS

Since the last reporting period, a graduate research assistant, Mr. Thomas S. David, has been added to the research effort. Mr. David is a master's degree candidate and came to Wichita State University from the University of Michigan. He is employed one-half time during the academic year and will be employed full time during the summer of 1967.

The effort during the past six months has led to the completion of a substantial portion of the fundamental goals of the research program. Under the direction of the principal investigator, a part-time graduate student, Mr. Thomas Gilley, has completed his thesis consisting of a study of the natural frequencies of the transverse vibrations of a spinning annular membrane with a frictionless hub. During this same period the principal investigator has completed the study of the spinning membrane which is fully clamped by a central hub. The intermediate case

of partial central clamping has been formulated and will be numerically evaluated for a few configurations to verify the supposition that it does in fact represent a case bounded on the two extremes by the frictionless hub and the fully clamped hub.

Additional effort has been expended toward the goal of extending the investigation to the study of the vibrations of shallow spherical membrane shells which are centrifugally loaded. The point of major interest has been the establishment of the proper shell stress distribution to be used for the equilibrium configuration about which vibrations occur. In particular, verification has been sought for the stress distribution used by Johnson (1) in his studies of the spinning shell problem. He uses a stress distribution which is the exact solution of his nonlinear shell theory in the membrane case. As he points out, only the case of the freely spinning shallow spherical shell can satisfy the membrane boundary conditions. The membrane solution shows that if the shell has any finite radius of curvature the radial stress vanishes. This theory will therefore not demonstrate a continuous convergence to the flat plate stress distribution in the case of shells with very large radii of curvature. It would thus seem that Johnson's results will not be valid for certain geometrical configurations.

In order to resolve this paradox the stress distribution in the freely spinning shell has been analyzed by utilizing the linear bending and membrane theory of E. Reissner (2,3). It has been found by numerical evaluation of solutions of his equations

that the stress distribution approaches that of linear membrane theory if the ratio of shell depth to shell thickness is sufficient. For small values of this ratio the stress distribution is nearly that of the spinning flat disk. Hence this theory shows a continuous variation from the membrane case to the flat disk case based on the ratio of shell depth to shell thickness. Johnson's theory can be expected to be valid only for the larger values of this ratio.

The behavior noted above has suggested a boundary layer effect at the free edge of the shell. To investigate this possibility Reissner's non-linear shallow shell equations (4,5,6) are being used and his approach to the boundary layer problem at the free edge is being followed. Initial investigations indicate that there is a boundary layer effect depending on the ratio of the shell depth to the shell thickness. For large values of this ratio the boundary layer is very thin and the direct stress distribution is essentially that predicted by membrane theory except in a narrow zone near the edge. For smaller values of this ratio the boundary layer expands and the solution shows a continuous variation into the flat disk stress distribution if this ratio continuously approaches zero.

Mr. David has been assigned the task of evaluating the stress distribution for the spinning shallow spherical shell with central clamping. He is currently checking out a computer program for this purpose which is based on Reissner's linear theory. It is anticipated that in this case a boundary layer effect will be observed at both the hub and outer edge.

FUTURE INVESTIGATIONS

The next six month period will be utilized mainly to refine and report on the investigations reported above. It is hoped the question of the spinning shell stress distribution will be sufficiently resolved to allow the formulation and solution of the equations for the free vibrations of the spinning shallow spherical shell based on linear membrane shell vibration theory.

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